



A layer potential approach to functional and clinical brain imaging

Masimba Nemaire, Paul Asensio, Jean-Michel Badier, Juliette Leblond,
Jean-Paul Marmorat

► To cite this version:

Masimba Nemaire, Paul Asensio, Jean-Michel Badier, Juliette Leblond, Jean-Paul Marmorat. A layer potential approach to functional and clinical brain imaging. Waves2022, Jul 2022, Palaiseau, France. hal-03934372

HAL Id: hal-03934372

<https://hal.science/hal-03934372>

Submitted on 11 Jan 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A layer potential approach to functional and clinical brain imaging

Masimba Nemaire^{1,2,*}, Paul Asensio², Jean-Michel Badier³, Juliette Leblond², Jean-Paul Marmorat⁴

¹Institut de Mathématiques de Bordeaux, Université de Bordeaux, Talence, France

²FACTAS, Inria Sophia Antipolis-Méditerranée, Valbonne, France

³Institut de Neurosciences des Systèmes, Aix-Marseille Université, Marseille, France

⁴Center of Applied Mathematics, Ecole des Mines ParisTech, Sophia Antipolis, France

*Email: masimba.nemaire@inria.fr

Abstract

We study the inverse problems of source recovery and cortical mapping for sEEG, EEG and MEG under the quasi-static approximation of Maxwell's equations. These problems are known to be ill-posed due to the existence of silent source, *i.e.*, those non-zero sources that generate null field. In addition the collected data are corrupted with noise that has to be corrected for, hence regularised Tikhonov problems will be solved. We make use of single and double layer potentials to express the electro-magnetic (EM) fields associated with a source which allows to simultaneously solve the source recovery and cortical mapping problems for a particular modality or coupled data. The expression of the electro-magnetic fields we use make the different couplings of sEEG, EEG and MEG data direct and complementary. Numerical results of experiments performed using meshes of realistic head geometries will be presented.

Keywords: ElectroEncephaloGraphy (EEG), stereo-ElectroEncephaloGraphy (sEEG), MagnetoEncephaloGraphy (MEG), inverse problems, brain imaging, cortical mapping

1 Introduction

We model the head as a layered conductor with different electric conductivities between layers which are constant in each layer. These homogeneous layers are the brain, skull and scalp, with the brain subdivided into the grey and white matter regions. Brain activity is modelled as vector-fields (sources) $\mu_0 \in [\mathcal{M}(\Sigma_0)]^3$, $\mathcal{M}(\Sigma_0)$ being a Banach space whose elements are supported on grey/white matter interface, the surface Σ_0 . Let Σ_i be the closed surfaces on which the electric conductivity is discontinuous, σ_i^- and σ_i^+ are the electric conductivities inside and outside Σ_i , respectively, and \mathcal{H}_i is the Hausdorff measure on Σ_i then the electric potential asso-

ciated with μ_0 at $x \in \mathbb{R}^3$ is:

$$\sigma(x)\phi(x) = \frac{1}{4\pi} \int \frac{(x-y)}{|x-y|^3} \cdot d\mu_0(y) - \sum_{i=1}^{n+1} \frac{\sigma_i^- - \sigma_i^+}{4\pi} \int_{\Sigma_i} \phi(y) \nu(y) \cdot \frac{(x-y)}{|x-y|^3} d\mathcal{H}_i(y). \quad (1)$$

The magnetic flux density associated with μ_0 at $x \in \mathbb{R}^3$ is:

$$\mathbf{B}(x) = \frac{\mu}{4\pi} \int \frac{(x-y)}{|x-y|^3} \times d\mu_0(y) - \mu \sum_{i=1}^{n+1} \frac{\sigma_i^- - \sigma_i^+}{4\pi} \int_{\Sigma_i} \nu(y) \times \frac{(x-y)}{|x-y|^3} \phi(y) d\mathcal{H}_i(y), \quad (2)$$

where the $\phi \in L^2(\Sigma_i)$ are as computed in (1).

2 Inverse Problems

We aim to solve problems of the following form with measurement in $D_1 \subset \mathbb{R}^3$ depending on the modalities:

Problem 1 Given data $f \in L^2(D_1)$ and parameters $\lambda, \lambda_i > 0, i = 1, 2, \dots, n+1$, find

$$(\mu_0, \phi_1, \phi_2, \dots, \phi_{n+1})_\lambda = \arg \inf_{(\mu_0, \phi_1, \phi_2, \dots, \phi_{n+1}) \in S} \mathcal{T}_{f,\lambda}(\mu_0, \phi_1, \phi_2, \dots, \phi_{n+1}),$$

where

$$\begin{aligned} \mathcal{T}_{f,\lambda} := & \|\mathcal{F}_1(\mu_0, \phi_1, \phi_2, \dots, \phi_{n+1}) - f\|_{L^2(D_1)}^2 \\ & + \|\mathcal{F}_2(\mu_0, \phi_1, \phi_2, \dots, \phi_{n+1})\|_{D_2}^2 \\ & + \lambda \|\mu_0\|_{TV} + \sum_{i=1}^{n+1} \lambda_i \|\phi_i\|_{L^2(\Sigma_i)}^2, \end{aligned}$$

where \mathcal{F}_1 corresponds to the EM data, \mathcal{F}_2 to the cortical mapping, D_2 is the Hilbert space

$$L^2(\Sigma_1) \times \dots \times L^2(\Sigma_n) \times L^2(\Sigma_{n+1}) \times L^2(\Sigma_{n+1})$$

which corresponds to the electric potential on the interfaces $\Sigma_i, i = 1, 2, \dots, n+1$ and S is the product Banach space

$$[\mathcal{M}(\Sigma_0)]^3 \times L^2(\Sigma_2) \times \dots \times L^2(\Sigma_n) \times L^2(\Sigma_{n+1}).$$

We propose to use an alternating minimisation procedure generating a sequence of solutions

$$\left\{(\mu_0^{\{k\}}, \phi_1^{\{k\}}, \phi_2^{\{k\}}, \dots, \phi_{n+1}^{\{k\}})\right\}_{k \in \mathbb{N}}$$

with some initial guess for $k = 0$ by solving the problems

$$\begin{aligned} \mu_0^{\{k+1\}} &= \arg \inf_{\mu_0 \in [\mathcal{M}(\Sigma_0)]^3} \mathcal{T}_{f,\lambda}(\mu_0, \phi_1^{\{k\}}, \dots, \phi_{n+1}^{\{k\}}) \\ (\phi_1^{\{k+1\}}, \phi_2^{\{k+1\}}, \dots, \phi_{n+1}^{\{k+1\}}) &= \\ &\arg \inf_{(\phi_1, \dots, \phi_{n+1}) \in D_2} \mathcal{T}_{f,\lambda}(\mu_0^{\{k\}}, \phi_1, \dots, \phi_{n+1}) \end{aligned}$$

to achieve a minimiser of $\mathcal{T}_{f,\lambda}$. Using the results of [1, 2] we conclude in both the continuous and discrete versions of Problem 1 that the sequence generated above converges to a minimiser of $\mathcal{T}_{f,\lambda}$ with the functional $\mathcal{T}_{f,\lambda}$ converging to its minimum linearly.

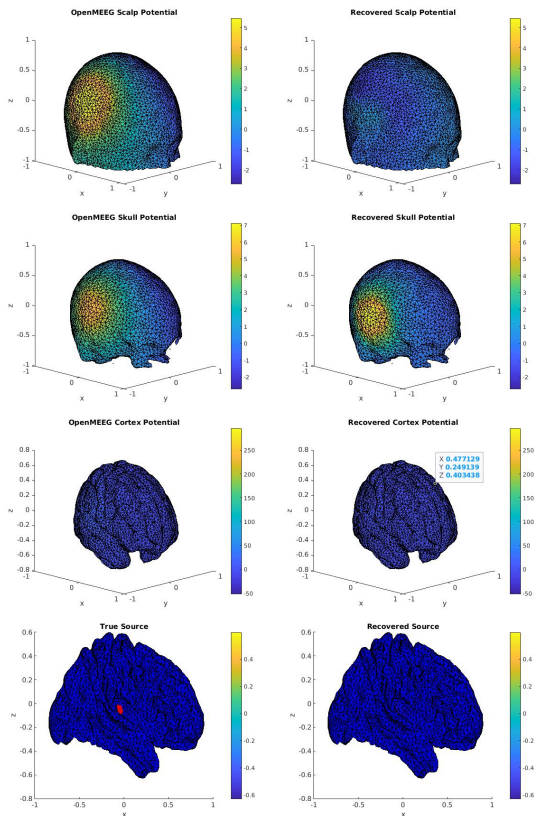


Figure 1: The synthetic data used here was generated in OpenMEEG. In these experiments we recover a dipolar source and perform the cortical mapping using the sEEG data. The *possible* dipole locations are assumed *a priori* to be the barycentres of the triangles of the grey/white matter interface mesh. The red dots represent the dipole locations.

3 Numerical Experiments

We require triangular meshes of the grey/white matter interface, cortex, skull and scalp. Note that (1) and (2) contain boundary integral operators that are defined on the cortex, skull and scalp. From [3] these boundary integrals can be computed exactly for triangular meshes and piecewise linear ϕ 's. When $\mathcal{M}(\Sigma_0)$ is the Banach space of Borel measures, we approximate μ_0 by a collection of dipoles. If the *possible* dipoles locations are *a priori* known and fixed, we use a FISTA algorithm with ℓ_1 penalisation to recover the dipole moments, see Figure 2 for sEEG with $D_1 \subset \text{Int}(\Sigma_1)$. Whereas if the dipoles locations are allowed to be arbitrarily located on Σ_0 , we use the algorithm in [4]. If $\mathcal{M}(\Sigma_0) = L^2(\Sigma_0)$ with μ_0 being normally oriented to Σ_0 , we model the magnitude and orientation of the vector-field as a piecewise linear $L^2(\Sigma_0)$ -function which results in Problem 1 being a least-squares problem. In this case a least-squares solution based on the Moore-Penrose pseudo-inverse can be obtained to solve the source recovery and cortical mapping problem together. The source is interpreted to be located in a neighbourhood of the extrema of the $L^2(\Sigma_0)$ function.

References

- [1] A. Beck, On the convergence of alternating minimization for convex programming with applications to iteratively reweighted least squares and decomposition schemes, *SIAM Journal on Optimization* **25** (2015), pp. 185–209.
- [2] J. Both, On the rate of convergence of alternating minimization for non-smooth non-strongly convex optimization in Banach spaces, *Optimization Letters* **05** (2021), pp. 1–15.
- [3] S. Nintcheu Fata, Explicit expressions for 3D boundary integrals in potential theory, *International Journal for Numerical Methods in Engineering* **78** (2009), pp. 32–47.
- [4] K. Bredies and H. K. Pikkarainen, Inverse problems in spaces of measure, *ESAIM:Control, Optimisation and Calculus of Variations* **19** (2013), pp. 190–218.