Surface identification through rational approximation of the back-scattering of an electromagnetic planar wave

Paul Asensio, Laurent Baratchart, Juliette Leblond, Martine Olivi, Fabien Seyfert, Martine Olivi

Surface identification through rational approximation of the back-scattering of an electromagnetic planar wave

Paul Asensio[1,*], **Laurent Baratchart**[1], **Juliette Leblond**[1], **Martine Olivi**[1], **Fabien Seyfert**[1]
[1]FACTAS, Inria Sophia Antipolis-Méditerranée, Valbonne, France
*Email: paul.asensio@inria.fr

Abstract

By measuring the scattered electromagnetic field produced by a plane wave on a smooth object at various frequencies, and then performing rational or meromorphic approximation of the transfer function, we consider the issue of identifying the shape of the object from the recovery of some characteristic singularities (which are poles because the object is smooth, but typically infinite in number). This technique can also be used to identify certain physical characteristics of the object for nondestructive testing. In this paper, we study the high-frequency behaviour of the scattered wave and more precisely the contribution of the 'optic' and 'creeping waves' parts of the field outside of the measured frequency band and thus in the reconstruction of the poles of the transfer function. These two parts are identified by their high-frequency behaviour.

Keywords: Scattering, High-frequency, Rational Approximation

1 Introduction

We are studying the scattering of a plane wave on an object as described in Figure 1. In this situation we measure the total electric field on a finite band of frequency at X_0 which is the point of emission of the sent plane wave (back-scattering configuration). The goal of this study is to be able to discriminate the object based on the poles of the transfer function:

$$F(k) = \frac{E_{sc}(X_0) \cdot E_0}{E_{inc}(X_0) \cdot E_0},$$

where k is the spatial pulsation (proportional to the frequency) and the incident electric field, which has a direction of propagation v, and the scattered electric field are respectively:

$$E_{inc}(X) = E_0 e^{ikv \cdot X},$$
$$E_{sc}(X) = E(X) - E_{inc}(X),$$

When the object is convex and smooth, this function is meromorphic with poles lying in the upper half-plane. The scattered electric field is solution to Helmholtz equation:

$$\Delta E_{sc} + k^2 E_{sc} = 0, \quad (1)$$
$$\nabla \cdot E_{sc} = 0, \quad (2)$$
$$\lim_{r\to\infty}(r(X \cdot \nabla - ik)E_{sc}) = 0, \quad (3)$$
$$(E_{sc} + E_{inc}) \times n = 0 \,\text{on}\, \partial\Omega. \quad (4)$$

In order to do the rational approximation of F, we need to describe the behaviour of the field E_{sc} at high frequency (outside the band of measured frequencies). In this text, we will mainly consider the case in which the object is a spherical PEC (Perfectly Electric Conductor) of radius a and in the back-scattering configuration ($X = -rv$) as in this case, we have an exact formula (Mie series) giving the solution [2]:

$$E_{sc}(-rv) = \frac{E_0}{kr}\sum_{n=1}^{\infty} i^n \left(n + \frac{1}{2}\right)\left[\frac{\hat{J}_n(ka)\hat{H}_n^{(2)}(kr)}{\hat{H}_n^{(2)}(ka)} - i\frac{\hat{J}_n{}'(ka)\hat{H}_n^{(2)\prime}(kr)}{\hat{H}_n^{(2)\prime}(ka)}\right],$$

where the $\hat{H}_n^{(2)}$ are the spherical Hankel functions of the second kind and the $\hat{J}_n$ are the spherical Bessel functions of the first kind.

As the partial sums of these series converge slowly when k increases, it is not a good formula to study its high-frequency behaviour.

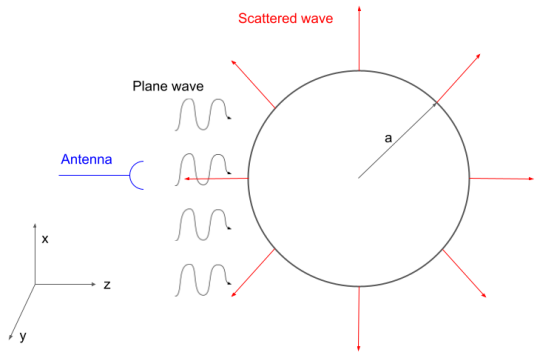

Figure 1: Setting of the scattering by a sphere.

2 Optical part

To study the high frequency behaviour of F, we choose a well-behaved ansatz as a Luneberg-Kline series [1]:

$$E_{sc}(k,X) = \sum_{n=0}^{N} \frac{A_n(X)}{(ik)^n} e^{ikS(X)} + o\left(\frac{1}{k^N}\right).$$

By substituting this form into (1) to (4) , we get an eikonal equation (5) and an infinite list of transport equations (6) that allow us to compute S and the $(A_n)_{n\in\mathbb{N}}$:

$$|\nabla S|^2 = 1, \tag{5}$$
$$(\Delta S + 2\nabla S \cdot \nabla) A_n = -\Delta A_{n-1}. \tag{6}$$

using the boundary conditions that for all $Y \in \partial\Omega$:

$$\begin{aligned} A_0(Y) \times Y &= -E_0 \times Y, \\ \forall n \in \mathbb{N}^*,\ A_n(Y) \times Y &= 0, \\ \forall n \in \mathbb{N},\ \nabla S \cdot A_n(Y) &= -\nabla \cdot A_{n-1}(Y), \\ S(Y) &= v \cdot Y. \end{aligned}$$

Using these equation, we compute the first terms of this series for a PEC and found that in the back-scattering case:

$$\begin{aligned} S &= (r-2a), \\ A_0 &= -\frac{a}{2r-a} E_0, \\ A_1 &= -2\frac{(r-a)^2}{(2r-a)^3} E_0, \\ A_2 &= 2\frac{(r-a)(2r^2-4ra+3a^2)}{(2r-a)^5} E_0. \end{aligned}$$

As shown in 4, this characterisation at high frequency is not enough for our purpose and we thus describe a method to get a more precise description of the field at high but finite frequencies.

3 Creeping wave

To simplify the use of the Mie series, we apply Watson transformation [3, 4] to it. It is a way to transform the sum into an integral using the residue theorem. We show that, far from the spherical object:

$$\begin{aligned} E_{sc}(-rv) &= P(\rho)\frac{\exp(ikr)}{kr} E_0, \\ P(\rho) &= P_c(\rho) + P_o(\rho), \end{aligned}$$

where the optical part P_o has the same high-frequency behaviour as computed before with the Luneberg-Kline ansatz and the creeping wave part P_c has the following behaviour:

$$P_c(\rho) \sim \frac{\tau^4 e^{i\frac{\pi}{3}}}{\beta_1 \mathrm{Ai}(-\beta_1)^2} \exp\left(i\pi\rho - e^{-i\frac{\pi}{6}}\tau\pi\beta_1\right),$$

where $\rho = ka$, $\tau = \tau(\rho) = \left(\frac{\rho}{2}\right)^{1/3}$ and β_1 is the first zero of the Airy function derivative Ai'

4 Numerical comparison

As P_c decreases exponentially when k goes to infinity, it is negligible in front of P_o. Nevertheless, let us study the importance of each of this terms when dealing with finite frequencies. We choose a frequency of 5 GHz for a sphere of radius $a = 0.15/2$ at a distance $r = 1$. We find that the three computed optics terms represent respectively 86.11%, 4.87% and 0.01% of the field while the creeping wave term represents 7.55% of the field. This shows that to reconstruct the poles of the transfer function, we will need to use the creeping wave part even if it is negligible at high-frequency.

References

[1] D. Bouche, F. Molinet and R. Mittra, *Asymptotic Methods in Electromagnetics*, 1st edition, Springer Berlin Heidelberg, Berlin, 1997.

[2] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*, 2nd edition, Wiley, London, 2001.

[3] T. Senior, Analytical and Numerical Studies of the Back Scattering Behavior of Spheres, *The University of Michigan Radiaton Laboratory Report* **7030-T** (1965).

[4] C. A. Valagiannopoulos, An overview of the Watson transformation presented through a simple example, *Progress In Electromagnetics Research* **75** (2007), pp. 137–152.