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# Spacetime Structure, Asymptotic Radiation, and Information Recovery for a Quantum Hybrid State

Yohan Potaux<sup>✉</sup> and Sergey N. Solodukhin

*Institut Denis Poisson UMR 7013, Université de Tours, Parc de Grandmont, 37200 Tours, France*

Debajyoti Sarkar<sup>✉</sup>

*Department of Physics, Indian Institute of Technology Indore, Khandwa Road, 453552 Indore, India*

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A hybrid quantum state is a combination of the Hartle-Hawking state for the physical particles and the Boulware state for the nonphysical ones (such as ghosts), as was introduced in our earlier work [Y. Potaux *et al.*, *Phys. Rev. D* **105**, 025015 (2022)]. We present a two-dimensional example, based on the Russo-Susskind-Thorlacius model, when the corresponding backreacted spacetime is a causal diamond, geodesically complete and free of the curvature singularities. In the static case it shows no presence of the horizon while it has a wormhole structure mimicking the black hole. In the dynamical case, perturbed by a pulse of classical matter, there appears an apparent horizon while the spacetime remains to be a regular causal diamond. We compute the asymptotic radiation both in the static and dynamical case. We define entropy of the asymptotic radiation and demonstrate that as a function of the retarded time it shows the behavior typical for the Page curve. We suggest interpretation of our findings.

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**Introduction.**—Since its inception by Hawking, the black hole (BH) information problem remained a classic, important, and unsolved problem. In the present note, in an attempt to answer these important questions, we follow the same route that we started in [1,2] some time ago. The key idea in this approach is that we first have to look, in a nonperturbative way, at the geometry that arises in the semiclassical description of the quantum gravity and that replaces the classical black hole geometry. The nonlocality present in the quantum effective action manifests itself in certain ambiguities, which are usually understood as some freedom in the choice of the quantum state. Some of the quantum states that have been discussed in the literature are as follows.

**The Hartle-Hawking state:** It contains thermal radiation at infinity and the stress-energy tensor is regular at the horizon. It describes a black hole in thermal equilibrium with the Hawking radiation.

**The Boulware state:** The stress-energy tensor is vanishing at infinity and there is no radiation there. However, being considered on a classical black hole metric, it is singular at the horizon.

If nonphysical particles are present, such as ghosts, the Boulware (*B*) state is the most appropriate one since it is

the only one where one does not detect the flow of the nonphysical particles at the asymptotic infinity of a classical black hole. Earlier discussions of the problem of ghosts appear in [3,4]. In fact, if both physical and nonphysical particles are present the adequate quantum state is a hybrid one [1]: the physical particles are in the Hartle-Hawking (HH) state while the nonphysical ones are in the Boulware state. The other observation, made on the case by case analysis for the hybrid state is that if at least some particles (either physical or nonphysical) are in the Boulware state, the respective backreacted geometry is horizonless and resembles that of a black hole mimicker, similarly to the one considered in [5]. This gives us an indication that hybrid state might provide a concrete example against the BH information paradox.

**The hybrid RST model.**—Many aspects of black holes may be tested in the two-dimensional case [6]. In the present note we give a complete analysis, within the two-dimensional Russo-Susskind-Thorlacius (RST) model [7], of the global structure of the backreacted geometry and of the radiation at the asymptotic infinity in a particular case when the number of the nonphysical fields exceeds the number of the physical fields. In the quantum case the classical action is modified by the terms that originate from integrating out the quantum conformal fields with the respective central charge  $\kappa$ . We here consider a case when there are two types of quantum fields with, respectively, positive and negative central charges,  $\kappa_1 > 0$  and  $\kappa_2 < 0$ , that corresponds to the physical and nonphysical fields. The complete action is a version of the two-dimensional RST

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action ( $W_0$  is a classical term that describes the string-inspired dilaton gravity coupled to a classical conformal field  $f$ , while  $W_1 + W_2$  is quantum;  $W_1$  is a local form of the Polyakov action),

$$\begin{aligned} W &= W_0 + W_1 + W_2, \\ W_0 &= \frac{1}{2\pi} \int \left\{ e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2} (\nabla f)^2 \right\}, \\ W_1 &= \sum_{i=1}^2 -\frac{\kappa_i}{2\pi} \int \left\{ \frac{1}{2} (\nabla\psi_i)^2 + \psi_i R \right\}, \\ W_2 &= -\frac{\kappa}{2\pi} \int \phi R, \quad \kappa = \kappa_1 + \kappa_2, \end{aligned} \quad (1)$$

where we introduced two different auxiliary fields  $\psi_i, i = 1, 2$ . They both satisfy the same equation  $\square\psi_i = R$  but with different boundary conditions. The difference is due to the fact that the physical fields will be in the HH state while the nonphysical fields will be in the  $B$  state. The metric variation  $T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} = 0$ , where (see [8] for some of the relevant equations used below)

$$\begin{aligned} T_{\mu\nu}^{(1)} &= \sum_{\kappa=1}^2 \frac{\kappa_i}{\pi} \left\{ \frac{1}{2} \partial_\mu \psi_i \partial_\nu \psi_i - \nabla_\mu \nabla_\nu \psi_i \right. \\ &\quad \left. + g_{\mu\nu} \left( \square\psi_i - \frac{1}{4} (\nabla\psi_i)^2 \right) \right\}, \\ T_{\mu\nu}^{(2)} &= \frac{\kappa}{\pi} (g_{\mu\nu} \square\phi - \nabla_\mu \nabla_\nu \phi). \end{aligned} \quad (2)$$

Another equation comes from variation of the action with respect to the dilaton. In the classical case,  $\kappa_1 = 0, \kappa_2 = 0$ , the solution is static,  $ds^2 = [dx^+ dx^- / (\lambda^2 x^+ x^- - M/\lambda)]$ , see [9]. It describes a black hole of mass  $\pi M$  and temperature  $T_H = \lambda/2\pi$ . The horizon is at  $x^+ x^- = 0$  and the curvature singularity at  $x^+ x^- = 0$ .

In the conformal gauge the metric  $ds^2 = -e^{2\rho} dx^+ dx^-$ , where  $\rho = \phi$  as in the classical case. In the complete quantum case, the appropriate analysis was done in [1]. The physical fields (with  $\kappa_1 > 0$ ) are in the Hartle-Hawking state (in the present context discussed in [10]), that is regular at the classical horizon ( $x^+ x^- = 0$ ). On the other hand, the nonphysical fields (with  $\kappa_2 < 0$ ) are in the Boulware state. In the absence of matter ( $f = 0$ ) the solution is static. The dilaton  $\phi(x^+ x^-)$  is found to be a solution of the master equation,

$$\Omega(\phi) = -\lambda^2 x^+ x^- - \frac{\kappa_2}{2} \ln |\lambda^2 x^+ x^-| + \frac{M}{\lambda}, \quad (3)$$

where  $\Omega(\phi) = e^{-2\phi} + \kappa\phi$ . In what follows we shall consider the case  $\kappa < 0$  as the solution in this case shows the most remarkable properties. The case of general  $\kappa$  will be considered in detail in an upcoming paper by the same authors.

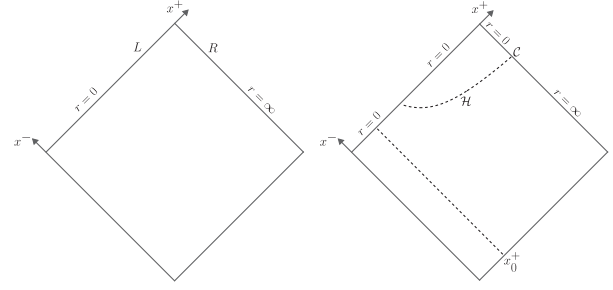


FIG. 1. Causal diamond structure of the static (left) and dynamic (right) solution.

*The global structure of the static solution for  $\kappa < 0$ .*—It corresponds to a totally regular spacetime, as we show below. We consider the causal diamond defined as  $\mathcal{M} \equiv (x^+ \geq 0, x^- \leq 0)$ . For any value of  $x^+ x^-$  in this domain there exists only one value of the dilaton  $\phi$ , i.e., it is a single valued function here. One finds that, in terms of variable  $r = e^{-\phi}$ ,  $r(\phi) \rightarrow +\infty$  when  $-x^+ x^- \rightarrow +\infty$  while  $r \rightarrow 0$  when  $-x^+ x^- \rightarrow 0$ . In both these limits the spacetime is asymptotically flat. For any null geodesic the boundary of  $\mathcal{M}$  lies at infinite values of the affine parameter. This proves that  $\mathcal{M}$  is geodesically complete.

The static spacetime metric admits a timelike Killing vector  $\xi = \lambda(x^+ \partial_+ - x^- \partial_-)$ . Its norm  $\xi^2 = \lambda^2 x^+ x^- e^{2\phi}$  is everywhere negative inside  $\mathcal{M}$ . The possible apparent horizon is defined by the condition that  $(\nabla\phi)^2$  vanishes there. One finds that  $(\nabla\phi)^2 = 0$  when  $x^+ x^- = 0$ . However, this happens on the border of the domain  $\mathcal{M}$  and hence it cannot define a boundary of the trapped region. Thus, we conclude that in the domain  $\mathcal{M}$  there exists neither Killing nor apparent horizons. The spacetime  $\mathcal{M}$  is geodesically complete and horizonless. It has the global structure of a causal diamond similar to the Minkowski spacetime.

Let us define the  $(tt)$  metric function  $g(x^+ x^-) = -\xi^2$  as minus the norm of the Killing vector  $\xi$ . As we have discussed above it never vanishes and hence is positive everywhere in  $\mathcal{M}$ . It develops a minimum

$$\min_{\mathcal{M}} g \simeq \left( \frac{2e}{\kappa_1} \right)^{-\kappa_1/\kappa_2} e^{-S_{\text{cl}}/|\kappa_2|}, \quad (4)$$

where  $S_{\text{cl}} = 2M/\lambda$  is the entropy of the classical black hole, that can be exponentially small for large values of  $M$ . This shows that the spacetime  $\mathcal{M}$  has the structure of a wormhole, the region where  $g$  takes its minimum is defined as the throat in which the time flows extremely slowly with respect to the time of an external observer (see [5,11,12]).

*The global structure of a dynamical solution for  $\kappa < 0$ .*—Let us consider the same domain  $\{x^+ > 0, x^- < 0\}$  with a shock wave of classical matter traveling along the geodesic  $x^+ = x_0^+$  in the  $x^-$  direction with mass  $m > 0$ ,  $\frac{1}{2}(\partial_+ f)^2 = (m/\lambda x_0^+) \delta(x^+ - x_0^+)$ . In this case one finds for the master equation, for  $x^+ \geq x_0^+$ ,

$$\Omega(\phi) = -\lambda^2 x^+ \left( x^- + \frac{m}{\lambda^3 x_0^+} \right) - \frac{\kappa_2}{2} \ln(-\lambda^2 x^+ x^-) + \frac{M+m}{\lambda}. \quad (5)$$

The spacetime above the line  $x^+ = x_0^+$  is no more static. It becomes evolving and an apparent horizon appears. The equation  $\partial_+ \phi = 0$  defines a curve

$$\mathcal{H}: -\lambda^2 x^+ \left( x^- + \frac{m}{\lambda^3 x_0^+} \right) = \frac{\kappa_2}{2} < 0, \quad x^+ \leq x_0^+ \quad (6)$$

which has a branch in the domain  $\{x^+ > 0, x^- < 0\}$ . This curve starts at the point  $[x^- = 0, x^+ = x_h^+ = (-\kappa_2)\lambda x_0^+/2m]$  and ends at the point  $[x^- = x_h^- = -(m/\lambda^3 x_0^+), x^+ = +\infty]$ .  $r(\phi)$  approaches 0 at the left ( $L$ ) future infinity both inside and outside the apparent horizon  $\mathcal{H}$  while at the right ( $R$ ) future infinity it approaches 0 inside  $\mathcal{H}$  and infinity outside  $\mathcal{H}$ . Scalar curvature approaches zero everywhere on the left and right future infinity except for the point  $\mathcal{C}$  at which horizon  $\mathcal{H}$  intersects asymptotic region  $R$ , where it takes a finite positive value. Thus, the curvature is not smooth on the future null infinity.

In the dynamic case, just like in the static case, all null and timelike geodesics can be continued indefinitely, the variation in the affine parameter is infinite approaching the future null infinity, both outside and inside the apparent horizon. Thus, the dynamical version of the spacetime  $\mathcal{M}$  is geodesically complete. The dynamical geometry in this case (as well as the static counterpart) has been illustrated in Fig. 1.

*Asymptotic energy density.*—We consider the stress-energy  $T_{\mu\nu}^{(12)}$  as the part due to the quantum matter while  $T_{\mu\nu}^{(0)}$  is purely geometrical, similar to the Einstein tensor in the 4D gravitational equations. In the hybrid quantum state and considered on the solution to the complete gravitational equations (2), this part reads

$$2\pi T_{\pm\pm}^{(12)} = 2(\kappa_1 + \kappa_2) \partial_{\pm}^2 \phi - \frac{\kappa_2}{(x^{\pm})^2}. \quad (7)$$

Clearly, it has two contributions: one, proportional to  $\kappa_1$ , is due to the physical fields and the other, proportional to  $\kappa_2$ , is due to the nonphysical fields. Some particular limits are of interest both in the static and the dynamic cases.

Static case: (i) Left future null infinity:  $T_{++}^{(12)} = 0$ , so that in this asymptotic region the out-going radiation of physical particles and nonphysical particles exactly compensate each other. (ii) Right future null infinity:  $2\pi T_{--}^{(12)} = [\kappa_1/(x^-)^2]$  (and  $T_{++}^{(12)} = 0$ ). Thus, only the physical particles contribute to the asymptotic energy density, which is positive. This is expected as the hybrid quantum state is designed to have precisely this property. This is the energy density of the thermal gas with temperature  $T = \lambda/2\pi$  as can be seen by passing to the asymptotic coordinates

$(\sigma^+, \sigma^-)$ :  $\lambda x^+ = e^{\lambda\sigma^+}$ ,  $\lambda x^- = -e^{-\lambda\sigma^-}$  in which metric is constant  $ds^2 = -d\sigma^+ d\sigma^-$ . Thus, despite the absence of a horizon an outside observer detects the thermal radiation.

Dynamical case: Compared to the static case the spacetime now is divided into two regions, inside and outside the apparent horizon. As a result, there are additional asymptotic limits. (i) Left future null infinity, both inside and outside apparent horizon: the stress energy vanishes in the same way as in the static case. (ii) Right future null infinity, inside apparent horizon ( $x^- > x_h^-$ ): the stress energy vanishes,  $T_{--}^{(12)} = 0$ . Effectively, inside the apparent horizon no outgoing radiation appears in any asymptotic region there. (iii) Right future null infinity, outside the apparent horizon ( $x^- < x_h^-$ ):

$$2\pi T_{--}^{(12)} = \frac{\kappa_1}{(\tilde{x}^-)^2} + \kappa_2 \left( \frac{1}{(\tilde{x}^-)^2} - \frac{1}{(x^-)^2} \right), \quad (8)$$

where  $\tilde{x}^- = x^- - x_h^-$ . For large negative values of  $x^-$  it approaches the positive thermal energy density as in the static case while as  $x^-$  increases there appears a negative contribution due to the nonphysical particles. (iv) At the point  $\mathcal{C}$  ( $x^- = x_h^-$ ,  $x^+ \rightarrow \infty$ ) on the apparent horizon  $T_{--}^{(12)}$  is negative and divergent.

Note that  $T_{++}^{(12)} = 0$  everywhere on the right future infinity.

*Entropy of radiation and the Page curve.*—Entropy of the radiation in the right asymptotic region can be defined by the differential equation,

$$\partial_- S = 2\pi(-x^-)T_{--}^{(12)}. \quad (9)$$

This equation is a differential form of the first law,  $dS = T^{-1}dE$ , where  $T = \lambda/2\pi$  and  $dE = T_{--}d\sigma^-$  is the energy defined in terms of the coordinates  $(\sigma^+, \sigma^-)$  introduced above. Using Eq. (8) one finds that for large negative values of  $x^-$  the entropy  $S$  is increasing. It is due to the positive energy density of the physical particles. Then there exists a point  $x_m^-$  at which the entropy has maximum (its derivative  $\partial_- S = 0$ ). This happens at  $x_m^- = (1 - \sqrt{\kappa/\kappa_2})^{-1}x_h^-$  so that  $x_m^- < x_h^-$ . Then, for values  $x^- > x_m^-$  the entropy is monotonically decreasing. In order to avoid the singularity at point  $\mathcal{C}$  we may consider the change in the entropy along the line of large but finite value of  $x^+$ . Then  $S(x^-)$  shows the behavior typical for the Page curve. The total change in the entropy along the line of constant  $x^+$  while  $x^-$  is changing in the interval  $(-\infty, 0)$  is given by integral,

$$\Delta S = 2\pi \int_{-\ell}^{-\epsilon} dx^- (-x^-) T_{--}^{(12)}, \quad x^+ = \text{const}, \quad (10)$$

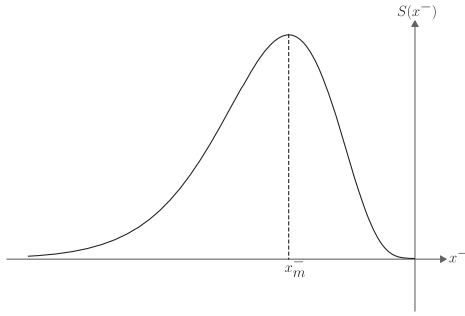


FIG. 2. Schematic behaviour of asymptotic radiation entropy  $S(x^-)$  as function of the retarded time  $x^-$ .

where we introduced two regulators,  $\ell$  and  $\epsilon$  to regularize the possible divergences for, respectively, large negative  $x^-$  and  $x^-$  close to 0, and  $T_{--}^{(12)}$  is given by (7). The computation shows that (10) does not depend on  $\epsilon$  so that one can safely take the limit  $\epsilon \rightarrow 0$ . On the other hand, the large  $\ell$  regulator should be kept till the end. The result (for large  $\ell$ ) is

$$\Delta S = \kappa_1 \ln(\lambda^2 x^+ \ell / e) + \frac{2M}{\lambda} + \frac{2m}{\lambda} \left(1 - \frac{x^+}{x_0^+}\right). \quad (11)$$

Notice that, rather surprisingly, it contains the two last terms that are completely classical and do not depend on  $\kappa_1$ . Computationally, they come from the upper limit in the integral (10). The other observation is that (11) does not depend on  $\kappa_2$  so that effectively only physical particles contribute to the total change in the entropy. Provided  $\kappa_1 = N/24$ , by passing to coordinates  $\sigma^+$ ,  $\sigma^-$  and introducing the respective regulator  $L$  (that effectively replaces  $\ell$ ), the first term can be rewritten as  $(\pi/3)NTL$ . This is precisely the thermal entropy (with temperature  $T = \lambda/2\pi$ ) of the outgoing modes. On the other hand, the second term is precisely the entropy of the classical static black hole. It is modified by the third term that is due to the perturbation and that is decreasing in the  $x^+$  direction. The Eq. (11) thus represents a balance in the entropy of two components of the system: thermal gas and the black hole. The general behavior of  $S(x^-)$ , illustrated in Fig. 2, is the behavior expected for the Page curve [13] (for a recent series of discussions on this direction see [14]). The characteristic Page time  $x_m^- = (1 - \sqrt{\kappa/\kappa_2})^{-1} x_h^-$  is a function of two parameters  $\kappa_1$  and  $\kappa = \kappa_1 + \kappa_2$ . It can be made macroscopic (large negative  $x_m^-$ ) by an appropriate choice of these parameters. This is consistent with the original Page analysis that showed that consistency with semiclassical physics requires the information to come out at macroscopic scales.

*Interpretation and conclusions.*—A possible interpretation of our result comes from considering a pair creation of physical and nonphysical particles, much like Hawking’s pair creation. In the static case, the spontaneously created

left-moving (physical and nonphysical) particles reach the left future null infinity and precisely compensate the energy of each other so that the total energy flux vanishes. A similar pair of the right-moving particles behaves differently. The physical particles go to the right future null infinity and form the asymptotic radiation there. On the other hand, their nonphysical counterpart stays in the bulk and does not reach the asymptotic region. They are in the Boulware quantum state that guarantees that the nonphysical particles are not visible in the asymptotic region in the static case. In the dynamical case there appears an apparent horizon that changes the behavior of the right-moving nonphysical particles. They now can reach the right asymptotic infinity but at much later retarded time  $x^-$  than when they were spontaneously created. For the entropy of the radiation this explains the declining part of the curve  $S(x^-)$ . Two lessons could be learned from the two-dimensional example considered here. First, when the backreaction is taken into account, the presence of particles in the Boulware quantum state may drastically change the global geometry of the spacetime, that classically describes a black hole. Here, we have presented the case when the backreacted spacetime is a completely regular and geodesically complete causal diamond, in which the classical horizon is replaced by a wormhole throat. Second, the particles in the Boulware quantum state may carry the important information, that is missing when only the particles in the Hartle-Hawking quantum state are considered. The ultimate release of the Boulware particles helps to reconstruct the complete information present in the system.

The analysis presented here should be compared with the earlier discussions of the Page curve [14]. Two principal differences have to be mentioned. First of all the earlier discussion of the Page curve was done in terms of the entanglement entropy while here we obtain the Page curve directly for the entropy of the asymptotic radiation. Since the radiation is directly accessible for an external observer this appears to have obvious observational advantages. The other point to be mentioned is that here we deal directly with the nonperturbative solution of the quantum gravitational equations while the earlier analysis appears to be perturbative.

The RST model is rather special and one may wonder how much of the present analysis would survive in a different two-dimensional model of gravity or, in fact, be relevant in four-dimensions. Although a detailed analysis is needed in each concrete cases, we do anticipate that in the principal aspects, the present picture will be relevant to other quantum gravitational models. The presence of the Boulware particles, such as ghosts, will necessarily modify the global structure of the spacetime, remove the classical horizon and replace it with a structure similar to that of the black hole mimicker. That it is the case in four-dimensions was demonstrated in [2]. The contribution of the Boulware



particles in the total energy and entropy balance should play an important role in the general understanding of the information recovery and the quantum unitarity.

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