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Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck. Computer-assisted enumeration and classification of multi-qubit doilies. Journées Informatique Quantique (JIQ'22), Nov 2022, Paris, France. hal-03932840

**HAL Id: hal-03932840**

**<https://hal.science/hal-03932840v1>**

Submitted on 10 Jan 2023

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# Computer-assisted enumeration and classification of multi-qubit doilies

Talk presented by A. Muller at the *Journées Informatique Quantique 2022*,  
organized by the CNRS working group *Informatique Quantique*  
at Paris (France), <https://qi.lip6.fr/jiq22/>

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November 14-15, 2022

## Abstract

For  $N \geq 2$ , an  $N$ -qubit doily is a doily living in the  $N$ -qubit symplectic polar space. These doilies are related to operator-based proofs of quantum contextuality. Following and extending the strategy of [SdBHG21] that focused exclusively on three-qubit doilies, we first bring forth several formulas giving the number of both linear and quadratic doilies for any  $N > 2$ . Then we present an effective algorithm for the generation of all  $N$ -qubit doilies. Using this algorithm for  $N = 4$  and  $N = 5$ , we provide a classification of  $N$ -qubit doilies in terms of types of observables they feature and number of negative lines they are endowed with.

The doily is the unique triangle-free self-dual finite geometry composed of 15 points and 15 lines, with three points on a line and, dually, three lines through a point.

This talk [MSG<sup>+</sup>22a] is about doilies labeled with  $N$ -qubit observables, to be called  *$N$ -qubit doilies* or *multi-qubit doilies*, that are related to Kochen–Specker operator-based proofs of quantum contextuality. We describe an algorithm for the enumeration of all  $N$ -qubit doilies, for any  $N \geq 2$ , and an optimized C implementation of this algorithm. We also present several classification criteria of  $N$ -qubit doilies, that are their types of observables, their linear or quadratic nature and their configurations of negative lines. As the numbers of  $N$ -qubit doilies increase rapidly with  $N$ , the enumeration program can in practice only be executed for small numbers of qubits. We use it to classify  $N$ -qubit doilies for

$N = 4$  and  $N = 5$ , according to these criteria. We thus produce precise tables for the number of doilies in each category. Before this work, the classification was only known for  $N = 3$  [SdBHG21]. The presented results are published in [MSG<sup>+</sup>22b].

## Acknowledgments

This work is supported by Agence Nationale de la Recherche - Plan France 2030 - EPiQ project (ANR-22-PETQ-0007), the EIPHI Graduate School (contract ANR-17-EURE-0002), and the Slovak VEGA Grant Agency, Project # 2/0004/20.

## References

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