



**HAL**  
open science

## Computer-assisted enumeration and classification of multi-qubit doilies

Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck

► **To cite this version:**

Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck. Computer-assisted enumeration and classification of multi-qubit doilies. Journées Informatique Quantique (JIQ'22), Nov 2022, Paris, France. hal-03932840

**HAL Id: hal-03932840**

**<https://hal.science/hal-03932840>**

Submitted on 10 Jan 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



**HAL**  
open science

## Computer-assisted enumeration and classification of multi-qubit doilies

Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck

► **To cite this version:**

Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck. Computer-assisted enumeration and classification of multi-qubit doilies. Journées Informatique Quantique, Nov 2022, Paris, France. hal-03932840

**HAL Id: hal-03932840**

**<https://hal.science/hal-03932840>**

Submitted on 10 Jan 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Computer-assisted enumeration and classification of multi-qubit doilies

Talk presented by A. Muller at the *Journées Informatique Quantique 2022*,  
organized by the CNRS working group *Informatique Quantique*  
at Paris (France), <https://qi.lip6.fr/jiq22/>

Axel Muller<sup>1</sup>, Metod Saniga<sup>2</sup>, Alain Giorgetti<sup>1</sup>, Henri de Boutray<sup>3</sup>,  
and Frédéric Holweck<sup>4,5,6</sup>

<sup>1</sup>FEMTO-ST institute, Univ. Bourgogne Franche-Comté, CNRS, France

<sup>2</sup>Astronomical Institute of the Slovak Academy of Sciences, SK-05960 Tatranská Lomnica, Slovakia

<sup>3</sup>ColibrITD, France

<sup>4</sup>Université de Technologie de Belfort-Montbéliard, F-90010 Belfort cedex, France

<sup>5</sup>Laboratoire Interdisciplinaire Carnot de Bourgogne (UMR 6303 - CNRS/ICB/UTBM), France

<sup>6</sup>Department of Mathematics and Statistics, Auburn University, Auburn, AL, USA

November 14-15, 2022

## Abstract

For  $N \geq 2$ , an  $N$ -qubit doily is a doily living in the  $N$ -qubit symplectic polar space. These doilies are related to operator-based proofs of quantum contextuality. Following and extending the strategy of [SdBHG21] that focused exclusively on three-qubit doilies, we first bring forth several formulas giving the number of both linear and quadratic doilies for any  $N > 2$ . Then we present an effective algorithm for the generation of all  $N$ -qubit doilies. Using this algorithm for  $N = 4$  and  $N = 5$ , we provide a classification of  $N$ -qubit doilies in terms of types of observables they feature and number of negative lines they are endowed with.

The doily is the unique triangle-free self-dual finite geometry composed of 15 points and 15 lines, with three points on a line and, dually, three lines through a point.

This talk [MSG<sup>+</sup>22a] is about doilies labeled with  $N$ -qubit observables, to be called  *$N$ -qubit doilies* or *multi-qubit doilies*, that are related to Kochen–Specker operator-based proofs of quantum contextuality. We describe an algorithm for the enumeration of all  $N$ -qubit doilies, for any  $N \geq 2$ , and an optimized C implementation of this algorithm. We also present several classification criteria of  $N$ -qubit doilies, that are their types of observables, their linear or quadratic nature and their configurations of negative lines. As the numbers of  $N$ -qubit doilies increase rapidly with  $N$ , the enumeration program can in practice only be executed for small numbers of qubits. We use it to classify  $N$ -qubit doilies for

$N = 4$  and  $N = 5$ , according to these criteria. We thus produce precise tables for the number of doilies in each category. Before this work, the classification was only known for  $N = 3$  [SdBHG21]. The presented results are published in [MSG<sup>+</sup>22b].

## Acknowledgments

This work is supported by Agence Nationale de la Recherche - Plan France 2030 - EPiQ project (ANR-22-PETQ-0007), the EIPHI Graduate School (contract ANR-17-EURE-0002), and the Slovak VEGA Grant Agency, Project # 2/0004/20.

## References

- [MSG<sup>+</sup>22a] Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, and Frédéric Holweck. Computer-assisted enumeration and classification of multi-qubit doilies. In *Journées Informatique Quantique 2022 - JIQ'22*, Paris, France, 2022. <https://quantcert.github.io/doilies/jiq2022handout.pdf>.
- [MSG<sup>+</sup>22b] Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, and Frédéric Holweck. Multi-qubit doilies: Enumeration for all ranks and classification for ranks four and five. *Journal of Computational Science*, 64:101853, 2022. <https://doi.org/10.1016/j.jocs.2022.101853>.
- [SdBHG21] Metod Saniga, Henri de Boutray, Frédéric Holweck, and Alain Giorgetti. Taxonomy of Polar Subspaces of Multi-Qubit Symplectic Polar Spaces of Small Rank. *Mathematics*, 9(18):2272, 2021. <https://doi.org/10.3390/math9182272>.