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From biological morphogenesis to engineering joint design: a bio-inspired algorithm

Kalenia Marquez-Florez, Santiago Arroyave-Tobon, Jean-Marc Linares

Aix Marseille Univ, CNRS, ISM, Marseille, France

Abstract

Nature has developed smart strategies to morph joint contact surfaces as a response to local mechanical stimuli. Hence, it is compelling to explore the advantages of mimicking these strategies to design mechanical pieces. Researchers and engineers have developed generative design strategies mostly focused on structural topological and shape optimization. However, these generative strategies are not well developed for joint contact surfaces, even though they are critical points in engineering structures and machinery. This work presents a computational tool for adaptive surface shaping in contact problems inspired by the morphogenesis of synovial joints. **The algorithm behind follows the cartilage growth rules, in which the hydrostatic and shear octahedral stresses either promote or inhibit an isotropic expansion of the local domain.** The implementation is based on the finite element method and it is freely available on GitHub. A parametric study was performed to tune the parameters of the algorithm and it was successfully tested for two elastic bodies in unilateral contact. It was evidenced that the cartilage growth rules adapted the geometry of the surfaces in such a way that the contact stresses got uniformly distributed. This study is a step forward in the scope of bio-inspired generative designs.

Keywords: joint morphogenesis, bio-inspiration, elastic contact, surface shaping, DoE

1. Introduction

Nature has served as a mentor for the development of innovative designs due to its energy-efficient, adaptable and sustainable processes [1]. Synovial joints offer a good example of how nature transforms its structures in such a manner that they become durable, adaptable, and efficient enough, thereby vertebrates can accommodate easily in different ecological environments.

Synovial joints, when healthy, are relatively highly wear-resistant structures designed to provide a smooth relative movement between bones. The capacity of these structures to bear a lifetime of loading cycles is, in part, due to the unique characteristics of the tissues, but mainly, to their morphology which distributes efficiently stresses within it. (author?) [2] demonstrated that regardless of the complex geometry of the articular surfaces of a lamb elbow, the

Email address: kalenia-maria.marquez-florez@univ-amu.fr (Kalenia Marquez-Florez, Santiago Arroyave-Tobon, Jean-Marc Linares)

10 contact zones are located at the same radius, minimizing the kinematic heterogeneousness and therefore the energy
11 loss [2]. Hence, it is compelling to apply synovial joint morphogenesis principles to the design of mechanical joints.
12 In this way, load-adapted designs can be generated with an enhanced lifespan.

13 Researchers and designers have tried before to employ nature principles to design mechanical joints. This inspira-
14 tion can either use the solution already given by nature (solution-based) or mimic the biological process that generates
15 said solution (processes-based) [3].

16 Several works have taken solution-based approaches to address problems related to force and movement transmis-
17 sion. (author?) [4]. followed by (author?) [5]. identified several joint mechanisms found in nature and described
18 their kinematics, actuators, and control. This inspired them to propose, first a two-degrees-of-freedom mechanism
19 that resembles the structure and motion of the knee [4]. The authors proposed later a shape-changing flexible mech-
20 anism without sliding elements, therefore reducing energy dissipation and lubrication requirement [5]. (author?) [6]
21 proposed a bushing bearing suited to misalignment, bio-inspired from a lamb elbow [6, 7]. They compared the wear
22 performance of the obtained design against a classical bush bearing. The bio-inspired bearing showed lower pres-
23 sures, an elliptic contact zone, and less wear rate than the classical one. Following this trend, (author?) [8] evaluated
24 numerically and for different loading conditions, the load-bearing performance of bio-inspired designs based on the
25 two morphologies that nature offers for one degree of freedom elbows, *i.e.* with and without a *trochlear sulcus* [8].
26 They found that the ones without *trochlear sulcus* had lower peak pressure values and larger contact areas, hence suit-
27 able for axial and combined loading; whereas the ones with *trochlear sulcus* were more suitable for bearing turnover
28 moments [8].

29 However, the bio-inspiration to design mechanical joints can be focused at a further level of biomimesis: A
30 processes-based approach. This, for instance, by adapting the strategies employed by nature to achieve their highly
31 functional synovial joints (joint morphogenesis process) to the specific needs of the mechanical design.

32 Nature's strategy to develop synovial joints initiates with an uninterrupted condensation of mesenchymal cells
33 (MSCs) within the developing limb, which later differentiate into chondrocytes (CCs) -cartilage cells, except at the
34 site of the future joint (Interzone) (fig. 1) [9]. Then, the synovial capsule develops following the physical separation
35 of the bone cartilaginous rudiments (cavitation). Joint morphogenesis takes place afterward regulated by a series
36 of tissue adaptations that depends on biochemical and mechanical stimuli (fig. 1). However, it is mainly due to the
37 proliferation of CCs that the cartilaginous bone rudiments grow and get their final shape [10, 11]. Simultaneously,
38 the endochondral ossification is starting with the onset of the primary ossification center (POC), and subsequently, the
39 onset of the secondary ossification center (SOC) [12]. These structures progressively ossify the cartilaginous bone
40 rudiments until there only remains a layered cartilaginous structure between them, the growth plate.

41 Designers have proposed generative tools for the conception of contact pieces such as shape optimization (SO) or
42 topological optimization (TO). These works have focused on obtaining a uniform contact pressure: SO by changing
43 the shape of the bodies in contact [13, 14, 15, 16, 17, 18, 19, 20, 21], and TO by optimizing the material layout within
44 a domain [22, 23, 24, 25]. Other designers have implemented frameworks that combine TO and SO to conceive

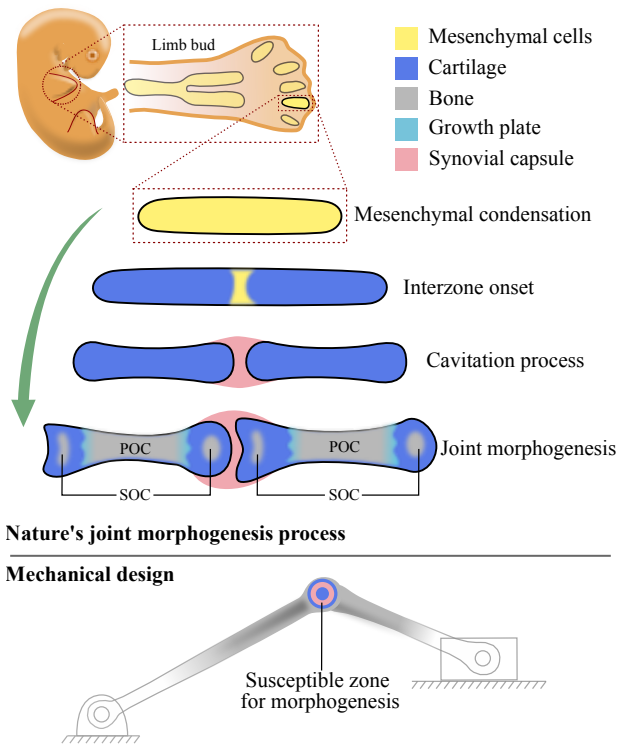


Figure 1: Representation of human upper limb development. Simplified scheme of the interphalangeal joint development process and the susceptible zones to bio-inspired morph in a mechanism. In nature, synovial joint development is marked by the onset of the Interzone within the condensation of mesenchymal cells in the limb bud. Subsequently, the physical separation of two cartilaginous bones takes place due to the cavitation process. Then, the joint morph thanks to the chondrocytes' response to mechanical and biochemical stimuli, which can promote either chondrocytes proliferation (growth and shaping) or chondrocytes hypertrophy (encouraging ossification, hence the onset of the POC and SOC). Within a mechanical design, the same principles on how the cartilage adapts to loading can be applied to morph a mechanical joint.

45 a two-component interlock, in which the contacting surfaces are modified as well as the material layout within the
 46 pieces [26]. However, these works only deal with the material distribution without modifying the shape of contact
 47 surfaces. (author?) [27] proposed a combined interface shape and material stiffness optimization method; in which
 48 both contact surfaces end up with the same geometry. Nevertheless, these methods (TO, SO, and (author?) [27]), do
 49 not seem to be potentially extrapolated for the design of mechanical pieces with relative movement between them (i.e.
 50 bushings and similar).

51 To date, only the authors' research group has made an initial approach to the application of the synovial joint
 52 morphogenesis principles to design mechanical pieces, demonstrating the feasibility of applying the cartilage growth
 53 laws in the automatic design of contact surfaces [28]. However, the way of establishing the process parameters and
 54 their influence on the final shape is not discussed. Apart from this previous work, the authors have not found other
 55 evidence in the literature in which joint morphogenesis has been applied to propose novel mechanical joint designs.

56 Hence, this work proposes a computational implementation of a bio-inspired algorithm for contact surface shap-
 57 ing and a parametric study followed by an optimization of these parameters. The algorithm is based on the biological
 58 principles of joint morphogenesis (section 2). The case of two elastic bodies in unilateral contact under static con-

59 ditions is studied (section 3.1). The biological joint morphogenesis process was translated to a mechanical setting
60 (section 3.2). The problem was formulated following a normalized approach (section 3.3) so that it could be solved
61 independently of the applied force and the dimensions of the initial geometry. Buckingham's Pi Theorem was used to
62 define the minimal set of dimensionless parameters that govern the bio-inspired process (section 3.4). The algorithm
63 was implemented using the finite element method (section 3.5). The most influential parameters of the algorithm were
64 optimized following a design of experiments (DoE) strategy (section 3.6). Using the optimized algorithm, a contact
65 profile was obtained in a generative way, mimicking the cartilage growth (section 4). Due to the stress-dependent
66 nature of the process, the obtained contact profile is well adapted to the applied load. This study is a stepping stone
67 for the development of a generative design framework inspired by nature's joint morphogenesis process. Additionally,
68 it can also draw some conclusions on how to develop more efficient mechanical pieces, and give some insights into
69 how nature adapts itself to the mechanical environment.

70 **2. On joint onset and morphogenesis**

71 This section carries out a literature review of the joint onset and morphogenesis processes is carried out. Also, an
72 exploration of computational models found in the literature that aimed to explain joint development-related processes
73 is performed. These models are the base of the morphogenesis-inspired process for designing contact profiles devel-
74 oped in this work. Finally, some conclusions that condensate the theories and computational models are portrayed.

75 *2.1. Overview of joint onset and morphogenesis*

76 The development of synovial joints initiates with an uninterrupted condensation of MSCs (skeletal blastema)
77 within the early limb bud. These MSCs then differentiate into CCs, however, on the site of the future joint the CCs
78 will de-differentiate to MSC; this place is known as the Interzone and is formed by packed layered condensation of
79 MSCs [29]. This first step of joint development is known as Interzone onset [30].

80 The definitive separation of the future bones (at this moment still cartilaginous), occurs with the cavitation process.
81 At this stage, a cavity is created between the cartilaginous molds which will be filled by synovial fluid. The cleavage
82 is due to the differentiation of the MSCs within the Interzone into other joint tissues (capsule, meniscus, ligaments,
83 etc.) [31], or even apoptosis (programmed cell death) [31, 32, 33]. The cartilaginous bones acquire their initial shape
84 during the Interzone onset and the cavitation process, either by CCs proliferation or due to the shape of the Interzone
85 [34, 35].

86 *2.2. Chondrocyte mechanical response*

87 In parallel, the muscles are developing from the base of the limb bud, and as they form the growing limbs increase
88 their movement, twitching, stretching, etc. [36]; in fact, studies have shown that the mechanical effects due to these
89 movements are necessary for a normal joint formation [37]. Thus, being the bones still in their cartilaginous state,

90 the joint morphing process is highly dependent on the response of the CCs to mechanical stimuli, mainly on their
91 proliferation, hypertrophy, and matrix production rates.

92 Carter and associates have proposed a theory of growth and ossification during the development of synovial joints
93 (known as Carter's theory) [38, 39, 40, 41, 42, 11]; in essence, they suggest that intermittent octahedral shear stress
94 accelerates chondrocyte hypertrophy (and subsequent ossification of the tissue), whereas cyclic compressive hydro-
95 static stress maintains cartilage in its current state, *i.e.* proliferative, which translates into growth of cartilaginous
96 tissue. This theory has been complemented by other experimental studies [43, 44, 45, 46, 47].

97 2.3. Biochemical factors

98 Undoubtedly, not only do mechanical stimuli affect joint morphogenesis but also biochemical factors. The bio-
99 chemical system between the Parathyroid Hormone-related Protein (PTHrP) and the Indian Hedgehog (Ihh) has rela-
100 tively high importance in the development of long bones [48]. Ihh is said to control CCs proliferation, maturation, and
101 hypertrophy [49]. PTHrP inhibits CCs hypertrophy, keeping them in a proliferative state [50]. The key point in the
102 PTHrP-Ihh system is that these molecules have a negative autoregulatory loop, which leads to a concentration gradient
103 that defines the proliferative zone size and rate. It should be recalled that it is the proliferation of the CCs that regu-
104 lates the growth of the cartilaginous tissue. PTHrP is highly concentrated at the periarticular perichondrium (where
105 the articular cartilage will be) of cartilaginous bones and diffuses to the growth plate. Then, where the concentration
106 of PTHrP drops below a critical threshold, the proliferation of CCs stops and they start to hypertrophy [48]. Ihh is
107 expressed by these early hypertrophic cells, which stimulates the synthesis of PTHrP.

108 2.4. Cartilage growth mathematical models

109 Computational models have been developed to simulate joint morphogenesis, in which the growth of the tissue
110 is regulated by the proliferation of the CCs and proportional to the CCs population density and the hydrostatic stress
111 [51, 52, 10].

112 The first model was developed by (author?) [51], who followed the Hueter-Volkmann law; the latter argues that
113 growth is reduced by an increased compression loading [53]. They modeled growth through a tensor $\mathbf{G} = \lambda \mathbf{I}$, where
114 λ is a scalar measure of the growth and \mathbf{I} is the identity tensor [51]. The scalar term $\lambda = k_b \psi_b + k_m \psi_m$ was a linear
115 combination of a mechanobiological stimulus ψ_m (which depended on the stress history) and a biological growth
116 stimulus ψ_b (function of the position along the long axis of the bone rudiment), where k_b and k_m are the relative
117 strength of the biological and mechanobiological stimuli. They obtained relatively congruent surfaces, however, it
118 must be considered that the Hueter-Volkmann law is a qualitative law that does not take into account the load history
119 and is mainly proposed for growth plates and bone growth [53].

120 Several years later, (author?) [52] proposed a model in which implemented a growth rate: $\dot{\epsilon} = \dot{\epsilon}_b + \dot{\epsilon}_m$, where $\dot{\epsilon}_b$
121 is the biological contribution to growth (considered in a similar manner that in (author?) [51]) and $\dot{\epsilon}_m$, the mechanical
122 contribution to growth, was proportional to the compressive hydrostatic stress [52]. They modeled growth as a thermal

123 isotropic expansion in which $\dot{\epsilon}$ was used as ‘temperature’. Additionally, they included a synovial capsule and evaluated
124 its effect on the joint morphogenesis process. They also obtained relatively more congruent shapes of the two joining
125 bones compared to the previous model.

126 Following (author?) [52], (author?) [10]. developed a computational model for the morphogenesis of an inter-
127 phalangeal joint. They considered a molecular pattern (PTHrP-Ihh) which dictated what zones of the cartilaginous
128 rudiments were proliferative (allowed to grow) [10]. As in (author?) [52], they also included an incompressible
129 synovial capsule. Regarding the mechanical growth rules, they considered that cyclic hydrostatic compressive stress
130 promoted growth; however, if the octahedral shear stress was above a threshold, the growth was inhibited (Carter’s
131 law). In that work, growth was modeled as an imposed strain: $\mathbf{d} = \alpha Cco^{-1} S_{\text{hyd}} \mathbf{I}$, which depended on a constant
132 cellular concentration Cco and the compressive hydrostatic stress S_{hyd} . They obtained a congruent joint similar to an
133 interphalangeal joint.

134 2.5. Literature review conclusions

135 Some conclusions might be drawn from the aforementioned models and the reviewed literature: (1) a biochemical
136 gradient function along the long axis of the rudiments is necessary to determine the proliferative zones that will add
137 to the overall growth of the bone; (2) compressive cyclic hydrostatic stress promotes growth; (3) high octahedral
138 shear stress inhibits cellular proliferation, hence growth; (4) the synovial capsule has an important role in the joint
139 morphogenesis. From these conclusions, a simplified biological process is proposed (fig. 2).

140 3. Materials and methods

141 The bio-inspired process was implemented using normalized variables to solve the problem independently of the
142 applied force and the dimensions of the initial geometry. The minimal set of dimensionless parameters governing the
143 bio-inspired process was identified by applying Buckingham’s Pi theorem. This set of parameters was optimized by
144 following a DoE strategy. Two contact profiles were obtained generatively (mimicking cartilage growth) using the
145 algorithm with the optimized parameters. The implementation and optimization of the algorithm are explained in the
146 next sections. The implementation is based on the finite element method and it is freely available on GitHub¹.

147 3.1. Unilateral contact problem

148 This study aims to morph the contact surface of two elastic bodies, one upper body, and one lower body, by
149 following nature’s rules of growth (fig. 3). The contact surface of the upper body starts as a hemispheric geometry of
150 constant radius ($R_u = \text{const}$), *i.e.* the local radius is constant regardless the point on the surface; for the lower body,
151 the contact surface starts as a semi-infinite half-space ($R_l = \infty$) (fig. 3 -left). An external distributed static force \mathbf{F} is
152 applied on the top of the upper body. Both bodies have the same material properties (E and ν). The upper body is

¹https://github.com/sarroyavet/BioDesign_joint_morphogenesis

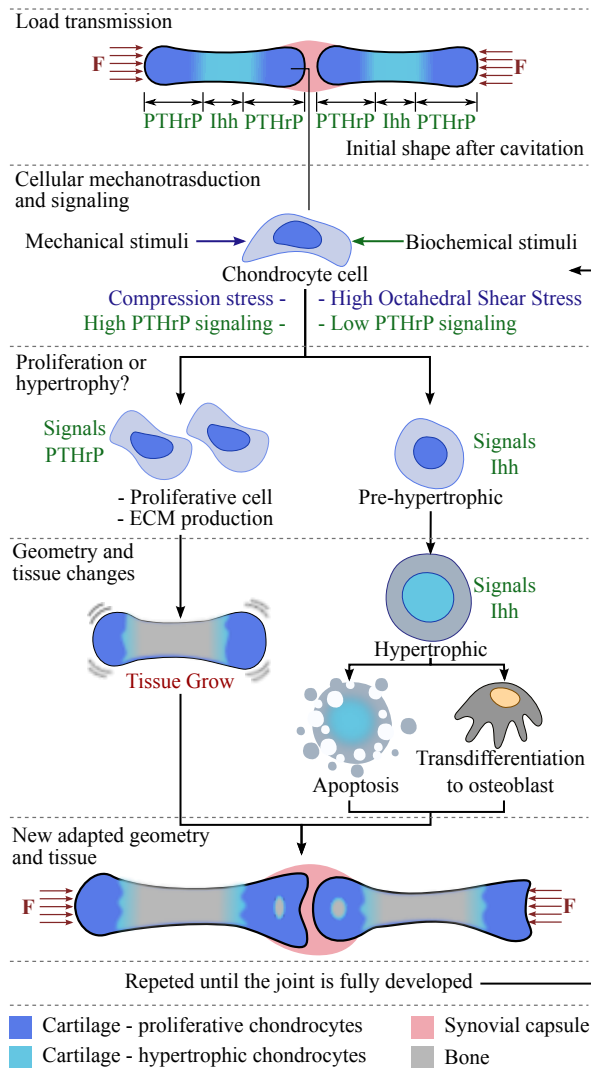


Figure 2: Chondrocyte cells adaptation rules. The cartilaginous bone rudiments are subjected to mechanical loading and, within it, an autoregulatory loop of Ihh-PTHrP. The CCs react to the mechanical and biochemical stimuli, depending on which these CCs can either proliferate or become hypertrophic. If the cell proliferates, means that the tissue (cartilage) must grow and morph; but if the cell becomes hypertrophic, then the tissue ossifies. This process continues until a full-grown ossified bone is achieved.

153 allowed to move along its vertical axis; the bottom boundary of the lower body can only move in the horizontal plane,
 154 except for the central point, which is fixed.

155 A bio-inspired morphogenesis process is then applied to both bodies (fig. 3 -center). The process promotes grow-
 156 ing (or expansion) at certain zones, allowing the parts to adapt to the mechanical loading. A couple of adapted surfaces
 157 are obtained at the end of the process; the contact profiles depend on the x coordinate (fig. 3 -right), *i.e.* $R_u = f(x)$ for
 158 the upper body, and $R_l = g(x)$ for the lower body.

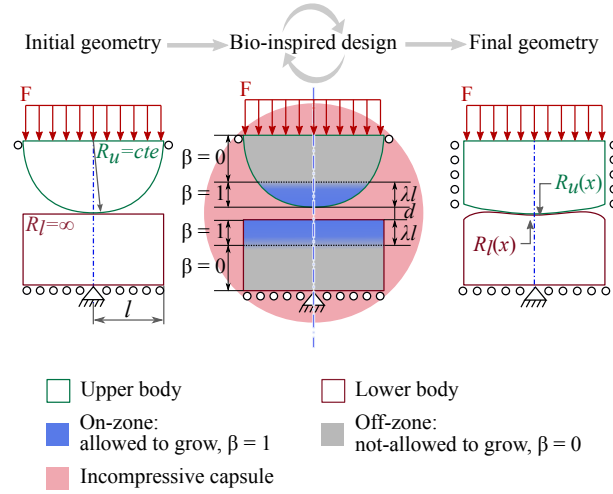


Figure 3: Application of the bio-inspired process for contact surface morphogenesis to a generic contact problem with elastic bodies and a static load (\mathbf{F}). Left: A hemispheric geometry of constant radius $R_u = \text{const}$ (upper body, green contour) and a semi-infinite half-space (lower body, red contour) in contact subjected to the joint morphogenesis process. Center: The pieces act as bone rudiments, and a capsule is included (pink region). The blue zone represents a bio-molecular gradient that acts as an on/off switch for growth. Right: Morphed contact surfaces; the upper and lower body shapes its contact profile depending on the x coordinate ($R_u = f(x)$ & $R_l = g(x)$).

159 3.2. From biological joint morphogenesis to mechanical design

160 Following the biological process described in section 2, one similar is proposed to emulate morphogenesis in
 161 mechanical pieces. The concepts considered within the bio-inspired process and its biological equivalent are presented
 162 in table 1 and are explained hereafter.

163 Mimicking the biological process, the mechanical pieces (equivalent to cartilaginous bone rudiments) are embed-
 164 ded in a structure that resembles the synovial capsule, *i.e.*, a spherical incompressible domain with material properties
 165 E_c and ν_c (fig. 3-center) (as done in (author?) [52, 10]).

166 Additionally, as in all biological joint morphogenesis models, there is a bio-molecular gradient involved, which
 167 can either add to the growth [51, 52] or act as an on/off switch for growth [10]. For the bio-inspired process, a switch
 168 was included through the *growth potential* (β), hence, $\beta = 1$ (blue zones within the bodies, fig. 3-center) when the
 169 zone is allowed to grow, and $\beta = 0$ when not (grey zones within the bodies, fig. 3-center). A smooth transition from
 170 the on-zone to the off-zone was implemented. Notice in fig. 3 that the zone allowed to grow is the one closer to the
 171 zone of interest, *i.e.*, the contacting surfaces. The length of these ‘epiphyses’, or zone allowed to grow, was defined as
 172 proportional to the initial width of the pieces through the ‘epiphyses’ length factor, λ (fig. 3-center).

173 In the biological process, hydrostatic compressive stress promotes the proliferation of chondrocytes [51, 52, 10], so
 174 tissue growth; therefore, within the bio-inspired process, the pieces will ‘grow’ proportionally to this same compres-
 175 sion hydrostatic stress (σ_{hyd}). In nature the octahedral shear stress (τ_{oct}), when high enough, inhibits the proliferation
 176 of chondrocytes, hence preventing growth. Within the bio-inspired process, the von Mises’ stress (σ_{vm}), which is an
 177 engineering equivalent proportional to τ_{oct} , was taken to inhibit the growth of the local domain: If σ_{vm} is above a
 178 threshold (σ_{lim}), the local domain cannot grow. From this, a *growth factor* (G_f) can be defined, which acts as a local

Table 1: From nature to mechanical design. Concepts equivalencies between the synovial joint morphogenesis process and the bio-inspired mechanical joint design process

Synovial joint morphogenesis	Bio-inspired joint design
<i>Cartilaginous bone rudiments</i> : Cartilaginous structures go through morphogenesis and endochondral ossification, and eventually, become long bones.	<i>Mechanical pieces</i> : Within the bio-inspired process, these pieces go through morphogenesis to give rise to an adapted contact surface profile.
<i>Synovial capsule</i> : Completely close structure filled with synovial fluid that surrounds the synovial joint.	<i>Incompressible capsule</i> : A spherical incompressible domain surrounding the pieces, that allows a uniform loading transmission between them while vouching for the continuity of the domain.
<i>Molecular pattern</i> : Biochemical distribution within the bone rudiments, that determines cellular behaviors (biochemical stimuli). The PTHrP-Ihh system is the one that most influences the development of long bones. This signaling can be influenced by other bio-molecular or mechanical stimuli.	<i>Growth potential β</i> : Variable which acts as an on/off switch, and defines the zones of the pieces domain that are susceptible to growth through the morphogenesis process.
<i>Growth factors</i> : Molecules produced by cells to signal (communicate) to their neighbor peers the stimuli, biochemical or mechanical, they are sensing. These signals trigger a chain of events; cells can amplify the signal or initiate processes (<i>i.e.</i> cell proliferation).	<i>Growth factor G_f</i> : Variable that indicates the amount of growth in function of the mechanical stimuli (σ_{hyd} , σ_{vm} , & σ_{lim}) and the growth potential (β).
<i>Cellular proliferation rate</i> : Tissue growth during joint morphogenesis is mainly due to cell proliferation; it is the proliferation rate and the direction of the cell division that influences the tissue proportions. This proliferation rate is defined by the maturation of the cell and, in the case of the chondrocytes, their level of hypertrophy.	<i>Growth coefficient α</i> : Parameter that indicates the rate at which the local domain expands with an increase of G_f . As in nature, this coefficient is set to evolve from a high value to a small one, to account for the ‘maturation’ of the domain of the mechanical pieces, <i>i.e.</i> , the domain growth coefficient diminishes at each growth step (g_s). The latter allows a large growth at the beginning and fine geometric adjustments at the end.
<i>Tissue growth</i> : As mentioned before, the morphogenesis of the bone rudiments is mainly due to cellular proliferation, without changing the cellular density within the tissue.	<i>Piece growth</i> : The pieces growth is due to a volumetric change proportional to the mechanical conditions.

179 signaling value for how the local domain will grow. G_f depends on β , σ_{hyd} , σ_{vm} and σ_{lim} (see eq. (1)).

$$G_f = \begin{cases} \beta|\sigma_{\text{hyd}}| & \text{if } \sigma_{\text{hyd}} \leq 0 \text{ \& } \sigma_{\text{vm}} < \sigma_{\text{lim}}, \\ 0 & \text{for other cases} \end{cases} \quad (1)$$

180 In nature, joints morph gradually until the SOC ossifies the entire epiphyses. Therefore, in the bio-inspired process,
181 the parameter *growth coefficient* (α) was defined. The growth coefficient, indirectly, involves the ‘maturation’ and
182 ‘ossification’ of the domain, hence, it should evolve from a high value, α_0 , to null in several given stages (steps).
183 Seen from a biological point of view, a high value of α means that the chondrocytes are highly proliferative whereas a
184 low α , indicates low proliferative cells (hypertrophic) and that most of the tissue has ossified. This parameter allows
185 having large growth rates (adaptations) at the beginning of the process and small adjustments by the end.

186 Growth can be simulated as a local volumetric expansion of the domain proportional to the G_f and α . The local
187 change of volume ($\Delta V/V$) can be expressed as follows (eq. (2)):

$$\frac{\Delta V}{V} = \alpha G_f \quad (2)$$

188 The structure of eq. (2) resembles the one of mechanical thermal expansion; being α the coefficient of thermal
 189 expansion and G_f the temperature change. As α depends on the number of growth steps g_s , *i.e.* $\alpha_i = s(i)$, where i
 190 is the current growth step (goes from 0 to g_s), the area under the curve $s(i)$ can be seen as the total rate of change
 191 volume used for growth. Then, it is possible to have the same outcome with two different functions of $s(i)$ as long as
 192 the integrals of the functions are the same.

193 3.3. Stress normalization

194 The stresses were normalized to make the morphogenesis process independent of the applied force \mathbf{F} and the
 195 initial dimension of the pieces. Then, at each growth step both analyzed stresses, σ_{hyd} and σ_{vm} , get normalized by the
 196 maximum of each within the domain:

$$\bar{\sigma}_{\text{hyd}} = \sigma_{\text{hyd}} (\sigma_{\text{hyd}}^{\text{max}})^{-1} \quad (3)$$

$$\bar{\sigma}_{\text{vm}} = \sigma_{\text{vm}} (\sigma_{\text{vm}}^{\text{max}})^{-1} \quad (4)$$

197 where $\bar{\sigma}_{\text{hyd}}$ and $\bar{\sigma}_{\text{vm}}$ are the normalized hydrostatic stress and von Mises' stress, respectively; σ_{hyd} and σ_{vm} are the
 198 local hydrostatic and von Mises' stresses; and $\sigma_{\text{hyd}}^{\text{max}}$ and $\sigma_{\text{vm}}^{\text{max}}$ are the maximum hydrostatic and von Mises' stresses
 199 within both the pieces in contact.

Table 2: Parameters the bio-inspired process.

<i>no</i>	<i>parameter name</i>	<i>letter</i>
-	External load	F
-	Radius of the upper piece	$R_s(l)$
-	Radius of the lower piece	$R_l(l)$
1	Pieces Poisson's ratio	ν
2	Capsule Poisson's ratio	ν_c
3	Capsule thickness	d
4	Pieces Width	l
5	'Epiphyses' length factor	λ
6	von Mises' Stress limit	σ_{lim}
7	Capsule Young's modulus	E_c
8	Pieces Young's modulus	E
9	Initial growth coefficient	α_0

200 3.4. Dimensionless analysis

201 Buckingham's Pi Theorem was used to define a minimal set of dimensionless parameters that govern the bio-
 202 inspired process. The set of parameters for the bio-inspired process is listed in table 2. The magnitude of the external
 203 load and the initial dimension of the pieces were not considered as the stresses are normalized (section 3.3). According
 204 to Buckingham's pi-theorem, the problem can be reduced from 9 to 7 dimensionless parameters. The chosen set of

205 dimensionless parameters is shown table 3. Notice that α has the inverse units G_f (see eq. (2)), however, since the
 206 stresses are normalized, α is also dimensionless.

Table 3: Dimensionless parameters for the bio-inspired process.

<i>no</i>	<i>parameter name</i>	<i>letter</i>
1	Bodies Poisson's ratio	ν
2	Capsule Poisson's ratio	ν_c
3	Capsule thickness factor	$\bar{d} = d/l$
4	'Epiphyses' length factor	λ
5	Normalized von Mises' Stress limit	$\bar{\sigma}_{\text{lim}}$
6	Young's ratio	$\theta = E/E_c$
7	Initial growth coefficient	α_0

207 3.5. Bio-inspired algorithm for contact surface morphogenesis

208 A bio-inspired algorithm for contact surface morphogenesis was proposed based on the equivalent biological pro-
 209 cess (see algorithm 1). This algorithm considers the mechanical equivalents of biological concepts, the normalization,
 210 and the dimensionless analysis explained in the previous sections. It was implemented in a computational framework
 211 based on the finite element method (FEM). For the implementation, the finite element analysis (FEA) software tool
 212 *EDF Code_Aster* was employed.

213 The algorithm has as input the magnitude of the loading $F = 100\text{N/mm}^2$ and the material properties of the pieces,
 214 $E = 100\text{GPa}$ and $\nu = 0.3$. The rest of parameters: The ones related to initial geometry, $l = 5\text{mm}$, $R_u = l$, and $R_l = \infty$;
 215 and those of the bio-inspired process are treated in section 3.6.

216 Then, some variables required for the process are initialized (line 1 - line 4). Once every parameter is defined, a
 217 recursive subroutine can start, which depends on the value of α (line 5 - line 34). Within the **while** loop (line 5), some
 218 geometrical tasks are performed; first, the mesh of the upper body, A_1 (line 6), and lower body A_2 (line 7), are created
 219 and located in such a way that the smallest distance **between their contact surfaces** is d . Then, the geometry and mesh
 220 of the capsule, A_3 , is generated (large enough to contain completely both meshes A_1 and A_2) (line 8).

Algorithm 1 Bio-inspired process algorithm

Require: F, E, ν **Ensure:** $l, R_l(l), R_u(l)$

```
1: Define  $\nu_c, \theta, \bar{\sigma}_{lim}, \lambda, \alpha_0, \bar{d}$ 
2:  $E_c \leftarrow E/\theta$ 
3:  $\alpha \leftarrow \alpha_0$ 
4:  $i \leftarrow 0$  ▷ Step counter
5: while  $\alpha > 0$  do
6:   Create the mesh of the upper body:  $A_1$ 
7:   Create the mesh of the lower body:  $A_2$ 
8:   Create the mesh of the capsule:  $A_3$ 
9:   Assign to  $A_1$  and  $A_2$ :  $E, \nu$ 
10:  Assign to  $A_3$ :  $E_c, \nu_c$ 
11:  Apply  $F$  and boundary conditions
12:  Get  $\sigma_{hyd}$  &  $\sigma_{vm}$  from FEA mechanical
13:   $\sigma_{hyd}^{max} \leftarrow \max(\sigma_{hyd})$  in  $A_1$  &  $A_2$ 
14:   $\sigma_{vm}^{max} \leftarrow \max(\sigma_{vm})$  in  $A_1$  &  $A_2$ 
15:  for each element  $j$  in  $A_{1,2}$  do
16:     $\bar{\sigma}_{hyd_j} \leftarrow \sigma_{hyd_j} (\sigma_{hyd}^{max})^{-1}$ 
17:     $\bar{\sigma}_{vm_j} \leftarrow \sigma_{vm_j} (\sigma_{vm}^{max})^{-1}$ 
18:     $\beta_j \leftarrow 0$  ▷ Initialize growth potential
19:     $G_{f_j} \leftarrow 0$  ▷ Initialize growth factor
20:    if  $\bar{\sigma}_{vm_j} \leq \sigma_{lim}$  then
21:      if  $\bar{\sigma}_{hyd_j} \leq 0$  then
22:         $\beta_j \leftarrow 1$ , if  $0 \leq y_j \leq \lambda l$ 
23:         $G_{f_j} \leftarrow \beta_j |\bar{\sigma}_{hyd_j}|$ 
24:      end if
25:    end if
26:  end for
27:  Use  $\mathbf{G}_f$  as temperature
28:  Use  $\alpha$  as expansion coefficient
29:  Get  $\Delta V/V$  from a FEA thermal
30:  Deform the  $A_1$  &  $A_2$  domains  $\leftarrow \Delta V/V$  ▷ Update geometry of  $A_1$  &  $A_2$ 
31:  Reset strains and stresses
32:   $i \leftarrow i + 1$  ▷ Update counter  $i$ 
33:   $\alpha \leftarrow -g_s^{-1} i + \alpha_0$  ▷ Update  $\alpha$ 
34: end while
```

221 Afterward, the mesh gets prepared to perform a mechanical FEA: The material properties, E and ν , are assigned
222 to elements that belong to bodies, conversely, E_c and ν_c to the elements of the capsule (line 9 - line 10). The force \mathbf{F}
223 and the boundary conditions are applied. From the FEA, the σ_{hyd} and σ_{vm} distributions are obtained (line 12); and the
224 maximums identified (line 13-line 14).

225 An elemental scan is performed. For each element of the bodies A_1 and A_2 , the stresses get normalized by the
226 σ_{hyd}^{max} and σ_{vm}^{max} (line 16 - line 17). Depending on the location of the elements, a value of β is assigned: If it is located
227 within the λl distance, then $\beta_j = 1$, else $\beta_j = 0$ (line 22).

228 Additionally, the elemental G_{f_j} is calculated following eq. (1). It considers the value of the elemental normalized
229 von Mises stress, $\bar{\sigma}_{vm_j}$; if it was over $\bar{\sigma}_{lim}$, then $G_{f_j} = 0$, else, if the elemental normalized hydrostatic stress, $\bar{\sigma}_{hyd_j}$, is
230 0 or negative, then $G_{f_j} = \beta \bar{\sigma}_{hyd_j}$ (line 20 - line 24).

231 As mentioned in section 3.2, growth can be modeled as a thermal expansion (as also done in (author?) [52]).
 232 Therefore, the model gets prepared for a thermal expansion FEA on the bodies A_1 and A_2 : G_f is given as temperature,
 233 and the reference temperature is taken as 0° ; α is given as the expansion coefficient of the material. From the thermal
 234 FEA, the changes in volume are calculated, and with it, the new stress-free geometries of the bodies A_1 and A_2 are
 235 updated (line 30). The loop finishes with an update of the growth step i and the parameter α (line 33).

236 As a remark, the value of α was assumed to evolve linearly (the function $s(i)$ is a linear function with negative
 237 slope), from α_0 to 0 in a number of steps g_s (eq. (5)). It should be recalled that it is possible to have the same outcome
 238 with two different functions $s(i)$ as long as their integrals are the same. The larger the number of steps, the more stable
 239 and smooth the growth, although, the computational time increases.

$$\alpha_i = -\frac{1}{g_s}i + \alpha_0 \quad (5)$$

240 3.6. Parametric study

241 A response surface (*RS*) methodology was followed to explore the space of parameters of the bio-inspired process
 242 and to tune them.

243 3.6.1. Monitored variable:

244 A contact FEA of the problem described in section 3.1 is performed on the geometry at each growth step i of the
 245 bio-inspired algorithm. The evolution of the maximum von Mises' stress is monitored and evaluated as a percentage
 246 of the maximum von Mises' stress of the initial geometry. This stress was considered since it is a metric directly
 247 related to the maximum contact pressure, they are proportional to each other; moreover, the von Mises' criterion is
 248 also used to determine some failure inceptions in contact mechanics of ductile materials [54].

249 The number of growth steps for each run of the *RS* design was set manually to 20, small enough so that the
 250 computational time is not high, but large enough so that the growth is stable. With the *RS* it is possible to find the
 251 optimal value of α_0 for $g_s = 20$. For more stable and smooth growth, the number of steps g_s may increase and α_0
 252 changed so that the area under the function in eq. (5) must be the same as the one found with the *RS*.

253 The simulations were performed in a 64-bit i7 processor @ 2.7 GHz, 16 GB RAM, Ubuntu 18.04 operating
 254 system. Each simulation of $g_s = 20$ took approximately 20 minutes. All the meshes were developed in GMSH [55].
 255 A convergence analysis allowed us to define a minimum mesh size of 16.67E-3 mm.

256 3.6.2. Factors of the response surface:

257 From the parameters mentioned in table 3, the Poisson's ratio of the capsule and that of the bodies, the 'epiphyses'
 258 length factor, and the capsule thickness factor were defined according to the following statements. The Poisson's ratio
 259 of the bodies, ν , was fixed at 0.3, the same as the original material; and for the capsule, since it is an incompressible
 260 material, ν_c was set at 0.49. The capsule thickness factor \bar{d} was set as small as possible so that there was at least one

261 line of mesh's elements between the pieces. The 'epiphyses' length factor, λ , was established following Saint-Venant's
 262 principle so that the zone allowed to grow was as long enough that reaches the distance where the stresses become
 263 uniform. Then, any growth of the domain at this distance would be uniform across the transverse area.

264 The remaining parameters, $\bar{\sigma}_{\text{lim}}$, θ , and α_0 were considered to have a great influence on the outcome of the
 265 response. The parameters were normalized within -1 and 1 and represented by the factors X_i (X_1 , X_2 , and X_3 ,
 266 corresponding to the parameters $\bar{\sigma}_{\text{lim}}$, θ , and α_0 respectively) (table 4). The intervals of the parameters were defined
 267 based on the experience of the researchers. Via several iterations, it was established that the optimal values of each
 268 parameter are within the chosen intervals.

269 3.6.3. Response surface function:

270 A *RS* of the selected parameters is proposed: $\widehat{Y}_{rs} = f(\bar{\sigma}_{\text{lim}}, \theta, \alpha_0)$. Since the parameters' intervals are relatively
 271 close to the optimum region, the curvature offered by a second-order polynomial is appropriated as an approximation
 272 function for the response surface [56]. The mathematical model for the response is described in eq. (6).

$$\widehat{Y}_{rs} = b_0 + \sum_{j=1}^n b_j X_j + \sum_{j=1}^n b_{jj} X_j^2 + \sum_{k=1}^{n-1} \sum_{j=k+1}^n b_{kj} X_k X_j \quad (6)$$

273 being \widehat{Y}_{rs} the approximated response; b_0 is the value of the *RS* when all factors are set at the level 0; b_j are the
 274 linear regression coefficients; b_{jj} , and b_{jl} are regression coefficients of double factor interactions; and n is the total
 275 number of factors ($n = 3$).

276 3.6.4. Experiment design:

277 The design matrix was chosen as a Box-Wilson Central Composite Design (central composite design) (fig. 4)
 278 since it is suitable for fitting a second-order *RS* [57]. Specifically, a face-centered composite design was selected due
 279 to its simplicity; it only requires 3 levels of each factor (-1 , 0 and 1) (table 4), and for 3 parameters a total of 15
 280 combinations of factors, *i.e.* runs, are needed (table 5).

Table 4: Factor ranges for the design of the response surface.

<i>parameter (factor)</i>	$X_i = -1$	$X_i = 0$	$X_i = 1$
ν (-)		0.3	
ν_c (-)		0.49	
\bar{d} (-)		3.40e-3	
λ (-)		2.1	
$\bar{\sigma}_{\text{lim}} (X_1)$	0.2	0.3	0.4
$\theta (X_2)$	500	5250	10000
$\alpha_0 (X_3)$	0.01	0.05	0.09

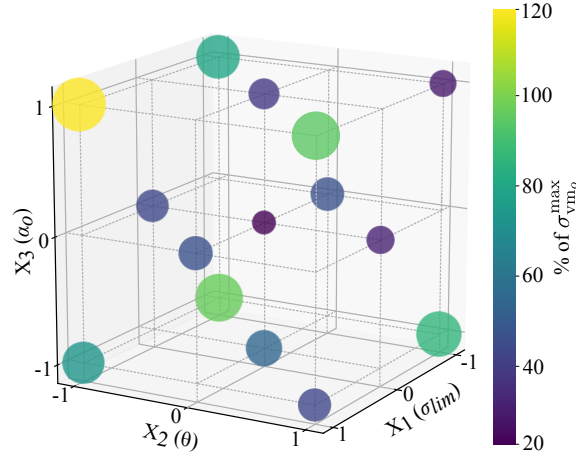


Figure 4: Bubble graphical representation of the outcome Y_c for all the computational runs. Each axis represents one normalized factor (X_1 , X_2 , and X_3). The size and color of each bubble represent the value of the outcome response Y_c .

281 4. Results

282 Table 5 shows the design matrix, the factors combination for each run c , the computational response Y_c , and
 283 regression coefficients b_j , b_{jj} and b_{kj} . In fig. 4 is shown a bubble graphical representation of the computational
 284 response for each run, which indicates that the minimum value of the response is within the ranges of the parameters.

Table 5: Response surface design matrix. Matrix of the face-centered composite design employed for the design of the RS , values of the computational measures, Y_c , and the obtained regression coefficients, b_j .

run	X_1	X_2	X_3	Y_c	coefficients	
1	-1	-1	-1	96.82%	b_0	$2.53e-1$
2	+1	-1	-1	73.19%	b_1	$3.90e-2$
3	-1	+1	-1	85.75%	b_2	$-1.27e-1$
4	+1	+1	-1	43.71%	b_3	$8.04e-3$
5	-1	-1	+1	79.91%	b_{12}	$9.77e-3$
6	+1	-1	+1	119.13%	b_{13}	$2.12e-1$
7	-1	+1	+1	29.35%	b_{23}	$-4.29e-2$
8	+1	+1	+1	94.79%	b_{11}	$2.08e-1$
9	-1	0	0	46.84%	b_{22}	$1.14e-1$
10	+1	0	0	46.85%	b_{33}	$2.03e-1$
11	0	-1	0	43.39%		
12	0	+1	0	31.52%		
13	0	0	-1	54.24%		
14	0	0	+1	38.58%		
15	0	0	0	22.61%		

285 The residual error e_c was estimated for each run through the arithmetic difference between the computational
 286 result, Y_c , and the estimated response, $\widehat{Y}_{rs}(X_1^c, X_2^c, X_3^c)$ (eq. (7)). The number of coefficients can be brought down by
 287 evaluating the standard deviation, σ_e , of the error e (eq. (8)); where N_e is the number of runs, N_f is the number of
 288 factors, and e_c is the error for each run. Then, the insignificant coefficients were identified with a Student's statistical
 289 test (t-test) and gradually removed until the standard deviation begins to augment; fig. 5 shows the evolution of σ_e .

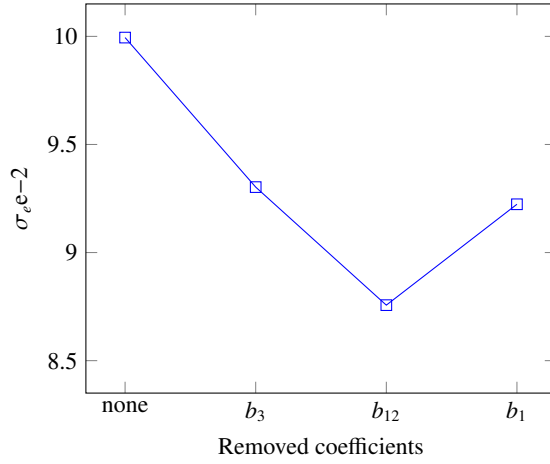


Figure 5: T-test performed to the coefficients of the response surface. The standard deviation of the error reduces as the coefficients b_3 and b_{12} are gradually removed, however, it increases when removing b_1 .

290 The t-test shows that the linear coefficient of the factor X_3 , b_3 , and the double factor interaction coefficient, b_{12} , were
 291 statistically insignificant, therefore from now on they are set to 0 in 6. The second-order response surface with the
 292 coefficients after significance check is shown in section 4.

$$e_c = Y_c - \widehat{Y}_{rs}(X_1^c, X_2^c, X_3^c) \quad (7)$$

$$\sigma_e = \sqrt{\frac{1}{N_e - N_f} \sum_{j=1}^{N_e} e_c^2} \quad (8)$$

$$\begin{aligned} \widehat{Y}_{rs} = & 0.2534 + 0.0390X_1 - 0.1273X_2 \\ & + 0.2128X_1X_3 - 0.0429X_2X_3 \\ & + 0.2082X_1X_1 + 0.1144X_2X_2 \\ & + 0.2039X_3X_3 \end{aligned} \quad (9)$$

293 4.1. Optimization of the response surface factors

294 The response surface (section 4) can be optimized through the minimization shown in section 4.1, which will give
 295 the values of the factors X_1 , X_2 , and X_3 .

$$\begin{aligned} \min_{X_i} \quad & \widehat{Y}_{rs} \\ \text{s.t.} \quad & -1 \leq X_i \leq 1 \quad \text{for } i = 1, 2, 3 \end{aligned} \quad (10)$$

296 The optimized value of $Y_{opt} = 21.28\%$ was obtained with the following values of the factors: $X_{1opt} = -0.1706$
 297 ($\bar{\sigma}_{lim} = 0.2829$), $X_{2opt} = 0.5849$ ($\theta = 8028.0646$) and $X_{3opt} = 0.1506$ ($\alpha_0 = 0.0560$). The confidence interval of the
 298 response surface $U(\widehat{Y}_{rs})$ was estimated through section 4.1, for a confidence of 95%. The $var(\widehat{Y}_{rs})$ was calculated
 299 considering the covariant matrix of the coefficients $\mathbf{Cov}(\mathbf{B})$, and the jacobian of the model (the jacobian is shown in
 300 section 4.1). The uncertainty for the optimal solution is then $U(Y_{opt}) = \pm 4.9572\%$. A graphical representation of the
 301 second-order RS is shown in fig. 6. The surfaces are shown per pair of factors, the remaining factor i was set at the
 302 optimum value (X_i^{opt}). The outcome of each run is shown as black dots, the optimum value is shown in green.

$$U(\widehat{Y}_{rs}) = 2 \sqrt{var(\widehat{Y}_{rs})} \quad (11)$$

$$var(\widehat{Y}_{rs}) = \mathbf{J} \mathbf{Cov}(\mathbf{B}) \mathbf{J}^T \quad (12)$$

$$\mathbf{J} = [X_1 \ X_2 \ X_{13} \ X_{23} \ X_1^2 \ X_2^2 \ X_3^2] \quad (13)$$

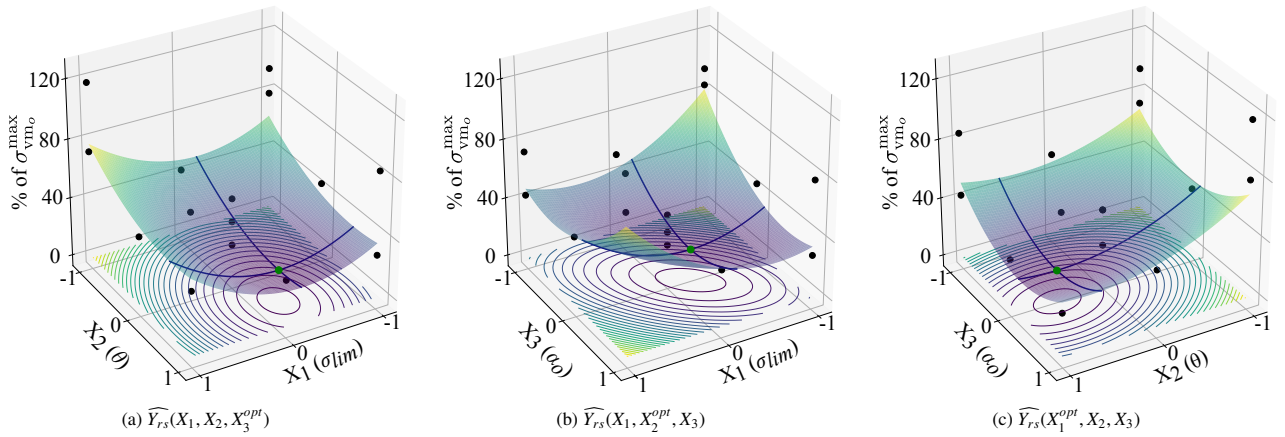


Figure 6: Diagram of the response surface. The graphic is divided into three surfaces in which the RS is calculated by pairing the factors X_1 , X_2 and X_3 , the remaining factor i was set at the optimum value (X_i^{opt}). The outcome of each run, Y_c , are shown as black dots, and the optimum is in green.

303 For the contact problem described in section 3.1, a new simulation of the bio-inspired algorithm was implemented
 304 setting the parameters at the values of the optimal response. The number of growth steps, g_s , was set at 20. After
 305 running the simulation, the maximum von Mises' stress of the last geometry was 25.8% of the initial; which is within
 306 the boundaries of the predicted value in section 4.1, $Y_{opt} \pm U(Y_{opt})$.

307 In fig. 7, the evolution of the von Mises' stress as a percentage of the one in the initial geometry, % of σ_0^{\max} , is
 308 shown at each step, as well as the profiles presented for each of the bodies at the initial, 6th, 12th, and 20th growth
 309 steps. The upper body is green-colored, whereas the lower one is reddish.

310 It should be pointed out that the algorithm appended material around the contact zones, increasing the area of
 311 contact, therefore reducing stress levels (fig. 7). The profiles were subjected to a smoothing process, as with other

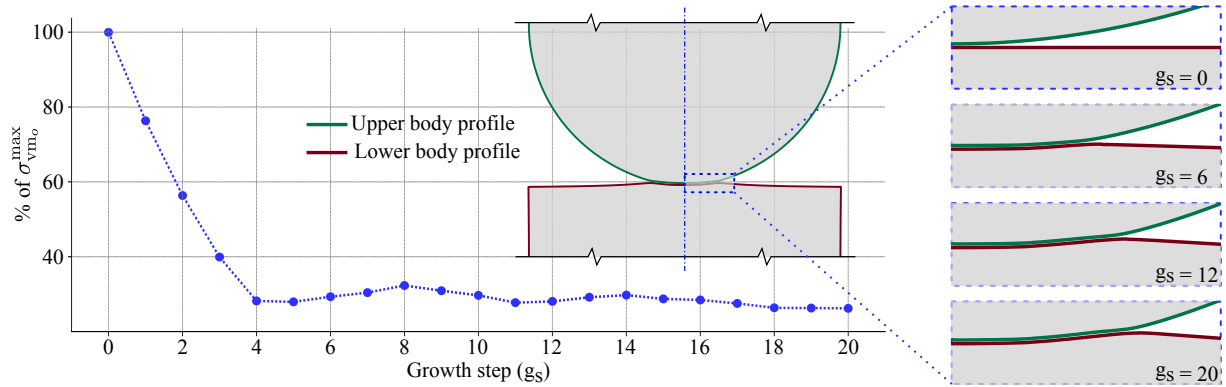


Figure 7: Evolution of the monitored variable. The evolution of the von Mises' stress as a percentage of the one in the initial geometry is shown at each step, and a scheme of the geometries at the 6th, 12th, and 20 steps are contrasted with the initial ($g_s = 0$). The upper body's profile is shown in green, conversely, the lower body's profile is shown in red.

312 generative tools, to diminish the effect of the discretized time and spatial growth. The smoothing was performed with
 313 500 growth steps (α_0 is scaled accordingly), and to each final profile, a 3rd order function was fitted (one for the upper
 314 and one for the lower body) to obtain smooth profiles.

315 In fig. 8 the obtained stresses are compared. The von Mises' (fig. 8a) and hydrostatic stress (fig. 8b) distributions
 316 are compared at the initial (fig. 8-i), last geometry after 20 steps (fig. 8-ii), and after the smoothing process (fig. 8-iii).
 317 The von Mises' stress was reduced to 25.8% of the initial geometry in the 20-step profile and 16.1% in the smoothed
 318 profile, still within the boundaries of the predicted value ($Y_{opt} \pm U(Y_{opt})$). Similarly, the hydrostatic stress was reduced
 319 to 27.9% of the one in the initial geometry in the 20-step profile, and to 15.5% in the smoothed profile.

320 The optimized profiles of the upper and lower bodies are shown in fig. 9 and compared with the initial geometry.
 321 The upper body's final profile morphed in such a way that its radius increased when compared with the initial one
 322 (fig. 9a). The lower body's profile shows greater changes, it adapted quickly to the profile of the upper body increasing
 323 the contact area between them (fig. 9b).

324 The similarity between the profiles was evaluated through a *euclidean distance-based similarity score*. The score
 325 considers the distance between the two objects and goes from zero to one, the closer to 1, the more similar the objects.
 326 To determine the similarity between the curves, they were first aligned considering their lower vertical value. The
 327 distance between them is calculated as shown in section 4.1, and the similarity score as in section 4.1. Based on this
 328 score, the profiles show a great similarity at small radial distances fig. 9c.

$$d(p_1, p_2) = \sqrt{(x_{p_1} - x_{p_2})^2 + (y_{p_1} - y_{p_2})^2} \quad (14)$$

$$Sim(p_1, p_2) = \frac{1}{1 + d(p_1, p_2)} \quad (15)$$

329 The changes in the contact pressure profile from the initial and the morphed geometry (smoothed) are shown in

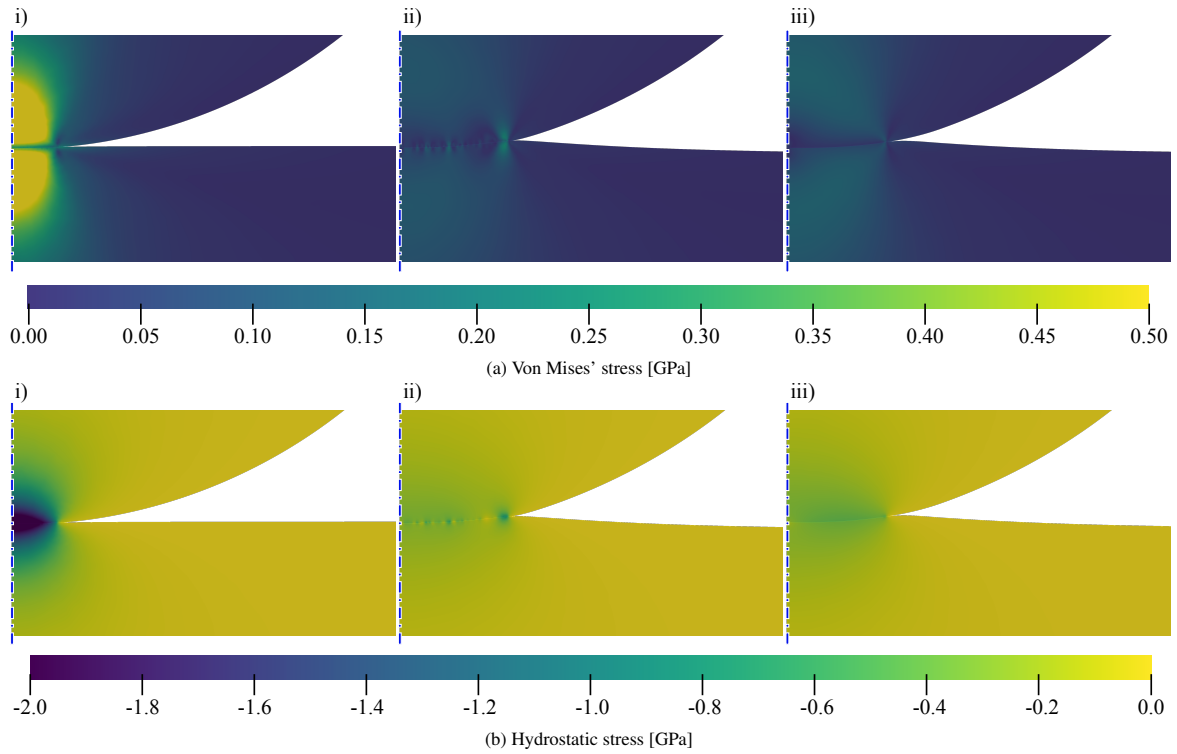


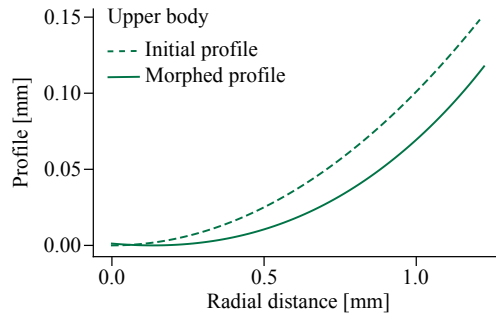
Figure 8: Stress distributions. Initial geometry (i), 20-steps (ii), after smoothing (iii).

330 fig. 9d. The pressure tends to become more uniform, and the maximum contact pressure is reduced to a 15% of the
 331 one in the initial geometry. This reduction in the overall contact pressure is related to the fact that the geometries of
 332 the contacting surfaces are more conformal between them, as in synovial joints. Moreover, the obtained profile shows
 333 the lower contact pressure at the center of the profile (at a radial distance of 0.0mm).

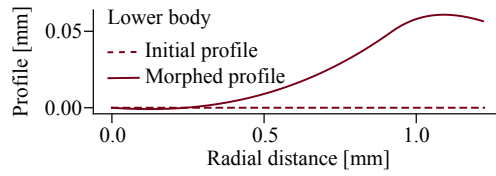
334 5. Discussion

335 A computational tool for surface shaping in unilateral contact problems inspired by the morphogenesis of synovial
 336 joints is presented in this work. The algorithm behind mimics the laws of cartilage growth (regulated by the mechan-
 337 ical environment). Its outcome was characterized by a parametric study. 7 dimensionless and normalized parameters
 338 were defined, from which 3 were identified as more influential in the outcome of the algorithm: $\bar{\sigma}_{lim}$, the normalized
 339 von Mises' stress that regulates growth, θ , which relates the material properties of the incompressible capsule and the
 340 pieces, and α_0 , which is the initial growth coefficient.

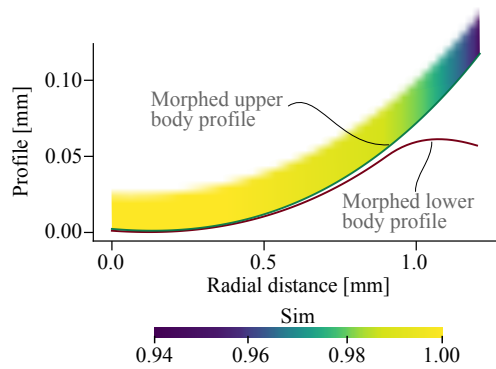
341 A DoE analysis was performed through the design of a *RS*, which allowed us to determine the behavior of the
 342 monitored variable (maximum von Mises' stress) as a function of the parameters. A second-order *RS* was obtained
 343 depending on the parameters $\bar{\sigma}_{lim}$, θ and α_0 . This response surface is valid within a normalized domain of the pa-
 344 rameters (table 4). The coefficients of the *RS* indicate the relative influence of each parameter and their interactions
 345 on the outcome. The *RS* shows relatively high values of the second-order coefficients showing that there is a great



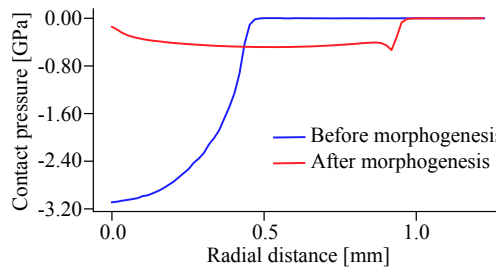
(a) Upper body profile changes



(b) Lower body profile changes



(c) Similarity score between the morphed upper and lower profiles



(d) Contact pressure before and after the morphogenesis

Figure 9: Contrasts among the profiles: Initial and final profiles. The upper and lower bodies' morphed profiles are compared with the initial ones, as well as the similarity score between the upper and lower morphed profiles in which the upper profile is color-mapped with the value of the similarity score. In these figures, the vertical axis is scaled 5 times the horizontal axis. Lastly, the contact pressure is shown before and after the morphogenesis.

346 interaction between the parameters, α_0 showed to have a large influence when interacting with the other two parame-
 347 ters, especially with $\bar{\sigma}_{lim}$. This shows that α_0 depends on the chosen value for σ_{lim} and θ . Moreover, since the linear
 348 coefficient of the parameter θ is relatively high (and negative), very small values of said parameter tend to increase

349 the monitored variable, whereas large values offer a better outcome. Conversely, its square coefficient, also large and
350 positive contributes to finding a minimum of the response function within the normalized domain of parameters.

351 It was found that the response surface had a minimum within the valid domain. Hence, an optimization of the
352 function was performed to find the values of $\bar{\sigma}_{lim}$, θ , and α_0 that offer the smallest value of the monitored variable.
353 Then, a run of the algorithm with the parameters set at the optimum level with the same number of growth steps (20)
354 used in the design of the *RS* was done. However, due to the nature of the method and the discretized growth of the
355 domain (both in time and space), a non-smooth stress distribution was found due to the geometry obtained profiles.
356 This effect was reduced by implementing a smooth transition between the on/off states given by $\bar{\sigma}_{lim}$ combined with
357 an increment in growth steps (500); α_0 was diminished proportionally. Moreover, 3rd-order functions were fitted to
358 the profiles to obtain a smooth geometry for stress analyses.

359 A profile of the surface was obtained for each of the pieces in contact. These profiles were obtained thanks to the
360 slow growth of the domain during a given number of steps. The algorithm does small adjustments to the profile by
361 adding gradually small portions of the material. The automatic shaping of the pieces followed a set of rules adopted
362 from the cartilage growth process. The algorithm gradually increased the contact area between the bodies which
363 ultimately translated to a reduction in the contact pressure as well as in the overall stresses. Interestingly, the obtained
364 profiles had very close shapes for intermediate radial distances, as shown by the similarity score. Conversely, for very
365 small distances the profiles seem to go smoothly in opposite ways, reducing slowly the contact pressure. The same
366 occurs for large radial distances, but not as smooth as for small radiuses, which generate small pressure concentrator
367 effects, nevertheless, the peak pressures are a lot smaller than in the initial geometry.

368 The obtained profiles can be scaled for any other similar contact problem by applying Buckingham's Pi Theorem.
369 In general, the unilateral contact problem has 5 parameters, including the equivalent Young's modulus E^* , the mag-
370 nitude of the applied force F , Poisson's ratio of the pieces $\nu_{1,2}$, and the width of the pieces l (the radiuses are not
371 considered since they are a function of the width of the pieces). The theorem reduces the problem to 3 dimensionless
372 numbers: $\nu_{1,2}$ and a number $\Phi = F/(l^2 E^*)$; for same materials in contact $\Phi = (2F(1 - \nu^2))/(l^2 E)$. The dimensionless
373 number Φ can then be used as a scale factor to obtain the profile for a given set of F , and materials in contact ($E_{1,2}$
374 and $\nu_{1,2}$). The structure of this scale factor is similar to the ones found in the literature to scale the logarithmic contact
375 surface proposed by Lundberg [58].

376 Other authors have performed shape optimization for contacting surfaces, in which they define a cost function
377 and aim to minimize it [15, 16, 14]. However, in those works, only the shape of one of the contacting surfaces
378 is optimized in the attempt of achieving a uniform contact pressure. Recently, in a work developed by (author?)
379 [27], a combination of the interface shape and the material stiffness optimization to obtain a uniform contact stress
380 distribution was done. They focused just on the contact interface and changed its shape equally for both surfaces
381 in contact. Conversely, in this paper, we found the profiles of the upper and lower bodies similar for small radial
382 distances.

383 The work presented here shows that by applying the basic laws of cartilage tissue adaptation to mechanical loading

384 an efficient profile can be achieved. In this new profile, the stresses are reduced as well as the contact pressure. Hence,
385 this work corroborates that nature distributes wisely the material within a domain so that the stress gets distributed as
386 uniformly as possible, contributing to a large lifespan of the joints.

387 The algorithm implemented here can be improved by including other biological processes in parallel with the
388 shaping of the contact surface, such as changes in the local material stiffness (during the ossification the tissue changes
389 its material properties gradually until fully ossified, adapting to the mechanical loading). Nevertheless, should be
390 considered the computational cost that this implies and the increase in the number of parameters of the algorithm to
391 be tuned. Moreover, an analysis of the influence of the type of function used for the growth coefficient α should be
392 performed in future works, as well as a way to make the growth less dependent on temporal and space discretization,
393 perhaps with a parametric curve mapping. Additionally, it should be considered that the DoE method with the design
394 of the *RS* does not assure the finding of a global optimum, since it is highly dependent on the ranges of the parameters.
395 Furthermore, experimental validations would be desirable to evaluate the manufacturing feasibility of the generated
396 profiles and the real performance in terms of contact load distribution.

397 **6. Conclusions**

398 Currently, mechanical joint design has not been approached by mimicking nature's joint morphogenesis process.
399 Based on this process, this work proposes an algorithm for the design of pieces in contact. The algorithm was
400 implemented in a FEM framework and the feasibility of obtaining an adapted joint surface was evaluated. The more
401 influential parameters of the bio-inspired process were identified and optimized through a response surface. The
402 results show that the algorithm, by mimicking the cartilage growth rules, adapts the geometry of the joint contact
403 surfaces in such a way that the contact stresses are distributed uniformly. Since the proposed bio-inspired algorithm
404 was implemented in a dimensionless fashion, the found solution for the contact profiles can be used for similar
405 problems. This work demonstrates that nature's laws for joint morphogenesis can be adapted for the generative design
406 of contact surfaces of mechanical joints. The latter opens the possibility to further develop the algorithm to address
407 more complex settings (combined loading, non-linear materials, dynamic setups, etc). This contribution is a step
408 forward in the scope of bio-inspired generative design.

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