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Hicham Alhajj Chehade[1,2], Daniel Dias[3,4], Marwan Sadek[2,5], Orianne Jenck[1], Fadi Hage Chehade[2]

[1] Univ. Grenoble Alpes, CNRS, Grenoble INP**, 3SR, F-38000 Grenoble, France

PhD student (*Corresponding author). email: hicham.alhajjchehade@3sr-grenoble.fr

Professor. email: daniel.dias@3sr-grenoble.fr

Assistant Professor. email: orianne.jenck@3sr-grenoble.fr

[2] Lebanese University, Doctoral School of Sciences and Technologies, Beirut, Lebanon

Professor. email: marwan.sadek@ul.edu.lb

Professor. email: fchehade@ul.edu.lb

[3] Antea Group, Antony, France

Geotechnical expert. email: daniel.dias@anteagroup.com

[4] School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei 23009, China,

Distinguished professor. email: daniel.dias@univ-grenoble-alpes.fr

[5] Univ. Lille, IMT Lille Douai, Univ. Artois, JUNIA Hauts-de-France, ULR 4515 - LGCgE, Laboratoire de Génie Civil et géo-Environnement, F-59000, Lille, France

Associate professor. email: Marwan.Sadek@polytech-lille.fr

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Abstract. This study concerns the seismic internal stability analysis of saturated reinforced soil retaining walls using the discretization technique and the limit analysis upper bound approach. The discretization technique permits to generate the potential failure mechanism of reinforced structures point by point. The seismic forces are represented based on the pseudo-dynamic approach. This latter is more realistic than the pseudo-static one which is commonly used. It allows accounting for the dynamic characteristics of the seismic loading. Knowing that the water presence is the main cause of most failure cases reported in the literature, the pore water effect within the backfill soil is considered together with a possible crack opening in cohesive soils. The reinforcement strength required to prevent the saturated reinforced soil wall failure is obtained through an optimization process. The developed approach is validated by comparison with the existing results obtained by Abd and Utili (2017) using the conventional limit analysis method. The presence of pore-water pressure leads to an increase in the reinforcement strength required to prevent the failure. Discussions are then carried out to point out the effects of the crack presence, the seismic loading and the soil properties on the structure stability. The cases of non-homogeneous and layered soils are investigated.

Keywords: Reinforced soil, Pore water effect, Limit analysis, Discretization technique, Pseudo-dynamic approach.

1 Introduction

Geosynthetic reinforced soil retaining walls are now a mature technology. They are structures which allow reinforcing a compacted backfill with horizontal reinforcement elements connected to the wall facing. The reinforcements significantly improve the global shear strength. The success of these structures is mainly due to their advantages compared to conventional retaining walls, concerning cost, time and space saving, in addition to their good performance due to their flexibility during strong earthquakes.

A compacted cohesionless granular soil is recommended as a backfill in the reinforced zone by most design codes (e.g., FHWA, NF P 94-270). The use of a good drainage system is also required. These requirements avoid the

development of interstitial pore pressures in the reinforced zone. However, the use of geosynthetic reinforcements, which do not have corrosion risk that can affect the metallic reinforcements, gives the opportunity to use poorly draining cohesive soils when granular soils are not available or are expensive (Guler et al. 2007). This soil type was successfully used in the reinforced earth wall construction (Riccio et al. 2014). Nevertheless, the use of these soils can lead to several problematical issues and therefore, reduce the system stability (Abd and Utili 2017). Among these problems, the pore-water pressures development and the crack formation in the backfill zone are the most dangerous ones. Koerner and Koerner (2018) presented 320 geosynthetic reinforced mechanically stabilized earth walls failure cases. They reported that the reinforced backfill soils used in 73% of these cases were cohesive ones. In addition, they reported that pore pressures developed in the reinforced backfill caused the failures of 63% of these walls.

The water presence in the cohesive backfill must be addressed due to its low permeability and the possible malfunction of the drainage system that could be caused by its clogging by fines. The system shear strength can then be significantly reduced. Furthermore, the cohesive soils present limited tensile strengths, and therefore, they can develop cracks on soil surface (Abd and Utili 2017). The cracks formation affects the reinforced cohesive soil retaining walls stability (Alhajj Chehade et al., 2019). This phenomenon has been observed at the surface of many reinforced cohesive slopes after earthquake events (Ling et al. 2001) in addition to the experimental studies (Porbaha and Goodings 1996).

The assessment of the seismic stability of reinforced soil structures could be performed using numerical approaches (finite differences-finite elements), limit equilibrium and limit analysis methods. The cracks introduce a discontinuity in the static and kinematic fields which induces a computationally-expensive cost to consider these discrete discontinuities in numerical methods (Utili and Abd 2016). Hence, to conduct a parametric study on the reinforced backfill retaining wall stability considering cracks formation, the use of numerical methods requires complex development. The most commonly used method to assess the unreinforced and reinforced slopes stability considering the cracks presence, in the literature, is the limit equilibrium one (Baker 1981; Chowdhury and Zhang 1991; Baker and Leshchinsky 2001, 2003). However, only the pre-existing crack case is considered through this method by the modification of the failure surface geometry. Alternatively, Chen et al. (1969) developed an efficient method to analyze slopes stability, known as the limit analysis method. Since then, this method has been used to assess the stability of all geotechnical structures including the reinforced soil walls. It permits to obtain more rigorous solutions considering the stress-strain relation of soils than the limit equilibrium does. In addition, it permits to include the process of crack propagation as a part of the failure mechanism. Recently, many studies assessed unreinforced slopes stability considering the cracks presence using the limit analysis upper bound approach (Michalowski 2013; Utili 2013; Zhao et al. 2016). In addition, Abd and Utili (2017) first investigated the effect of cracks on the stability of geosynthetic reinforced slopes. A homogeneous backfill soil was considered and two cracks types are examined, pre-existing cracks and cracks that can be formed as a part of the collapse mechanism.

In the reinforced soil wall stability analysis, a rotational toe log-spiral failure surface is generally considered as the most critical failure mechanism (Abd and Utili 2017; Alhajj Chehade et al. 2019, 2020). Mollon et al. (2011) developed a discretization method coupled with the kinematic approach of limit analysis, in the framework of face stability assessment of circular tunnels. The discretization technique permits to overcome the conventional kinematic approach limitations by generating the failure mechanism point by point. It allows the consideration of non-uniform soils and the use of the pseudo-dynamic approach to represent the seismic loading instead of the pseudo-static one, which is more realistic. The good agreement of the results with the field measurements in the work of Mollon et al. (2011), inspired many researchers to extend the discretization-based kinematic analysis to assess the stability of different geotechnical structures (Pan and Dias 2015; Qin and Chian 2018; Sun et al. 2018). In this context, Alhajj Chehade et al. (2019) successfully extended it to generate a toe log-spiral failure mechanism in the framework of reinforced soil walls stability assessment.

In this study, the limit analysis upper bound theorem is employed for the seismic stability assessment of geosynthetic reinforced soil retaining walls with saturated non-uniform backfill soils. A rotational failure mechanism generated point by point through the discretization technique is considered. This technique gives the possibility to represent the seismic loading through the pseudo-dynamic approach as well as the possibility of considering non-homogeneous backfill soil. A saturated cohesive backfill soil is considered. The influences of the pore water pressure, the cracks presence, the soil heterogeneity and the seismic loading on the seismic internal stability of reinforced soil retaining walls are assessed and discussed.

2 Upper bound theorem of limit analysis

The upper bound theorem of limit analysis is based on the balance between the internal and external forces of a kinematically admissible velocity field. An advantage of this method is the consideration of the soil stress-strain relationship through the concept of yield criterion and its associated flow rule. Neverthless, among the disadvantages of this method is that the shape of the failure surface is imposed. In addition, the soil behavior is assumed to be an ideal rigid, perfectly plastic body with an associated normality rule based on the Coulomb yield condition, which is not rational.

In this study, the kinematic theorem is applied to analyze the seismic internal stability of geosynthetic reinforced soil retaining walls in saturated poorly draining backfills. A rigid block assumption is considered and the soil is considered as an ideal rigid perfectly plastic material obeying associated flow rule. This theorem gives a rigorous lower bound to the required reinforcement tensile strength that prevents the failure of the reinforced soil walls. The reinforced wall fails if the external work rate exceeds the internal energy dissipation rate for any assumed kinematically admissible failure mechanism. A rotational log-spiral failure mechanism passing through the wall toe is assumed (Abd and Utili

2017; Alhajj Chehade et al. 2019a, 2020). The geosynthetic reinforcements only resist tension without any bending or compression resistance. These reinforcements are finite in number, having a uniform length, placed with a uniform vertical spacing, providing therefore a uniform tensile strength distribution over depth. They are placed horizontally and provide forces that correspond to their tensile strength or pullout resistance.

3 Discretization technique

The discretization technique was proposed for the first time by Mollon et al. (2011) to generate a rotational failure mechanism, in the framework of the stability analysis of circular tunnels. Alhajj Chehade et al. (2019) extended this technique for reinforced soil retaining walls stability assessment. It allows to overcome the limitations of the traditional kinematic approach which can only be used in case of homogeneous soils and can only represents the seismic loading by the pseudo-static approach (Alhajj Chehade et al., 2019). This is because when the sliding surface is generated point-by-point through this technique, the soil properties and the seismic loading could be easily specified for each infinitesimal part of the failure surface. In this paper, a log-spiral failure mechanism passing by the wall toe is generated using this technique as illustrated in Fig. 1. A level backfill is considered (Horizontal surface), H denotes the wall height and β is the angle formed between the horizontal and the wall facing.

The failure surface $ABCD$ delimits the moving soil area and the soil at rest. It is formed by the log-spiral surface BC and the potential formed crack CD. Along the first surface, the soil fails purely in shear while it can also fail in tension along the crack. The failure surface is defined by the parameters r_B, θ_B and ξ. r_B corresponds to the length of OB, θ_B corresponds to the angle between the vertical direction and the (OB) and ξ is the ratio of the crack depth to the wall height. Two cracks' cases can be involved in the failure mechanism. Pre-existing cracks in the reinforced soil due to weathering, desiccation and other climate phenomenon, and cracks that are formed during the collapse mechanism process. For simplicity in this study and similar to various studies (Michalowski 2013; Utili 2013; Zhao et al. 2016), the cracks considered are assumed to be vertical even though they can presented curved shapes as reported by Hu et al. (2010). For rigid blocks assumption, the soil block $ABCD$ is rotating about the center O with an angular velocity Ω. The center of rotation O can be defined by the mechanism parameters r_B and θ_B. The discretization technique aims at defining the failure surface by a series of points B_j iteratively i.e. each point is derived from the previous point, knowing that the wall toe is the starting point of the generation process. The normality condition should be respected in order to obtain a kinematically admissible failure mechanism. Therefore, the angle between each segment $[B_jB_{j+1}]$ formed by two consecutive points on the failure surface, and the velocity vector should be equal to the soil friction angle φ_i. This angle becomes equal to δ along the vertical crack CD as depicted in Fig.2. The angle $\delta\theta$, known as the discretization angle, denotes the angle between two consecutives radial lines, χ_j and χ_{j+1}. The accuracy of the failure

mechanism is affected by this angle. A closer match to the log-spiral failure mechanism is obtained with small values of this angle. The detailed steps and equations used to generate the failure mechanism are presented in (Alhajj Chehade et al., 2019).

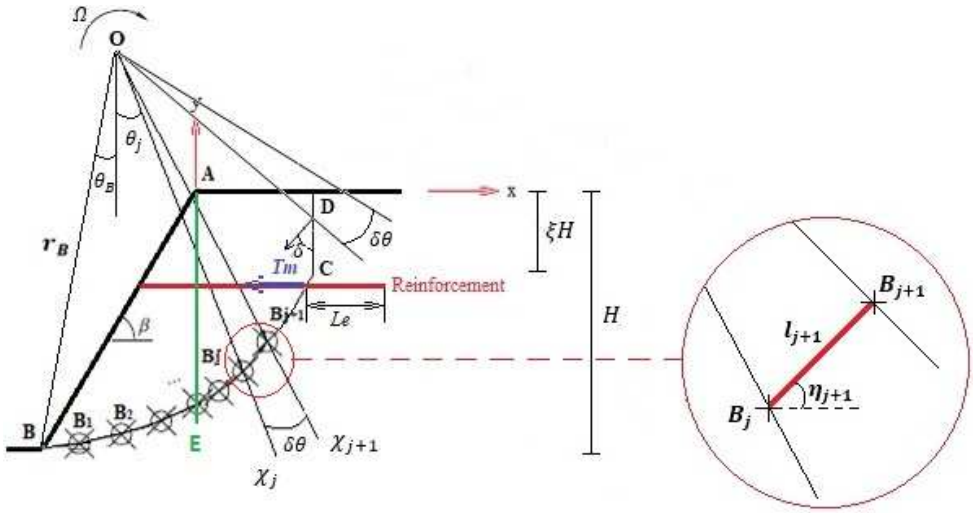

Fig.1. Generation of the potential failure surface by the discretization technique

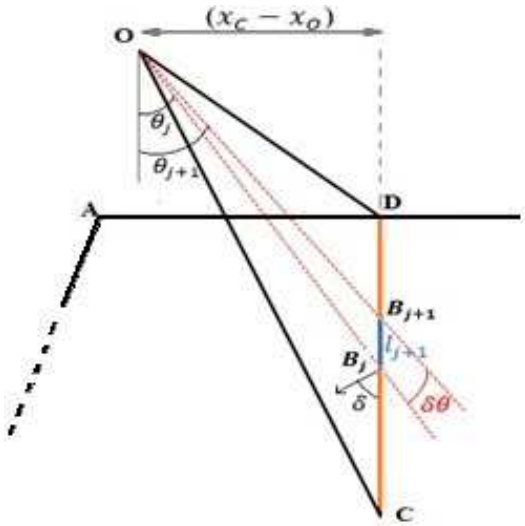

Fig.2. Generation of the crack

4 Pseudo-dynamic approach

The use of the real-time acceleration history to represent a seismic event is always the best choice. Such approach requires a high computational effort using numerical analysis. However, the seismic loading is commonly represented

using the pseudo-static approach when the former approach is not justifiable. In the framework of this approach, the seismic accelerations are considered constants. Alternatively, Steedman and Zeng (1990) proposed a more realistic approach, the pseudo-dynamic one, to represent the seismic loading through the limit equilibrium method. It represents a good compromise between the pseudo-static accelerations and the real-time acceleration history. It accounts for the time and spatial variation of the seismic shaking. The discretization method allows the use of this approach with the kinematic theorem of limit analysis to represent the seismic loading which is not applicable in case of conventional limit analysis method (Qin and Chian 2017, 2019).

The pseudo-dynamic analysis considers finite shear and primary wave velocities acting within soil medium, while the pseudo-static one assumes infinite values. The soil shear modulus G is assumed to remain constant during the shaking in the whole soil medium. This means that the shear wave velocity V_s and the primary wave velocity V_p that depend on G, remain also constant. On the other hand, the accelerations magnitudes and phases vary in both directions along the backfill depth. The ratio $V_p/V_s = 1.87$, used for soils characterized with a Poisson's ratio $\upsilon = 0.3$, is considered in this paper.

In reality, as mentioned before, the real time history acceleration with a wide frequency content is the best way to consider an earthquake. For simplicity, similar to most previous studies (Choudhury and Nimbalkar, 2006; Nimbalkar et al. 2006; Ahmad and Choudhury 2008; Basha and Sivakumar Babu 2011; Qin and Chian 2017, 2019; Alhajj Chehade et al. 2019a, 2020), this work adopt a sinusoidal acceleration. The accelerations magnitudes in both directions are assumed to vary linearly along the reinforced backfill depth. If the accelerations are amplified by a factor f at the ground surface related to the accelerations at the wall base, the horizontal and vertical acceleration expressions at any depth y and time t can be expressed as:

$$\begin{cases} a_h = \left[f + \dfrac{y}{H}(f-1)\right].k_h g \sin\dfrac{2\pi}{T}\left(t - \dfrac{H+y}{V_s}\right) \\ a_v = \left[f + \dfrac{y}{H}(f-1)\right].k_v g \sin\dfrac{2\pi}{T}\left(t - \dfrac{H+y}{V_p}\right) \end{cases} \quad (1)$$

where H is the wall height, g the acceleration due to gravity, T the lateral and vertical shakings period, k_h and k_v the horizontal and vertical seismic coefficients respectively.

5 Discretization-based kinematic analysis

The limit analysis kinematic approach is based on the balance between internal energy dissipation $\dot{D}$, and the work rates of the external forces, $\dot{W}$, in any kinematically admissible failure mechanism. Its application gives a lower bound to the required tensile reinforcement strength T_m, which ensures the reinforced soil wall stability.

The internal energy dissipation during the failure process takes place in the geosynthetic reinforcing elements by tensile failure or/and along the reinforcing elements by pullout failure. In addition, based on the rigid blocks assumption, an internal energy is dissipated along the discontinuity surface due to the soil plastic deformation. When the failure mechanism involves the cracks presence, the internal energy dissipation along the crack CD must be accounted only in the case of crack that forms during the failure mechanism process since an amount of energy is required for its formation (Michalowski, 2013). On the other hand, for a pre-existing crack that is already present in the soil, $\dot{D_{CD}}$=0.

On the other hand, the external work rates involve the soil block weight $ABCD$ work rate, the inertia forces work rates representing the seismic loading in both directions and the pore water pressure one.

The required tensile reinforcement strength to maintain the reinforced soil wall equilibrium, can be determined by equating the external work rates and the internal energy dissipations, as stated by the kinematic theorem of limit analysis. This approach provides a lower bound for the required strength through an optimization procedure. The energy balance equation is given by:

$$\dot{W_\gamma} + \dot{W_{kh}} + \dot{W_{kv}} + \dot{W_u} = \dot{D_{BC}} + \dot{D_{CD}} + \dot{D_T} \tag{2}$$

Where $\dot{W_\gamma}$ is the soil weight work rate ; $\dot{W_{kh}}$ and $\dot{W_{kv}}$ are the seismic forces work rates ; $\dot{W_u}$ is the pore-water pressure work rate; $\dot{D_{BC}}$, $\dot{D_{CD}}$ and $\dot{D_T}$ are the internal energy dissipated along the rotational part of the sliding surface, along the vertical crack and, in and along the reinforcements respectively.

Due to the discretization, the computations of the external work rates are performed by summation of the elementary work rates of the elementary trapezoidal surface $B'_jB_jB'_{j+1}B_{j+1}$ shown in Fig. 3 and are expressed as:

$$\dot{W_\gamma} = \sum_j (\gamma . A_j) . \left(\Omega . (x_{Gj} - x_O)\right) \tag{3}$$

$$\dot{W_{kh}} = \sum_j \left(\gamma . A_j . \left[f + \frac{y_{Gj}}{H}(f-1)\right] . k_h \sin\frac{2\pi}{T}\left(t - \frac{H + y_{Gj}}{V_s}\right)\right) . \left(\Omega . (y_O - y_{Gj})\right) \tag{4}$$

$$\dot{W_{kv}} = \sum_j \left(\gamma . A_j . \left[f + \frac{y_{Gj}}{H}(f-1)\right] . k_v \sin\frac{2\pi}{T}\left(t - \frac{H + y_{Gj}}{V_p}\right)\right) . \left(\Omega . (x_{Gj} - x_O)\right) \tag{5}$$

where A_j is the infinitesimal area of the trapezoidal element $B'_jB_jB'_{j+1}B_{j+1}$; x_{Gj} and y_{Gj} are the coordinates of its gravity center G_i; x_O and y_O are the center of rotation coordinates of the rotational failure mechanism O.

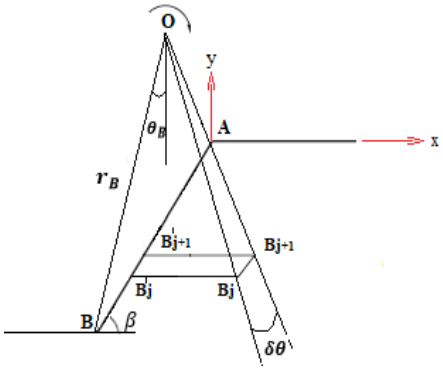

Fig.3. Elementary trapezoidal surface

The complete derivation of these work rates equations are presented in Alhajj Chehade et al. (2019). On the other hand, the work rate calculation of the pore-water pressure is presented in detail in the next paragraph.

For saturated soils, the effect of water is taken into consideration through introducing the external work rate of the pore water pressure. It is obtained by summation of the pore water pressure work rate applied to the soil skeleton and the external water pressure work rate on the boundary surface. As presented in Viratjandr and Michalowski (2006), the work of the seepage and buoyancy forces are both included in the analysis. Bishop and Morgenstern (1960) introduced the so-called pore-water pressure coefficient r_u to consider the pore water pressure. The distribution of the pore-water pressure u below the water table, is then given by $r_u \gamma h$ where γ is the soil unit weight and h the depth of the considered point below the ground surface (water table). Therefore, in the balance equation, the pore-water pressure work rate $\dot{W}_u$ can be expressed as a summation of both forces work rates as follows:

$$\dot{W}_u = -\gamma_w \int_V \frac{\partial h}{\partial x_i} . v_i dV + \gamma_w \int_V \frac{\partial Z}{\partial x_i} . v_i dV \tag{6}$$

where the first term represents the seepage force work rate over the entire collapse volume and the second one is the buoyancy force work rate, γ_w is the water unit weight, h is the hydraulic head, Z is the elevation head, v_i the velocity vector at velocity discontinuities and V is the moving block volume. Alternatively, Viratjandr and Michalowski (2006) by developing the derivative $\frac{\partial}{\partial x_i}$ of the product uv_i, showed that the expression of the water pressure work rate can be written by summation of the pore pressure work rate applied to the soil skeleton and on the boundary surface as follows:

$$\dot{W}_u = -\int_V u . \dot{\varepsilon_u} dV - \int_S u . n_i . v_i dS \tag{7}$$

where $\dot{\varepsilon_u}$ is the volumetric strain rate, S the failure block boundary surface and n_i the corresponding outward normal unit vector normal to this latter.

The first term in Eq. (7) is assumed to be zero since a rigid block is considered. In the framework of the failure surface generation using the point-to-point method, the kinematical admissibility condition must be satisfied and therefore, the velocity vector must be inclined by to the failure surface by an angle equal the soil friction angle φ. Hence the angle between the vector n_i and the velocity vector is equal to $\frac{\pi}{2}+\varphi$. The velocity vector of the collapse block at a point B_j of the failure surface is equal to the product of the angular velocity Ω by the length of $[OB_j]$, L_j.The pore-water pressure work rate along the failure surface is then given by the summation of elementary work rates as follows:

$$\begin{cases} \dot{W}_u = \sum_j r_u.\gamma.\left(x_j.\tan\beta - y_j\right).l_j.\Omega.L_j.\sin\varphi & \text{along the boundary surface } BE \\ \dot{W}_u = \sum_j r_u.\gamma.\left(-y_j\right).l_j.\Omega.L_j.\sin\varphi & \text{along the boundary surface } ED \end{cases} \quad (8)$$

where x_j and y_j are the coordinates of point B_j and l_j is the length of $[B_{j-1}B_j]$.

Since rigid blocks assumption is considered, the internal energy dissipation occurs along the sliding surface that separates the failure region and the soil at rest, and along the reinforcements. The internal energy dissipated along the sliding surface is divided into two parts: along the log-spiral part and along the vertical crack. Eq. 9 gives the expression of $\dot{D_{BC}}$, as a summation of the internal work rate per infinitesimal length as follows:

$$\dot{D_{BC}} = \sum\left(c_j.\left(L_j.\Omega.\cos\varphi_j\right).l_j\right) \quad (9)$$

Where L_j and l_j are respectively the lengths of the segments $[OB_j]$ and $[B_{j-1}B_j]$, c_j and φ_j are respectively the soil cohesion and the soil friction angle at the point B_j.

Eq. 10 gives the internal energy dissipation along the vertical crack CD for soil tensile cut-off (zero tensile strength) which is generally assumed for safety reasons. On the other hand, Eq. 11 gives the energy dissipated in the reinforcement $\dot{D_T}$ due to its tensile failure.

$$\begin{cases} \dot{D_{CD}} = \sum c_j\Omega L_j\cos\varphi_j\dfrac{1-\sin\delta_j}{1-\sin\varphi_j}l_j & \text{Crack formation (tension cut} - \text{off)} \\ \dot{D_{CD}} = 0 & \text{Pre} - \text{existing crack} \end{cases} \quad (10)$$

$$\begin{cases} \dot{D_T} = \sum\left(\dfrac{n_r T_t}{H}.l_j.\sin\eta_j.\Omega.L_j.\cos(\theta_B + j.\delta\theta)\right) & \text{along the boundary BC} \\ \dot{D_T} = \sum\left(\dfrac{n_r T_t}{H}.l_j.\Omega.L_j.\cos(\theta_B + j.\delta\theta)\right) & \text{along the vertical crack CD} \end{cases} \quad (11)$$

where η_j is the angle that forms $[B_{j-1}B_j]$ with the horizontal axis, δ_j is the angle between the vertical crack CD and the velocity vector, T_t is the reinforcement tensile strength and n_r is the reinforcement layers number.

In most previous studies, the required reinforcement strength is calculated considering only this reinforcement failure type. Concerning the reinforcement length, it is determined through an optimization process in a such a way that when considering the combined failure mode (rupture in some reinforcement layers and pullout in the others), the required reinforcement strength remains the same as obtained previously when considering only the tensile failure mode. However, this method can lead to unpractical reinforcement lengths in many cases. Alhajj Chehade et al. (2019, 2020) consider for the first time the combined reinforcement failures modes (tensile and pullout failures) simultaneously, with a pre-fixed reinforcement length. The internal energy dissipated along a single reinforcement due to pullout failure mode, is expressed as:

$$d\dot{D}_p = T_p \Omega Y_i \tag{12}$$

where T_p is the pullout resistance of the considered reinforcement layer, Y_i is the vertical distance between the geosynthetic layer and the rotation center O.

Note that the pullout force calculation for the internal energy dissipation along the reinforcements, should account for the water presence. It can then be written as:

$$T_p = 2\gamma z^*(1 - r_u)L_e f^* \tag{13}$$

where z^* is the overburden depth; L_e is the length of the reinforcement beyond the failure surface, known as the effective length; f^* is the apparent friction coefficient at the soil/reinforcement interface.

A lower bound to the required reinforcement is determined using a two-step genetic algorithm proposed by Guo et al.(2018) to perform an optimization process with respect to four variables, namely $[r_B, \theta_B, \xi, t]$. The length of $[OB]$, r_B, the angle between (OB) and the vertical direction, θ_B, and the normalized crack depth, ξ, are the geometrical parameters that define the failure surface (Fig.1a), while t is the time involved in the pseudo-dynamic approach. The interested readers are referred to Guo et al. (2018) for detailed description of the two-step genetic algorithm.

6 Comparison with the existing solutions

In order to validate the robustness of the proposed discretization mechanism, the obtained results are compared with those by Abd and Utili (2017) obtained using the conventional limit analysis method. Abd and Utili (2017) used the conventional upper bound approach of limit analysis in order to evaluate the reinforcement strength required to ensure the internal stability of geosynthetic reinforced slopes under static conditions. The effect of pre-existing crack is investigated. Only the reinforcement tensile failure is considered to determine the reinforcement strength while the

reinforcement length is calculated in such a way that the required reinforcement strength do not need to be increased when considering reinforcement tensile and pullout failures. They assumed both linear (uniform) and triangular reinforcement distribution. The pore water pressure effect on the required reinforcement tensile strength was analyzed by considering the same approach used here through the so-called pore-water pressure coefficient r_u.

In order to validate the application of the discretization method in this study, the results of the discrete method are obtained under the same conditions used in the work of Abd and Utili (2017). The case of the crack that is formed as a part of the failure mechanism is considered. The soil is considered without tensile strength (tension cut-off). The comparisons of results are depicted in Fig. 4. for different walls inclinations β, and coefficients r_u. The good agreement between the two methods shows the efficiency of the proposed method.

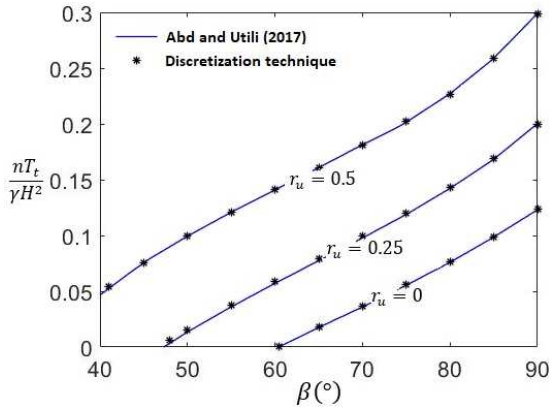

Fig. 4. Comparison of the conventional limit analysis and the discretization technique ($\varphi = 20°$ and $c/\gamma H = 0.1$)

7 Numerical results

7.1 Required reinforcement strength for reinforced homogeneous soil retaining wall

To investigate the pore-water pressure effect, tension crack and seismic loading, the results in terms of required reinforcement strength in a normalized form are presented for two wall inclinations $\beta = 75°$ and $\beta = 90°$. The two reinforcement failures modes are considered simultaneously. The following parameters are considered: H=7 m, γ=18 kN/m^3, φ=25°, c=12.6 kPa, λ=0.5, L_r=1.2 H, n_r=10, f_0^*=1.2, f_1^*=0.6, f=1.2, V_S=150 m/s, V_P=280.5 m/s, and T=0.3 s, where λ=k_v/k_h, c the soil cohesion, L_r the reinforcement length, n the reinforcement number, f_0^* and f_1^* are the initial and the minimum apparent friction coefficients at the soil/strip interface.. The discretization angle $\delta\theta$ is considered to be equal to 0.01° in this paper, since this value represents a good compromise between accuracy and time calculation according to Alhajj Chehade et al. (2021).

According to fig. 5, it is observed that the required reinforcement strength increases with the horizontal seismic coefficient, the pore-water ratio and the wall inclination (β). Considering the crack formation as a part of the failure

mechanism leads to larger required reinforcement strength values than the ones obtained when considering an intact soil.

It is evident that increasing the horizontal seismic coefficient k_h, increases the required reinforcement tensile strength. Moreover, the pore-water pressure is found to be unfavorable to the reinforced soil wall stability. The presence of cracks leads to larger required reinforcement strength. In particular, when the value of the pore-water pressure or of the seismic loading is important, the required reinforcement strength increase is more obvious. Hence, it is critical to consider these effects when poorly draining cohesive soils are used, as backfill materials, in seismic zones for economic reasons.

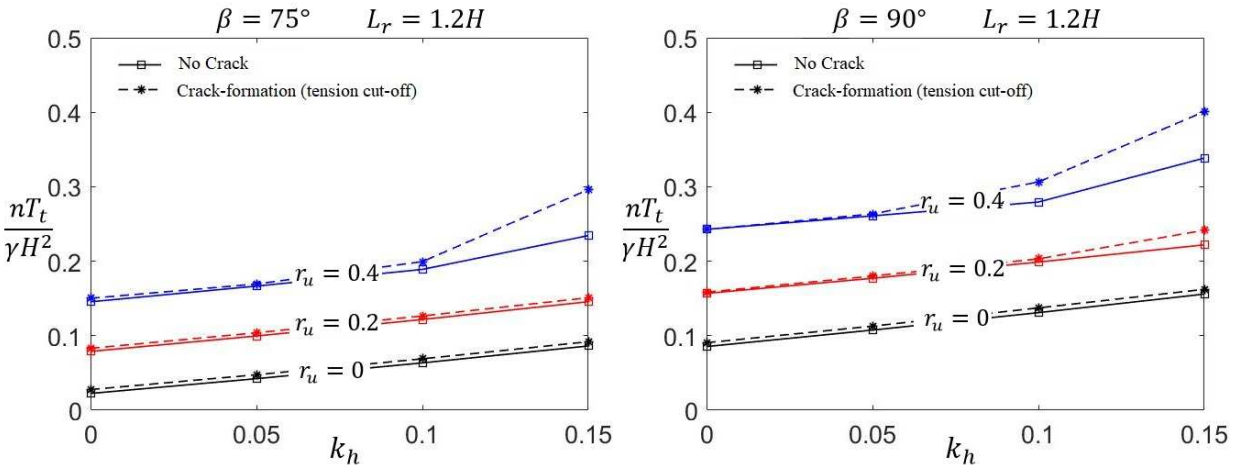

Fig. 5. Required reinforcement strength against seismic coefficient for intact and cracked backfills

The reinforcement strength required to prevent the failure of geosynthetic reinforced soil retaining wall, for different values of horizontal seismic coefficient, pore-water ratio and wall inclination (β) are also presented in Table1. Two crack cases are considered. Table 1 presented also the normalized crack depth ξ in case of crack formation as a part of the failure mechanism. The results showed that the normalized crack depth ξ of the critical failure surface generally decreased with the increase of β and k_h. However, when the pore-water ratio is high ($r_u = 0.4$), some exceptions observed.

Table 1 Required reinforcement strength and normalized crack depth for different values of r_u and k_h.

r_u	k_h		$\frac{nT_t}{\gamma H^2}$		Normalized crack depth ξ	
			$\beta = 75°$	$\beta = 90°$	$\beta = 75°$	$\beta = 90°$
0	**0**	**No crack**	0.025	0.085	-	-
		Crack formation	0.028	0.091	0.241	0.222

	0.05	**No crack**	0.042	0.108	-	-
		Crack formation	0.048	0.113	0.220	0.207
	0.1	**No crack**	0.064	0.131	-	-
		Crack formation	0.069	0.137	0.204	0.196
	0.15	**No crack**	0.086	0.156	-	-
		Crack formation	0.092	0.162	0.190	0.187
0.2	**0**	**No crack**	0.079	0.157	-	-
		Crack formation	0.083	0.159	0.321	0.277
	0.05	**No crack**	0.100	0.177	-	-
		Crack formation	0.104	0.180	0.282	0.252
	0.1	**No crack**	0.122	0.199	-	-
		Crack formation	0.127	0.203	0.254	0.232
	0.15	**No crack**	0.146	0.222	-	-
		Crack formation	0.151	0.242	0.230	0.164
0.4	**0**	**No crack**	0.146	0.243	-	-
		Crack formation	0.150	0.243	0.597	0
	0.05	**No crack**	0.167	0.261	-	-
		Crack formation	0.169	0.263	0.466	0.003
	0.1	**No crack**	0.189	0.279	-	-
		Crack formation	0.199	0.306	0.318	0.350
	0.15	**No crack**	0.234	0.339	-	-
		Crack formation	0.296	0.401	0.510	0.450

Fig.6 shows the soil cohesion effect on the reinforcement strength required to prevent the reinforced soil wall failure, considering different pore-water coefficients values for a homogeneous backfill soil. The horizontal seismic coefficient is equal to 0.1. The pore pressure coefficient increase leads to a significantly increase of the normalized reinforcement strength. It is clear that the soil cohesion improves wall stability. When the soil cohesion increases, the normalized required reinforcement strength decreases for different values of r_u, for both reinforced earth walls inclinations. The influence of the soil cohesion is more pronounced in the case of a pore-water coefficient r_u equal to 0.4. The required reinforcement strength is slightly greater for the crack presence case. The discrepancy between the curves corresponding to the cases of crack formation and no crack is more noticeable when the soil cohesion increases.

For the inclined retaining wall with $\beta = 75°$, for a soil cohesion c higher than 19 kPa and a pore-water coefficient r_u equal to zero, the required reinforcement strength which ensure the reinforced soil wall stability is zero. The is no need to reinforced the backfill area with geosynthetic elements.

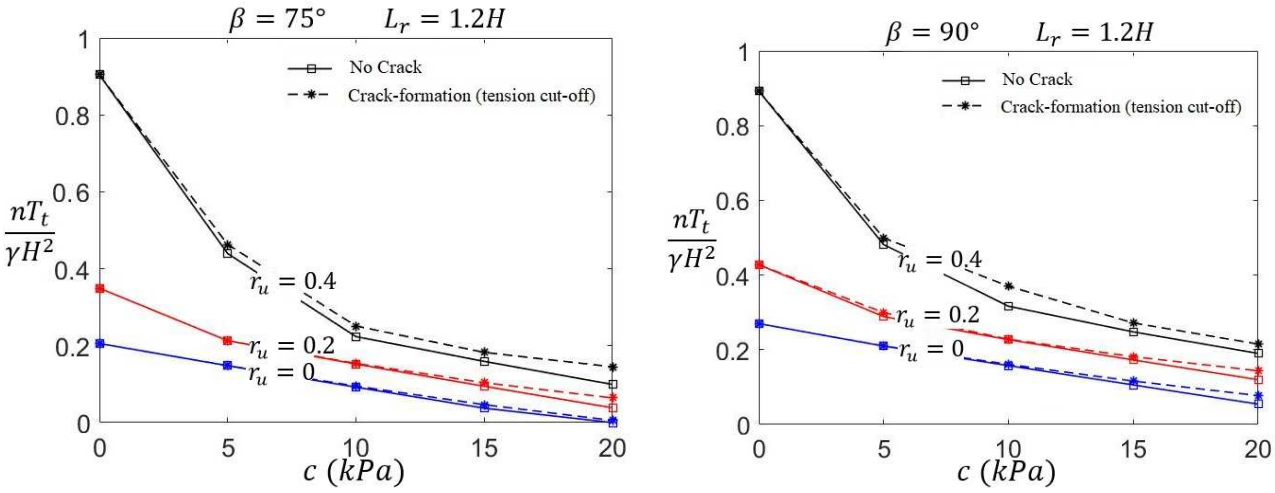

Fig. 6. Required reinforcement strength against soil cohesion for intact and cracked backfill

The soil friction influence on the tensile strength of the geosynthetic reinforcements used to maintain the reinforced soil wall stability is presented in Fig. 7 for different pore pressure coefficients. An uniform backfill soil and a horizontal seismic coefficient equal to 0.1 are considered. It is clear that higher friction angle implies better quality backfills and therefore, that the required reinforcement strength is higher in case lower friction angle values for both reinforced soil structures inclinations and different values of r_u, considering or not the presence of cracks. The effect of the soil friction angle φ increases with the increases of the values of r_u. The required reinforcement strength is slightly greater for the crack presence case. It is important to note that this difference between the results obtained considering or not the crack presence decreases when increasing the soil friction angle φ as well as when decreasing the pore-water coefficient r_u and the wall inclination.

For the inclined retaining wall with $\beta = 75°$, the required reinforcement strength that assure the reinforced soil wall stability is equal to zero for a soil friction angle higher than 37° and a zero pore-water coefficient r_u value.

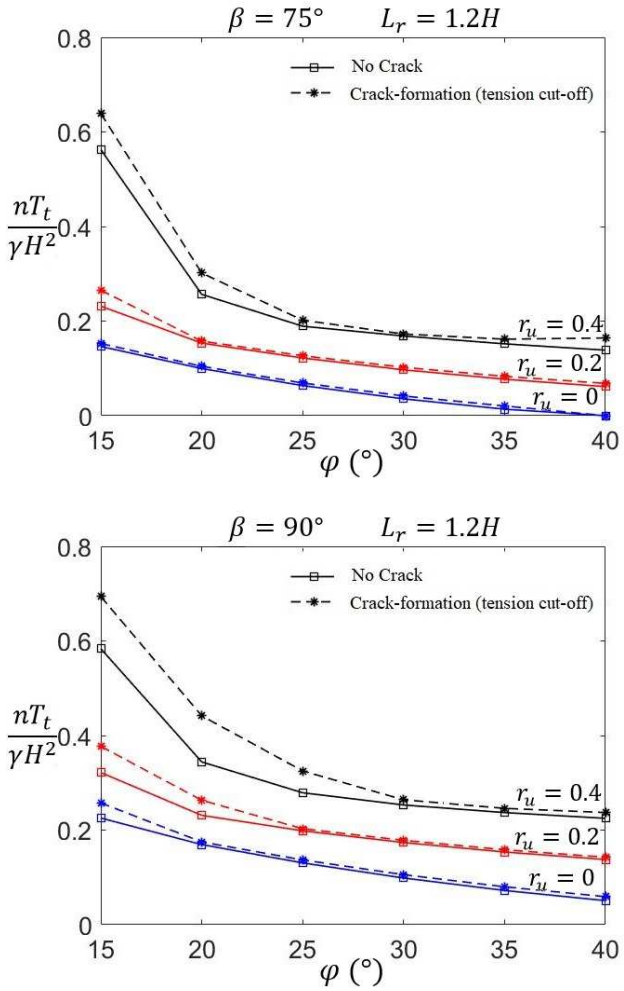

Fig. 7. Required reinforcement strength against soil friction angle for intact and cracked backfill

7.2 Required reinforcement strength for reinforced non-homogeneous soil retaining walls

Most previous studies on the reinforced soil retaining walls stability assume a homogeneous soil. However, soils are non-homogeneous in nature and theirs properties show spatial variability (Pan and Dias, 2015). This heterogeneity affects the reinforced soil retaining walls stability, hence the importance of its consideration in the design. The conventional kinematic theorem can only be used in cases of homogeneous soils. The discretization-based kinematic analysis method gives the ability to overcome this limitation, and to consider the soil heterogeneity. The variation of the soil strength parameters, the soil friction angle and cohesion, are considered. On the other hand, the soil unit weight γ is considered constant in the whole domain. In this study, for convenience and simplicity, two types of heterogeneity are considered: the soil properties increase linearly with depth and layered backfill soil profile.

7.2.1 Linearly increased soil strength profile

The soil strength parameters are assumed to vary linearly only in the vertical direction (Fig. 8). This means that they are assumed to be constant in the horizontal plane, i.e. at the same depth. The soil unit weight variation is neglected in

the whole domain. c_1 and φ_1 are respectively the soil cohesion and friction angle at the ground surface, while c_2 and φ_2 are those at the wall toe level.

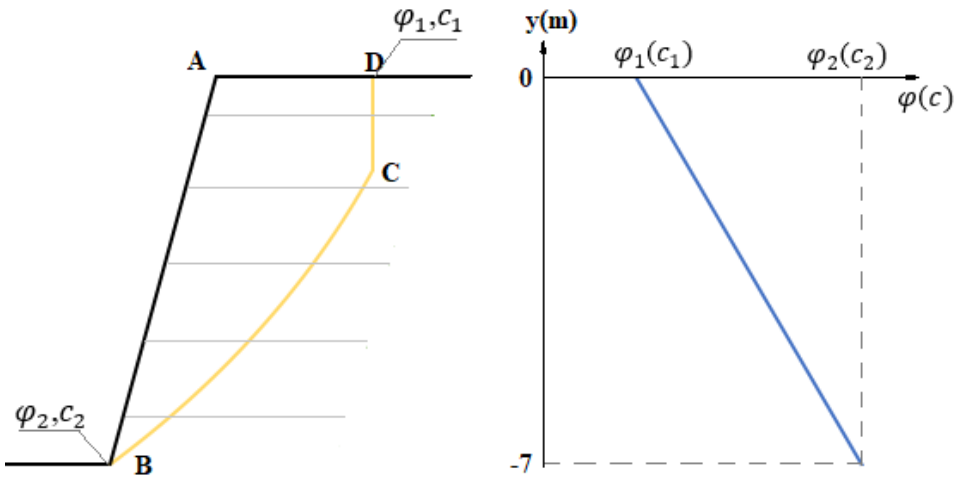

Fig.8. Distribution of the soil properties with depth

The effect of each soil strength parameters variability is investigated separately. When analyzing the influence of a parameter, the second one is taken as constant. An inclined reinforced earth wall with β=75° is considered. The other parameters are kept the same as the previous section: H=7 m, γ=18 kN/m^3, $\delta\theta$=0.01^o, k_h=0.1, λ=0.5, L_r=1.2 H, n=10, f_0^*=1.2, f_1^*=0.6, f=1.2, V_S=150 m/s, V_P=280.5 m/s, and T=0.3 s

First, the influence of the soil cohesion variability is investigated. The required normalized reinforcement strength is plotted against different ground surface soil cohesion values c_1 (Fig. 9). The soil cohesion at the wall toe level is kept equal to c_2=12.6 kPa. The soil friction angle is considered constant across the whole field φ_1=φ_2=25°.

A decrease of the required reinforcement strength is observed when c_1 increases from 2.5 to 22.5 kPa for both cases of intact or cracked soil. The required reinforcement strength obtained in case of crack formation is higher than the case of an intact soil. The difference between the two cases is more pronounced for greater values of c_1. The required reinforcement strength decreases linearly with c_1 for the case of water-pressure coefficients r_u equals to 0 and 0.2. However, for $r_u = 0.4$, the rate of decrease is greater for c_1 smaller than 7.5 kPa. This can be explained by the reinforcements pullout failure that occurs in some layers under these conditions.

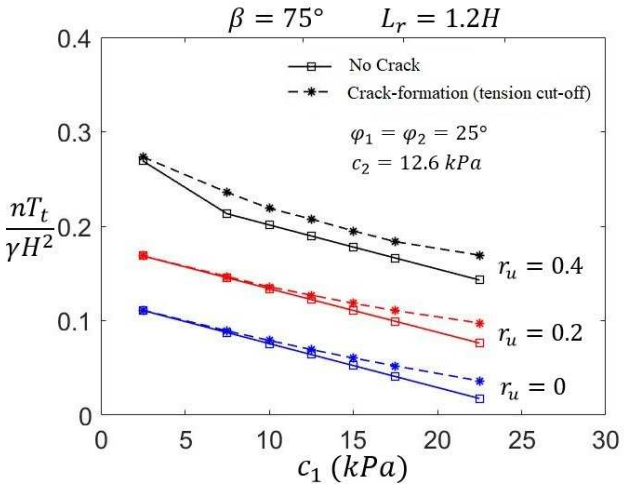

Fig. 9. Required reinforcement strength against non-uniform soil cohesion for intact and cracked backfill

The influence of the soil friction angle variability is also investigated apart. The required normalized reinforcement strength is plotted against different ground surface soil friction angles φ_1 (Fig. 10). The soil friction angle at the wall toe level is kept equal to φ_2=25°. The soil cohesion is considered constant across the whole field c_1=c_2=12.6 kPa.

The increase of φ_1 from 15° to 35° decreases slightly the required normalized reinforcement strength and therefore, this increase slightly enhances the reinforced earth wall stability for both intact and cracked soils and different values of r_u. The required reinforcement strength is slightly greater for the crack presence case except the case of pore-water pressure r_u equal to 0.4, and precisely for soil friction angle φ_1 at the wall toe smaller than 25°. Wherein that case, the difference between the intact soil case and the cracked soil one becomes significant, and increases with the soil friction angle φ_1 decrease. This is because the reinforcements pullout failure in some layers in the case of cracked soils.

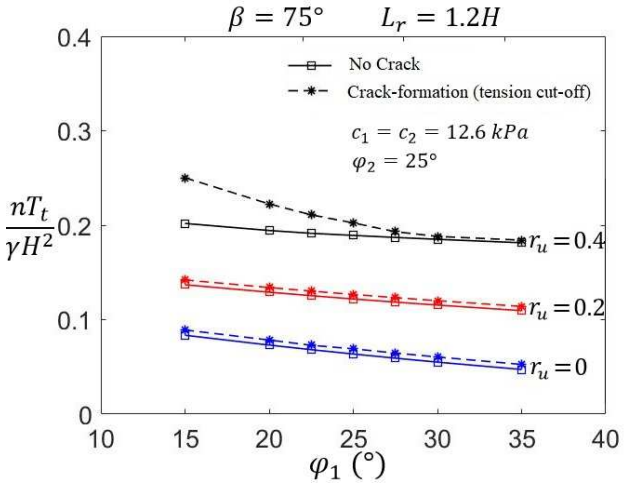

Fig. 10. Required reinforcement strength against non-uniform soil friction angle for intact and cracked backfill

7.2.2 *Layered soil profile*

In most of the real cases, the soil profile is a layered one. In this section, a reinforced soil retaining wall with a stratified backfill composed of two soil layers with different soil strength parameters (see Fig. 11), is analyzed using the developed method. A coefficient m is defined to distinguish the stratified condition as the ratio of the upper layer height to the wall height. The soil strength parameters are φ_1 and c_1 in the upper layer and φ_2 and c_2 in the lower one.

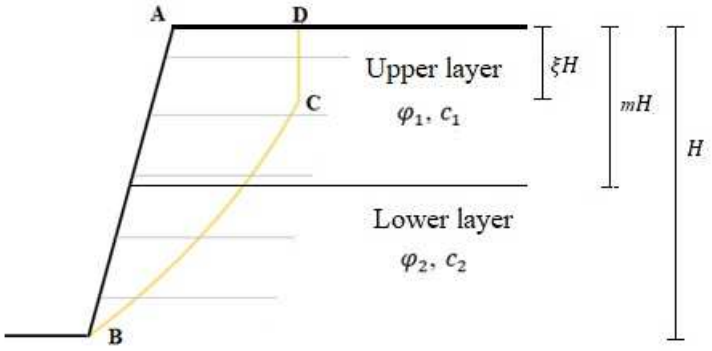

Fig. 11. Heterogeneous backfill with two soil layers

The reinforced soil wall considered is inclined with β=75° and all the parameters are kept the same as the previous sections except the soil strength parameters. Fig. 12 shows the pore-water pressure coefficient r_u and soil cohesion c_2 of the lower layer influences on the normalized required reinforcement strength. The analysis without the presence of cracks as well as the case of crack formation are both considered. A two-layered backfill required higher reinforcement strength in order to prevent the structure failure when the coefficient r_u increased and when the soil cohesion c_2 in the lower layer decreased.

The required reinforcement strength is slightly greater in case of the crack presence in the failure surface. An increase in the lower layer soil cohesion c_2 from 10 to 20 kPa leads to 35.37% and 36.73% reduction of the normalized required reinforcement strength for intact and cracked soil respectively, for a pore-water pressure coefficient equal to 0.4. These reductions become respectively equal to 57.21% and 56.35% for intact and cracked soils when the pore-water pressure does not exist ($r_u = 0$).

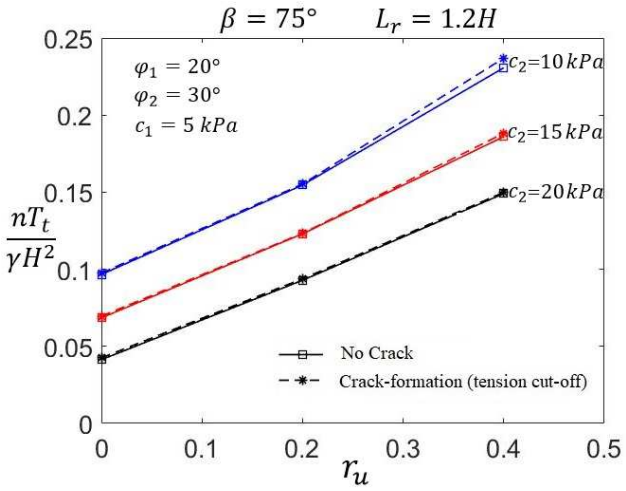

Fig. 12. Required reinforcement strength against r_u and soil cohesion c_2 for a two-layered backfill

Fig. 13 illustrates the influence of the pore-water pressure coefficient r_u and lower layer soil friction angle φ_2 on the normalized required reinforcement strength. The same two crack cases are considered. The stability of the reinforced soil wall is improved when the coefficient r_u decreased and when the soil friction angle φ_2 in the lower layer increased. This is logical since the water has a destabilizing effect and the soil friction angle increase enhances the wall stability by providing an additional resistance.

Considering the crack formation through the failure mechanism leads to a higher required reinforcement strength. A decrease in the lower layer soil friction angle φ_2 from 35 to 25° leads to respectively 75.8% and 77.12% of normalized required reinforcement strength increase for intact and cracked soil, for a zero pore-water pressure coefficient. These increases become respectively equal to 25.73% and 29.84% for intact and cracked soil when the pore-water pressure coefficient is equal to 0.4.

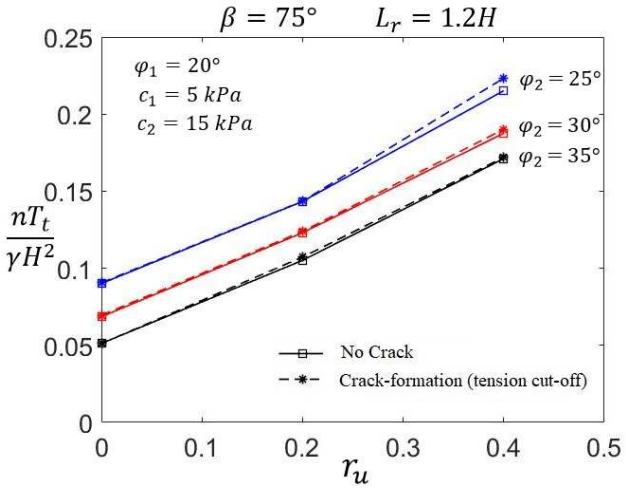

Fig. 13. Required reinforcement strength against r_u and soil friction angle φ_2 for a two-layered backfill

8 Conclusion

This paper investigates the seismic internal stability of geosynthetic reinforced saturated soil wall considering soil cracks, using the kinematic theorem of limit analysis combined with the discretization technique. The results are presented in terms of required reinforcement strength that prevent wall failure. The discretization technique consists to generate the rotational sliding surface of the reinforced soil wall point by point based on the associated flow rule. This method represents a suitable tool to overcome the limitations of the conventional upper bound limit analysis method. It gives the ability to easily integrate the soil spatial variability properties for nonhomogeneous or multi-layered soils. In addition of the benefits of considering the soil heterogeneity, it allows the implementation of the pseudo-dynamic approach which takes into account the aspects neglected by the pseudo-static approach adopted in the conventional kinematic analysis. The effect of the buoyancy forces and seepage forces are considered. It is included as an additional external force in the limit analysis upper bound method.

In order to validate the proposed approach, the obtained results are verified by comparison with those obtained by Abd and Utili (2017), who evaluated the reinforced slopes stability, using the conventional upper bound approach of limit analysis. Pre-existing crack as well as crack formation through the collapse mechanism are considered without consideration of the seismic loading. A good consistency between the two approaches proves the developed method effectiveness.

The effects of the crack presence, seismic loading and the soil strength parameters on the required reinforcement strengths are assessed. Three scenarios are considered for the backfill soil strength parameters, a homogeneous soil, non-homogeneous soil with a linearly increased strength parameters or a layered backfill.

The consideration of the crack formation is found to have a destabilizing effect on the reinforced soil retaining walls stability. This is more pronounced for lower soil friction angle values, greater values of soil cohesion and pore water coefficient.

The pore water pressure has also a negative effect on these structures. In addition, the required reinforcement strength is highly dependent on the soil strength parameters. The soil friction angle and cohesion enhance the reinforced soil wall stability.

The discretization-based kinematic analysis have shown its effectiveness for the seismic internal stability analysis of reinforced saturated soil retaining walls with crack presence. The key advantage of this method is its capacity to deal with multi-layered or nonhomogeneous soil profiles. In addition, it allows the implementation of the pseudo-dynamic approach which gives the benefits of considering the dynamic characteristic of the seismic loading.

Numerical calculations or physical models will be necessary to evaluate the reliability of the developed method in this paper.

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