



**HAL**  
open science

# Joint production and energy supply planning of an industrial microgrid

Zoe Fornier, Dorian Grosso, Vincent Leclere

► **To cite this version:**

Zoe Fornier, Dorian Grosso, Vincent Leclere. Joint production and energy supply planning of an industrial microgrid. 2023. <hal-03927692>

**HAL Id: hal-03927692**

**<https://hal.science/hal-03927692v1>**

Preprint submitted on 6 Jan 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# Joint production and energy supply planning of an industrial microgrid.

Zoe Fornier<sup>1</sup>, Dorian Grosso<sup>3</sup>, and Vincent Leclere<sup>2</sup>

<sup>1</sup>METRON - CERMICS, France, zoe.fornier@enpc.fr

<sup>2</sup>CERMICS, France, vincent.leclere@enpc.fr

<sup>3</sup>METRON, France, dorian.grosso@metronlab.com

January 6, 2023

## Abstract

We consider the problem of jointly optimizing the daily production planning and energy supply management of an industrial complex, with manufacturing processes, renewable energies and energy storage system. It is naturally formulated as a mixed-integer multistage stochastic problem. This problem is challenging for three main reasons: there is a large number of time steps (typically 24), renewable energies are uncertain and uncontrollable, and we need binary variables modeling hard constraints. We discuss various solution strategies, in particular Model Predictive Control, Dynamic Programming, and heuristics based on the Stochastic Dual Dynamic Programming algorithm. We compare these strategies on two variants of the problem: with or without day-ahead energy purchases.

## 1 Introduction

The latest Intergovernmental Panel on Climate Change (IPCC) warns us yet again on the consequences of climate change, and incites governments, industries and citizens to change accordingly. The COP26, held on November 2021, set up a clear objective of securing global net-zero emissions by mid-century. Therefore the industry, counting for one fifth of global emissions (5th IPCC report), must take strong actions to reduce them. In this respect, the Clean Energy Ministerial Industrial Deep Decarbonisation Initiative (IDDI) calls out for a change in the energy supply, as industry consumes fuel massively to produce local energy, especially steel and cement production. To put things in perspective, renewable generation represent only 11.2% of electricity generation in the industrial sector in 2020, which is far less than its share in global electricity generation, up to 28% in 2020, according to the International Energy Agency (IEA), see their Tracking Industry 2021 report [Intb] and their Global Energy Review 2021 report [Inta]. For instance, micro-grids are an alternative energy supply model. They are defined (see *e.g.*, [HPG18]) as a small-scale power grid that can operate independently or collaboratively with the power grid. Generally, they are compounds of Energy Storage Systems (ESS) and renewable energy generation units (wind turbines, solar panels).

However, incorporating renewable energies in the supply mix is challenging as they are intermittent, unpredictable and uncontrollable. To counteract these defects it is often suggested to add an ESS (we refer to [Geo+21] for an overview of the available ESS). Indeed, an ESS allows transferring energy across time-steps, making it controllable and compensating intermittency. Unpredictability of the renewable production

19 requires to go from a deterministic formulation to a stochastic formulation. Indeed, a classical determin-  
 20 istic problem is often misleading and optimistic about the potential of the ESS. Unfortunately, multistage  
 21 stochastic problem are known to be numerically challenging (see *e.g.*, [Sha06]). Starting from a standard  
 22 scheduling industrial problem, we consider using an onsite micro-grid to provide an alternative energy supply  
 23 to the main grid. We obtain a mixed-integer multistage stochastic problem optimizing jointly the production  
 24 planning and the energy supply management of an industrial facility with in advance and intraday energy  
 25 purchases.

## 26 1.1 The industrial microgrid management problem

27 We consider a facility with  $I$  machines that manufactures up to  $J$  types of products that can be stored (see  
 28 fig. 1a). Our goal is to provide the facility with a joint production and energy supply planning, on a discrete  
 29 horizon  $t \in [T]$ . The planning should minimize the total expected cost (economic, environmental and labour)  
 30 while satisfying production target and technical constraints.

31 Depending on the facility at hand, many technical constraints need to be satisfied. We can classify them in  
 32 three types. First, *physical constraints* are induced by the machines at hand. For example most machines, as  
 33 grinders or plastic extruders, require warming up before being operational. Another straightforward example  
 34 comes from the food industry, where machines need to be cleaned up to reconfigure the production line.  
 35 Second, *process constraints* which correspond to precedence constraints mandating sequential execution of  
 36 some tasks (usually called flow-shop problems). For instance, in a chocolate factory, every batch production  
 37 will follow in order cleaning, roasting, shell removing, grinding and conching. Finally, *implied constraint*  
 38 model decision-maker preferences or human resources constraints. For example, the decision maker may  
 39 limit the number of re-starts to limit wear-off, if a machine is hard to access or for human power reasons.

40 In this paper we consider a problem with bounded production and set-up costs. In addition, we consider  
 41 shared resources constraints such that some products cannot be produced simultaneously. Factory energy  
 42 needs, proportional to production, are met with electricity from a main grid or produced onsite by a micro-grid  
 43 consisting in solar panels coupled with an energy system storage (ESS) see fig. 1b.

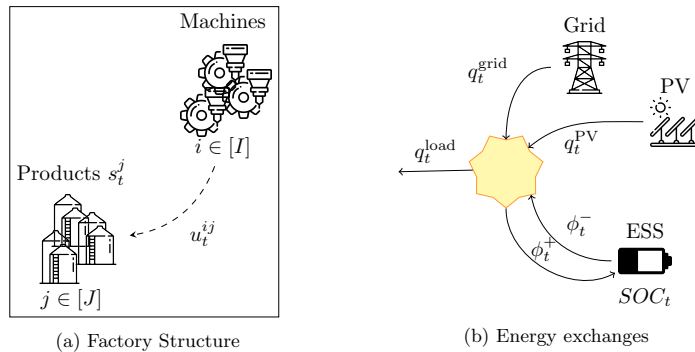


Figure 1: Industrial Management Problem

44 Electricity from the main grid can be purchased through two different contracts, usually cumulated: Intra-  
 45 day contract where prices are fixed annually, the factory pays the energy extracted from the main grid at  
 46 a given time  $t$ ; In-advance contract where the factory buys energy blocks in advance (*e.g.*, a day ahead of

47 production) at a preferential rate. Decisions are made adjusting energy purchases based on intra-day rates  
48 in real time.

## 49 1.2 Literature review

50 We consider a problem coupling production planning and energy supply management. Taken separately,  
51 each problem has been widely studied, but considering them simultaneously is less common, especially when  
52 taking into account uncertainty, leading to large multistage stochastic optimization problem. In this section,  
53 we review the state-of-the-art of energy-aware production planning under uncertainties.

### 54 1.2.1 Production planning and Scheduling problems under uncertainties

55 A typical angle for energy-aware production systems is to minimize energy waste, see the reviews [Bän+21],  
56 [BG16] and references therein. This part of the literature looks for production plan, or scheduling, that are  
57 more energy efficient, adapting tools from well studied problems like single or parallel machine scheduling,  
58 job-shop, flow-shop or lot-sizing.<sup>1</sup> However, few papers discuss the economic impact of integrating renewable  
59 energy sources onsite: indeed, the industrial energy supply is traditionally guaranteed by an external grid.  
60 In their survey [Bän+21], Bänisch et al. count 8 articles (out of 192) that consider an onsite energy genera-  
61 tion and an ESS. The literature lacks research on industrial problems with distributed generation systems,  
62 though, they are widely studied on their own. We refer to the review [Alo+22] where Alonso-Travesset et al.  
63 focus on recent studies on models under uncertainties in distributed generation systems. They highlight the  
64 necessity of properly taking into account uncertainties in those problems, in particular regarding renewable  
65 energy generation. There are two main ways of handling uncertainty: stochastic optimization and robust  
66 optimization.

67 In the first paradigm, we model uncertain variables as random variables with known distribution, usually  
68 represented by a scenario tree. Further, as uncertainties are revealed step by step, stochastic problems are  
69 often multistage problems which are known to be challenging, while there exists various methods to tackle  
70 2-stage problems *e.g.*, based on Bender’s decomposition (see [BL97]). As a result, multistage problems are  
71 classically relaxed into 2-stage problems: all decision variables, except the first stages variable, are assumed  
72 to be taken with the full knowledge of the uncertainty. This is the strategy adopted by Golari, Fan, and Jin  
73 in [GFJ16] to optimize the production planning of interconnected factories each connected to a micro-grid.  
74 Biel et al. take this approach as well in [Bie+18] to solve a flow-shop problem under uncertainties regarding  
75 wind energy generation. In another article ([WMG20]), Wang, Mason, and Gangammanavar study a similar  
76 problem with multi-objectives (total completion time and energy costs), where selling an energy excess to  
77 the main grid is allowed. They propose an epsilon-constraint algorithm integrated with the L-shaped method  
78 ([Bir85]), which is a Benders decomposition adapted to 2-stage stochastic programs.

79 In the second paradigm, robust optimization, we consider the worst case in possible uncertainty realizations.  
80 This is the choice made by Ruiz Duarte, Fan, and Jin in [RFJ20], where they evaluate the renewable energy  
81 integration with an ESS in a factory while optimizing the production planning. This is modeled by a 2-stage  
82 problem: in the first stage, a production plan is defined whereas in the second stage the decisions regarding  
83 the energy management system are made to minimize its energy costs under the worst-case energy generation  
84 scenario. The robust uncertainty set is determined by statistical tools. Bridging both worlds, Shahandeh,  
85 Motamed Nasab, and Li propose in [SML19] to divide random variables into two categories: static and

---

<sup>1</sup>The job-shop problem, see *e.g.*, [Man60], looks for an optimal scheduling plan for  $n$  jobs, consisting of operations with precedence constraints, on  $m$  machines. The flow-shop problem is a variant of the job-shop problem with a strict order of all operations on all jobs. Finally, a lot-sizing problem optimizes the production quantities of each item at each time step.

86 dynamic variables. The idea is to apply robust optimization on one variable category and then stochastic  
87 optimization on the other, considering a scenario tree. This results into two hybrid algorithms, mixing robust  
88 and stochastic optimization to solve a multistage problem with different uncertainty types.

89 Furthermore, in these industrial problems, the solution is not only affected by renewable energies' variabil-  
90 ity: costs and demands are other known uncertainty sources. If some articles consider time-of-use (TOU)  
91 electricity rates ([Bie+18], [MP13], [Li+17] and [WMG20]), which are fixed prices in contract depending  
92 on consumption's times, others consider variable prices. In that respect, Bohlayer et al. ([Boh+20]) and  
93 Ierapetritou et al. ([Ier+02]) both study mixed-integer multistage stochastic problems under energy prices  
94 uncertainty. See also Fazli Khalaf and Wang ([FW18]) who solve a 2-stage stochastic scheduling problem  
95 considering both electricity prices and energy generation as random variables. Finally, in lot-sizing problems,  
96 the product demand is often random: Higle and Kempf consider a multistage stochastic program in [HK10]  
97 to solve a production planning problem under demand uncertainty, trying to avoid cumulating stocks.

### 98 1.2.2 Strategical decision problems for Microgrid

99 We have covered stochastic considerations for operational or tactical production planning problems. We now  
100 discuss strategical decisions like investing in renewable energies and ESS, with question of size, technologies  
101 and number of ESS and energy generation units. To adapt their energy mix, factories need to design  
102 what distributed generation system is suited for their production. In [FMH21], Fattahi, Mosadegh, and  
103 Hasani focus on the planning in mining supply chains with renewable energy investment: at each stage,  
104 warehouse or generation systems can be installed. They propose a multistage stochastic minimization model  
105 yielding strategic decisions, costs include energy consumption and production costs, holding inventory and  
106 transport costs as well as investment costs. Furthermore, they suggest a methodology evaluating the social  
107 and environmental impact both of transportation and system life-cycle.

108 The growing interest in microgrids is driven by environmental concerns. Thus, instead of minimizing energy  
109 waste, a more direct approach consists in integrating environmental objectives into costs. For example Li  
110 et al., in [Li+17], assess wind and solar generation deployment costs in order to achieve net-zero carbon.  
111 They also investigate a renewable generation energy system's resilience: can it answer the energy demand  
112 despite its inherent uncertainty? They answer this question through a multistage stochastic micro-grid sizing  
113 problem, with given energy demand. The flexible demand aspect is taken into account by Pham et al., who  
114 extend Golari, Fan, and Jin's work in [Pha+19], considering both stochastic demand and the micro-grid  
115 sizing. Their goal is to determine if it is economically viable to provide the system with only renewable  
116 energies: the objective is to minimize energy bought from the main-grid, not factory costs.

117 Investing in micro-grids doesn't require only sizing but also investigating the different existing technologies  
118 and their characteristics. In [Tsi+21], Tsianikas et al. ask the following questions: is it necessary to extend  
119 the factory's storage capacity? If yes, when and what storage quantity should they install? What type of  
120 storage technology should they choose? Answering those questions allows determining an optimal investment  
121 strategy, and is key to optimizing interconnected micro-grids in the long run. An interesting take on the  
122 subject is given in [HBF15]: when most micro-grid investment model consider the ESS sizing at the beginning,  
123 Hajipour, Bozorg, and Fotuhi-Firuzabad propose to extend the storage capacity and invest in renewable  
124 generation units at different times, leading to a multistage stochastic problem. This model allows life-cycle  
125 constraints or decreasing technology efficiency to have an impact on results.

### 1.3 Contributions

Our contribution in this paper lies in four aspects. First, we propose an optimization model for a coupled management problem with both production and energy supply planning. We take into account the multistage structure of the problem, the uncertainties due to onsite renewable energy generation and binary variables modeling physical production constraints. In particular, we model shared resources constraints: a choice has to be made between different products at each time. Therefore, it is crucial, when reducing the problem to stage  $t$  with dynamic programming, to have visibility on the consequences of choosing a product at  $t$ . Second, we consider both on-demand supply with TOU pricing and in-advance energy purchasing. The latest brings complexity to the multistage problem with first stage variables impacting the whole horizon costs. Third, we discuss multiple solution strategies based on well known and new methodology: a deterministic approach know as Model Predictive Control (MPC); Stochastic Dynamic Programming (SDP); and an approach solving linear multistage stochastic problems, Stochastic Dual Dynamic Programming (SDDP). Finally, as there does not exist an efficient algorithm to solve large mixed-integer multistage stochastic problems, we propose heuristic methods relying on the approximated cost-to-go function given by SDDP. We highlight theoretical and practical limits of these solution strategies on numerical examples.

The remainder of the paper is laid out as follows. Section 2 introduces the problem formulation and define two quantities assessing the sensitivity of the problem to stochasticity. We present in section 3 dynamic programming methods to solve multistage mixed-integer stochastic problems. Those methods being unsatisfactory for the problem at hand, we then proceed to detail different heuristics in section 4. Finally, section 5 presents numerical results.

### 1.4 Notations

To facilitate understanding, we go through some notation used in this paper. We denote  $[a : b] := \{a, \dots, b\}$  the set of integers between  $a$  and  $b$ , and  $[T] := [1 : T]$  the set of non-null integers smaller than  $T$ . Accordingly,  $X_{[n]}$  denote the collection  $X_{[n]} := \{X_i\}_{i \in [n]}$ . Generally speaking, we denote the state variables  $x$ , the control variables  $u$  and the noise  $\xi$ . All random variables are in bold characters, further if  $\xi$  is a random variable then  $\xi$  denotes a realization of this variable. Finally,  $\sigma(\xi_{[t]})$  represents the  $\sigma$ -algebra generated by  $\{\xi_\tau\}_{\tau \in [t]}$ .

## 2 Multistage stochastic problems

In the considered problem we can often distinguish between strategical and operational decisions. The strategical decision (like sizing of elements, or in-advance purchasing of electricity) are then seen as fixed parameter for the operational management problem. Here, in section 2.1 we describe the operational management problem with given strategical decision  $\theta$ , while section 2.2 presents the global problem considering both the strategical decision costs as well as the operational costs. Finally, in section 2.3 we discuss the so-called *anticipative framework* for multistage program and the Value of Stochastic Solution (VSS) and Expected Value of Perfect Information (EVPI) indicators which assess the sensitivity of the problem to uncertainties.

### 2.1 Parametrized multistage problem

We consider a *controlled dynamic system*, that is a sequence of random vector  $\mathbf{x}_{[T]}$  that follows a dynamic (see eq. (1b)), affected by a sequence of noises  $\xi_{[T]}$ . Each noise  $\xi_t$  takes value in a finite set  $\Xi_t$ , and we denote  $\Omega := \prod_{t \in [T]} \Xi_t$ . We assume that these noises represent all the uncertainty in the problem at hand, with known probability distribution, resulting in a probability space  $(\Omega, \mathcal{A}, \mathbb{P})$ . We call *scenario* a sequence  $\xi_{[T]}$  of

165 realization of the noise at each time step. Although there exists a finite number of scenarios  $S = |\Omega|$ , it can  
 166 be very large. For example, if we consider a problem with hourly time steps, each noise taking 10 different  
 167 values *i.e.*,  $|\Xi_t| = 10$ , then  $S = 10^{24}$ .

168 With this definition, for a given parameter  $\theta$  (representing here battery size or day-ahead purchase...), we  
 169 consider the following parametrized multistage stochastic problem:

$$(P_\theta) \quad V(x_0; \theta) := \min_{(\mathbf{u}_t, \mathbf{x}_t)_{t \in [T]}} \mathbb{E} \left[ \sum_{t=1}^T L_t^\theta(\mathbf{x}_{t-1}, \mathbf{u}_t, \boldsymbol{\xi}_t) \right] \quad (1a)$$

$$\text{s.t.} \quad \mathbf{x}_t = D_t^\theta(\mathbf{x}_{t-1}, \mathbf{u}_t, \boldsymbol{\xi}_t) \quad \forall t \in [T], \quad (1b)$$

$$\mathbf{x}_t \in X_t^\theta \quad \forall t \in [T], \quad (1c)$$

$$\mathbf{u}_t \in \mathcal{U}_t^\theta(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t) \subset U_t^\theta \quad \forall t \in [T], \quad (1d)$$

$$\sigma(\mathbf{u}_t) \subset \sigma(\boldsymbol{\xi}_1, \dots, \boldsymbol{\xi}_t) \quad \forall t \in [T], \quad (1e)$$

170 where  $\mathbf{x}_t$  is a vector of continuous state variables;  $\mathbf{u}_t$  represent mixed-integer control variables, binary vari-  
 171 ables are necessary to model physical constraints such as sharing resources constraints; and  $\boldsymbol{\xi}_t$  are exogeneous  
 172 finitely supported random variables. We take the classical risk neutral approach where we aim at minimizing  
 173 expected costs in eq. (1a) given as a sum of instantaneous costs given by function  $L_t^\theta : X_{t-1}^\theta \times U_t^\theta \times \Xi_t \rightarrow \mathbb{R}$   
 174 depending on current state  $\mathbf{x}_{t-1}$ , control  $\mathbf{u}_t$  and noise  $\boldsymbol{\xi}_t$ . Further, eq. (1b) describes the dynamic of the  
 175 controlled system, that is how the next state  $x_t$  is obtained from current state  $x_{t-1}$ , control  $u_t$  and noise  $\xi_t$   
 176 through  $D_t^\theta : X_{t-1}^\theta \times U_t^\theta \times \Xi_t \rightarrow X_t^\theta$ . Equation (1c) represent constraints on the state variable  $x_t$ , and eq. (1d)  
 177 constrains the admissible control variables. Note that we can only enforce constraints linking variables at  $t$   
 178 and  $t + 1$ . Finally, the last constraint (eq. (1e)), commonly known as *non-anticipativity* constraint, represent  
 179 the information available when taking decision  $\mathbf{u}_t$ . In particular, in this framework, we observe the random  
 180 variable  $\boldsymbol{\xi}_t$  realization, before making decision  $\mathbf{u}_t$ , with no knowledge of future random realizations from  $t + 1$   
 181 to  $T$ .

182 Technically, solutions of eq. (1) are random variables of  $(\Omega, \mathcal{A}, \mathbb{P})$ , meaning they are functions of the scenario  
 183  $\xi_{[T]}$ , *i.e.*, there exists functions  $\psi_{[T]}^\theta$ , such that, for all  $t \in [T]$ ,  $u_t = \psi_t^\theta(\xi_{[T]})$  and consequently, resulting  
 184 recursively from the dynamic  $D_t^\theta$ , there exist a function  $\Psi_t$  such that  $x_t = \Psi_t^\theta(\xi_{[T]})$ . We call *noise-based*  
 185 *policy* a sequence of such functions  $\psi_{t \in [T]}^\theta$ , with, for each  $t \in [T]$ , function  $\psi_t^\theta : \prod_{\tau=1}^T \Xi_\tau \rightarrow U_t^\theta$  returning  
 186 a control  $u_t$  for any scenario  $\xi_{[T]}$ . Therefore, given a scenario  $\xi_{[T]}$  and initial state  $x_0$ , a noise-based policy  
 187 computes a *trajectory*, which is the resulting sequence of state and control variables  $(u_{[T]}, x_{[T]})$  depending on  
 188 the scenario, whose cost is given by eq. (2a). Then, averaging over all scenarios, we compute the noise-based  
 189 policy expected cost  $V^\psi(x_0; \theta)$  in eq. (2b).

$$\hat{V}^\psi(x_0, \xi_{[T]}; \theta) := \sum_{t=1}^T L_t^\theta(x_{t-1}, \psi_t(\xi_{[T]}), \xi_t) \quad (2a)$$

$$\text{where } x_t = D_t^\theta(x_{t-1}, \psi_t(\xi_{[T]}), \xi_t), \quad \forall t \in [T].$$

$$V^\psi(x_0; \theta) := \mathbb{E}_\xi \left[ \hat{V}^\psi(x_0, \boldsymbol{\xi}_{[T]}; \theta) \right]. \quad (2b)$$

191 We say that a noise-based policy  $\psi$  is *non-anticipative* if eq. (1e) is satisfied. Further, a non-anticipative  
 192 noise-based policy is *admissible* if eqs. (1c) and (1d) are satisfied. The non-anticipativity constraint ensures  
 193 that random variable  $\mathbf{u}_t$  is measurable with respect to the  $\sigma$ -algebra generated by  $\xi_{[t]}$ . In other words, it  
 194 ensures that  $\psi_t$  only depends on the first  $t$  elements of  $\xi_{[T]}$ , more precisely, with a slight abuse of notation:

$$\psi_t(\xi_{[t]}) := \psi_t(\xi_{[t]}, \xi'_{t+1}, \dots, \xi'_T) = \psi_t(\xi_{[t]}, \xi''_{t+1}, \dots, \xi''_T), \quad \forall \xi', \forall \xi'', \forall t.$$

195 **Remark 1** (Scenario trees point of view). *A classical approach in stochastic programming consists in rep-*  
 196 *resenting the probability space through a scenario tree  $\mathcal{T}$ , such that the extensive formulation of  $(P_\theta)$  has as*  
 197 *many variables as nodes in  $\mathcal{T}$ . More precisely, a node  $\nu \in \mathcal{T}$ , of depth  $t$ , represent the realization of all noises*  
 198 *up to time  $t$ ,  $\xi_{[t]}$ . Then a node  $\nu$  corresponding to noise realization  $\xi_{[t]}$  has  $|\Xi_{t+1}|$  children. In this setting, a*  
 199 *random process  $(\mathbf{u}_t, \mathbf{x}_t)_{t \in [T]}$  is non anticipative (satisfying (1e)), if and only if, for all  $t \in [t]$ ,  $(\mathbf{u}_t, \mathbf{x}_t)$  is a*  
 200 *function of the nodes of depth  $t$  in  $\mathcal{T}$ .*

201 **Remark 2** (Relatively complete recourse assumptions). *Ensuring admissibility in a multistage framework*  
 202 *can be difficult, as constraints in future stages can induce constraint in the current stage. To characterize*  
 203 *these dependencies, we introduce the complete and relatively complete recourse assumptions. We say that*  
 204 *Problem (1) has complete recourse if all control are admissible, more precisely if every sequence of control*  
 205  *$u_t \in U_t^\theta$  results in an admissible trajectory. This is a strong assumption, usually obtained by penalizing*  
 206 *most constraints. A less stringent requirement, is the relatively complete recourse which guarantees that*  
 207 *there always exists an admissible control. Relatively complete recourse ensures that we can, from any state*  
 208  *$x_t \in X_t^\theta$ , and for any scenario, construct an admissible trajectory with a non-anticipative policy.*

209 **Remark 3** (Estimating noise-based policy cost through Monte Carlo). *Bear in mind that, for multistage*  
 210 *problem with horizon  $T$  larger than a few unit, exactly computing the expected cost to a given noise-based policy*  
 211 *might not be numerically tractable. For example, consider that each noise  $\xi_t$  can take 10 different values,*  
 212 *then computing  $V^\psi(x_0, \theta)$  requires  $O(10^T)$  operations. However, if exact computation is not reachable, it can*  
 213 *always be estimated through standard Monte-Carlo simulations.*

214 *This approach consists in drawing a number  $N$  of independent scenarios  $\{\xi_{[T]}^n\}_{n \in [N]}$ , where typically  $N \geq$*   
 215 *1000. We then evaluate the cost of the noise-based policy along each of the  $N$  scenario,  $\hat{V}^\psi(x_0, \xi_{[T]}^n; \theta)$ , and*  
 216 *estimate the expected cost  $V^\psi(x_0; \theta)$  as*

$$V^\psi(x_0; \theta) \approx \frac{1}{N} \sum_{n=1}^N \hat{V}^\psi(x_0, \xi_{[T]}^n; \theta). \quad (3)$$

217 *Under weak assumptions, the Central Limit Theorem (CLT) (see e.g., Section 27 [Bro83]) give some control*  
 218 *over the error made by this estimation. For the remain of the paper it is enough to keep in mind that, roughly*  
 219 *speaking, for  $N$  large, the difference between the true expected cost and its estimation is smaller than  $2\sigma/\sqrt{N}$*   
 220 *with probability at least 95%, where  $\sigma$  is the standard deviation of  $\{\hat{V}^\psi(x_0, \xi_{[T]}^n; \theta)\}_{n \in [N]}$ .*

## 221 2.2 The strategic design problem

222 We now consider the strategic problem aiming at deciding the best parameter  $\theta \in \Theta$ , taking into account the  
 223 strategic decision cost  $I(\theta)$ , and the expected operational management cost  $V_1^\theta(x_0)$ . We model it as follows:

$$(P) : \min_{\theta \in \Theta} I(\theta) + V_1(x_0; \theta) \quad (4)$$

224 One could ask if the strategic Problem (4) can simply be seen as an operational Problem (1), with an  
 225 additional first stage  $t = 0$  with decision  $u_0 = \theta$ . Unfortunately, it is not as straightforward as it seems: in  
 226 eq. (1d) the constraints on the control  $u_t$  only depend on the current state  $x_t$  (and noise  $\xi_t$ ), and cannot  
 227 depend on past control  $u_0$ . The solution consists in considering an extended state  $(x_t, \theta)$ , where the second  
 228 part is decided at stage  $t = 0$  and then carried on from stage to stage by the dynamics of the system. With  
 229 this additional time-step, and extended state, we indeed fall back to the classical setting of eq. (1). However,  
 230 the Dynamic Programming based methods presented in section 3 do not scale well with the dimension of the  
 231 state. Thus, considering an extended state  $(x_t, \theta)$  is not a numerically efficient solution if  $\theta$  is multidimensional  
 232 (for example if  $\theta$  is a day-ahead electricity plan).

233 We end the section with some tools to evaluate policies for either the strategic or operational problem.

### 234 2.3 Deterministic tools to study multistage problem

235 To characterize the impact of uncertainty on a stochastic problem, we present two indicators: the *Expected*  
 236 *Value of Perfect Information* (EVPI) and the *Value of Stochastic Solution* (VSS).

237 A natural lower-bound for stochastic problems comes from relaxing the non-anticipativity constraint (1e).  
 238 We are then in the anticipative, or perfect information, framework which consists in assuming that we can  
 239 look into the future and know the noises realization (*e.g.*, how much renewable energy is available at any  
 240 given time). More precisely, the anticipative noise-based policy  $\psi_{ant}$  returns, for each scenario  $\xi_{[T]}$  a solution  
 241 perfectly adapted to this scenario, that is an optimal solution to the deterministic problem:

$$\hat{V}^{\psi_{ant}}(x_0, \xi_{[T]}; \theta) := \min_{(u_t, x_t)_{t \in [T]}} \sum_{t=1}^T L_t^\theta(x_{t-1}, u_t, \xi_t) \quad (5a)$$

$$\text{s.t. } x_t = D_t^\theta(x_{t-1}, u_t, \xi_t) \quad \forall t \in [T], \quad (5b)$$

$$x_t \in X_t^\theta \quad \forall t \in [T], \quad (5c)$$

$$u_t \in \mathcal{U}_t^\theta(x_{t-1}, \xi_t) \subset U_t^\theta \quad \forall t \in [T]. \quad (5d)$$

242

243 Note that, obviously, this noise-based policy is usually not admissible for Problem (1) as it requires unavailable  
 244 information. However, by definition, the value:

$$V^{\psi_{ant}}(x_0; \theta) := \mathbb{E}[\hat{V}^{\psi_{ant}}(x_0, \xi_{[T]}; \theta)], \quad (6)$$

245 is a lower bound of the cost incurred by any admissible noise-based policy on this scenario. The expected  
 246 value of perfect information (EVPI) is defined as:

$$EVPI = V(x_0; \theta) - V^{\psi_{ant}}(x_0; \theta), \quad (7a)$$

247 which characterizes what we could gain if we had perfect prediction. Note that EVPI is given as an expectation  
 248 over all scenarios, which is usually untractable, but can be estimated by Monte-Carlo (see remark 3) by solving  
 249 a reasonable number of deterministic programs.

250 We have obtained a lower bound by relaxing the information constraint, we now go the other way by consid-  
 251 ering a noise-based policy which does not adapt to new information. More precisely, we solve a deterministic  
 252 problem where noises are replaced by their expected value, yielding a deterministic sequence of control  
 253  $(u_t^{EV})_{t \in [T]}$ .

254 The *expected value* noise-based policy  $\psi^{EV}$  is such that for every scenario  $\xi_{[T]}$ , and all  $t \in [T]$ ,  $\psi_t^{EV}(\xi_{[T]}) =$   
 255  $u_t^{EV}$ . Beware that, unless we are in a complete recourse framework, the EV noise-based policy is often  
 256 non-admissible, leading to infinite cost. For example, for an industrial microgrid without external grid, if  
 257 the control  $u_t^{EV}$  is a production plan at  $t$  requiring solar energy  $\mathbb{E}[\xi_t]$ , then this production plan is not valid  
 258 on any scenario  $\xi_{[T]}$  such that  $\xi_t < \mathbb{E}[\xi_t]$ . Generally speaking, the expected cost of the EV noise-based  
 259 policy  $V^{\psi^{EV}}(x_0; \theta) = \mathbb{E}[\hat{V}^{\psi^{EV}}(x_0, \boldsymbol{\xi}_{[T]}; \theta)]$  is an upper bound of the problem  $(P_\theta)$ 's value  $V(x_0; \theta)$ . Again,  
 260  $V^{\psi^{EV}}(x_0; \theta)$  is defined as a sum over all scenarios in  $\Omega$  but can be estimated by Monte Carlo (see remark 3).

261 Finally, we introduce the value of the stochastic solution (VSS), see [Bir82], as the difference between the  
 262 EV noise-based policy expected cost and the optimal expected cost:

$$VSS = V^{\psi^{EV}}(x_0; \theta) - V(x_0; \theta) \geq 0. \quad (7b)$$

### 263 3 Dynamic Programming approaches

264 Assuming that the noises are finitely supported, a multistage stochastic problem like Problem1 can always be  
 265 cast as large scale deterministic problem (see *e.g.*, [BL97]). The Stochastic Programming literature then offer  
 266 dedicated algorithm exploiting the special structure of such problems. However, the size of these deterministic  
 267 equivalent is linear in the number of scenarios, which is often exponential in the horizon. For example, if we  
 268 consider 10 possible realizations per stage, and  $T = 24$ , we have  $10^{24}$  scenarios, rendering such approaches  
 269 intractable.

270 A solution consists in compressing the information required to take a decision. To this end we make a crucial  
 271 stagewise independence assumption, and turn to Dynamic Programming tools, presented here. Those are  
 272 exact methods, although they are not enough to tackle our industrial problem, and implementable heuristics  
 273 derived from these ideas are presented in section 4.

274 Recall that we can always go from  $(P_\theta)$  to  $(P)$  by including  $\theta$  in the state. Going forward, we then alleviate  
 275 notational burden by dropping the  $\theta$  in all notations.

#### 276 3.1 Stochastic Dynamic Programming

277 Our goal is to have a stochastic approach to resolve our problem: we resort to Stochastic Dynamic Program-  
 278 ming (SDP). Consider the problem  $(P_t)$  on sub-horizon  $[t : T]$ ,

$$(P_t) \quad V_t(x_{t-1}) := \min_{(\mathbf{u}_t, \mathbf{x}_t)_{t \in [t:T]}} \mathbb{E} \left[ \sum_{\tau=t}^T L_\tau(\mathbf{x}_{\tau-1}, \mathbf{u}_\tau, \boldsymbol{\xi}_\tau) \right] \quad (8a)$$

$$\text{s.t.} \quad \mathbf{x}_\tau = D_\tau(\mathbf{x}_{\tau-1}, \mathbf{u}_\tau, \boldsymbol{\xi}_\tau) \quad \forall \tau \in [t:T], \quad (8b)$$

$$\mathbf{x}_\tau \in X_\tau \quad \forall \tau \in [t:T], \quad (8c)$$

$$\mathbf{u}_\tau \in \mathcal{U}_\tau(\mathbf{x}_{\tau-1}, \boldsymbol{\xi}_\tau) \subset U_\tau \quad \forall \tau \in [t:T], \quad (8d)$$

$$\sigma(\mathbf{u}_\tau) \subset \sigma(\boldsymbol{\xi}_1, \dots, \boldsymbol{\xi}_\tau) \quad \forall \tau \in [t:T]. \quad (8e)$$

279 With the stage-wise independence assumption, the Dynamic Programming principle ensures that the value  
 280 functions follow the following recursive equations:

$$\hat{V}_t(x, \xi) = \min_{u_t \in \mathcal{U}_t(x, \xi)} \underbrace{L_t(x, u_t, \xi)}_{\text{instantaneous cost}} + \underbrace{V_{t+1}(D_t(x, u_t, \xi))}_{\text{cost-to-go}}, \quad (9a)$$

$$V_t(x) = \mathbb{E} [\hat{V}_t(x, \boldsymbol{\xi}_t)], \quad (9b)$$

$$V_{T+1}(x) = 0. \quad (9c)$$

281 Solving Problem (P<sub>θ</sub>) is equivalent to computing  $V_1(x_0)$ . However, for any  $x \in X_{T-3}$ , computing  $V_{T-2}(x)$   
 282 requires full knowledge of  $V_{T-1}$ . With continuous state, it is usually impossible. Indeed, for a given point  
 283  $x \in X_{T-2}$ , we can compute the exact value  $V_{T-1}(x)$  by solving  $|\Xi_{T-1}|$  deterministic problems defined in  
 284 eq. (9a). We cannot however obtain the value of  $V_{T-1}$  for all  $x \in X_{T-2}$ , as there are a non-finite number of  
 285 them, unless we are in a very specific case where the solution can be obtained analytically (*e.g.*, unconstrained  
 286 linear-quadratic case). Thus, even computing exactly the value of  $V_{T-2}$  at a single point is not possible.

287 Therefore, to accomodate for inexact value functions, we introduce the bellman operators which generalize  
 288 eq. (9) so the dynamic equations hold for any given function  $R$  approximating the cost-to-go  $V_{t+1}$ .

$$\text{Backward operators} \quad \begin{cases} \hat{\mathcal{B}}_t(R) : x, \xi \mapsto \min_{u_t \in \mathcal{U}_t(x, \xi)} L_t(x, u_t, \xi) + R(D_t(x, u_t, \xi)), \\ \mathcal{B}_t(R) : x \mapsto \mathbb{E} [\hat{\mathcal{B}}_t(R)(x, \boldsymbol{\xi}_t)]. \end{cases} \quad (10a)$$

$$\text{Forward operators} \quad \begin{cases} u_t^* \in \arg \min_{u_t \in \mathcal{U}_t(x, \xi)} L_t(x, u_t, \xi) + R(D_t(x, u_t, \xi)), \\ \hat{\mathcal{F}}_t(R) : x, \xi \mapsto D_t(x, u_t^*, \xi). \end{cases} \quad (10b)$$

289 The backward operator  $\mathcal{B}_t$ , defined in eq. (10a), returns an approximation, at a given state  $x$ , of the cost-to-go  
 290  $V_t$  starting from time  $t$ , given an approximation of the cost-to-go starting from time  $t+1$ . Assume that we  
 291 have an approximation  $\tilde{V}_{t+1}$  of the cost-to-go from  $t+1$ . We can select a finite number of point  $\{x_{t-1}^k\}_{k \in K}$  at  
 292 which we compute, through the Bellman Operator, an approximation of  $V_t$ . To get an approximation on  $X_{t-1}$ ,  
 293 we need to interpolate these values. Thus, given a discretization of each state space  $X_t$ , and an interpolation  
 294 method we can, recursively, compute an approximation of every cost-to-go function see algorithm 2.

295 The forward operator, defined in eq. (10b), returns the optimal next state  $x_t$ , given a starting state  $x$ , a  
 296 noise  $\xi$  and an approximation of the cost-to-go from  $t+1$ . Note that, in practice, computing  $\hat{\mathcal{B}}_t(R)(x, \xi)$

297 or  $\hat{\mathcal{F}}_t(R)(x, \xi)$  consists in solving the same deterministic problem. Nevertheless, if the backward operator is  
 298 well defined, the forward operator requires a choice if the optimal solution is not unique. To be completely  
 299 rigorous, we should say that a forward operator defines a selection of the optimal solution set.

---

**Algorithm 2:** Stochastic Dynamic Programming

---

```

1 Input :  $x_0$ , discretization grids  $\mathcal{X}_t^D$ , interpolation operator.
2 Output : approximated value function  $\tilde{V}_t$ 
3  $\tilde{V}_{T+1} = 0$ .
4 for  $t : T \rightarrow 1$  do
5   for  $x_{t-1}^D \in \mathcal{X}_{t-1}^D$  do
6     // We discretize  $X_t$ 
7     for  $\xi_t \in \Xi_t$  do
8       Solve the one-stage deterministic optimization problem:
9        $\tilde{V}_t(x_{t-1}^D, \xi_t) = \hat{\mathcal{B}}_t(\tilde{V}_{t+1})(x_{t-1}^D, \xi_t)$ .
10       $\tilde{V}_t(x_{t-1}^D) = \sum_{\xi_t \in \Xi_t} \pi_{\xi_t} \tilde{V}_t(x_{t-1}^D, \xi_t)$ ; // expected value
10   Define  $\tilde{V}_t$  for any  $x \in X_{t-1}$  by interpolation on  $\{(x_{t-1}^D, \tilde{V}_t(x_{t-1}^D))\}_{x_{t-1}^D \in \mathcal{X}_{t-1}^D}$ .
```

---

300

301 Through the stagewise independence assumption, Dynamic Programming ensures that we can compress  
 302 information, by looking for an optimal solution as a function of the current state instead of past noises. More  
 303 precisely, the noise-based policy  $\psi_{[T]}$ , defined in section 2.1, can be replaced by a sequence of functions  $\pi_{[T]}$ ,  
 304 called *state-based feedback*, such that each function  $\pi_t$  depends only on the current state and actual noise  
 305 realization  $\pi_t : X_{t-1} \times \Xi_t \mapsto U_t$ . For any approximated cost-to-go functions, we have a *state-based feedback*,  
 306 see eq. (10b). Further, any scenario  $\xi_{[T]}$ , yields a state trajectory:

$$x_t := \hat{\mathcal{F}}_t(\tilde{V}_t)(x_{t-1}, \xi_t) \quad \forall t \in [T], \quad \text{with } x_0 \text{ given.} \quad (11)$$

307 **Remark 4** (Curse of dimensionality). *Note that algorithm 2 solves  $O(T \cdot |X_t^\theta| \cdot |\Xi|)$  deterministic, one-stage,*  
 308 *problem. Thus, Dynamic Programming is a powerful tool as the multistage problem considers  $(|\Xi|^T)$  scenarios*  
 309 *and turns the exponential complexity in the horizon  $T$  into a linear one.*

310 *However, Dynamic Programming is limited by what is known as the curse of dimensionality. Indeed, we have*  
 311 *to solve, for each time step,  $|X_t^\theta| \cdot |\Xi|$  problem. A discretization of  $X_t$  usually requires a number of points*  
 312 *exponential in the dimension of  $X_t$ . Thus, in practice, Dynamic Programming cannot be used for state with*  
 313 *more than 5 dimensions.*

314 *Therefore, Dynamic Programming may be a solution for the operational problem presented in section 2.1, if*  
 315 *the state is reasonable, but is unrealistic for the strategic problem of section 2.2. Indeed, recall that not only*  
 316 *do we have all stocks  $x_t$ , but also design parameter  $\theta$  as state variables, and  $\theta$  can naturally be of dimension*  
 317 *24 for in-advance purchases.*

### 3.2 Trajectory following dynamic programming algorithms

To counteract the dynamic programming computational issues, a class of *Trajectory Following Dynamic Programming* (TFDP) algorithms (see [FL22] for a recent overview) has been developed. The crux of these algorithms is to iterate between forward phases that compute state trajectories, and backward phases that improve cost-to-go estimations.

In stochastic dynamic programming (section 3.1), we do only one backward phase, on a given grid discretizing the state space. The approximate cost-to-go function  $\tilde{V}_t$  is defined outside of this grid through an interpolation operator. By contrast, TFDP algorithms automatically, and iteratively, generate the trial points at which we estimate the value function. Further, to extend the definition of  $\tilde{V}$  outside of the trial points, they leverage problem-specific properties (*e.g.*, convexity, monotonicity, Lipschitz-continuity...) instead of generic interpolation operators.

More specifically, in a forward phase of a TFDP algorithm, a state trajectory is computed using the current cost-to-go estimations (see eq. (11)). Then in a backward phase, the cost-to-go estimations are refined around the state trajectory computed in the forward phase. These approximations are generally given as maximum of elementary functions called *cuts*. The TFDP algorithms depends on different types of cuts with various assumptions. For example, the well-known Stochastic Dual Dynamic Programming (SDDP) algorithm [PP91] defines classical linear cuts (Bender’s cut), obtained through linear programming duality, assuming the problem is convex and continuous. In line with SDDP, the Stochastic Dual Dynamic Integer Programming (SDDiP), [ZAS19], assumes that all state variables are binary, that there exists some continuous recourse ensuring relatively complete recourse assumption, and derives specific linear cuts. As one can always represent bounded integer variables, and approximate continuous variables, through binaries, the algorithm is theoretically applicable for a large number of settings, including ours, but is limited in practice as each step requires solving a MILP, and as the convergence is generally slow. Another algorithm, the Mixed Integer Dynamic Approximation Scheme (MIDAS) (see [PWB20]) assumes monotonicity of the cost-to-go functions, and uses piecewise constant cuts to approximate them. Finally, the Stochastic Lipschitz Dynamic Programming (see [ACF22]), simply assumes Lipschitz regularity of the cost-to-go functions, and uses reverse norm cuts. SDDiP, MIDAS and SLDP might be applicable to the industrial microgrid setting, but are generally slow to converge without additional, problem-specific, cuts.

We now present more precisely the SDDP algorithm, dedicated to convex continuous problem, and link it with the continuous relaxation of (P).

### 3.3 Stochastic Dual Dynamic Programming (SDDP)

For linear multistage stochastic problems with stagewise independence, the SDDP algorithm has proven to be an efficient tool, widely used in the energy community in particular for long term hydro-management. It is the most well known and studied example of TFDP algorithm, relying on Benders’ cut obtained through linear programming duality. In particular, it requires continuous variables.

We therefore consider the continuous relaxation of (P), denoted  $(P^r)$ . It is the same problem as Problem (1) but we assume all binary variables are in  $[0, 1]$  instead of  $\{0, 1\}$ , represented by  $\mathbf{u}_t^r \in \mathcal{U}_t^r(\mathbf{x}_{t-1}, \boldsymbol{\xi}_t)$ . Accordingly, we denote  $V_t^r$  the cost-to-go functions of  $(P^r)$ . They are linked through a Bellman backward operator  $\mathcal{B}_t^r$  defined by eq. (10a), but minimizing variables  $\mathbf{u}_t^r$  over  $\mathcal{U}_t^r$  instead of  $\mathcal{U}_t$ .

357

---

**Algorithm 3:** Stochastic Dual Dynamic Programming
 

---

```

// Initialization
1  $k = 0, V_t^{r,0} = LB.$ 
2 for  $k : 0, \dots$  do
3   Simulate a scenario  $\{\xi_t^k\}_{t \in [T]}$ .
   // Forward phase
4    $x_0^k = x_0.$ 
5   for  $t : 1 \rightarrow T$  do
6      $x_t^k = \hat{\mathcal{F}}_t^r(V_t^{r,k})(x_{t-1}^k, \xi_t^k).$ 
   // Backward phase
7    $V_{T+1}^{r,k} = 0$ 
8   for  $t : T \rightarrow 1$  do
9     // Cut computation
10    for  $\xi$  realization of  $\xi_t$  do
11      Solve  $\hat{\mathcal{B}}_t^r(V_{t+1}^{r,k})(x_{t-1}^k, \xi)$  and obtain coefficients  $\hat{\alpha}_t^k(\xi)$  and  $\hat{\beta}_t^k(\xi)$  such that:
12      
$$\hat{\alpha}_t^k(\xi)^T x + \hat{\beta}_t^k(\xi) \leq \hat{\mathcal{B}}_t^r(V_{t+1}^{r,k})(x, \xi) \quad \forall x.$$

13      Define  $\alpha_t^k = \mathbb{E}[\hat{\alpha}_t^k(\xi_t)]$  and  $\beta_t^k = \mathbb{E}[\hat{\beta}_t^k(\xi_t)].$ 
14      Define  $V_t^{r,k} : x \mapsto \max_{\kappa \leq k} (\alpha_t^\kappa^T x + \beta_t^\kappa).$ 

```

---

358 Leveraging the convexity of  $(P^r)$ , the SDDP algorithm 3, approximates each  $V_{t+1}^r$  as a maximum of affine  
 359 functions. More precisely, at iteration  $k$ , we first compute a trial trajectory  $(x_t^k)_{t \in [T]}$ . Then, in the back-  
 360 ward phase, we can compute  $\mathcal{B}_t^r(V_{t+1}^r)(x_{t-1}^{k+1})$  by solving  $|\Xi_t|$  linear problems. Linear programming duality  
 361 yields a sub-gradient of  $\mathcal{B}_t^r(V_{t+1}^{r,k+1})$  at  $x_{t-1}^{k+1}$ , which in turn defines an affine function which under estimates  
 362  $\mathcal{B}_t^r(V_{t+1}^{r,k+1}) \leq \mathcal{B}_t^r(V_{t+1}^r) = V_t^r$ . In particular, at iteration  $K$ , the approximate cost-to-go functions  $V_t^{r,K}$  are  
 363 given as a maximum of affine cuts, *i.e.*,

$$V_t^{r,K} : x \mapsto \min_x \gamma_k \tag{12a}$$

$$\gamma_k \geq \alpha_{t,k} + \beta_{t,k}^T x \quad \forall k \in [K]. \tag{12b}$$

364 Recall that, given any approximated cost-to-go function, the forward Bellman operator (see section 3.1),  
 365 produces a state-based feedback, satisfying in particular the binary constraints. Thus, it seems natural to  
 366 use the functions  $V_t^{r,K}$  as approximated cost-to-go, leading to a state-based feedback  $\pi_t^{\text{SDDP}}$ . The main limit  
 367 of this approach is that we are quite greedy in the way we repair the binary constraints. Indeed,  $V_t^{r,K}$  does  
 368 not account for binary constraints, and the forward operator only considers their impact on one time-step.  
 369 We illustrate the limit of this approach on the following toy example.

370 **Example 1** (Limit of continuous relaxation.). *Consider a production unit that produces two products  $j =$*   
 371  *$A, B$ , over  $T = 2$  time steps and one machine. The shared resource constraint, modeled through binary*

372 variables  $b_t^j$ , implies that we must decide which product to produce at  $t = 1$ , and which at  $t = 2$ . We look  
 373 for the production plan minimizing costs while satisfying a demand  $D = 1$  in both products at the end of the  
 374 horizon. The problem is formalized as follows.

$$\min \quad 3u_1^A + 2u_1^B + (u_2^A + u_2^B) \quad (13a)$$

$$\text{s.t.} \quad u_1^j + u_2^j \geq D \quad j = A, B, \quad (13b)$$

$$0 \leq u_t^j \leq 2b_t^j \quad j = A, B \quad t = 1, 2, \quad (13c)$$

$$b_t^A + b_t^B \leq 1 \quad t = 1, 2, \quad (13d)$$

$$b_t^j \in \{0, 1\}, \quad u_t^j \geq 0 \quad j = A, B \quad t = 1, 2. \quad (13e)$$

375 For the true problem, it is optimal to produce  $B$  in the first period and  $A$  in the second period, resulting in  
 376 an optimal cost of 3. However, in the continuous relaxation of Problem (13),  $b_t^j \in [0, 1]$ , and producing both  
 377 products at the same time is allowed. For instance, producing both product at time  $t = 2$  (with  $b_2^A = b_2^B = 0.5$ )  
 378 is admissible for the relaxed problem, yielding an optimal cost of 2.

379 Let  $V_2^r$  be the relaxed cost-to-go function given by:

$$V_2^r(u_1^A, u_1^B) = \min_{u_2^A, u_2^B, b_2^A, b_2^B} \quad u_2^A + u_2^B \quad (14a)$$

$$\text{s.t.} \quad u_1^j + u_2^j \geq D \quad j = A, B, \quad (14b)$$

$$0 \leq u_2^j \leq 2b_2^j \quad j = A, B, \quad (14c)$$

$$b_2^A + b_2^B \leq 1, \quad (14d)$$

$$b_2^j \geq 0, \quad u_2^j \geq 0 \quad j = A, B. \quad (14e)$$

380 Now, using the cost-to-go approximation  $V_2^r$  to determine optimal decisions of the mixed-integer problem at  
 381  $t = 1$ , we solve:

$$\min_{u_1^A, u_1^B, b_1^A, b_1^B} \quad 3u_1^A + 2u_1^B + V_2^r(u_1^A, u_1^B) \quad (15a)$$

$$\text{s.t.} \quad b_1^A + b_1^B \leq 1, \quad (15b)$$

$$0 \leq u_1^j \leq 2b_1^j \quad j = A, B, \quad (15c)$$

$$b_1^j \in \{0, 1\} \quad j = A, B. \quad (15d)$$

382 Note that, when solving Problem15, we make decisions at  $t = 1$  considering the cost impact at  $t = 2$ , but  
 383 not knowing what decisions are attached to this cost. In dynamic programming, infeasibility is supposed to be  
 384 propagated through costs: in this example, with the real cost-to-go function,  $V_2(0, 0) = +\infty$  and the solution  
 385  $u_1^A = u_1^B = 0$  would never be chosen. However, if we use the relaxed cost-to-go function, the infeasible solution  
 386  $u_1^A = u_1^B = 0$  has a cost  $0 + V_2^r(0, 0) = 2$  and is chosen rather than the optimal solution  $u_1^A = 1; u_1^B = 0$ ,  
 387 whose cost is  $2 + V_2^r(0, 1) = 3$ .

388 We address this limit in section 4.4 through a look-ahead heuristic that consider more than one time-step.

## 4 Heuristics for multistage problems

Let sum up. We have at hand: a stochastic algorithm with unreasonable computational time and a stochastic algorithm solving a continuous relaxation of our problem. Those are exact methods, but will not allow us to solve the problem in a satisfactory manner. Could we come up with heuristics taking into account uncertainties, using SDDP, and solving mixed-integer problems such as ours?

### 4.1 The Expected Value (EV) heuristic

One of the challenges is to take into account random variables. A common simplification consists in reducing the problem to its deterministic version, by replacing the random variable by our current best estimation.

However, we are not in a complete recourse setting, meaning that the deterministic production and energy plan computed is not necessarily admissible. Therefore, a first heuristic consists in computing the deterministic solution fixing part of control variables, and then, adjusting the rest of the variables to actual random variable realization. In our particular microgrid problem, we fix production variables and then adjust energy flows to actual solar energy produced. We opt for a simple strategy described in fig. 2.

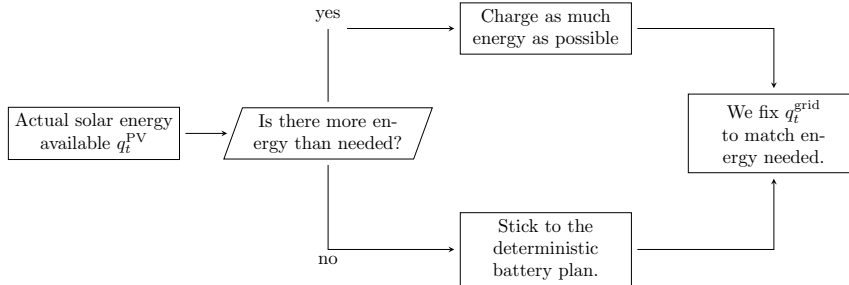


Figure 2: Corrected EV heuristic algorithmic scheme

This strategy has no flexibility, which is needed in a system subjected to uncertainties. It serves as a benchmark for stochastic solution. Note that this heuristic consists in repairing the solution of the Expected Value problem detailed in section 2.3.

### 4.2 Model Predictive Control

To add flexibility to the previous approach, we present the Model Predictive Control (MPC) approach, as a first adaptive approach. To use MPC we need some *forecast* methodology, that takes available information to predict the values of the random variables  $\{\xi_t\}_{t \in [T]}$ . The algorithm then consists in solving successive deterministic sub-problems (see algorithm 4). Step after step, it applies the decision of the first control obtained, reveals the realization of the next random variable, and recomputes all other decisions, updating forecasted values if possible.

As long as we can get a solution to the MILPs in reasonable time, MPC is an easy option to implement. However, this method yields no performance guarantee, and does not really take randomness into account, as the solution is computed for a single possible realization, but simply recomputes the solution as more

---

**Algorithm 4: Model Predictive Control**


---

1 **Input** :  $x_0$ , initial forecast  $\{\xi_\tau^0\}_{\tau \in [T]}$ .

2 **for**  $t : 1 \rightarrow T$  **do**

3     Update forecasted values  $\{\xi_\tau^0\}_{\tau \in [T]}$ .

$$u_t^*, \dots, u_T^* = \arg \min \quad L_t(x_{t-1}, u_t, \xi_t) + \sum_{\tau > t}^T L_\tau(x_{\tau-1}, u_\tau, \xi_\tau^0)$$

$$\text{s.t.} \quad x_\tau = D_\tau(x_{\tau-1}, u_\tau) \quad \forall \tau \in [t : T],$$

$$u_\tau \in \mathcal{U}_\tau(\mathbf{x}_{\tau-1}, \xi_\tau^0) \quad \forall \tau \in [t : T],$$

$$x_\tau \in X_\tau \quad \forall \tau \in [t : T].$$

$$x_t := D_t(x_{t-1}, u_t^*)$$


---

416 information becomes available. Consequently, the quality of the solution provided by MPC depends mainly  
417 on the quality of the forecasted values, the flexibility of the problem and the sensitivity of the problem  
418 to uncertainty. To quantify this sensitivity we can use the *Value of Stochastic Solution* (VSS) defined in  
419 section 2.3.

420 On a simple example, we show that MPC does not take into account the stochasticity of the problem, and  
421 can be largely suboptimal in case of asymmetry of the cost.

422 **Example 2.** Consider a production unit with  $J = 1$  product over  $T = 2$  time steps with random demand  $\mathbf{d}$   
423 at  $t = 2$ . We assume  $\mathbf{d} = 0$  with probability  $\frac{1}{2}$  and  $\mathbf{d} = 2$  with probability  $\frac{1}{2}$ , then  $\mathbb{E}[\mathbf{d}] = 1$ . We obtain the  
424 following problem:

$$\min \quad \mathbb{E}[u_1 + 10\mathbf{u}_2] \quad (16a)$$

$$\text{s.t.} \quad u_1 + \mathbf{u}_2 \geq \mathbf{d}, \quad (16b)$$

$$u_1, \mathbf{u}_2 \geq 0, \quad (16c)$$

$$\sigma(\mathbf{u}_2) \subset \sigma(\mathbf{d}). \quad (16d)$$

425 The MPC method returns solution  $u_1 = 1$ , and then adapts  $u_2 = 0$  if  $\mathbf{d} = 0$  and  $u_2 = 1$  if  $\mathbf{d} = 2$ , leading  
426 to an expected cost of  $V^{\psi_{EV}} = \frac{1}{2} + \frac{11}{2} = 6$ . On the other hand, as the recourse cost is high, the two-stage  
427 approach chooses to produce enough for both scenario at stage 1, i.e.,  $u_1 = 2$ , leading to an expected cost  
428  $V = 2$ .

429 Thus, the stochastic solution is better than the MPC solution, which is emphasized by the *Value of Stochastic*  
430 *solution*  $VSS = V^{\psi_{EV}} - V = 4$ . Further, the anticipative expected cost is  $V^{\psi_{ant}} = 0 + \frac{2}{2} = 1$ , so that the  
431 expected value of perfect information  $EVPI = V - V^{\psi_{ant}}$  is 1.

### 4.3 2-stage stochastic programming

433 The strategic design problem (P) balances the design cost  $I(\theta)$  and the operational cost  $V(x_0, \theta)$ . The  
434 2-stage stochastic programming consists in relaxing the non-anticipativity constraint for all operational de-

435 cisions, which amounts to approximating  $V(x_0, \theta)$  by  $V^{\phi_{ant}}(x_0, \theta)$ . Hence, the design problem becomes a  
 436 two-stage stochastic program, where the first stage decision is the strategic decision  $\theta$  and the recourse are  
 437 the operational decisions, *i.e.*,

$$\min_{\theta \in \Theta} I(\theta) + \mathbb{E} [\hat{V}^{\psi_{ant}}(x_0, \boldsymbol{\xi}_{[T]}; \theta)] \quad (17)$$

438 Recall that  $\hat{V}^{\psi_{ant}}(x_0, \boldsymbol{\xi}_{[T]}; \theta)$  is the optimal value of the operational problem knowing the full scenario real-  
 439 ization  $\boldsymbol{\xi}_{[T]}$ . Thus, computing the exact value of  $V^{\phi_{ant}}(x_0, \theta) = \mathbb{E} [\hat{V}^{\psi_{ant}}(x_0, \boldsymbol{\xi}_{[T]}; \theta)]$  would require to solve a  
 440 deterministic operational problem for each possible scenario  $\boldsymbol{\xi}_{[T]} \in \Omega$ . There is usually far too many scenario  
 441 to consider, thus, we resort to Sample Average Approximation, which is the 2-stage extension of Monte Carlo  
 442 methods (see remark 3). We draw  $S_{MC}$  scenarios, and approximate problem (17) as:

$$V^{2S_{MC}}(x_0) := \min_{\theta \in \Theta} \min_{(x_t^s, u_t^s)_{s \in [S_{MC}], t \in [T]}} I(\theta) + \sum_{s=1}^{S_{MC}} \frac{1}{S_{MC}} \left[ \sum_{t=1}^T L_t^\theta(x_{t-1}^s, u_t, \xi_t^s) \right] \quad (18a)$$

$$\text{s.t.} \quad x_t^s = D_t^\theta(x_{t-1}^s, u_t^s, \xi_t^s) \in X_t^\theta \quad \forall t \in [T], \forall s \in [S_{MC}], \quad (18b)$$

$$u_t^s \in \mathcal{U}_t^\theta(x_{t-1}^s, \xi_t^s) \subset U_t \quad \forall t \in [T], \forall s \in [S_{MC}]. \quad (18c)$$

443 All the approaches presented in this section up to this point relax non-anticipativity constraints but keep  
 444 binary constraints by solving MILPs. In section 3.3, we saw that SDDP solves problem (P) with non-  
 445 anticipativity constraints but relaxing binary constraints. We now look for a trade-off between information  
 446 relaxation and integrity relaxation.

#### 447 4.4 Look-ahead heuristic

448 Were the forward operator (see eq. (10b)) to have more visibility on the future variable possibilities (or  
 449 impossibilities), we have the intuition that the algorithm would perform better. Indeed, as it is defined, the  
 450 operator takes the best decision possible at  $t$  by optimizing a one-stage problem minimizing the current cost  
 451 at  $t$  plus an approximate cost-to-go function from  $t + 1$ . Details of the problem complexity are thus only  
 452 represented over one stage, and the impact of decision at time  $t$  on the next stage should all be taken into  
 453 account by the approximate cost-to-go function.

454 To have a better representation of the problem, we can consider  $\tau$ -stage problems with a final cost-to-go  
 455 function  $\tilde{V}_{t+\tau}$  instead of one-stage problems (with final cost-to-go function  $\tilde{V}_{t+1}$ ). More precisely we define a  
 456  $\tau$ -look-ahead Bellman operator  $\mathcal{B}_t^\tau$  as:

$$\hat{\mathcal{B}}_t^\tau(R) : x, \xi \mapsto \min_{u \in \mathcal{U}_t(x, \xi)} L_t(x, u, \xi) + \min_{(\mathbf{x}_{t'}, \mathbf{u}_{t'})_{t' \in [t+1:t+\tau]}} \mathbb{E} \left[ \sum_{t'=t+1}^{t+\tau} L_{t'}(\mathbf{x}_{t'-1}, \mathbf{u}_{t'}, \xi_{t'}) + R(\mathbf{x}_{t+\tau}) \right] \quad (19a)$$

$$\text{s.t.} \quad \mathbf{x}_t = D_t(x, u, \xi), \quad (19b)$$

$$\mathbf{x}_{t'} = D_{t'}(\mathbf{x}_{t'-1}, \mathbf{u}_{t'}, \xi_{t'}) \quad t' \in [t+1 : t+\tau], \quad (19c)$$

$$\mathbf{u}_{t'} \in \mathcal{U}_{t'}(\mathbf{x}_{t'-1}, \xi_{t'}) \quad t' \in [t+1 : t+\tau], \quad (19d)$$

$$\sigma(\mathbf{u}_{t'}) \subset \sigma(\xi_{[t+1:t']}) \quad t' \in [t+1 : t+\tau]. \quad (19e)$$

$$\mathcal{B}_t^\tau(R) : x \mapsto \mathbb{E} \left[ \hat{\mathcal{B}}_t^\tau(R)(x, \xi) \right] \quad (19f)$$

457 In this setting, the first-stage decisions are optimized knowing the impact they have on the next  $\tau - 1$  stages,  
 458 thanks to eqs. (19b) to (19e), and a cost-to-go function  $R$  from  $t + \tau + 1$ . However, the  $\tau$ -stage decisions are  
 459 taken without any visibility on the future except a given cost-to-go function. For this reason, when solving  
 460 each  $\tau$ -stage problem  $\hat{\mathcal{B}}_t^\tau(R_{t+\tau+1})(x_{t-1})$ , we only store the first-stage variables  $u_t$  and then move along to  
 461 the next sub-problem  $\hat{\mathcal{B}}_{t+1}^\tau(R_{t+\tau+2})(x_t)$ .

462 In a sense, we allow the operators to look ahead of time to choose their decision at  $t$ , and call this method the  
 463 look-ahead heuristic. We associate to the backward operator  $\hat{\mathcal{B}}_t^\tau$  a forward operator  $\hat{\mathcal{F}}_t^\tau(R) : X_{t-1} \times \Xi_t \rightarrow X_t$   
 464 which returns  $x_t^* = D_t(x, u, \xi)$  where  $u_t^*$  is an optimal value from the first min in (19a).

465 For clarity, we explicitly give the 2-look-ahead Bellman operator:

$$\hat{\mathcal{B}}_t^2(\mathcal{R})(x, \xi_t) = \min_{\substack{x_t, (x_{t+1}^s)_{s \in |\Xi_{t+1}|} \\ u_t, (u_{t+1}^s)_{s \in |\Xi_{t+1}|}}} L_t(x, u_t, \xi_t) + \sum_s \mathbb{P}(\xi_{t+1} = \xi_{t+1}^s) \left[ L_{t+1}(x_t, u_{t+1}^s, \xi_{t+1}^s) + \mathcal{R}(x_{t+1}^s) \right] \quad (20a)$$

$$\text{s.t.} \quad x_t = D_t(x, u_t, \xi_t), \\ x_{t+1}^s = D_{t+1}(x_t, u_{t+1}^s, \xi_{t+1}^s) \quad \forall s \in |\Xi_{t+1}|,$$

$$\mathcal{B}_t^2(\mathcal{R})(x) = \mathbb{E} \left[ \hat{\mathcal{B}}_t^2(\mathcal{R})(x, \xi_t) \right]. \quad (20b)$$

466 Note that this 2-look-ahead Bellman operator considers the exact cost at  $t$  and  $t + 1$ , and uses  $\mathcal{R}$  as an  
 467 estimation of expected cost-to-go from  $t + 2$  to  $T$ . In particular, due to the new information, we must  
 468 consider as many decisions  $u_{t+1}^s$  as there are realizations for random variable  $\xi_{t+1}$ .

469 Combining these new operators with the approximated cost-to-go functions computed by SDDP (see sec-  
 470 tion 3.3), we get a heuristic where the non-anticipativity constraints hold at any time, and the integrity  
 471 constraints are kept on  $\tau$  time steps. Unfortunately, increasing the look-ahead horizon *i.e.*,  $\tau$ , greatly in-  
 472 creases the complexity of the sub-problems we solve. For instance, with  $|\Omega_t| = 10$ , the backward operator  $\hat{\mathcal{B}}_t^\tau$   
 473 at  $t$  solves a problem with  $10^{\tau-1}$  times more variables than  $\hat{\mathcal{B}}_t$ .

## 474 5 Numerical results

475 We now present a study case from our industrial partner on which we evaluate the numerical methods  
 476 presented above. In section 5.1 we detail the study case, intraday results, given in section 5.2, show that the

477 MPC method is most adapted to our study, it is then used for the day-ahead problem in section 5.3 where  
 478 SDDP shows its advantages.

## 479 5.1 Study case

480 The problem ( $P_\theta$ ) introduced in section 1 and formalized in section 2.1 is motivated by a cement factory in  
 481 South Korea. We solve the problem for an hourly planning on one day, with  $T = 24$  time steps.

482 In the Republic of Korea, electricity rates are fixed for the industry and depend on different time slots and  
 483 the season. We took the rates given by the Korea Electricity Power Corporation website [Newa] and thus  
 484 obtain  $\{p_t^{\text{ID}}\}_{t \in [T]}$ . We consider that buying energy in advance is cheaper and fix the day-ahead rates to 90%  
 485 of the real-time rates.

486 Then we collect data for solar generation on [Newb]. Each dataset provides a year of hourly data on horizontal  
 487 radiations  $\mathbf{I}_t$ . Then the available solar energy is given as  $\mathbf{q}_t^{\text{PV}} = \eta_{\text{PV}} \mathcal{A}_{\text{cell}} n_{\text{cell}} C \mathbf{I}_t$  where  $\eta_{\text{PV}} = 0.16$  is the  
 488 solar panel efficiency factor,  $\mathcal{A}_{\text{cell}} = 0.0232 \text{ m}^2$  is the surface area of a cell,  $n_{\text{cell}} = 72$  is the number of cells  
 489 per panel,  $n_{\text{panel}}$  is the number of solar panels,  $C = \frac{1}{3600}$  is the conversion factor (here MJ to MWh) and  $\mathbf{I}_t$   
 490 in  $\text{W.m}^{-2}$  is the horizontal solar radiation.

491 From this data, we use a forecast algorithm to predict a daily solar energy generation: the model is trained on  
 492 the last 72 hours data to produce generation scenario over the next 24 hours. From this model we estimate,  
 493 at each time step  $t$ , 9 quantiles. We finally assume that the noise is stagewise independent, leading to  $9^T$   
 494 scenarios.

495 The factory owns  $I = 3$  mills and produces  $J = 3$  different cements (F32, F40, CPV). We consider binary  
 496 variables,  $(\mathbf{b}_t^{ij})_{t,i,j}$ , deciding which cement we produce on which mill at each given time and continuous  
 497 positive variables,  $(\mathbf{u}_t^{ij})_{t,i,j}$ , representing the quantity of cement produced.

$$\sum_j \mathbf{b}_t^{ij} \leq 1 \quad \forall i, t, \quad (21a)$$

$$\max_i \mathbf{b}_t^{i1} + \max_i \mathbf{b}_t^{i3} \leq 1 \quad \forall t, \quad (21b)$$

$$u_t^{i \min} \mathbf{b}_t^{ij} \leq \mathbf{u}_t^{ij} \leq u_t^{i \max} \mathbf{b}_t^{ij} \quad \forall i, j, t, \quad (21c)$$

$$\mathbf{q}_t^{\text{load}} = \sum_{i,j} \alpha^{i,j} \mathbf{u}_t^{ij} + \beta^{i,j} \mathbf{b}_t^{ij} \quad \forall t, \quad (21d)$$

$$\mathbf{b}_t^{ij} \in \{0, 1\} \quad \forall i, j, t. \quad (21e)$$

498 One mill can produce only one cement at a time (21a), and two of them (F40 and CPV) sharing resources,  
 499 cannot be produced simultaneously (21b). Bounds are given by the industrial on the production of each  
 500 cement for each mill (21c). An analysis on the factory's data leads us to model a mill's energy consumption,  
 501 on the range  $[u_t^{i \min}, u_t^{i \max}]$ , as an affine function of its cement production (21d).

502 The factory owns solar panels and a battery. Thus the energy supply is a mix of solar energy available  $\mathbf{q}_t^{\text{PV}}$ ,  
 503 of charge  $\phi_t^+$  and discharge  $\phi_t^-$  from the battery, and of energy bought from the main grid  $\mathbf{q}_t^{\text{grid}}$ . With these  
 504 elements, we need to ensure that the energy supply exceeds the energy demand  $\mathbf{q}_t^{\text{load}}$ , leading to the following  
 505 control constraints (representing eq. (1d)).

$$\mathbf{q}_t^{\text{PV}} + \phi_t^- - \phi_t^+ + \mathbf{q}_t^{\text{grid}} \geq \mathbf{q}_t^{\text{load}} \quad \forall t, \quad (21\text{f})$$

$$0 \leq \phi_t^+ \leq \frac{SOC_{max}}{4} \quad \forall t, \quad (21\text{g})$$

$$0 \leq \phi_t^- \leq \frac{SOC_{max}}{4} \quad \forall t, \quad (21\text{h})$$

$$0 \leq \mathbf{q}_t^{\text{grid}}. \quad (21\text{i})$$

506 Hence, the state of the system is described by the cements and battery stocks. The stocks of cement are  
 507 modeled with state variables  $(\mathbf{s}_t^j)_{t,j}$ . The demand at time  $t$  is modeled as a deterministic vector  $(d_t^j)_{j \in J}$ ,  
 508 estimated from factory production data. Initial stocks are empty. Then the stock variables follow dynamic  
 509 equations and bounding constraints given by

$$\mathbf{s}_t^j = \mathbf{s}_{t-1}^j - d_t^j + \sum_i \mathbf{u}_t^{ij} \quad \forall t, j, \quad (21\text{j})$$

$$\mathbf{s}_t^j \geq 0 \quad \forall t, j, \quad (21\text{k})$$

$$s_0^j = 0. \quad (21\text{l})$$

510 Indeed, for each time  $t$  and product  $j$ , the factory has to satisfy a demand  $d_t^j$ , which is ensured by the positivity  
 511 of stocks requirement (see eq. (21k)). Further, the quantity of energy stored in the battery,  $(SOC_t)_t$ , is also  
 512 modeled as a state variable:

$$SOC_t = SOC_{t-1} - \frac{1}{\rho} \phi_t^- + \rho \phi_t^+ \quad \forall t, \quad (21\text{m})$$

$$SOC_{min} \leq SOC_t \leq SOC_{max} \quad \forall t. \quad (21\text{n})$$

513 The battery size is proportional to the installed renewable capacity. We study three cases, where  $SOC_{max}$   
 514 is equal to the quantity of energy the solar panels can produce in 0.5, 3 or 6 hours. We also fix  $\phi_{max}^+$  and  
 515  $\phi_{max}^-$  to a quarter of the battery's capacity per time-step and the efficiency factor  $\rho$  to 0.9. Note that the  
 516 dynamics in eq. (1b) are here represented by eqs. (21j) and (21m), and the state constraints with eqs. (21k)  
 517 and (21n).

518 In section 2.2 we describe a strategic problem where a parameter  $\theta$  design the operational problem ( $P_\theta$ ).  
 519 Here we consider that energy can be either bought in advance (*e.g.*, on a day-ahead market), or in real  
 520 time through industrial contract with fixed price. Therefore, we have strategic continuous positive variables  
 521  $\theta = \{v_t^{\text{DA}}\}_{t \in [T]}$  which model the energy bought in advance to the main grid at price  $p_t^{\text{DA}}$ .

522 We can decompose the energy bought from the grid  $\{\mathbf{q}_t^{\text{grid}}\}_{t \in [T]}$  (see eq. (22a)) into energy bought in advance  
 523  $\{v_t^{\text{DA}}\}_{t \in [T]}$  plus energy bought during the day, represented by continuous positive variables  $\{\mathbf{v}_t^{\text{ID}}\}_{t \in [T]}$ .

$$\mathbf{q}_t^{\text{grid}} = v_t^{\text{DA}} + \mathbf{v}_t^{\text{ID}} \quad \forall t \in [T], \quad (22\text{a})$$

$$v_t^{\text{DA}}, \mathbf{v}_t^{\text{ID}} \geq 0 \quad \forall t \in [T]. \quad (22\text{b})$$

524

525 The production and energy plan thus aim to minimize the following stochastic optimization problem:

$$\min \quad \underbrace{\sum_t p_t^{\text{DA}} v_t^{\text{DA}}}_{I(\theta)} + \mathbb{E} \left[ \sum_t p_t^{\text{ID}} v_t^{\text{ID}} \right] \quad (23a)$$

$$s.t. \quad \text{eqs. (21) and (22),} \quad (23b)$$

$$\sigma(\mathbf{b}_t, \mathbf{u}_t, \mathbf{q}_t^{\text{grid}}, \phi_t^+, \phi_t^-) \subset \sigma(\boldsymbol{\xi}_{[t]}) \quad \forall t \in [T]. \quad (23c)$$

526

## 527 5.2 Intraday results

528 In this section we present and analyze the results obtained when solving problem (23) on instances in which  
 529 energy can only be bought in real time, which is equivalent to fixing  $\theta := 0$  *i.e.*,  $v_t^{\text{DA}} = 0$  for all  $t$ . Further,  
 530 we only consider a demand at the end of the day:  $d_t^j > 0$  only for  $t = T$ .

531 On a given day, for various renewable size ( $n_{\text{panel}} \in \{100, 200, 400, 600\}$ ) and battery sizing ( $SOC_{\text{max}}$  repre-  
 532 sents 0.5, 3 or 6 hours of maximum renewable production), we test the different strategies, evaluating them  
 533 over 500 common scenarios drawn from our statistical model. More precisely, we compare:

- 534 1. the elementary strategy, described in section 4.1, which solves the EV problem then adapt energy  
 535 variables following a deterministic procedure as noises are revealed;
- 536 2. the MPC strategy, see section 4.2, which consists in solving deterministic sub-problems at each stage,  
 537 with updated information, to adjust the solution trajectory accordingly;
- 538 3. and the Look-Ahead (LA), with  $\tau = 2$ , explained in section 4.4, strategy which computes a solution  
 539 with dynamic programming using an under-approximation of future costs given by SDDP.

540 To evaluate a strategy's performance over a given scenario, we define the *anticipative regret* of admissible  
 541 noise-based policy  $\pi$ , on a scenario  $\xi_{[T]}$ , as the relative gap between its cost and the anticipative lower bound:

$$AR^\pi(\xi_{[T]}) = \frac{\hat{V}^\pi(x_0, \xi_{[T]}; \theta) - \hat{V}^{\pi_{\text{ant}}}(x_0, \xi_{[T]}; \theta)}{|\hat{V}^{\pi_{\text{ant}}}(x_0, \xi_{[T]}; \theta)|}. \quad (24)$$

542 In fig. 3 we report the anticipative regret of each strategy. The results clearly show MPC's superiority on these  
 543 instances. On the one side, the EV heuristic yields unsatisfactory results in comparison to MPC: its expected  
 544 anticipative regret is always higher, and its expected cost as well. Further, except on the first column, which  
 545 corresponds to instances with few uncertainties (*i.e.*, a solar factor of 0.5), and the first instance of the  
 546 second column (a more uncertain instance but with a small battery), the EV heuristic performs worse than  
 547 the look-ahead heuristic. As uncertainties grow (from left to right), the costs of the EV heuristic are farther  
 548 and farther away from the anticipative lower bound, showing that a purely deterministic procedure is not  
 549 relevant for our problem.

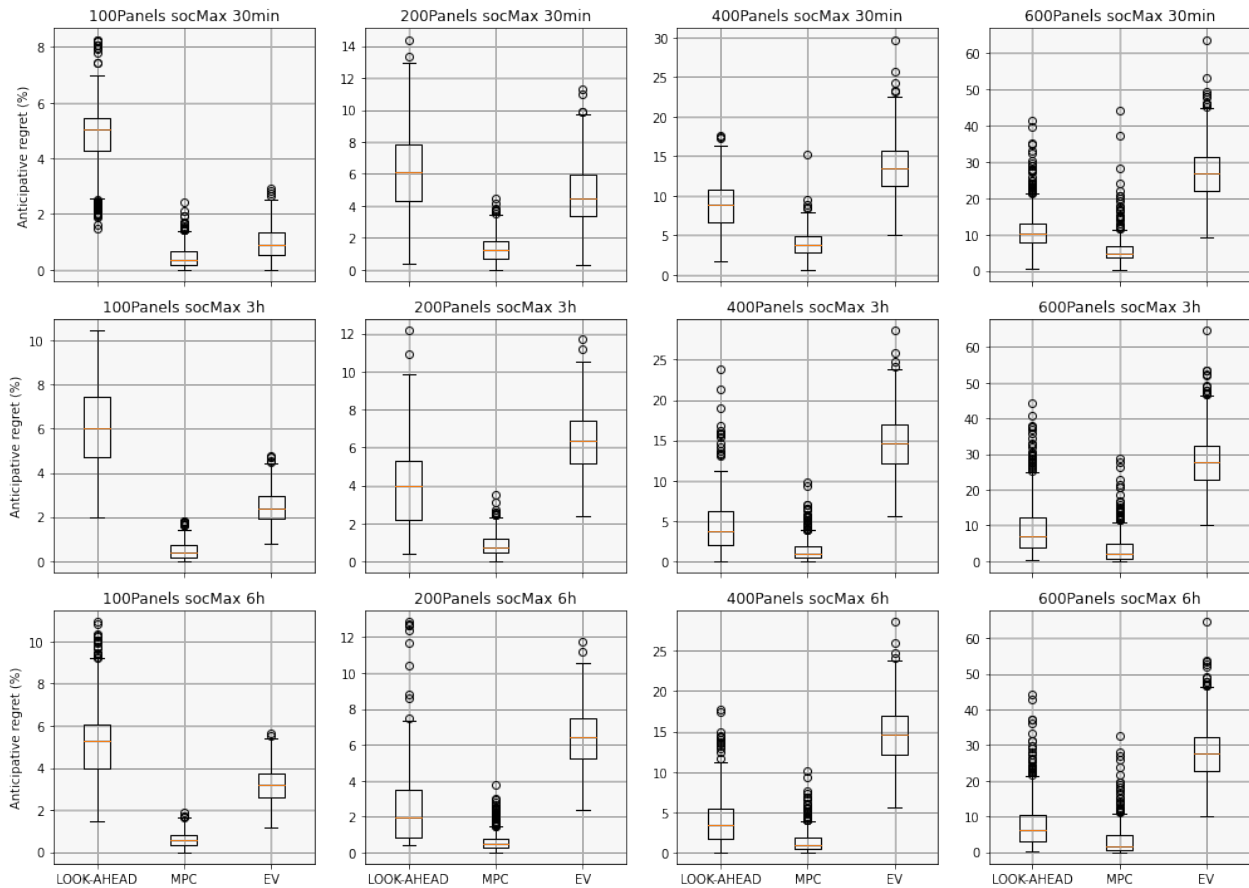


Figure 3: Anticipative regret (AR) in percentage for different solar park capacity and ESS capacity: increasing solar energy (and thus variability) from left to right, and increasing battery storage capacity (proportional to solar energy available) from top to bottom.

550 On the other side, the look-ahead heuristic, properly taking uncertainties into account with a stochastic  
 551 procedure, but relaxing some integrity constraint, does not perform as well as MPC. Indeed, the latter,  
 552 adjusting the solution trajectory to uncertainties, yields solutions close to their anticipative lower bound:  
 553 even for the most volatile instances (*i.e.*, the ones with a solar factor of 3, all on fig. 3's fourth column), the  
 554 anticipative regret is lower than 5% and in most cases insignificant. These performances can be explained  
 555 by the problem structure: the uncertainty source does not impact significantly future costs, in case of solar  
 556 energy variations at  $t$ , MPC foresees the cost impact and adapts accordingly. Furthermore, for industrial  
 557 problems with renewable generation, we confirm the necessity of installing an ESS to make the system flexible.  
 558 In fig. 4, we plot the optimal expected cost of the various methods on instances with growing ESS capacity.  
 559 Clearly, the expected optimal expected cost decreases as the ESS capacity increases, although the marginal  
 560 impact of the ESS capacity is decreasing.

561 Whereas MPC results are better, we call attention to its limits: on table 1 we can see that MPC takes longer

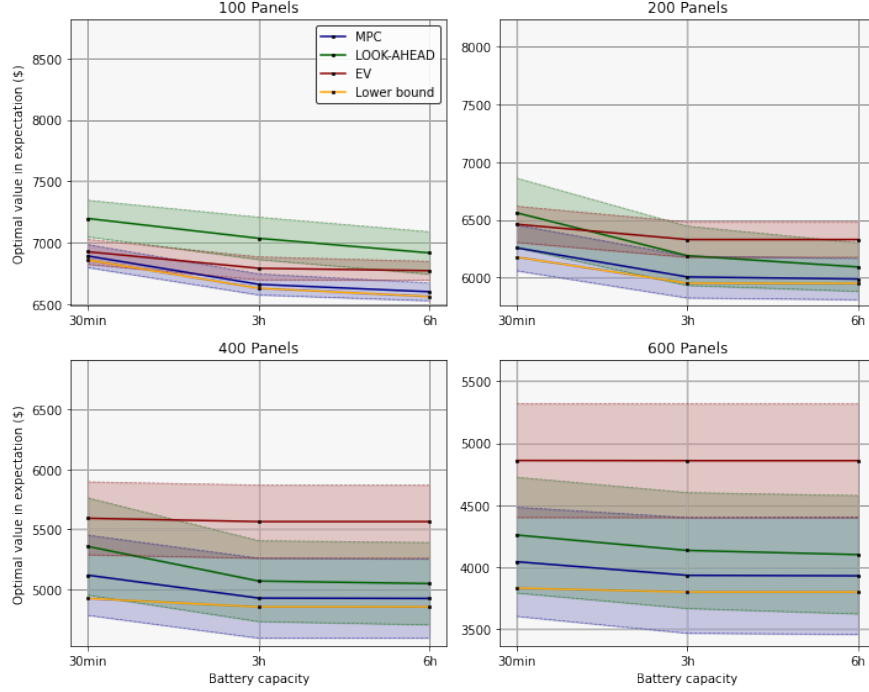


Figure 4: Expected value of strategies with 95% confidence interval.

562 in computation time than the look-ahead heuristic, even more so on instances with the most variability. On  
 563 these instances, it remains reasonable (a few seconds per problem at the most for an hour step time problem),  
 564 but with larger instances, more constraints, it could be unsuitable. Note that SDDP converges after only a  
 565 100 iterations, taking approximately 250s per instance.

### 566 5.3 Day-ahead results

567 We now consider the full problem 23 with strategic and operational decisions. In particular, we consider an  
 568 initial time step ( $t = 0$ ), where the industrial buys in advance energy quantities for the whole horizon. To  
 569 our knowledge, this type of contract does not exist yet in South Korea, but they could be interesting for the  
 570 regulator to encourage certain consumption scheme. It can also model the access to energy markets for large  
 571 consumers or consumers aggregated through virtual power plants. We fix the in advance prices at 90% of  
 572 intra-day prices.

573 The problem (P), see eq. (4), can be decomposed in two parts: first a strategical problem with variable  $\theta$ ,  
 574 here day-ahead energy purchases, and constraints  $\theta \in \Theta$ ; then an operational sub-problem ( $P_\theta$ ), see eq. (1),  
 575 parametrized by  $\theta$ . Our intuition is that a deterministic method might not be flexible enough because first  
 576 stage decisions impact the whole horizon. Note that the parametrized problem ( $P_\theta$ ) corresponds to the  
 577 intraday problem we solve in section 5.2. Here we saw that the most efficient method to solve ( $P_\theta$ ) is MPC,  
 578 in this section we determine through different methods the best strategical decision  $\theta^*$  and then run MPC  
 579 on the problem ( $P_{\theta^*}$ ).

$SOC_{max}$ $n_{panel}$	0.5h			3h			6h		
	MPC	L-A	SDDP	MPC	L-A	SDDP	MPC	L-A	SDDP
100	21	6.5	277	12	7.6	268	25	20	262
200	26	8.0	213	4.4	2.9	225	38	18	238
400	254	11	249	136	26	234	193	24	260
600	248	10	266	125	22	250	135	23	261

Table 1: Expected computation time (in seconds) for different solar park capacity and ESS capacity.

580 We assume that the demand is only positive at the end of the day  $d_T^j > 0$  and we test various renewable  
581 size ( $n_{panel} \in \{100, 200, 400, 600\}$ ). In section 5.2, we tested different battery size, and results showed that  
582 extending the battery capacity, to a certain point, improves costs and the system flexibility. Consequently,  
583 we now fix the battery capacity to 3 hours of maximum renewable production.

584 To optimize  $\theta$ , we test 3 methods evaluated over 1000 common scenarios:

- 585 1. the Expected Value strategy, see section 2.3, which solves a deterministic (P ) replacing random vari-  
586 ables by their expected value;
- 587 2. the 2–stage strategy, detailed in section 4.3, which takes the decision  $\theta$  minimizing the expected cost  
588 over  $S_{MC} = 10$  scenarios  $(\xi_{[T]}^s)_{s \in [S_{MC}]}$ . As  $S_{MC}$  is small, compared to the noise space, for computational  
589 reasons, we consider the median scenario with probability  $\frac{1}{2}$ ;
- 590 3. the SDDP strategy in section 3.3 solves  $(P^r)$ , the continuous relaxation of the problem, and yields a  
591 solution taking into consideration the uncertainties on the whole horizon, but relaxing integrity.

$n_{panel}$	OPT			AR (in %)		
	EV	2stage	SDDP	EV	2stage	SDDP
100	6067	<b>6023</b>	6038	1.6	<b>0.9</b>	1.1
200	5471	5483	<b>5451</b>	2.1	2.3	<b>1.7</b>
400	4552	4553	<b>4481</b>	4.2	4.2	<b>2.5</b>
600	3714	3691	<b>3641</b>	8.7	7.9	<b>6.7</b>

Table 2: Expected Cost (Opt) and Anticipative Regret (AR) of the solution obtained when finding  $\theta$  with the different methods (EV, 2–stage, SDDP); parametrizing the operational problem with this  $\theta$ ; then solving the parametrized operational problem with MPC.

$n_{panel}$	EV			2stage			SDDP		
	$I(\theta_{EV})$	$V(x_0; \theta_{EV})$	Opt	$I(\theta_{2S})$	$V(x_0; \theta_{2S})$	Opt	$I(\theta_r)$	$V(x_0; \theta_r)$	Opt
100	6002	65	6067	5830	193	<b>6023</b>	5659	379	6038
200	5369	102	5471	5123	360	5483	5102	349	<b>5451</b>
400	4357	195	4552	4073	480	4553	4043	438	<b>4481</b>
600	3394	320	3714	2965	726	3691	3094	548	<b>3642</b>

Table 3: We obtain  $\theta_{EV}$ ,  $\theta_{2S}$ ,  $\theta_r$  by solving the problem respectively with the EV strategy, 2–stage programming and SDDP; then we parametrize and solve the operational problem with MPC for each  $\theta$ .

592 From table 2, reporting simulated cost and anticipative regret of the various heuristics, we observe that,  
 593 except for the instance with less uncertainties (first line), the day-ahead energy purchases determined with  
 594 SDDP yield a lower expected cost as well as a lower anticipative regret than those determined with 2-stage  
 595 programming or the EV strategy. As uncertainties grow (from top to bottom on the table), the anticipative  
 596 regret increases and the gap between the AR of EV and the one of SDDP gets wider. Indeed, in the instance  
 597 with a solar factor equal to 1, the anticipative regret is 0.4% lower for SDDP whereas it is 2% lower for the  
 598 instance with more uncertainties (factor equal to 3).

599 On table 3 we separate design costs  $I(\theta)$  from operational costs  $V(x_0; \theta)$  for all instances solved. Whereas  
 600 the EV strategy essentially pays energy in advance, the two-stage and SDDP strategies have lower design  
 601 costs and buy more energy in real time. This can be explained because a stochastic approach is looking for a  
 602 trade-off between initial and recourse decisions. Assume that we have more energy than predicted, this extra  
 603 energy comes for free and we better not have bought too much energy in advance, forcing us to throw this  
 604 extra energy away (we can't charge the battery more than what is allowed). On the contrary, if we have less  
 605 energy than predicted, we must either adapt the production plan (which might be possible) or buy energy in  
 606 real time which is not that much more expensive than if we bought it in advance (110% of day-ahead prices).  
 607 Thus, we understand that in this problem, it is more efficient to underestimate the quantity of energy to buy  
 608 from the main grid, as we have more to gain if the solar realization exceeds its prediction than we have to  
 609 lose in the opposite case.

## 610 References

- 611 [ACF22] Shabbir Ahmed, Filipe Goulart Cabral, and Bernardo Freitas Paulo da Costa. “Stochastic Lips-  
 612 chitz Dynamic Programming”. In: *Mathematical Programming* 191.2 (Feb. 2022).
- 613 [Alo+22] Àlex Alonso-Travesset et al. “Optimization Models under Uncertainty in Distributed Generation  
 614 Systems: A Review”. In: *Energies* 15.5 (Mar. 2022).
- 615 [Bän+21] Kristian Bänisch et al. “Energy-Aware Decision Support Models in Production Environments: A  
 616 Systematic Literature Review”. In: *Computers & Industrial Engineering* 159 (Sept. 2021).
- 617 [BG16] Konstantin Biel and Christoph H. Glock. “Systematic Literature Review of Decision Support  
 618 Models for Energy-Efficient Production Planning”. In: *Computers & Industrial Engineering* 101  
 619 (Nov. 2016).
- 620 [Bie+18] Konstantin Biel et al. “Flow Shop Scheduling with Grid-Integrated Onsite Wind Power Using  
 621 Stochastic MILP”. In: *International Journal of Production Research* 56.5 (Mar. 2018).
- 622 [Bir82] John R. Birge. “The Value of the Stochastic Solution in Stochastic Linear Programs with Fixed  
 623 Recourse”. In: *Mathematical Programming* 24.1 (Dec. 1982).
- 624 [Bir85] John R. Birge. “Decomposition and Partitioning Methods for Multistage Stochastic Linear Pro-  
 625 grams”. In: *Operations Research* 33.5 (Oct. 1985).
- 626 [BL97] John Birge and François Louveaux. *Introduction to Stochastic Dynamic Programming*. Jan. 1997.
- 627 [Boh+20] Markus Bohlayer et al. “Energy-Intense Production-Inventory Planning with Participation in  
 628 Sequential Energy Markets”. In: *Applied Energy* 258 (Jan. 2020).
- 629 [Bro83] Gavin Brown. *Probability and measure*. 1983.
- 630 [FL22] Maël Forcier and Vincent Leclère. “Convergence of Trajectory Following Dynamic Programming  
 631 Algorithms for Multistage Stochastic Problems without Finite Support Assumptions”. In: *Opti-  
 632 mization Online* (2022).

- 633 [FMH21] Mohammad Fattahi, Hadi Mosadegh, and Aliakbar Hasani. “Sustainable Planning in Mining  
634 Supply Chains with Renewable Energy Integration: A Real-Life Case Study”. In: *Resources Policy*  
635 74 (Dec. 2021).
- 636 [FW18] Alireza Fazli Khalaf and Yong Wang. “Energy-Cost-Aware Flow Shop Scheduling Considering  
637 Intermittent Renewables, Energy Storage, and Real-Time Electricity Pricing”. In: *International*  
638 *Journal of Energy Research* 42.12 (Oct. 2018).
- 639 [Geo+21] Ramy Georgious et al. “Review on Energy Storage Systems in Microgrids”. In: *Electronics* 10.17  
640 (Jan. 2021).
- 641 [GFJ16] Mehdi Golari, Neng Fan, and Tongdan Jin. “Multistage Stochastic Optimization for Production-  
642 Inventory Planning with Intermittent Renewable Energy”. In: *Production and Operations Man-*  
643 *agement* 26 (Sept. 2016).
- 644 [HBF15] Ehsan Hajipour, Mokhtar Bozorg, and Mahmud Fotuhi-Firuzabad. “Stochastic Capacity Expans-  
645 ion Planning of Remote Microgrids With Wind Farms and Energy Storage”. In: *IEEE Trans-*  
646 *actions on Sustainable Energy* 6.2 (Apr. 2015).
- 647 [HK10] Julia L. Higle and Karl G. Kempf. “Production Planning Under Supply and Demand Uncertainty:  
648 A Stochastic Programming Approach”. In: *Stochastic Programming*. 2010.
- 649 [HPG18] Adam Hirsch, Yael Parag, and Josep Guerrero. *Microgrids A Review of Technologies, Key Drivers,*  
650 *and Outstanding Issues*. 2018.
- 651 [Ier+02] M. G. Ierapetritou et al. “Cost Minimization in an Energy-Intensive Plant Using Mathematical  
652 Programming Approaches”. In: *Industrial & Engineering Chemistry Research* 41.21 (Oct. 2002).
- 653 [Inta] International Energy Agency. *Global Energy Review 2021*. <https://www.iea.org/reports/global-energy-review-2021>. Online; Accessed: 2022-09-13.
- 654 [Intb] International Energy Agency. *Tracking Industry 2021*. <https://www.iea.org/reports/tracking-industry-2021>. Online; Accessed: 2022-09-13.
- 655 [Li+17] Binbin Li et al. “Toward Net-Zero Carbon Manufacturing Operations: An Onsite Renewables  
656 Solution”. In: *Journal of the Operational Research Society* 68.3 (Mar. 2017).
- 657 [Man60] Alan S. Manne. “On the Job-Shop Scheduling Problem.” In: *Operations Research* (1960).
- 658 [MP13] Joon-Yung Moon and Jinwoo Park. “Smart Production Scheduling with Time-Dependent and  
659 Machine-Dependent Electricity Cost by Considering Distributed Energy Resources and Energy  
660 Storage”. In: *International Journal of Production Research* 52 (Dec. 2013).
- 661 [Newa] New Energy and industrial technology Development Organization. <https://home.kepco.co.kr/kepco/EN/main.do>. Online; Accessed: 2022-09-13.
- 662 [Newb] New Energy and industrial technology Development Organization. <https://appww1.infoc.nedo.go.jp/appww/index.html?lang=2>. Online; Accessed: 2022-09-13.
- 663 [Pha+19] An Pham et al. “A Multi-Site Production and Microgrid Planning Model for Net-Zero Energy  
664 Operations”. In: *International Journal of Production Economics* 218 (Dec. 2019).
- 665 [PP91] M. V. F. Pereira and L. M. V. G. Pinto. “Multi-Stage Stochastic Optimization Applied to Energy  
666 Planning”. In: *Mathematical Programming* 52.1-3 (May 1991).
- 667 [PWB20] A. B. Philpott, F. Wahid, and J. F. Bonnans. “MIDAS: A Mixed Integer Dynamic Approximation  
668 Scheme”. In: *Mathematical Programming* 181.1 (May 2020).
- 669 [RFJ20] José Luis Ruiz Duarte, Neng Fan, and Tongdan Jin. “Multi-Process Production Scheduling with  
670 Variable Renewable Integration and Demand Response”. In: *European Journal of Operational*  
671 *Research* 281.1 (Feb. 2020).
- 672 [Sha06] Alexander Shapiro. “On Complexity of Multistage Stochastic Programs”. In: *Operations Research*  
673 *Letters* 34.1 (Jan. 2006).

- 678 [SML19] Hossein Shahandeh, Farough Motamed Nasab, and Zukui Li. “Multistage Stochastic Capacity  
679 Planning of Partially Upgraded Bitumen Production with Hybrid Solution Method”. In: *Opti-  
680 mization and Engineering* 20.4 (Dec. 2019).
- 681 [Tsi+21] Stamatis Tsianikas et al. “A Storage Expansion Planning Framework Using Reinforcement Learn-  
682 ing and Simulation-Based Optimization”. In: *Applied Energy* 290 (May 2021).
- 683 [WMG20] Shasha Wang, Scott J. Mason, and Harsha Gangammanavar. “Stochastic Optimization for Flow-  
684 Shop Scheduling with on-Site Renewable Energy Generation Using a Case in the United States”.  
685 In: *Computers & Industrial Engineering* 149 (Nov. 2020).
- 686 [ZAS19] Jikai Zou, Shabbir Ahmed, and Xu Andy Sun. “Stochastic Dual Dynamic Integer Programming”.  
687 In: *Mathematical Programming* 175.1-2 (May 2019).