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Juan Vargas, Cedric Thienot, Xavier Lagrange. Analytical Calculation of the User Threshold for the Switching between Unicast and Broadcast in Cellular Networks. NOMS 2022: IEEE/IFIP Network Operations and Management Symposium, Apr 2022, Budapest (HO), Hungary. 10.1109/noms54207.2022.9789770 . hal-03927310

**HAL Id: hal-03927310**

**<https://hal.science/hal-03927310>**

Submitted on 30 Jan 2023

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# Analytical Calculation of the User Threshold for the Switching between Unicast and Broadcast in Cellular Networks

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January 6, 2023

## Abstract

The ever increasing demand for high quality multimedia content in mobile networks requires the implementation of techniques for efficient spectrum management. Broadcast transmission is the preferred solution for scenarios in which the same content is transmitted to many users at the same time. The objective of this study is to propose an analytical method to calculate the user threshold to switch from unicast to broadcast in order to reduce radio resource utilization. We compute the probability of coverage of a given location in a Multicast Broadcast Single Frequency Network (MBSFN) area using tools from stochastic geometry. This expression is general and valid for a point not necessarily in the center of the MBSFN area. We use it to develop a method to calculate the user threshold analytically.

Index Terms - MBSFN, SC-PTM, Broadcast, Unicast, Mobile Networks, Stochastic Geometry.

## 1 Introduction

Mobile devices capable of displaying Ultra High-Definition (UHD) video and bandwidth-hungry services as 360-degree video and virtual/augmented reality, are expected to become more popular in the following years [1]. This arises the necessity for techniques to improve radio resource utilization efficiency since spectrum is limited and expensive. Broadcast transmissions improve efficiency by reducing bandwidth consumption, compared to unicast, in scenarios where many users demand access to the same multimedia content at the same time in the same geographical area. This is the case of mobile TV, video streaming of popular events and group communications in mission critical scenarios.

In unicast transmission mode User Equipments (UEs) receive a bitrate according to their channel quality thanks to link adaptation techniques. The better the channel quality, the higher the bitrate. If each user in a certain service area is demanding a different content, unicast transmission mode should be used. However, if many users demand the same information, a broadcast transmission mode may be more efficient. There exist two broadcast transmission modes in cellular networks, Multicast Broadcast Single Frequency Network (MBSFN) and Single-Cell Point-To-Multipoint (SC-PTM). In MBSFN mode, a group of synchronized Base Stations (BS) located in the same geographical area transmit the same information to a group of users. These BS form the MBSFN area. On the receivers side, the UEs combine the signals as if they were copies of the same signal generated by multipath propagation. The waveform configuration for the MBSFN transmission is selected by the BSs based on the coverage target. Therefore, the users with the lowest Signal-to-Interference-plus-Noise Ratio (SINR) in the broadcast group restrict the data rate of the transmission. On the other hand, SC-PTM refers to broadcast transmissions in one cell. There is no need for synchronization which simplifies deployment and configuration. However, SC-PTM transmissions do not benefit from interference reduction as MBSFN.

The problem we address in this paper is the switching between unicast and broadcast transmissions. There exist a number of users per cell in the service area from which the broadcast transmission becomes more efficient than unicast. This is called the user threshold [2]. Since the users are dynamic in terms of their location and content demands, the number of users demanding the same service in a geographical area changes with time. There exist an standardized mechanism to inform the Broadcast Multicast Service Center (BMSC) how many users are demanding a certain content, called Consumption Reporting [2]. However, the standard does not provide techniques to calculate the user threshold. In our previous work we proposed a model to calculate the user threshold and showed that it changes according to the characteristics of the scenario: BS density,

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beamforming configuration in unicast, size of the MBSFN area, etc. [3]. Nevertheless, these results were based on Monte Carlo simulations which are time consuming and not tractable. Thus, in this paper we consider a new approach based on stochastic geometry results.

Stochastic geometry refers to the study of random spatial patterns, the most important one being the Poisson Point Process (PPP). In cellular network analysis, modeling the location of Base Stations (BS) as a PPP enables stochastic geometry concepts and theorems to be used. This approach allows the development of mathematical expressions to estimate performance indicators as the probability of coverage or the data rate [4][5]. Furthermore, the results obtained when considering a PPP distribution for the BS are pessimistic compared to actual cellular network deployments [4]. This allows to guarantee a minimum performance level.

In this study we provide an analytical method to calculate the user threshold based on previous stochastic geometry results and theorems and our user threshold model. Particularly, we use the expression to calculate the coverage probability for a user in unicast mode developed in [4] and propose a new version of the equation to calculate the coverage probability for a user receiving in MBSFN mode presented in [5]. The new equation takes into consideration the location of the user. Then, we integrate these expressions with the user threshold model and propose a method to calculate it analytically.

The remainder of this paper is structured as follows. Section II presents the research work that set the basis of our study, section III presents the derivation of the location dependant probability of coverage in MBSFN mode. In section IV we present the method to calculate the user threshold analytically and simulation results are presented in section V. Finally, conclusions are presented in section VI.

## 2 Related Work

Researchers have developed expressions to calculate the probability of coverage of a user when receiving in unicast and MBSFN transmission modes under certain assumptions using stochastic geometry. Hereafter we present briefly the two findings that set the basis for our study.

### 2.1 Probability of coverage in unicast

Andrews, Baccelli and Ganti developed in [4] an expression to calculate the probability of coverage of a user in unicast mode. They consider a UE placed at the origin of the plane and base stations located according to a PPP of density  $\lambda$ . The path loss model is based on Okumura-Hata-Cost231 with fading. The path loss exponent is denoted as  $\alpha$  and the path loss factor as  $k$ . Rayleigh fading is modeled as an exponential random variable (r.v.)  $h$  with unit rate and shadowing is not considered. The probability of coverage is the SINR Complementary Cumulative Distribution Function (CCDF) such that

$$p_{c_{UC}}(T) = \mathbb{P}[S > T], \quad (1)$$

where  $S$  is the r.v. representing the SINR and  $T$  is the SINR threshold. Under these assumptions, the probability of coverage in unicast ( $p_{c_{UC}}$ ) is calculated as

$$p_{c_{UC}}(T) = \pi\lambda \int_0^\infty e^{-\pi\lambda v(1+\rho(T,\alpha))-TP_N v^{\alpha/2}} dv, \quad (2)$$

where

$$\rho(T, \alpha) = T^{2/\alpha} \int_{T^{-2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}} du, \quad (3)$$

where  $P_N = \frac{N}{P_{tx}k}$  is the normalized noise power.  $P_{tx}$  denotes the BS transmission power and  $N$  the noise power.

### 2.2 Probability of coverage in MBSFN

Most recently, Sahu, Chaurasia and Gupta found an expression to calculate the probability of coverage for a user receiving in MBSFN mode [5]. The propagation model is the same than in [4]. They consider PPP-distributed BS located in a circular surface of radius  $R$  and a UE located at the origin of the plane. The model assumes that all the BS belong to the MBSFN area but only those located at a distance shorter or equal to  $R_s$  provide useful signal power. BS at a distance larger than  $R_s$  generate interference.  $R_s$  is called the connectivity radius. Under this assumptions, the probability of coverage ( $p_{c_{SFN}}$ ) is calculated as

$$p_{c_{SFN}}(T) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{1}{t} \text{Im} \left[ e^{-jtTP_N} e^{-2\pi\lambda t^{2/\alpha} (M(t,T) + jN(t,T))/(2\alpha)} \right] dt, \quad (4)$$

where

$$M(t, T) = T^{2/\alpha} \left[ B\left(\frac{1}{\alpha}, -\frac{1}{\alpha} + 1; \frac{1}{1 + t^2 T^2 R_s^{-2\alpha}}\right) - B\left(\frac{1}{\alpha}, -\frac{1}{\alpha} + 1; \frac{1}{1 + t^2 T^2 R_s^{-2\alpha}}\right) \right] - B\left(\frac{1}{\alpha}, -\frac{1}{\alpha} + 1; \frac{1}{1 + t^2 R_s^{-2\alpha}}\right), \quad (5)$$

$$N(t, T) = T^{2/\alpha} \left[ B\left(\frac{1}{\alpha} + \frac{1}{2}, -\frac{1}{\alpha} + \frac{1}{2}; \frac{1}{1 + t^2 T^2 R_s^{-2\alpha}}\right) - B\left(\frac{1}{\alpha} + \frac{1}{2}, -\frac{1}{\alpha} + \frac{1}{2}; \frac{1}{1 + t^2 T^2 R_s^{-2\alpha}}\right) \right] + B\left(\frac{1}{\alpha} + \frac{1}{2}, -\frac{1}{\alpha} + \frac{1}{2}; \frac{1}{1 + t^2 R_s^{-2\alpha}}\right), \quad (6)$$

and  $B(x, y; z)$  is the incomplete Beta function which is defined as

$$B(x, y; z) = \int_0^z u^{x-1} (1-u)^{y-1} du. \quad (7)$$

This expression considers a user at the center of the MBSFN area. This is an ideal case in MBSFN since the BSs surrounding the user provide useful signal power but, in reality, we can find UEs located at the border of the MBSFN area. In that case, the interference, generated by the BSs located at a distance  $r > R_s$  from the origin, increases and the SINR for the user is lower.

### 3 Location dependant probability of coverage in MBSFN

In MBSFN transmission mode, the modulation and coding scheme (MCS), and therefore the data rate, are fixed based on the users in the broadcast group with the lowest SINR and the coverage target. Therefore, it is important to find an expression to calculate the probability of coverage in MBSFN mode for a UE located anywhere in the MBSFN area. This allows to extend the performance analysis, notably to consider the case of a user at the border of the MBSFN area.

We consider that the MBSFN area is formed by the BSs located at a distance  $r \leq R_s$  from the origin. The UE can be located at any distance  $\xi R_s$  from the origin, such that  $0 \leq \xi \leq 1$  as shown in Fig. 1. The model in [5] considers only the case  $\xi = 0$ . However, the assumption that the signals from all the BS inside the MBSFN area are useful for a user at the border ( $\xi = 1$ ) is valid for values of  $R_s < 2.5\text{km}$  when using the OFDM extended cyclic prefix. This is because  $c * T_{CP} = 5\text{ km}$ , where  $c$  denotes the speed of electromagnetic waves and  $T_{CP} = 16.67\text{ }\mu\text{s}$  the CP length. This length of  $R_s$  is useful for scenarios with a coverage area smaller than  $20\text{ km}^2$ .

We can also think of  $\xi R_s$  as the radius of a target service area. It is the size of the area where the users are located. In that case, UEs located at a distance  $\xi R_s$  from the origin have the lowest SINR.

As presented in [5], to calculate the probability of coverage of a user in a MBSFN network, we need to calculate the Laplace transform of the interference ( $I$ ) and the Laplace transform of the useful signal power ( $P$ ) such that

$$p_{c_{SFN}}(T) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{1}{t} \text{Im} [\mathcal{L}_I(jtT) e^{-jtTP_n} \mathcal{L}_P(-jt)] dt. \quad (8)$$

The objective is to calculate the probability of coverage in MBSFN mode for a user located at a distance  $\xi R_s$  from the origin. The Laplace transform of  $P$  is calculated as a function of the distance between a BS and the UE. The location of a BS can be represented as a vector  $x$  in Cartesian coordinates, such that  $\|x\| = r \leq R_s$ , see Fig. 1. When considering the user located at the origin of the plane,  $\|x\| = r$  is the BS-UE distance. In our case, we denote the UE location (vector) as  $y$  and the distance between the BS and the UE is  $\|x - y\|$ . Therefore, the Laplace transform of  $P$  can be calculated as

$$\mathcal{L}_P(s) = \exp \left( -\lambda \int_{\mathbb{R}^2} \left( 1 - \mathbb{E}_h \left[ e^{-sh\|x-y\|^{-\alpha}} \right] \right) dy \right). \quad (9)$$

To solve the expected value in (9) we use the moment generating function of the exponential r.v.  $h$  which is calculated as

$$\begin{aligned} M_h(a) &= \mathbb{E}[e^{ah}] \\ &= \frac{1}{1-a}. \end{aligned} \quad (10)$$

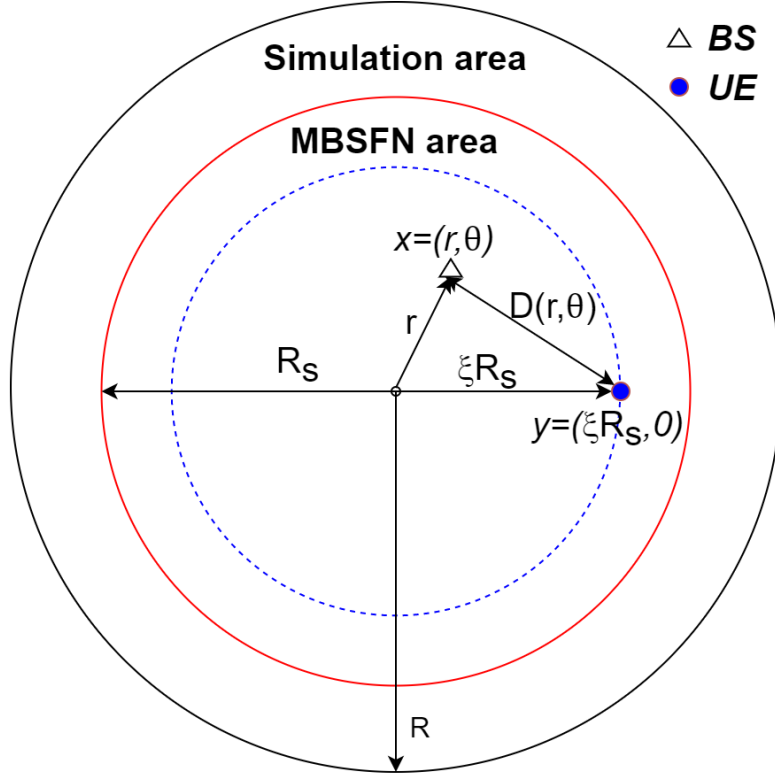


Figure 1: Extended model for the coverage probability in MBSFN mode

Afterwards, we express the integral over  $\mathbb{R}^2$  in polar coordinates. The distance between the UE and the BS,  $D(r, \theta)$  in Fig. 1, is given by

$$\begin{aligned} D(r, \theta) &= ||x - y|| \\ &= ((\xi R_s)^2 + r^2 - 2\xi R_s r \cos(\theta))^{\frac{1}{2}}. \end{aligned} \quad (11)$$

Then, the Laplace transform of  $P$  in polar coordinates is calculated as

$$\mathcal{L}_P(s) = \exp \left( -\lambda \int_0^{R_s} \int_0^{2\pi} \frac{sr}{s + D(r, \theta)^\alpha} d\theta dr \right). \quad (12)$$

Similarly, the Laplace transform for  $I$  can be calculated as

$$\mathcal{L}_I(s) = \exp \left( -\lambda \int_{R_s}^R \int_0^{2\pi} \frac{sr}{s + D(r, \theta)^\alpha} d\theta dr \right), \quad (13)$$

the only difference being the integration limits for  $r$  since interfering BS are at a distance  $r$  such that  $R_s < r < R$ . By replacing (12) and (13) in (8) with the respective value of  $s$ , we obtain the probability of coverage for a UE located in the point  $(\xi R_s, 0)$  in MBSFN mode.

## 4 User Threshold Calculation

In our previous work [3], we defined the user threshold as the number of users per cell from which broadcast transmission becomes more efficient than unicast.

### 4.1 Unicast to MBSFN

MBSFN transmission is more efficient than unicast in terms of resource utilization if

$$N_u W_{UE} > N_{SFN} W_{SFN}, \quad (14)$$

where  $N_u$  is the number of users demanding the broadcast service,  $W_{UE}$  is the average bandwidth used by each user,  $N_{SFN}$  is the average number of cells in the MBSFN area, and  $W_{SFN}$  is the bandwidth used by

each cell during MBSFN transmission.  $W_{UE}$  and  $W_{SFN}$  are expressed in terms of SINR based on the Shannon theorem. The capacity of a system ( $C$ ) in bits per second (bits/s) is calculated as  $C = W \log_2(1 + S)$ , where  $W$  denotes the system bandwidth and  $S$  the SINR. Therefore,

$$W_{UE} = C \mathbb{E} \left[ \frac{1}{\log_2(1 + S)} \middle| S \geq S_{mUC} \right] = C\Gamma_{UC}, \quad (15)$$

and

$$W_{SFN} = C \left[ \frac{1}{\log_2(1 + S_{mSFN})} \right] = C\Gamma_{SFN}, \quad (16)$$

where  $S_{mUC}$  and  $S_{mSFN}$  are the minimum SINR to be covered in each transmission mode.  $\Gamma_{UC}$  and  $\Gamma_{SFN}$  are the ratio between bandwidth and bitrate [Hz/bps], in each transmission mode. They are determined based on the coverage target. Notice that in unicast mode we take the expected value to account for link adaptation while in MBSFN mode, only the minimum SINR is considered. Then, the user threshold in units of UE/cell is calculated as

$$U_{TSFN} = \frac{\Gamma_{SFN}}{\Gamma_{UC}}. \quad (17)$$

To calculate  $\Gamma_{SFN}$  we use equation (8). We calculate numerically the value of SINR threshold ( $T$ ) at which the coverage probability ( $p_{cSFN}$ ) is equal to the target, typically 95% [6] [7]. The obtained SINR value is  $S_{mSFN}$ . On the other hand,  $\Gamma_{UC}$  can be calculated using the law of the unconscious statistician: the expected value of a measurable function of  $X$ ,  $g(X)$ , given that  $X$  has a probability density function (PDF)  $f(x)$ , is given by:

$$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)f(x) dx. \quad (18)$$

The PDF of  $X$ ,  $f(x)$ , can be obtained from its Cumulative Distribution Function (CDF),  $F(x)$ , as  $f(x) = \frac{d}{dx}F(x)$ . In our case  $X = S$  and  $F(S)$  can be calculated using (2) since  $F(S) = 1 - p_{cUC}(S)$ . Furthermore, the function  $g(S) = \frac{1}{\log_2(1+S)}$ , therefore

$$\Gamma_{UC} = \int_{S_{mUC}}^{S_{MUC}} g(S) \left( \frac{d}{dS}F(S) \right) dS, \quad (19)$$

where  $S_{MUC}$  is the maximum SINR value expected in unicast mode. Equation (19) has the form of a Riemann integral. Then, since  $F(S)$  is continuously differentiable, (19) can be transformed into a Riemann-Stieltjes integral such that

$$\Gamma_{UC} = \int_{S_{mUC}}^{S_{MUC}} g(S) dF(S). \quad (20)$$

Solving the Riemann-Stieltjes integral in (20) by parts, we can calculate  $\Gamma_{UC}$  as

$$\Gamma_{UC} = g(S_{MUC})F(S_{MUC}) - g(S_{mUC})F(S_{mUC}) - \int_{S_{mUC}}^{S_{MUC}} F(S) dg(S), \quad (21)$$

where

$$dg(S) = \frac{-\ln(2)}{(1+S)(\ln(S+1))^2} dS. \quad (22)$$

The value of  $S_{mUC}$  can be calculated numerically using (2) and the coverage target. Finally, the user threshold to switch from unicast to MBSFN is obtained by replacing (21) and the value of  $\Gamma_{SFN}$  in (17).

## 4.2 Unicast to SC-PTM

SC-PTM transmission is more efficient than unicast in terms of resource utilization if

$$N_u W_{UE} > N_{SC} W_{SC}, \quad (23)$$

where  $N_{SC}$  is the number of cells that transmit in SC-PTM mode and  $W_{SC}$ , the bandwidth used by each cell, is calculated as

$$W_{SC} = C \left[ \frac{1}{\log_2(1 + S_{mUC})} \right] = C\Gamma_{SC}. \quad (24)$$

Table 1: Simulation parameters

Parameter	Value
Simulation area radius ( $R$ )	10 km
System Bandwidth ( $W$ )	10 MHz
Carrier Frequency ( $f_c$ )	700 MHz
BS Transmission Power ( $P_{tx}$ )	46 dBm
Noise Power ( $N$ )	-95 dBm
Path Loss Exponent ( $\alpha$ )	3.76
Path Loss Factor ( $k$ )	$1.3804 * 10^{-12}$
Target coverage probability	95%

Therefore, we can calculate the User Threshold for SC-PTM as

$$U_{T_{SC}} = \frac{\Gamma_{SC}}{\Gamma_{UC}}. \quad (25)$$

As seen in (24), to calculate  $\Gamma_{SC}$  we need  $S_{m_{UC}}$  which can be obtained using (2) and the coverage target. On the other hand  $\Gamma_{UC}$  is calculated using (21).

## 5 Simulation Results

We analyse the performance of our model and compare it with the model in [5] for a 700-MHz system, which is typical for public safety communications [8]. More precisely, we study the impact of the variations of each parameter ( $\xi$ ,  $R_s$ ,  $\lambda$  and  $\alpha$ ) in the user threshold calculation. We developed both, numerical computation based on formulae of sections 2, 3 and 4 and Monte-Carlo simulations. We compare the results obtained with the two approaches.

Simulations use the 3GPP macro cell propagation model for urban area presented in [9], which is based on Okumura-Hata-Cost231, for a 700 MHz carrier frequency. The remaining simulation parameters are presented in table 1.

A comparison between the outage probability (SINR Cumulative Distribution Function) obtained using the analytical expression in (8) and Monte Carlo simulations is presented in Fig. 2. The first observation from these results is that the analytical and simulation curves match almost exactly. The 95% confidence intervals were calculated but they are not presented since they are small. This confirms the analytical expression in (8). Now, the most pessimistic scenario for MBSFN is  $\xi = 1$  because the interference generated by the cells outside the MBSFN area is high for a user at the border. Actually, it is possible that the BS providing the highest signal power is not in the MBSFN area. On the other hand,  $\xi = 0$  is the most optimistic scenario since the user is in the middle of the MBSFN area. It is the model considered in [5]. We can observe that for an outage probability of 0.05 and  $R_s = 2.5$  km, the SINR difference between the optimistic and pessimistic scenarios is 36 dB. A good compromise is to take the median value. Assuming that in a real deployment the UEs are uniformly distributed in the entire broadcast surface, half of the users are in the inner circle of radius  $R_s/\sqrt{2}$ . The SINR difference between the optimistic approach and the median approach for  $R_s = 2.5$  km is 6 dB. Furthermore, we consider two radius of MBSFN area,  $R_s = 0.6$  km and  $R_s = 2.5$  km. Notice that for the same  $\xi$  value, the outage probability decreases when increasing  $R_s$ . This is because more cells participate in the MBSFN transmission and therefore the SINR increases. However, this difference is smaller when  $\xi = 1$  since the interference from cells outside the broadcast area increases drastically in both cases.

Fig. 3 presents the ratio between bandwidth and bitrate in unicast mode ( $\Gamma_{UC}$ ) vs the path-loss exponent ( $\alpha$ ). In the case of Monte Carlo results, we present the 95% confidence interval for  $\Gamma_{UC}$  obtained for each  $\alpha$  value. Notice that the curve for the analytical calculation of  $\Gamma_{UC}$  is slightly below Monte Carlo results. This is because the expression in (2) considers an infinite surface and we consider  $R = 10$  km. However, this confirms the analytical expression in (21).

In SC-PTM mode, the user threshold does not depend on  $\lambda$ ,  $R_s$  or  $\xi$ . The user threshold vs the path-loss exponent ( $\alpha$ ) for two different values of  $\lambda$  is presented in Fig. 4. The number of UE per cell to switch from unicast to SC-PTM is between 8.61 and 8.85 in all cases. Therefore, if 9 UE/cell demand the same multimedia content, SC-PTM is more efficient than unicast.

The user threshold to switch from unicast to MBSFN vs  $\xi$  is presented in Fig. 5. These results were obtained using the analytical expressions in (2), (8) and (21). The coverage target is 95% and we set  $S_{M_{UC}} = 30$  dB. We consider two values for the BS density,  $\lambda = 4$  BS/km<sup>2</sup> and  $\lambda = 12$  BS/km<sup>2</sup>. The first observation is that for the same  $R_s$  value the user threshold is lower for the higher BS density. This is because the SINR provided by a MBSFN transmission increases with higher BS densities [5][3]. On the other hand, the probability of

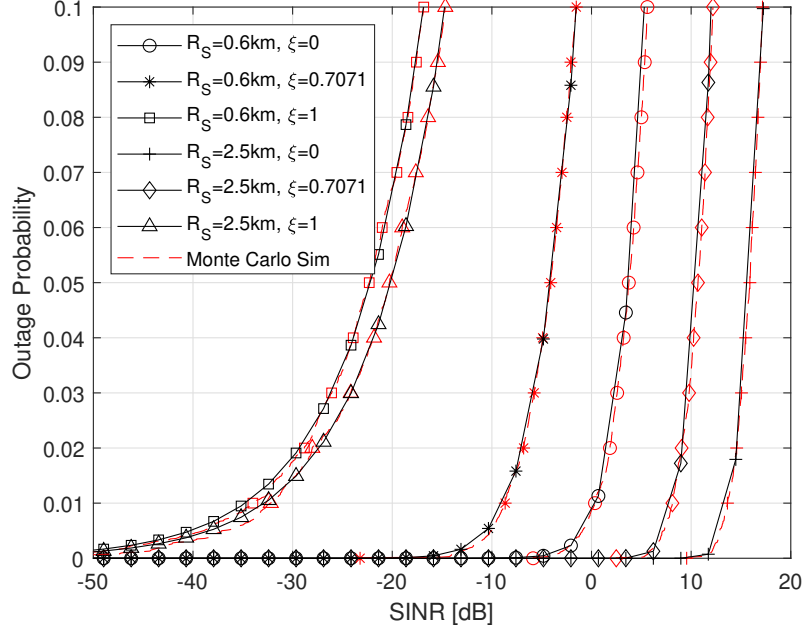


Figure 2: Outage probability for a user receiving in MBSFN mode considering different values of  $\xi$ ,  $R_s$  and  $\lambda = 12$  BS/km<sup>2</sup>. Curves obtained via the Analytical expression in (8) and Monte Carlo simulations.

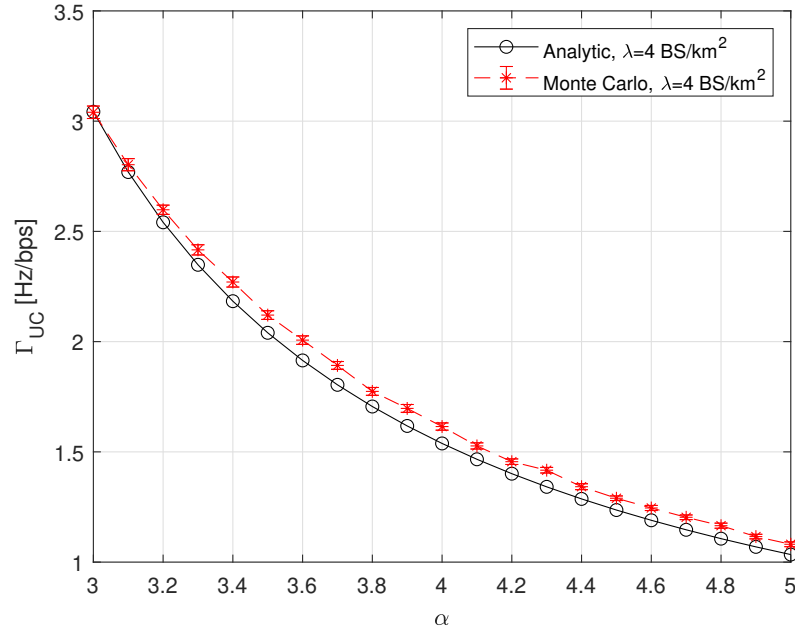


Figure 3:  $\Gamma_{UC}$  vs path-loss exponent  $\alpha$ . Analytical calculation vs Monte Carlo simulation.

coverage in unicast does not depend on the BS density [4]. Therefore, for the set of parameters considered in this study, the value of the user threshold depends on the MBSFN network configuration. Furthermore, a high user threshold means that many users are needed for MBSFN to be more efficient than unicast. Thus, when deploying a MBSFN network it is better to keep a low user threshold. In Fig. 5 we observe that the value of user threshold remains almost constant for  $R_s = 2.5$  km in the interval  $0 < \xi < 0.5$ , particularly for the higher BS density. As consequence, the expression (4) provided in [5] could be used if the radius of the MBSFN area is large and twice the radius of the target service area. A bigger MBSFN area would not drastically improve MBSFN performance. On the other hand, if a small MBSFN area is considered or  $\xi > 0.5$  or a higher precision in the user threshold calculation is required, the expression proposed in this paper should be used. Furthermore, deployments in which users are located at a distance larger than  $0.9R_s$  from the origin should be avoided since the user threshold increases drastically in this region and the MBSFN mode may never be activated. Finally,



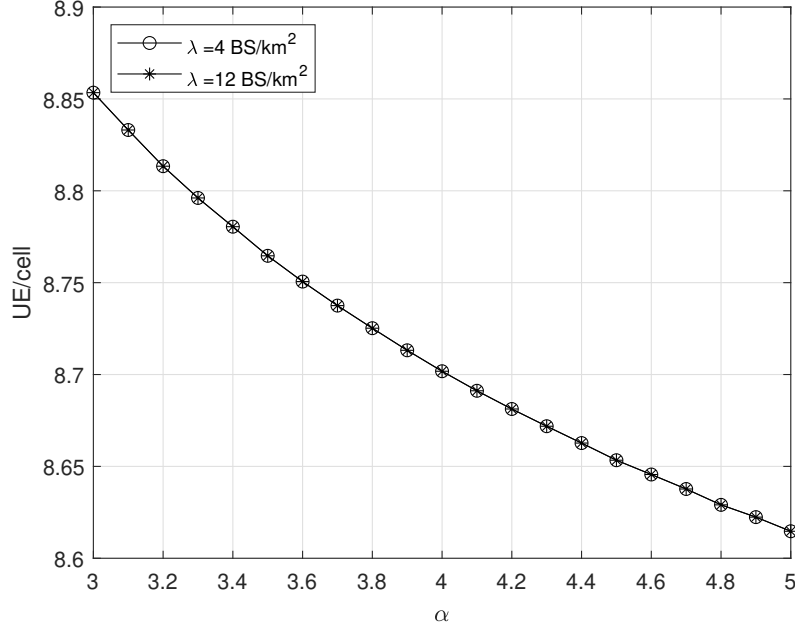


Figure 4: User threshold to switch from unicast to SC-PTM mode vs  $\alpha$  considering different  $\lambda$  values. Curves are obtained via the analytical expression in (25).

observe that SC-PTM is a better solution when the MBSFN area is small, the BS density is low and users are located close to the border of the MBSFN area.

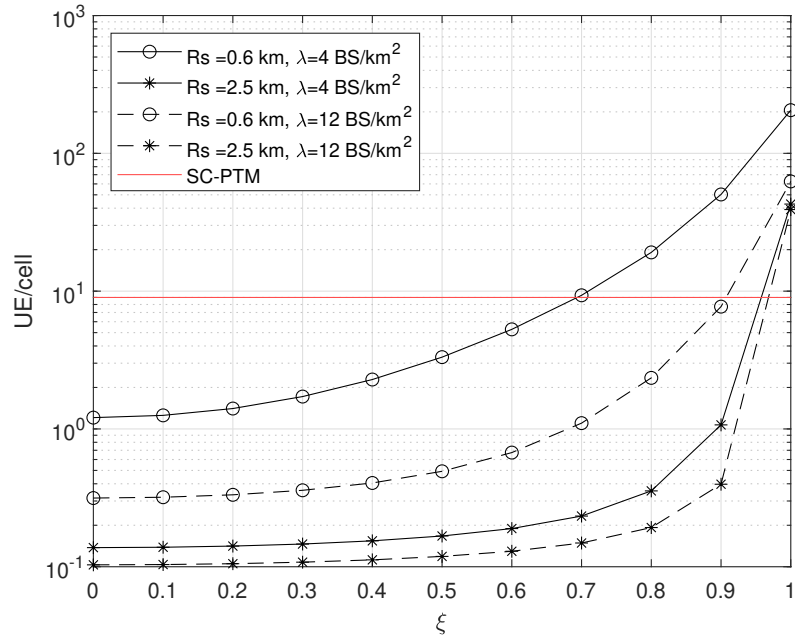


Figure 5: User threshold to switch from unicast to MBSFN mode vs  $\xi$  considering different  $R_s$  and  $\lambda$  values. Curves are obtained via the Analytical expression in (8).

In Fig. 6 we present the user threshold vs the radius of the MBSFN area ( $R_s$ ) for different values of  $\xi$  and  $\lambda$ . We appreciate that the value of  $\xi$  has a more important impact on the user threshold than the BS density. Observe that the user threshold is higher when  $\lambda = 12$  BS/km<sup>2</sup> and  $\xi = 0.9$  than when  $\lambda = 4$  BS/km<sup>2</sup> and  $\xi = 0.7071$ . This means that it is better to have an MBSFN area significantly larger than the service area with a low BS density than an MBSFN area with a similar size than the service area with a high BS density. However, for the same value of  $\xi$ , MBSFN does benefit from a higher BS density.

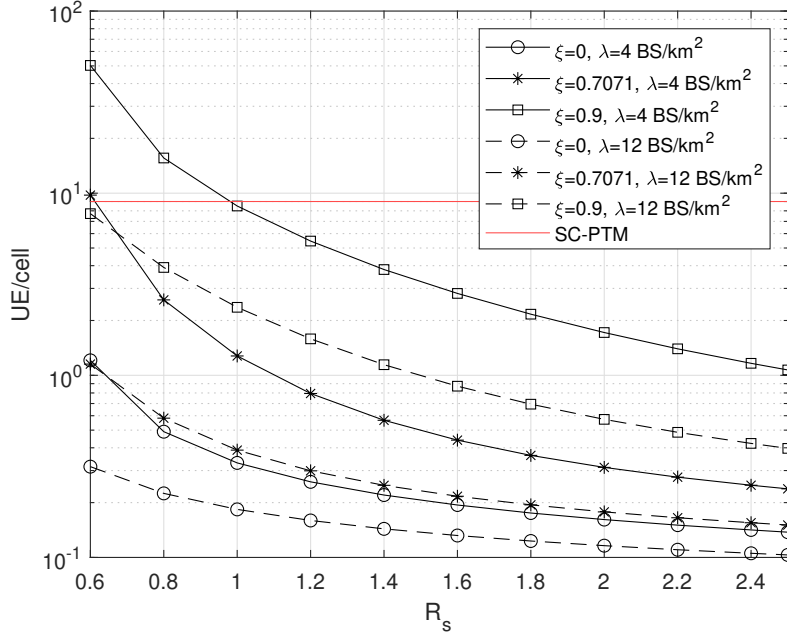


Figure 6: User threshold to switch from unicast to MBSFN mode vs  $R_s$  considering different  $\xi$  and  $\lambda$  values. Curves are obtained via the Analytical expression in (8).

## 6 CONCLUSION

This paper presents an analytical expression to calculate the probability of coverage for a user located at any point between the origin of the plane and the border of the MBSFN area when receiving in MBSFN mode. Furthermore, we use this expression and the formula for the probability of coverage in unicast mode, to develop an expression to calculate the user threshold to switch from unicast to MBSFN mode or SC-PTM mode.

Our approach can be used to make an accurate estimation of the user threshold. Either in real time by obtaining the location of the user further away from the origin (closer to the border of the MBSFN area) or beforehand by knowing the radius of the service area, where the users are expected to be. In both cases, the distance should be smaller than the radius of the MBSFN area ( $R_s$ ). As future work we intend to explore machine learning techniques and data from actual cellular networks deployments to calculate the user threshold.

## References

- [1] Cisco White Paper. *Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2017–2022*. White Paper. ENST Paris, 2019.
- [2] 3GPP. *Technical Specification Group Services and System Aspects; Multimedia Broadcast/Multicast Service (MBMS); Architecture and functional description*. TS 23.246. Version 16.1.0. Sept. 2019.
- [3] J. Vargas et al. “Broadcast-Multicast Single Frequency Network versus Unicast in Cellular Systems”. In: *2020 16th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob)*. Oct. 2020, pp. 1–6.
- [4] Jeffrey G. Andrews, Francois Baccelli, and Radha Krishna Ganti. “A Tractable Approach to Coverage and Rate in Cellular Networks”. In: *IEEE Transactions on Communications* 59.11 (Nov. 2011), pp. 3122–3134.
- [5] Reena Sahu, Kanchan K. Chaurasia, and Abhishek K. Gupta. “SINR and Rate Coverage of Broadcast Networks using Stochastic Geometry”. In: *2020 International Conference on Signal Processing and Communications (SPCOM)*. ISSN: 2474-915X. July 2020, pp. 1–5.
- [6] 3GPP. *Evolved Universal Terrestrial Radio Access (E-UTRA); Study on single-cell point-to-multipoint transmission for E-UTRA*. TR 36.890. Version 13.0.0. June 2015.
- [7] Letian Rong, Olfa Ben Haddada, and Salah-Eddine Elayoubi. “Analytical Analysis of the Coverage of a MBSFN OFDMA Network”. In: *IEEE GLOBECOM 2008 - 2008 IEEE Global Telecommunications Conference*. Nov. 2008, pp. 1–5.
- [8] Abhaykumar Kumbhar et al. “A Survey on Legacy and Emerging Technologies for Public Safety Communications”. In: *IEEE Communications Surveys Tutorials* 19.1 (2017), pp. 97–124.
- [9] 3GPP. *Evolved Universal Terrestrial Radio Access (E-UTRA); Radio Frequency (RF) system scenarios*. TR 36.942. Version 15.0.0. June 2018.