

Towards a unified view of unsupervised non-local methods for image denoising: the NL-Ridge approach

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Overview

We propose a **unified view** to reconcile **state-of-the-art unsupervised non-local denoisers**. We derive **NL-Ridge algorithm** which leverages local linear combinations of noisy similar patches.

- ▷ We show that non-local denoisers are characterized by the **family of functions** (f_Θ) used to process groups of similar noisy image patches.
- ▷ The optimal parameters Θ^* for each group are found by minimizing an approximation of the ℓ_2 **risk** via a two-step algorithm.
- ▷ NL-Ridge linearly combines noisy similar patches. Our closed-form aggregation weights are computed, in the second step, through **multivariate Ridge regressions**.
- ▷ NL-Ridge **outperforms** well established state-of-the-art unsupervised denoisers, including BM3D [1] and NL-Bayes [2], as well as recent unsupervised deep learning methods [5, 6, 7], while being simpler conceptually.

Parameter optimization

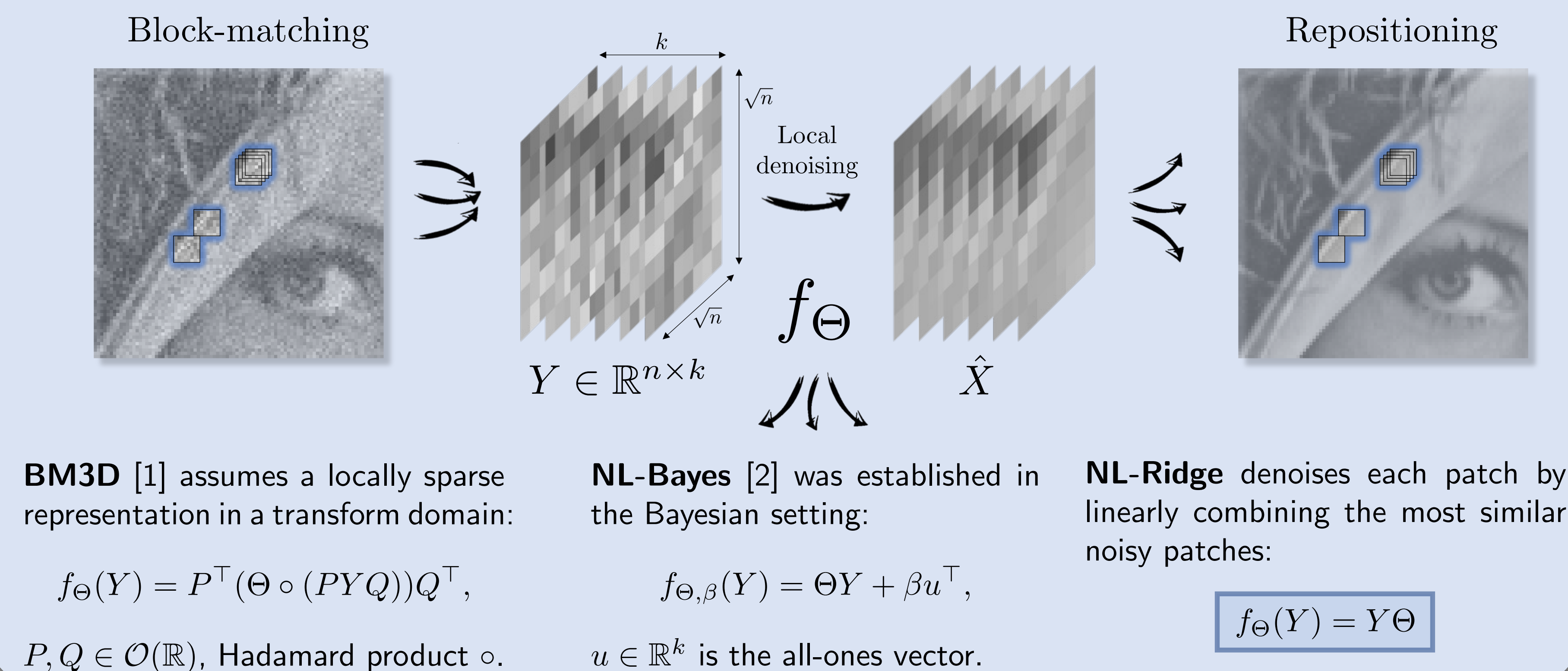
The optimal parameters Θ^* for the local denoiser f_Θ are found by minimizing the ℓ_2 **risk**:

$$R_\Theta(X) = \mathbb{E} \|f_\Theta(Y) - X\|_F^2.$$

As Θ^* requires the knowledge of X which is unknown, we propose the following two-step algorithm:

- Step 1: Minimize **Stein's unbiased risk estimate** (SURE) [3] to approximate Θ^* , reposition all the denoised patches and compute the \hat{I}_1 image aggregating estimators at each pixel.
- Step 2: Θ is improved via "**internal adaptation**" [4] with the pilot image \hat{I}_1 .

General principle of non-local denoisers



NL-Ridge

Step 1: SURE

Proposition 1

An unbiased estimate of the risk $R_\Theta(X)$ is **Stein's unbiased risk estimate** $\text{SURE}_\Theta(Y)$:

$$\begin{aligned} & -kn\sigma^2 + \|f_\Theta(Y) - Y\|_F^2 + 2\sigma^2 \text{div} f_\Theta(Y) \\ & = -kn\sigma^2 + \|Y\Theta - Y\|_F^2 + 2n\sigma^2 \text{tr}(\Theta) \end{aligned}$$

▷ Substituting this estimate for the risk $R_\Theta(X)$:

$$\arg \min_{\Theta} R_\Theta(X) \approx \arg \min_{\Theta} \text{SURE}_\Theta(Y) = \hat{\Theta}_1$$

$$\hat{\Theta}_1 = I_k - n\sigma^2(Y^\top Y)^{-1}$$

Step 2: Internal adaptation

Proposition 2

The quadratic risk $R_\Theta(X)$ is:

$$R_\Theta(X) = \|X\Theta - X\|_F^2 + n\sigma^2\|\Theta\|_F^2$$

minimized for (**multivariate Ridge regression**):

$$\Theta^* = (X^\top X + n\sigma^2 I_k)^{-1} X^\top X$$

▷ Substituting \hat{X}_1 for X in the risk expression:

$$\arg \min_{\Theta} R_\Theta(X) \approx \arg \min_{\Theta} R_\Theta(\hat{X}_1) = \hat{\Theta}_2$$

$$\hat{\Theta}_2 = (\hat{X}_1^\top \hat{X}_1 + n\sigma^2 I_k)^{-1} \hat{X}_1^\top \hat{X}_1$$

Results

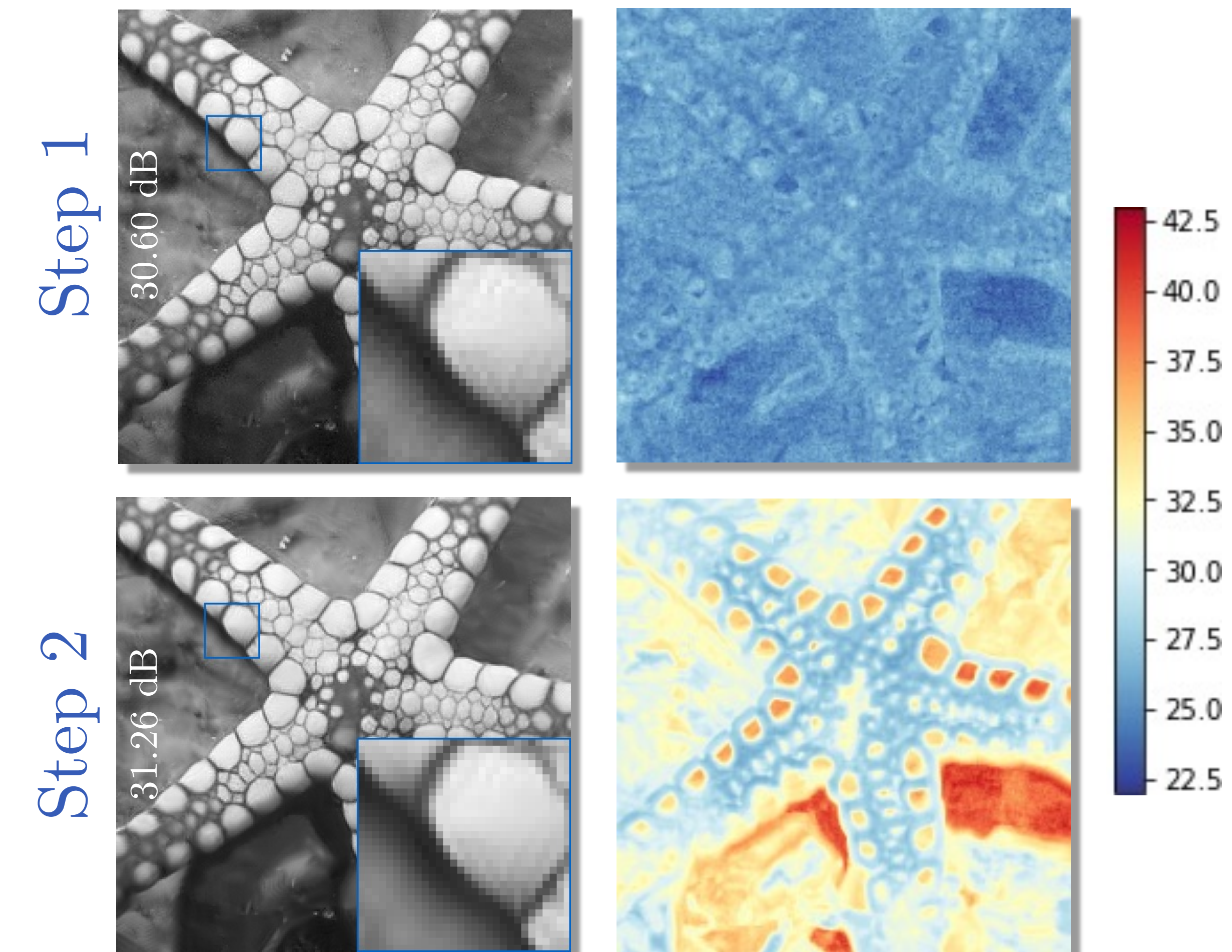


Fig. 2: Denoised images after each step ($\sigma = 15$) and PSNR colormaps (in dB) of denoised similarity matrices associated with each noisy patch.

Methods		Set12			BSD68			Urban100		
		Noisy	24.61 / 20.17 / 14.15		24.61 / 20.17 / 14.15			24.61 / 20.17 / 14.15		
Unsupervised	Traditional	BM3D	32.37 / 29.97 / 26.72		31.07 / 28.57 / 25.62			32.35 / 29.70 / 25.95		
		NL-Bayes	32.25 / 29.88 / 26.45		31.16 / 28.70 / 25.58			31.96 / 29.34 / 25.56		
		NL-Ridge	<u>32.46</u> / <u>30.00</u> / <u>26.73</u>		<u>31.20</u> / <u>28.67</u> / <u>25.67</u>			<u>32.53</u> / <u>29.90</u> / <u>26.29</u>		
	Deep learning	GBNL-Ridge	<u>32.71</u> / <u>30.24</u> / <u>26.86</u>		<u>31.42</u> / <u>28.87</u> / <u>25.78</u>			<u>33.00</u> / <u>30.37</u> / <u>26.49</u>		
		WNNM	32.70 / <u>30.26</u> / <u>27.05</u>		31.37 / 28.83 / <u>25.87</u>			32.97 / <u>30.39</u> / <u>26.83</u>		
Supervised	Deep learning	DIP	30.12 / 27.54 / 24.67		28.83 / 26.59 / 24.13		- / - / -	- / - / -		
		Noise2Self	31.01 / 28.64 / 25.30		29.46 / 27.72 / 24.77		- / - / -	- / - / -		
		Self2Self	<u>32.07</u> / <u>30.02</u> / <u>26.49</u>		<u>30.62</u> / <u>28.60</u> / <u>25.70</u>		- / - / -	- / - / -		
		DnCNN	32.86 / 30.44 / 27.18		31.73 / 29.23 / 26.23			32.68 / 29.97 / 26.28		
		FFDnet	32.75 / 30.43 / 27.32		31.63 / 29.19 / 26.29			32.43 / 29.92 / 26.52		
		LIDIA	32.85 / 30.41 / 27.19		31.62 / 29.11 / 26.17			32.80 / 30.12 / 26.51		

Table. 1: PSNR (dB) results on various datasets corrupted with Gaussian noise ($\sigma = 15, 25, 50$). Best among each category is in bold. Best among each subcategory is underlined.



Fig. 1: PSNR results on Barbara corrupted with additive white Gaussian noise ($\sigma = 20$).

References

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