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## Model Free Control = PI Control

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#### Abstract

This note shows that Model Free Control strategy is simply a PI control strategy. The note shows also the influence of the estimated values of the plant gain upon the performance of the control system.

*Keywords:* PI control, Model free control, Youla-Kucera observer, continuous time, discrete-time

#### List of Acronyms

MFC - Model Free Control PI - Proportional + Integral Controller YK - Youla Kucera

#### 1. Continuous Time Case

In MFC [4] it is assumed that the output of the plant to be controlled an be expressed as:

$$y'(t) = F(t) + \alpha u(t) \tag{1}$$

where u(t) is the plant input, y(t) is the output and F(t) is a disturbance. In fact, it is assumed that the output of the plant can be represented by an integrator with the gain  $\alpha$ subject to an input disturbance F(t). In MFC papers [..] F(t) is termed "ultra local model" of the plant. However F(t) is not a model of the plant because this dependence on u(t) is not taken into account. In MFC, F(t) is theoretically estimated through

$$\hat{F}(t) = y'(t) - \alpha u(t) \tag{2}$$

and the theoretical control law is:

$$u(t) = -[\hat{F}(t) - y^{*'}(t) + K_p(y(t) - y^{*}(t))]g$$
(3)

where  $y^*$  is the reference and  $g = 1/\alpha$ .

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#### 2. Discrete time case

Unfortunately in practice y'(t) can not be exactly obtained and one should use time discretization. In addition the gain  $\alpha$  is not exactly known and should be replaced by its estimation. Why it is relatively easy to get an order of magnitude of  $\alpha$ , getting a very precise estimation is difficult without using plant model identification techniques. Therefore one has to take into account for building the control, that one has just an estimation of  $\alpha$ which will be denoted  $\hat{\alpha}$ .

The plant equation becomes in discrete time [1]:

$$y(t+1) = y(t) + F(t+1) + \alpha u(t)$$
(4)

where F in discrete time is the continuous time F multiplied by  $T_s$  ( $=FT_s$ ) where  $T_s$  is the sampling period. Eq. (2) becomes in discrete time:

$$\hat{F}(t) = y(t) - y(t-1) - \hat{\alpha}u(t-1) = (1 - q^{-1})y(t) - \hat{\alpha}q^{-1}u(t)$$
(5)

This is just a Youla - Kucera disturbance observer used in many control schemes and in particular in adaptive regulation (see [5], [2]). The control law given in Eq. (3) becomes in discrete time:

$$u(t) = -[\hat{F}(t) - (y^*(t+1) - y^*(t))]\hat{g} - K_p[y(t) - y^*(t)]\hat{g}$$
(6)

where  $y^*$  is the reference which is known one step ahead (we know at time t the value which we want to attain at t + 1) and  $\hat{g} = \frac{1}{\hat{\alpha}}$ . Combining Eq. (5) with Eq. (6) the diagram of the control scheme shown in Fig. 1 is obtained.



Figure 1: Model Free Control Diagram

Taking into account Fig.1, Eq.(6) can be rewritten as:

$$S(q^{-1})u(t) = T(q^{-1})y^*(t+1) - R(q^{-1})y(t)$$
(7)
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where:

$$S(q^{-1}) = 1 - q^{-1} = 1 + s_1 q^{-1}$$
(8)

$$T(q^{-1}) = [1 - (1 - K_p)q^{-1}]\hat{g} = t_0 + t_1 q^{-1}$$
(9)

$$R(q^{-1}) = [1 + K_p - q^{-1}]\hat{g} = r_0 + r_1 q^{-1}$$
(10)

The controller is a digital PI controller in R-S-T form (see [3]) The corresponding equivalent diagram is shown in Fig.2.

The poles of the closed loop are given by:



Figure 2: Model Free Control Diagram - R-S-T form.

$$P = AS + BR = (1 - q^{-1})(1 - q^{-1}) + \alpha q^{-1}[1 + K_p - q^{-1}]\hat{g} = 1 - [2 - \hat{g}\alpha(1 + K_p)]q^{-1} - (1 - \hat{g}\alpha)q^{-2}$$
(11)

where  $A = 1 - q^{-1}$ ,  $B = \alpha q^{-1}$  and the transfer operator from y\* to y is given by:

$$S_{yy*} = \frac{TB}{P} = \frac{[1 - (1 - K_p)q^{-1}]\hat{g}\alpha q^{-1}}{1 - [2 - \hat{g}\alpha(1 + K_p)]q^{-1} - (1 - \hat{g}\alpha)q^{-2}}$$
(12)

The I/O steady state gain is given by

$$S_{yy*}(1) = \frac{K_P \hat{g}\alpha}{2 - \hat{g}\alpha(2 + K_p)} \tag{13}$$

For  $\hat{g} = \frac{1}{\alpha}$  the I/O transfer operator has a unit steady state gain. Of course it is possible to obtain a unit gain also for  $\hat{\alpha} \neq \alpha$  by taking  $T(q^{-1}) = R(q^{-1})$  or T = R(1).

In the case  $\hat{g} = \frac{1}{\alpha}$  the poles of the closed lop are given by:

$$P = 1 - (1 - K_p)q^{-1}; \quad for \quad \hat{g} = 1/\alpha \tag{14}$$

the stability of the closed loop is assured for  $0 < K_p < 2$  (or  $0 < K_p < 1$  if one wants a pole with an a periodic response) while this is not true when  $\hat{\alpha} \neq \alpha$ . In this later case  $K_p$  has to be chosen such the P given in Eq.(11) has its roots inside the unit circle (one can use well known stability criteria). It is also possible solving a Bezout equation to find the values of the coefficients of  $R(q^{-1})$  for given desired closed loop poles [3].

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