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► **To cite this version:**

| Ioan Doré Landau. Model Free Control = PI Control. 2023. hal-03923535v1

HAL Id: hal-03923535

<https://hal.science/hal-03923535v1>

Preprint submitted on 4 Jan 2023 (v1), last revised 21 Feb 2023 (v3)

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Model Free Control = PI Control

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Abstract

This note shows that Model Free Control strategy is simply a PI control strategy. The note shows also the influence of the estimated values of the plant gain upon the performance of the control system.

Keywords: PI control, Model free control, Youla-Kucera observer, continuous time, discrete-time

List of Acronyms

MFC - Model Free Control

PI - Proportional + Integral Controller

YK - Youla Kucera

1. Continuous Time Case

In MFC [4] it is assumed that the output of the plant to be controlled can be expressed as:

$$y'(t) = F(t) + \alpha u(t) \quad (1)$$

where $u(t)$ is the plant input, $y(t)$ is the output and $F(t)$ is a disturbance. In fact, it is assumed that the output of the plant can be represented by an integrator with the gain α subject to an input disturbance $F(t)$. In MFC papers [...] $F(t)$ is termed "ultra local model" of the plant. However $F(t)$ is not a model of the plant because the dependence on $u(t)$ is not taken into account. In MFC, $F(t)$ is theoretically estimated through

$$\hat{F}(t) = y'(t) - \alpha u(t) \quad (2)$$

and the theoretical control law is:

$$u(t) = -[\hat{F}(t) - y^{*'}(t) + K_p(y(t) - y^*(t))]g \quad (3)$$

where y^* is the reference and $g = 1/\alpha$.

2. Discrete time case

Unfortunately in practice $y'(t)$ can not be exactly obtained and one should use time discretization. In addition the gain α is not exactly known and should be replaced by its estimation. Why it is relatively easy to get an order of magnitude of α , getting a very precise estimation is difficult without using plant model identification techniques. Therefore one has to take into account for building the control, that one has just an estimation of α which will be denoted $\hat{\alpha}$.

The plant equation becomes in discrete time [1] :

$$y(t+1) = y(t) + F(t+1) + \alpha u(t) \quad (4)$$

where F in discrete time is the continuous time F multiplied by T_s ($= FT_s$) where T_s is the sampling period. Eq. (2) becomes in discrete time:

$$\hat{F}(t) = y(t) - y(t-1) - \hat{\alpha}u(t-1) = (1 - q^{-1})y(t) - \hat{\alpha}q^{-1}u(t) \quad (5)$$

This is just a Youla - Kucera disturbance observer used in many control schemes and in particular in adaptive regulation (see [5], [2]). The control law given in Eq. (3) becomes in discrete time:

$$u(t) = -[\hat{F}(t) - (y^*(t+1) - y^*(t))]\hat{g} - K_p[y(t) - y^*(t)]\hat{g} \quad (6)$$

where y^* is the reference which is known one step ahead (we know at time t the value which we want to attain at $t+1$) and $\hat{g} = \frac{1}{\hat{\alpha}}$. Combining Eq. (5) with Eq. (6) the diagram of the control scheme shown in Fig. 1 is obtained.

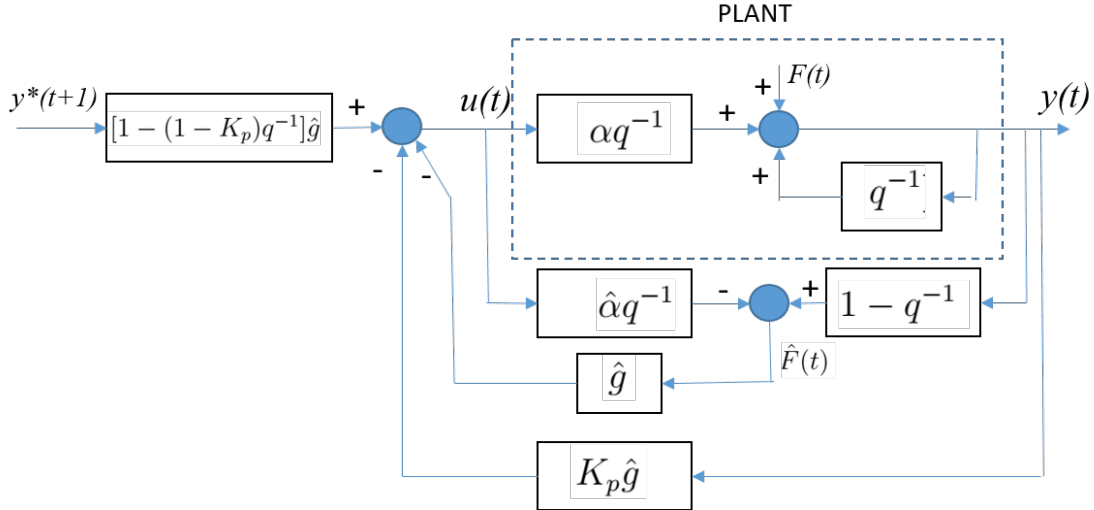


Figure 1: Model Free Control Diagram

Taking into account Fig.1, Eq.(6) can be rewritten as:

$$S(q^{-1})u(t) = T(q^{-1})y^*(t+1) - R(q^{-1})y(t) \quad (7)$$

where:

$$S(q^{-1}) = 1 - q^{-1} = 1 + s_1 q^{-1} \quad (8)$$

$$T(q^{-1}) = [1 - (1 - K_p)q^{-1}]\hat{g} = t_0 + t_1 q^{-1} \quad (9)$$

$$R(q^{-1}) = [1 + K_p - q^{-1}]\hat{g} = r_0 + r_1 q^{-1} \quad (10)$$

The controller is a digital PI controller in R-S-T form (see [3]) The corresponding equivalent diagram is shown in Fig.2.

The poles of the closed loop are given by:

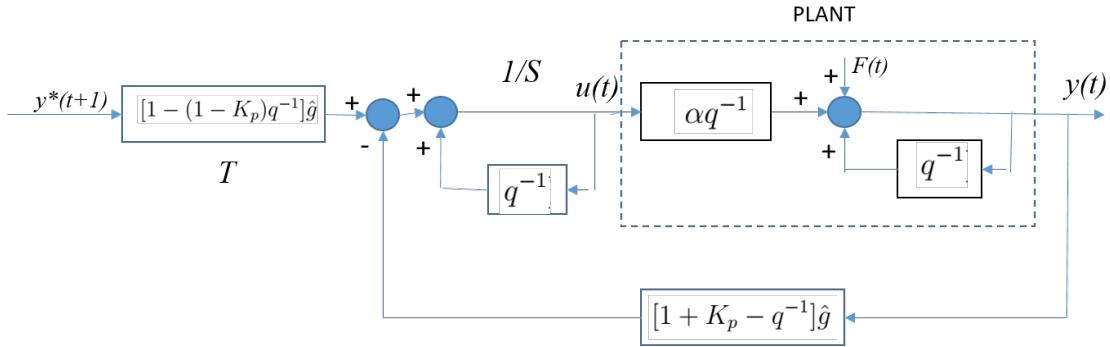


Figure 2: Model Free Control Diagram - R-S-T form.

$$P = AS + BR = (1 - q^{-1})(1 - q^{-1}) + \alpha q^{-1}[1 + K_p - q^{-1}]\hat{g} = 1 - [2 - \hat{g}\alpha(1 + K_p)]q^{-1} - (1 - \hat{g}\alpha)q^{-2} \quad (11)$$

where $A = 1 - q^{-1}$, $B = \alpha q^{-1}$ and the transfer operator from y^* to y is given by:

$$S_{yy^*} = \frac{TB}{P} = \frac{[1 - (1 - K_p)q^{-1}]\hat{g}\alpha q^{-1}}{1 - [2 - \hat{g}\alpha(1 + K_p)]q^{-1} - (1 - \hat{g}\alpha)q^{-2}} \quad (12)$$

The I/O steady state gain is given by

$$S_{yy^*}(1) = \frac{K_p \hat{g} \alpha}{2 - \hat{g} \alpha (2 + K_p)} \quad (13)$$

For $\hat{g} = \frac{1}{\alpha}$ the I/O transfer operator has a unit steady state gain. Of course it is possible to obtain a unit gain also for $\hat{a} \neq \alpha$ by taking $T(q^{-1}) = R(q^{-1})$ or $T = R(1)$.

In the case $\hat{g} = \frac{1}{\alpha}$ the poles of the closed loop are given by:

$$P = 1 - (1 - K_p)q^{-1}; \quad \text{for } \hat{g} = 1/\alpha \quad (14)$$

the stability of the closed loop is assured for $0 < K_p < 2$ (or $0 < K_p < 1$ if one wants a pole with an a periodic response) while this is not true when $\hat{a} \neq \alpha$. In this later case K_p has to be chosen such the P given in Eq.(11) has its roots inside the unit circle (one can use well known stability criteria). It is also possible solving a Bezout equation to find the values of the coefficients of $R(q^{-1})$ for given desired closed loop poles [3].

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