

Robust Computation of Stress Intensity factor within the eXtended Finite Element Method Context.

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Outline

Convergence analysis of the XFEM method.

- Influence of the integration Scheme
- Influence of the Enrichment zone on convergence and conditionnement.
- Higher order.

Stress Intensity factor computation.

- 4 methods compared on 3D bench mark.
- Convergence results.

3D examples.

Convergence

Stress Intensity
Factors

3D Examples

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X-FEM for crack

$$\begin{aligned} \mathbf{u}(\mathbf{x}) = & \sum_{I \in \mathcal{N}} N_I(\mathbf{x}) \mathbf{u}_I + \sum_{I \in \mathcal{N}_{\text{crack}}} N_I(\mathbf{x}) H(\text{ls}_n(\mathbf{x})) \mathbf{a}_I \\ & + \sum_{I \in \mathcal{N}_{\text{front}}} \sum_{\alpha} N_I(\mathbf{x}) F_{\alpha}(\text{ls}_n(\mathbf{x}), \text{ls}_t(\mathbf{x})) \mathbf{a}_{I\alpha} \end{aligned}$$

$$[F_{\alpha}] = \left[\sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \frac{\theta}{2} \sin \theta, \sqrt{r} \cos \frac{\theta}{2} \sin \theta \right]$$

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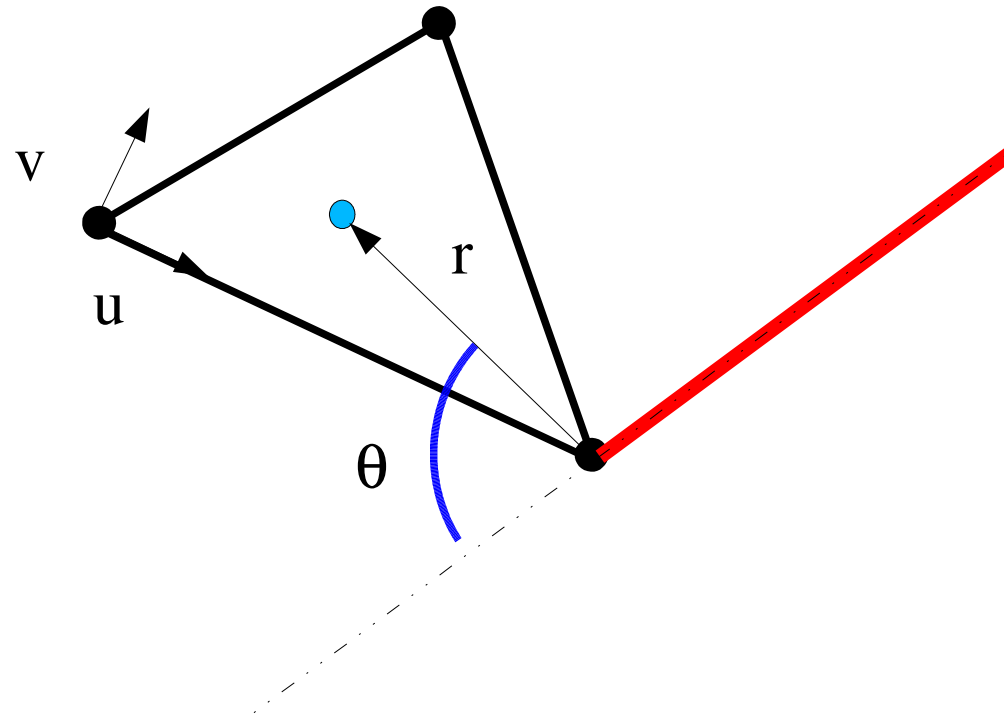
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Integration problem.

Terms in $1/r$, $1/\sqrt{r}$ and \sqrt{r} to integrate for the stiffness matrix



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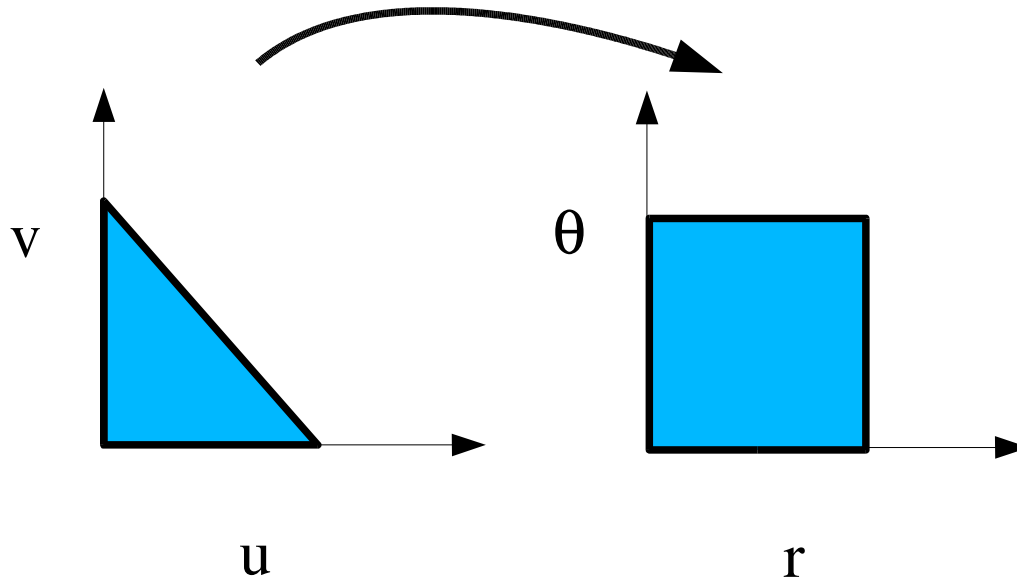
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Integration problem.

$$\int_T f dudv = \int_T f r dr d\theta$$



+ change of variables $r \rightarrow y^2$

All the term of the stiffness are now polynomials in $y \rightarrow$ Gauss rule

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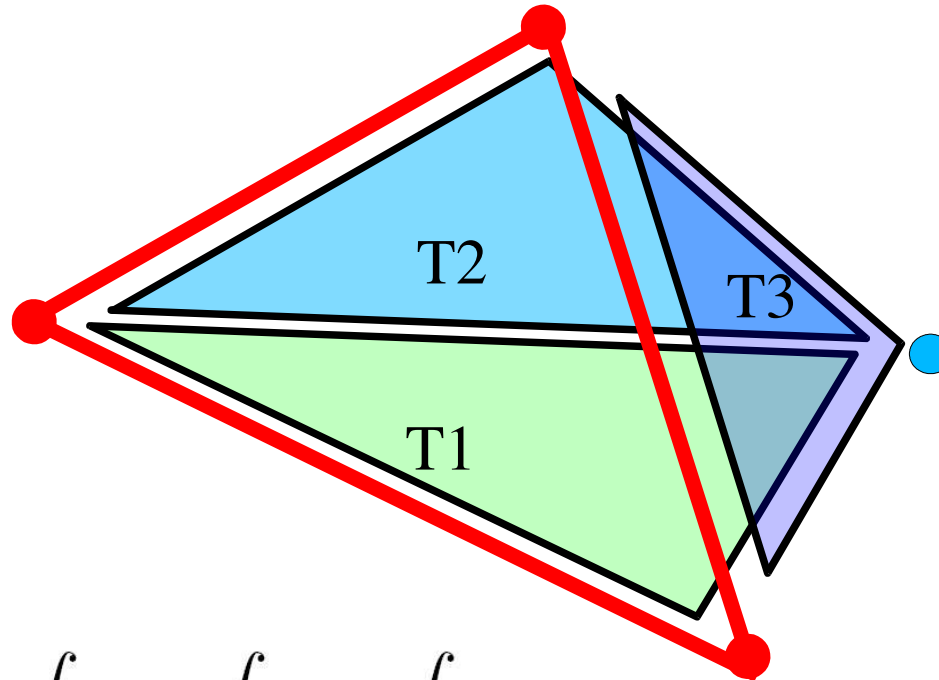
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Integration problem.

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$$\int_T = \int_{T1} + \int_{T2} - \int_{T3}$$

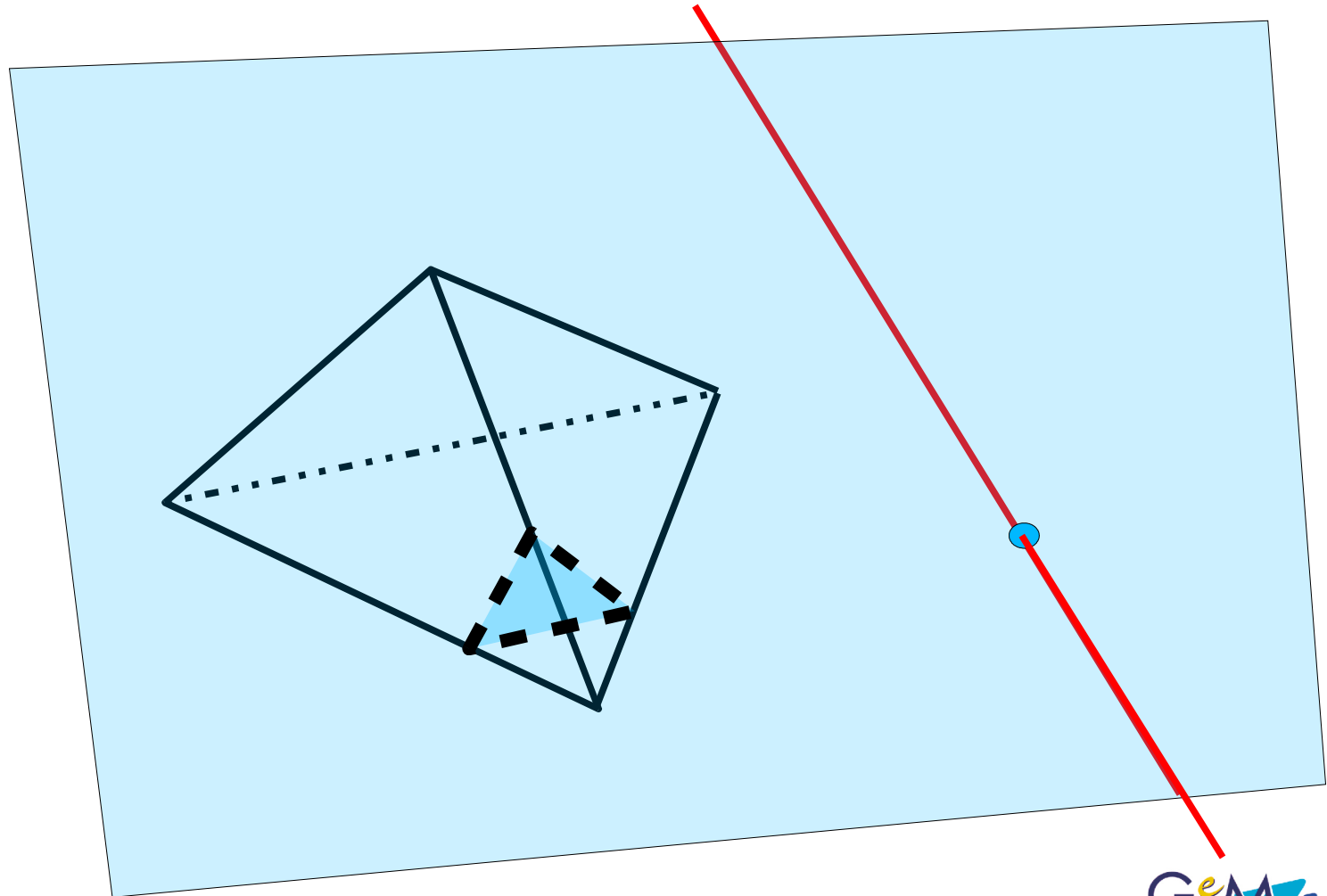
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Integration problem : 3D case

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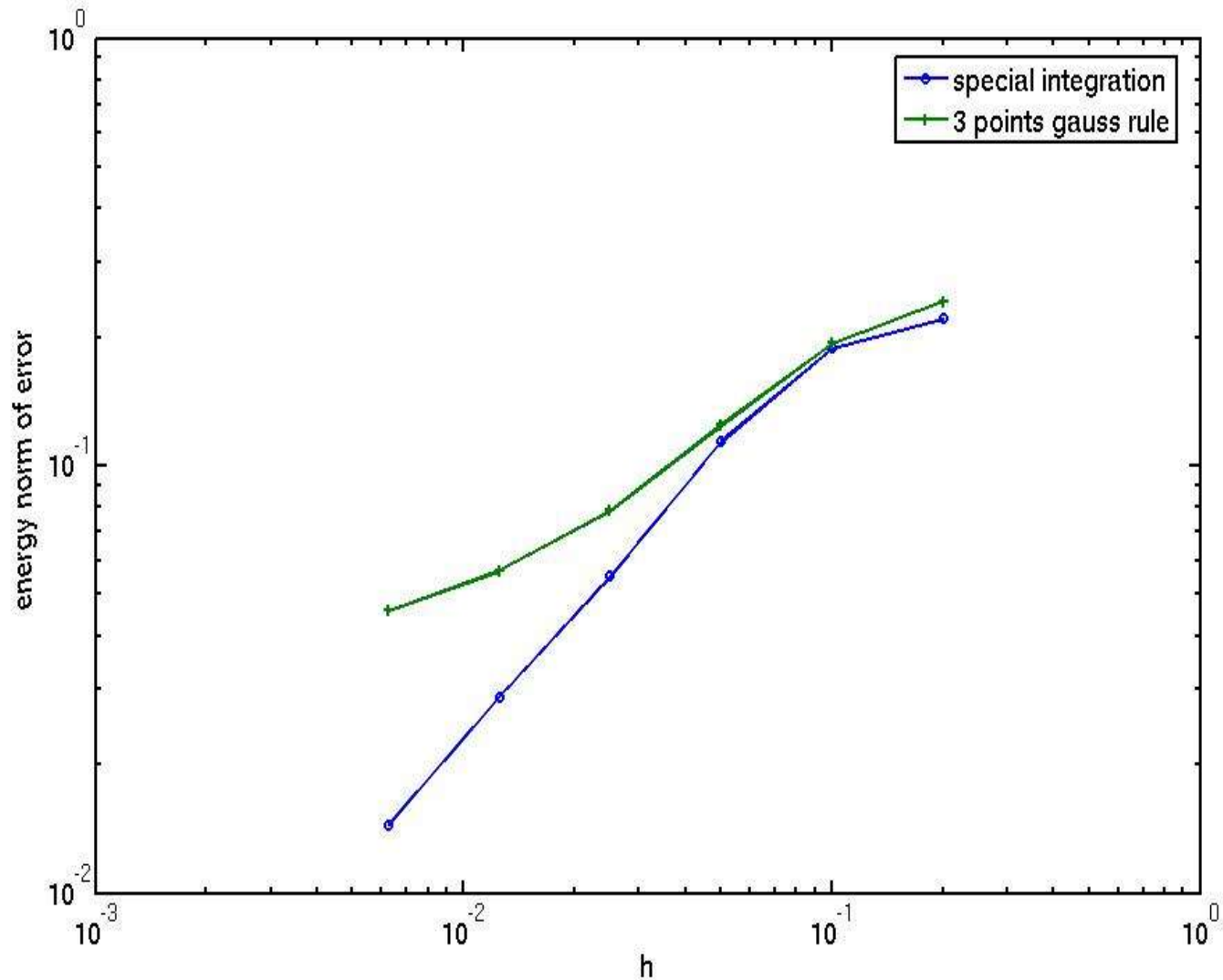
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Integration problem : Convergence results



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Integration problem

Good :

- Integration converge with few points.
- Only needed on enriched zone.

Bad :

- Difficult to implement efficiently
- Change of variable only valid for a given singularity.

Alternative : a recursive/ auto adaptative scheme on subdivision of the elements.

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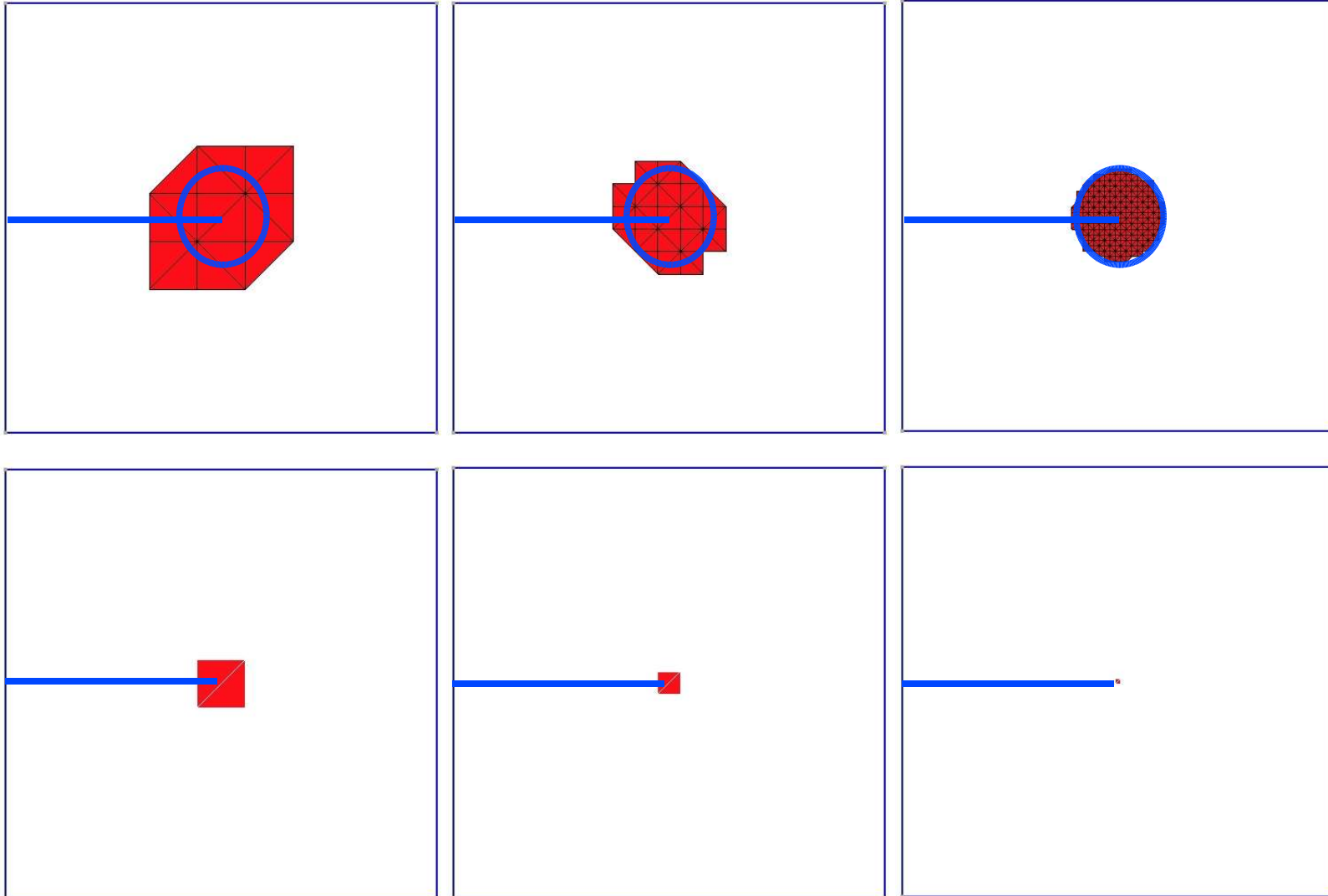
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Topologic / Geometric enrichment

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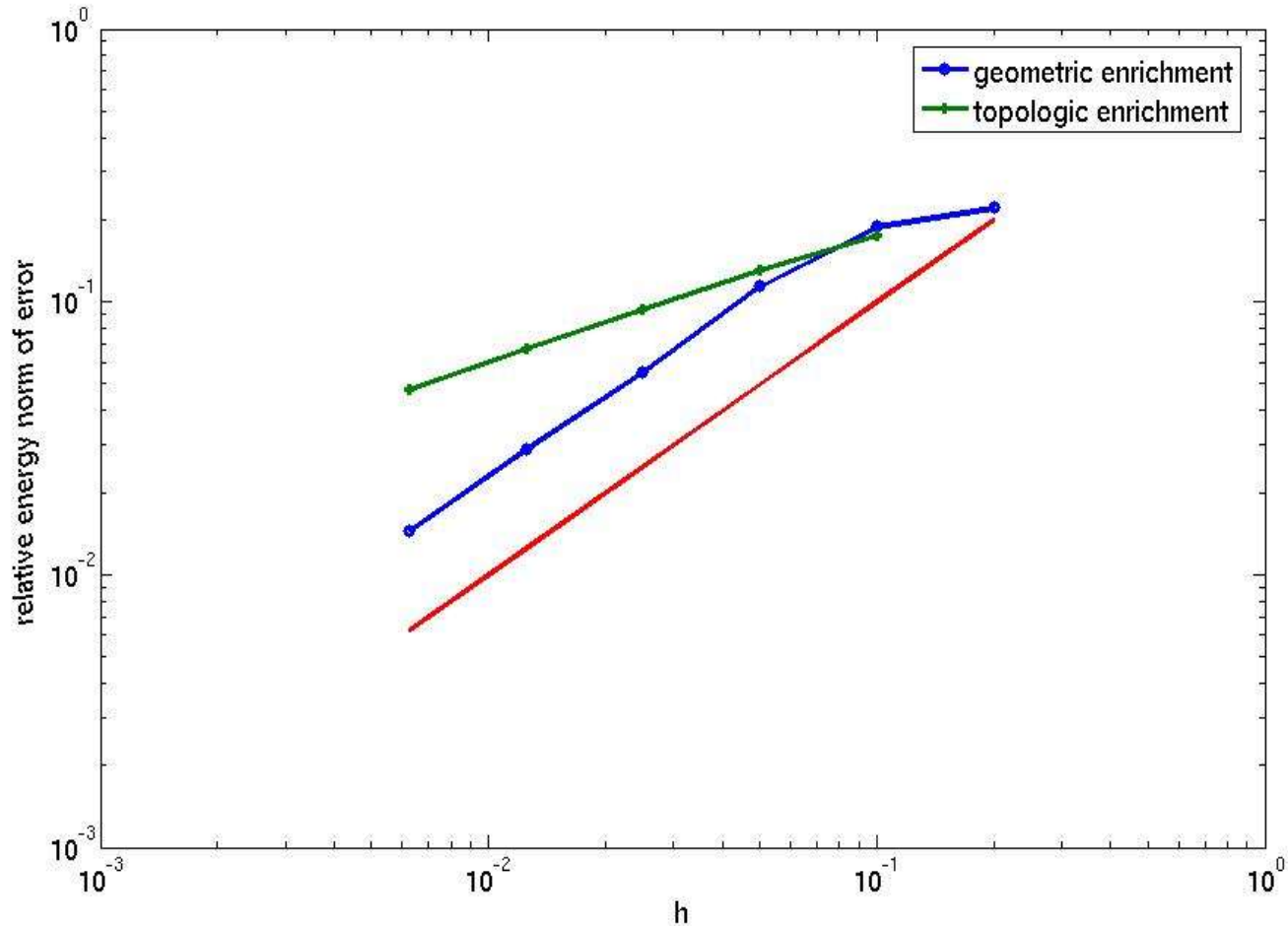
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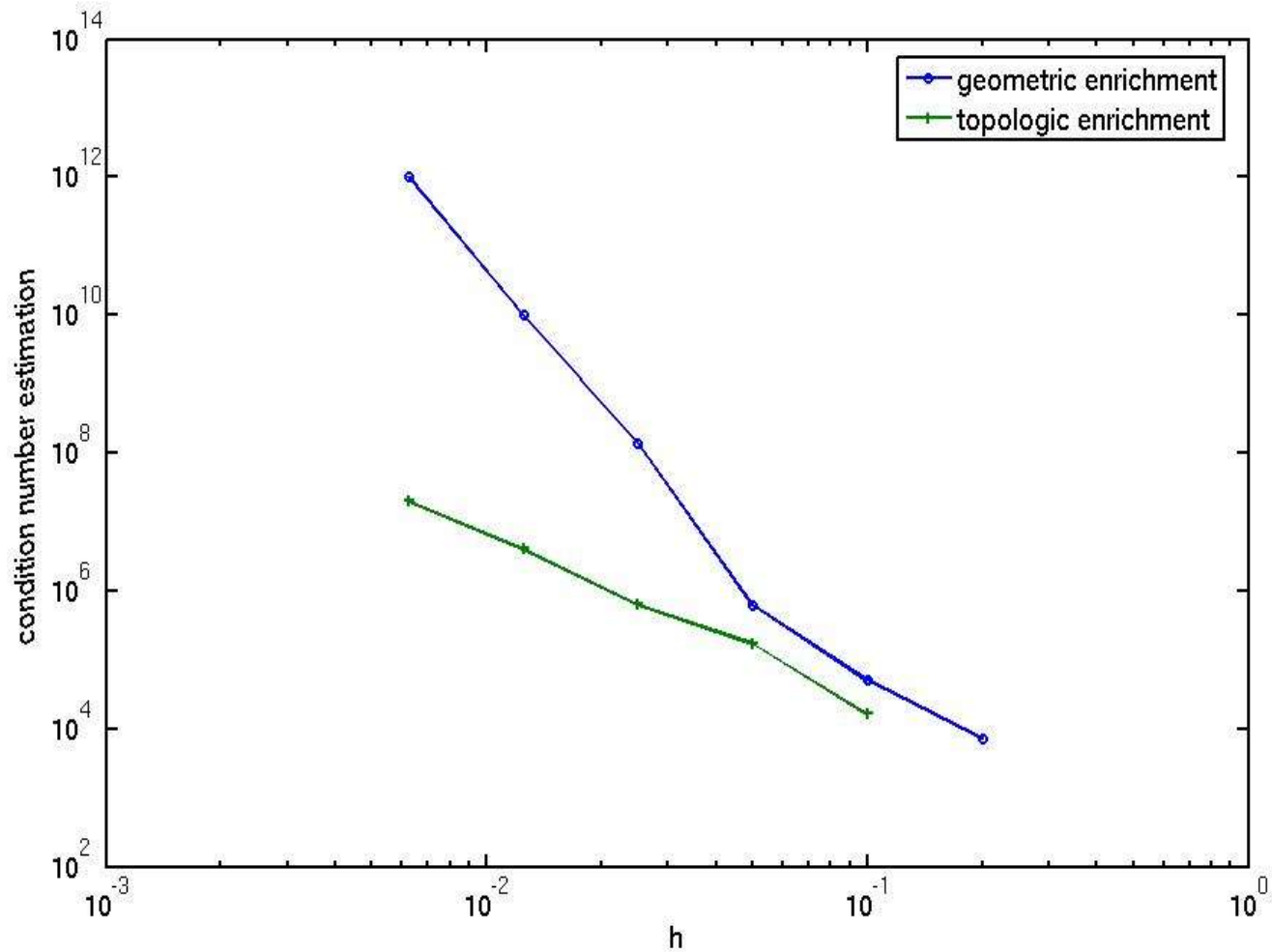
Topologic / Geometric enrichment

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Nice, but « there is no free lunch »



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Conditioning problem

Possible solutions :

- Global enrichment with a cut-off function (Laborde et al.)
 - In 2d, the number of enrichment function is the same whatever the mesh size
 - In 3d it grow like h (versus h^3 for geometric enrichment)
 - Condition number is greatly improved.
 - BUT : Sparsity of the matrix is broken

- Preconditioning by bloc diagonalisation of the enriched dofs attached to each node. (Minnebo, Bechet, Moes).
 - Conditionnement is greatly improve.
 - Sparse structure is untouched.
 - Verify fast (<1% of solver cpu time)

Convergence

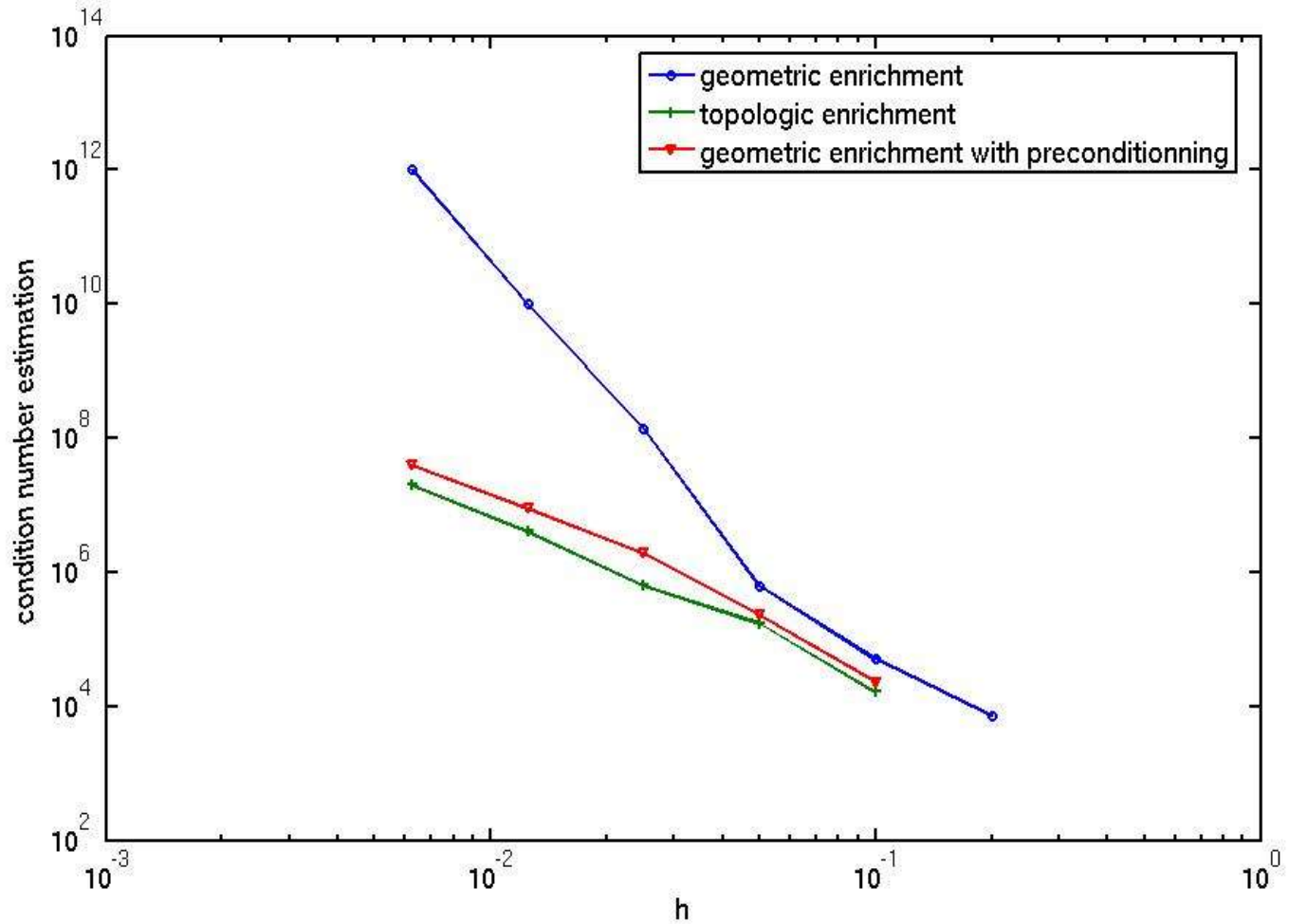
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Conditioning problem

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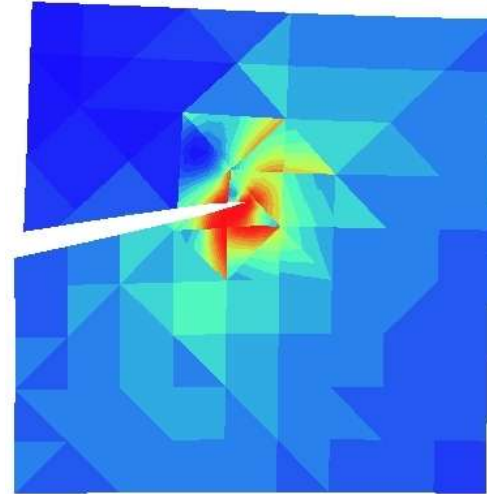
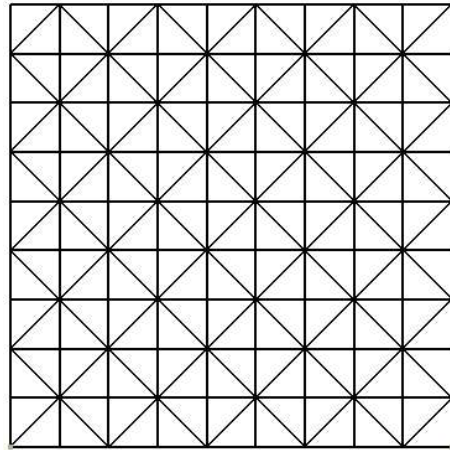
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Higher order ?

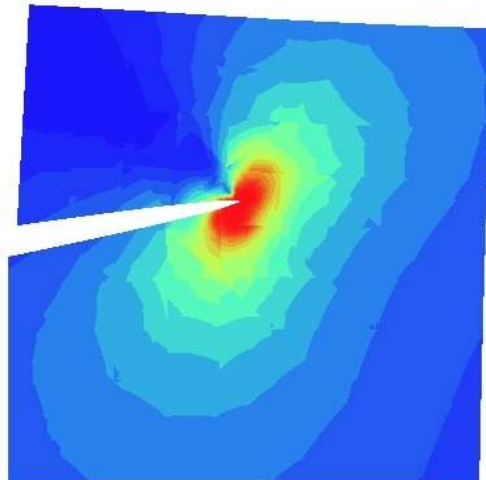
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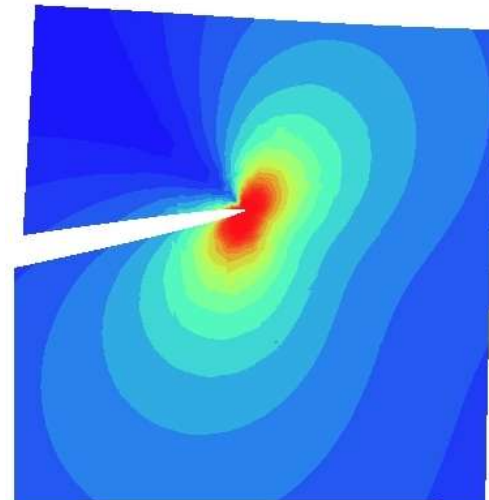
3D Examples



$p=1$



$p=2$

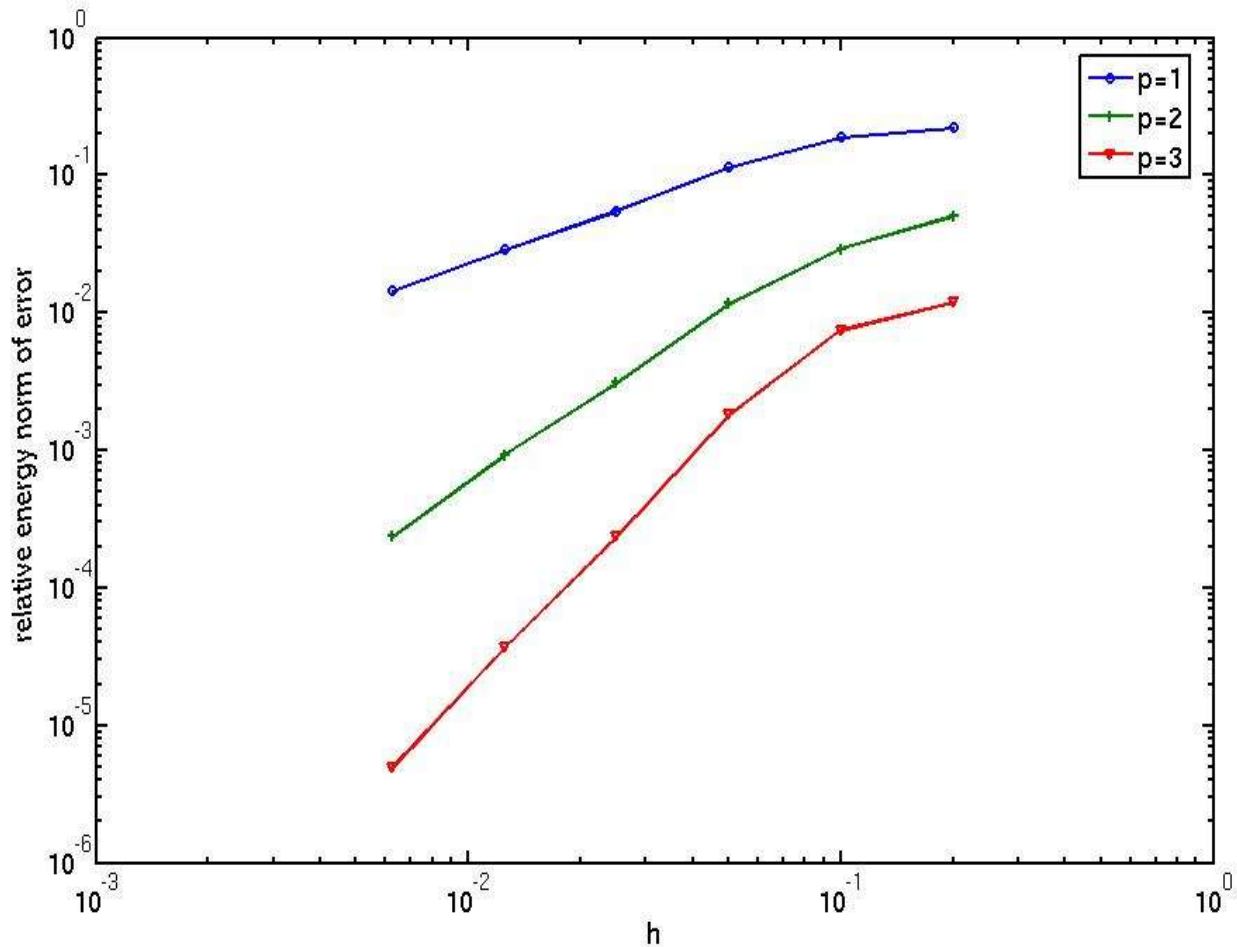


$p=3$

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p-version convergence results

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Only the classical function are higher order.

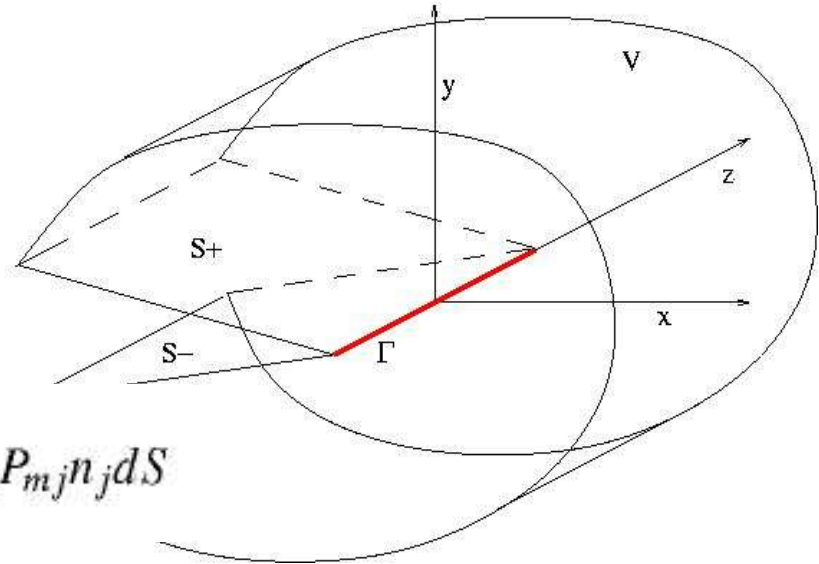
Stress Intensity factor computation

- Eshelby Tensor

$$P_{mj} = \frac{1}{2} \sigma_{kl} \epsilon_{kl} \delta_{mj} - \sigma_{ij} u_{i,m}$$

- J domaine Integral :

$$J = \int_V q_{m,j} P_{mj} dV + \int_{S^+ \cup S^-} q_m P_{mj} n_j dS$$



- Physical meaning :

$$J = \int_{\Gamma} \mathbf{G} \cdot \mathbf{q} d\Gamma$$

- Interaction integral based method to compute the SIFS.

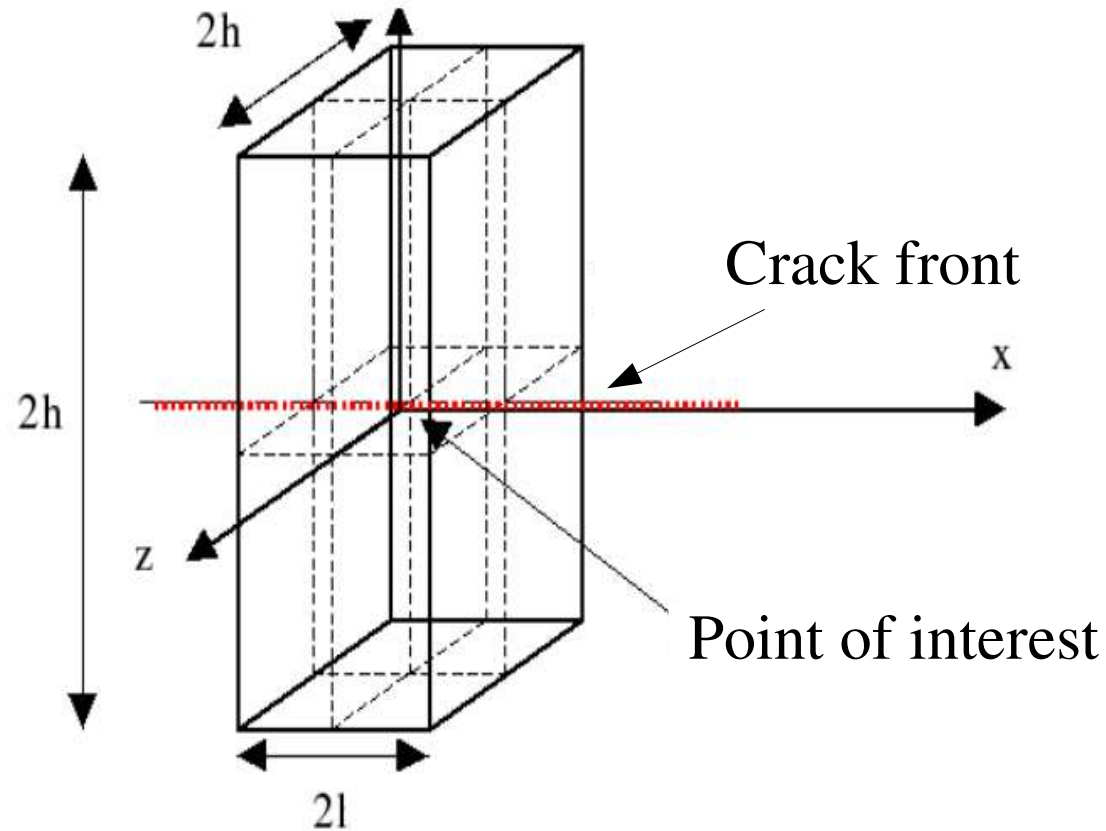
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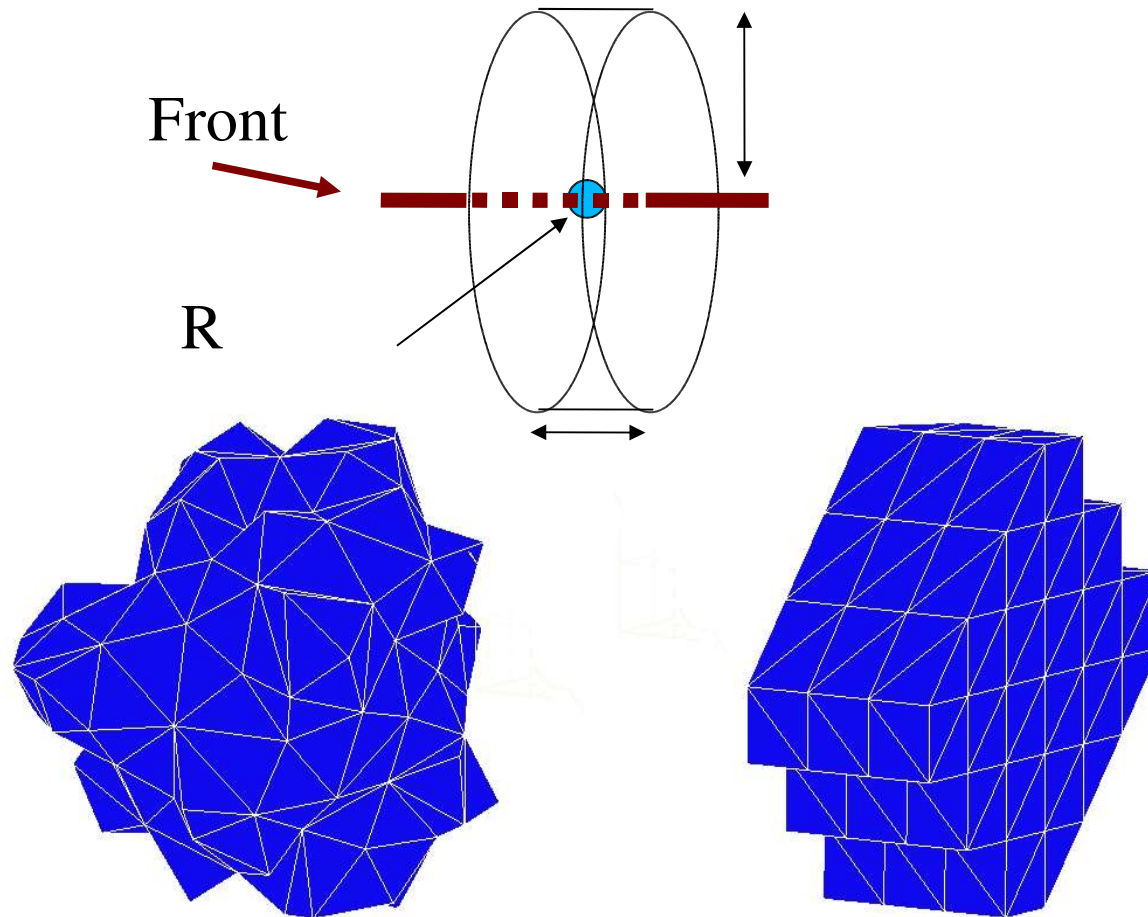
Local boxes



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Local zone depending on the mesh

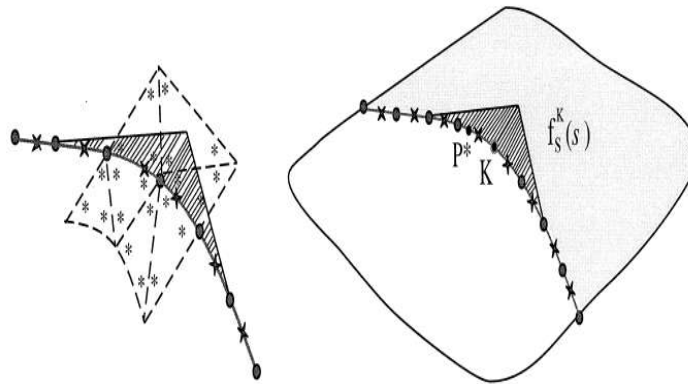


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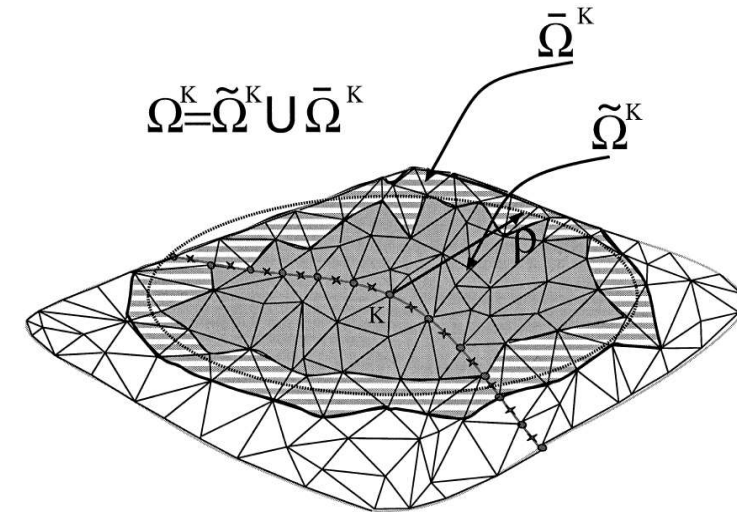
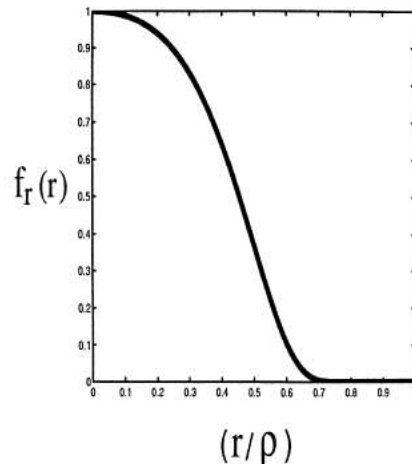
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Globale Methode (adapted from parks)

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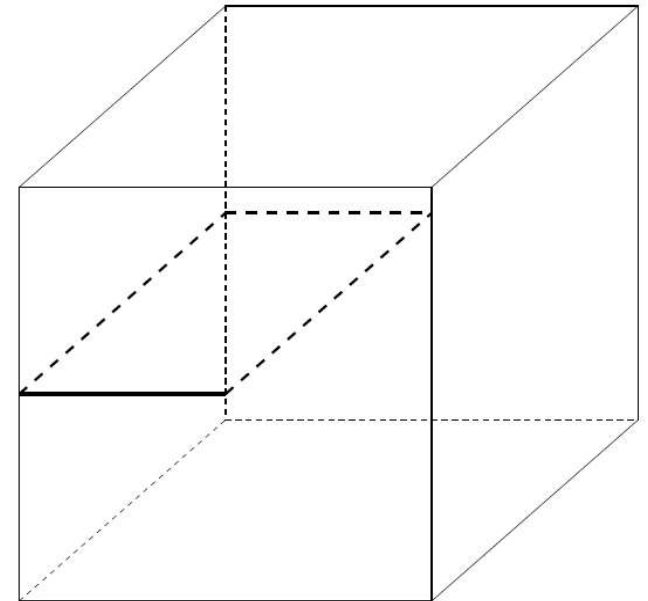
$$f_r(r) = \exp\{-[r/(r-\rho)]^2\}$$



- Build an approximation of the stress intensity field along the crack tip and solve a global problem

Bench- mark

- Plan crack on an infinite domain.
- Computation domain : a cube, with stress from the exact solution applied on the faces.
- Mesh type :
 - structured
 - unstructured
 - conform or not to the crack



Convergence

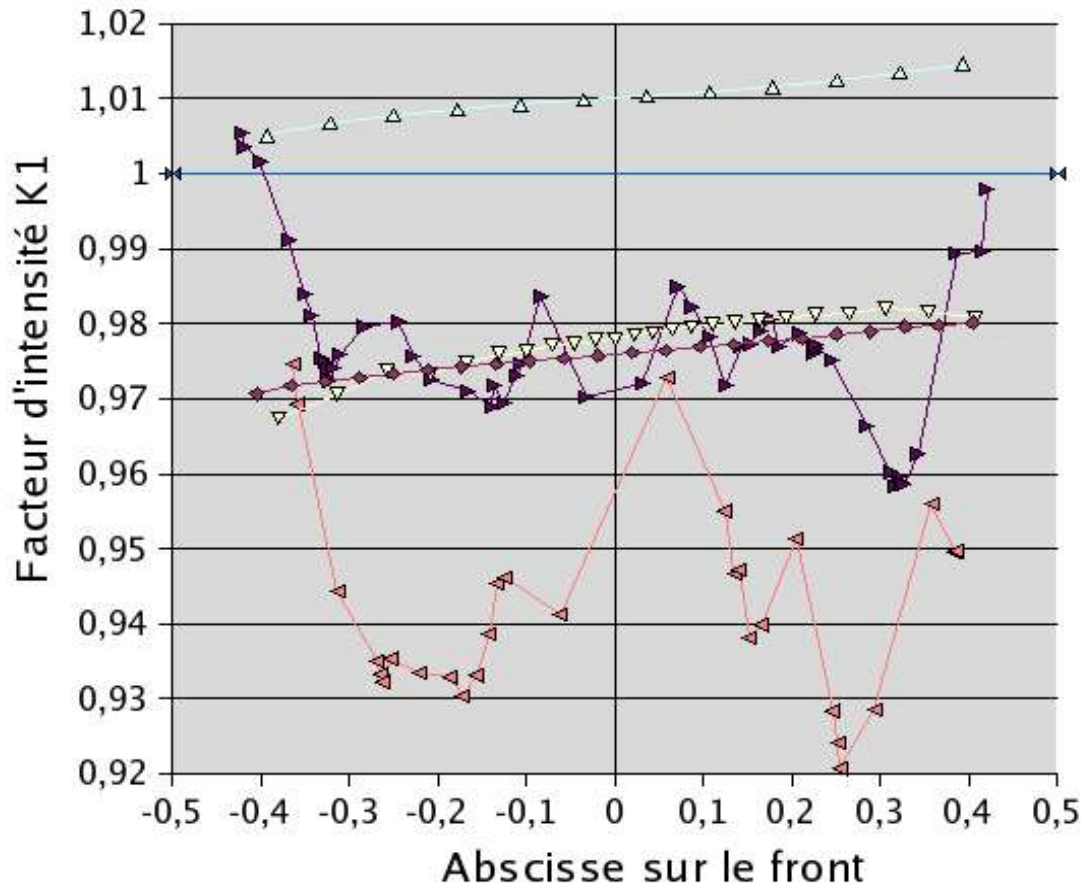
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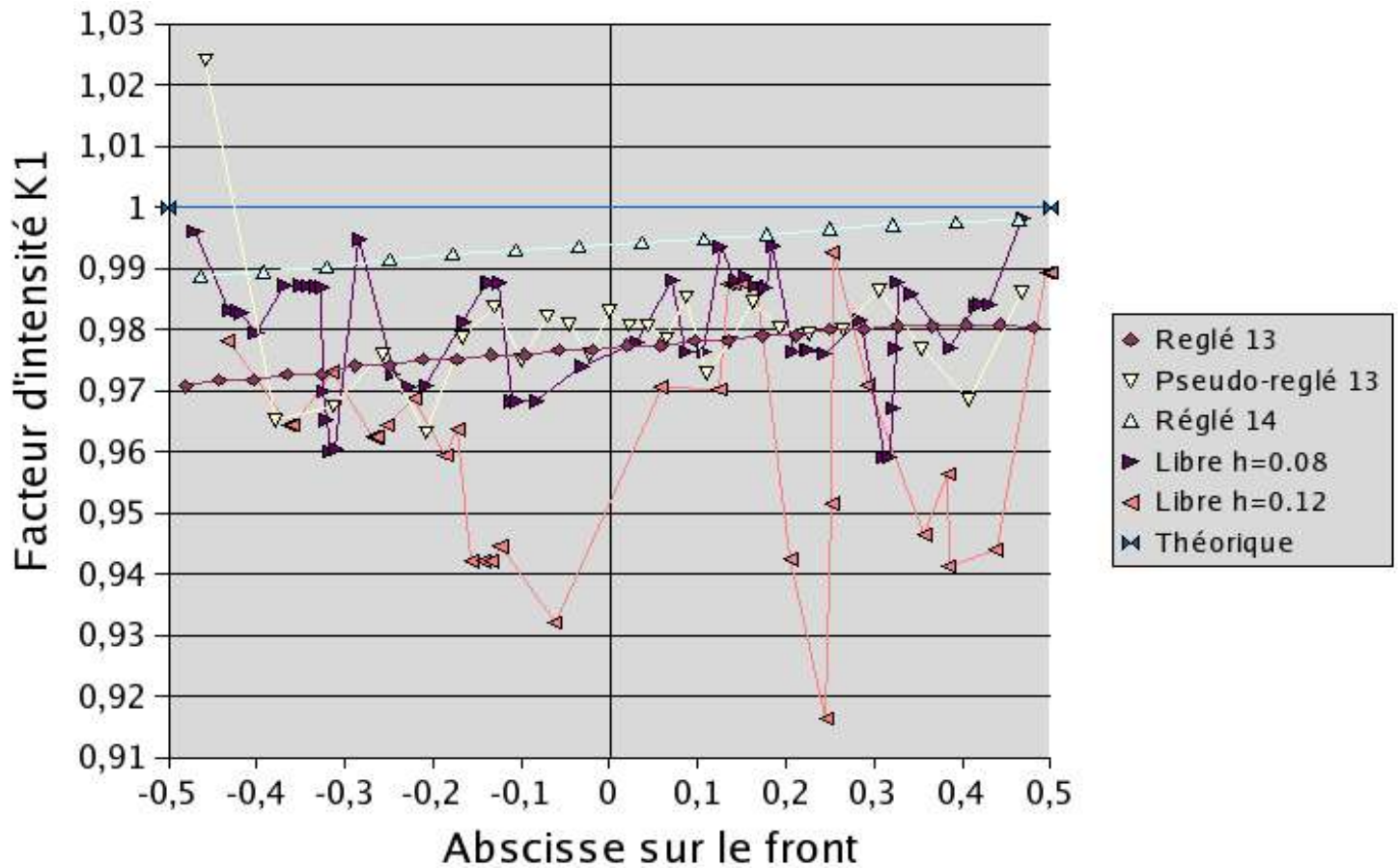
Local Boxes

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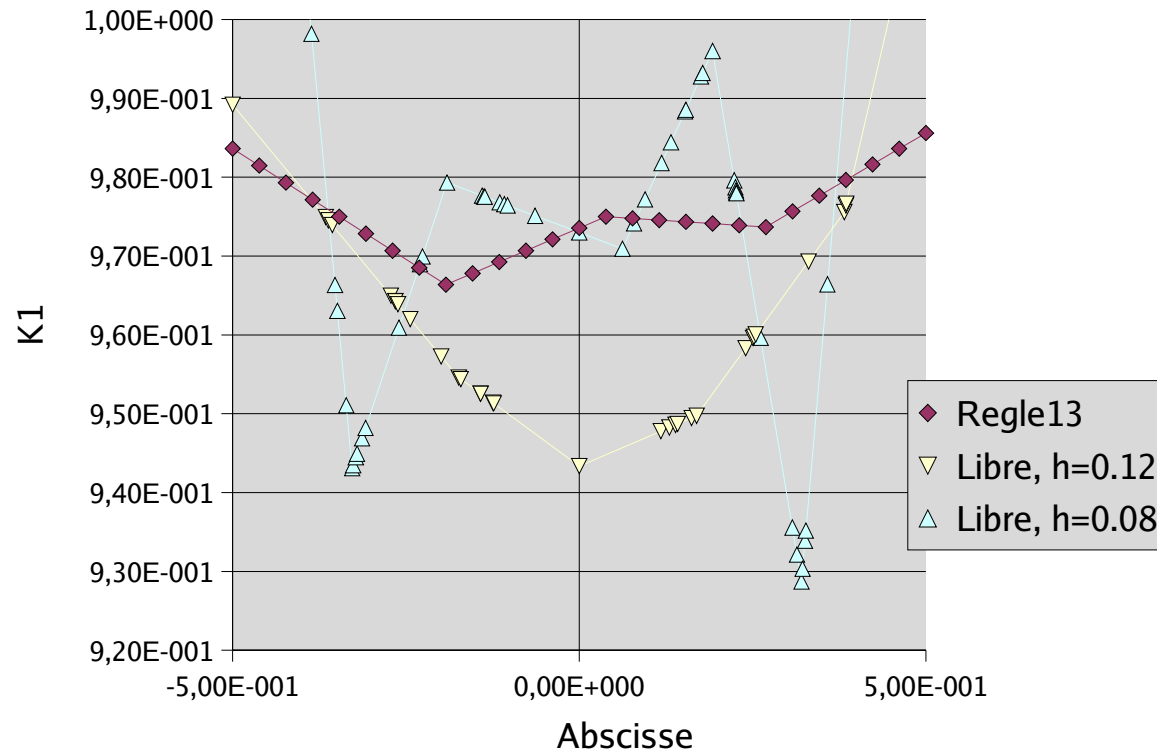
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Local zone



Global Method

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Conclusion on the 3 methods

Local boxes

- Good results on structured mesh.
- Oscillations on unstructured meshes.
- Implementation is difficult (need to integrate properly on the intersections of the boxe and the mesh)

Local Zone

- Same type of results.
- Easier implementation.

Global Method

- Computation time shorter (only one integrale),
- but difficult to implement efficiently (Integration over the intersection of the mesh and the extention to the domain of finite element representation of the stress intensity factor).
- Oscillation are reduced, but still there.

Global modal method

- Instead of a finite element representation of the stress intensity field, use a C infini representation.
 - Advantage = integration is possible directly on the mesh
- The C infini representation is written in terms of orthogonal functions (Legendre or Fourier series for closed cracks).
 - The system to solve is diagonal.
 - Direct acces to mean value and results are easy to filtrate.
 - Interpolation of the sifs defined every where, no need for extension step prior to a propagation algorithm.
- To further stabilise the result, a topological volume that tend to zero with h is removed from the integration domain.

Convergence

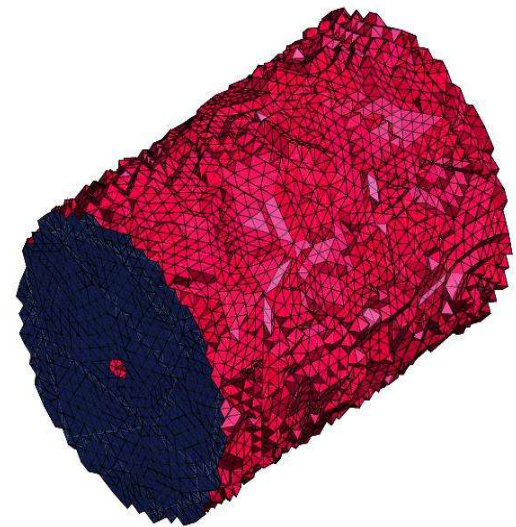
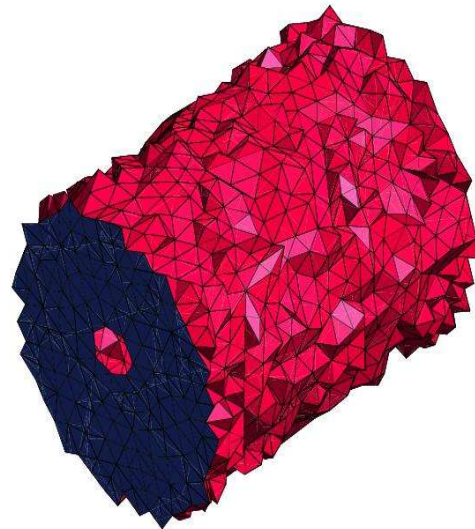
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Global modal method

mesh	nodes	elements	dofs
1	1024	4975	4695
2	7452	39800	33813
3	56423	318400	255e3
4	438125	2547200	2e7



Convergence

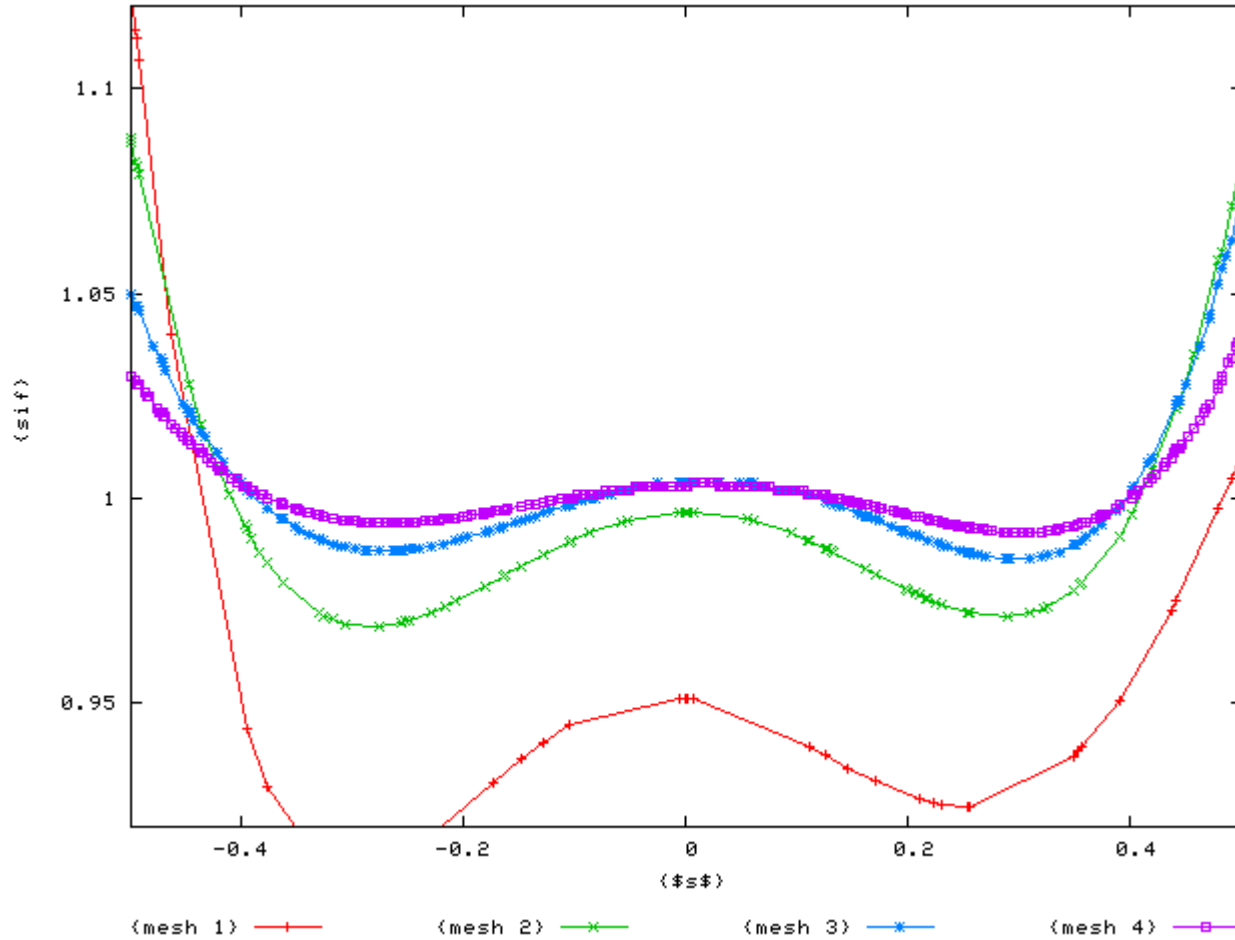
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Global modal method

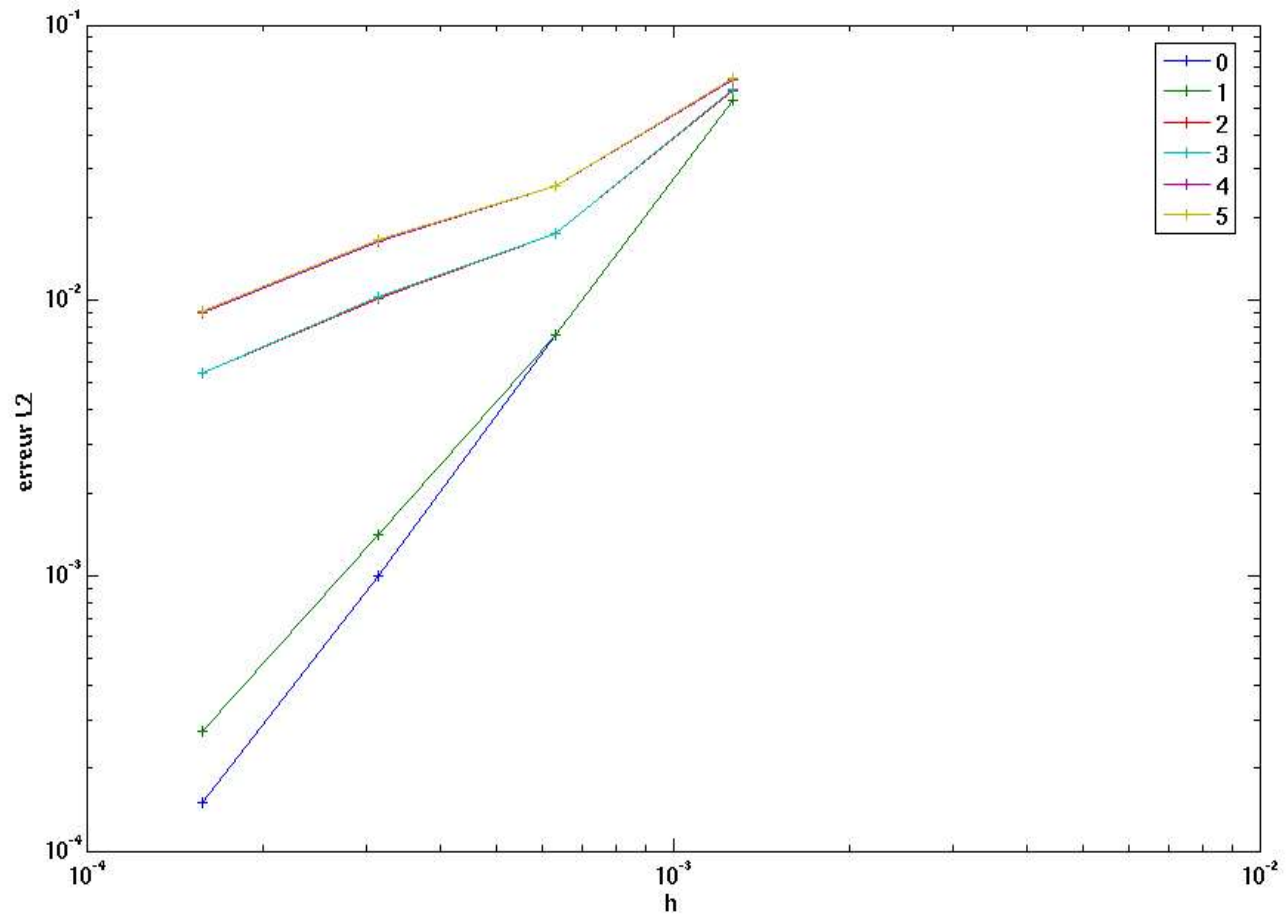
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Global modal method

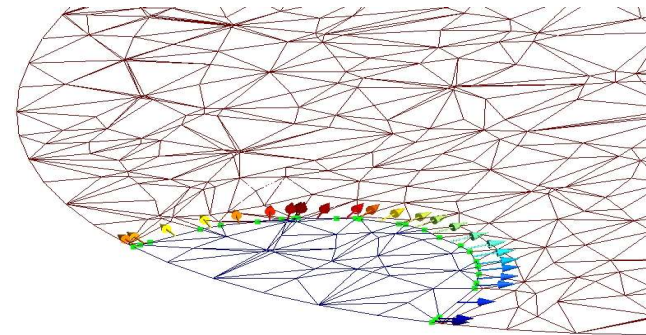
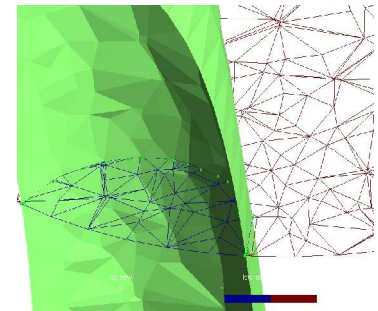
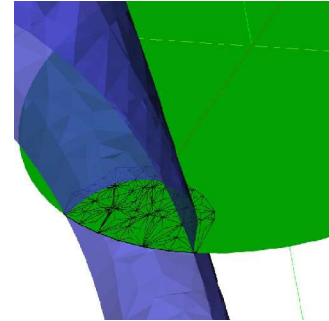
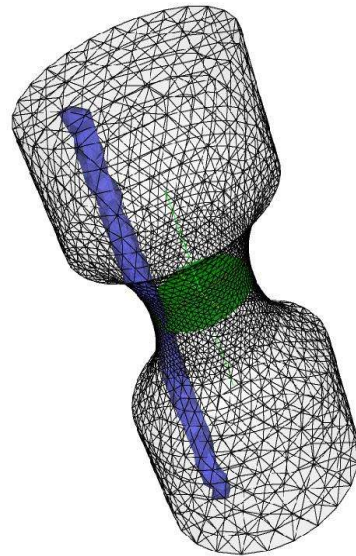
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L2 error on K1 for each term of the serie

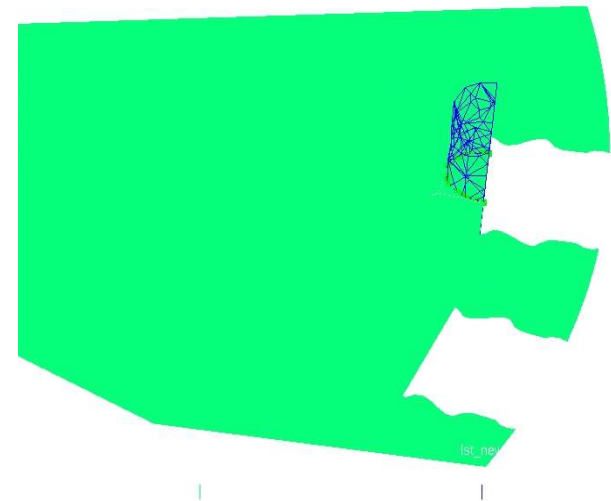
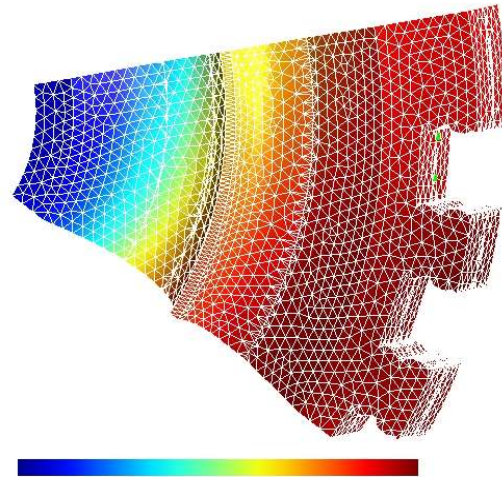
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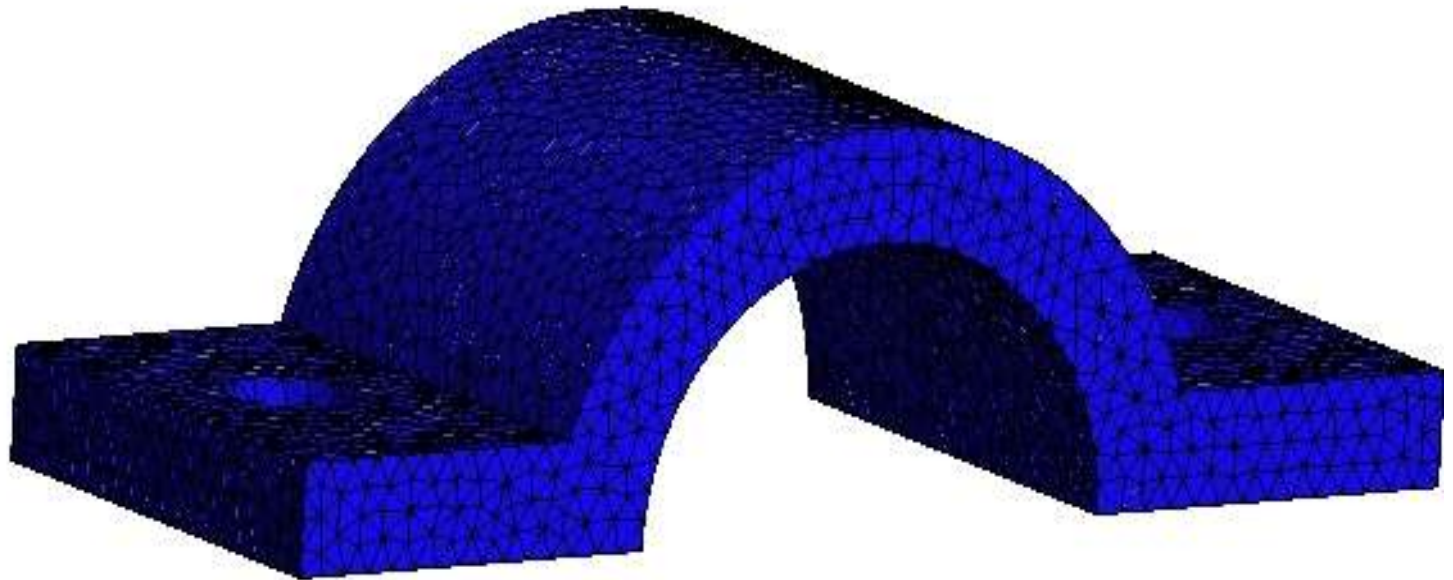
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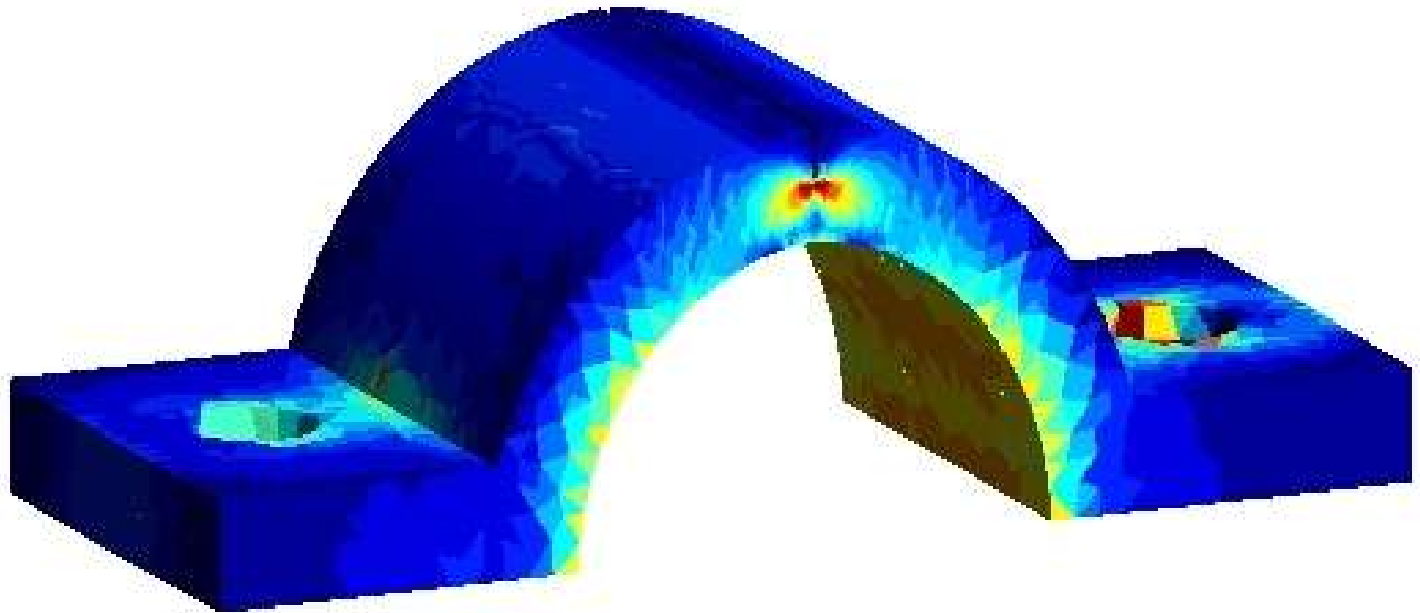
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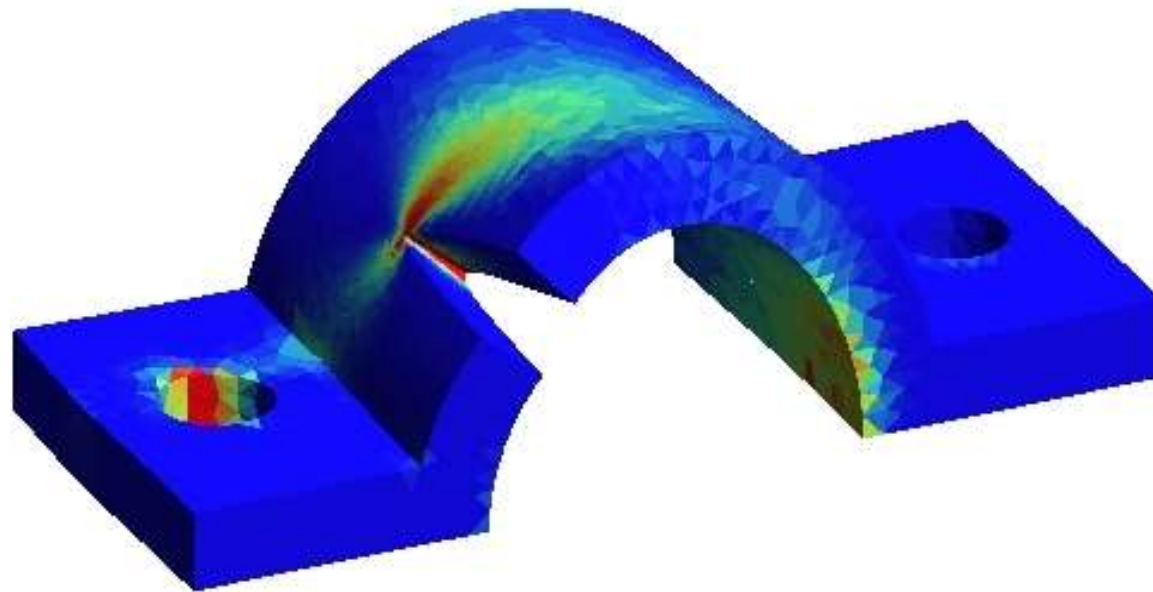
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3D examples

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Thanks to X. Oyarzabal for
this computation

Conclusion

- How to get optimal order of convergence without sacrificing conditioning.
- Convergence results for p-version of the XFEM.
- A robust method for extraction of stress intensity factor.
- Ready to work on complex geometries.

Convergence

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