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When are Google data useful to nowcast GDP?

An approach via preselection and shrinkage

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Abstract

Alternative data sets are widely used for macroeconomic nowcasting together with machine learning-based tools. The latter are often applied without a complete picture of their theoretical nowcasting properties. Against this background, this paper proposes a theoretically grounded nowcasting methodology that allows researchers to incorporate alternative Google Search Data (GSD) among the predictors and that combines targeted preselection, Ridge regularization, and Generalized Cross Validation. Breaking with most existing literature, which focuses on asymptotic in-sample theoretical properties, we establish the theoretical out-of-sample properties of our methodology and support them by Monte-Carlo simulations. We apply our methodology to GSD to nowcast GDP growth rate of several countries during various economic periods. Our empirical findings support the idea that GSD tend to increase nowcasting accuracy, even after controlling for official variables, but that the gain differs between periods of recessions and of macroeconomic stability.

Keywords: Nowcasting, Big data, Google search data, Targeted preselection, Ridge regularization.

1 Introduction

Currently, practitioners widely use large sets of alternative data – such as web-scraped data, Google data (*i.e.* Trends, Correlate, or Search), scanner data, or satellite

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data – for short-term macroeconomic forecasting and nowcasting purposes (see *e.g.* Ng, 2017). The main research questions related to alternative data sets are: (A) when such data improve nowcasting accuracy, and (B) whether they are useful even after controlling for official variables generally used by forecasters (*e.g.* opinion surveys, production). Answering these questions requires the use of appropriate machine learning/econometrics methods. Recent macroeconomic empirical literature has seen an explosion of various methods to account for the specificity of alternative data, but for most of them we do not know their out-of-sample (OOS) theoretical properties, which matter the most for forecasting purposes. This paper puts forward a new methodology to deal with Google Search Data (GSD) for nowcasting purposes and establishes OOS large-sample properties for the proposed method. The challenging feature of GSD as a whole is their high dimension. GSD differ from Google Trends mainly in that GSD are volume variations of Google queries with respect to the first value while Google Trends provides the ratio between the search shares for a particular keyword/category over a given sub-period of time and the maximum search share for the same keyword/category over a larger chosen period. Macroeconomic nowcast based on GSD have been proposed in *e.g.* Goetz and Knetsch (2019) while Google Trends have been used in, for example, Choi and Varian (2009, 2012), Scott and Varian (2015), Vosen and Schmidt (2011), D’Amuri and Maruccci (2017), Niesert et al. (2020), Borup and Schütte (2022), and references therein. However, unlike our study, all these papers do not include theoretical contributions.

Our proposed nowcasting method, which we call *Ridge after Model Selection*, is a two-step approach: (i) first, GSD variables are preselected, conditional on the official variables, by targeting the macroeconomic aggregate to be nowcast, and (ii) second, a Ridge regularization is applied to those preselected GSD and official variables. The Ridge tuning parameter is chosen by Generalized Cross Validation (GCV). The main theoretical contributions of this paper are threefold. First, we prove that our targeted preselection retains all the variables in the true model with probability approaching one

(sure screening property). Second, we establish an upper bound for both the in-sample and OOS prediction error associated with the Ridge after model selection estimator. Third, we evaluate optimality of GCV to choose the regularization parameter α of the Ridge regularization for OOS prediction. To the best of our knowledge, previous literature has established in-sample optimality of the GCV in the setting of Ridge regularization but not OOS optimality, see *e.g.* Li (1986) and Carrasco and Rossi (2016).

Giannone et al. (2008) popularized the concept of macroeconomic nowcasting, which differs from standard forecasting approaches in the sense that it aims at evaluating current macroeconomic conditions on a high-frequency basis. The idea is to provide policy makers with a real-time evaluation of the state of the economy ahead of the release of official Quarterly National Accounts, which come out with a delay. For example, the New York Fed and the Atlanta Fed have recently developed new tools to evaluate US GDP quarterly growth on a high-frequency basis (see <https://www.newyorkfed.org/research/policy/nowcast> and <https://www.frbatlanta.org/cqer/research/gdpnow.aspx>). A large literature has developed that deals with nowcasting GDP growth for different countries, see *e.g.* Aruoba et al. (2009), Doz et al. (2011), Aastveit and Trovik (2012), and Ferrara and Marsilli (2019).

To clarify the presentation of our nowcasting methodology, we present an example. Suppose the variable one wants to nowcast is the quarterly GDP growth rate. The predictors are made of two subsets: a set of official variables and a set of the weekly GSD variables. Our nowcasting model is based on linear regression models that incorporate predictors sampled over different frequencies (*e.g.*, monthly and weekly) and released with various reporting lags so that the relevant information set for calculating the nowcast evolves within the quarter. To explicitly account for the information set available at a specific time within the quarter, a forecaster will consider a different set of predictors for each week, the same frequency of the higher-frequency variable considered in our example.

To estimate the models, we use our aforementioned *Ridge after Model Selection* procedure. The first step, preselection, is based on the t-statistics associated with each GSD variable in a regression that includes the official predictors as well, see *e.g.* Bai and Ng (2008) and Barut et al. (2016). In the second step, a Ridge regularization is applied to the linear regression model incorporating the official variables and the GSD variables that have been preselected. The regularization parameter α is set equal to the minimum of the GCV criterion. Prior literature has proposed forecasting approaches based on Ridge regression to deal with dense models with a large number of predictors, *e.g.* De Mol et al. (2008) and Carrasco and Rossi (2016). We go beyond this literature by considering models in which the dimension can be ultra-high and that can be either sparse or dense.

For our two-step procedure, we derive the three theoretical properties mentioned previously: the sure screening property of our preselection procedure, upper bounds for the in-sample and OOS prediction errors associated with the Ridge after model selection estimator, and optimality of the GCV for OOS prediction. The upper bounds are functions of the number of predictors N , the number of time-series observations T , and the Ridge regularization parameter α . With regard to the GCV, we know from the previous literature that GCV has optimality properties for in-sample prediction, see *e.g.* Li (1986), Andrews (1991) and Carrasco and Rossi (2016). We complete this result by showing that the minimizer of the GCV is as good as the minimizer of the conditional mean squared prediction error for OOS prediction. The latter is the objective of nowcasting for central bankers.

We study finite sample properties of our procedure using a Monte Carlo exercise. Our study analyzes how the dimension of the problem, N and T , the degree of sparsity s in the model, and the correlation among the predictors affect the performance of our method compared with other widely used methods in macroeconomic nowcast, like Lasso, Ridge without preselection, and Principal Component Analysis estimators. We

show that when the true data generating process is sparse with a large number of active predictors, our *Ridge after Model Selection* procedure outperforms all the considered competitors for OOS prediction.

Finally, we conduct an empirical study to answer our two research questions (A) and (B) stated above for GSD with respect to GDP growth nowcast for three countries/areas: the euro area, the United States, and Germany. Common GDP nowcasting tools integrate standard official macroeconomic information stemming from, for instance, national statistical institutes, central banks, and international organizations. Two types of official data are typically considered: (i) hard data (production, sales, employment ...) and (ii) opinion surveys (households or companies are asked about their view on current and future economic conditions). Sometimes, financial markets information, generally available on high frequency basis, is also integrated into the information set. In our study, we include official data (i) and (ii) together with the alternative GSD into our information set. In addition, we consider financial market information for robustness check.

We analyze three time periods: a period of cyclical stability (2014q1 – 2016q1), a period that exhibits a sharp downturn in GDP growth rate (2017q1 – 2018q4), and a period of recession (the Great Recession period from 2008q1 to 2009q2). Overall, empirical results show that GSD are useful when nowcasting GDP growth. At the beginning of the quarter, when no official information is available about the current state of the economy, we show that using only Google data leads to reasonable mean squared forecasting errors (MSFEs), sometimes only slightly higher than those obtained at the end of the quarter when the information set is complete. As soon as we integrate official macroeconomic information, starting from the fifth week of the quarter, MSFEs decrease, reflecting the importance of this type of data in nowcasting. Overall, combining macroeconomic variables and GSD variables in the same model appears to be fruitful.

A striking result of our empirical analysis is that, on the one hand, the preselection

step is crucial in the first two periods considered, as it generates better outcomes than nowcasting procedures that have no preselection. This result confirms previous findings from the nowcasting literature, see *e.g.* Bai and Ng (2008) and Boivin and Ng (2006) for dynamic factor models. On the other hand, recession periods present specific patterns as a model that only contains GSD, without any preselection step, tends to be preferred in terms of nowcasting accuracy. This result is quite robust over the three countries/areas that we consider in the study.

The rest of the paper is organized as follows. In Section 2, we describe the nowcasting model and our *Ridge after Model Selection* procedure, for which theoretical results are provided in this section and in Section 3. The Monte Carlo exercise is in Section 4, and the empirical application is in Section 5. Section 6 concludes with a summary. Additional material and proofs are in the Supplementary Appendix.

2 Methodology

2.1 The nowcasting equation

To nowcast any series of interest Y_t , we focus on linear bridge equation models, which allow us to construct Y_t nowcasts by using predictors available at different frequencies. To fix ideas, suppose the frequency of Y_t is quarterly. We include three types of predictors: *soft* variables, such as opinion surveys, *hard* variables, such as industrial production or sales, and variables stemming from GSD. GSD are available on a weekly basis, and the other predictors are available on a monthly base. Let t denote a given quarter of interest identified by its last month, for example the first quarter of 2005 is dated by $t = \text{March}2005$. The following is a bridge equation model to nowcast Y_t for a specific quarter t , for $t = 1, \dots, T$:

$$Y_t = \beta_0 + \beta'_s x_{t,s} + \beta'_h x_{t,h} + \beta'_g x_{t,g} + \varepsilon_t, \quad \mathbf{E}[\varepsilon_t | x_{t,s}, x_{t,h}, x_{t,g}] = 0, \quad (2.1)$$

where $x_{t,s}$ is the N_s -vector containing *soft* variables, $x_{t,h}$ is the N_h -vector containing *hard* variables, $x_{t,g}$ is the N_g -vector of variables coming from GSD, and ε_t is an unobservable shock. In general, $x_{t,s}$, $x_{t,h}$, and $x_{t,g}$ are sampled over different frequencies and released with various reporting lags, so the relevant information set for calculating the nowcasts evolves within the quarter. In practice, one must consider a model for each new available information set, which arises at the frequency of the highest frequency variable in the model, see Section 5.1 for more details on this point. Model (2.1) is estimated by the *Ridge after model selection* procedure, which is summarized in Algorithm 1 and explained in details in Sections 2.2 and 2.3.

2.2 Step 1: preselection of Google Search data

The recent literature on nowcasting and forecasting with large data sets concludes that using the largest available data set is not necessarily the optimal approach when nowcasting a specific macroeconomic variable such as GDP, at least in terms of nowcasting accuracy (see *e.g.* Boivin and Ng, 2006; Barhoumi et al., 2010). The problem arises because we have too many variables and using all the variables would only add noise in the estimation process. As shown in Bai and Ng (2008), an empirical way to circumvent this issue is to more accurately target the choice of predictors.

As we explain in Section 5, the GSD have a very high dimension N_g compared with T , and in our empirical analysis we find that using all the variables in the GSD is not always a good strategy because we pay the price of dealing with ultra-high dimensionality without increasing the nowcasting accuracy as measured by the MSFE. In fact, it might be that the series Y_t to be nowcast is highly predictable by a subset of the GSD and that this subset is specific to Y_t . For this reason, before estimating model (5.1) we preselect GSD by retaining the most relevant variables for Y_t nowcasting, capturing much of the variability in it. We refer to this approach as a targeted preselection.

Preselection is based on the procedure proposed in Bai and Ng (2008) and Barut

Algorithm 1: Ridge after model selection

Data: Training sample $\{Y_t, x_{t,s}, x_{t,h}, x_{t,g}; t = 1, \dots, T\}$.

Step 1:

- (1) **for** $j = 1, \dots, N_g$ **do**
 - regress** Y_t on a constant, $x_{t,s}$, $x_{t,h}$, and $x_{t,g,j}$;
 - compute** the t-statistics t_j associated with $x_{t,g,j}$;
- (2) select the Google variables that have the largest absolute value $|t_j|$ compared with a given threshold $\lambda > 0$: set $\widehat{M}_g := \widehat{M}_g(\lambda) := \{1 \leq j \leq N_g : |t_j| > \lambda\}$.

Step 2:

- (1) Construct $\widehat{M} = \widehat{M}(\lambda) := \{1, \dots, N_1\} \cup \{N_1 + j; j \in \widehat{M}_g(\lambda)\}$, $x_{t,g,\widehat{M}_g} := \{x_{t,g,j}; j \in \widehat{M}_g\}$, and $X_{t,\widehat{M}} := (1, x'_{t,s}, x'_{t,h}, x'_{t,g,\widehat{M}_g})'$;
- (2) for every α in a grid \mathcal{A} :
 - (a) compute

$$\widehat{\beta}_{\widehat{M}}(\alpha) = \left(\frac{1}{T} \sum_{t=1}^T X_{t,\widehat{M}} X'_{t,\widehat{M}} + \alpha I_{\widehat{M}} \right)^{-1} \frac{1}{T} \sum_{t=1}^T X'_{t,\widehat{M}} Y_t,$$

where $I_{\widehat{M}}$ is the $|\widehat{M}|$ -dimensional identity matrix;

- (b) compute

$$GCV(\alpha) := \frac{\sum_{t=1}^T (Y_t - X'_{t,\widehat{M}} \widehat{\beta}_{\widehat{M}}(\alpha))^2}{T \left(1 - \text{tr}(X_{t,\widehat{M}} (T^{-1} \sum_{t=1}^T X_{t,\widehat{M}} X'_{t,\widehat{M}} + \alpha I_{\widehat{M}})^{-1} X'_{t,\widehat{M}} / T) / T \right)^2},$$

where $\text{tr}(\cdot)$ denotes the trace operator;

- (3) select $\widehat{\alpha} = \arg \min_{\alpha \in \mathcal{A}} GCV(\alpha)$.

Forecasting:

- Data:** $X_{\tau,\widehat{M}}$ for $\tau > T$;
 - return** $X'_{\tau,\widehat{M}} \widehat{\beta}_{\widehat{M}}(\widehat{\alpha})$.
-

et al. (2016) which, in our framework, works as follows. We begin with the standard linear regression equation (2.1). Then, by denoting with $x_{t,g,j}$ the j -th GSD variable, we apply the following approach:

- (1) for each $j = 1, \dots, N_g$, regress Y_t on a constant, $x_{t,s}$, $x_{t,h}$, and $x_{t,g,j}$, and compute the corresponding t-statistics t_j associated with $x_{t,g,j}$;
- (2) select the Google variables that have the largest absolute value $|t_j|$ compared with a given threshold $\lambda > 0$: $\widehat{M}_g := \widehat{M}_g(\lambda) := \{1 \leq j \leq N_g : |t_j| > \lambda\}$.

The basic idea of this approach is that a GSD variable is retained depending on its contribution for predicting Y_t after controlling for a set of official variables. Associated with each λ there is a selected submodel $\widehat{M}_g := \widehat{M}_g(\lambda)$. In practice, we take λ as the $(1 - \tau)$ -quantile of a $\mathcal{N}(0, 1)$ distribution with $\tau \in \{20\%, 10\%, 5\%, 2.5\%, 1\%, 0.5\%\}$. The parameter τ must be interpreted as the percentage of false positives that can be tolerated. In contrast to Barut et al. (2016), we do not assume that the conditional variance of Y_t is known, and we estimate it to construct the t-statistics. This partially modifies the proof of the sure screening property in Theorem 2.1 below. This property is not established in Bai and Ng (2008).

Let $N_1 := 1 + N_s + N_h$, $N := N_1 + N_g$, $X_t := (1, x'_{t,s}, x'_{t,h}, x'_{t,g})'$, $X_{t,O} := (1, x'_{t,s}, x'_{t,h})'$, $X_{t,O,j} := (X'_{t,O}, x_{t,g,j})'$ for every $j \in \{1, \dots, N_g\}$, and $\beta := (\beta_0, \beta'_s, \beta'_h, \beta'_g)'$. We introduce the following assumption.

ASSUMPTION A.1. *Assume that : (i) $Y_t = \beta'_* X_t + \varepsilon_t$, $t = 1, \dots, T$, with β_* the true value of β , $\mathbf{E}[\varepsilon|X_t] = 0$, and $\mathbf{E}[\varepsilon\varepsilon'|X_t] = \sigma^2 I$, where $\varepsilon = (\varepsilon_1, \dots, \varepsilon_T)'$; (ii) $\beta_{*j} \neq 0$, $\forall j \leq N_1$ and for $1 \leq s_g^* \leq N_g$, $\beta_{*g} = (\beta_{*g,1}, \dots, \beta_{*g,s_g^*}, \mathbf{0})'$, where $\mathbf{0}$ is a $(N_g - s_g^*)$ -vector of zeros and $\beta_{*g,j} \neq 0$ for all $j = 1, \dots, s_g^*$; (iii) $\varepsilon_t|X_t$, $t \geq 1$ are independent zero-mean sub-Gaussian random variables.*

Assumption A.1 (i) states that the true model is linear. Assumption A.1 (ii) states that the subvector of the true β_{*g} corresponding to the Google variables is s_g^* -sparse and that the true sparse model is $M^* := \{1, \dots, N_1\} \cup \{N_1 + j; j \in M_g^*\}$, where $M_g^* := \{1 \leq j \leq N_g : \beta_{*g,j} \neq 0\}$ is the subset of the true sparse model containing only the indices of the active Google variables with size $s_g^* = |M_g^*|$. Assumption A.1 (iii) assumes sub-Gaussianity of the errors conditional on the covariates X_t . This assumption is more general than assuming Gaussianity of $\varepsilon_t|X_t$ and allows for distributions whose tails are dominated by the tails of a Gaussian distribution.

For every $j \in \{1, \dots, N_g\}$, define $\widetilde{\beta}_{O,j} := \arg \min_{\beta_{O,j}^1, \beta_{g,j}} \mathbf{E}(Y_t - X'_{t,O}\beta_{O,j}^1 - x_{t,g,j}\beta_{g,j})^2$, which is the pseudo-true value of $\beta_{O,j} := (\beta_{O,j}^1, \beta_{g,j})' \in \mathbb{R}^{N_1+1}$ in the j -th misspeci-

fied model and define $\sigma_{O,j}^2 := \mathbf{E}[(Y_t - X'_{t,O,j}\tilde{\beta}_{O,j})^2]$. Misspecification arises because, in general, $\tilde{\beta}_{O,j}$ differs from the corresponding coefficients in β_* . Finally, $\mathcal{B} := \{\beta_{O,j}, j = 1, \dots, N_g; |\beta_{O,j,1}| \leq B, \dots, |\beta_{O,j,N_1}| \leq B, |\beta_{O,j,N_1+1}| \leq B\}$ for a large positive constant B is the set over which the Least Squares estimates in step (1) are searched. The next assumption allows us to control for the estimated $\sigma_{O,j}^2$ in the construction of the t-statistics.

ASSUMPTION A.2. *Assume that: (i) there exists a constant $c > 0$ such that $\mathbf{E}[\|x'_{t,g}\beta_{*g}\|_2^2] \leq c$; (ii) $\{(x'_{t,s}, x'_{t,h}, x'_{t,g})'\}_{t \geq 1}$ is a zero mean strictly stationary sequence with values in \mathbb{R}^{N-1} ; (iii) for every $j \in \{1, \dots, N_g\}$, $\mathbf{E}[\tilde{\varepsilon}_{t,j}^4 | X_{t,O,j}]$ is bounded, where $\tilde{\varepsilon}_{t,j} := (y_t - X'_{t,O,j}\tilde{\beta}_{O,j})$; (iv) there exist two constants $0 < \underline{C}_x^2 < \overline{C}_x^2 < \infty$ such that*

$$\underline{C}_x^2 \leq \min_{1 \leq j \leq N_g} \lambda_{\min}(\mathbf{E}[X_{t,O,j}X'_{t,O,j}]) \leq \max_{1 \leq j \leq N_g} \lambda_{\max}(\mathbf{E}[X_{t,O,j}X'_{t,O,j}]) \leq \overline{C}_x^2,$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix; (v) there exist two constants $0 < \underline{\sigma}_O^2 < \overline{\sigma}_O^2 < \infty$ such that $\underline{\sigma}_O^2 \leq \min_{1 \leq j \leq N_g} \sigma_{O,j}^2 \leq \max_{1 \leq j \leq N_g} \sigma_{O,j}^2 \leq \overline{\sigma}_O^2$.

The first three parts of the next assumption are the same as Conditions 1 and 2 in Barut et al. (2016) but made specific to our framework; we refer to that paper for comments about it. Parts (iv) and (v) of the next assumption allow us to control the deviation of an empirical process without assuming independency. In Sections C.5.1–C.5.2 of the Supplementary Appendix we verify parts (iv) and (v) of Assumption A.3 for the i.i.d. case. For a function h , define $\mathbb{G}_T[g(u_t)] := \frac{1}{\sqrt{T}} \sum_{t=1}^T [g(u_t) - \mathbf{E}(g(u_t))]$.

ASSUMPTION A.3. *Assume that: (i) for $j \in M_g^*$, there exist two positive constants $c_1, c_2 > 0$ and $0 < \kappa < 1/2$ such that $|\text{cov}(Y_t, x_{t,g,j} | X_{t,O})| \geq c_1 T^{-\kappa}$, and uniformly in $j \in \{1, \dots, N_g\}$: $\mathbf{E}[x_{t,g,j}^2] \leq c_2$; (ii) there exists a sufficiently large constant κ_T such that for $\epsilon_T := 16\kappa_T K_T(1 + \ell)\sqrt{(N_1 + 1)/(T\underline{C}_x^4)}$ with \underline{C}_x^2 given in Assumption A.2 (iv), ℓ a*

positive constant, and K_T an arbitrarily large constant, it holds for all $j \in \{1, \dots, N_g\}$:

$$\sup_{\beta \in \mathcal{B}; \|\beta - \tilde{\beta}_{O,j}\| \leq \epsilon_T} \left| \mathbf{E} \left(\left[(Y_t - X'_{t,O,j} \beta)^2 - (Y_t - X'_{t,O,j} \tilde{\beta}_{O,j})^2 \right] \mathbb{1}_{\Omega_{T,O,j}^c}(X_{t,O,j}, Y_t) \right) \right| \leq o(N_1/T),$$

where $\Omega_{T,O,j}^c = \{|Y_t| > m_0 K_T^\rho / s_0\} \cup_{l=1}^{N_1+1} \{|X_{t,O,j,l}| > K_T\}$; (iii) there exist positive constants m_0, m_1, s_0, s_1 , and ρ such that for sufficiently large τ , $P(|X_{t,j}| > \tau) \leq m_1 \exp\{-m_0 \tau^\rho\}$ for all $j \in \{2, \dots, N\}$, and $\mathbf{E}[\exp\{2\beta'_* X_t s_0\} + \exp\{-2\beta'_* X_t s_0\}] \leq s_1$; (iv) for every $\ell > 0$,

$$P \left(\sup_{\beta \in \mathcal{B}; \|\beta - \tilde{\beta}_{O,j}\| \leq \epsilon_T} \left| (\beta + \tilde{\beta}_{O,j})' \frac{1}{\sqrt{T}} \mathbb{G}_T [X_{t,O,j} X'_{t,O,j} \mathbb{1}_{\Omega_{T,O,j}}] (\beta - \tilde{\beta}_{O,j}) - 2 \frac{1}{\sqrt{T}} \mathbb{G}_T [Y_t X'_{t,O,j} \mathbb{1}_{\Omega_{T,O,j}}] (\beta - \tilde{\beta}_{O,j}) \right| \geq 4\epsilon_T \kappa_T K_T \sqrt{\frac{N_1+1}{T}} \ell \right) \leq e^{-2\ell^2},$$

where ϵ_T is defined as previously and $\mathbb{1}_{\Omega_{T,O,j}} := \mathbb{1}_{\Omega_{T,O,j}}(X_{t,O,j}, Y_t)$ with $\Omega_{T,O,j} := \{\|X_{t,O,j}\|_\infty \leq K_T, |Y_t| \leq m_0 K_T^\rho / s_0\}$, $\|\cdot\|_\infty$ the maximum norm, and m_0, ρ, K_T as introduced previously; (v) for every positive constants ℓ, b_1 , and K_T the arbitrarily large constant introduced previously and $\mathbb{1}_{T,O,j} := \mathbb{1}\{\|X_{t,O,j}\|_\infty \leq K_T\}$:

$$P \left(\left\| \frac{1}{T} \sum_{t=1}^T (X_{t,O,j} X'_{t,O,j} - \mathbf{E}[X_{t,O,j} X'_{t,O,j} \mathbb{1}_{T,O,j}]) \right\|_{op} > b_1 T^{-\ell} \middle| \Omega_{T,O,j} \right) \leq 2(N_1+1) \exp \left\{ -\frac{T^{1-2\ell} b_1^2}{4(N_1+1)^2 K_T^4 \max\{b_1, 1\}} \right\}.$$

The next theorem establishes the sure screening property of our selection procedure.

THEOREM 2.1. *Suppose that Assumptions A.1–A.3 hold. Let $\kappa_T := K_T B(N_1+1) + m_0 K_T^\rho / s_0$, with K_T and ρ given in Assumption A.3 (ii)–(iii). Assume that $T^{1-2\kappa} / (\kappa_T K_T)^2 \rightarrow \infty$ and that $T^{-\kappa/2} K_T^{\rho/2} = \mathcal{O}(1)$ with $\kappa < 1/2$ given in Assumption A.3 (i). Then, by taking $\lambda = c_0 T^{1/2-\kappa}$ for some constant $c_0 > 0$, it holds that*

$$P \left(M_g^* \subset \widehat{M}_g(\lambda) \right) \geq 1 - 8s_g^*(N_1+1) \exp \left\{ -\frac{\min\{c_2, b_1^2/4, 1/4\}}{\kappa_T^2 K_T^2} T^{1-2\kappa} \right\} - 6s_g^* T m_2 e^{-m_0 K_T^\rho} - 2s_g^* T \exp \left\{ -\frac{K_T^\rho}{4CK_1^2} \right\} - 2s_g^* \exp \left\{ -c_1 T^{1-2\kappa} \min \left\{ \frac{c_\epsilon^2}{4K_1^2}, \frac{c_\epsilon}{2K_1} \right\} \right\},$$

where b_1, C, c_ϵ are positive constants, $c_2 := \frac{C^2 c_1^2}{(256 N_1)}$, $m_2 := (N_1 m_1 + \sqrt{s_1} \sqrt{\mathbf{E}[\exp\{4C_m s_0^2 \|\varepsilon\|_{\psi_2}^2\}]})$ for some positive constants c_1, C_m and with $\|\varepsilon\|_{\psi_2} := \max_t \sup_{p \geq 1} p^{-1/2} (\mathbf{E}[|\varepsilon_t|^p | X_t])^{1/p}$, $K_1 := \max_{j \in M_g^*} \max_t \|\tilde{\varepsilon}_{t,j}^2\|_{\psi_1}$, with $\|\cdot\|_{\psi_1}$ denoting the sub-exponential norm.

We present a proof of this theorem in Appendix ???. The result of the theorem is similar to Barut et al. (2016, Theorem 3). The difference, between our result (and proof) and theirs, is the presence of additional terms in the lower bound of the probability. These terms are present in our result due to the variance estimation in our approach (to construct the t-statistic t_j and \widehat{M}_g). Instead, Barut et al. (2016) assume the variance to be known. If $1/(4CK_1^2) \geq m_0$, $\log(s_g^*) = o(\min\{T^{1-2\kappa}/(\kappa_T K_T)^2, K_T^\rho, T^{1-2\kappa}/(\max\{K_1^2, K_1\})\})$, $\log(N_1) \frac{(\kappa_T K_T)^2}{T^{1-2\kappa}} < \min\{c_2, b_1^2/4, 1/4\}$, and $\log(Tm_2) < m_0 K_T^\rho$, then Theorem 2.1 establishes that the selected submodel includes the true model M_g^* with probability approaching one. Moreover, if $\underline{c} < \min\left\{\frac{c_\epsilon^2}{K_1^2}, \frac{c_\epsilon}{K_1}\right\} < \bar{c}$, for two positive constants \underline{c}, \bar{c} , then the last two terms in the rate are negligible with respect to the other ones, and if we take the optimal order $K_T \asymp T^{(1-2\kappa)/A}$, where $A := \max\{4 + \rho, 2 + 3\rho\}$, then $P\left(M_g^* \subset \widehat{M}_g(\lambda)\right) \gtrsim 1 - s_g^* T m_2 \exp\{-CT^{(1-2\kappa)\rho/A}\}$. In this case, it follows from Lemma B.1 in the Supplementary Appendix that with our methodology we can deal with an N_g such that $\log(N_g) = o(T^{(1-2\kappa)\rho/A})$. Similarly, we can deal with an s_g^* and an m_2 such that: $\log(s_g^*) = o(T^{(1-2\kappa)\rho/A})$ and $\log(m_2) = o(T^{(1-2\kappa)\rho/A})$, which means that N_1 and $\|\varepsilon\|_{\psi_2}$ are allowed to increase at a certain rate. If $X_{t,j}$ are sub-Gaussian, then Assumption A.2 (iv) is satisfied with $\rho = 2$, which gives $\log(N_g) = o(T^{(1-2\kappa)/4})$.

2.3 Step 2: Ridge regression

Because GSD have a very high dimension, with the number of variables being much larger than the number of observations, even after the preselection in Step 1 the number of selected Google variables may still be large compared with the time dimension T . To deal with this large number of preselected covariates, in Step 2 we use Ridge regularization. Let $\widehat{M} = \widehat{M}(\lambda) := \{1, \dots, N_1\} \cup \{N_1 + j; j \in \widehat{M}_g(\lambda)\}$ and denote

$X_{t,\widehat{M}} := (1, x'_{t,s}, x'_{t,h}, x'_{t,g,\widehat{M}_g})'$, where $x_{t,g,\widehat{M}_g} := \{x_{t,g,j}; j \in \widehat{M}_g\}$ is the vector containing only the preselected Google variables. We estimate the parameter β in equation (2.1) by minimizing a penalized residuals sum of squares in which the penalty is given by the squared Euclidean norm $\|\cdot\|_2$, and we define the *Ridge after Model Selection* estimator as: $\widehat{\beta} := \widehat{\beta}(\alpha)$, where

$$\widehat{\beta}(\alpha) := \underset{\beta \in \mathbb{R}^N; \beta_{g,j}=0, j \in \widehat{M}_g^c}{\operatorname{argmin}} \left\{ \frac{1}{T} \sum_{t=1}^T (Y_t - \beta_0 - \beta'_s x_{t,s} - \beta'_h x_{t,h} - \beta'_g x_{t,g})^2 + \alpha \|\beta\|_2^2 \right\}, \quad (2.2)$$

and $\alpha > 0$ is a regularization parameter that tunes the amount of shrinkage. Without loss of generality, we can assume that the selected elements of $x_{t,g}$ corresponding to the indices in \widehat{M}_g are the first elements of the vector. Let $\mathbf{0}$ be the $(N_g - |\widehat{M}|)$ -dimensional column vector of zeros. Then, we can write $\widehat{\beta}$ as $\widehat{\beta} := (\widehat{\beta}'_{\widehat{M}}, \mathbf{0})'$, where

$$\widehat{\beta}_{\widehat{M}} = \widehat{\beta}_{\widehat{M}}(\alpha) = \left(\frac{1}{T} \sum_{t=1}^T X_{t,\widehat{M}} X'_{t,\widehat{M}} + \alpha I_{\widehat{M}} \right)^{-1} \frac{1}{T} \sum_{t=1}^T X'_{t,\widehat{M}} Y_t,$$

and $I_{\widehat{M}}$ is the $|\widehat{M}|$ -dimensional identity matrix with $|\widehat{M}| = N_1 + |\widehat{M}_g|$.

Empirical choice of the parameter α is based on the generalized cross-validation (GCV) technique (see Li, 1986, 1987), the idea of which is to choose a value for α for which the MSFE is as small as possible. Carrasco and Rossi (2016) recently used this technique in an in-sample forecasting setting. To complement their study, we show in Section 3.2 that GCV performs well for out-of-sample prediction as well. With GCV, the researcher selects the value of α that minimizes the following quantity:

$$GCV(\alpha) := \frac{\sum_{t=1}^T (Y_t - X'_{t,\widehat{M}} \widehat{\beta}_{\widehat{M}}(\alpha))^2}{T \left(1 - \operatorname{tr}(X_{t,\widehat{M}} (T^{-1} \sum_{t=1}^T X_{t,\widehat{M}} X'_{t,\widehat{M}} + \alpha I_{\widehat{M}})^{-1} X'_{t,\widehat{M}} / T) / T \right)^2},$$

where T denotes the number of quarters in the training sample and $\operatorname{tr}(\cdot)$ denotes the trace operator. We denote by $\widehat{\alpha}$ the value of α that minimizes $GCV(\alpha)$. In the next

section, we establish the theoretical properties of $\widehat{\beta}$ and $\widehat{\alpha}$.

3 Theoretical Properties

3.1 In-sample and Out-of-sample Prediction Error

In this section, we study the convergence to zero of the in-sample and OOS prediction error associated with the *Ridge after Model Selection estimator*. Asymptotic properties for the OOS prediction error associated with the Ridge estimator without model selection have been analysed in De Mol et al. (2008) and Carrasco and Rossi (2016) for dense models while asymptotic properties for the in-sample prediction error are well known in the inverse problems literature; for example see Carrasco et al. (2007) and for a Bayesian interpretation of the Ridge estimator see Florens and Simoni (2012, 2016).

To the best of our knowledge, extant literature has not established the theoretical properties of the Ridge estimator coupled with a targeted selection. Here, we fill this gap and establish an upper bound for both the in-sample and OOS prediction error for sparse models. This upper bound gives the rate of convergence as $N, T \rightarrow \infty$. It also gives the rate for dense models – that is, when $s_g^* = N_g$.

Let $X := (X_1, \dots, X_T)'$ be a $(T \times N)$ matrix. Let M^{*c} denote the complementary set of $M^* := \{1 \leq j \leq N : \beta_{*j} \neq 0\}$ in $\{1, \dots, N\}$ with $s^* := |M^*|$. For a vector $\beta \in \mathbb{R}^N$ and an index set $M \subset \{1, \dots, N\}$, denote $\beta_M := (\beta_{M,j})_{j=1}^N$ with $\beta_{M,j} := \beta_j \mathbb{1}\{j \in M\}$, and for a $(T \times N)$ matrix X , denote by X_M the $(T \times |M|)$ matrix made of the columns of X corresponding to the indices in M , and denote by $X_{t,M}$ the transpose of the t -th row of X_M . Thus, β_M has zero outside the set M . For a vector $\delta \in \mathbb{R}^N$ and given covariates X_t , $t = 1, \dots, T$, define the squared prediction norm of δ as $\|\delta\|_{2,T}^2 := \delta' X' X \delta / T$, the ℓ_0 -norm of δ as $\|\delta\|_0 := \sum_{j=1}^N \mathbb{1}\{\delta_j \neq 0\}$ and the Euclidean norm as $\|\delta\|_2 := \sqrt{\delta' \delta}$.

Next, we introduce an assumption known in the literature as a restricted sparse eigenvalue condition on the empirical Gram matrix $X'X/T$ (see *e.g.*, Belloni and Cher-

nozhukov, 2013), and it is an extension of the restricted isometry condition (e.g., Candes and Tao, 2007). The quantity m in the assumption restricts the number of nonzero components outside the set M^* of the vectors $\delta \in \mathbb{R}^N$ considered. The larger m is, the more restrictive the first part of the assumption is.

ASSUMPTION A.4. For a given $m < T$, for a $\delta \in \mathbb{R}^N$, with probability $1 - o(1)$, $\underline{\varphi}(m)^2 := \min_{\|\delta_{M^*c}\|_0 \leq m, \delta \neq 0} \frac{\|\delta\|_{2,T}^2}{\|\delta\|_2^2} > 0$ and $\overline{\varphi}(m) := \max_{\|\delta_{M^*c}\|_0 \leq m, \delta \neq 0} \frac{\|\delta\|_{2,T}^2}{\|\delta\|_2^2} > 0$.

Chernozhukov et al. (2021, Theorem B.2) provide primitive conditions on the population covariance matrix ensuring that this assumption holds for covariates X_t with temporal dependence. We define the condition number associated with the empirical Gram matrix $(X'_{\widehat{M}} X_{\widehat{M}})/T$: $\mu(\widehat{m}) = \frac{\sqrt{\overline{\varphi}(\widehat{m})}}{\underline{\varphi}(\widehat{m})}$, where $\widehat{m} := |\widehat{M} \setminus M^*| \mathbb{1}\{\widehat{M} \supseteq M^*\}$ is the number of incorrect covariates selected. Similarly, define $\widehat{k} := |M^* \setminus \widehat{M}| \mathbb{1}\{M^* \not\subseteq \widehat{M}\}$. We start by establishing an upper bound on the in-sample prediction error. For a proof see the Supplementary Appendix C.1.

THEOREM 3.1 (In-sample prediction error). Suppose that Assumptions A.1 (i)–(ii) and A.4 are satisfied and that $\varepsilon_t|X_t$ is Gaussian. Let \widehat{M} be the model selected in the first step. Let $\widehat{\beta}$ be the Ridge estimator defined in equation (2.2). Then, for every $\epsilon > 0$, there is a constant K_ϵ independent of T such that with probability at least $1 - \epsilon$,

$$\begin{aligned} \|\widehat{\beta} - \beta_*\|_{2,T} \leq & \left(K_\epsilon \sqrt{\frac{(\widehat{m} + s^*)[\log(N) + \log(e^2 \mu(\widehat{m}))]}{T}} + 2\alpha \|\beta_*\|_2 \frac{1}{\underline{\varphi}(\widehat{m})} \right) \mathbb{1}\{M^* \subseteq \widehat{M}\} + \\ & \left(\frac{K_\epsilon \sigma}{\sqrt{T}} \sqrt{(\widehat{k} + \widehat{m}) (\log(s^* + \widehat{m}) + \log(e^2 \mu(\widehat{k} + \widehat{m})))} + \frac{2\alpha}{\underline{\varphi}(\widehat{m})} \|\beta_*\|_2 + \|\beta_{*, M^* \setminus \widehat{M}}\|_{2,T} \right) \mathbb{1}\{M^* \not\subseteq \widehat{M}\}. \end{aligned}$$

As noted in the discussion following Theorem 2.1, $P(M^* \subset \widehat{M}) \rightarrow 1$ under some conditions (and so is the probability of $\{M^* \subseteq \widehat{M}\}$). We remark that on the event $\{M^* \not\subseteq \widehat{M}\}$, instead, we get a bias term given by $\|\beta_{*, M^* \setminus \widehat{M}}\|_{2,T}$, which is intuitive because the second-step Ridge estimator is always biased for the components in $M^* \setminus \widehat{M}$.

The next corollary establishes an upper bound for the Euclidean norm of $(\widehat{\beta} - \beta_*)$.

Corollary 3.1 (Coefficient estimation). Suppose that Assumptions A.1 (i)–(ii) and A.4 are satisfied and that $\varepsilon_t|X_t$ is Gaussian. Let \widehat{M} be the model selected in the first step.

Let $\widehat{\beta}$ be the Ridge estimator defined in equation (2.2). Then, for every $\epsilon > 0$, there is a constant K_ϵ independent of T such that with probability at least $1 - \epsilon$,

$$\begin{aligned} \|\widehat{\beta} - \beta_*\|_2 &\leq \left(K_\epsilon \sqrt{\frac{(\widehat{m} + s^*)[\log(N) + \log(e^2 \mu(\widehat{m}))]}{T \underline{\varphi}(\widehat{m})^2}} + 2\alpha \|\beta_*\|_2 \frac{1}{\underline{\varphi}(\widehat{m})^2} \right) \mathbb{1}\{M^* \subseteq \widehat{M}\} + \\ &\left(\frac{K_\epsilon \sigma}{\underline{\varphi}(\widehat{m}) \sqrt{T}} \sqrt{(\widehat{k} + \widehat{m}) (\log(s^* + \widehat{m}) + \log(e^2 \mu(\widehat{k} + \widehat{m})))} + \frac{2\alpha}{\underline{\varphi}(\widehat{m})^2} \|\beta_*\|_2 + \frac{\|\beta_{*, M^* \setminus \widehat{M}}\|_{2, T}}{\underline{\varphi}(\widehat{m})} \right) \mathbb{1}\{M^* \not\subseteq \widehat{M}\}. \end{aligned}$$

Compared with the upper bound for the in-sample prediction error, every term in the upper bound in Corollary 3.1 has an additional factor of $1/\underline{\varphi}(\widehat{m})$. As seen in Assumption A.4, $\underline{\varphi}(\widehat{m})$ must be interpreted as the smallest restricted eigenvalue of the empirical Gram matrix and so it can be small when N is large. Therefore, the upper bound in Corollary 3.1 can be larger than the upper bound in Theorem 3.1.

In the next theorem, we establish an upper bound for the OOS prediction error. Let $(Y_\tau, X'_\tau)'$, $\tau > T$, be a new copy of $(Y_t, X'_t)'$ that satisfies Assumption A.1 (i)-(ii) and that is independent of (Y, X) with $Y := (Y_1, \dots, Y_T)'$.

Corollary 3.2 (OOS prediction error). *Let the assumptions and the notations of Theorems 2.1 and 3.1 hold. Let $\widehat{\beta}$ be the Ridge estimator defined in equation (2.2). Take $K_T \asymp T^{(1-2\kappa)/A}$, where $A := \max\{4 + \rho, 2 + 3\rho\}$, and let $c < \min\left\{\frac{c_\epsilon^2}{K_1^2}, \frac{c_\epsilon}{K_1}\right\} < C$ for two positive constants c, C , $\log(N_g m_2) = o(T^{(1-2\kappa)\rho/A})$. Then there is a constant K_ϵ independent of T such that with probability converging to 1, conditional on X_τ ,*

$$X'_\tau(\widehat{\beta} - \beta_*) \leq \|X_{\tau, \widehat{M}}\|_2 \left(K_\epsilon \sqrt{\frac{(\widehat{m} + s^*)[\log(N) + \log(e^2 \mu(\widehat{m}))]}{T \underline{\varphi}(\widehat{m})^2}} + 2\alpha \|\beta_*\|_2 \frac{1}{\underline{\varphi}(\widehat{m})^2} \right).$$

The upper bound for the OOS prediction error is larger than the upper bound for the in-sample prediction error because X_τ has dimension N , which is large. Preselection allows us to reduce this dimension from N to $(\widehat{m} + s^*)$, and we do not need to assume that $\|X_\tau\|_2 = O_p(1)$ as, for example, in Carrasco and Rossi (2016).

3.2 Out-of-sample evaluation of the selection of α

In this section, we evaluate the performance of $\hat{\alpha}$, the minimizer of the GCV as described in Section 2.3, for OOS prediction which is the objective of nowcasting for central bankers. Previous research (Li, 1986; Carrasco and Rossi, 2016) has established optimality of GCV minimization for in-sample prediction in the setting of Ridge regularization, but these studies do not consider optimality for OOS prediction. Although Leeb (2008) does so in a setting different from Ridge regularization to evaluate the performance of model selection our proofs depart entirely from their proofs.

Consider a new copy $(Y_{T+1}, X'_{T+1})'$ of $(Y_t, X'_t)'$ that satisfies Assumption A.1 (i)–(ii) and that is independent of $(Y^{(T)}, X^{(T)})$, where $Y^{(T)} := (Y_1, \dots, Y_T)'$ and $X^{(T)} := (X_1, \dots, X_T)'$, that is, $Y_{T+1} = \sum_{j=1}^{s^*} X_{T+1,j} \beta_{*,j} + \varepsilon_{T+1} = \sum_{j=1}^{\hat{m}+s^*} X_{T+1,j} \beta_{*,j} + \varepsilon_{T+1}$ with $\beta_{*,j} = 0$ for every $j \in \{s^*+1, \dots, \hat{m}+s^*\}$, $\mathbf{E}[\varepsilon_{T+1}|X_{T+1}] = 0$, and $\text{Var}(\varepsilon_{T+1}|X_{T+1}) = \sigma^2$. When researchers aim to forecast, they want that the $\hat{\alpha}$ selected by GCV (based on the sample (Y, X)) be optimal for OOS prediction. The OOS performance of a selected value $\hat{\alpha}$ is evaluated by considering the conditional mean squared prediction error given by

$$\rho^2(\alpha; Y^{(T)}, X^{(T)}) := \mathbf{E}[(Y_{T+1} - \hat{\beta}(\alpha)' X_{T+1})^2 | Y^{(T)}, X^{(T)}, \mathcal{A}],$$

where $\hat{\beta}(\alpha)' X_{T+1} = \sum_{j=1}^{\hat{m}+s^*} X_{T+1,j} \hat{\beta}_j(\alpha)$, $\hat{\beta}(\alpha)$ is the *Ridge after Model Selection* estimator defined in equation (2.2), and $\mathcal{A} := \{M^* \subseteq \widehat{M}\}$. Theorem 3.2 provides the rate at which the GCV criterion converges to $\rho^2(\alpha; Y^{(T)}, X^{(T)})$ uniformly over α in a given region as $T \rightarrow \infty$. To this end, we introduce the following notation: for a vector $\beta \in \mathbb{R}^N$ and an index set $M \subset \{1, \dots, N\}$, denote $\underline{\beta}_M := (\beta_j)_{j \in M} \in \mathbb{R}^{|M|}$, $\widehat{\Sigma}_M := X'_M X_M / T$, $\Sigma_M := \mathbf{E}[X_{t,M} X'_{t,M}]$, and $\mathcal{B}(M) := \left\{ X^{(T)}; \left\| \widehat{\Sigma}_M - \Sigma_M \right\|_{op} \leq C \sqrt{\frac{|M|}{T}} \right\}$ with $0 < C < \infty$ as a universal constant. For a matrix A , $\|A\|_{op}$ denotes its operator norm.

ASSUMPTION A.5. *Assume that: (i) $\mathbf{E}[\varepsilon_t^4] < C_1$ for some constant $0 < C_1 < \infty$; (ii) for every t , $\mathbf{E}[X_{t,\widehat{M}} X'_{t,\widehat{M}} | X^{(t-1)}, \mathcal{A}] = \Sigma_{\widehat{M}}$, and $\Sigma_{\widehat{M}}$ is bounded almost surely; (iii)*

$P(M^* \notin \widehat{M}) = o(r_{\mathcal{A},T})$, where $r_{\mathcal{A},T}$ is a non-stochastic sequence independent of α that converges to zero as $T \rightarrow \infty$; (iv) $P(\mathcal{B}(\widehat{M})^c) = o(r_{\mathcal{B},T})$, where $r_{\mathcal{B},T}$ is a non-stochastic sequence independent of α that converges to zero as $T \rightarrow \infty$; and (v) for any index set $M \subset \{1, \dots, N\}$, there exist a $w(M) \in \mathbb{R}^{|M|}$ and $\gamma > 0$ such that $\underline{\beta}_{*,M} = \Sigma_M^{\gamma/2} w(M)$ and $\|w(M)\|_2 < \infty$ (source condition).

Assumption A.5 (iii) is satisfied by our preselection method in Step 1 under Assumptions A.1–A.3. Theorem 2.1 gives an explicit expression for $r_{\mathcal{A},T}$ under those assumptions. Assumption A.5 (iv) is satisfied, for example, if $X_{t,\widehat{M}}$ are independent sub-Gaussian random vectors with sub-Gaussian norm bounded by a constant K . In this case, C in the definition of \mathcal{B} is equal to $4K^2$ and $r_{\mathcal{B},T} = 2\mathbf{E}[\exp\{2\log(\widehat{m} + s^*) - c \min\{\sqrt{\frac{\widehat{m}+s^*}{T}}, \frac{\widehat{m}+s^*}{T}\}T\}]$ for a numerical constant $c > 0$ by the Bernstein’s inequality for sub-exponential random variables (see, e.g. Vershynin, 2018, Theorem 1.4.1).

THEOREM 3.2. *Let Assumptions A.1 (i)–(ii) and A.5 hold. Let $\varepsilon_t|X_t$, $t \geq 1$ be independent zero-mean random variables, $(\widehat{m} + s^*)/T < 1$ for every $T \geq 1$ with probability 1, and $\tilde{\gamma} := \min\{\gamma, 2\}$. Then, for every $\alpha > 0$,*

$$\begin{aligned} & |\rho^2(\alpha; Y^{(T)}, X^{(T)}) - GCV(\alpha)| \\ &= \mathcal{O}_p\left(\alpha^{\tilde{\gamma}} + \frac{1}{\alpha\sqrt{T}} + \frac{1}{\sqrt{T}} + \frac{(\widehat{m} + s^*)}{T}\alpha^{(\tilde{\gamma}-2)}\right) + \mathcal{O}_p(\max\{r_{\mathcal{A},T}, r_{\mathcal{B},T}\}). \end{aligned}$$

Moreover, for any constants $0 < \underline{\alpha} < \infty$ and $0 < \underline{u} < 1/2$, and for a sequence $\bar{\alpha}_T \rightarrow 0$ as $T \rightarrow \infty$ such that: $T^{-1/2+\underline{u}}/\bar{\alpha}_T \rightarrow 0$, it holds that

$$\sup_{\alpha \in [\underline{\alpha}T^{-1/2+\underline{u}}, \bar{\alpha}_T]} |\rho^2(\alpha; Y^{(T)}, X^{(T)}) - GCV(\alpha)| = \mathcal{O}_p(r_T) + \mathcal{O}_p(\max\{r_{\mathcal{A},T}, r_{\mathcal{B},T}\}),$$

where $r_T := \bar{\alpha}_T^{\tilde{\gamma}} + T^{-\underline{u}} + \frac{(\widehat{m}+s^*)}{T}T^{(2-\tilde{\gamma})(1-2\underline{u})/2}$.

This theorem establishes the rate at which the conditional mean squared prediction error converges to the *GCV* criterion. If $\frac{(\widehat{m}+s^*)}{T^{2\underline{u}T^{\tilde{\gamma}(1/2-\underline{u})}}} \rightarrow 0$, then the convergence of the two

criteria is uniform over a shrinking interval. The next theorem establishes the OOS optimality of the GCV-minimizer $\hat{\alpha}$.

THEOREM 3.3. *Consider the minimizers of $GCV(\alpha)$ and $\rho^2(\alpha; Y^{(T)}, X^{(T)})$:*

$$\hat{\alpha} = \arg \min_{\alpha \in [\underline{\alpha}T^{-1/2+\underline{u}}, \bar{\alpha}_T]} GCV(\alpha)$$

and $\alpha^* = \arg \min_{\alpha \in [\underline{\alpha}T^{-1/2+\underline{u}}, \bar{\alpha}_T]} \rho^2(\alpha; Y^{(T)}, X^{(T)})$, where $\underline{\alpha}$, \underline{u} , and $\bar{\alpha}_T$ are as defined in Theorem 3.2. Then, in the setting of Theorem 3.2, (i) $\hat{\alpha}$ is as good as α^* for OOS prediction in the sense that

$$|\rho^2(\hat{\alpha}; Y^{(T)}, X^{(T)}) - \rho^2(\alpha^*; Y^{(T)}, X^{(T)})| = \mathcal{O}_p(r_T) + \mathcal{O}_p(\max\{r_{A,T}, r_{B,T}\}),$$

where r_T is defined in Theorem 3.2, and (ii) the OOS predictive performance can be consistently estimated in the sense that

$$|GCV(\hat{\alpha}) - \rho^2(\hat{\alpha}; Y^{(T)}, X^{(T)})| = \mathcal{O}_p(r_T) + \mathcal{O}_p(\max\{r_{A,T}, r_{B,T}\}).$$

4 Monte Carlo exercise

This section presents the results of a simulation exercise. We are interested in understanding how the dimension of the problem, N and T , the degree of sparsity s^* in the model, and the correlation among the predictors affect the nowcasting performance of our estimation method in finite sample. To this end, we conduct two exercises. The first consists in comparing our *Ridge after Model Selection* procedure with the most commonly used methods in the macroeconomic nowcasting/forecasting literature. For the description and results of this exercise, see Section D in the Supplementary Appendix. Here, we show the results of the second exercise, in which we examine the effect of varying N, T, s^* on the in-sample and OOS prediction error to confirm the theoretical

results in Section 3.1.

The data are simulated according to the following DGP: $t = 1, \dots, T$,

$$\begin{aligned} y_t &= \gamma' z_t + \beta' x_t + v_t, & z_t = (z_{1,t}, z_{2,t})' &\sim \mathcal{N}_2 \left(0, \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \right), \\ \underset{(N \times 1)}{x_t} &= \underset{(N \times 1)}{\delta'} z_t + u_t, \end{aligned} \tag{4.1}$$

where $\gamma = (1, 2)'$, $u_t \sim \mathcal{N}_N(0, \Psi)$, Ψ is an $(N \times N)$ -full rank covariance matrix, and $v_t \sim \mathcal{N}(0, 1)$. In the DGP for y_t , we have two sets of covariates: z_t , which we are sure is in the model, and x_t , which must be preselected. The possible sparsity of the model only affects x_t . Specification of β governs the sparsity of the model, δ determines the correlation between x_t and z_t , and Ψ affects the correlation among covariates in x_t that are included in the model and those that are not. We consider a sparse structure: $\beta_j \sim \mathcal{N}(0, 1)$ for $j \leq s^*$ and $\beta_j = 0$ for $j > s^*$.

For the parameter vector δ , we use the specifications: $\delta = 0.2\iota$ and $\delta = 0.8\iota$ with ι denoting a $(2 \times N)$ matrix of ones. For the covariance matrix Ψ , we consider two cases: (I) *uncorrelated*: $\Psi = I_N$ and (II) *decreasing correlation*: $\Psi = (|0.5|^{j-k})_{j,k}$. For N, T, s , we consider: $(N = 150, T = 100, s = 105)$, $(N = 200, T = 150, s = 105)$, $(N = 200, T = 150, s = 110)$, and $(N = 200, T = 100, s = 110)$. We adjust split between in-sample and OOS to keep the same fraction of observations in the two samples.

We present the results in Tables 1-2 in Appendix A as ratios with respect to the case $N = 150, T = 100, s = 105$. That is, we show the in-sample mean squared error (MSE) of each triplet (N, T, s) relative to the in-sample MSE for $N = 150, T = 100, s = 105$, labeled ‘‘MSER’’, and the OOS MSE of each triplet (N, T, s) relative to the OOS MSE for $N = 150, T = 100, s = 105$, labeled ‘‘MSFER’’. The threshold parameter λ is set equal to the $\{80\%, 90\%, 95\%, 97.5\%, 99\%, 99.5\%\}$ -quantiles of a $\mathcal{N}(0, 1)$ distribution and is indicated in the first column of Tables 1 and 2. This choice corresponds to a false discovery rate (FDR) equal to 20%, 10%, 5%, 2.5%, 1%, 0.5%, respectively.

We see that when T increases from 100 to 150, both the MSE (in-sample error) and the MSFE (OOS error) decrease even if N increases as well (first two columns in Tables 1-2). When s is also increasing (going from $s = 105$ to $s = 110$), we see a reduction in the MSE and MSFE, even if it is smaller (third and fourth columns in Tables 1-2), and for the values of λ corresponding to a FDR of 20% (and also 10% in the case $\delta = 0.2\iota$), only the MSFE decreases. The conclusions are similar for the two structures of Ψ . Instead, when N and s increase but T remains fixed (last two columns in Tables 1-2), the MSE is decreasing in most of the cases but the MSFE is increasing. This illustrates our extra term $\|X_{\tau, \widehat{M}}\|_2$ in the upper bound for the OOS prediction error in Corollary 3.2.

5 Empirical Study

In this section, we present the empirical results obtained by applying our *Ridge after Model selection* procedure to nowcast GDP growth rate with weekly GSD for three countries/areas: the euro area (EA, hereafter), the United States, and Germany. We present our OOS results for three various phases of the business cycle: a calm period (2014–2016), a period with a sudden downward shift in GDP growth (2017–2018) and a recession period with large negative growth rates (2008–2009). We consider 2014–2016 a calm period because it does not show any strong GDP movements, excepting a decline in oil prices starting from mid-2014. The period 2017–2018 is interesting in that although the global economy was recovering in 2017 at a faster pace than economists expected the trade war initiated by the Trump administration came by surprise, leading to a sharp slowdown in global GDP growth amidst rising uncertainties around global trade. Last, all the considered countries/areas experienced a large drop in the level of GDP during the Great Recession of 2008–2009, in the wake of the global financial crisis. During these three periods, the quality and sample size of GSD may have improved, for instance, due to the population’s improved access to internet. While we implicitly assume

that no structural change in the quality of GSD occurred over the sample considered, interpretation of our empirical findings should take this assumption into account.

We compute the empirical results against a background of pseudo real-time analysis; that is, we account for the release dates of official variables, but we do not use vintages of data.

5.1 Design of the empirical analysis

The objective of this empirical application is to nowcast on a high-frequency basis quarterly GDP growth rate (variable Y_t in model (2.1)) for three countries/areas. Official GDP data come from Eurostat for EA as a whole, from Destatis for Germany, and from the Bureau of Economic Analysis for the U.S. The official monthly macroeconomic series that we use as regressors $x_{t,h}$ and $x_{t,s}$ are, respectively, the growth rate of the industrial production index, which is the measure of hard data most used by practitioners, denoted by IP_t , and a composite index of opinion surveys from various sectors as a proxy for soft variables, denoted by S_t . As regards Germany and EA, we use for S_t sentiment indexes computed by the European Commission, and we use the well known ISM index for the U.S. economy.

GSD are weekly data received and made available by the European Central Bank every Tuesday. GSD are data related to queries performed with Google search machines. Google assigns queries to particular categories using natural language processing methods. GSD are not the same as Google Trends data but rather are indexes of weekly volume changes of Google queries grouped by category and by country. Data are normalized at 1 at the first week of January 2004, the first week of their availability. Then, the following values indicate the deviation from the first value. The GSD we use for our study cover weekly Google searches for the six main EA countries: Belgium, France, Germany, Italy, the Netherlands, and Spain, as well as for the United States, ranging from the January 1, 2004, to December 31, 2018. For each country, we have at our

disposal a total of $N_g = 296$ variables (categories). When dealing with the EA as the whole, we account for information conveyed by the six main countries, meaning that we have access to a total of $N_g = 1,776$ GSD variables.

Treating weekly data is particularly challenging as the number of entire weeks present in every quarter is not always the same, and a careful analysis must be done when incorporating these data. Original data are not seasonally adjusted, thus we take the growth rate over 52 weeks to eliminate the seasonality within the data, a standard procedure when dealing with weekly data (see Lewis et al., 2020). To account for the variation over a quarter, we add a second differentiation over 13 weeks. Consequently, models are estimated on a recursive basis starting the last week of March 2005.

In addition to the challenge of frequency mismatch in the data, we must address the fact that data on official series and GSD are released with various reporting lags, which leads to an unbalanced information set at each point in time within the quarter. In the literature, this issue is referred to as a *ragged-edge database* (see Angelini et al., 2011).

Because of the different frequencies of $x_{t,s}$, $x_{t,h}$, and $x_{t,g}$ – monthly and weekly, respectively – and of the various reporting lags, the relevant information set for calculating the nowcasts evolves within the quarter. We assume that a given quarter is made up of thirteen weeks. Thus, by letting $x_{t,j,(v)}$ designate the vector of series in $x_{t,j}$ released at week v of period t and $x_{t,j,(v),i}$ the i -th series in $x_{t,j,(v)}$, we denote by $x_{t,j,i}^{(w)}$ a summary of $\{x_{t,j,(v),i}, 1 \leq v \leq w\}$. Then, we define the relevant information set at week w of a quarter t as $\Omega_t^{(w)} := \{x_{t,j,i}^{(w)}, i = 1, \dots, N_j, j \in \{s, h, g\}\}$. For simplicity, we keep in $\Omega_t^{(w)}$ only the observations relative to the current quarter t and do not consider past observations. While the series in $x_{t,g}$ are in $\Omega_t^{(w)}$ for every $w = 1, \dots, 13$, the other variables are in the relevant information set only for the weeks corresponding to (and after) their release, and so the data set is unbalanced.

To explicitly account for the relevant information set at each week of the quarter, we replace model (2.1) by a model for each week w denoted by $M_{(w)}$ and defined as:

$\forall t = 1, \dots, T, \forall w = 1, \dots, 13,$

$$M_{(w)} : \quad \mathbf{E}[Y_t | \Omega_t^{(w)}] = \beta_{0,w} + \beta'_{s,w} x_{t,s}^{(w)} + \beta'_{h,w} x_{t,h}^{(w)} + \beta'_{g,w} x_{t,g}^{(w)}, \quad (5.1)$$

where $x_{t,g}^{(w)} := (x_{t,g,i}^{(w)})_{i=1,\dots,N_g}$, $\beta_{j,w,i} = 0$ if $x_{t,j,i}^{(w)} \notin \Omega_t^{(w)}$. For instance, as the first observation of EA-industrial production relative to the current quarter t is released in week 9, we set the corresponding $\beta_{h,w} = 0$ for every $w = 1, \dots, 8$. The bridge equation (5.1) exploits weekly information to obtain more accurate nowcasts of Y_t . The idea of having 13 models is that a researcher aiming to nowcast the current-quarter values of Y_t will use the model corresponding to the current week of the quarter. For instance, to nowcast the current-quarter value of Y_t at the end of week 2, we would use model $M_{(2)}$.

With regards to macroeconomic series, we mimic the exact release dates as published by statistical offices. Within the EA, for instance, the first survey of the quarter, referring to the first month, typically arrives in week 5. Then, the second survey of the quarter, related to the second month, is available in week 9. The IP_t for the first month of the quarter is available about 45 days after the end of the reference month, which is generally in week 11. Finally, the last survey, related to the third month of the quarter, is available in week 13 (see Figure 1 in the Supplementary Appendix). For the U.S. economy, survey data are released at the same dates as for the EA, but industrial production data are released about four weeks earlier.

To construct the vector $x_{t,g}^{(w)}$ in equation (5.1) containing GSD variables, we take for each Google variable the sample average of its observations over the period week 1 to week w of quarter t . That is, we construct $x_{t,g}^{(w)} = \sum_{v=1}^w x_{t,g,(v)} / w$. Take, for instance, $w = 3$ (*i.e.* model 3 which is used at week 3), then $x_{t,g}^{(3)}$ is equal to $(x_{t,g,(1)} + x_{t,g,(2)} + x_{t,g,(3)}) / 3$.

For the survey S_t in $x_{t,s}^{(w)}$ and the industrial production IP_t in $x_{t,h}^{(w)}$, we impose the following structure, which mimics the data release explained previously in the case of the EA. The variable $x_{t,s}^{(w)}$ is not present in models 1 to 4 because the current S_t is not available in the first four weeks of the quarter, so that $\beta_{s,1} = \beta_{s,2} = \beta_{s,3} = \beta_{s,4} = 0$.

Then, for models $w \in \{5, \dots, 8\}$, $x_{t,s}^{(w)}$ is the value of S_t for the first month of the quarter: $x_{t,s}^{(w)} = S_{t,1}$, where $S_{t,i}$ denotes the variable S_t referring to the i -th month of quarter t . In models $w \in \{9, \dots, 12\}$, $x_{t,s}^{(w)}$ is equal to the average of the survey data available at the end of the first and second months of the quarter: $x_{t,s}^{(w)} = (S_{t,1} + S_{t,2})/2$. Lastly, in model 13, $x_{t,s}^{(w)}$ is the average of the survey data over the quarter: $x_{t,s}^{(w)} = (S_{t,1} + S_{t,2} + S_{t,3})/3$. Similarly, the variable $x_{t,h}^{(w)}$ is not present in models $w \in \{1, \dots, 10\}$ (so that $\beta_{h,1} = \dots = \beta_{h,10} = 0$), and in models $w \in \{11, \dots, 13\}$, $x_{t,h}^{(w)}$ is the value of IP_t for the first month of quarter t . For Germany and the United States we use a similar structure that mimics the specific data release for these countries.

We split our data set in two non-overlapping subsamples: the training set and the OOS set. The latter starts at 2014q1, 2017q1, or 2008q1, depending on the period we are considering, and the training sample always finishes two quarters before the beginning of the OOS period to take into account the delay due to the release of GDP figures (see Section E.5 in the Supplementary Appendix). We use a recursive scheme method, that is, we re-estimate the parameters at each new nowcasting quarter using all the past information available until the penultimate quarter before the nowcasting one.

In the implementation of the preselection step of our procedure, we can take into account the problem of frequency mismatch among the different sets of variables and make each monthly/weekly series comparable to the quarterly Y_t series in terms of frequency by replacing each predictor with either its value available at a given week during the quarter t or with the average of its observations over the quarter t . In the second case, we replace $x_{t,s}$, $x_{t,h}$, and $x_{t,g}$ by $\sum_{m=1}^3 x_{t,s,(m)}/3$, $\sum_{m=1}^3 x_{t,h,(m)}/3$, and $x_{t,g}^{(13)} := \sum_{w=1}^{13} x_{t,g,(w)}/13$, respectively, with $x_{t,s,(m)}$, $x_{t,h,(m)}$ and $x_{t,g,(v)}$ denoting the vectors of soft, hard, and Google variables released at month m and week v of quarter t , respectively. In this application, we carry out the preselection step only once by using the training sample available for the first nowcast origin and the average values of the observations on the covariates over each quarter (see Section E.4 in the Supplementary Appendix for a detailed expla-

nation). For the choice of the tuning parameter α , we choose it for each model w by minimizing the GCV criterion $GCV(\alpha)$.

5.2 Overall evaluation of GSD

This subsection presents our empirical results for the three countries/areas and the three economic periods as explained previously to answer questions (A) and (B) in the Section 1. We estimate the following models: (a) the nowcasting models $M_{(1)}, \dots, M_{(13)}$ accounting for the full set of information (GSD, hard data, and soft data) without preselection, (b) the nowcasting models $M_{(1)}, \dots, M_{(13)}$ accounting for the full set of information (GSD, hard data, and soft data) with the preselection of Step 1 (i.e. Ridge after Model selection approach), (c) the nowcasting models $M_{(1)}, \dots, M_{(13)}$ by using only GSD (i.e. $\beta_{s,w} = \beta_{h,w} = 0$ for every $w = 1, \dots, 13$ in Equation (5.1)), and (d) the models that only account for hard and soft data (i.e., without GSD, $\beta_{g,w} = 0$ for every $w = 1, \dots, 13$ in Equation (5.1)). Root MSFE (RMSFE) results are presented in Tables 3–11 in Appendix A, each row corresponding to those four models. The complete tables containing the RMSFEs for six values of τ are in Section E.7 in the Supplementary Appendix. From there we see how the choice of λ impacts the nowcasting accuracy, and for policy-makers, we recommend to use small values of τ (1% or 0.5%).

By looking at rows 2–4 of Tables 3–11, we can compare nowcasts obtained with and without GSD in a pseudo real-time exercise to assess: (i) if GSD are informative when no official data are available for the forecaster and (ii) the extent to which GSD remain informative when official data become available. The first stylized fact that we observe in our results is the downward sloping evolution of RMSFEs throughout the quarter stemming from models $M_{(1)}, \dots, M_{(13)}$ with full information (second row). This result is in line with what could be expected from nowcasting exercises when integrating increasingly more information throughout the quarter (see Angelini et al. (2011)). Second, when using only Google information (third row), we still observe a decline of

RMSFEs throughout the quarter but to a much lower extent. Third, when focusing on the beginning of the quarter, models that only integrate Google information provide very reasonable RMSFEs, which are slightly higher than those obtained at the end of the quarter. This means that using only Google data at the beginning of the quarter, when no other official information is available about the current state of the economy, is a pertinent strategy for economists aiming at tracking GDP.

Let us now focus on the gain from using our *Ridge after Model Selection* strategy. During both calm and sudden shift periods (namely, 2014–2016 and 2017–2018), this estimation strategy applied to the whole data set (hard data, soft data, and GSD) generally tends to provide the lowest RMSFEs (second row of Tables 3–11 in the Appendix, lowest values in boldface). This result, which is robust over the different countries/areas, does not hold during the recession period (2008–2009). Thus, we have here few important results. First, by comparing the first and second rows of Tables 3–11, we can conclude that, outside recession periods, our *Ridge after Model Selection* strategy outperforms a strategy that would skip the data preselection step. Second, when comparing the second, third and fourth rows of Tables 3–11, we observe that, outside recession periods, combining information (i.e. macroeconomic and Google data) generally leads to more accurate nowcasts than those based solely on either pure macroeconomic information or pure Google information. These findings are robust to the size of the training sample: we also used a smaller training sample of the same length as the one used for 2008q1–2009q2.

Third, recession periods exhibit a very specific pattern because preselecting data does not necessarily generate lower RMSFEs during those phases of the business cycle. Indeed, for almost all weeks within the quarter, the Ridge model that only integrates Google data without preselection outperforms other models (third rows of Tables 5, 8, 11). Thus, our results suggest that during a recession, (i) forecasters do not have to preselect data and (ii) GSD provide more accurate information than official macroeconomic

data. This latter result can also be observed during the COVID-19 recession, during which research shows that using new sources of high-frequency data is an effective way to get more reactive and accurate nowcasts of the economic activity (Lewis et al. (2020)).

5.3 Robustness checks

Next, we present the results of two robustness checks we carried out for the EA. The first concerns a true real-time analysis executed by using vintages of GDP and industrial production. The second robustness check consists in controlling for additional official macroeconomic series other than industrial production and opinion surveys.

5.3.1 A true real-time analysis

We perform the first analysis over the period 2014–2016 for which we use vintages of data for EA GDP and industrial production (survey data are generally not revised) and account for the observed timeline of data release as provided by Eurostat. When available, we also include lagged GDP growth among the explanatory variables of the nowcasting models (see Table 25 in the Supplementary Appendix, which gives the exact weeks in the OOS period 2014q1–2016q1 for which the lagged GDP is included in the real-time analysis).

Tables 36–38 in the Supplementary Appendix present the RMSFEs values that compare the same four models (a)–(d) as in the previous pseudo real-time analysis. Overall, the results that we have illustrated herein for the pseudo real-time exercise still hold in true real-time. In particular, they show that (i) nowcasting accuracy improves throughout the quarter, (ii) GSD provide valuable information at the beginning of the quarter when no official information is available, (iii) combining macroeconomic information with Google information improves the results, and (iv) our Ridge after Model Selection strategy outperforms the other approaches in terms of nowcasting accuracy. These results increase our confidence about the reliability of the real-time use of GSD when

nowcasting EA GDP.

5.3.2 Controlling for additional macroeconomic variables

So far, we have controlled for economic surveys, S_t , and industrial production, IP_t , as official series. This choice is motivated by the fact that practitioners consider both series the two most important variables to assess the EA economic state. Here, we aim to check the robustness of our evaluation about GSD to a richer macroeconomic information set. For this purpose, we include additional macroeconomic series among the covariates $x_{t,s}^{(w)}$ and $x_{t,h}^{(w)}$ in model (5.1) (see Table 26 in the Supplementary Appendix for a description of the 22 macroeconomic variables used). Studies in the nowcasting literature commonly use these types of macroeconomic series (*e.g.*, sales, exports, unemployment rate) which are continuously monitored by policy makers and market participants. We refer to this richer set of variables as a *Big Official Set* and to the set made of the previously considered variables, IP_t and S_t , as a *Small Official Set*. The robustness check covers two periods: 2014q1 – 2016q1, which exhibited cyclical stability, and 2017q1 – 2018q4, which exhibited a downward shift in the EA-GDP growth series.

As a performance measure, we take the ratios between the RMSFEs obtained by using GSD together with the *Small Official Set* of data (*resp.* with the *Big Official Set* of data) in the numerator (*resp.* in the denominator). A ratio larger than one indicates that including additional official series improves nowcasting accuracy, and conversely.

Results shown in Table 45 of the Supplementary Appendix highlight that in the period of cyclical stability (2014q1–2016q1) the inclusion of additional macroeconomic variables when using our Ridge after Model Selection strategy does not generally improve the nowcasting accuracy except for week 4 and, to a lesser extent week 2. This result still holds when no preselection of data occurs and when we only account for macroeconomic variables. This result provides a strong indication of the robustness of our findings in that it means that using only industrial production and a survey provides a good estimate

when nowcasting EA GDP. During a downward shift in the GDP, as in the period 2017q1 – 2018q4, it seems worth including a larger set of macroeconomic variables in weeks 4 and 6 to 10 (row 4 of Table 45). For the first three weeks and last three weeks, the *Small Official* data set is preferred. However, if we compare the results obtained without preselection of GSD (row 5), inclusion of additional macroeconomic variables does not improve the nowcasting accuracy. A possible explanation is that the uncertainty generated by the trade war does not have a strong common impact on all macroeconomic variables and does not adversely affect economic activity across the board but it directly affects the GSD.

6 Conclusions

Large data sets arising from alternative sources of information have gained popularity among macroeconomists whose goal is to assess the current state of the economy on a high-frequency basis. The design of an appropriate econometric methodology for macroeconomic nowcasting must take into account the specific structure of the alternative data used. This paper deals with the particular structure of GSD and proposes an econometric approach for nowcasting macroeconomic quantities based on GSD together with official data when available. Our proposed *Ridge after Model Selection* approach consists of two step: in the first step, GSD variables are preselected, conditionally on the official variables, by targeting the macroeconomic aggregate to be nowcast. In the second step, a Ridge regularization is applied to those preselected GSD and official variables, with Ridge tuning parameter chosen by GCV.

Our theoretical contribution consists in showing different types of optimality properties of our proposed procedure. First, we prove that our targeted preselection retains all the active variables in the true model with probability approaching one as $N, T \rightarrow \infty$. Second, we derive an upper bound for both the in-sample and OOS prediction errors

associated with the *Ridge after Model Selection* estimator as well as for the parameter estimator. Finally, we are the first to demonstrate optimality of GCV for OOS prediction in the setting of Ridge regularization.

We illustrate our procedure through numerical studies and an empirical application in which we nowcast GDP growth rates for the EA as whole, the United States, and Germany by combining standard macroeconomic variables and alternative GSD. Empirical results show that GSD contain valuable information about the current economic state and that combining standard macroeconomic variables with GSD variables is generally fruitful. They also suggest that the preselection step is crucial, as it consistently leads to better outcomes. However, recession periods present specific patterns as, during those phases of the business cycle, models that only contain non-preselected GSD tend to outperform in terms of nowcasting accuracy.

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Supplementary Material: The Supplementary material consists of an online Supplementary Appendix that contains additional theoretical results, all the proofs of the results in the paper, additional simulations, figures, tables, and the discussion of macroeconomic series used in the empirical study.

References

- K. Aastveit and T. Trovik. Nowcasting Norwegian GDP: the role of asset prices in a small open economy. *Empirical Economics*, 42(1):95–119, 2012.
- D. W. Andrews. Asymptotic optimality of generalized CL, cross-validation, and generalized cross-validation in regression with heteroskedastic errors. *Journal of Econometrics*, 47(2):359–377, 1991.
- E. Angelini, G. Camba-Mendez, D. Giannone, L. Reichlin, and G. Ruenstler. Short-term forecasts of euro area GDP growth. *Economic Journal*, 14:C25–C44, 2011.
- S. B. Aruoba, F. X. Diebold, and C. Scotti. Real-time measurement of business conditions. *Journal of Business & Economic Statistics*, 27(4):417–427, 2009.
- J. Bai and S. Ng. Forecasting economic time series using targeted predictors. *Journal of Econometrics*, 146(2):304 – 317, 2008.
- K. Barhoumi, O. Darne, and L. Ferrara. Are disaggregate data useful for forecasting French GDP with dynamic factor models ? *Journal of Forecasting*, 29:132–144, 2010.
- E. Barut, J. Fan, and A. Verhasselt. Conditional sure independence screening. *Journal of the American Statistical Association*, 111(515):1266–1277, 2016.
- A. Belloni and V. Chernozhukov. Least squares after model selection in high-dimensional sparse models. *Bernoulli*, 19(2):521–547, 05 2013.
- J. Boivin and S. Ng. Are more data always better for factor analysis? *Journal of Econometrics*, 132:169–194, 2006.
- D. Borup and E. C. M. Schütte. In search of a job: Forecasting employment growth using google trends. *Journal of Business & Economic Statistics*, 40(1):186–200, 2022.

- E. Candes and T. Tao. The dantzig selector: Statistical estimation when p is much larger than n . *The Annals of Statistics*, 35(6):2313–2351, 12 2007.
- M. Carrasco and B. Rossi. In-sample inference and forecasting in misspecified factor models. *Journal of Business & Economic Statistics*, 34(3):313–338, 2016.
- M. Carrasco, J.-P. Florens, and E. Renault. Linear inverse problems in structural econometrics estimation based on spectral decomposition and regularization. volume 6, Part B of *Handbook of Econometrics*, pages 5633 – 5751. Elsevier, 2007.
- V. Chernozhukov, W. K. Härdle, C. Huang, and W. Wang. LASSO-driven inference in time and space. *The Annals of Statistics*, 49(3):1702 – 1735, 2021.
- H. Choi and H. Varian. Predicting initial claims for unemployment insurance using Google trends. *Google Technical Report*, 2009.
- H. Choi and H. Varian. Predicting the present with Google trends. *Google Technical Report*, 2012.
- F. D’Amuri and J. Marcucci. The predictive power of Google searches in forecasting unemployment. *International Journal of Forecasting*, 33:801–816, 2017.
- C. De Mol, D. Giannone, and L. Reichlin. Forecasting using a large number of predictors: Is Bayesian shrinkage a valid alternative to principal components? *Journal of Econometrics*, 146(2):318–328, 2008.
- C. Doz, D. Giannone, and L. Reichlin. A two-step estimator for large approximate dynamic factor models based on Kalman filtering. *Journal of Econometrics*, 164(1): 188–205, 2011.
- L. Ferrara and C. Marsilli. Nowcasting global economic growth: A factor-augmented mixed-frequency approach. *The World Economy*, pages 846–875, March 2019.
- J.-P. Florens and A. Simoni. Nonparametric estimation of an instrumental regression: A quasi-Bayesian approach based on regularized posterior. *Journal of Econometrics*, 170(2):458–475, 2012.
- J.-P. Florens and A. Simoni. Regularizing priors for linear inverse problems. *Econometric Theory*, 32(1):71–121, 2016.
- D. Giannone, L. Reichlin, and D. Small. Nowcasting: the real-time informational content of macroeconomic data. *Journal of Monetary Economics*, 55(4):665–676, 2008.
- T. Goetz and T. Knetsch. Google data in bridge equation models for german GDP. *International Journal of Forecasting*, 35(1):45–66, 2019.
- H. Leeb. Evaluation and selection of models for out-of-sample prediction when the sample size is small relative to the complexity of the data-generating process. *Bernoulli*, 14(3):661 – 690, 2008.
- D. J. Lewis, K. Mertens, and J. H. Stock. Monitoring Real Activity in Real Time:

- The Weekly Economic Index. Liberty street economics, Federal Reserve Bank of New York, 2020.
- K.-C. Li. Asymptotic optimality of C_L and generalized cross-validation in ridge regression with application to spline smoothing. *The Annals of Statistics*, 14(3):1101–1112, 1986.
- K.-C. Li. Asymptotic optimality for C_p , C_L , cross-validation and generalized cross-validation: Discrete index set. *The Annals of Statistics*, 15(3):958–975, 1987.
- S. Ng. *Opportunities and Challenges: Lessons from Analyzing Terabytes of Scanner Data*, volume 2 of *Econometric Society Monographs*, page 1–34. 2017.
- R. F. Niesert, J. A. Oorschot, C. P. Veldhuisen, K. Brons, and R.-J. Lange. Can Google search data help predict macroeconomic series? *International Journal of Forecasting*, 36(3):1163–1172, 2020.
- S. Scott and H. Varian. Bayesian variable selection for nowcasting economic time series. In A. Goldfarb, S. Greenstein, and C. Tucker, editors, *Economic Analysis of the Digital Economy*, pages 119–135. NBER, 2015.
- R. Vershynin. Four lectures on probabilistic methods for data science. In *The mathematics of data*, pages 231–271, 2018.
- S. Vosen and T. Schmidt. Forecasting private consumption: Survey-based indicators vs. Google trends. *Journal of Forecasting*, 30(6):565–578, 2011.

A Appendix

$\delta = 0.2\iota$						
	$N = 200, T = 150, s = 105$	$N = 200, T = 150, s = 110$	$N = 200, T = 100, s = 110$			
FDR	MSER	MSFER	MSER	MSFER	MSER	MSFER
$\Psi = I_N$						
20%	0.9526	0.8104	1.0206	0.8478	0.7831	1.1369
10%	0.9620	0.8412	1.0061	0.8863	0.9226	1.0240
5%	0.9414	0.8364	0.9797	0.8794	0.9667	1.0041
2.5%	0.9343	0.8611	0.9770	0.9030	0.9825	1.0019
1%	0.9130	0.8753	0.9538	0.9220	1.0001	1.0188
0.5%	0.9089	0.8781	0.9485	0.9324	1.0013	1.0146
$\Psi = ((0.5)^{ j-k })_{j,k}$						
20%	0.9643	0.7904	1.0066	0.8154	0.9051	1.1521
10%	0.9657	0.8325	1.0053	0.8724	0.9492	1.0860
5%	0.9236	0.8387	0.9654	0.8812	0.9805	1.0222
2.5%	0.9178	0.8503	0.9642	0.8905	0.9996	1.0216
1%	0.9011	0.8593	0.9477	0.9025	1.0262	1.0287
0.5%	0.8865	0.8607	0.9403	0.9097	1.0301	1.0427

Table 1: Effect of N, T, s on the in-sample and OOS prediction error. In-sample (MSER) and OOS MSE (MSFER) are expressed as ratios with respect to the case $N = 150, T = 100, s = 105$. FDR denotes the percentage of false positives that can be tolerated.

$\delta = 0.8\iota$						
	$N = 200, T = 150, s = 105$	$N = 200, T = 150, s = 110$	$N = 200, T = 100, s = 110$			
FDR	MSER	MSFER	MSER	MSFER	MSER	MSFER
$\Psi = I_N$						
20%	0.9481	0.8047	1.0185	0.8433	0.7856	1.11799
10%	0.9476	0.8438	0.9955	0.8885	0.9184	1.01455
5%	0.9275	0.8280	0.9718	0.8747	0.9767	0.99942
2.5%	0.9338	0.8477	0.9768	0.8932	1.0001	1.00466
1%	0.9136	0.8666	0.9547	0.9149	0.9993	1.02664
0.5%	0.9097	0.8694	0.9487	0.9248	0.9995	1.01358
$\Psi = ((0.5)^{ j-k })_{j,k}$						
20%	0.9488	0.7826	0.9952	0.8103	0.8957	1.14796
10%	0.9511	0.8283	0.9941	0.8686	0.9463	1.07847
5%	0.9081	0.8314	0.9555	0.8779	0.9928	1.02038
2.5%	0.9018	0.8368	0.9499	0.8777	1.0081	1.01674
1%	0.8987	0.8490	0.9452	0.8942	1.0303	1.03002
0.5%	0.8866	0.8518	0.9373	0.9010	1.0321	1.04643

Table 2: Effect of N, T, s on the in-sample and OOS prediction error. In-sample (MSER) and OOS MSE (MSFER) are expressed as ratios with respect to the case $N = 150, T = 100, s = 105$. FDR denotes the percentage of false positives that can be tolerated.

EA – Nowcasting during 2014q1 – 2016q1

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.4467	0.4816	0.3897	0.3659	0.3239	0.3829	0.3901	0.3609	0.3427	0.3422	0.3103	0.3142	0.3111
Sel + Ridge (Google+S+IP)	0.2889	0.2607	0.2400	0.2493	0.1747	0.1706	0.1695	0.1608	0.1641	0.1668	0.2222	0.2178	0.2082
Sel + Ridge (Google)	0.3026	0.2769	0.2841	0.3008	0.3052	0.3107	0.3001	0.2974	0.2984	0.2975	0.2964	0.2867	0.2880
No Google					0.1807				0.1897		0.1928		0.2017

Table 3: RMSFEs corresponding to the nowcasting period 2014q1 – 2016q1. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without pre-selection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).

EA – Nowcasting during 2017q1 – 2018q4

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.5592	0.5956	0.5713	0.5642	0.3604	0.3605	0.3186	0.3064	0.5100	0.4836	0.4181	0.4527	0.4788
Sel + Ridge (Google+S+IP)	0.3505	0.3306	0.3341	0.3227	0.2330	0.2664	0.2501	0.2415	0.2720	0.2451	0.1316	0.1340	0.1314
Sel + Ridge (Google)	0.3760	0.3431	0.3262	0.3276	0.3230	0.3167	0.3051	0.2875	0.2894	0.2856	0.2795	0.2763	0.2700
No Google					0.4340				0.4841		0.2871		0.3177

Table 4: RMSFEs corresponding to the nowcasting period 2017q1 – 2018q4. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).

EA – Nowcasting during recession periods

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	1.4601	1.2693	1.2268	1.0596	1.0458	0.9831	0.9340	1.0843	1.1047	1.1047	0.9632	0.9401	0.9101
Sel + Ridge (Google+S+IP)	1.5481	1.4771	1.5257	1.6215	1.5581	1.6184	1.6345	1.6313	1.6344	1.6677	1.0953	1.0468	1.0622
Ridge (Google)	1.4601	1.2693	1.2268	1.0596	0.7745	0.8267	1.0072	1.0732	1.0415	1.0042	0.9962	0.9735	0.9657
No Google					1.5269				1.4241		1.6351		1.2888

Table 5: RMSFEs corresponding to the nowcasting period 2008q1 – 2009q2. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Ridge (Google)” refers to model (5.1) with only Google data estimated without preselection, and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).

U.S. – Nowcasting during 2014q1 – 2016q4

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.5843	0.5177	0.5771	0.5769	0.4588	0.4735	0.4207	0.4213	0.4315	0.4313	0.4479	0.4488	0.4786
Sel + Ridge (Google+S+IP)	0.4889	0.4792	0.4647	0.4670	0.4101	0.4277	0.3957	0.3922	0.3948	0.3933	0.4233	0.4273	0.4509
Sel + Ridge (Google)	0.4873	0.4833	0.4829	0.4816	0.4777	0.4740	0.4750	0.4751	0.4745	0.4746	0.4753	0.4749	0.4703
No Google					0.4062		0.4061		0.4156		0.4260		0.4466

Table 6: RMSFEs corresponding to the nowcasting period 2014q1 – 2016q4. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₅ in Table 24 in the Supplementary Appendix).

U.S. – Nowcasting during 2017q1 – 2018q4

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.4746	0.4090	0.4625	0.4549	0.2762	0.3021	0.2401	0.2407	0.2963	0.2506	0.2555	0.1791	0.2081
Sel + Ridge (Google+S+IP)	0.3639	0.3601	0.3092	0.3181	0.1735	0.1685	0.1347	0.1330	0.1042	0.0991	0.1187	0.1081	0.1320
Sel + Ridge (Google)	0.3482	0.3335	0.3177	0.3270	0.3168	0.3103	0.3061	0.3055	0.2949	0.2833	0.2770	0.2816	0.2757
No Google					0.2598		0.2255		0.3604		0.3510		0.2979

Table 7: RMSFEs corresponding to the nowcasting period 2017q1 – 2018q4. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₅ in Table 24 in the Supplementary Appendix).

U.S. – Nowcasting during recession periods

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	1.0156	1.0507	1.0506	1.0282	1.0967	1.1191	1.1185	1.1986	1.2123	1.1884	1.1738	1.1801	1.1217
Sel + Ridge (Google+S+IP)	1.0320	1.0611	1.0590	1.0521	1.2030	1.2029	1.1929	1.1893	1.1815	1.1751	1.1319	1.1307	1.0396
Ridge (Google)	1.0156	1.0507	1.0506	1.0282	0.9744	0.9320	0.9731	0.9991	1.0061	1.0158	1.0204	1.0196	1.2224
No Google					0.8439		1.1286		1.0580		1.0659		0.7828

Table 8: RMSFE corresponding to the nowcasting period 2008q1 – 2009q2. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with SIS pre-selection, “Ridge (Google)” refers to model (5.1) with only Google data estimated without pre-selection, and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₅ in Table 24 in the Supplementary Appendix).

Germany – Nowcasting during 2014q1 – 2016q1

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.3316	0.3225	0.3296	0.3365	0.3137	0.2905	0.2687	0.2690	0.2695	0.2684	0.2713	0.2754	0.2698
Sel + Ridge (Google+S+IP)	0.2619	0.2454	0.2219	0.2306	0.2378	0.2406	0.2373	0.2382	0.2429	0.2460	0.2717	0.2794	0.2754
Sel + Ridge (Google)	0.2265	0.2266	0.2484	0.2436	0.2433	0.2341	0.2310	0.2296	0.2362	0.2372	0.2380	0.2470	0.2453
No Google					0.3977				0.4208		0.2914		0.4325

Table 9: RMSFEs corresponding to the nowcasting period 2014q1 – 2016q1. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).

Germany – Nowcasting during 2017q1 – 2018q4

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	0.4889	0.4894	0.4837	0.4777	0.4352	0.4800	0.4649	0.4441	0.5038	0.4927	0.4382	0.4059	0.4181
Sel + Ridge (Google+S+IP)	0.3814	0.3794	0.3751	0.3769	0.3831	0.3826	0.3876	0.3898	0.3238	0.3181	0.3141	0.3088	0.2917
Sel + Ridge (Google)	0.3812	0.3800	0.3788	0.3757	0.3711	0.3680	0.3712	0.3584	0.3184	0.3097	0.3141	0.3187	0.3140
No Google					0.6532				0.6241		0.3632		0.6433

Table 10: RMSFEs corresponding to the nowcasting period 2017q1 – 2018q4. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Sel + Ridge (Google)” refers to model (5.1) with only Google data (preselected), and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).

Germany – Nowcasting during recession periods

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13
Ridge (Google+S+IP)	1.9279	1.9100	1.9123	1.9212	2.2660	2.3237	2.3365	2.3162	2.2939	2.2790	2.7585	2.7372	2.6581
Sel + Ridge (Google+S+IP)	1.9289	1.8990	1.9053	1.9018	2.1467	2.1477	2.1469	2.1461	2.1353	2.1382	2.6461	2.6538	2.5886
Ridge (Google)	1.9279	1.9100	1.9123	1.9212	1.9477	2.0123	2.0200	1.9972	1.9793	1.9603	1.9440	1.9173	1.9054
No Google					2.1580				2.0660		2.6107		1.9803

Table 11: RMSFEs corresponding to the nowcasting period 2008q1 – 2009q2. “Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and Industrial Production Index (IPI) estimated without preselection, “Sel + Ridge (Google+S+IP)” refers to model (5.1) with Google data, Survey, and IPI estimated with our *Ridge after Model Selection* procedure, “Ridge (Google)” refers to model (5.1) with only Google data estimated without preselection, and “No Google” refers to models without Google data (*NoGoogle*₁ - *NoGoogle*₄ in Table 23 in the Supplementary Appendix).