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## Treating inequality constraint in a continuous medium with a coupled X-FEM/Level-Set strategy

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### Introduction

The aim of this study is to propose a new approach to deal with continuum media subject to a kinematic inequality constraint  $\gamma(\underline{K}) \leq 0$ , where  $\underline{K}$  may be any kinematic second-order tensor. This kind of problem is similar to a constrained minimization problem with inequality constraints. It is equivalent to :

$$\inf_{\underline{u} \in S} \Pi(\underline{u})$$

with  $S = \{\underline{u} \mid \gamma(\underline{K}) \leq 0\}$  and  $\Pi(\underline{u})$  the total potential energy of the system. The common approaches to imposing the inequality constraints classically the Lagrange multiplier method [1], the penalty method [2], and the augmented Lagrangian method [3]. Excluding the penalty method, all these approaches lead to variational inequalities because we do not know a priori the shape of the active constraint and unactive constraint zone in the medium. To take into account such a constraint the elastic strain energy function  $\psi$  can be split into two distinct parts :

$$\psi(\underline{C}) = \psi^r(\underline{C}) + I_-(\gamma(\underline{K}))$$

with  $\psi^r$ , the parent or regular potential,  $\underline{C}$ , the right Cauchy-Green tensor, and  $I_-(x)$ , the characteristic function of  $\mathbb{R}^-$  such that :

$$I_-(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ +\infty & \text{else} \end{cases}$$

### The level-set approach

The strategy proposed here is to split the whole domain  $\Omega$  into two distinct subdomains separated by  $\Gamma$ . The first domain denoted by  $\Omega_c$  is the constrained zone or active zone where an equality constraint is imposed. The second domain denoted by  $\Omega_l$  is the unconstrained or unactive zone, where no constraints are imposed.

The two subdomains are separated by the interface  $\Gamma$ , located by the iso-zero of a level-set function [4]. On each subdomains, we have:

$$\psi(\underline{C}) = \psi^r(\underline{C}) + I_0(\gamma(\underline{K})) \quad \forall \underline{x} \in \Omega_c$$

$$\psi(\underline{C}) = \psi^r(\underline{C}) \quad \forall \underline{x} \in \Omega_l$$

with,

$$I_0(x) = \begin{cases} 0 & \text{if } x = 0 \\ +\infty & \text{else} \end{cases}$$

The location of the interface  $\Gamma$  becomes an unknown of the problem and, thus, we are looking for the exact position of the interface such that the total potential energy is stationary :

$$\inf_{\underline{u} \in S'} \Pi(\underline{u}, \Gamma) \text{ avec } S' = \{\underline{u} \mid \gamma(\underline{K}) = 0 \text{ sur } \Omega_c\} \quad (1a)$$

$$D_{\delta q_n} \Pi(\underline{u}(\Gamma), \Gamma) = 0 \quad \forall \delta q_n \quad (1b)$$

where  $D_{\delta q_n} \Pi$  indicates the directional derivative of  $\Pi$  in the direction of  $\delta q_n$ , where  $\delta q_n$  denotes a velocity field modifying the location of the interface  $\Gamma$ .

Starting from an initial position  $\Gamma_0$  of the interface, we search the exact position iteratively. At each step of the algorithm a velocity is computed from the stationary condition (1) and a level-set equation is solved to find the new location of the interface.

## First results and future prospects

We have applied the method described above to solve a simple one dimensional problem in the small strain elastostatic setting. We wish to limit the uniaxial strain of a truss by an arbitrary value  $\alpha$  under a prescribed load. Since we know the exact solution, we can compare it to the numerical one. The first results are promising since the presented strategy provides a convergence rate equals to the optimal rate of convergence in spite of the lack of regularity of the exact solution. The level-set localization of the interface and a correct enrichment of the finite element approximation [5] permits the optimal convergence. Compared to an augmented Lagrangian method coupled with an active-set algorithm [6], we obtain a significant gain in accuracy. Preliminary results have been obtained for two dimensional cases and will be presented.

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