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Computing with Manuscripts: Time between Mean and True Syzygies in John of Lignères' *Tabule magne**

Matthieu HUSSON

Introduction

General questions

When performing computations, ancient and medieval astronomers worked with astronomical tables accessible to them in manuscripts. Not only did they perform their computations 'by hand', but also the numbers they manipulated were 'read' from handwritten documents, with all the potential complexity of this act. Manuscript transmission produces variability in many ways, but, for the purposes of our research, we will treat it here primarily on two different levels. On one level, scribal variants can occur when numbers are copied or when the rows and columns of tables are inadvertently shifted. On a second, more structural, level, variations arise, especially in Latin sources, in the ways tables and related canons are assembled in manuscripts. This produces not only variants in different manuscript witnesses of a given table, but also deeper variants in the ways tables can be combined in astronomical procedures. Computation, on the other hand, is sensitive to differences in numbers and procedures. Even small procedural variations in rounding and truncation for elementary arithmetical operations might yield different results. How does the variability inherent to manuscript transmission affect the process of computing and the results produced by computation? Can we build tools to isolate these effects? What kind of critical edition and analysis of the tabular material can be constructed in order to grasp these effects? How can a digital information system on astronomical tables be helpful in this respect? How can such a set of tools then inform us on what 'reading' a table, 'precision', and 'errors' may have meant for historical actors? Is it possible, for instance, to collect supporting evidence indicating that some particularly skilled 'readers' were able to adjust scribal variants in the manuscript they were using in a computation to avoid results they would

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consider inappropriate? How does the layout and presentation of a table set influence its possible use in computations or the frequency of specific types of scribal variants? What level of accuracy in computation could historical actors produce with their computation tools? How different can these results be when faced with two different manuscript versions of the ‘same’ table set? What are the relations between procedures described in canons and the actual astronomical tables found in manuscripts? These are the kinds of questions this chapter asks in the context of a specific case study: determination of the time between mean and true syzygies¹ with the computational tools presented by the known manuscript tradition of John of Lignères’ *Tabule magne*.²

Brief introduction to syzygy computations

‘Syzygy’ is a general term pointing to the conjunction or opposition of the sun and the moon in ecliptic longitude. Computing syzygy is thus, from a mathematical perspective, a pursuit problem. Two points are moving on a given circle and the moment they coincide or are diametrically opposed is to be determined. Two aspects make this computation complex in the context of ancient astronomy. First, the velocities of the sun and moon are not constant. Second, the mathematical tools of ancient astronomy were generally designed to compute heavenly positions at a fixed time. Syzygy computation requires the opposite: find the time at which a given configuration of heavenly objects is satisfied. Ancient astronomers usually address these issues by decomposing the computation into two steps. In the first step, they compute what they call ‘mean syzygy’, that is the point and moment of conjunction or opposition of the sun and moon if they are considered to move at a constant mean pace along the ecliptic. The second step starts from the result of the first. It takes into account the ‘true’ position and changing velocities of the sun and moon at the time of ‘mean syzygy’ by considering their respective ‘equations’ (correction terms for the varying paces of the luminaries) and determines from this the time of ‘true syzygy’. Many different procedures were proposed by ancient actors to address this second step: some use tables of the solar and lunar motions directly, others rely on different types of tables specifically designed for this purpose. Most

¹ The literature on this topic is large. Cf. Chabás and Goldstein, ‘Computational Astronomy: Five Centuries’; Kremer, ‘Thoughts on John of Saxony’s’ Method’; Chabás and Goldstein, *A Survey of European Astronomical Tables*, pp. 139–54. My selection of syzygy computations to explore these general questions is inspired by this scholarship, which also analyses the mathematical properties and astronomical foundations of different historical procedures.

² Chabás and Goldstein, *The Alfonsine Tables*; Chabás and Goldstein, *A Survey of European Astronomical Tables*; North, ‘The Alfonsine Tables in England’; Pouille, ‘John of Lignères’; Saby, *Les canons de Jean de Lignères*; Husson, ‘Ways to Read a Table’. For an overview of the *Tabule magne* and the identification of the manuscripts containing them, see Chabás, *Computational Astronomy in the Middle Ages*, pp. 199–206.

procedures are iterative and propose successive evaluations of the time of true syzygy. This second step is by far the more complex of the two. The present chapter is concerned with the way in which the manuscript tradition of John of Lignères' *Tabule magne* framed the computation of the second step of syzygy computation: finding the time between mean and true syzygy.

The *Tabule magne* and true syzygy computations

The *Tabule magne* is a set of astronomical tables with their canons. It was created by John of Lignères and dedicated in 1325 to Robert the Lombard, Dean of Glasgow, with two other texts on astronomical instruments (a *saphea* and an *equatorium*).³ The *Tabule magne* are concerned with computations of syzygies (mean and true) and planetary positions (mean and true). A particular feature of the *Tabule magne* is related to planetary positions, for which double-argument equation tables are used for the first time in the tradition of Latin sources. The *Tabule magne* also have a strong link to the various versions of the Oxford Tables and were important in the transmission of Parisian Alfonsine astronomy to England.

In the four known manuscript witnesses of the canons, a chapter with the title *Tempus vere coniunctionis et oppositionis solis et lune invenire* ('To find the time of the true conjunction or opposition of the sun and the moon') is found.⁴ This canon describes a method to compute the time from mean to true syzygy. The procedure in this canon instructs the reader to compute two astronomical quantities at the time of mean syzygy: first the distance between the true sun and the true moon; and second the difference between the velocities of the sun and the moon (the *superatio*). In order to compute these quantities, one needs to rely on tables that are not specified and only implied in the context of this particular chapter. Then these two quantities are used as the arguments of a specific table. This table is described in the canons with the name *tabula longitudinis horarum* ('table of the longitudes of hours'). It simply tabulates the quotient of the first argument by the second.⁵ The result of the reading of this table is then taken as an approximation of the time between mean and true syzygy. The process is to be iterated until one estimates that the true sun and true moon are equal.

³ Husson, *Les domaines d'application*.

⁴ Cambridge, Gonville and Caius College, MS 110, pp. 1–5; Erfurt, UFB, Amplon. Q 366, 28r–32v (see for a digital copy: https://dnhb.thulb.uni-jena.de/receive/ufb_cbu_00022114); Paris, BnF, MS 7281, 201v–205v (see for a digital copy: <https://gallica.bnf.fr/ark:/12148/btv1b525030045>); Paris, BnF, lat. 10263, 70r–78r (see for a digital copy: <https://gallica.bnf.fr/ark:/12148/btv1b9072582f>).

⁵ Tables of this type for the computation of true syzygies are related to a tradition that can be traced back to the twelfth-century Andalusian astronomer Ibn al Kammād (Chabás and Goldstein, *A Survey of European Astronomical Tables*, pp. 145–46).

Scholarship has identified fifteen manuscript witnesses of the tables of the *Tabule magne*.⁶ Six witnesses include specific tables for the computation of true syzygy.⁷ Among these, five contain the same group of four tables presenting respectively the solar and lunar equations and velocities. This group is always presented in a single grid⁸ and allows one to carry out the first part of the procedure described by the canons.⁹ In this paper I will refer to it as the ‘equations and velocities grid’. Two among the six witnesses present the division table described in the canons.¹⁰ However, the format of the table in the manuscripts does not correspond exactly to the description of the table in the canons. The canon’s description implies that the table is presented in a single grid, where the table’s arguments and entries have no specific order of magnitude. Only the procedure in the canons fixes the rule according to which these orders of magnitude are determined. In the manuscripts, the table is divided into two distinct grids: one in which the first argument (i.e., the distance between the sun and the moon) is in degrees, the other where the first argument is in minutes. In both cases the units of the entries are also specified directly in the grid

⁶ Paris, BnF, Latin 7300A, 94v–112r (<https://gallica.bnf.fr/ark:/12148/btv1b100271684>); Bernkastel-Kues, Cusanusstift, MS 210, 89v, 103r–117r; Bernkastel-Kues, Cusanusstift, MS 212, 91v–92r and 93r (tables for radices and syzygies); Cambridge, Gonville and Caius College, MS 110, pp. 7–18; Erfurt, UFB, Amplon. F 376, 30v–53v; Erfurt, UFB, Amplon. F 388, 1r–42v; Lisbon, Biblioteca da Ajuda, MS 52-XII-35, 67r–92v; London, British Library, Add. 24070, 24v–42v; Paris, BnF, lat. 7286C, 10v–11r, 23v–24r (see for a digital copy: <https://gallica.bnf.fr/ark:/12148/btv1b10035226k>); Paris, BnF, lat. 10264, 1r–28v (see for a digital copy: <https://gallica.bnf.fr/ark:/12148/btv1b10036926k>); Segovia, Biblioteca de la Catedral, MS 84, pp. 680–91; Vatican, BAV, Pal. lat. 1367, 60v–62r (see for a digital copy: https://digi.ub.uni-heidelberg.de/diglit/bav_pal_lat_1367); Vatican, BAV, Pal. lat. 1374, 26r–27v, 51v (see for a digital copy: https://digi.ub.uni-heidelberg.de/diglit/bav_pal_lat_1374); Vatican, BAV, Pal. lat. 1376, 46r, 102r–130r (see for a digital copy: https://digi.ub.uni-heidelberg.de/diglit/bav_pal_lat_1376); Vatican, BAV, Pal. lat. 1412, 102r–116v (see for a digital copy: https://digi.ub.uni-heidelberg.de/diglit/bav_pal_lat_1412). Note that table sets are often very mixed in manuscripts from the Latin tradition. I here give extensive folio ranges in which tabular material from the *Tabule magne* are found along with tabular material from different origins.

⁷ Cambridge, Gonville and Caius College, MS 110; Erfurt, UFB, Amplon. F 388; Paris, BnF, lat. 10264; Vatican, BAV, Pal. lat. 1367; Vatican, BAV, Pal. lat. 1374; Vatican, BAV, Pal. lat. 1376, 46r, 102r–130r; Vatican, BAV, Pal. lat. 1412.

⁸ In this paper I use the word ‘table’ in order to point to a set of arguments and entries (mathematically) related to each other, and I use the word ‘grid’ to point to the particular layout in which tables are written. A single table can be displayed in several grids. Several tables can be grouped into a single grid.

⁹ This arrangement is probably linked directly to John of Lignères and a very similar type of tables arrangement is found in his tables of 1321 (Saby, *Les canons de Jean de Lignères*), except with argument every 6° instead of every 1°.

¹⁰ Cambridge, Gonville and Caius College, MS 110; Paris, BnF, lat. 10264.

displaying the table. Finally, three manuscript witnesses among the six present a double-argument table by John of Murs relying on a completely different logic for the computation of true syzygies.¹¹ While the canons of the *Tabule magne* describe a unique and quite simple procedure to compute true syzygy, the manuscript tradition of the table set shows that users of the *Tabule magne* did not feel compelled by the canons, and made the computations along diverse and different lines.¹²

This situation, which was quite typical for the respective traditions of canons and table sets, demonstrates that the relation between the tables described or implied by the canons and those actually found in table sets was not straightforward. It also shows that the relation between the procedures described in the canons and those that can actually be performed with the tables found in table sets is likewise not straightforward. This complexity of course has a documentary aspect: it depends heavily on what is usually described as the ‘accident of transmission’. This points to the history of each individual manuscript, which can, to some extent, be analysed and partly recovered from a careful examination of the document’s material, graphical and intellectual dimensions. Like scribal variants, some of these ‘accidents’ are completely unintended or the result of events that occurred long after the manuscripts were actually used by medieval astronomers. However, some aspects of these ‘accidents’ are consequences of the status of these writings and the way they were used, among other things in computations, by historical actors. In any case, the complexity also reflects, at least partially, the conditions under which these actors could perform computations.

In light of this complexity I will focus here on the equation and velocity grid and on the *Tabula longitudinis horarum* as they are found in the manuscripts. I will study their mathematical properties in the context of the algorithm described by the related canon in the *Tabule magne*. In particular, I will not explore the full range of possibilities the tables offer for syzygy computations, which could rely also on other tables or on other procedures. This focused approach has two main steps. First, I will offer a critical edition of the true syzygy tables in John of Lignères’ *Tabule magne*. I will transcribe and study each manuscript witness and then conduct an astronomical and mathematical analysis of the tables. Second, I will use this critical edition to propose tools that can help address general issues of computing with manuscripts by

¹¹ Erfurt, UFB, Amplon. F 388, Vatican, BAV, Pal. lat. 1367, and Vatican, BAV, Pal. lat. 1376, 46r, 102r–130r contain a copy of John of Murs’ *Tabule permanentes*. The Erfurt manuscript includes this in the same set of folios bearing the *Tabule magne*. The *Tabule permanentes* are another set of tables designed for the same purpose and produced also in Paris by John of Murs; see Richard Kremer’s contribution to this volume.

¹² It will be shown that numerical performance may also be an important aspect of the limited success of the *Tabula longitudinis horarum* among Latin astronomers.

hand by reconstructing the computations of medieval astronomers with these tables and analysing their properties of convergence and robustness within the procedure described by the canons.¹³

Critical edition of the tables

Figure 1 lists the five manuscripts that contain the equation and velocity grid and the *tabula longitudinis horarum*. Their details and *sigla* are as follows:

- **C**: Cambridge, Gonville and Caius College, MS 110
- **P**: Paris, BnF, Latin 10264
- **V1**: Vatican, BAV, Pal. lat. 1367
- **V2**: Vatican, BAV, Pal. lat. 1374
- **V3**: Vatican, BAV, Pal. lat. 1412

There are many ways to build critical editions from this material. A common aim of a critical edition is to propose a version of the table as close as possible to what could have been the intention of its original compiler. This usually requires a careful study of variants in order to build a stemma on which the critical edition will rely. In this study, my aim is different. I need to build a critical edition that supports an analysis of the practice of computing with manuscripts. In particular, I want to understand how scribal agency is related to the uses of tables for computation. The way to achieve this begins with a careful description of the presentation of the tables in the different manuscripts. I also want to identify scribal variants in witnesses that have different kinds of numerical effects on the computational procedure or its result. The main tool for this will be a mathematical and astronomical analysis of the tables. Manuscript descriptions, mathematical and astronomical analysis of tables, and the critical edition are the three main parts of this section.

Description of the manuscripts

Scribal variants in a copy and, more generally, the way a given manuscript can be used in a computation depend on different diplomatic features of the copy. By diplomatic features, I mean those aspects of the copy that depend mainly on scribal agency, intentional or not, constrained by the context of scribal work. Exactly which diplomatic features may have an impact on the table copy as a computational tool remains a new and mostly open set of questions in research.¹⁴ I will take the opportunity of this edition to explore these issues and hope to contribute the beginnings of answers as well as more accurate formulations of the questions involved.

¹³ Husson, 'Astronomers' Elementary Computations'.

¹⁴ See the articles by Montelle and van Dalen in this volume.

Tables	\ Sigla	C	P	V1	V2	V3
Solar equation		X	X	X	X	X
Lunar equation		X	X	X	X	X
Solar velocity		X	X	X	X	X
Lunar velocity		X	X	X	X	X
<i>Tabula longitudinis horarum</i>		X	X			

Figure 1: The different versions of the tables to be critically edited.

For these purposes I am relying on a general distinction between ‘digit’, ‘number’ and ‘quantity’ that can be most easily understood in a specific instance. In the equation and velocity grid the maximum value of the solar equation can be read as 2;10 arc degrees for arguments 92 to 95. This is a specific astronomical quantity. Among other things, it is expressed using the units ‘degree’ and ‘minute’ and the number ‘2;10’. This number in turn is expressed with the digits ‘2’ and ‘10’. The ways these different elements are related and presented on the page are essential features of the general layout of tables with respect to their use in computations. This will be a first point of attention in my descriptions. A second important point of attention will be to describe those aspects that point to the use of a group of tables in a single procedure. These aspects include, for instance, the positioning of the tables on the pages and their display in different types of grid, their positioning in the manuscript quires, their possible titles, headings and accompanying paratexts, etc.

The manuscript tradition of the *Tabule magne*, at least for the tables here under consideration, is not very original. The scribes responsible for the copies I will describe below have not created new designs for grids or new ways to express astronomical quantities in tabular format. The purpose of my description is not to identify originality. It is rather to understand how, in particular situations, tables are presented as a tool for computation. The fact that most of the diplomatic features I describe here are common to many tables may just attest to the fact that they are part of a common body of tacit knowledge related to how astronomical tables are to be presented and read. It is interesting to point out that on many occasions aspects of this tacit knowledge are shared across linguistic domains and are the object of a specific transmission process.¹⁵ I also hope that these descriptions will be useful in discussions about the diplomatic features of table copies that need to be taken into account in the diplomatic transcription of tables.

¹⁵ Husson, ‘Remarks on Two Dimensional Array Tables’; Li Liang, ‘Tables with “European” Layout’.

Cambridge, Gonville and Caius College, MS 110

Cambridge, Gonville and Caius College, MS 110 (C) is a vellum composite manuscript containing five codicological units of the fourteenth and fifteenth centuries. Although composite, the manuscript is intellectually coherent as it contains only material related to astral sciences and especially to mathematical astronomy.¹⁶ The initial and final leaves of the manuscript are remnants of a fourteenth-century document. They contain astronomical material produced in the 1320s around the Parisian faculty of arts, viz., John of Lignères's *Tabule magne* from pp. 1 to 18 and a fragment of John of Saxony's almanac from pp. 363 to 368.¹⁷ This witness of Lignères' *Tabule magne* is the only known one where canons and tables are found together in a set of quires that results from a single production act.

Several features of C's copy of the equation and velocity tables point to their common use in a single computation. First, these tables are presented on two pages facing each other. The four tables only occupy the top three quarters of the page and two other sets fill the remaining space. The equation and velocity table is presented under a unified title *Tabula equationis solis et lune et ad inveniendum motus solis et lune in una hora* ('Table of the equation of the sun and moon and for finding the motion of the sun and moon in one hour'). This title is repeated on pp. 16 and 17. The use of the singular for table is interesting for a grid that gathers two equation and two velocity tables. A further feature unifying the four tables into a single computational tool in C is the arrangement of the arguments and entries. Similar to what is usually done for planetary equation tables, all four tables share the same argument headed *linee numeri communis*. Under this heading, two columns are found; the first runs from 1 to 30 with a step of one, the second starts symmetrically at 29 and runs down to 0 with a step of one. After this we read three times the same set of four headings: *equatio solis, equatio lune, motus solis in una hora, motus lune in una hora*. With the symmetry of the argument this covers six zodiacal signs on each page, thus the full zodiac on pp. 16 and 17. These column headings identify the astronomical quantities that are tabulated. The *tabula longitudinis horarum* is split into two grids. The first is at the bottom of page 16, the other on page 18 presented in a landscape format. The title of the table appears only at the top of the first grid on page 16. Neither on page 16 nor on page 18 do headings indicate the type of astronomical quantities to read for the first argument in the first row or for the second argument in the first column. The type of astronomical quantity that one reads in the entries is not mentioned either. The two grids display difference columns that are distinguished, among other

¹⁶ James, *A Descriptive Catalogue*, vol. I, pp. 114–15. I am in debt also to Sebastian Falk for providing me with pictures of the relevant manuscript folios for my study.

¹⁷ Chabás and Goldstein, *A Survey of European Astronomical Tables*, pp. 91–92.

things, by the heading *differentia*. Thus, the only heading of this table points to a mathematical property of the number presented. The use of the table and its interpretation can only rely on a good comprehension of the procedure of syzygy computation in general and in particular of the one described in the canons to the *Tabule magne*.

A second set of diplomatic features of C is related to the ways astronomical quantities are expressed by sexagesimal numbers and used in computations. In the equation and velocity grid, the first two columns under the heading *linee numeri communis* represent different kinds of astronomical quantities depending on the specific table for which they will be used as an argument. For instance, if read in relation to the *equatio solis* the number in the *linee numeri communis* refers to the mean argument of the sun; if read in relation to the *motus solis in una hora* the same number refers to the true solar longitude. In each case, however, the value is an arc of the zodiac. The zodiacal signs are implicitly expressed by the layout of the table on the facing pages 16 and 17. Only the numbers of degrees are written, but no unit for them is explicitly mentioned. The situation with respect to the astronomical quantities displayed in the entries of the tables is different. Under each table heading that marks the quantities, you have either three columns (for the *equatio solis* and the *equatio lune*) or two columns (for the *motus solis* and the *motus lune*). Each of these columns stands for a particular unit of the sexagesimal numbers expressing the astronomical quantity. For instance, the *equatio lune* is expressed in degrees, minutes and seconds marked by their abbreviations ('g' for *gradus*, 'm' for *minuta*, and '2' for *secunda*) as sub-headings in the first row of the table. This shows that what each column means in such a table and the way columns work together to constitute astronomical quantities is very different for columns used as arguments and those used as entries.

The unit sub-headings are nicely and systematically written on p. 17 but with less rigor on p. 16. This variation shows that these features are truly 'diplomatic', i.e., they depend on scribal agency. There is no use of colours in this manuscript to mark the numbers (numbers are marked in black, the table ruling is in red). However, the scribe used another feature to reduce his labour and produce a readable page. For all the tabulated astronomical quantities the first sub-column (in degrees for the two equations, in minutes for the two velocities) is highly repetitive. For instance, on p. 17 the first column of the *equatio solis* should have a sequence of twenty-four '2's for the digit of the degree of the equation. The scribe decided to write only every second, third or fourth digit in the sequence. He used this strategy almost exclusively for the first column of each astronomical quantity. This produces a visual effect that underlines the astronomical quantities to the reader. On the other hand, this way to omit 'repetitive' numerical information in specific columns of the tables amplifies the risk of introducing shift variants into the copy. Because

these potential shift variants will occur in the column that has the greatest order of magnitude, their impact on the computation will be significant. Some instances of this will be identified later.

At the bottom of the columns for the *equatio solis* and *equatio lune* one finds the word *adde* to indicate how the entries are to be managed in the larger procedure to find the true longitude of the sun or moon. It is also related to the symmetry of the *linee numeri communis* columns. If the table is read from top to bottom the equations will be subtracted, if the table is read from bottom to top the equations will be added.

The two grids displaying the *tabula longitudinis horarum* treat the arguments and entries differently. The single digit arguments are not marked with a heading or unit. The first argument ranges from 27 to 34 in both grids. The second argument ranges from 1 to 8 in the first grid and from 1 to 34 in the second grid. Differences and actual entries are also treated in distinct ways. Units of the entries are expressed using abbreviations and top column headings similar to the grid for the equations and velocities. In the first grid on p. 16, these units are hours, minutes and seconds. In the second grid on p. 18, these units are minutes and seconds. In terms of layout, the first argument is treated as a kind of table heading in both grids. The differences have no explicit units. In the first grid on p. 16, each cell contains a two-position number, e.g., 4 46, which the context allows to be identified as 4 minutes 46 seconds. Thus while quantities are separated in different cells for the entry, they are gathered in one cell for the differences. Because of the very small differences between two successive entries, this particular diplomatic feature does not appear in the second grid on p. 18.

Paris, BnF, Latin 10264

P is a fifteenth-century paper manuscript of 286 folios. It is closely related to Paris, BnF, Latin 10263 as both are linked to the fifteenth-century printer and humanist Arnaud of Brussels.¹⁸ This coupling of manuscripts is interesting because Latin 10263 is one of the four witnesses of the canons to the *Tabule magne*. The close relationship between the witnesses of the table set and of the canons might explain why P has, like C, a table set coherent with the canons concerning syzygy computations. P has an interesting intellectual profile: along with the *Tabule magne*, it contains cosmological, cosmographical and geographical texts of Alfarгани and Albertus Magnus. The way the five tables are arranged in this manuscript is almost identical to that found in C (cf. Plate 13). Only three relevant variants will be mentioned here. First, the two grids of the *tabula longitudinis horarum* are copied together on fol. 30v. Thus, in this

¹⁸ Poulle, *La bibliothèque scientifique*.

Parisian witness, the two steps of the procedure correspond to two different openings of the manuscript. Second, in the equation and velocity grid the zodiacal signs are marked with numbers from 0 to 5 at the top of the grid and from 6 to 11 in reverse order at the bottom of the grid. Third, the scribe of this copy did not use the technique of omitting repetitive numbers in columns, as was done by the scribe of C.

Vatican, BAV, Pal. Lat. 1367

The three Vatican witnesses only have the equation and velocity grid. V1 is a fifteenth-century composite manuscript with the first 84 folios of parchment and the last 79 of paper. Despite the composite nature of the manuscript, its intellectual profile is coherent with texts of astronomy, astrology and medicine. The parchment section contains only astronomical tables. It is opened by a version of the Parisian Alfonsine Tables as described in John of Saxony's canons of 1327¹⁹ and is completed with material from other sets, including parts of the *Tabule magne*. The equation and velocity grid is spread over three pages (70v–71v). The layout of the grid is similar to that of C but some relevant variants can be noted. The first striking diplomatic aspect is the use of colours. Black and red are linked to numerical quantity; all digits that need to be read together in order to form a number and signify an astronomical quantity are of the same colour. Colours separate the columns of the tables. The argument columns common to all four tables are repeated for each zodiacal sign on 70v.

Vatican, BAV, Pal. Lat. 1374

V2 is a paper manuscript of 126 folios copied in Prague in 1407. It contains exclusively astronomical tables. Like in V1, the equation and velocity grid is spread over three pages. However, the use of colours is different and not systematic in this manuscript. The integrity of quantities is not respected. For instance, the *motus solis* is written in red for the minutes and in black for the seconds. This use of colour enhances the possibility of shifts when copying the table column by column.

Vatican, BAV, Pal. Lat. 1412

The last witness, V3, is a 138-folio paper manuscript copied in Paris in 1453–54. It is entirely concerned with mathematical astronomy and includes texts like the *Theorica planetarum gerardi*, various canons of John of Lignères and different tables from the Alfonsine traditions. The equation and velocity grid is spread over three pages. The diplomatic features that are of interest in this

¹⁹ Poulle, *Les tables alphonsines*.

study are almost identical to those of V1. One may, however, note that on fol. 111r the scribe has chosen to copy the *linee numeri communis* only once in the middle of the page.

Scribal agency affects the ways a given manuscript can be used in a computation. The association of tables in grids and the distribution of these grids on pages is one important aspect. The organisation of these grids is also relevant especially with respect to the relation between arguments and entries and with respect to the symmetry of the tables. Scribal agency also affects the way digits, numbers and units are related to the astronomical quantities they express. The practice of avoiding the copy of repetitive numbers and use of colours that does not respect the integrity of astronomical quantities are two interesting instances. These different diplomatic features also may induce specific types of column shift variants in the copy. Finally, the above survey has confirmed the manuscript grouping that was already apparent from the contents of the manuscripts with respect to syzygy computation. The Cambridge and Paris manuscripts are distinct from the three Vatican witnesses especially with respect to the use of colours.

Mathematical and astronomical analysis of the tables

Astronomical and mathematical understanding of the tables is enhanced by 'recomputing' the table according to historically pertinent methods. 'Understanding' here means identifying the astronomical models, parameters and mathematical methods on which the tables rely. Different contributions in this volume illustrate this type of inquiry on original tables. Recomputations can be done at different levels of accuracy depending on the evidence of the manuscript, the precision of the table, and the aim of the study. In the context of this chapter, I need to recompute values that will give me a point of comparison from which the manuscript variants can be analysed and a critical edition established. In particular, the recomputed values need to help me identify those scribal variants that could have been identified also by especially 'skilled' table users.

Note that the tables analysed here are either already fairly well known or mathematically simple, so that it will not be necessary for me to develop this part of the analysis very far. The *Tabula longitudinis horarum* is a division table. The top row argument, i.e., the *superatio* or velocity difference between the moon and the sun, is divided by the left column argument, i.e., the elongation. The first grid of this table has the elongation running up to 8° (somewhat more than the maximum elongation in a half-day). The second grid of this table has the elongation running up to 34 arcminutes (enough to let the result of the division between the *superatio* and the elongation reach 60 minutes). At the level of precision required for this analysis the recomputation of

this table is trivial.²⁰ The solar and lunar equations are those of the Parisian Alfonsine Tables as described for instance in John of Saxony's canons of 1327 and printed in the 1483 *editio princeps*. It is not necessary, for my purpose, to elaborate further on the existing literature on these equations.²¹ The solar and lunar velocities are also well documented.²² However, their recomputation presents interesting methodological aspects in a simple situation. The cases of the sun and moon are fairly similar. Thus, I will here present only the recomputation of the lunar velocity. The minimum value of the lunar velocity in the table is 0;30,18 and the maximum value 0;36,04. This pair of values indicates that the table is related to the corpus of the Toledan Tables.²³ Thus the first logical step in exploring how this table could have been computed is to recompute the lunar velocity starting from the Toledan Tables. One may apply the following formula, where $\bar{\alpha}$ is the mean anomaly of the moon, ν is the lunar velocity, m is the mean lunar motion expressed in degrees per hour, m_a is the mean lunar motion in anomaly in degrees per hour and c is the lunar equation of anomaly:²⁴

$$\nu(\bar{\alpha}) = m + m_a(c(\bar{\alpha} + 1) - (c(\bar{\alpha}))).$$

Figure 2 displays the differences in seconds (the precision of the velocity tabulated in the manuscripts) between the results obtained using this formula²⁵ and the values in manuscript C. The results obtained with the other manuscript witnesses are qualitatively identical. A pattern is apparent in this diagram that shows that probably an interpolation grid was used in computing the velocity table.

²⁰ In the recomputation I have performed an exact division and have rounded the result to the precision of the recomputed table. I do not study specifically the practice of division among astronomers of the late medieval period in the Latin tradition.

²¹ For a short state of the art and excellent starting point for the scholarship on the solar and lunar equations in European traditions, see Chabás and Goldstein, *A Survey of European Astronomical Tables*, pp. 63–73.

²² See Chabás and Goldstein, *A Survey of European Astronomical Tables*, pp. 95–99.

²³ Pedersen, *The Toledan Tables*. From the point of view of astronomical theory the table set is not homogenous. In particular, it is not possible to derive the velocity tables from the equation tables as could be expected. The former rely on Alfonsine parameters while the latter rely on different Toledan parameters. This situation is not uncommon in table sets that circulated in Latin sources during the latter part of the Middle Ages.

²⁴ Different formulas are possible especially with respect to the use of c but it will not be necessary to explore them here. See Goldstein, 'Lunar Velocity in the Ptolemaic tradition'.

²⁵ I used the lunar equation of anomaly from Pedersen, *The Toledan Tables*, vol. IV, pp. 1453–58: 0;32,56,0 °/h for the lunar mean motion in longitude and 0;32,40,0 °/h for the mean motion in anomaly.

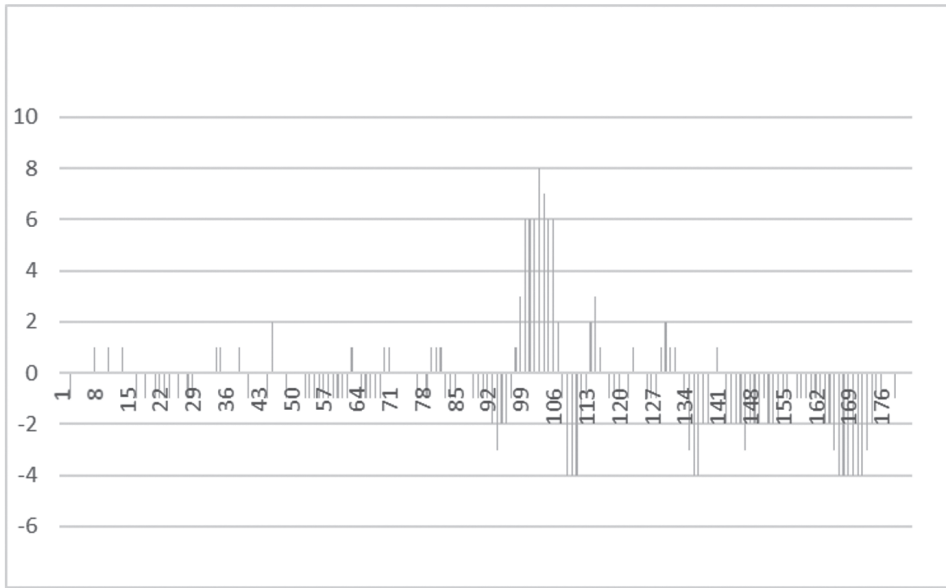


Figure 2: Differences in seconds between C and results obtained with the first recomputation scenario.

The lunar velocity table usually presented in the Toledan Tables is given with a step of 6 degrees for the argument. A close comparison of the values of the tables edited by Pedersen²⁶ and those for multiples of 6 degrees in the manuscripts here under consideration shows that both sets of values coincide exactly except in three cases:

- For argument 30, Pedersen's edition has 30,55, while John of Lignères' version reads 30,36.
- For argument 102, Pedersen's edition has 33,17, while John of Lignères' version reads 30,27.
- For argument 168, Pedersen's edition has 35,58 while John of Lignères' version reads 35,54.

Thus, the possibility that the table in our manuscripts was computed using interpolation in between nodes at every 6 degrees taken from the Toledan Tables (with the adjustments listed above) is worth exploring. Figure 3 shows the differences in seconds between the lunar velocity table in C and the results of such a recomputation. The agreement is obviously much better. And the results obtained with the other manuscripts are qualitatively similar. I have chosen to stop my recomputation effort at this point.²⁷

²⁶ Pedersen, *The Toledan Tables*, vol. IV, p. 1412.

²⁷ Recomputed values for all tables are provided in the critical apparatus of the edition of the tables in Appendix A.

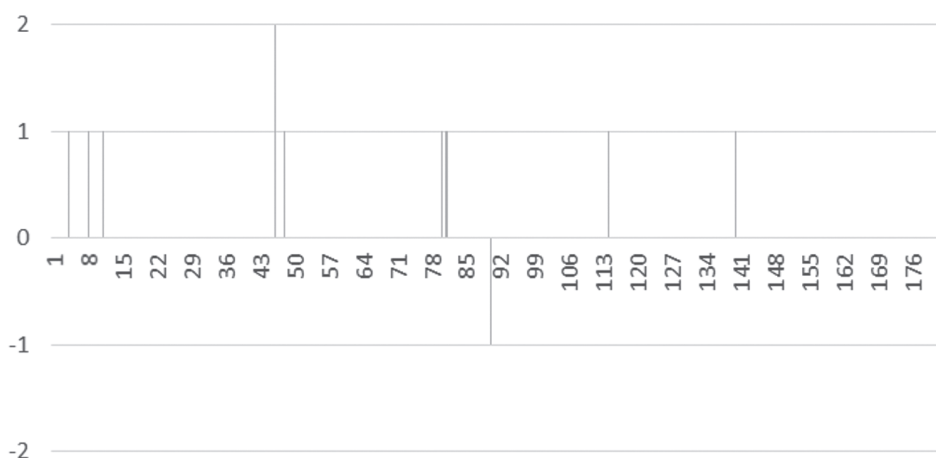


Figure 3: Differences in seconds between C and the results obtained with a second recomputation scenario.

Critical edition

I have two types of material for each table: first, manuscript transcriptions from each witness of the table; second, recomputed or expected values for each table. In the context of this study, the goal of the critical edition is not to restore a version of the table intended by John of Lignères, thus I do not need to study the interrelations of the manuscripts and provide a stemma. Rather, the goal of the edition is to provide a reference point for the analysis of manuscript variants and a tool to analyse the effect of these variants on the computations and their results. As a consequence of the first goal, the edition must be neutral with respect to the different manuscripts, because if the edition is by construction closer to one manuscript than to the others, the specificities of computing with this particular manuscript will not be in sufficient contrast with those of computing with the critical edition. The second goal of the edition implies that the edition must be close enough to the expected values so that computations made from the edition do not potentially lead to the computational effects I want to isolate in each particular manuscript. Finally, the computational effects we need to isolate are small in most cases, thus the critical edition must also remain close to the manuscripts. In the end, the edition needs to be a middle term of some sort between the different manuscript versions and the expected or recomputed version.

In light of these requirements, I have adopted a simple algorithmic rule to construct the critical edition. Each value is determined according to a majority rule in which the expected values are weighted with a coefficient two and the manuscript witnesses are weighted with a coefficient one. This ensures that the expected values have more weight than any manuscript, that all the manu-

scripts have the same weight, and, because the weights given to each source are close to each other, the critical edition will remain close enough to the manuscript witnesses. Following such a procedure has the supplementary advantage that the resulting critical edition is entirely transparent as no *ad hoc* emendations are made. Since the computation of the *tabule longitudinis horarum* is trivial I have chosen to give a weight of three to the recomputed values and kept the two manuscript versions with a weight of one. This implies that the edited version is identical with the recomputed table.

In the next section, this particular choice will allow the comparison of various means of division: modern division, exact division using the algorithmic rules of ancient actors, division using an exact version of the table (i.e. with values rounded to seconds), division using the quotient tables as they appear in the manuscripts with their variant. More generally, the choice of the above algorithmic rule to construct the critical edition will prove efficient with respect to our goal: it will be possible, for instance, to distinguish between ‘obvious’ variants that any ‘skilled’ user would have ‘corrected’ during or before the computation and ‘non-obvious’ variants that would silently go into the computation. It will also be possible to isolate various kinds of variants and their effects on the computation by comparing the manuscript version to the critical edition.

As, to my knowledge, this is the first time such a procedure has been used to generate a critical edition of a table with respect to a given research objective, it is not clear if the set of weights I have selected is the only or even the best to achieve these goals and I make no claim about this. I do note that this kind of weighted procedures could easily be implemented as a tool to generate a critical edition directly from queries to a database of astronomical tables. In this context such issues could be investigated systematically. These weighted procedures are in principle flexible enough to fit many different kinds of research goals for critical editions, including the classical stemma-oriented type of critical edition. Naturally these types of ‘computer-assisted’ editions are not to replace the expertise of the historian. Even if he decides to use such a tool, the choice of the scholarly goal of the edition, the weight to be given to the various sources, and whether or not to follow the result of the weighted procedure in individual cases will remain up to the researcher.

A typology of scribal variants in tables

My edition of the five tables is given in Appendices A to E and the conventions used to mark the variants are specified there. In order to explore how manuscript variants affect results of computations and practices with tables I distinguish two directions along which they can be analysed.²⁸ First, some vari-

²⁸ An interesting analysis of manuscript variants in a tradition of a table set involving Arabic and Latin sources is found in van Dalen and Pedersen, ‘Re-editing the Tables’.

ants affect one number at a time while others affect in correlated ways a group of numbers. These phenomena are connected in different ways to diplomatic choices when copying the table and may point in some cases to specific mathematical practices. These two categories of variants will affect the computation with tables differently since variants where sets of numbers are changed in correlated ways are likely to have a greater impact. A second manner of analysing variants relies on the specific way quantities are shaped from numbers in astronomical tables. Quantities are expressed as a set of numbers (i.e., digits) each having a different order of magnitude in sexagesimal arithmetic. The numbers at each of these orders of magnitude will usually have different diplomatic properties. The number of degrees will change very slowly and thus parts of the column will be repetitive, the number of seconds will usually behave more randomly except for some arithmetical tables where cycles are likely to appear that will help reading and copying, etc. A variant affecting the number of seconds in a quantity will presumably have a smaller impact on the computation than a variant affecting the number of degrees in the same quantity. Thus one might consider that from a computational perspective a table does not usually present one type of variant but has as many as there are positions in its entries.

Among variants affecting one number at a time, some remain inscrutable. For instance, in the table for the solar equation, V3 gives the value 2;7,18 for argument 104. However, all the other witnesses and the recomputation give a '14' instead of the final '18'. There is no obvious palaeographical or mathematical explanation for this variant. On the other hand, some variants clearly have a palaeographical cause. For instance, in the table for the solar equation the '29' in the number of minutes for argument 138 varies as '20' in V1 and V2 and as '39' in P. These types of variants are linked to the script used to denote the number. They are more and more frequent as the order of magnitude of the digit is smaller and control of the value of the number from local parsing becomes more difficult. There are practically no errors of this type affecting the number of degrees in the table set here under analysis. Some variants affect the last position of a number by a value of plus or minus one. In some of those cases, no palaeographical explanation is available. For instance, in the table for the lunar velocity the value for argument 65 was most probably interpolated in between the values for arguments 60 and 66. All versions except V3 give the value '36' in the last position, V3 gives '37' instead. There is no simple Latin palaeographical explanation for a '7' instead of a '6', so that a 'rounding effect' is much more likely to account for this variant. This suggests that some actors along the transmission of the table did recompute the interpolated values and thus produced small variant traditions of the tables because of variations in their rounding practices.

There are different types of variants that affect sets of numbers in correlated ways. The most common type shifts a block of values vertically. For instance,

the table for the solar equation in V2 has a shift affecting the numbers of minutes and the numbers of seconds between arguments 45 and 48.²⁹ Overall, this type of shift is more frequent in the three Vatican manuscripts than in the two others. This might be related to the specific use of colours made by the Vatican group of manuscripts. Some other types of variants similar to this block shift also occur, and they especially affect the numbers of degrees. They are linked to the repetitive character of the table for this order of magnitude. For instance, in the Cambridge manuscript, the degree column in the lunar equation table shows a block of '3's instead of '2's for arguments 146 to 150. Similar effects on the numbers of degrees are attested in V2 for the solar equation table for arguments 115 to 120 and arguments 151 to 153. Another curious effect of repetitive character affecting a block of values is seen in the solar velocity table in V3. For arguments 151 to 159 a '23' replaces '32' in the second column. Finally, some mathematical effects also produce variants that create block shifts. A clear instance is given in the table for the solar velocity in V3. For arguments 160 to 170, V3 gives '32' as the number of seconds while all the other manuscripts give '33'. One might remember that for arguments 174 and 180, which are interpolation nodes, all manuscripts read 2'32" instead of the expected 2'33". Thus in all manuscripts but V3 the solar velocity shows an unexpected decrease from 2'33" to 2'32" in the seconds' column. One of the actors along the chain of transmission that produced V3 felt this was to be corrected. However, instead of changing the value of the two last interpolation nodes to 2'33", he changed the value of the two preceding interpolation nodes to 2'32" instead of 2'33". All the values dependent on these nodes are then affected.

Computing ΔT with manuscript tables of the *Tabule magne*

Preparing the computations

In order to explore the practice of computing the time between mean and true syzygy (hereafter ΔT) with these manuscripts we need to consider at least three different types of variability that must be addressed: manuscript variability, procedural variability, and arithmetical variability. The first sections of this paper were devoted to building the tools needed to manage manuscript variability especially for the critical edition of the tables.

As far as procedural variability is concerned, as noted above, the table set of the *Tabule magne* could be used to compute the position between mean and true syzygy in many ways. In this section I want to focus on the variability stemming from manuscripts' variants. Thus, procedural variability is to be con-

²⁹ It is not clear in this specific case how the entry for argument 49, namely 1;34,34, was generated: simple linear interpolation would produce 1;34,36 or 1;34,37.

trolled rather than explored. The canons of the *Tabule magne* are here helpful because they describe only one method for computing the time between mean and true syzygy, which I will thus follow closely:

1. Starting from the mean anomaly of the sun and moon compute the difference in longitude of the true sun and true moon.
2. Compute the difference of the velocities of the sun and moon.
3. Divide the first by the second result and obtain an estimation of ΔT .
4. If the true sun and moon are at syzygy after that estimation of ΔT (i.e. their true positions are equal or 180° removed at the desired level of accuracy), stop; if not, iterate.

Following this procedure closely allows me to avoid dependence on tables that are not in the set I have critically edited above, and thus to keep a good control over manuscript variability. In particular, starting the procedure directly from the mean anomaly of the sun and moon allows me to avoid depending on a set of mean motion tables from which to derive a set of mean conjunctions and oppositions for the sun and moon for a time period that would also need to be justified. There is a second consequence of the choice to closely follow the canons and the table set. After the first iteration the sun and moon are not at mean syzygy any more. In principle the first and second lunar model are then not equivalent, and the second lunar model should be used to compute the moon's true position and its velocity. However, there is no lunar equation of centre in the table set here under consideration, as would be necessary to use the second lunar model, and we have seen that the lunar velocity table is also dependent on the first lunar model drawn from the Toledan Tables. Thus computing the time between mean and true syzygy according to the table proposed in the *Tabule magne* set implies that one should follow the first lunar model. In concrete terms, I will compute ΔT according to the above procedure for every pair of mean anomaly of the sun and moon in the range 0 to 360 with a step of 10 degrees.

I am fully conscious that it would be important and interesting to explore other possible procedural scenarios with the same accuracy, and to compute ΔT or even true syzygy times for other table sets. These other ways to use the tables (and to associate them with other velocity, equation and mean motion tables) will likely lead to different values of ΔT .³⁰ However, I am not trying to compare the results found in that way to 'exact Alfonsine results' or to explore this procedural variability. My aim is only to understand what kind of variability in the computational procedure and in its results derive from the manu-

³⁰ The impact of this kind of alternatives on the computed time between mean and true syzygy has been investigated for the case of John of Saxony in Kremer, 'Thoughts on John of Saxony's Method'.

script variants in this corpus. In this respect, the results obtained in this study are likely to be qualitatively valid also for other table sets and procedures as the manuscript variability will be of the same kind for other procedures and so too will be the arithmetical practices on which the computations rely.

The last type of variability to consider is arithmetical variability. Arithmetical practices and their variability are an important factor here because their order of magnitude is likely to be of the same order as that produced by manuscript variants. In order to control this variability I have made two different choices.

First, a competent table user presumably corrects some manuscript variants, especially those that affect the number of degrees in an entry. It is thus important to see how these skilled corrections affect the computation and its results. For this reason, I have repeated my computations for four different situations:

1. The table set as found in the manuscripts.
2. The table set corrected for variants that only affect degrees.³¹
3. The table set corrected for variants that affect degrees and minutes.
4. The critically edited table set.

A comparison of the results produced in these situations will help me understand how a competent table user is able to improve the accuracy of results by amending some easily spotted variants in his tables.

Second, the arithmetic of historical actors is not that of modern computational software. I have thus also explored different arithmetical algorithms in my computations:

1. Computation with floating numbers and modern arithmetic.
2. Simulated computations of historical actors, in which all numbers are converted to integer multiples of the smallest sexagesimal unit used (here the second) and regular integer arithmetic along with a specified type of rounding is used.³²
3. The same as situation 2) but with the use of the *tabula longitudinis horarum* for the final division.

Thus, multiple manuscript versions and their corrected versions are combined with different possible arithmetical algorithms in an iterative procedure. The space of computations explored is huge and in some cases, different scenarios

³¹ Here 'corrected' means that the number is set to be identical to the one chosen in the critical edition: I eliminate the variant. Consequently, 'corrected' does not mean that the number is set to be equal to the one I would expect from a recomputation.

³² Husson, 'Astronomers' Elementary Computations'.

lead to the same results. I will not fully describe this space here.³³ Rather I will explore two aspects of the computation of the time between mean and true syzygy with this set of manuscripts. First, it is important to see if the computations actually produce a result. In other words, I will consider how the convergence of the iterative process is affected by variants in the manuscripts and by arithmetical practices. For this, I will consider manuscript versions one by one and, after each iteration, I will inspect the difference in ecliptic longitude between the true sun and true moon. If the procedure converges, this distance should approach zero. Once the convergence issue is clarified, I will be in a position to analyse how manuscript variants propagate iteration after iteration, and compare the results produced by the different manuscript versions. In other words, I will ask the question: are the ΔT values produced by different manuscripts the same? And if different, by how much and why? When asking this second question, I will compare the actual values of ΔT produced by different manuscripts after different numbers of iterations. For each of these two questions, the properties and results of computation with the edited version of the table will be the paradigm against which the phenomena linked to the different manuscript versions are identified. All effects that I will point out are local: they are caused by individual variants in the manuscripts and influence only the ΔT found for specific solar and lunar positions.

Computing with one manuscript: convergence issues

In this section I will present three different results in order to grasp how convergence occurs when computing with the tables attested in the corpus of the *Tabule magne*. The first thing is to measure the effect of arithmetic variability on the convergence. The second is to measure the effect of manuscript variability on the speed of the convergence. The third and last case will be a curious situation of non-convergence.

In order to isolate the particular effect of arithmetic variability on the convergence, three iterations of the process were made using the critically edited table set with the three different types of arithmetic here considered. As can be seen in Figure 4, the distance between the true sun and true moon after three iterations is zero everywhere when floating number arithmetic is used.

³³ I have used spreadsheets to explore the space of computations because I needed to follow each step of the computations with accuracy. These spreadsheets are available on demand. Soon the DISHAS platform (see the introduction of this volume) will allow researchers to manipulate tables and different related procedures directly in a Python environment. This will allow a much more efficient exploration of the space of computations created by historical tables and procedures. It will also foster collaborative research on these topics as the research data and procedures will be publicly available.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	0	0	0	0	0	0	0	0	0	0	0
180	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	0	0	0	0	0	0	0	0	0	0	0
240	0	0	0	0	0	0	0	0	0	0	0	0	0
270	0	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	0	0	0	0	0	0
330	0	0	0	0	0	0	0	0	0	0	0	0	0
360	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4: Distance between the true sun and true moon in arcsecs after three iterations when using floating number arithmetic and the critically edited tables.

If the arithmetic of an historical actor is used, this same convergence occurs after three iterations with residual arithmetical noise. Figure 5 shows that when using integer arithmetic with rounding to seconds this noise has a maximum magnitude of ± 2 seconds. This noise persists also after six iterations.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	-1	-2	-1	-1	0	0	0	0	0	0	1	0
30	1	0	0	0	0	0	1	0	1	0	0	0	1
60	-1	-1	-2	1	0	0	0	-1	0	1	0	0	-1
90	1	1	1	1	0	1	1	0	-1	0	-1	2	1
120	1	0	-1	0	0	0	1	0	0	0	2	0	1
150	0	0	0	0	0	0	1	0	0	0	1	1	0
180	0	0	0	0	0	-1	0	1	0	0	0	0	0
210	1	-1	-1	0	0	0	-1	0	0	0	0	0	0
240	0	0	-2	0	0	0	-1	0	0	0	1	0	-1
270	-1	-2	1	0	1	0	-1	0	0	-1	-1	-1	-1
300	0	0	0	-1	0	1	0	0	0	-1	2	1	1
330	-1	0	0	0	-1	0	-1	0	0	0	0	0	-1
360	0	-1	0	0	0	0	0	0	0	0	0	1	0

Figure 5: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic and the critically edited tables.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	14	6	-5	-10	0	-5	-12	-22	31	-16	0
30	9	28	-17	-28	-28	26	21	8	-5	-27	29	-13	9
60	-1	27	-4	-12	-8	30	15	-18	17	-16	-12	18	-1
90	28	2	-23	29	-25	9	-8	8	29	-10	-1	-22	28
120	8	-23	1	-7	-18	-20	26	-7	29	-4	1	28	8
150	26	-21	-9	-22	-20	-24	-22	29	18	-1	-7	8	26
180	0	3	7	-4	-10	-6	0	6	10	4	-7	-3	0
210	8	-8	7	1	-18	-29	22	24	20	22	9	21	-26
240	6	-28	-1	4	-28	7	-26	20	18	7	-1	23	-8
270	-20	22	1	10	-29	-8	8	-9	25	-29	23	-2	-28
300	17	-18	12	16	-17	18	-15	-30	8	12	4	-27	1
330	25	13	-29	27	5	-8	-21	-26	28	28	17	-28	-9
360	0	16	-31	22	12	5	0	-5	-12	-22	31	-16	0

Figure 6: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic, the *tabula longitudinis horarum* and the critically edited tables.

However, if the *tabula longitudinis horarum* is used for the final division, the magnitude of the arithmetical noise is around fifteen times larger with a maximum of ± 28 seconds (Figure 6). This noise remains robust even after six iterations. This is already a significant result because the effects of the various arithmetical choices available to the actors show that even on arithmetical grounds they must have had a nuanced understanding of convergence.

These arithmetical effects are robust enough to be preserved across the manuscript variability. Arithmetical variability and manuscript variability do not compound and appear to be, in the situation analysed here, independent. Whatever manuscript version is used to compute ΔT , it will not be possible to go below the arithmetical noise that was isolated above. When using integer arithmetic there will always be a residual arithmetical noise of up to two arcsecs and when using the *Tabula longitudinis horarum* the residual noise can be as large as half an arcmin. This arithmetical noise, especially as shown in the last case, may seem large. However, most ephemerides were computed to the nearest minute during the late medieval period in Europe, thus these computations met a standard that was state of the art at the time.³⁴

Now that this arithmetical effect is known, I will give results only using integer arithmetic provided that it is easy to conceive what the result would be if other arithmetics were used. The second effect I want to consider is that of manuscript variability on convergence speed. In this respect, it is important

³⁴ Personal discussion with Richard Kremer (January 2019).

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	-1	-2	-1	-1	0	0	0	0	0	0	1	0
30	1	0	1	0	0	0	1	0	0	2	0	1	1
60	-1	-1	-2	1	0	0	0	-1	0	1	-1	0	-1
90	1	1	1	1	0	1	1	0	-1	0	-1	2	1
120	1	0	-1	0	0	0	1	0	0	0	2	0	1
150	0	0	1	0	0	0	1	0	0	0	1	1	0
180	0	0	0	0	0	-1	0	1	0	0	0	0	0
210	1	-1	-1	0	0	0	-1	0	0	0	-1	0	0
240	0	0	-2	0	0	0	-1	0	0	0	1	0	-1
270	-1	-2	1	0	1	0	-1	-1	0	-1	-1	-1	-1
300	0	0	1	-1	0	1	0	0	0	-1	2	1	1
330	-1	-1	0	-2	0	0	-1	0	0	0	-1	0	-1
360	0	-1	0	0	0	0	0	0	0	0	0	1	0

Figure 7: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic and manuscript P.

to understand that the first two steps of the iterative procedure proposed by the canons to the *Tabule magne* are complex processes requiring in most cases several readings in tables, linear interpolations and multiple elementary arithmetical operations on the numbers composing the astronomical quantities. Depending on the situation and the skills of the computer, adding one iteration to the computation will require between five to fifteen minutes of arithmetical work on paper. In order to isolate and illustrate this effect I compare manuscripts P and V3. Figure 7 shows that after three iterations with the tables in manuscript P the distance between the true sun and true moon are everywhere within the arithmetical noise proper to the use of integer arithmetic. Thus with this manuscript, just as with the critical edition, we obtain a final value for ΔT after three iterations.

The situation is different when we use V3 instead of P. Figure 8 shows that after three iterations the distance between the true sun and true moon can rise in some cases up to almost 10 arcmins. It is only after two more iterations that these worst-case scenarios are finally settled, as is shown in Figure 9. In other words, an historical actor's computing with V3 rather than P would in most cases find a stable value of ΔT in three iterations but could in some cases need as many as five. This certainly makes a significant difference. In this respect the computation of ΔT with V2 is qualitatively similar to that of V3.

It is interesting to note that the effects of manuscript variability disappear when the errors in the degree columns of V3 are corrected, as is shown in Figure 10. A user able to make these 'skilled corrections' to the tables would avoid the inconvenience of having to do the additional iterations.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	-1	-2	-1	-1	0	0	0	0	0	0	1	0
30	1	0	0	0	0	33	1	0	1	0	0	0	1
60	9	-7	-6	-9	-3	355	13	-18	-10	-8	-16	-16	9
90	-3	2	1	1	0	70	-3	0	-1	0	-1	2	-3
120	1	0	-1	-1	0	71	1	0	0	0	2	0	1
150	0	0	0	0	0	32	1	0	0	0	1	1	0
180	0	0	0	0	0	-1	0	1	0	0	0	0	0
210	1	-1	-1	0	0	0	-1	-32	0	0	0	0	0
240	0	0	-2	0	0	0	-1	-71	0	1	1	0	-1
270	2	-2	1	0	1	0	3	-70	0	-1	-1	-2	3
300	-20	16	16	8	10	18	-13	-355	3	9	6	7	-9
330	-1	0	0	0	-1	0	-1	-33	0	0	0	0	-1
360	0	-1	0	0	0	0	0	0	0	0	0	1	0

Figure 8: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic and manuscript V3.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	0	1	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	2	1	0	0	0	0	0	0
60	1	0	0	0	0	1	0	0	0	0	0	0	1
90	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	-1	0	0	0	0	0	0	0
150	0	0	0	0	0	2	0	0	0	0	1	0	0
180	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	-1	0	0	0	0	-2	0	0	0	0	0
240	0	0	0	0	0	0	0	1	0	0	0	0	0
270	0	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	0	0	-1	0	0	0	0	-1
330	-1	0	0	0	0	0	-1	-2	0	0	0	0	0
360	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 9: Distance between the true sun and true moon in arcsecs after five iterations when using integer arithmetic and manuscript V3.

With respect to convergence speed, the case of V1 also shows an interesting mathematical phenomenon. The distance between the true sun and true moon in arc-seconds after three iterations using integer arithmetic is presented in Figure 11. It shows a set of results within the expected arithmetical noise.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	-1	-2	-1	-1	0	0	0	0	0	0	1	0
30	1	0	0	0	0	0	1	0	1	0	0	0	1
60	0	-1	-1	1	0	0	0	0	-1	1	-1	1	0
90	-3	2	1	1	0	2	-3	0	-1	0	-1	2	-3
120	1	0	-1	-1	0	0	1	0	0	0	2	0	1
150	0	0	0	0	0	0	1	0	0	0	1	1	0
180	0	0	0	0	0	-1	0	1	0	0	0	0	0
210	1	-1	-1	0	0	0	-1	0	0	0	0	0	0
240	0	0	-2	0	0	0	-1	0	0	1	1	0	-1
270	2	-2	1	0	1	0	3	-2	0	-1	-1	-2	3
300	1	-1	1	-1	1	0	0	0	0	-1	1	1	0
330	-1	0	0	0	-1	0	-1	0	0	0	0	0	-1
360	0	-1	0	0	0	0	0	0	0	0	0	1	0

Figure 10: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic and a version of manuscript V3 in which discrepancies in the degrees are corrected.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	-1	-2	-1	-1	0	0	0	0	0	0	1	0
30	1	0	0	0	0	0	1	0	1	0	0	0	1
60	-1	-1	-1	0	-1	0	0	-1	0	1	0	0	-1
90	-3	1	1	1	0	2	-3	0	-1	0	-1	2	-3
120	1	0	-1	0	0	0	1	0	0	0	2	0	1
150	0	0	0	0	0	0	1	0	0	0	1	1	0
180	0	0	0	0	0	-1	0	1	0	0	0	0	0
210	1	-1	-1	0	0	0	-1	0	0	0	0	0	0
240	0	0	-2	0	0	0	-1	0	0	0	1	0	-1
270	2	-2	1	0	1	0	3	-2	0	-1	-1	-1	3
300	0	0	0	-1	0	1	0	0	1	0	1	1	1
330	-1	0	0	0	-1	0	-1	0	0	0	0	0	-1
360	0	-1	0	0	0	0	0	0	0	0	0	1	0

Figure 11: Distance between the true sun and true moon in arcsecs after three iterations when using integer arithmetic and manuscript V1.

However, the situation at the fourth iteration becomes even better, as shown in Figure 12. It remains stable at least until the sixth iteration. Thus, in the grid of the distance between the true sun and true moon after the third iteration, some of the non-zero values are caused by the manuscript variants, but

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	0	-1	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	-1	0	0	0	0	0	0
60	0	1	0	0	0	0	0	-1	0	0	0	0	0
90	0	1	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	0	0	0	0	0	0	0	0	-1	0	0
180	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	1	0	0	0	0	0	0	0	0	0	0
240	0	0	0	0	0	0	0	0	0	0	0	0	0
270	0	0	0	0	0	0	0	0	0	0	0	-1	0
300	0	0	0	0	0	1	0	0	0	0	0	-1	0
330	1	0	0	0	0	0	1	0	0	0	0	0	0
360	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 12: Distance between the true sun and true moon in arcsecs after four iterations when using integer arithmetic and manuscript V1.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	0	1	0	0	0	0	0	0	0	0	0
30	0	0	1	0	0	0	1	0	0	0	0	0	0
60	0	1	0	0	0	0	0	1	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	675	1860	929	0	0	0	0	0	-1	0	0
180	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	1	0	0	0	0	0	-929	-1860	-675	0	0
240	0	0	0	0	0	0	0	0	0	0	0	0	0
270	-1	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	-1	0	0	0	0	0	-1	0
330	-1	0	0	0	0	0	-1	0	0	0	-1	0	0
360	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 13: Distance between the true sun and true moon in arcsecs after five iterations when using integer arithmetic and manuscript C.

these values are at the level of the arithmetical noise. Most of these are eliminated by a further iteration. It should be noted that no table in V1 shows discrepancies from the critical edition in the numbers of degrees. Thus, this effect is produced only by variants in minutes and seconds and probably difficult to avoid even for a ‘skilled’ user.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0	0	0	-1	0	0	0	0	0	0	0	0	0
30	0	0	-1	0	0	0	-1	0	0	0	0	0	0
60	0	-1	0	0	0	0	0	-1	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0
150	0	0	-674	-1861	-929	0	0	0	0	0	1	0	0
180	0	0	0	0	0	0	0	0	0	0	0	0	0
210	0	0	-1	0	0	0	0	0	929	1861	674	0	0
240	0	0	0	0	0	0	0	0	0	0	0	0	0
270	1	0	0	0	0	0	0	0	0	0	0	0	0
300	0	0	0	0	0	1	0	0	0	0	0	1	0
330	1	0	0	0	0	0	1	0	0	0	1	0	0
360	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 14: Distance between the true sun and true moon in arcsecs after six iterations when using integer arithmetic and manuscript C.

In the different situations presented so far, the procedure converges and the manuscripts produce a value for ΔT . However, this is not always the case. Figure 13 shows the distance between the true sun and moon after five iterations using C and integer arithmetic. Most of the values are within the arithmetical noise (and this was already the case at the third iteration, just as it was for P and V1), but six values show a distance of up to 30 arcmins.

The next iterations do not improve this situation, as shown in Figure 14. We get the same six outliers with the same order of magnitude, except that their sign is reversed. This oscillation between two sets of outliers continues endlessly as further iterations are performed.

Probably the most skilled users would have tried to correct the values in the table. This attitude would have been rewarded because the effect disappears with a version of C corrected for the variants in the degrees. This in turn shows that this oscillation is truly an effect of manuscript variants.

This first set of results clearly shows that not all manuscript versions have the same arithmetical features. Some versions, because of their variants, require additional iterations to obtain convergence in certain cases. There are even cases where no result is obtained. The largest effects of manuscript variants, such as oscillation instead of convergence, are corrected when variants in the largest order of magnitude are amended, but some effects remain even when variants only in minutes and seconds are kept. Finally, the amplitude of the arithmetical noise is related to specific computational methods. This noise usually swamps the effects of manuscript variants somewhere between the third and fourth iteration and implies that the practical notion of convergence and

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0,00	0,00	0,00	0,00	129,42	0,00	0,00	0,00	-129,42	0,00	0,00	0,00	0,00
30	0,00	0,00	0,00	0,00	128,04	0,00	0,00	0,00	-128,04	0,00	0,00	0,00	0,00
60	0,00	0,00	0,00	0,00	124,50	0,00	0,00	0,00	-124,50	0,00	0,00	0,00	0,00
90	0,00	0,00	0,00	0,00	119,14	0,00	0,00	0,00	-119,14	0,00	0,00	0,00	0,00
120	0,00	0,00	0,00	0,00	113,39	0,00	0,00	0,00	-113,39	0,00	0,00	0,00	0,00
150	0,00	0,00	0,00	0,00	108,98	0,00	0,00	0,00	-108,98	0,00	0,00	0,00	0,00
180	0,00	0,00	0,00	0,00	107,20	0,00	0,00	0,00	-107,20	0,00	0,00	0,00	0,00
210	0,00	0,00	0,00	0,00	108,98	0,00	0,00	0,00	-108,98	0,00	0,00	0,00	0,00
240	0,00	0,00	0,00	0,00	113,39	0,00	0,00	0,00	-113,39	0,00	0,00	0,00	0,00
270	0,00	0,00	0,00	0,00	119,14	0,00	0,00	0,00	-119,14	0,00	0,00	0,00	0,00
300	0,00	0,00	0,00	0,00	124,50	0,00	0,00	0,00	-124,50	0,00	0,00	0,00	0,00
330	0,00	0,00	0,00	0,00	128,04	0,00	0,00	0,00	-128,04	0,00	0,00	0,00	0,00
360	0,00	0,00	0,00	0,00	129,42	0,00	0,00	0,00	-129,42	0,00	0,00	0,00	0,00

Figure 15: Differences in minutes between ΔT produced by V2 and by the critical edition after one iteration using integer arithmetic.

zero distance between the sun and moon must have been nuanced for the historical actors if only on arithmetical grounds.

This study also allows to propose manuscript groupings, based on computation performance. In this respect V2 and V3 are put together as less efficient versions requiring in specific cases five or six iterations while C, P, and V1 all have similar convergence speeds reasonably close to that of the edited version.³⁵

Comparing computational features of manuscripts: variant propagation and coherency issues

The preceding section portrayed an individual historical actor computing with one manuscript at a time. In this final section, I shall consider a collective of historical actors and compare results given by the different manuscript versions. More precisely, the values of ΔT given by each manuscript version are compared with those given by the edited version.

The case of V2 is representative of the different phenomena I was able to identify with respect to this question. Figure 15 shows the differences in minutes between the values of ΔT produced with V2 and those produced with the critical edition after one iteration. In most cases the two values of ΔT agree, except for two large sets of discrepancies of around 2 hours.

If the degrees of the values in V2 are ‘corrected’ to match those of the critical edition these two large sets of discrepancies disappear, thus showing that they are an effect of manuscript variability at this level.

³⁵ If we exclude the very specific oscillation case of C.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0,00	0,00	0,00	0,00	129,24	0,00	0,00	0,00	-129,28	0,00	0,00	0,00	0,00
30	0,00	0,00	0,00	0,00	128,00	-13,21	0,00	0,00	-127,61	0,00	0,00	0,00	0,00
60	0,00	0,00	0,00	0,00	112,00	-31,28	0,00	0,00	-123,27	0,00	0,00	0,00	0,00
90	1,37	0,33	0,00	0,00	99,28	-36,57	1,35	0,00	-117,59	0,00	0,00	0,00	1,37
120	0,00	0,00	0,00	0,00	99,78	-29,28	0,00	0,01	-112,36	0,00	0,01	0,01	0,00
150	0,00	0,00	0,00	0,00	109,13	-12,56	0,00	0,00	-108,62	0,00	0,00	0,00	0,00
180	0,00	0,00	0,00	0,00	107,46	0,00	0,00	0,00	-107,46	0,00	0,00	0,00	0,00
210	0,00	0,00	0,00	0,00	108,62	0,00	0,00	12,56	-109,13	0,00	0,00	0,00	0,00
240	0,00	-0,01	-0,01	0,00	112,36	-0,01	0,00	29,28	-99,78	0,00	0,00	0,00	0,00
270	-1,37	0,00	0,00	0,00	117,59	0,00	-1,35	36,57	-99,28	0,00	0,00	-0,33	-1,37
300	0,00	0,00	0,00	0,00	123,27	0,00	0,00	31,28	-112,00	0,00	0,00	0,00	0,00
330	0,00	0,00	0,00	0,00	127,61	0,00	0,00	13,21	-128,00	0,00	0,00	0,00	0,00
360	0,00	0,00	0,00	0,00	129,24	0,00	0,00	0,00	-129,28	0,00	0,00	0,00	0,00

Figure 16: Differences in minutes between ΔT produced by V2 and by the critical edition after two iterations using integer arithmetic.

	0	30	60	90	120	150	180	210	240	270	300	330	360
0	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
30	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
60	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
90	1,37	0,33	0,00	0,00	0,00	0,23	1,35	0,00	0,00	0,00	0,00	0,00	1,37
120	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,00	0,01	0,01	0,00
150	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
180	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
210	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
240	0,00	-0,01	-0,01	0,00	-0,01	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00
270	-1,37	0,00	0,00	0,00	0,00	0,00	-1,35	-0,23	0,00	0,00	0,00	-0,33	-1,37
300	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
330	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
360	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Figure 17: Differences in minutes between ΔT produced by V2 with values corrected to match the critical edition in degrees and minutes and the critical edition after two iterations using integer arithmetic.

On the other hand, adding one iteration to the computation does not smoothen the differences between the critical edition and V2. As can be seen in Figure 16, not only are the two columns of large differences maintained at the same order of magnitude, but new discrepancies appear of a smaller but yet significant magnitude of more than 30 minutes. For the second iteration new

values from the tables are used. This explains the appearance of new discrepancies. Discrepancies appearing at one stage of the iteration are usually maintained in the following iterations. This is linked to the fact that this iterative process is generally convergent. If the convergence target of a manuscript version is slightly distinct from that of an edited text, successive iterations will only confirm the tendency and thus discrepancies will be conserved.

Finally, it is interesting to see that even when V2 is 'corrected' so that its degree values match those of the critical edition, these minor discrepancies between values of ΔT remain although their magnitude is much smaller and in most cases less than a minute, as can be seen in Figure 17.

Conclusion

In this article my aim was to explore the practice of computing with manuscripts that contain variant entries. For this I have selected a specific astronomical issue and a single set of astronomical tables.

From a methodological perspective, this goal required a complete astronomical understanding of the table set and I thus relied on standard recomputation approaches. I have associated in specific ways these recomputation approaches with that of diplomatic description of tables and variants typology. I also designed my critical edition not as research of a hypothetical genuine version of the tables but as a tool to explore variants and potential 'skilled corrections' of these variants by table users. Finally, I developed new tools to 'restore' astronomical tables as computational devices by attending to their accompanying canons.

The results obtained are encouraging. My approach allows us to isolate computational effects proper from the effects of manuscript variants and to compare differing arithmetical procedures. Analysis suggests that historical actors engaged with these types of tables and computations could probably have detected some of these effects. For instance, some manuscript versions generate results that converge more rapidly than others do; and some tables, like the *tabula longitudinis horarum*, have arithmetical performances significantly different from that of other ways to perform the computation.

I hope that some of the approaches and tools developed and used here manually and on a small scale could be further refined and integrated into tool boxes for digital humanities that are being developed for the history of the astral sciences. For instance, the various alternative arithmetical algorithms may be useful in other contexts as well. The use of weighted majority rules to produce a specific type of critical edition could also be usefully implemented as a general digital tool. Finally, research on those diplomatic features of astronomical tables that might have an impact on the way manuscripts can be used in computations, if only by being linked to certain kinds of manuscript vari-

ants, ought also to be pursued. In the end, these tools might enable the design of digital diplomatic transcriptions of astronomical tables that could help us explore the use of tables in computations and as such provide more insight into actors' practices than does a simple digital *facsimile* of a manuscript.

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Appendix A: Critical edition of the tables

A1. Conventions

Digits with variants are bold in the table. In the critical apparatus, variant positions are identified in parentheses using the line and column marks written around the grid. Variant digits are given after the sigla of the witnesses that include them. Variants are listed column by column, from top to bottom.

In all tables, arguments and headings are marked in grey shaded cells.

A2. Edition of the solar equation

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>	<i>O</i>
	argument		Solar Equation			argument		Solar Equation			argument		Solar Equation		
	d	d	d	m	s	d	d	d	m	s	d	d	d	m	s
<i>1</i>	1	179	0	2	10	31	149	1	4	46	61	119	1	51	51
<i>2</i>	2	178	0	4	19	32	148	1	6	37	62	118	1	52	56
<i>3</i>	3	177	0	6	27	33	147	1	8	28	63	117	1	54	9
<i>4</i>	4	176	0	8	36	34	146	1	10	19	64	116	1	55	6
<i>5</i>	5	175	0	10	44	35	145	1	12	9	65	115	1	56	9
<i>6</i>	6	174	0	12	53	36	144	1	13	58	66	114	1	57	11
<i>7</i>	7	173	0	15	2	37	143	1	15	41	67	113	1	58	2
<i>8</i>	8	172	0	17	10	38	142	1	17	24	68	112	1	58	52
<i>9</i>	9	171	0	19	19	39	141	1	19	6	69	111	1	59	41
<i>10</i>	10	170	0	21	28	40	140	1	20	48	70	110	2	0	26
<i>11</i>	11	169	0	23	36	41	139	1	22	29	71	109	2	1	16
<i>12</i>	12	168	0	25	45	42	138	1	24	10	72	108	2	2	2
<i>13</i>	13	167	0	27	53	43	137	1	25	50	73	107	2	2	41
<i>14</i>	14	166	0	30	1	44	136	1	27	29	74	106	2	3	21
<i>15</i>	15	165	0	32	8	45	135	1	29	8	75	105	2	3	59
<i>16</i>	16	164	0	34	16	46	134	1	30	46	76	104	2	4	36
<i>17</i>	17	163	0	36	23	47	133	1	32	23	77	103	2	5	16
<i>18</i>	18	162	0	38	30	48	132	1	33	59	78	102	2	5	48
<i>19</i>	19	161	0	40	37	49	131	1	35	30	79	101	2	6	17
<i>20</i>	20	160	0	42	43	50	130	1	37	0	80	100	2	6	45
<i>21</i>	21	159	0	44	49	51	129	1	38	30	81	99	2	7	12
<i>22</i>	22	158	0	46	55	52	128	1	39	58	82	98	2	7	37
<i>23</i>	23	157	0	48	59	53	127	1	41	57	83	97	2	8	2
<i>24</i>	24	156	0	51	4	54	126	1	42	54	84	96	2	8	27
<i>25</i>	25	155	0	53	4	55	125	1	44	14	85	95	2	8	45
<i>26</i>	26	154	0	55	2	56	124	1	45	34	86	94	2	9	1
<i>27</i>	27	153	0	57	1	57	123	1	46	53	87	93	2	9	17
<i>28</i>	28	152	0	58	59	58	122	1	48	10	88	92	2	9	32
<i>29</i>	29	151	1	0	57	59	121	1	49	28	89	91	2	9	45
<i>30</i>	30	150	1	2	54	60	120	1	50	45	90	90	2	9	57

(D28) V3:59 (E4) V1:38 (E7) P:3 (I11) V1:24 (I12) P:23 (I15) V2:30 (I16) V2:32 (I17) V1:30; V2:33 (I18) V1, V2:32 (I19) V2:34 (I24) PV2,V3:43 (I28) V3:46 (J11) V1,V2,V3:10 (J15) V2,V3:46 (J16) V2:23 (J17) V1:46; V2:59 (J18) V1,V2:23 (J19) V2:34 (J23) E:27 (J27) C,P:52 (J28) V2:18 (O3) V3:0 (O6) P:12 (O11) V1,V2,V3:13 (O12) P:3 (O22) V2,V3:36

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>	<i>O</i>
	argument		Solar Equation			argument		Solar Equation			argument		Solar Equation		
	d	d	d	m	s	d	d	d	m	s	d	d	d	m	s
<i>1</i>	91	89	2	9	59	121	59	1	53	46	151	29	1	5	1
<i>2</i>	92	88	2	10	0	122	58	1	52	35	152	28	1	2	54
<i>3</i>	93	87	2	10	0	123	57	1	52	24	153	27	1	0	57
<i>4</i>	94	86	2	10	0	124	56	1	50	15	154	26	0	58	40
<i>5</i>	95	85	2	9	57	125	55	1	48	59	155	25	0	56	33
<i>6</i>	96	84	2	9	51	126	54	1	47	46	156	24	0	54	25
<i>7</i>	97	83	2	9	36	127	53	1	46	20	157	23	0	52	17
<i>8</i>	98	82	2	9	20	128	52	1	44	53	158	22	0	50	9
<i>9</i>	99	81	2	9	2	129	51	1	43	26	159	21	0	48	11
<i>10</i>	100	80	2	8	45	130	50	1	41	57	160	20	0	45	54
<i>11</i>	101	79	2	8	25	131	49	1	40	27	161	19	0	43	44
<i>12</i>	102	78	2	8	6	132	48	1	38	57	162	18	0	41	35
<i>13</i>	103	77	2	7	41	133	47	1	37	25	163	17	0	39	26
<i>14</i>	104	76	2	7	14	134	46	1	35	53	164	16	0	37	16
<i>15</i>	105	75	2	6	46	135	45	1	34	20	165	15	0	35	6
<i>16</i>	106	74	2	6	18	136	44	1	32	46	166	14	0	32	51
<i>17</i>	107	73	2	5	48	137	43	1	31	12	167	13	0	30	35
<i>18</i>	108	72	2	5	18	138	42	1	29	37	168	12	0	28	19
<i>19</i>	109	71	2	4	42	139	41	1	27	50	169	11	0	26	1
<i>20</i>	110	70	2	4	5	140	40	1	26	3	170	10	0	23	42
<i>21</i>	111	69	2	3	37	141	39	1	24	16	171	9	0	21	22
<i>22</i>	112	68	2	2	37	142	38	1	22	28	172	8	0	19	1
<i>23</i>	113	67	2	1	45	143	37	1	20	40	173	7	0	16	40
<i>24</i>	114	66	2	0	51	144	36	1	18	51	174	6	0	14	19
<i>25</i>	115	65	1	59	53	145	35	1	17	0	175	5	0	11	58
<i>26</i>	116	64	1	58	55	146	34	1	15	8	176	4	0	9	36
<i>27</i>	117	63	1	57	57	147	33	1	13	16	177	3	0	7	12
<i>28</i>	118	62	1	56	57	148	32	1	11	13	178	2	0	4	48
<i>29</i>	119	61	1	55	57	149	31	1	9	10	179	1	0	2	24
<i>30</i>	120	60	1	54	57	150	30	1	7	0	180	0	0	0	0

(C25) V2:2 (C26) V2:2 (C27) V2:2 (C28) V2:2 (C29) V2:2 (C30) V2:2 (D22) V3:1 (E7) V1:56 (E11) C,V1:26 (E14) V3:18 (E18) V2,V3:16 (E23) V3:47 (I2) P,V1:53 (I3) P,E:51 (I16) C,P:33 (I18) V1,V2:20 (J1) V1:48 (J4) E:12 (J6) E:40 (J7) P:30 (J8) V1:52 (J12) C:53; V2:53 (J16) V2:26 (J17) V2:13 (J18) P:39; V2:40 (J24) V2:11 (J30) E:7 (M1) V2,V3:0 (M2) V1,V3:0 (M3) V1,V2:0 (N12) p:42 (O8) C:11; P:2 (O9) C,P:9; E:1 (O10) E:53 (O12) P:38 (O16) V3:50 (O28) V1,V2,V3:28

A3. Edition of the lunar equation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	argument		Lunar equation			argument		Lunar equation			argument		Lunar equation		
	d	d	d	m	s	d	d	d	m	s	d	d	d	m	s
1	1	179	0	4	46	31	149	2	21	46	61	119	4	7	47
2	2	178	0	9	31	32	148	2	25	55	62	118	4	10	27
3	3	177	0	14	15	33	147	2	30	5	63	117	4	13	3
4	4	176	0	19	0	34	146	2	34	12	64	116	4	15	35
5	5	175	0	23	44	35	145	2	38	17	65	115	4	18	3
6	6	174	0	28	28	36	144	2	42	21	66	114	4	20	27
7	7	173	0	33	11	37	143	2	46	22	67	113	4	22	47
8	8	172	0	37	54	38	142	2	50	19	68	112	4	25	2
9	9	171	0	42	27	39	141	2	54	14	69	111	4	27	12
10	10	170	0	47	19	40	140	2	58	7	70	110	4	29	18
11	11	169	0	52	0	41	139	3	1	58	71	109	4	31	20
12	12	168	0	56	41	42	138	3	5	46	72	108	4	33	18
13	13	167	1	1	20	43	137	3	9	31	73	107	4	35	11
14	14	166	1	5	59	44	136	3	13	13	74	106	4	36	59
15	15	165	1	10	38	45	135	3	16	51	75	105	4	38	43
16	16	164	1	15	15	46	134	3	19	26	76	104	4	40	23
17	17	163	1	19	51	47	133	3	23	59	77	103	4	41	58
18	18	162	1	25	24	48	132	3	27	30	78	102	4	43	28
19	19	161	1	29	0	49	131	3	30	57	79	101	4	44	53
20	20	160	1	33	32	50	130	3	34	20	80	100	4	46	13
21	21	159	1	38	3	51	129	3	37	40	81	99	4	47	26
22	22	158	1	42	33	52	128	3	40	57	82	98	4	48	35
23	23	157	1	46	1	53	127	3	44	19	83	97	4	49	38
24	24	156	1	51	27	54	126	3	47	20	84	96	4	50	41
25	25	155	1	55	52	55	125	3	50	26	85	95	4	51	38
26	26	154	2	0	15	56	124	3	53	29	86	94	4	52	28
27	27	153	2	4	37	57	123	3	56	30	87	93	4	53	11
28	28	152	2	8	57	58	122	3	59	26	88	92	4	53	50
29	29	151	2	13	14	59	121	4	2	17	89	91	4	54	25
30	30	150	2	17	29	60	120	4	4	5	90	90	4	54	58

(C28) C,P:1 (D7) C:32 (D14) P:15 (D16) V3:19 (D18) C:22; P:27; E:24 (D23) E:47 (E9) E:37 (E12) V2,V3:14 (E18) C:34; E:27 (E19) V1:1 (E25) C,P:51 (E28) C,P:59 (E30) C,P:39 (H29) V3:3 (H30) V3:3 (I8) V2:30 (I11) V1,V2:5 V3:2 (I15 V2,V3:19 (I16) E:20 (I17) V1,V2:19 (I18) V1,V2:23 (I19) V1,V2:27 (I20) V1,V2:30 (I21) V1,V2:34 (I22) V1,V2:47 (I30) V3,E:5 (J1) E:43 (J11) V1,V2,V3:46 (J15) V2,V3:26 (J1è) C,P:29; V1,V2:26 (J18) V1,V2: 59 (J19) V1,V2:30 (J20) V1,V2:57 (J21) V1,V2:20 (J22) V1,V2:40 (J25) V2:27 (J30) C:2; E:4 (N9) V1:57 (N23) V3:45 (N28) V2:54 (O2) V1:37 (O5) V3:30 (O7) C,P:57 (O8) C,P:20 (O14) V2,V3:50 (O18) C,P,V1:48 (019):V2:51 (O23) p:28 (O26) V2:38 (O29) P:35 (O30) E:54

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>	<i>O</i>
	argument		Lunar equation			argument		Lunar equation			argument		Lunar equation		
	d	d	d	m	s	d	d	d	m	s	d	d	d	m	s
<i>1</i>	91	89	4	55	18	121	59	4	24	38	151	29	2	34	52
<i>2</i>	92	88	4	55	37	122	58	4	22	7	152	28	2	30	6
<i>3</i>	93	87	4	55	49	123	57	4	19	38	153	27	2	25	16
<i>4</i>	94	86	4	55	55	124	56	4	16	58	154	26	2	20	23
<i>5</i>	95	85	4	56	0	125	55	4	14	13	155	25	2	15	26
<i>6</i>	96	84	4	55	56	126	54	4	11	23	156	24	2	10	26
<i>7</i>	97	83	4	55	43	127	53	4	8	28	157	23	2	5	22
<i>8</i>	98	82	4	55	25	128	52	4	5	31	158	22	2	0	17
<i>9</i>	99	81	4	55	4	129	51	4	2	30	159	21	1	55	9
<i>10</i>	100	80	4	54	41	130	50	3	59	29	160	20	1	49	58
<i>11</i>	101	79	4	54	12	131	49	3	56	5	161	19	1	44	44
<i>12</i>	102	78	4	53	38	132	48	3	52	47	162	18	1	39	27
<i>13</i>	103	77	4	52	59	133	47	3	49	23	163	17	1	34	9
<i>14</i>	104	76	4	52	14	134	46	3	45	52	164	16	1	28	49
<i>15</i>	105	75	4	51	22	135	45	3	42	17	165	15	1	23	26
<i>16</i>	106	74	4	50	22	136	44	3	38	37	166	14	1	18	1
<i>17</i>	107	73	4	49	17	137	43	3	34	53	167	13	1	12	34
<i>18</i>	108	72	4	48	10	138	42	3	31	3	168	12	1	7	6
<i>19</i>	109	71	4	46	54	139	41	3	27	10	169	11	1	1	36
<i>20</i>	110	70	4	45	33	140	40	3	23	12	170	10	0	56	5
<i>21</i>	111	69	4	44	7	141	39	3	19	9	171	9	0	50	32
<i>22</i>	112	68	4	42	34	142	38	3	15	2	172	8	0	44	58
<i>23</i>	113	67	4	40	56	143	37	3	10	50	173	7	0	39	23
<i>24</i>	114	66	4	39	15	144	36	3	6	35	174	6	0	33	47
<i>25</i>	115	65	4	37	29	145	35	3	2	15	175	5	0	28	10
<i>26</i>	116	64	4	35	37	146	34	2	57	51	176	4	0	22	33
<i>27</i>	117	63	4	33	41	147	33	2	53	23	177	3	0	16	56
<i>28</i>	118	62	4	31	34	148	32	2	48	51	178	2	0	11	18
<i>29</i>	119	61	4	29	20	149	31	2	44	15	179	1	0	5	40
<i>30</i>	120	60	4	27	0	150	30	2	39	35	180	0	0	0	0

(D5) V1,V2,V3:55 (D20) P:55 (E4) C,P:56; V1:51 (E9) C:41 (E19) C:59 (H28) C:3 (H:29) C:3 (H30) C:3 (I12) V1,V2,V3:53 (I21) V1:10 (I21) V1:10 (J2) V3,E:11 (J10) C,E:20 (J14) V2:53 (J22) V3:8 (J23) C,P:35 (J24) C,P:15; V1:25 (J25) C,P:51 (J26) C,P:23 (J27) C,P:51 (J28) C,P:14 (N8) VA,V2:9 (O1) C,P:0 (O7) V3:25 (O10) V2,V3:59 (O11) V3:24 (O18) V1:16 (O25) P:50

A4. Edition of the solar velocity

	<i>A</i>		<i>B</i>		<i>C</i>		<i>D</i>		<i>E</i>		<i>F</i>		<i>G</i>		<i>H</i>		<i>I</i>		<i>J</i>		<i>K</i>		<i>L</i>	
	argument		solar velocity				argument		solar velocity				argument		solar velocity									
	d	d	m	s	d	d	m	s	d	d	m	s	d	d	m	s	d	d	m	s	d	d	m	s
<i>1</i>	1	179	2	23	31	149	2	24	61	119	2	25												
<i>2</i>	2	178	2	23	32	148	2	24	62	118	2	25												
<i>3</i>	3	177	2	23	33	147	2	24	63	117	2	26												
<i>4</i>	4	176	2	23	34	146	2	24	64	116	2	26												
<i>5</i>	5	175	2	23	35	145	2	24	65	115	2	26												
<i>6</i>	6	174	2	23	36	144	2	24	66	114	2	26												
<i>7</i>	7	173	2	23	37	143	2	24	67	113	2	26												
<i>8</i>	8	172	2	23	38	142	2	24	68	112	2	26												
<i>9</i>	9	171	2	23	39	141	2	24	69	111	2	26												
<i>10</i>	10	170	2	23	40	140	2	24	70	110	2	26												
<i>11</i>	11	169	2	23	41	139	2	24	71	109	2	26												
<i>12</i>	12	168	2	23	42	138	2	24	72	108	2	26												
<i>13</i>	13	167	2	23	43	137	2	24	73	107	2	26												
<i>14</i>	14	166	2	23	44	136	2	24	74	106	2	26												
<i>15</i>	15	165	2	23	45	135	2	25	75	105	2	27												
<i>16</i>	16	164	2	23	46	134	2	25	76	104	2	27												
<i>17</i>	17	163	2	23	47	133	2	25	77	103	2	27												
<i>18</i>	18	162	2	23	48	132	2	25	78	102	2	27												
<i>19</i>	19	161	2	23	49	131	2	25	79	101	2	27												
<i>20</i>	20	160	2	23	50	130	2	25	80	100	2	27												
<i>21</i>	21	159	2	23	51	129	2	25	81	99	2	27												
<i>22</i>	22	158	2	23	52	128	2	25	82	98	2	27												
<i>23</i>	23	157	2	23	53	127	2	25	83	97	2	27												
<i>24</i>	24	156	2	23	54	126	2	25	84	96	2	27												
<i>25</i>	25	155	2	23	55	125	2	25	85	95	2	27												
<i>26</i>	26	154	2	23	56	124	2	25	86	94	2	27												
<i>27</i>	27	153	2	24	57	123	2	25	87	93	2	28												
<i>28</i>	28	152	2	24	58	122	2	25	88	92	2	28												
<i>29</i>	29	151	2	24	59	121	2	25	89	91	2	28												
<i>30</i>	30	150	2	24	60	120	2	25	90	90	2	28												

(H15) V2:24 (L2) C,P:26 (L14) V2:27

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>
	argument		solar velocity		argument		solar velocity		argument		solar velocity	
	d	d	m	s	d	d	m	s	d	d	m	s
<i>1</i>	91	89	2	28	121	59	2	30	151	29	2	32
<i>2</i>	92	88	2	28	122	58	2	30	152	28	2	32
<i>3</i>	93	87	2	28	123	57	2	30	153	27	2	32
<i>4</i>	94	86	2	28	124	56	2	30	154	26	2	32
<i>5</i>	95	85	2	28	125	55	2	30	155	25	2	32
<i>6</i>	96	84	2	28	126	54	2	30	156	24	2	32
<i>7</i>	97	83	2	28	127	53	2	30	157	23	2	32
<i>8</i>	98	82	2	28	128	52	2	30	158	22	2	32
<i>9</i>	99	81	2	29	129	51	2	30	159	21	2	33
<i>10</i>	100	80	2	29	130	50	2	30	160	20	2	33
<i>11</i>	101	79	2	29	131	49	2	30	161	19	2	33
<i>12</i>	102	78	2	29	132	48	2	30	162	18	2	33
<i>13</i>	103	77	2	29	133	47	2	30	163	17	2	33
<i>14</i>	104	76	2	29	134	46	2	30	164	16	2	33
<i>15</i>	105	75	2	29	135	45	2	30	165	15	2	33
<i>16</i>	106	74	2	29	136	44	2	30	166	14	2	33
<i>17</i>	107	73	2	29	137	43	2	30	167	13	2	33
<i>18</i>	108	72	2	29	138	42	2	32	168	12	2	33
<i>19</i>	109	71	2	29	139	41	2	32	169	11	2	33
<i>20</i>	110	70	2	29	140	40	2	32	170	10	2	33
<i>21</i>	111	69	2	29	141	39	2	32	171	9	2	32
<i>22</i>	112	68	2	29	142	38	2	32	172	8	2	32
<i>23</i>	113	67	2	29	143	37	2	32	173	7	2	32
<i>24</i>	114	66	2	29	144	36	2	32	174	6	2	32
<i>25</i>	115	65	2	29	145	35	2	32	175	5	2	32
<i>26</i>	116	64	2	29	146	34	2	32	176	4	2	32
<i>27</i>	117	63	2	29	147	33	2	32	177	3	2	32
<i>28</i>	118	62	2	29	148	32	2	32	178	2	2	32
<i>29</i>	119	61	2	29	149	31	2	32	179	1	2	32
<i>30</i>	120	60	2	29	150	30	2	32	180	0	2	32

(H1) E:29 (H2) E:29 (H14) E:31 (H15) E:31 (H16 E:31 (H17) E:32 (H18) C,P:30 (H19) C,P:30 (H20 C,P:30 (H21) C,P:30 (H22) C,P:30 (H23) C,P:30 (H24) C,P:30 (L1) V3:23 (L2) V3:23 (L4) V3:23 (L5) V3:23 (L6) V3:23 (L7) V3:23 (L8) V3:23 (L9) V3:23 (L10) V1,V2:33; V3:23 (L11) V2:32; V3:23 (L12) V3:32 (L13) V3:32 (L14) V3:32 (L15) V3:32 (L16) V3:32 (L17) V3:32 (L18) V3:32 (L19) V3:32 (L20) V3:32 (L21) E:33

A5. Edition of the lunar velocity

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>
	argument		lunar velocity		argument		lunar velocity		argument		lunar velocity	
	d	d	m	s	d	d	m	s	d	d	m	s
<i>1</i>	1	179	30	18	31	149	30	37	61	119	31	26
<i>2</i>	2	178	30	18	32	148	30	38	62	118	31	29
<i>3</i>	3	177	30	18	33	147	30	40	63	117	31	31
<i>4</i>	4	176	30	19	34	146	30	41	64	116	31	33
<i>5</i>	5	175	30	19	35	145	30	42	65	115	31	36
<i>6</i>	6	174	30	19	36	144	30	43	66	114	31	38
<i>7</i>	7	173	30	20	37	143	30	44	67	113	31	41
<i>8</i>	8	172	30	20	38	142	30	46	68	112	31	43
<i>9</i>	9	171	30	20	39	141	30	47	69	111	31	46
<i>10</i>	10	170	30	21	40	140	30	48	70	110	31	48
<i>11</i>	11	169	30	21	41	139	30	50	71	109	31	51
<i>12</i>	12	168	30	21	42	138	30	51	72	108	31	53
<i>13</i>	13	167	30	22	43	137	30	53	73	107	31	56
<i>14</i>	14	166	30	22	44	136	30	54	74	106	31	58
<i>15</i>	15	165	30	23	45	135	30	58	75	105	32	1
<i>16</i>	16	164	30	23	46	134	30	58	76	104	32	3
<i>17</i>	17	163	30	24	47	133	31	0	77	103	32	6
<i>18</i>	18	162	30	24	48	132	31	1	78	102	32	8
<i>19</i>	19	161	30	25	49	131	31	3	79	101	32	11
<i>20</i>	20	160	30	25	50	130	31	5	80	100	32	14
<i>21</i>	21	159	30	26	51	129	31	7	81	99	32	17
<i>22</i>	22	158	30	27	52	128	31	8	82	98	32	19
<i>23</i>	23	157	30	27	53	127	31	10	83	97	32	22
<i>24</i>	24	156	30	28	54	126	31	12	84	96	32	25
<i>25</i>	25	155	30	29	55	125	31	14	85	95	32	28
<i>26</i>	26	154	30	31	56	124	31	16	86	94	32	31
<i>27</i>	27	153	30	32	57	123	31	18	87	93	32	34
<i>28</i>	28	152	30	33	58	122	31	20	88	92	32	36
<i>29</i>	29	151	30	35	59	121	31	22	89	91	32	38
<i>30</i>	30	150	30	36	60	120	31	24	90	90	32	42

(D4) C:19 (D7) E:19 (D10) E:20 (D26) E:30 (G16) V1,V2,V3:31 (G17) E:30
(H4) V2,V3:42 (H5) V2,V3:43 (H6) V2,V3:44 (H7) V2,V3:45 (H15) E:56 (H16)
V1,V2,V3:0 (H17) E:59 (L5) V3:37 (L8) V3:42 (L16) V2:2 (L19) C,P:15 (L25)
V2:26 (L29) E:39

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>
	argument		lunar velocity		argument		lunar velocity		argument		lunar velocity	
	d	d	m	s	d	d	m	s	d	d	m	s
<i>1</i>	91	89	32	45	121	59	34	17	151	29	35	33
<i>2</i>	92	88	32	48	122	58	34	20	152	28	35	35
<i>3</i>	93	87	32	51	123	57	34	23	153	27	35	37
<i>4</i>	94	86	32	53	124	56	34	26	154	26	35	39
<i>5</i>	95	85	32	56	125	55	34	29	155	25	35	41
<i>6</i>	96	84	32	59	126	54	34	32	156	24	35	43
<i>7</i>	97	83	33	4	127	53	34	35	157	23	35	45
<i>8</i>	98	82	33	8	128	52	34	38	158	22	35	46
<i>9</i>	99	81	33	13	129	51	34	41	159	21	35	48
<i>10</i>	100	80	33	18	130	50	34	43	160	20	35	49
<i>11</i>	101	79	33	22	131	49	34	46	161	19	35	51
<i>12</i>	102	78	33	27	132	48	34	49	162	18	35	52
<i>13</i>	103	77	33	29	133	47	34	52	163	17	35	52
<i>14</i>	104	76	33	30	134	46	34	54	164	16	35	53
<i>15</i>	105	75	33	32	135	45	34	57	165	15	35	53
<i>16</i>	106	74	33	33	136	44	34	59	166	14	35	53
<i>17</i>	107	73	33	35	137	43	35	2	167	13	35	54
<i>18</i>	108	72	33	36	138	42	35	4	168	12	35	54
<i>19</i>	109	71	33	39	139	41	35	7	169	11	35	55
<i>20</i>	110	70	33	42	140	40	35	9	170	10	35	57
<i>21</i>	111	69	33	46	141	39	35	11	171	9	35	58
<i>22</i>	112	68	33	49	142	38	35	13	172	8	35	59
<i>23</i>	113	67	33	53	143	37	35	16	173	7	36	1
<i>24</i>	114	66	33	55	144	36	35	18	174	6	36	2
<i>25</i>	115	65	33	58	145	35	35	20	175	5	36	2
<i>26</i>	116	64	34	1	146	34	35	22	176	4	36	3
<i>27</i>	117	63	34	5	147	33	35	25	177	3	36	3
<i>28</i>	118	62	34	8	148	32	35	27	178	2	36	3
<i>29</i>	119	61	34	11	149	31	35	29	179	1	36	4
<i>30</i>	120	60	34	14	150	30	35	31	180	0	36	4

(D10) V2:16; V3:19 (D16) V3:32 (D23) V1,vé:54; E:52 (H5) V2:20 (H6) V2:22 (H19) E:6 (H27) V2:22 (L1) V1:34 (L8) V2,V3:47 (L9) V1:47 (L10) V1:48 (L19) V3:57

A6. Edition of the Tabula longitudinis horarum

<i>I</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>	<i>R</i>			<i>S</i>			<i>T</i>			<i>U</i>			<i>V</i>			<i>W</i>			<i>X</i>			<i>Y</i>		
																		h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s
2	1	2	13	20	2	8	34	2	4	8	2	0	0	1	56	8	1	52	30	1	49	5	1	45	53																
3	2	4	26	40	4	17	9	4	8	17	4	0	0	3	52	15	3	45	0	3	38	11	3	31	46																
4	3	6	40	0	6	25	43	6	12	25	6	0	0	5	48	23	5	37	30	5	27	16	5	17	39																
5	4	8	53	20	8	34	17	8	16	33	8	0	0	7	44	31	7	30	0	7	16	22	7	3	32																
6	5	11	6	40	10	42	51	10	20	41	10	0	0	9	40	39	9	22	30	9	5	27	8	49	25																
7	6	13	20	0	12	51	26	12	24	50	12	0	0	11	36	46	11	15	0	10	54	33	10	35	18																
8	7	15	33	20	15	0	0	14	28	58	14	0	0	13	32	54	13	7	30	12	43	38	12	21	11																
9	8	17	46	40	17	8	34	16	33	6	16	0	0	15	29	2	15	0	0	14	32	44	14	7	4																

(C3) C,P: 12; (U10) C:37; (V8) C: 32; (X10) C,P:3; (Y3) C:54;

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1																	
		27		28		29		30		31		32		33		34	
2	m	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s
3	1	2	13	2	9	2	4	2	0	1	56	1	52	1	49	1	46
4	2	4	27	4	17	4	8	4	0	3	52	3	45	3	38	3	32
5	3	6	40	6	26	6	12	6	0	5	48	5	38	5	27	5	18
6	4	8	53	8	34	8	17	8	0	7	45	7	30	7	16	7	4
7	5	11	7	10	43	10	21	10	0	9	41	9	23	9	5	8	49
8	6	13	20	12	51	12	25	12	0	11	37	11	15	10	55	10	35
9	7	15	33	15	0	14	29	14	0	13	33	13	8	12	44	12	21
10	8	17	47	17	9	16	33	16	0	15	29	15	0	14	33	14	7
11	9	20	0	19	17	18	37	18	0	17	25	16	53	16	22	15	53
12	10	22	13	21	26	20	41	20	0	19	21	18	54	18	11	17	39
13	11	24	27	23	34	22	46	22	0	21	17	20	38	20	0	19	25
14	12	26	40	25	43	24	50	24	0	23	14	22	30	21	49	21	11
15	13	28	53	27	51	26	54	26	0	25	10	24	23	23	38	22	56
16	14	31	7	30	0	28	58	28	0	27	6	26	15	25	27	24	42
17	15	33	20	32	9	31	2	30	0	29	2	28	8	27	16	26	28
18	16	35	33	34	17	33	6	32	0	30	58	30	0	29	5	28	14
19	17	37	47	36	26	35	10	34	0	32	24	31	53	30	55	30	0
20	18	40	0	38	34	37	14	36	0	34	50	33	45	32	44	31	46
21	19	42	13	40	43	39	19	38	0	36	46	35	38	34	33	33	32
22	20	44	27	42	51	41	23	40	0	38	43	37	30	36	22	35	18
23	21	46	40	45	0	43	27	42	0	40	39	39	23	38	11	37	4
24	22	48	53	47	9	45	31	44	0	42	35	41	15	40	0	38	44
25	23	51	7	49	17	47	35	46	0	44	31	43	8	41	49	40	35
26	24	53	20	51	26	49	39	48	0	46	27	45	0	46	38	42	21
27	25	55	33	53	34	51	43	50	0	48	23	46	53	45	27	44	7
28	26	57	47	55	43	53	48	52	0	50	19	48	45	47	16	45	53
29	27	60	0	57	51	55	52	54	0	52	15	50	38	49	5	47	39
30	28			60	0	57	56	56	0	54	12	52	30	50	55	49	25
31	29				60	0	58	0	56	8	54	23	52	44	51	11	
32	30					60	0	58	4	56	15	54	33	52	56		
33	31							60	0	58	8	56	22	54	42		
34	32										60	0	58	11	56	28	
35	33												60	0	58	14	
36	34														60	0	

(C6) C:52; (C27) P:32; (E14) P:42; (K20) p:20; (K21) p:48; (K27) C, P:24; (K28) C,P:20; (K29) C,P:16; (M5) C, P:37; (M7) C,P:22; (M9) C, P:52; (M11) C, P:37; (M13) C, P:22; (M15) C, P:7; (M17) C, P:52; (M18) p:44; (M19) C, P:37; (M21) C, P:22; (M23) C,P:7; (M25) C:52; (M27) C, P:37; (M29) C,P:22; (M31) C, P:7; (P24) C, P:39; (Q17) C, P:26; (Q34) P:26;