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# **An extension of the Tobler's law?**

## **Influence of the metric to compute the distance between an optimal facility and its demand points**

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This research work presents an extension of the Tobler's first law (TFL), that includes the metric used to measure the distance between a center and its demand points, in an optimal facility location problem framework (Hakimi, 1964). It provides a short overview of the works derived from the TFL and opens on the definition of a second law, less geographical and more mathematical. Indeed, its seems the distance, that is to say in our case a  $L_p$ -norm (a generalized Minkowski distance) considered to estimate a proximity between two points, has a strong weight on the meaning and on the shape of the TFL.

We organise our talk in several steps. We first recall the foundations and the geographical meaning of the Tobler's first law (1970, 1971) which is still considered as a solid and reliable base in many disciplines (for instance, see the introduction in Cressie & Moores, 2021). This led to many citations of this law, which is indeed a geographically verified assumption rather than a demonstrated law.

We first draw a short history of this law (Cauvin & Reymond, 1991, Miller, 2004, Sui, 2004), provide a few examples of its usages (Grasland, 2009, Hecht & Moxley, 2009) and its extensions, especially in the context of the Santa-Barbara's research group (Couclelis, 1996, Montello et al. 2003). We list and summarize several attempts to design a complementary or alternative second law in quantitative geography (Goodchild in Sui, 2004, Hecht & Moxley, 2009, Foresman & Luscombe, 2017, Josselin et al., 2017).

Then, in a second section, we move on transport and mobility concerns. We define the different purposes (efficacy, equity) of the principal centers used in geography (Beguin, 1989) and optimal location problem solving:  $k$ -median and  $k$ -center notably, to which we add the  $k$ -barycenter (modelling equality), that has interesting intermediate properties. We link these centers to their well-known objective functions.

In a third section, we focus on the 1-facility optimal location, a refined formal framework to explain how we generalize the optimal center through  $L_p$ -norms (Peeters & Thomas, 2000). Using sensitivity analysis (Drezner & Hamacher, 2004), we find out by computation a generalized fonction linking center location sensitivity, demand point influence and distance between the center and the demands (Ciligot-Travain & Josselin, 2013, Josselin et al., 2016).

We use this function to draw new balanced optimal locations, respectively with  $p=1.5$  and  $p=3$  in the  $L_p$ -norm. For those, we map the demand point spatial influence of the center location. We develop more deeply the  $L_3$ -norm, which catches our attention: it shows a particular property of a linear relation between the demand points weights and their distance to the optimal center. As we did previously with well-known  $k$ -median and  $k$ -center metrics, we associate new optimal centers a semantic, respectively “effility” and “equaquity” to those two compromises.

In conclusion, we suggest how, in practical urban planning, our theoretical results can be potentially used to tune the effect of the distance on the demand point influence on the center. Finally, we draw up a comparison between our geographical law and the TFL: is our law an extension of the TFL or a second complementary law? We open a discussion about the proximity and distance collapse or reinforcement, as H. Couclelis put forward in 1996 and which is still a hot issue, from our perspective.

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