

# A Mass Conserving Implicit Volume Penalty Method for Moving-Body Flows

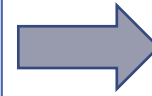
*Tsetoglou Iason*<sup>1</sup>, *Cailler M.*<sup>2</sup>, *Bénard P.*<sup>1</sup>, *Moureau V.*<sup>1</sup>, *Lartigue G.*<sup>1</sup> and *Reveillon J.*<sup>1</sup>



<sup>1</sup>CORIA, CNRS UMR6614, Normandie Université, INSA and University of Rouen, France, [iason.tsetoglou@coria.fr](mailto:iason.tsetoglou@coria.fr)

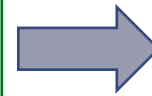
<sup>2</sup>Safran Tech Digital Sciences and Technologies Department, France, [melody.cailler@safrangroup.com](mailto:melody.cailler@safrangroup.com)

- Engineering applications involve fluid flows around solids
  - Stationary bodies
  - Moving bodies



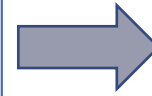
- Use of Computational Fluid Dynamics (CFD)
  - Simulate numerically the problem
  - Examine the physical phenomena

- Examples:
  - Biological flows
  - Turbomachinery
  - Aircraft design
  - Wind turbines
  - And many more..



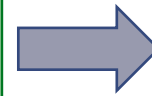
- ❖ Complex geometries
  - Need for unstructured grids
  - Moving boundaries
  - Need for re-meshing
- ❖ Complex flows
  - Unsteady 3D effects
  - Need for highly predictive flow solvers

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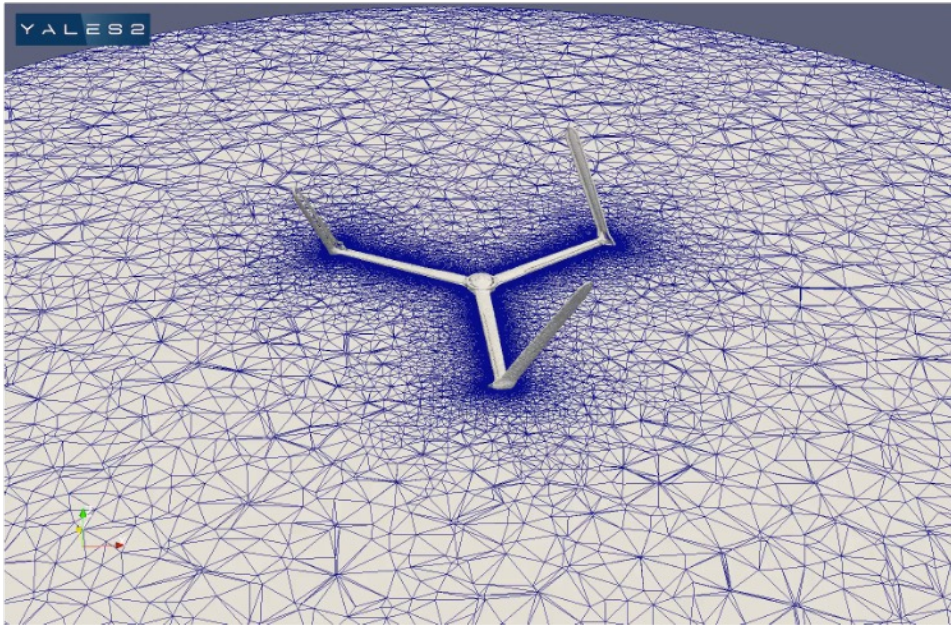


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What are the state-of-the-art rotor modelling approaches for unsteady high-fidelity simulations?

## Body-fitted (Sliding/Overset mesh methods) [1]

- + Geometry well represented in calculation domain
- + Aerodynamic forces calculated directly
- Mesh movement/adaptation needed to follow the solid
- Very expensive; unaffordable for regular use

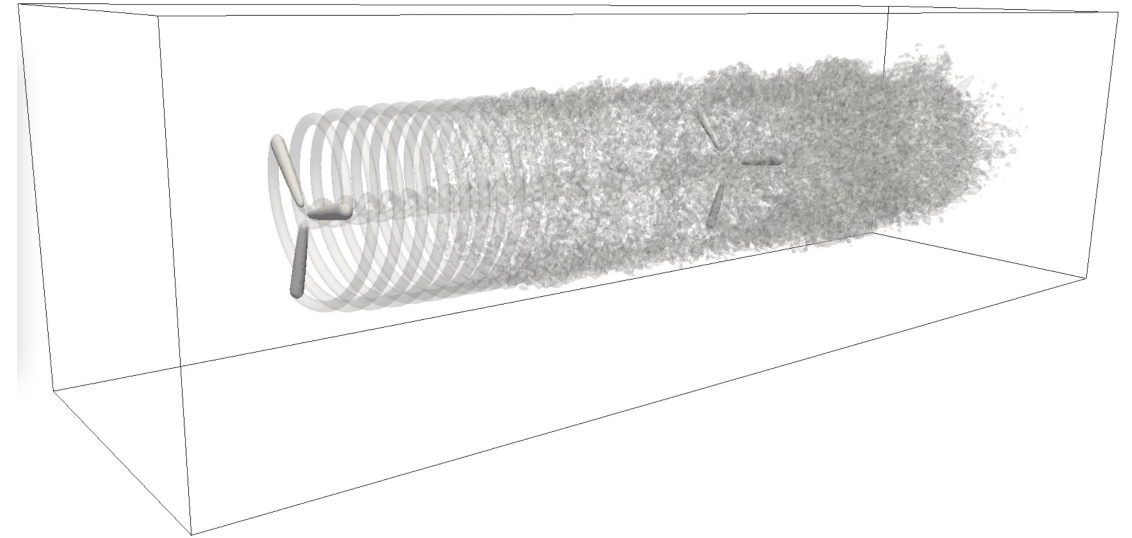


$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \nu \nabla^2(\mathbf{u})$$

$$\mathbf{u}_{\text{boundary\_node}} = \mathbf{u}_{\text{wall\_imposed}}$$

## Actuator Line Method (ALM) [1]

- + Geometry represented as a single line of points
- + Affordable cost for regular use
- Experimental data the aerodynamic coefficients needed
- Unrealistic fields near the blades

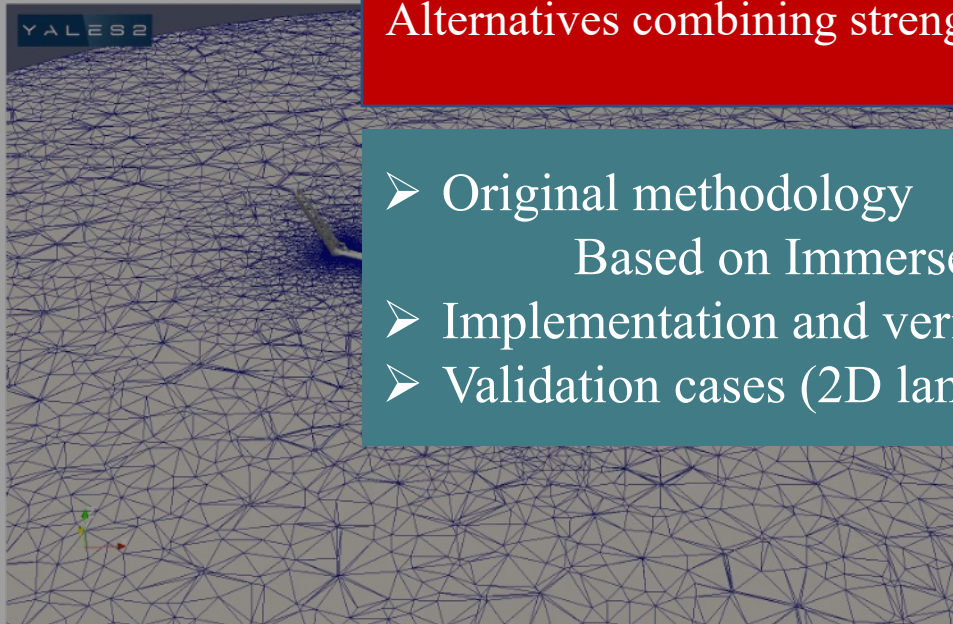


$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \nu \nabla^2(\mathbf{u}) + \mathbf{f}_{\text{ALM}}$$

$$\mathbf{f}_{\text{ALM}} = f(C_L^{\text{exp}}, C_D^{\text{exp}}, \dots)$$

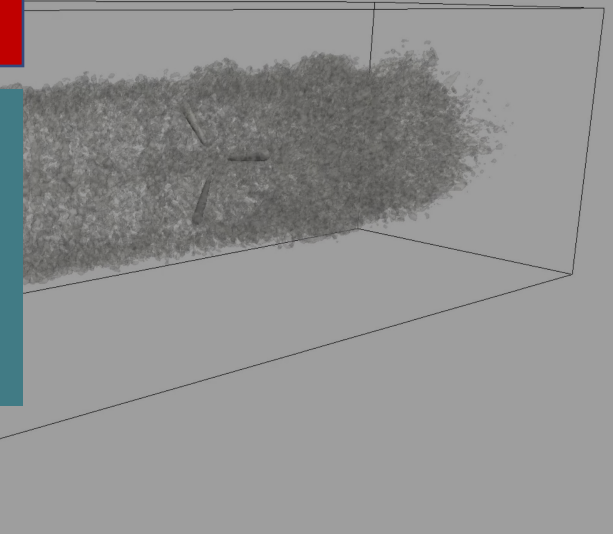
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Alternatives combining strengths of Body-fitted methods and ALM?

- Original methodology
  - Based on Immersed Boundary Methods
- Implementation and verification
- Validation cases (2D laminar flows)

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \nu \nabla^2(\mathbf{u})$$

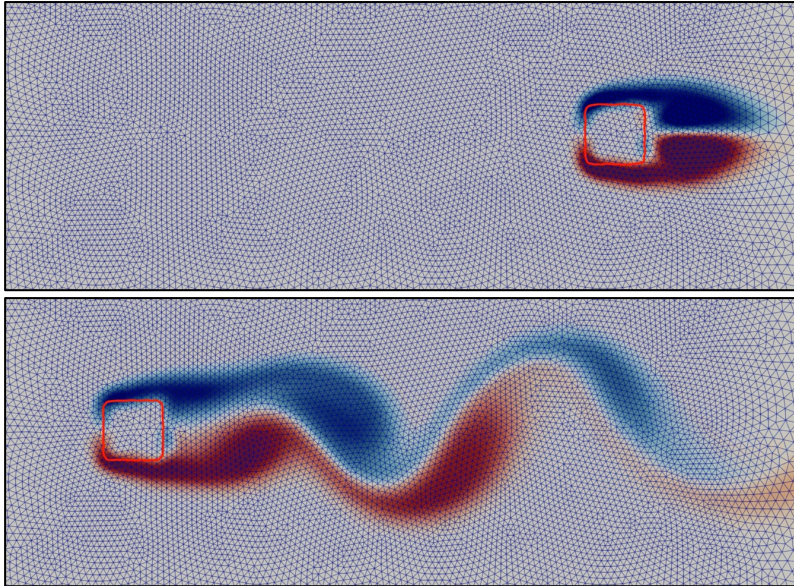
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## Immersed Boundary Methods (IBM) [1]

- + Geometry well represented in calculation domain
- + Aerodynamic forces calculated directly
- + No re-meshing required
- +/- Variety of models with different strengths and weaknesses [2]
- Mass/momentum conservation not always guaranteed [3]



$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \nu \nabla^2(\mathbf{u}) + \mathbf{f}_{\text{IBM}}$$
$$\mathbf{f}_{\text{IBM}} = f(\mathbf{u}_f, \mathbf{u}_s, \phi_s, \dots)$$



## Proposition: VOS-Implicit Volume Penalty IBM

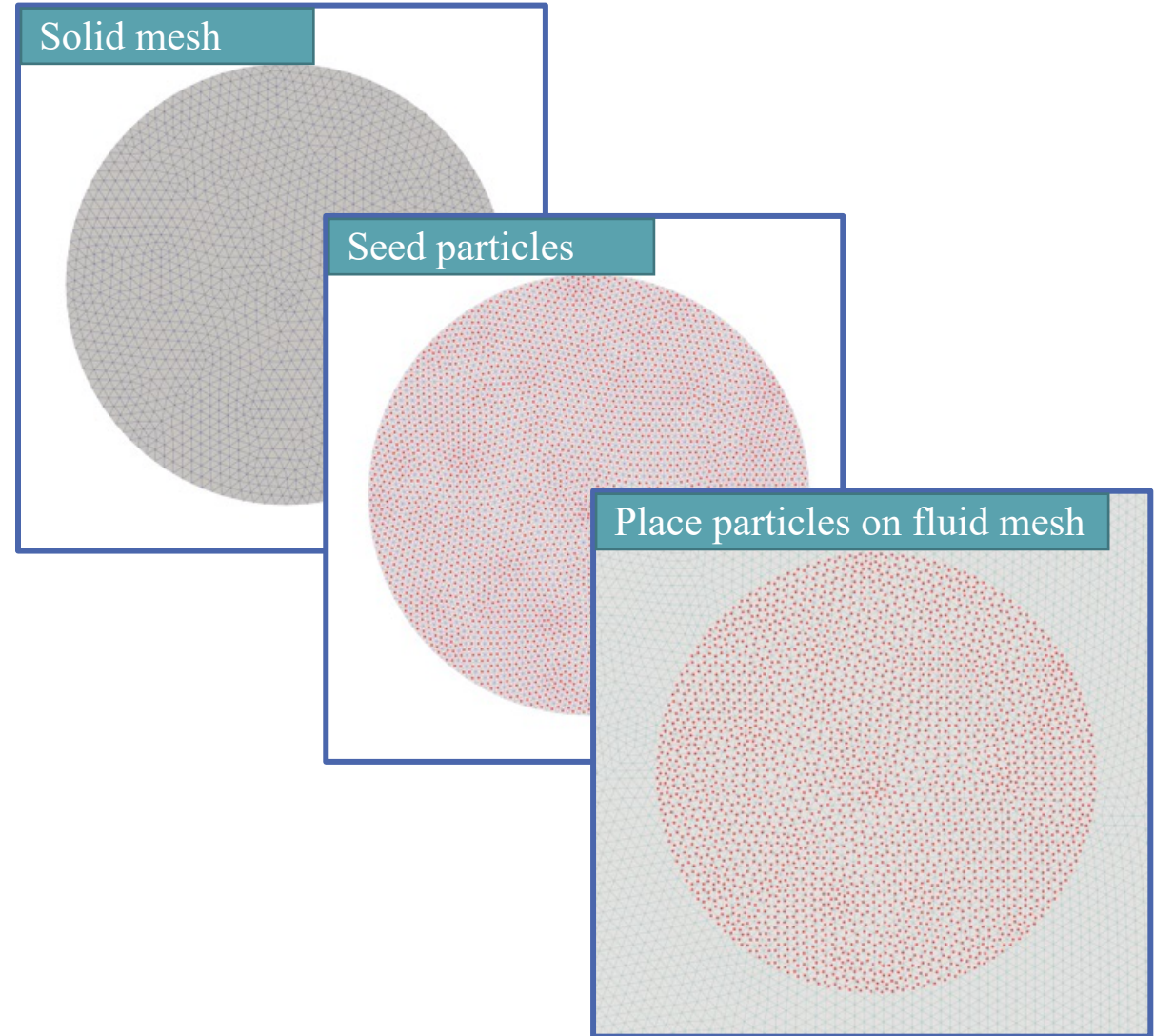
Target characteristics:

- Appropriate description of the moving solid
- Solid/fluid volume preservation
- Correct influence of solid to the fluid's dynamics
- Accurate prediction of aerodynamic loads
- Efficient, robust and affordable

# Volume of Solid - Implicit Volume Penalty (VOS-IVP) IBM

## VOS-IVP Ingredients

Lagrangian particles for the solid displacement



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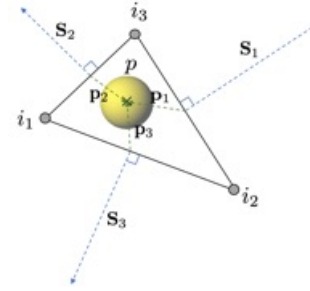
Lagrangian particles for the solid displacement

Volume of Solid (VOS) approach [1]

$$\frac{\partial}{\partial t} (\phi_f) + \nabla \cdot (\phi_f \mathbf{u}_f) = 0$$

$$\frac{\partial}{\partial t} (\phi_f \mathbf{u}_f) + \nabla \cdot (\phi_f \mathbf{u}_f \mathbf{u}_f) = -\frac{1}{\rho} \nabla P + \frac{1}{\rho} \nabla \cdot (\phi_f \boldsymbol{\tau})$$

## Linear Interpolation



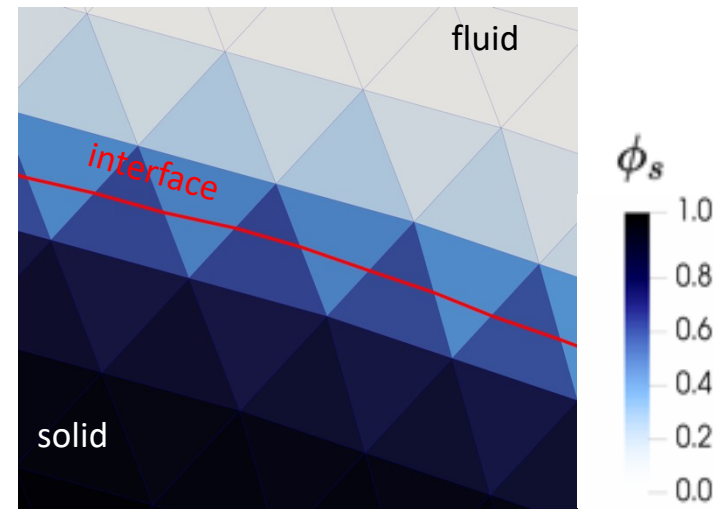
$$V_{s,i} = \sum_{p | \mathbf{x}_p \in E_i} V_p W_{i,p}$$

$$W_{i,p} = \frac{|\mathbf{x}_p - \mathbf{x}_{fi}| \cdot \mathbf{S}_{fi}}{\sum_{i' \in \mathcal{N}(E_p)} |\mathbf{x}_p - \mathbf{x}_{fi'}| \cdot \mathbf{S}_{fi'}}$$

$$\phi_{s,i} = \frac{V_{s,i}}{V_i}$$

$\phi_s = 1$ , in solid region

$\phi_f = (1 - \phi_s) = 1$ , in fluid region





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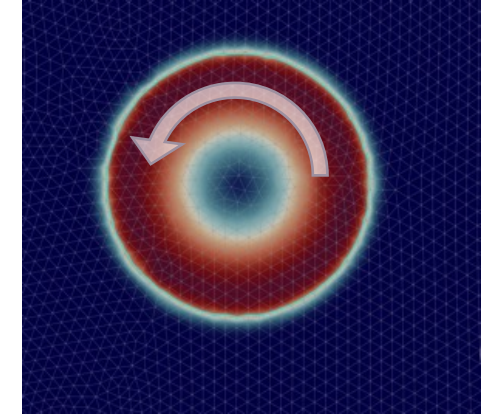
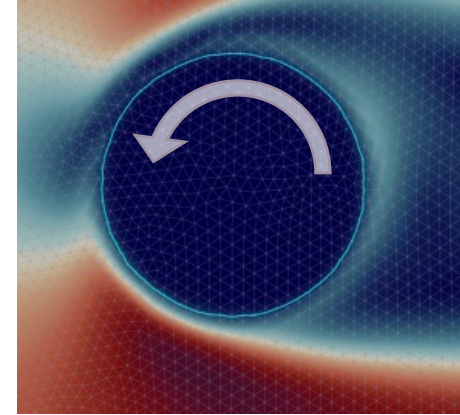
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Composite velocity

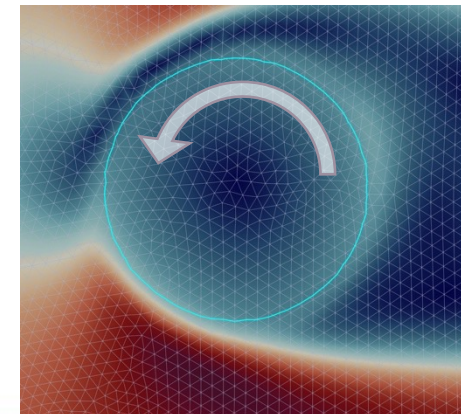
$$\mathbf{u} = \phi_f \mathbf{u}_f + \phi_s \mathbf{u}_s$$

**Example: Rotating cylinder (Magnus Effect)**

Fluid velocity  $\phi_f \mathbf{u}_f$       Solid velocity  $\phi_s \mathbf{u}_s$



Composite velocity  $\mathbf{u} = \phi_f \mathbf{u}_f + \phi_s \mathbf{u}_s$



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Composite velocity

$$\mathbf{u} = \phi_f \mathbf{u}_f + \phi_s \mathbf{u}_s$$

Penalization IBM [2]

$$\mathbf{F}^{s \rightarrow f} = \frac{\chi_s}{\eta} (\mathbf{u}_s - \mathbf{u})$$

$$\chi_s = \mathbb{H}(\phi_s), \quad \eta = \alpha \Delta t$$

# Volume of Solid - Implicit Volume Penalty (VOS-IVP) IBM

## VOS-IVP governing equations

- Continuity equation:  $\nabla \cdot \mathbf{u} = \frac{\partial}{\partial t} (\phi_s) + \nabla \cdot (\phi_s \mathbf{u}_s) \quad \mathcal{L}_s$
- Momentum equation:  $\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\frac{1}{\rho} \nabla P + \frac{1}{\rho} \nabla \cdot (\phi_f \boldsymbol{\tau}) + \frac{\chi_s}{\eta} (\mathbf{u}_s - \mathbf{u})$   
 $+ \frac{\partial}{\partial t} (\phi_s \mathbf{u}_s) + \nabla \cdot (\phi_s \mathbf{u}_s \mathbf{u}_s) \quad \mathcal{P}_s$

## Implicit Volume Penalty approach

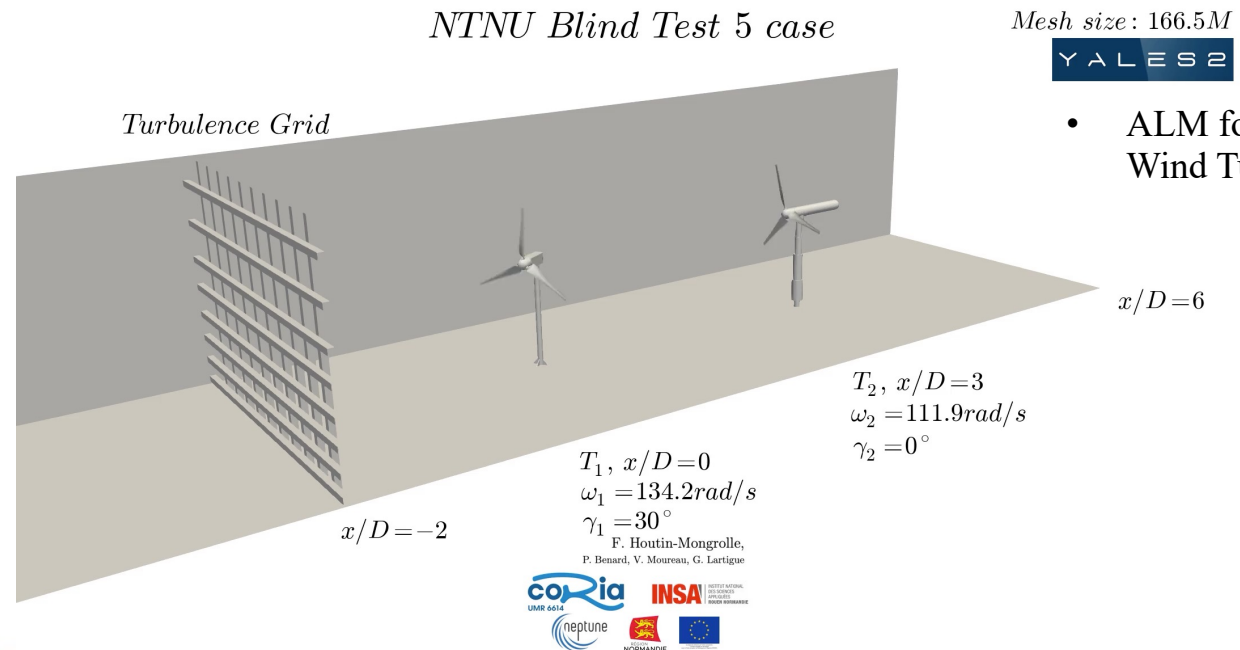
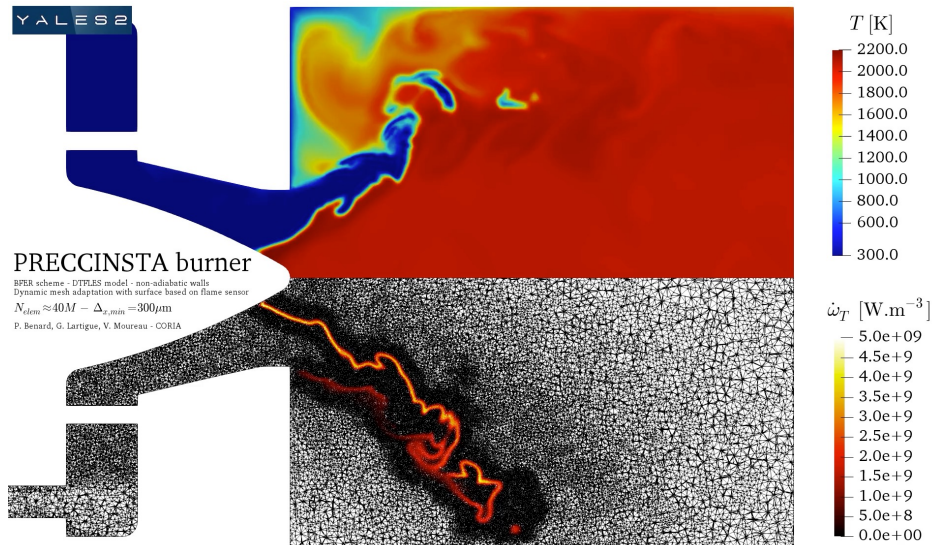
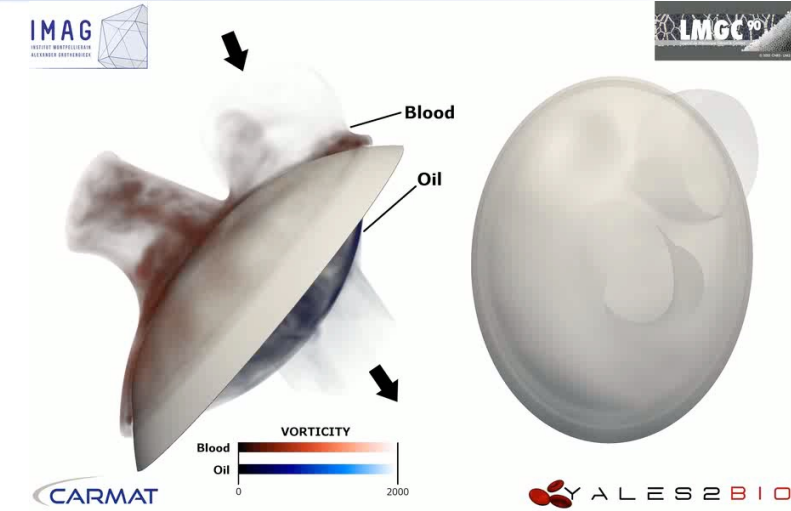
- Penalizing the predicted velocity:  $\mathbf{u}^* = \frac{1}{\gamma} [\mathbf{u}^n + \Delta t \mathcal{R}] + \frac{\gamma - 1}{\gamma} \mathbf{u}_s^{n+1}$
- Penalizing the final velocity:  $\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho} \nabla P^{n+1/2} + \frac{\chi}{\eta} (\mathbf{u}^* - \mathbf{u}^{n+1})$
- Modified Pressure Poisson equation (PPE):  $\nabla \cdot \left( \frac{1}{\gamma \rho} \nabla P \right) = \frac{\nabla \cdot \mathbf{u}^*}{\Delta t} - \frac{\nabla \cdot \mathbf{u}^{n+1}}{\Delta t}$

Remark:  $\gamma = \left(1 + \frac{\chi}{\eta} \Delta t\right)$  the penalty density factor varies in space and time:  $\gamma(\mathbf{x}, t) = 1$  in fluid,  $\gamma(\mathbf{x}, t) > 1$  in solid

# The CFD platform [1,2]: YALES2

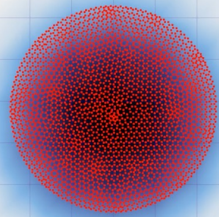
## • Features

- Large Eddy Simulations
- **Unstructured meshes** and adaptive grid refinement
- **Low-Mach** number Navier-Stokes equations (incompressible and variable density)
- Double-domain decomposition and hybrid OpenMP/MPI communications
- Highly efficient solvers for linear system inversion (PCG, DPCG)
- 4th-order central finite-volume method and 4th-order time integration
- Suited for **massively parallel computing** (>32 000 procs)
- Used by 250+ scientists/engineers in academia and industry

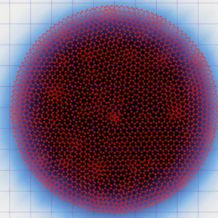


# Solid Volume Conservation

Fluid Mesh  
Solid in oscillation



$\Delta x = D/5$



$\Delta x = D/10$

At each iteration:

1. Move the particles
2. Interpolate particles' volume & compute solid volume fraction

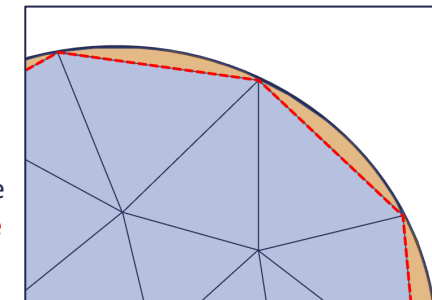
Numerical volume:  $V_s(t) = \int_{\Omega} \phi_s(t) dV$

Theoretical volume:  $V_{\text{theory}} = \frac{1}{4} \pi D^2$

Error in volume:  $\varepsilon_s(t) = \frac{|V_s(t) - V_{\text{theory}}|}{V_{\text{theory}}}$

$D/\Delta x$	$\langle \varepsilon_s \rangle [\%]$	$\varepsilon_s^{rms} [\%]$
1	0.16	$7.35 \cdot 10^{-14}$
5	0.16	$2.11 \cdot 10^{-14}$
10	0.16	$1.38 \cdot 10^{-14}$

- The mean error remains constant
- Depends on discretization errors of initial solid mesh

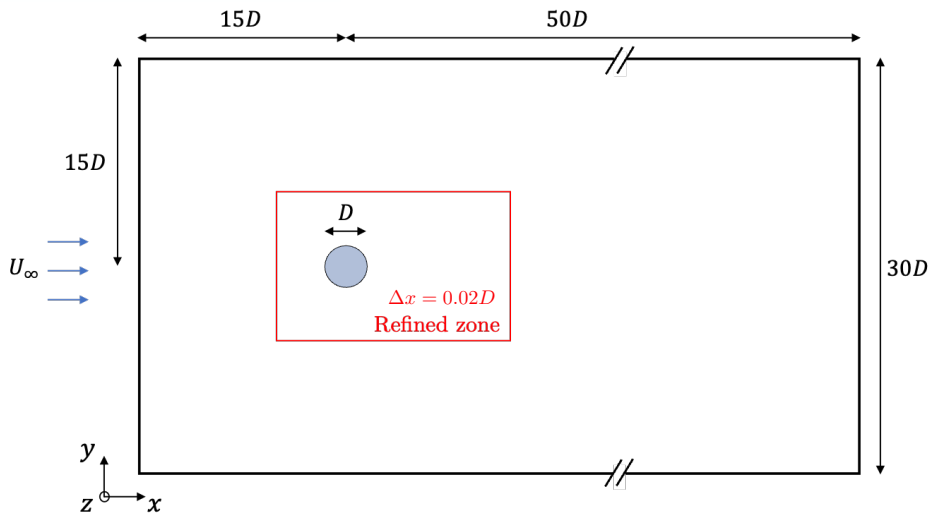


Theoretical solid surface  
Discretized solid surface  
Lost solid volume

- The mean fluctuation error is essentially negligible
- Diminishes with finer grid resolution → linked to interpolation errors

**The solid volume is conserved in time and is independent of the fluid mesh resolution!**

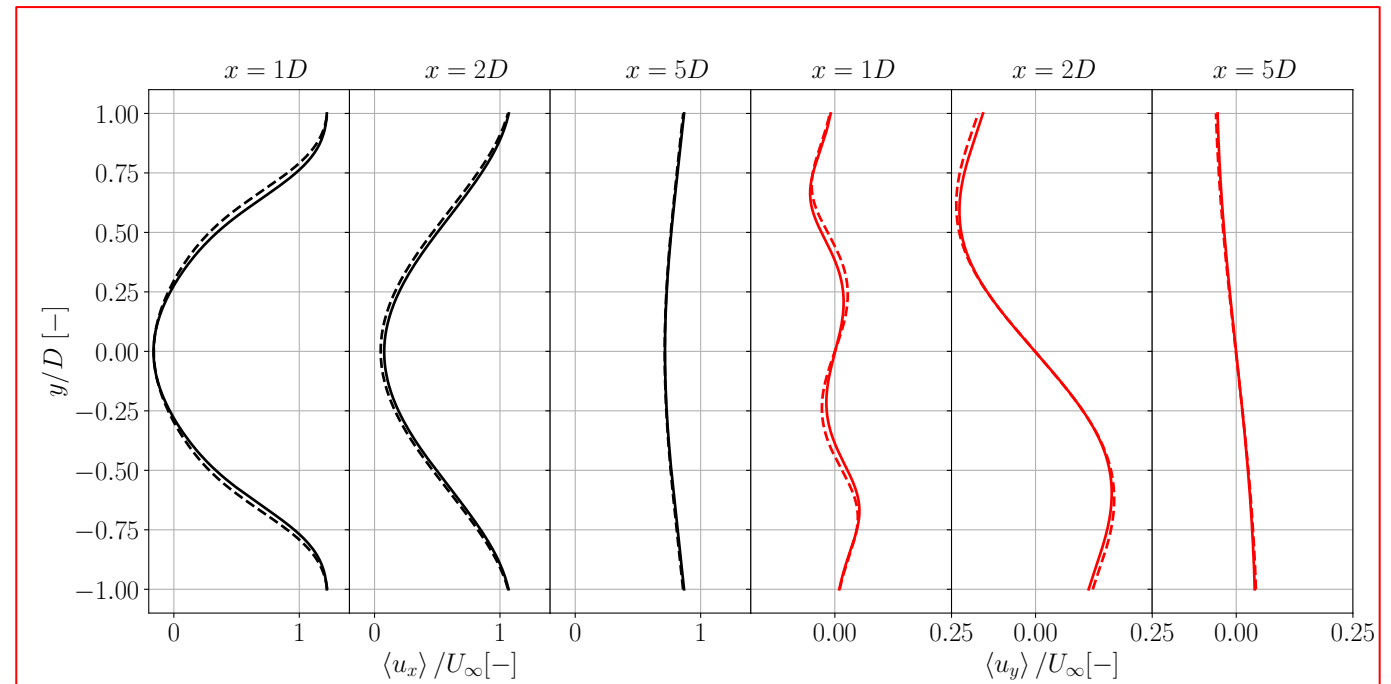
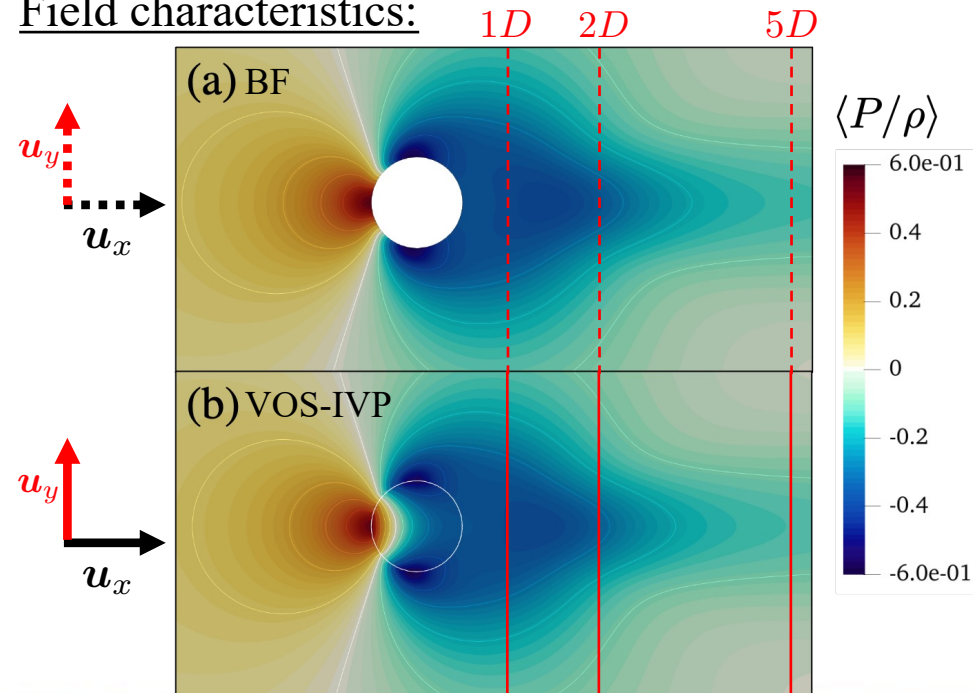
# 2D Validation: Flow over stationary cylinder at $Re=100$



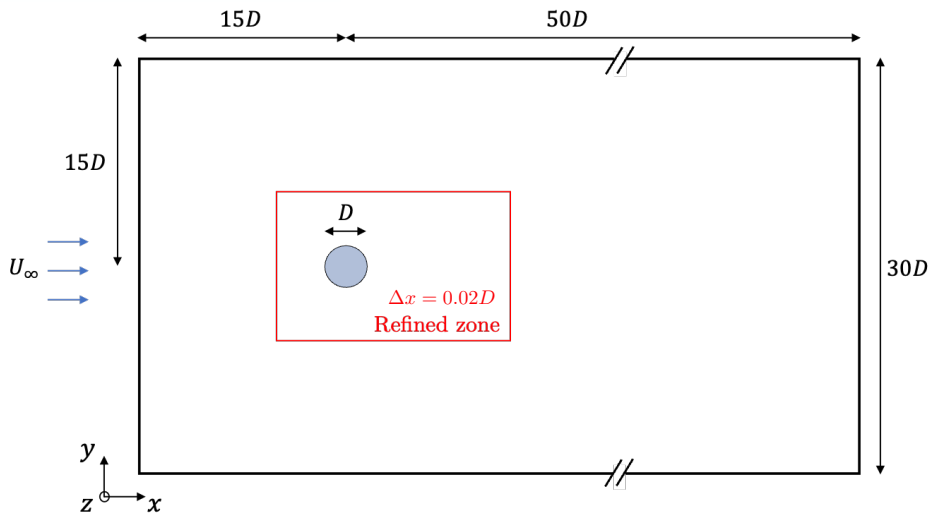
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Qu et al. [1]	1.31	0.22	0.16	1.41
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VOS-IVP	1.30	0.22	0.17	1.32

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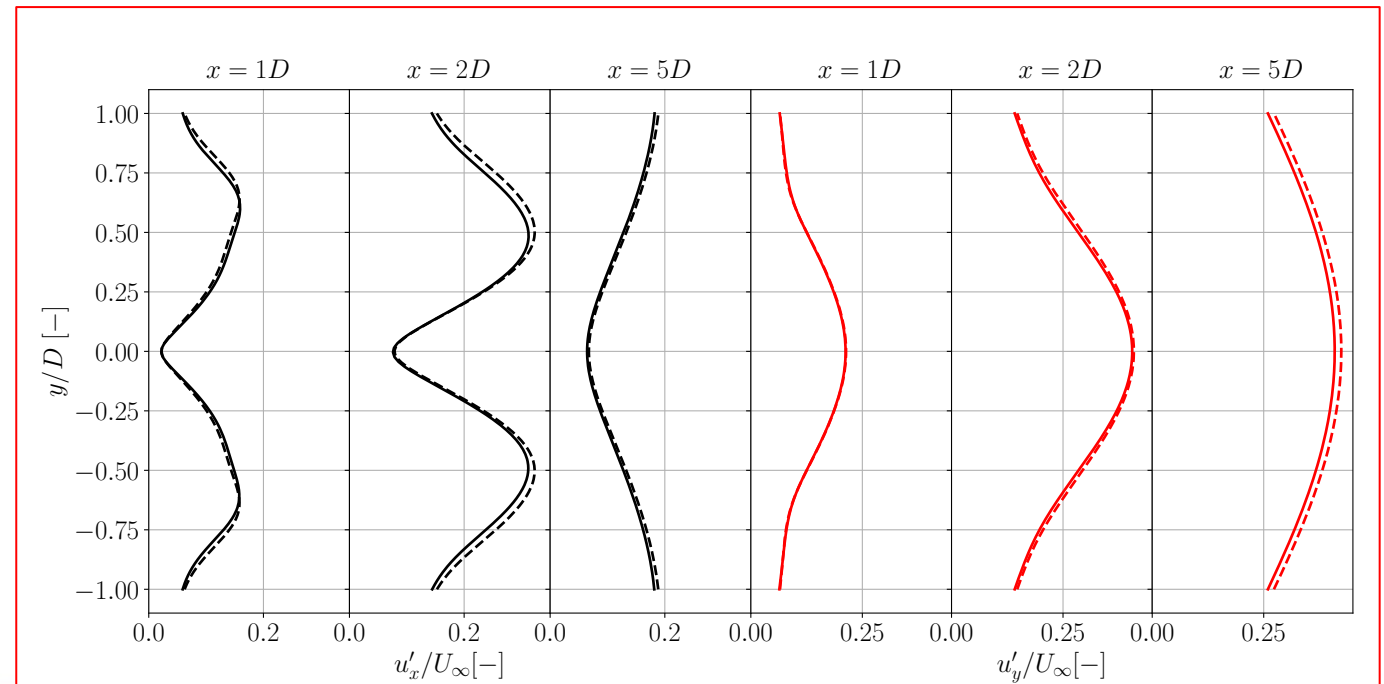
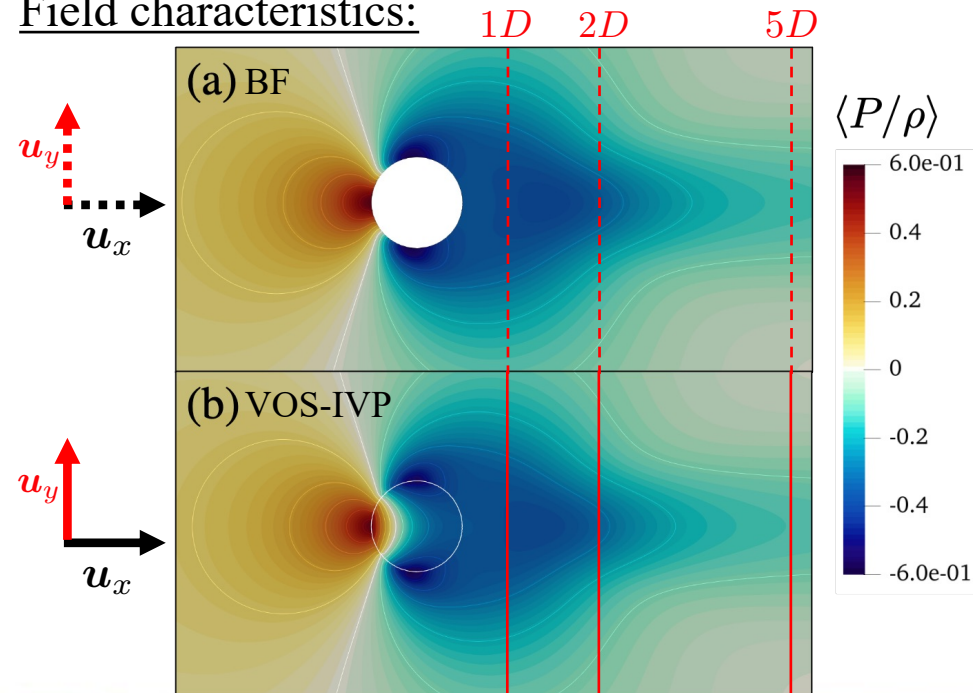
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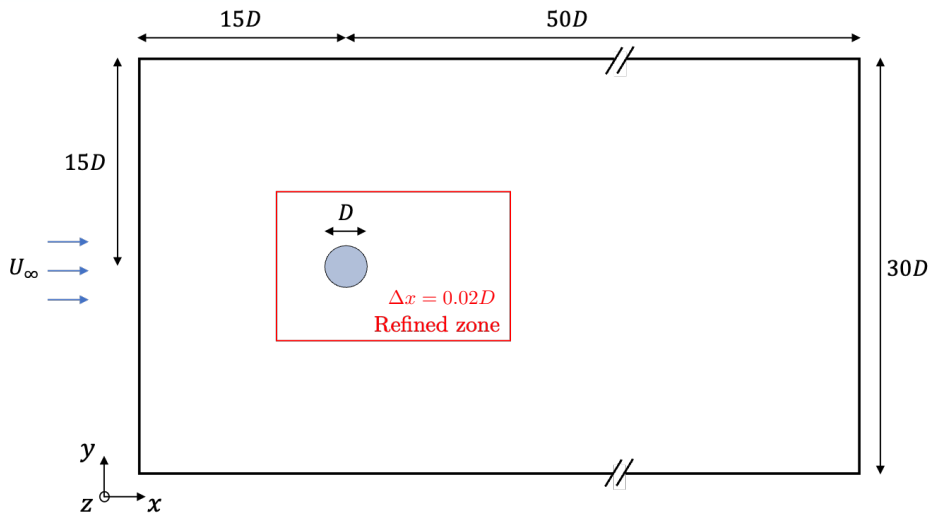
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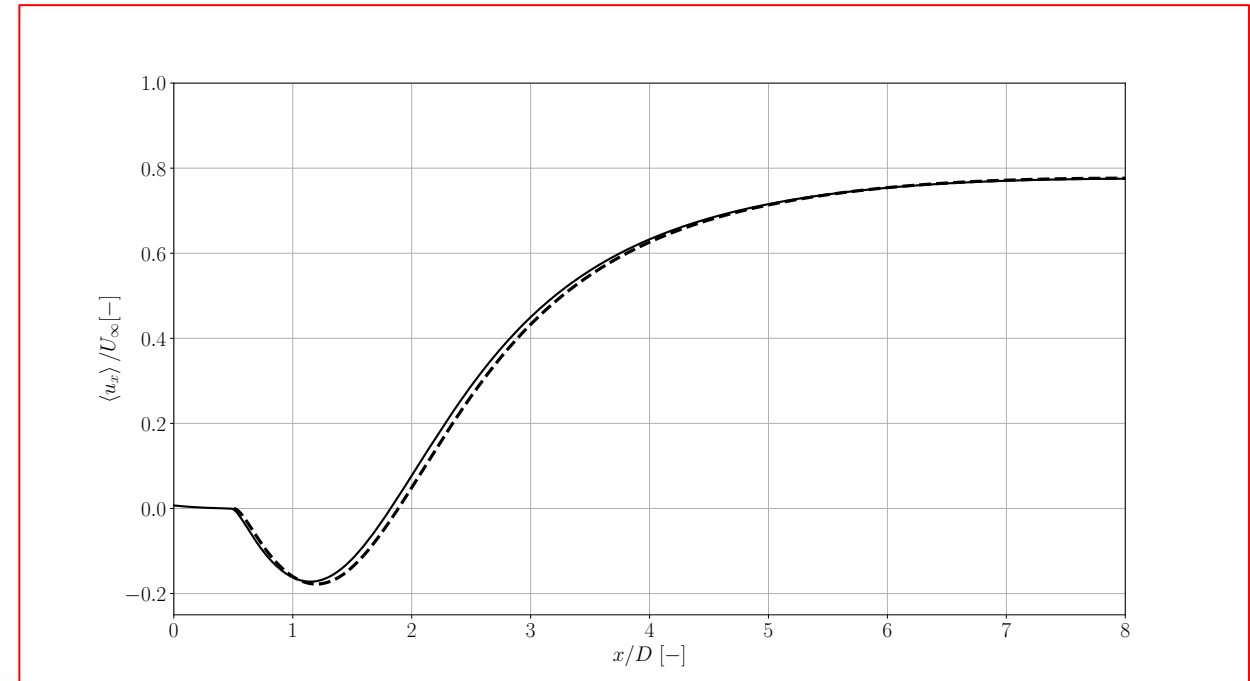
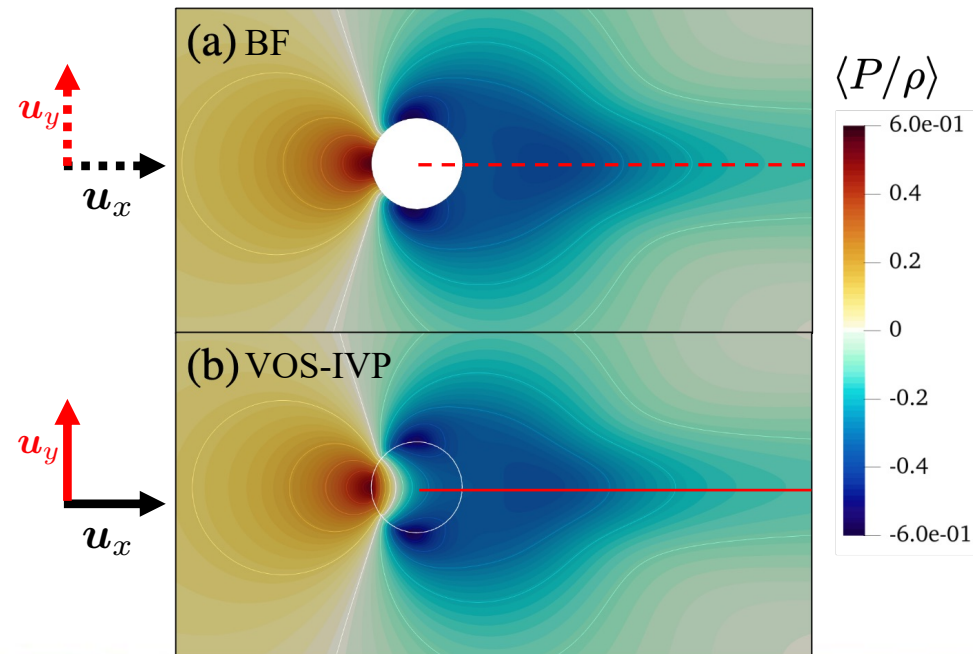
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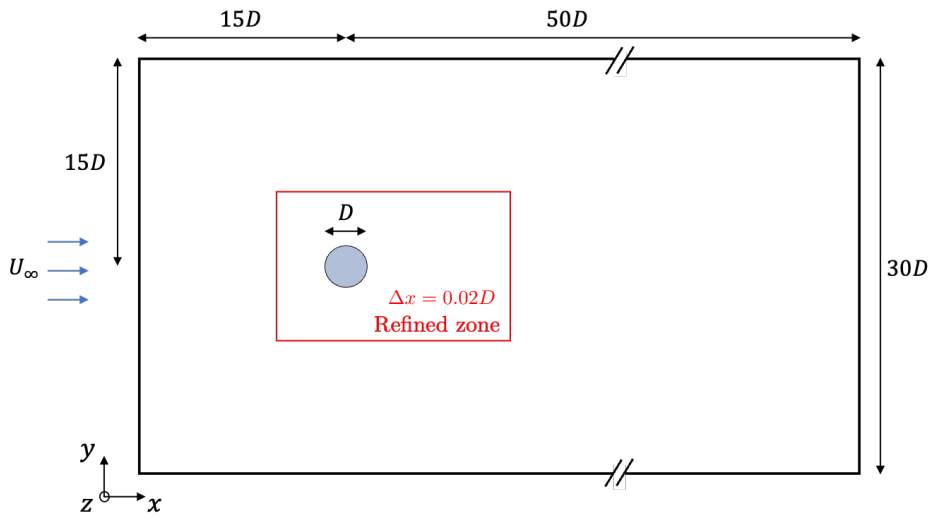
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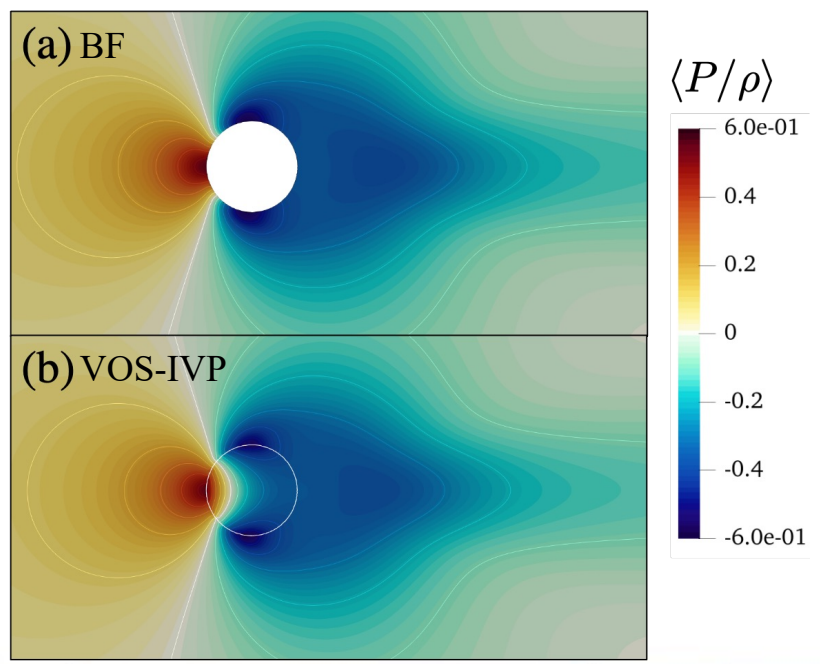
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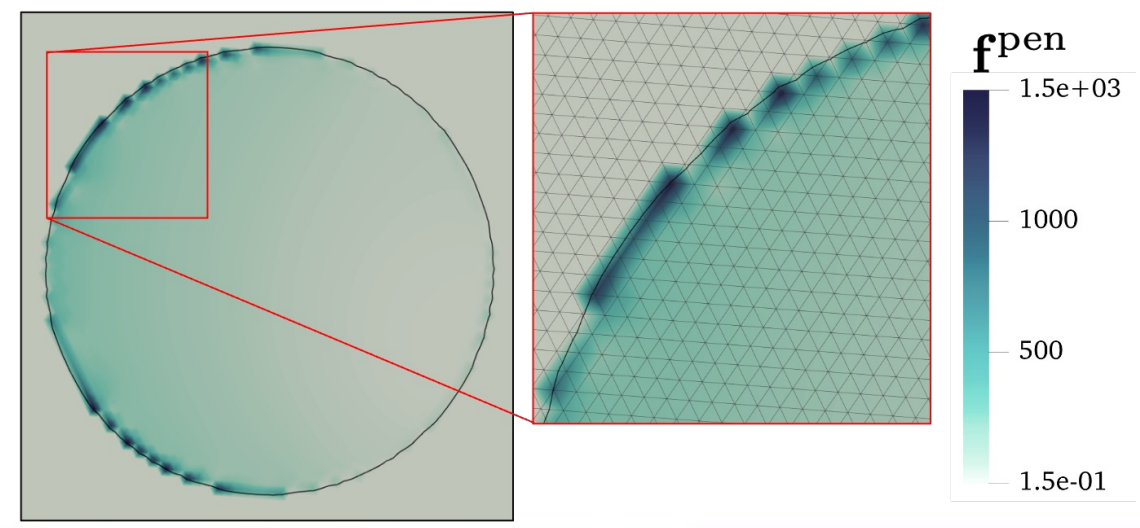
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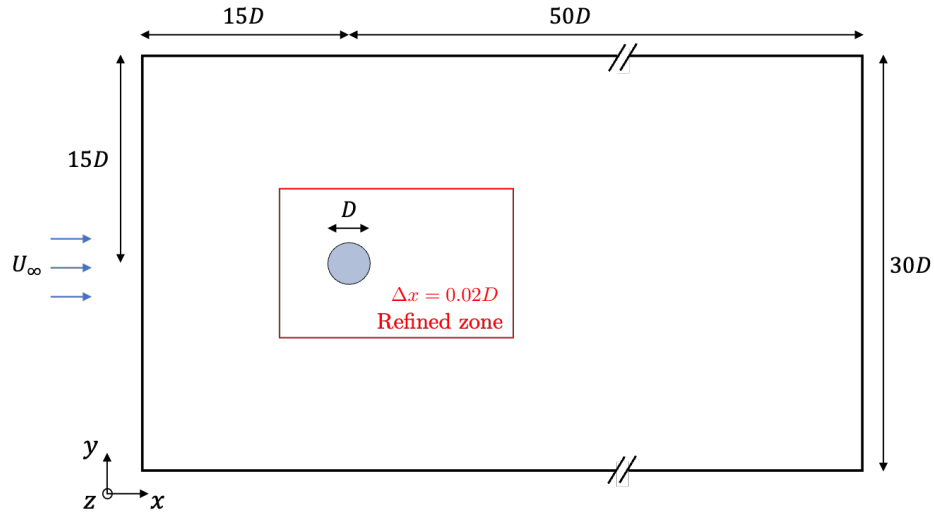
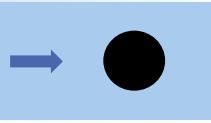


## Local forcing points:

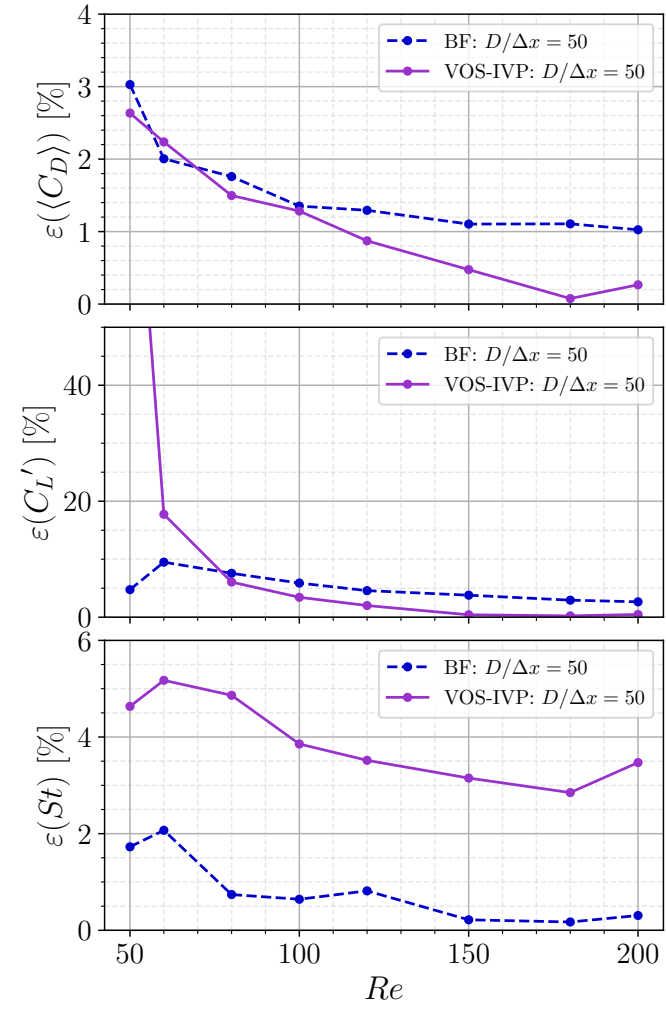
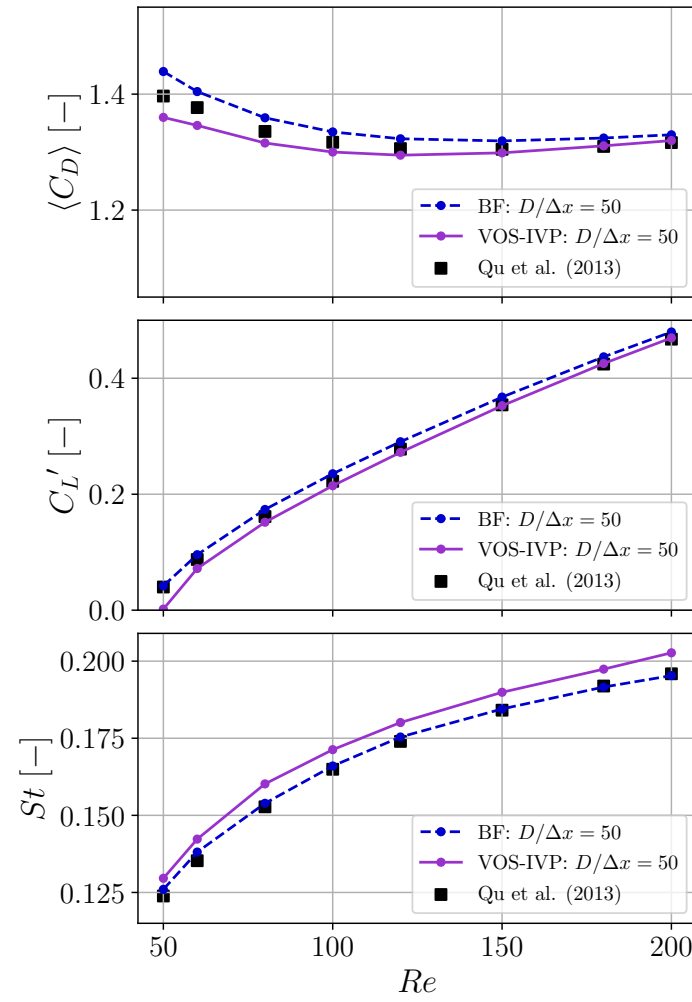
Sharp penalty mask:  $\chi_s = \begin{cases} 1 & \text{if } \phi_s > 0.5 \\ 0 & \text{else.} \end{cases}$



# 2D Validation: Flow over stationary cylinder at $Re=50-200$



## Results:



- VOS-IVP results match very well with BF and reference data [1]
- Drag relative error remains less than 3%
- Lift relative error less than 10%
- Strouhal relative error less than 6%

# 2D Validation: Oscillating cylinder in quiescent fluid



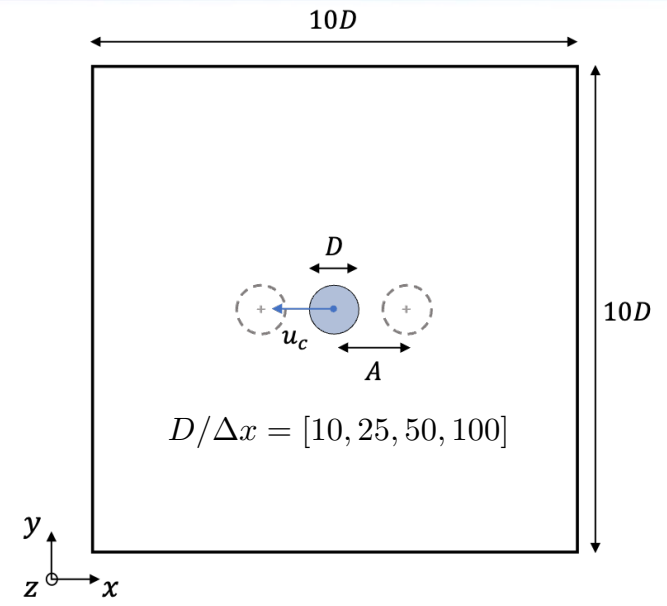
## Set-up:

$$\mathbf{u}_c(t) = -2\pi f A \cos(2\pi f t)$$

$$f = 0.2 \text{ Hz}$$

$$Re = 100$$

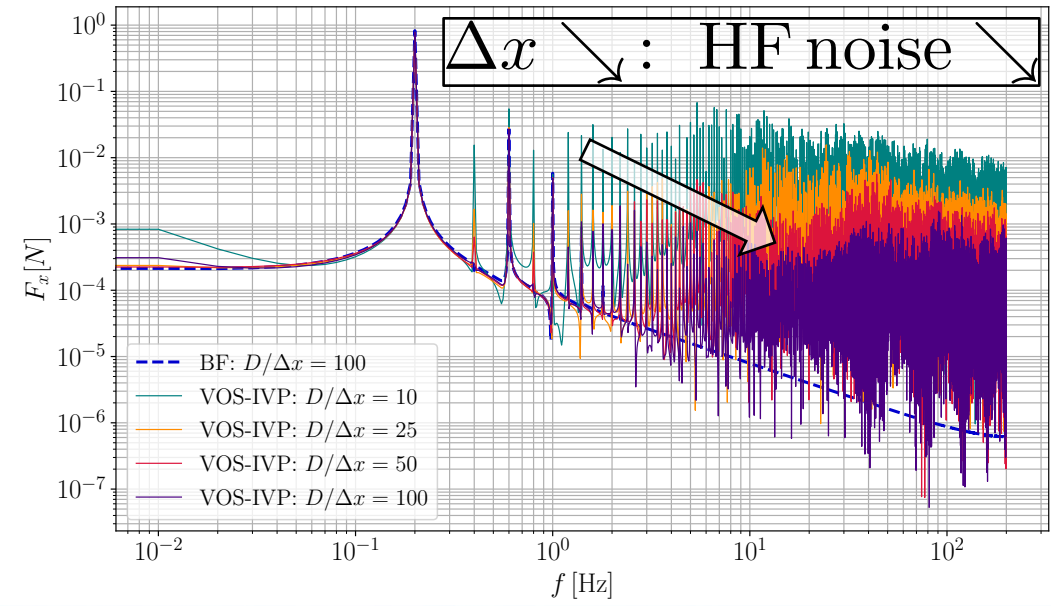
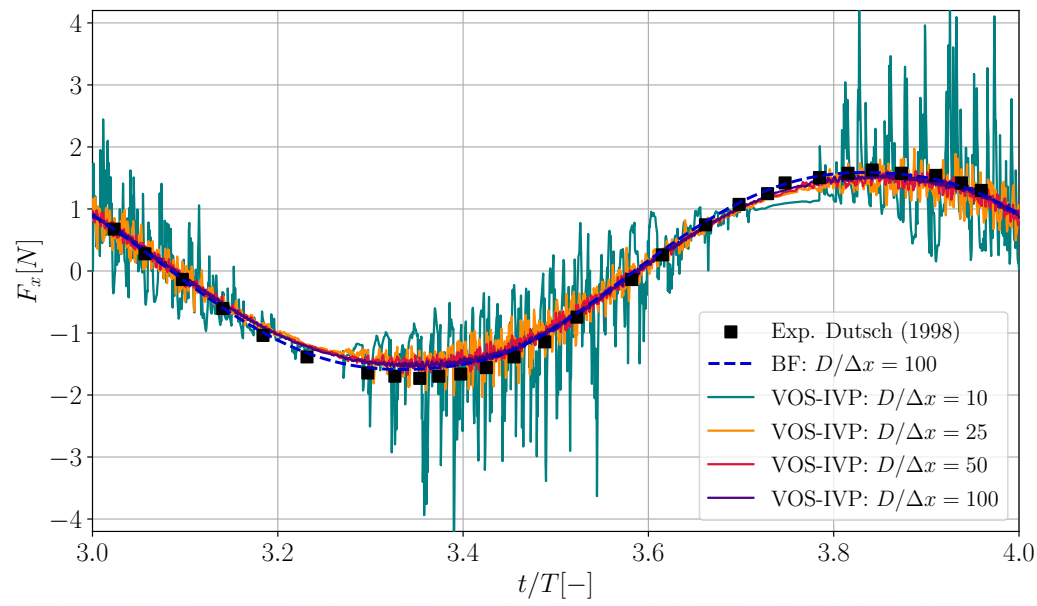
$$KC = \frac{U_{\max}}{fD} = 5$$



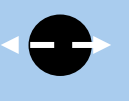
## Force & frequency:

Case	$F'_x [N]$	$\varepsilon(F'_x) [\%]$
* BF $D/\Delta x = 100$	1.14	—
VOS-IVP $D/\Delta x = 10$	1.24	8.77
VOS-IVP $D/\Delta x = 25$	1.07	6.14
VOS-IVP $D/\Delta x = 50$	1.08	5.26
VOS-IVP $D/\Delta x = 100$	1.11	2.63

\* Fixed boundary with moving frame of reference



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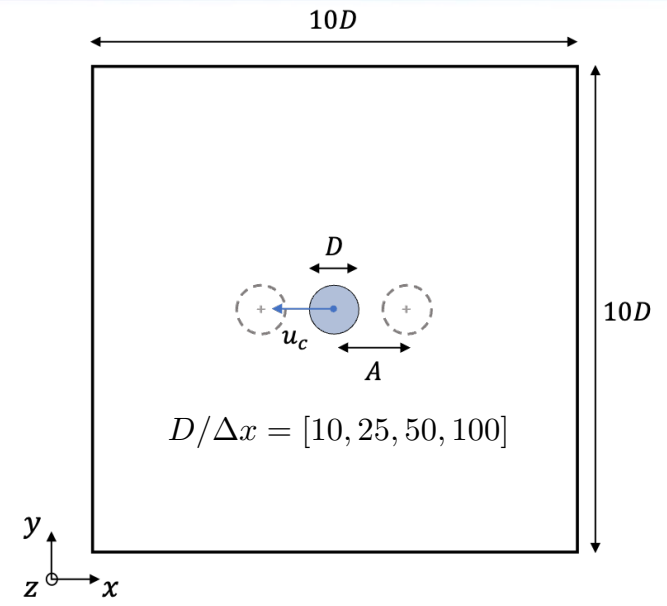
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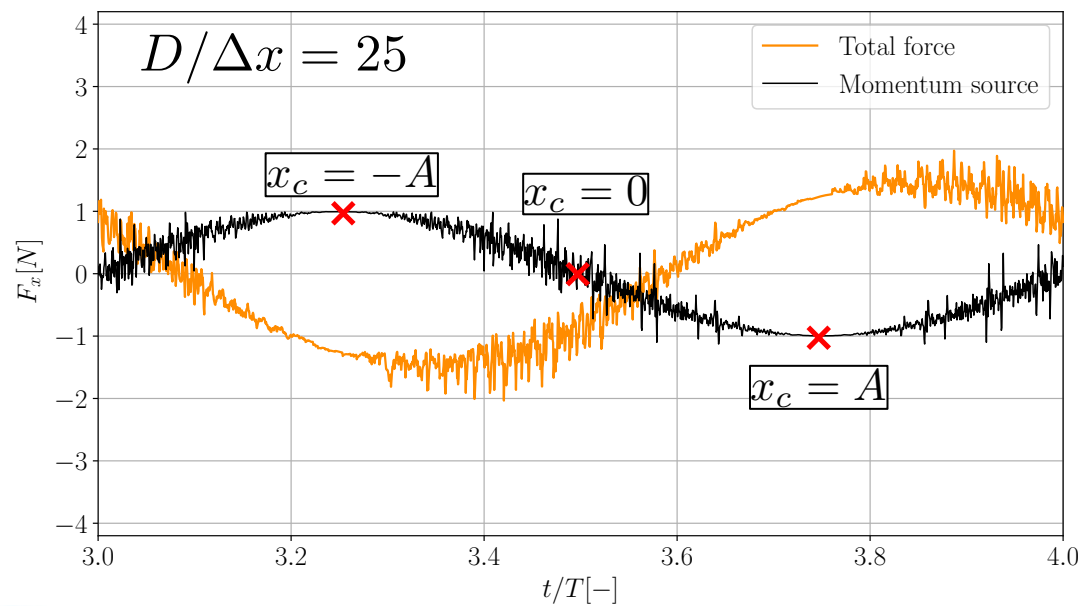
$$Re = 100$$

$$KC = \frac{U_{\max}}{fD} = 5$$



## Force & frequency:

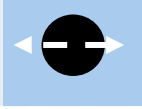
Case	$F'_x [N]$	$\varepsilon(F'_x) [\%]$
BF $D/\Delta x = 100$	1.14	—
VOS-IVP $D/\Delta x = 10$	1.24	8.77
VOS-IVP $D/\Delta x = 25$	1.07	6.14
VOS-IVP $D/\Delta x = 50$	1.08	5.26
VOS-IVP $D/\Delta x = 100$	1.11	2.63



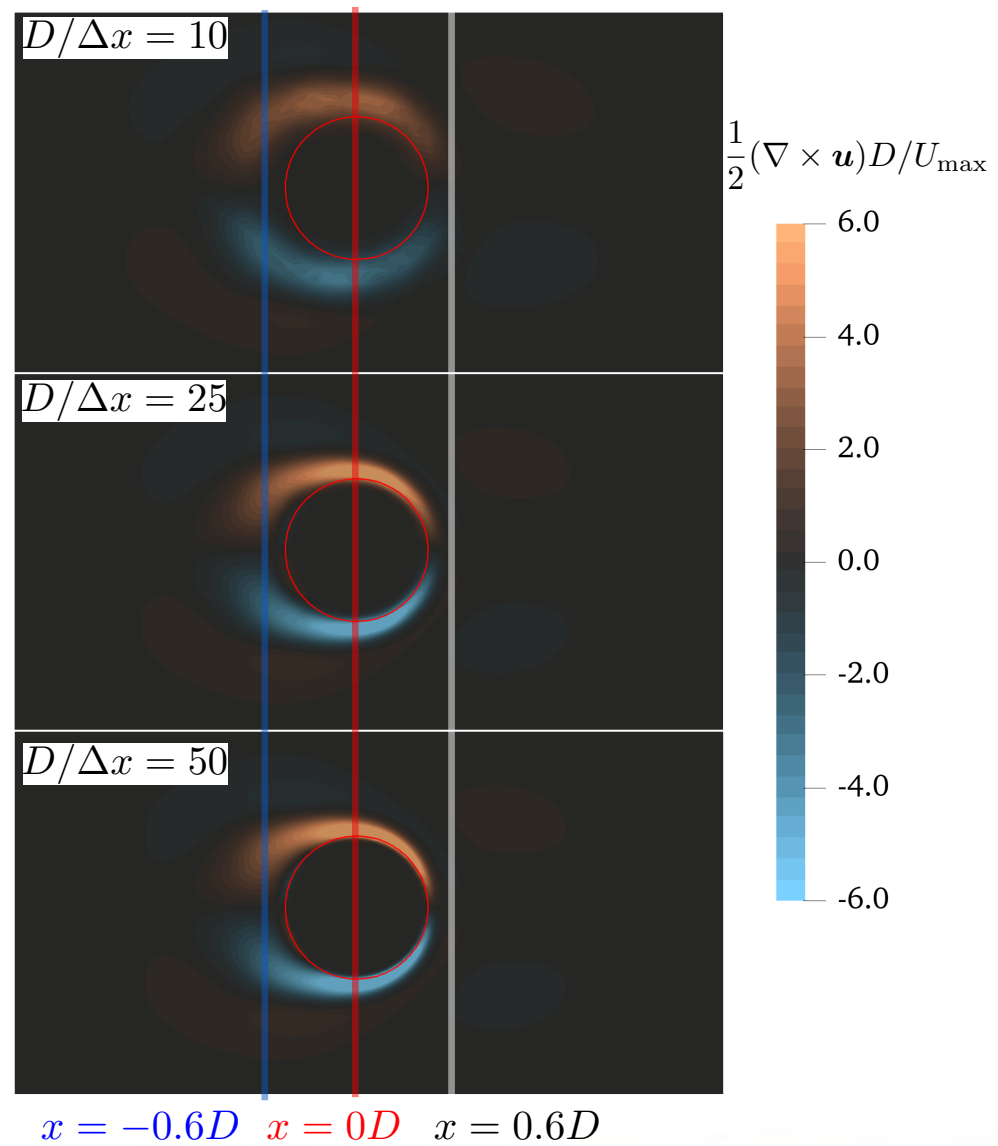
- The noise comes from the momentum source term  $\mathcal{P}$
- Specifically, there are local overshoots at the immersed body's surface for the time derivative:

$$\frac{\partial}{\partial t} (\phi_s \mathbf{u}_s) |_{x=x_i} : \frac{\phi_{s,i}^{n+1} \mathbf{u}_{s,i}^{n+1} - \phi_{s,i}^n \mathbf{u}_{s,i}^n}{\Delta t}$$

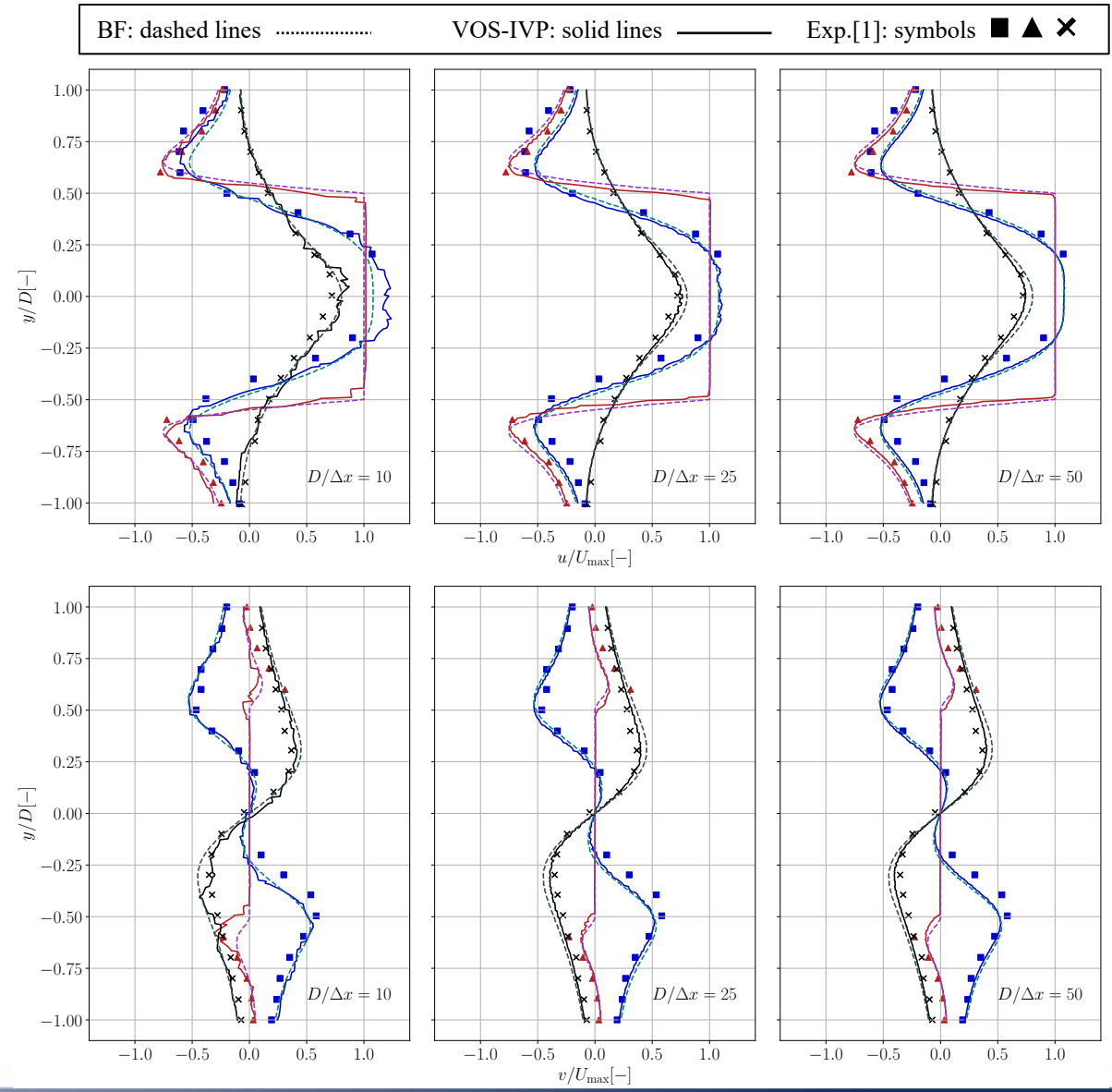
# 2D Validation: Oscillating cylinder in quiescent fluid



Field characteristics:



Velocity profiles at phase angle  $180^\circ$ :



# Computational performances

Performance measurements:

➤ Reduced computational cost:  $RCT = WCT \times N_{\text{cores}}/\text{iteration}/\text{node}$  [ $\mu\text{s}/\text{iter}/\text{node}$ ]



Stationary solid:  $N_{\text{particles}} = 18377$

Case	$N_{\text{elements}}$	$N_{\text{cores}}$	$\Delta t \times 10^{-4}$	$RCT$	$hCPU$
BF	$371 \times 10^3$	20	1.012	7.9	40.1
VOS-IVP	$373 \times 10^3$	20	1.072	8.2	39.7

❖ Identical cost per iteration



Moving solid:  $N_{\text{particles}} = 73508$

Case	$N_{\text{elements}}$	$N_{\text{cores}}$	$RCT$	$RCT(\text{particles})$	$RCT(\text{PPE})$	$hCPU$
BF $D/\Delta x = 100$	$584 \times 10^3$	8	11.8	—	3.2	76.7
VOS-IVP $D/\Delta x = 100$	$602 \times 10^3$	8	24.5	4.3	11.5	164

- ❖ Higher cost per iteration for VOS-IVP
- ❖ PPE cost increases due to variable Laplace operator
- ❖ Iso-grid, the new method costs in total two times more (x2)

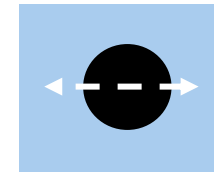
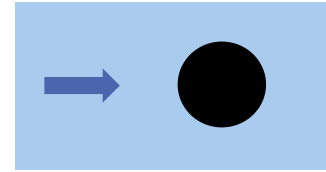
## Conclusions

### Volume of Solid – Implicit Volume Penalty method

Target characteristics:

- Appropriate description of the moving solid
- Solid/fluid volume preservation
- Correct influence of solid to the fluid's dynamics
- Accurate prediction of aerodynamic loads
- Efficient, robust and affordable

Validated on 2D laminar cases:



# Conclusions & perspectives

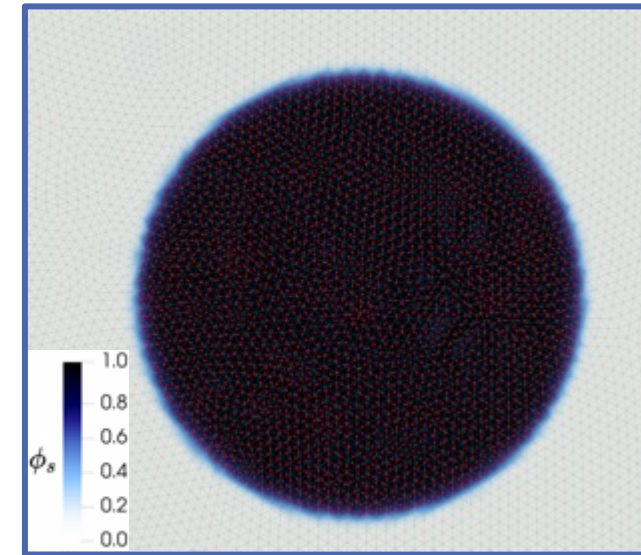
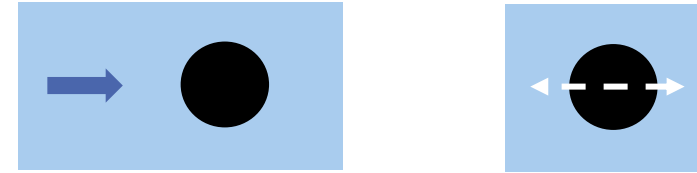
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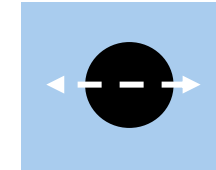
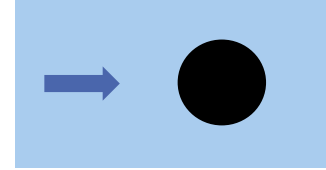
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## Validated on 2D laminar cases:



$D/\Delta x$	$\langle \varepsilon_s \rangle [\%]$	$\varepsilon_s^{rms} [\%]$
1	0.16	$7.35 \cdot 10^{-14}$
5	0.16	$2.11 \cdot 10^{-14}$
10	0.16	$1.38 \cdot 10^{-14}$

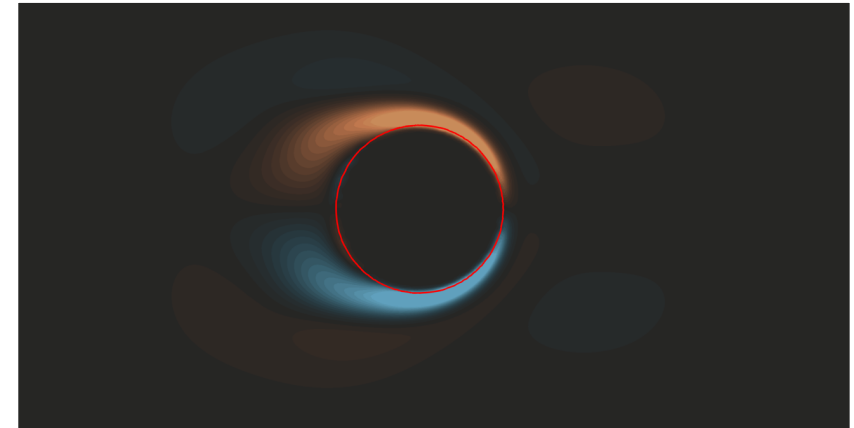
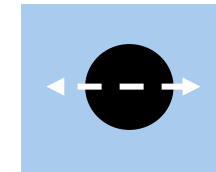
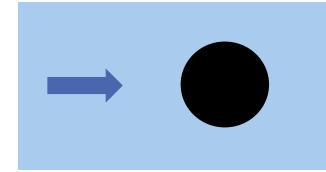
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# Conclusions & perspectives

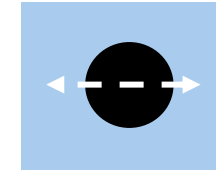
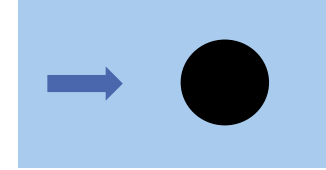
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x2

# Conclusions & perspectives

## Conclusions

### Volume of Solid – Implicit Volume Penalty method

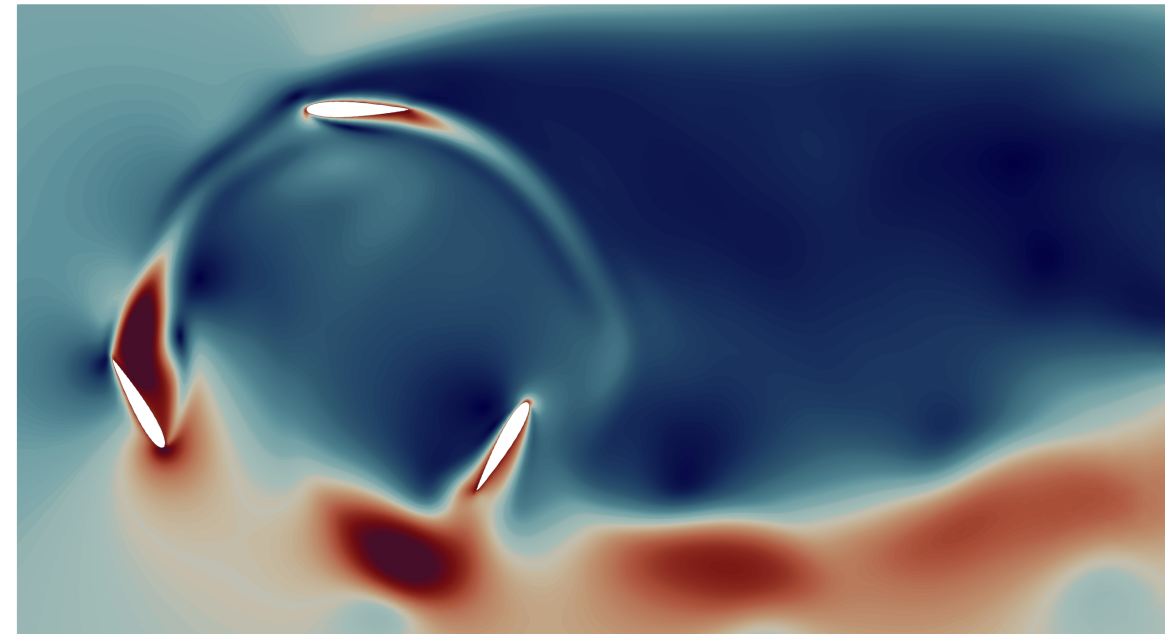
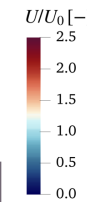
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## Perspectives

- Tame HF noise in force estimation
- In-depth performance study and code optimization
- Turbulent flow validation cases & Wind energy applications (VAWT, HAWT)

## Validated on 2D laminar cases:



# Conclusions & perspectives

## Conclusions

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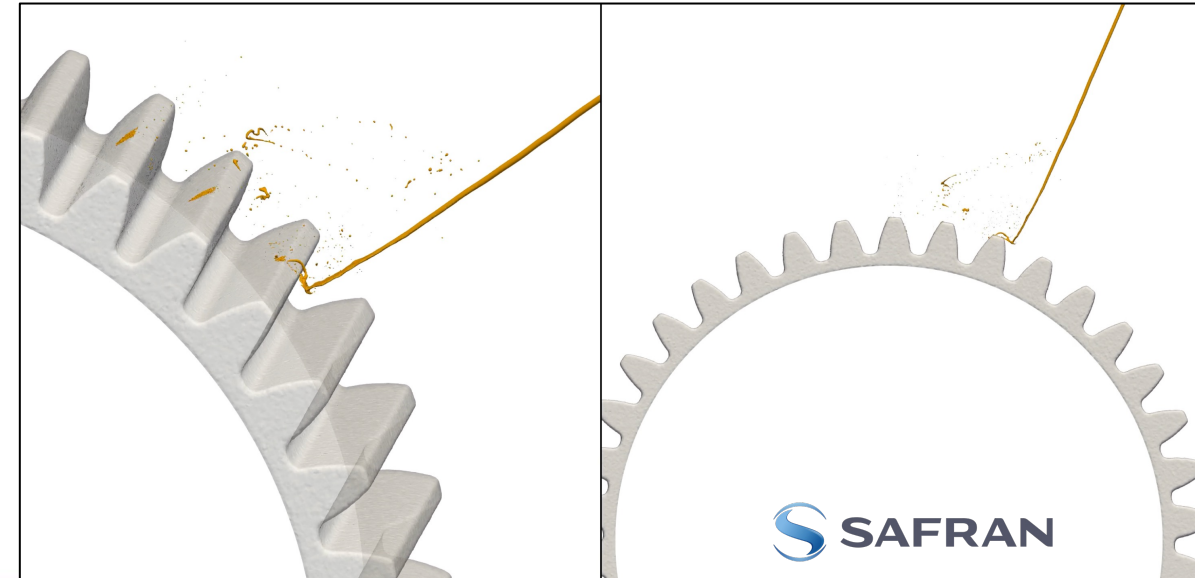
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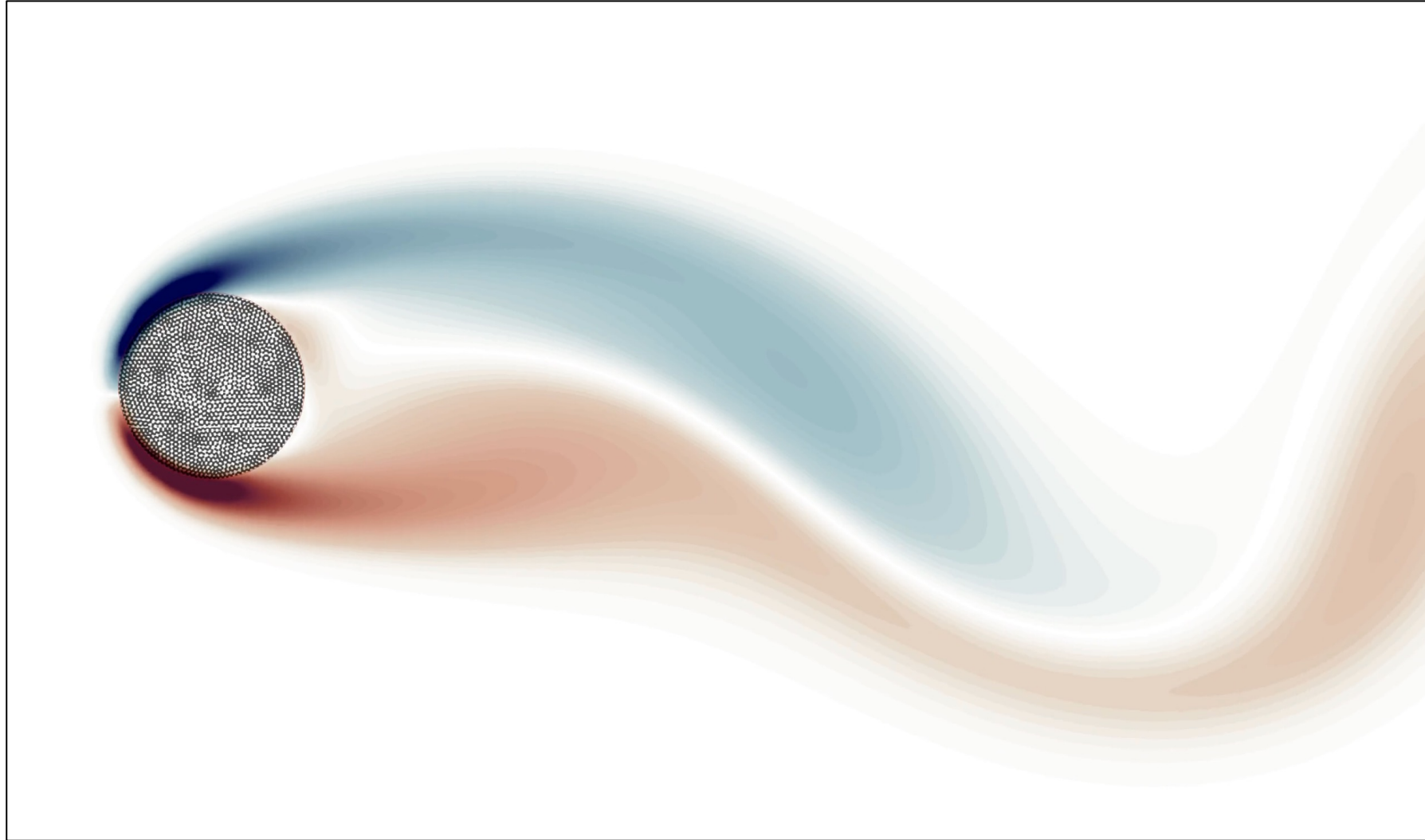


## Industrial application: SAFRAN Tech

- Use in transmission gearbox simulations
- Video credit: Mélody Cailler



# Thank you for your attention!



# BACK UP

# Implicit Volume Penalty (Detailed)

Advancing velocity field with implicit penalty method

Momentum equation

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = -\nabla \cdot (\mathbf{u}^n \mathbf{u}^n) - \frac{1}{\rho} \nabla P^{n+1/2} + \frac{\chi}{\eta} (\mathbf{u}_s^{n+1} - \mathbf{u}^{n+1})$$

$$\gamma = \left( 1 + \frac{\chi}{\eta} \Delta t \right)$$

Predicted penalized velocity

$$\mathbf{u}^* = \frac{1}{\gamma} \mathbf{u}^n + \frac{1}{\gamma} \Delta t \left[ -\nabla \cdot (\mathbf{u}^* \mathbf{u}^n) - \frac{1}{\rho} \nabla P^{n-1/2} \right] + \frac{\gamma - 1}{\gamma} \mathbf{u}_s^{n+1}$$

$$\mathbf{u}^{**} = \mathbf{u}^* + \frac{\Delta t}{\gamma \rho} \nabla P^{n-1/2}$$

Final penalized velocity

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^{**}}{\Delta t} = -\frac{1}{\rho} \nabla P^{n+1/2} + \frac{\gamma - 1}{\Delta t} (\mathbf{u}^{**} - \mathbf{u}^{n+1})$$

Pressure Poisson

$$\nabla \cdot \left( \frac{1}{\gamma \rho} \nabla P \right) = \frac{\nabla \cdot \mathbf{u}^{**}}{\Delta t} - \frac{\nabla \cdot \mathbf{u}^{n+1}}{\Delta t}$$

Remark: the penalty factor varies in space and time:

$$\gamma(\mathbf{x}, t) = 1 \text{ in fluid, } \gamma(\mathbf{x}, t) > 1 \text{ in solid}$$

Computing penalty forcing term

Penalty force

$$\begin{aligned} \mathbf{F}^{s \rightarrow f} &= \frac{\chi}{\eta} (\mathbf{u}_s^{n+1} - \mathbf{u}^{n+1}) \\ &= \frac{\chi}{\eta} (\mathbf{u}_s^{n+1} - \mathbf{u}^{\text{prediction}}) + \frac{\chi}{\eta} (\mathbf{u}^{\text{prediction}} - \mathbf{u}^{n+1}) \\ &= \mathbf{f}^{\text{prediction}} + \mathbf{f}^{\text{correction}} \end{aligned}$$

$$\mathbf{f}^{\text{prediction}} = \frac{\chi}{\eta} (\mathbf{u}_s^{n+1} - \mathbf{u}^{**})$$

$$\mathbf{f}^{\text{correction}} = \frac{\chi}{\eta} (\mathbf{u}^{**} - \mathbf{u}^{n+1})$$



## Force estimation

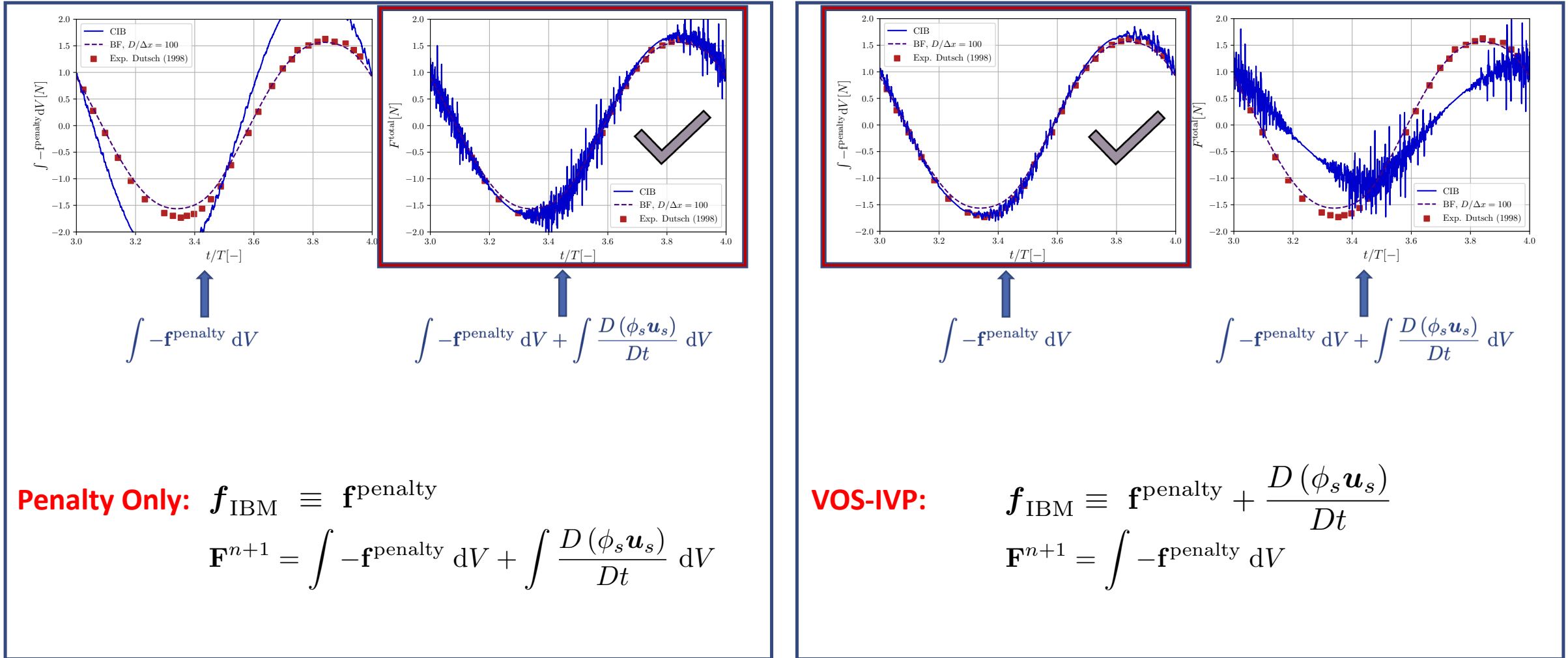
- For mobile immersed bodies, the negative sum of the IB forcing terms is equal to the total force, which includes both the forces from external fluid and the internal forces (needed to move the fluid inside the solid region) [1,2]

$$\mathbf{F} = \underbrace{-\sum_{\Omega} \rho \mathbf{f}_{\text{IB}} \Delta V}_{\mathbf{F}_{\text{tot}}} + \underbrace{\sum_{\Omega_s} \rho \frac{d\mathbf{u}}{dt} \Delta V}_{\mathbf{F}_{\text{in}}}$$

- In our case the internal force is already supplied to the system via the solid momentum source term whose role is to move the fluid inside the solid, so the external force can be computed by the negative sum of the penalty forces over the domain.

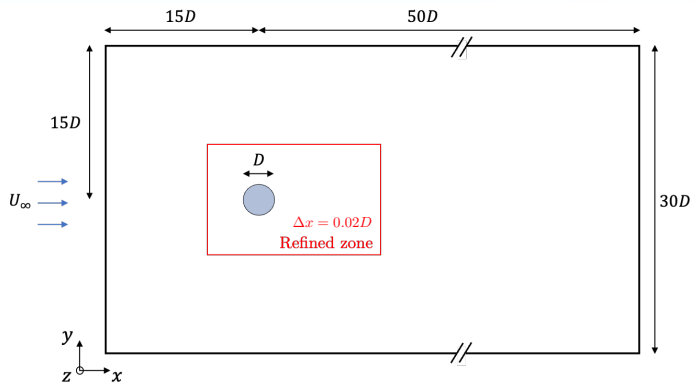
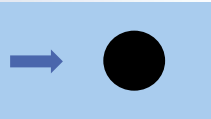
$$\mathbf{F} = -\sum_{\Omega} \rho \mathbf{f}^{\text{pen}} \Delta V = \sum_{\Omega} \rho (\mathbf{f}^{\text{prediction}} + \mathbf{f}^{\text{correction}}) \Delta V$$

Inline force compared to **experimental data** and **resolved body-fitted simulation** ( $y^+=0.5$ ) w/ moving reference frame:



- If the solid momentum is included to the N.S. equation, the total force has way less high frequency noise.
- This noise is also reduced by increasing the mesh resolution.
- Very good agreement with reference cases.

# 2D Convergence: Flow over stationary cylinder at Re=100

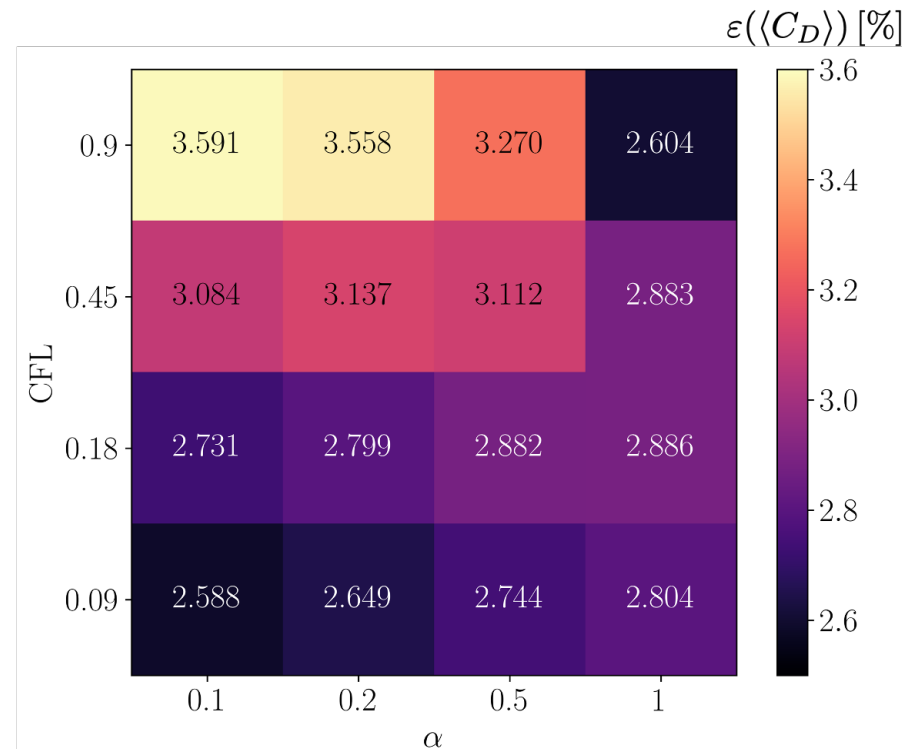
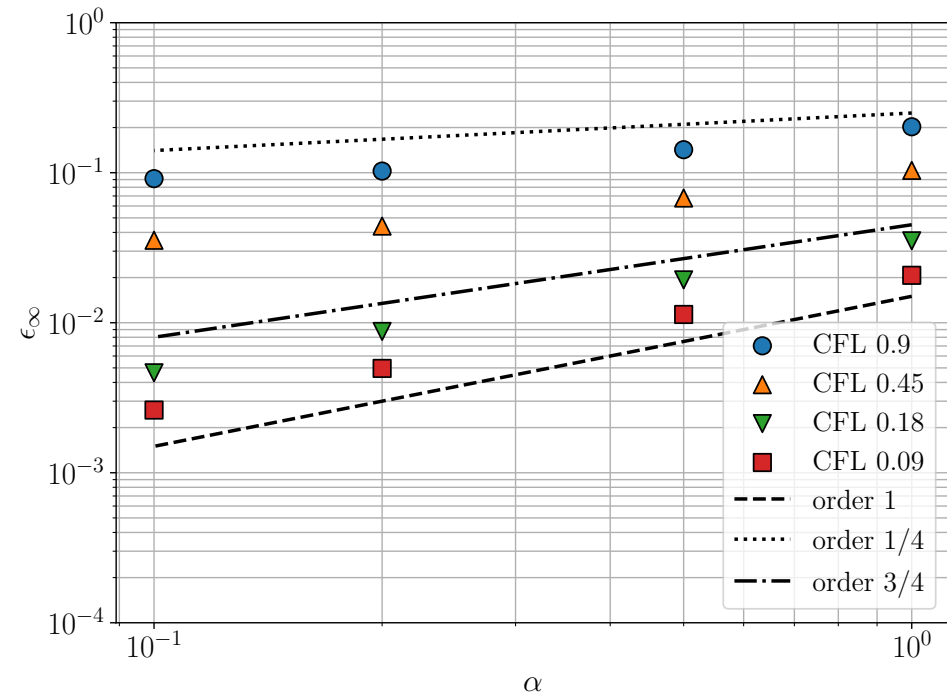


➤ Study of time-step (CFL) and penalty time parameter ( $\alpha$ ),

$$\text{CFL} = |\mathbf{u}| \frac{\Delta t}{\Delta x} \quad \alpha = \frac{\eta}{\Delta t}$$

on the error of maximum velocity inside the solid and mean drag coefficient (comp. to BF)

$$\epsilon_\infty = \max(\chi_s \phi_s \|\mathbf{u}\|_2) \quad \epsilon(\langle C_D \rangle) = 100 \times \frac{|\langle C_D \rangle_{\text{VOS-IVP}} - \langle C_D \rangle_{\text{BF}}|}{\langle C_D \rangle_{\text{BF}}} \%$$



➤ But the average costs:

$$hCPU_{\text{CFL}0.9} = 20$$

$$hCPU_{\text{CFL}0.09} = 80$$