



HAL
open science

Invariances and transformations

Thomas Delavande

► **To cite this version:**

Thomas Delavande. Invariances and transformations: Essay on gravitation, electromagnetism, nuclear interactions, from Kepler to pentaquarks. 2022. hal-03913341

HAL Id: hal-03913341

<https://hal.science/hal-03913341>

Submitted on 26 Dec 2022

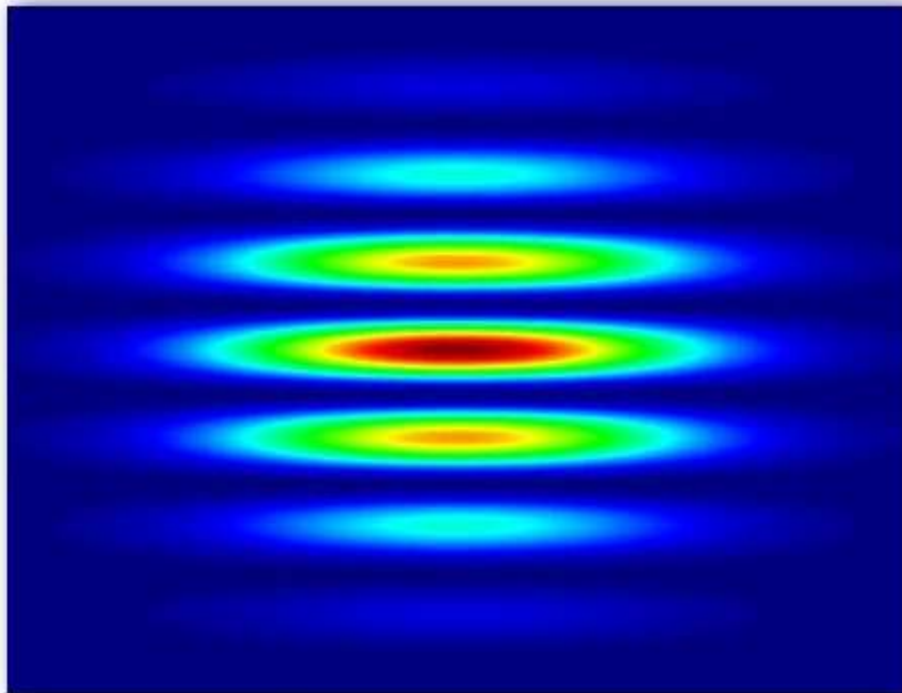
HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Thomas Delavande

Invariances and transformations

**Essay on gravitation, electromagnetism, nuclear interactions, from Kepler
to pentaquarks**



December 2022 release

Essential point of the essay: source field equations for gravitation analogous to those of electromagnetism

(Equations proposed in Memoir 3, chapters III, IV and V)

On the model of the couple magnetic field \vec{B} and electric field $\vec{E}l$, we propose to interpret the pulsation $\vec{\Omega}$ and the wave vector K as a couple pulsation field $\vec{\Omega}$ and field wave vector \vec{K} , couple which will intervene in the source field equations proposed for gravitation.

Beforehand, let us recall the well-known Maxwell source field equations in electromagnetism and their analogue in gravitation (with $\vec{E}S$ electrostatic field, $\vec{E}l$ electric field and $\vec{G}r$ gravitational field). Let us observe that there is no a priori analogue known to Maxwell Ampère for gravitation:

	Electromagnetism	Gravitation
Gauss	$\frac{\rho}{\varepsilon_0} = \text{div}\vec{E}S$ $\frac{\rho}{\varepsilon_0} = \text{div}\vec{E}l$	$-4\pi G\rho = \text{div}\vec{G}r$
Ampère	$\mu_0\vec{j} = \text{rot}\vec{B} - \mu_0\varepsilon_0 \frac{\partial\vec{E}l}{\partial t}$	

Let us rewrite these equations by restricting ourselves to 2 dimensions of Space, this simplification having the objective of better understanding the analogies thereafter:

	Electromagnetism	Gravitation
Gauss	$\frac{\rho}{\varepsilon_0} = \frac{\partial E_{S_x}}{\partial x} + \frac{\partial E_{S_y}}{\partial y}$ $\frac{\rho}{\varepsilon_0} = \frac{\partial E_{l_x}}{\partial x} + \frac{\partial E_{l_y}}{\partial y}$	$-4\pi G\rho = \frac{\partial G_{r_x}}{\partial x} + \frac{\partial G_{r_y}}{\partial y}$
Ampère	$\mu_0 j_y = \frac{\partial B_z}{\partial x} - \mu_0\varepsilon_0 \frac{\partial E_{l_y}}{\partial t}$	

Applying Ampère's theorem (i.e., following an analogy with Maxwell Ampère's source field equation) to the pulsation field $\vec{\Omega}$ and to the wave vector field \vec{K} , then substituting the density sources of charge ρ and electric current j_y by mass densities ρ and momentum \dot{p}_y , we will obtain several source field equations applying to gravitation. The table below gives the analogues that will be proposed:

	Electromagnetism	Gravitation
Gauss	$\frac{\rho}{\varepsilon_0} = \frac{\partial E_{S_x}}{\partial x} + \frac{\partial E_{S_y}}{\partial y}$ $\frac{\rho}{\varepsilon_0} = \frac{\partial E_{l_x}}{\partial x} + \frac{\partial E_{l_y}}{\partial y}$	$-4\pi G\rho = \frac{\partial G_{r_x}}{\partial x} + \frac{\partial G_{r_y}}{\partial y}$ $-4\pi G\rho = \frac{\partial 2c^2 K_x}{\partial x} + \frac{\partial 2c^2 K_y}{\partial y}$
Ampère	$\mu_0 j_y = \frac{\partial B_z}{\partial x} - \mu_0\varepsilon_0 \frac{\partial E_{l_y}}{\partial t}$	$-\frac{1}{\hbar}\dot{p}_y = \frac{\partial \Omega_z}{\partial x} - \frac{\partial K_y}{\partial t}$

By dividing the 1st term of Ampère by the 1st term of Gauss, one finds a group velocity (or similar), as well for the electromagnetism as for the gravitation, with the condition of checking a relation between the constants of the source field equations concerned:

Group velocity	$v_g = \mu_0 \varepsilon_0 \frac{j_y}{\rho} = c^2 \frac{\partial B_z}{\partial E l_x}$	$v_g = \frac{1}{4\pi G \hbar} \frac{\dot{p}_y}{\rho} = \frac{\partial \Omega_z}{\partial K_x}$
Condition on constants	$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	$c = \sqrt{\frac{G \hbar}{c l_p^2}}$

Let us transcribe these equations with the notations usually used in this essay, notations which aim to underline the analogies on the one hand between gravitation and electromagnetism, on the other hand between Space and Time:

	Electromagnetism	Gravitation
Gauss	$\frac{\vec{\rho}_{xy}^t}{\varepsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{E}_{yt}^{1/x}}{\partial y}$ $\frac{\vec{\rho}_{xy}^t}{\varepsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{E}_{yt}^{1/x}}{\partial y}$	$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial \vec{G}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{G}_{yt}^{1/x}}{\partial y}$ $-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial 2c^2 \vec{K}_{yt}^{1/x}}{\partial y}$
Ampère	$\mu_0 \vec{j}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}_{ty}^{1/x}}{\partial t}$	$-\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$
Group velocity	$v_g = \mu_0 \varepsilon_0 \frac{j_{xt}^y}{\rho_{xy}^t} = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$	$v_g = \frac{1}{4\pi G \hbar} \frac{\dot{p}_{xt}^y}{\rho_{xy}^t} = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$
Condition on constants	$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	$c = \sqrt{\frac{G \hbar}{c l_p^2}}$

Observe that the Ampere Gravitation equation corresponds to:

- on the first term on the right to Newton's fundamental principle of dynamics (Newton's second law):

$$\left(\frac{d\vec{p}}{dt} \right)_{xt}^y = \frac{-\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x} = \frac{-\partial E p}{\partial x}$$

- on the second term on the right to the relation momentum wavelength (or wave vector) $p = \frac{h}{\lambda} = \hbar K$ of Albert Einstein and Louis de Broglie:

$$\left(\frac{d\vec{p}}{dt} \right)_{xt}^y = \frac{\partial \hbar \vec{K}_{ty}^{1/x}}{\partial t}$$

Finally, let us emphasize that the quotient of the 2 gravitation source field equations makes it possible to find a group velocity $v_g = \frac{\partial \Omega}{\partial K}$, according to a condition relating to the Planck length $l_p = \sqrt{\frac{G \hbar}{c^3}}$. This is, as we will see later, the stumbling block and the fundamental argument in favour of the source field equations proposed here for gravitation.

Introduction: why are the laws of gravity and electricity so similar?

Anyone interested in the laws of gravitation and electricity may be surprised at the similarity between these laws. Often, to a concept used in gravitation (or more generally in mechanics), corresponds a similar concept used in electricity.

We can cite pell-mell:

- the Newtonian gravitational potential and the Coulombian electric potential,
- energy, power in mechanics and energy, power in electricity,
- the mass and the electric charge, with however an important difference, since we observe two natures of electric charge and a single nature of mass,
- mechanical forces (gravitational force, centrifugal force, Coriolis force) and electrical forces (electrostatic force, Lorentz magnetic force, etc.). There is, however, a notable difference between electric force and gravitational force: in an electric field, the acceleration of a body depends on its specific electric charge, whereas in a gravitational field, the acceleration of a body does not depend on its mass.

These resemblances between gravitation and electricity have often been underlined and exploited by physicists. Mechanical theories influenced electrical theories, which in turn influenced mechanical theories.

For example, in the 17th century Christiaan Huygens points out the resemblance between mechanical waves and optical theory to develop an early wave theory of light. In the 19th century, Michael Faraday relied on mechanical considerations to develop the notions of magnetic field and electric field. As a return of things, his ideas inspire the notion of gravitational field, with an action no longer at a distance, but which gradually spreads in space. In the 1920s, Louis de Broglie and Erwin Schrödinger exploited the analogy with standing mechanical waves to formulate the wave equation of the electron.

These similarities between mechanics and electricity will serve as a red thread for the reflections carried out in this essay. They will nevertheless remain enigmatic here. Indeed, only a few leads to the deep reasons for these similarities will be mentioned.

This essay is composed of 6 Memoirs. In the 1st Memoir, we will focus on the changes of reference frames and the forces of inertia. The starting point will be the following question of A. Einstein. Why in classical physics are there two types of forces: "normal" forces and inertial forces (sometimes called pseudo forces) that occur in the same equations? We will try to answer this question by assimilating any force to a force of inertia.

The study of the forces of inertia will be an opportunity to approach the model of the Bohr electron. Model proposed in 1913 by Niels Bohr and which involves centrifugal inertial force and electrostatic force. Through the notions of Gauge invariances and transformations introduced in 1918 by Hermann Weyl, Gauge invariances that can be compared to a conservation of the Laws of Nature, Gauge transformations that can be compared to changes of frames of reference, we will study the wave equations of E. Schrödinger (1925), Wolfgang Pauli (1927) and Paul Dirac (1928). This will address the main wave functions of quantum physics in the 1920s.

The 2nd Memoir will offer more conjectures than the 1st. We will come back to the analogies between fluid mechanics and electromagnetism, analogies for example underlined by Henri Poincaré in 1893. From these analogies, we will propose the notions of pulsation field $\vec{\Omega}$ and wave vector field \vec{K} . These two fields will apply respectively in a spatial plane and in a spatiotemporal plane. Thereafter, we will try to build all the forces of classical physics on the model of the Coriolis inertial force, with a quantity preserved during the change of reference frames and a quantity cancelled which is similar to a "generalized" rotation vector or to a field.

The study of fields and forces will be an opportunity to return to 19th century physics, physics which was particularly involved in electricity and electromagnetism. Among others, Charles-Augustin Coulomb's law for electrostatics (1785), the Siméon-Denis Poisson equation (1813) (from which A.

Einstein was largely inspired for his theory of General Relativity), the work of M. Faraday on magnetic induction (from 1821), the magnetic force of Pierre-Simon Laplace (from 1820) macroscopic precursor of the magnetic force of Hendrik Lorentz, as well as the force of inertia of Gustave -Gaspard Coriolis (1835).

In the 3rd Memoir, we will examine the source field equations of James Clerk Maxwell, equations involving charge density and electric current density, magnetic and electric fields, and applying to electromagnetism. Following an electricity gravitation analogy, we will propose source field equations involving mass density and momentum density, pulsation and wave vector fields, and applying to gravitation. We will touch on quantum gravity, when to obtain a group velocity of a wave identical to the speed of a mass body, we will obtain the condition over an infinitesimal distance: $dx = \sqrt{\frac{Gh}{c^3}}$, distance identical to the Planck length. We will also suggest the Maxwell-Faraday analog for gravitation.

This Memoir will focus on André-Marie Ampère's research, in particular the observation that a magnet and a current loop produce identical effects: a magnetic field (from 1820), then Maxwell's equations proposed by J. C. Maxwell in 1865 (and transcribed using partial derivatives in 1884 by Oliver Heaviside and Willard Gibbs). We will discuss the idea of L. de Broglie (1924) of an electron that is both wave and particle. In passing, we will deal with a bit of Newtonian mechanics (*the Principia* by I. Newton published in 1687), a bit of Hamiltonian mechanics (proposed by William Rowan Hamilton in 1833), a bit of Einsteinian mechanics (with $E = mc^2$ stated by A. Einstein in 1905 in the context of Special Relativity), and Compton scattering (1922). We will also mention the gravitational waves suggested by A. Einstein in 1916 in the context of General Relativity and confirmed experimentally in 2015 by researchers from the LIGO (Laser Interferometer Gravitational-Wave Observatory).

The 4th Memoir will deal with elementary particle waves and weak and strong nuclear interactions. It will be a synthetic course on the main theories developed in the 20th century, which describe these nuclear interactions. We will mention the work of Werner Heisenberg on isospin (1932), Enrico Fermi's theory of weak interaction (1933), Hideki Yukawa's model for nuclear interactions (1935). Yukawa's model involves a mediating particle of the photon type, but having a mass (the particle is baptized mesotron by H. Yukawa).

Then, we will talk about the work of Murray Gell-Mann (1960s) on quarks and the strong interaction, then the electroweak model of Sheldon Glashow-Sтивен Weinberg-Abdus Salam (1960s). The Electroweak Model brings together the electromagnetic, weak and hyper interactions. We will briefly discuss the Higgs mechanism or BEH (mechanism postulated independently in 1964 by Robert Brout and François Englert, and by Peter Higgs) in order to assign a mass to particle waves. These works from the 1950s and 1970s are grouped together in what is now called the Standard Model.

The 5th Memoir will always be interested in elementary particle waves and will again be speculative. We will try to build a bridge between the Standard Models and some gravitation source field equations proposed in the 3rd memory. We will associate these source field equations with 4 quantum numbers qualified as source field (spin S, isospin I, strangeness St and baryonic number Ba) and with 4 electromagnetic interactions, hyper, weak and strong, which all appear in the Standard Model. Towards the end of the Memoir, a track will be exposed explaining the similarities between electricity and gravitation. This track will present the gravitational interaction as formed of two constituents: one the electromagnetic interaction, the other the hyper interaction.

This Memoir will begin with a parallel between J. Kepler's law of areas (1609), the notions of angular momentum, then of quantum spin proposed by Samuel Goudsmit and George Uhlenbeck (1925). We will discuss the proposals for new quantum numbers that are the hypercharge Y or the strangeness St (Kazuhiko Nishijima and M. Gell-Mann in the 1950s). We will also come back to the Quark Model (M. Gell-Mann and George Zweig from 1961 to 1964). Thereafter, we will approach the angle of Nicola Cabibbo (1963), an angle allowing to describe from the quarks up and down the weak neutral currents. We will also deal with the S. Glashow-Jean Illiopoulos-Luciano Maiani mechanism, a mechanism which imagines the existence of a 4th quark, and which involves weak neutral currents. Through the work of Bruno Pontecorvo and Ziro Maki (from the 1960s), we will also be interested in

neutrino oscillations: hypothesis of several families of neutrinos, existence of an angle (now called similar Pontecorvo angle for leptons to that of Cabibbo for quarks). Finally, we will mention the pentaquarks (first detection in 2003 and confirmation in 2015).

The 6th Memoir will undoubtedly be the most speculative of all – some may rightly consider that we are out of Science here –. We will wonder about the differences between the Time felt (Time oriented like an arrow) and the Time used in physics (Time often described mathematically as a spatial dimension).

To answer this question, we will study the most varied concepts. John Wheeler and Richard Feynman's idea of an electron going back in time (concept mentioned in R. Feynman's Nobel Prize acceptance speech in 1965). Works of Rudolf Clausius on entropy (1865), Ludwig Boltzmann on statistical entropy (around 1870), Claude Shannon on information entropy (1950s). Experiments by Chien-Shiung Wu (1957) on parity violation in the beta decay of cobalt-60. General relativity (A. Einstein around 1915). First big-bang models resulting from it, models proposed by Willem de Sitter, Alexandre Friedmann and Georges Lemaître in the 1920s. Two experimental discoveries confirm these models: that in 1920 by Edwin Hubble of an expanding universe, then that in 1965 by Arno Penzias and Robert Wilson of low temperature electromagnetic radiation.

Finally, we will broaden our reflections to fields other than physics, such as the phenomenological approach of Edmund Husserl (early 20th century) or the Gaia hypothesis of James Lovelock (1979).

Preface to the reader, what can this essay bring him?

This essay favours two major approaches: a historical approach and a speculative approach. On the one hand, it wants to summarize the great ideas of physics from the pioneers of the 17th century until today. Without claiming to be exhaustive, these ideas are the ones that seemed to us the most remarkable or those that interested us the most. On the other hand, it wishes to bring back the fruit of our physical reflections for a little more than fifteen years.

By following these two approaches, historical and speculative, this essay has a triple objective: to learn (or to initiate), to debate and to inspire the reader.

Learn

While writing this essay, we frequently wondered what reader profile we were aiming for. Of course, we are open to all readers. However, we have often imagined a young, curious, intelligent mind, who would not yet know much about physics, and who, thanks to this essay, could quickly *learn* about its major concepts.

Obviously, it is by reading varied and often contradictory works that one progresses the most. However, to our knowledge, there are few works including mathematical equations, and which bring together in a condensed way the main questions of physics of the 17th, 18th, 19th and 20th centuries.

We tried to write a book that we would have dreamed of reading at 25, when we became deeply interested in theoretical physics. A book intended to save precious time for a young reader in understanding the main ideas of physics yesterday and today.

Debate

We also wanted to write an essay that provokes in the reader the desire to *debate*. I sincerely hope that this essay will be "debatable", that is to say that certain speculative ideas proposed here are worth debating, in order to enrich the current scientific debate.

However, we count on the indulgence of the reader, and we apologize beforehand for having sometimes left in the speculative parts, a certain number of unfinished calculations, that is to say not demonstrative. Indeed, it seemed interesting to us to indicate the paths or tracks followed, even if these did not always lead to the desired destination.

Inspire

Finally, our main objective is to *inspire* the reader, to generate new ideas at home in agreement or even in contradiction with the ideas and the tracks proposed here. Our model is the PhD of L. de Broglie which inspired E. Schrödinger, enabled him to develop the equation bearing his name, and brought together (even unified) physics and chemistry.

Our wildest dream is that this essay will inspire a few "new schrödingers" who will manage to unify the different interactions of physics, then to explain qualitatively and quantitatively the enigmatic properties of waves particles (such as their quantum numbers, their electric charges, or their masses, etc.).

We also hope that this essay inspires some Carlo Rubbia or Simon van der Meer, the discoverers in 1983 of the W^\pm and Z^0 bosons, predicted by the Electroweak Model of the 1960s. Another boson predicted by the Electroweak Model is evoked, the hitherto undiscovered B boson. As additional characteristics, it will be proposed that the B boson is massless like the photon, and that it carries an entire isospin. The most important thing remains, to detect it and confirm these hypotheses.

To end this preface, we thank in advance all readers who will kindly send us their criticisms, their comments, as well as any errors or ambiguities that have crept into the text.

This is a work of synthesis dealing with very varied fields of physics. Like any work of synthesis, it has its faults, and in certain areas, faults and serious errors may have slipped into it. So, thank you to all the experts, who in their respective fields, will take the time to read through this essay and report any errors. These remarks and corrections will, we hope, improve the present text for a future edition.

Explain to my youngest daughter Mycènes (14 years old) the essay Invariances and Transformations

My last daughter Mycenae, 14 years old (almost 15 years old), already has some good knowledge of physics. For example, she learned the delicate notions of energy, mechanical energy, kinetic energy and potential energy. She understood that energy measures the capacity of a system to be able to modify a movement (or in a broader sense, to modify a state).

Recently, as part of a presentation, she became interested in the phenomena of electromagnetic induction, that is to say these surprising and "magical" phenomena, where for example when a magnet is moved near an electric circuit, there appears an electric current.

I tried to explain to my daughter the phenomena of electromagnetic induction and thereby the spirit of the Invariances and Transformations essay which, in my opinion, allows her to understand these "magical" phenomena more intuitively. With my daughter, I do not use the title of Invariances and Transformations, but that of Time Light Theory, a title that I find prettier, more poetic, even if the link between Time and Light remains confused.

I remember reading in the work of A. Einstein (unfortunately, I no longer know where or what exactly, so I'm probably transforming a little) that even a 5-year-old child should be able to understand physical equations. 5 years seems a little young to me, but I agree with A. Einstein in the sense that the "good" physical theories are undoubtedly also the simplest. Invariances and Transformations is not a simple essay. The main reason is, I believe, that he does not solve his first question: that of the resemblance between gravitation and electromagnetism. It only suggests clues. However, I have tried to explain in the few lines that follow, the main principles of this long essay, in the most intuitive and simple way possible.

Like forces

Mycenae, do the following different thought experiments with me.

First, imagine yourself in a car moving in a straight line on the highway. Suddenly, this car accelerates. You then feel thrown backwards, pinned to your chair, as if subject to a force that takes you backwards.

Always imagine yourself in a car moving in a straight line on the highway. Suddenly, this car decelerates. You feel projected forward, as if subject to a force that takes you forward.

Now imagine yourself on a rotating carousel. You feel projected towards the outside of the carousel, as if subjected to a force that would make you "flee" from the carousel. In Physics, we also speak of centrifugal drive inertial force. I give you its equation: $m(\vec{\omega} \wedge \vec{\omega}M) \wedge \vec{\omega}$, with $\vec{\omega}$ the carousel rotation speed.

Always imagine yourself on a rotating carousel, and that the rotation of the carousel is accelerating. A bit like in the car accelerating on the highway, you feel yourself flattened backwards. You have the feeling of a force whose direction is backwards and whose direction is perpendicular to the radius of the carousel. In Physics, we also speak of driving inertia, due to the variation in the speed of rotation of the carousel. I give you its equation: $m\vec{\omega}M \wedge \frac{d\vec{\omega}}{dt}$.

Always imagine yourself on a rotating carousel, and that the rotation of the carousel is decelerating. A bit like in a car on the highway, you feel projected forward, you have the feeling of a force whose direction is forward and whose direction is perpendicular to the radius of the carousel. In Physics, we always speak of the driving inertial force due to the variation in the speed of rotation of the carousel.

Imagine yourself again on a rotating carousel (at constant speed), and that you are moving towards the center of the carousel (note that your speed thus decreases). You feel carried away towards the direction of rotation of the carousel (the front of the previous experiences), you have the feeling of a

force whose direction is that of the rotation of the carousel and whose direction is perpendicular to your relative speed on the carousel. In physics, we talk about the Coriolis inertial force. I give you its equation: $m\vec{v}_r \wedge 2\vec{\Omega}$.

Imagine yourself again on a rotating carousel (at constant speed), and that you are moving towards the outside of the carousel (note that this increases your speed). You feel carried towards the direction opposite to the rotation of the carousel (the back of the previous experiences), you have the feeling of a force whose direction is opposite to the rotation of the carousel and whose direction is perpendicular to your relative speed on the carousel. In physics, we always talk about Coriolis inertial force.

For all these thought experiments, "purist" physicists do not really speak of forces, but of forces of inertia or pseudo-forces, which are added to Newton's fundamental principle of dynamics (Newton's second law), when passing from a so-called Galilean reference frame (the term inertial reference frame is also often used) to a non-Galilean reference frame. The notion of a Galilean frame of reference is not easy to grasp. You can see it as a frame of reference where there are no inertial forces to add to Newton's fundamental principle of dynamics. But it must be recognized, by this definition, the fish bites its tail a little.

The fundamental idea of the Invariance and Transformation essay is to reduce any force (gravitational, electric, magnetic) to a force of inertia added during a change of frames of reference.

Intuitively understand the phenomena of electromagnetic induction, analogies between mechanics and magnetism

Go back to the phenomena of electromagnetic inductions and try to understand them intuitively using previous thought experiments on the carousel.

You can see the magnet and the magnetic field it generates, like a carousel for electric charges. When an electric charge or an electric circuit are in a magnetic field, it is as if they were on a carousel. Of course, you don't see this carousel, and important point: this carousel does not apply to masses like that of the previous thought experiments, but to electric charges. For the rest, you will see, they are very similar.

When you move a magnet over time, you vary the magnetic field created by that magnet. It's as if you were varying the speed of the carousel, it's as if the electric charges (or the electric circuit) were on a carousel whose speed of rotation varies, it's as if they were subjected to a force of training due to the variation in the speed of rotation of the carousel.

This induces, what is called in physics, an electromotive field, which integrated along the electric circuit, gives an electromotive force and therefore the appearance of an electric current in this circuit. In physics, this is called the Neumann case of electromagnetic induction. I give you his equation: $q\vec{O}M \wedge \frac{d\vec{B}}{2dt}$. You see that this equation looks like $m \frac{d\vec{\Omega}}{dt} \wedge \vec{O}M$.

When you make an electric circuit move in a (constant) magnetic field, it is as if the electric charges were moving on a carousel, it is as if they were subjected to a Coriolis force. This induces an electromotive field, which integrated along the electric circuit, gives an electromotive force and therefore the appearance of an electric current in this circuit. In physics, this is called the Lorentz case of electromagnetic induction. I give you its equation: $q\vec{v}_r \wedge \vec{B}$. You see that this equation looks like $m\vec{v}_r \wedge 2\vec{\Omega}$.

What you must understand is that the analogue of what generates the electric current in a circuit is not the speed of rotation of the carousel, but in the case of Neumann: the variations in time of the speed of rotation of the carousel, and in the case of Lorentz: the existence of a relative speed (of the electric circuit) on the rotating carousel.

Intuitively understand the gravitational force and the electric force, analogy between Space and Time

The fundamental advance of 20th century Physics is undoubtedly to have understood that Time (in fact, Time in its complex form it) is a dimension identical to the 3 dimensions of Space x , y and z .

20th century Physics was thus able to introduce the notions of 4-dimensional Space-Time, quadrivectors, etc.

Why is Time in Physics like this? It remains a great mystery. Be that as it may, this makes it possible to mathematically define a rotation in a spatiotemporal plane x , it , in total analogy with a rotation in a spatial plane x , y .

This makes it possible to propose in this essay that gravitational field (or rather what is called wave vector field) and electric field are "generalized" rotations in spatiotemporal planes. This makes it possible to propose that gravitational force and electric force are like analogues in Time of the Coriolis force (which is based on a rotation in Space), that is to say inertial forces to be added when one places oneself in the frame of reference where one cancels the "generalized" speed of rotation of the "spatiotemporal" carousel and where one "moves temporally" on this spatiotemporal carousel.

This "temporal displacement" on the spatiotemporal carousel will no doubt seem surprising to you, since it is not a question of a speed (or a flow of Time), but of a mass in the case of the gravitational force and of an electric charge in the case of electric force. I give you their equations proposed in this essay: $\vec{m}^t \wedge \vec{G}r$ and $\vec{q}^t \wedge \vec{E}l$. You see that these equations also look like $\vec{v}_r \wedge 2\vec{\Omega}$.

Bibliographic confidences, tribute to A. Einstein and L. de Broglie

« To study any question, it is always better to read the original Memoirs, because one is better able to assimilate a nascent science »

James Clerk Maxwell in *A treatise on electricity and magnetism*

My initial questioning about gravity and electricity comes from my high school years. How to explain the strong mathematical similarities between the laws of electricity and those of gravitation? Conversely, how can we explain that gravity and electricity, despite their similarities, are ultimately so distinct from each other?

During my years of higher education, I acquired the basics of classical physics: Newtonian mechanics and 19th century electromagnetism. However, I have studied relatively little 20th century physics, touched upon Special Relativity and the Schrödinger equation. At that time, I knew nothing or very little of the major historical issues and controversies in physics.

A few years after finishing my electrical engineering studies, I became particularly interested in theoretical physics. This was done via a course from my wife (who is a physics teacher) on the experiments of Albert Abraham Michelson. These experiments concluded that the speed of light was always measured equal to c , regardless of the speed of the experimenter, thus contrary to the principle of Galilean relativity.

To tell the truth, I vaguely remembered that A. Einstein had proposed a solution to this paradox, but I couldn't remember which one in detail. I had once gone through some academic books reproducing the theories of A. Einstein, without arousing any great curiosity in me. I then wanted to read the original works of A. Einstein. Like many, I was amazed by his elegant ideas, by his style comparable to an author of detective novels, by the original questions he asked and the elegant answers he offered.

In his scientific writings, A. Einstein frequently starts from major principles that he sets as objectives. He combines them with the experimental discoveries of his time and deduces often very audacious consequences. For example, in his 1905 article on Special Relativity, he questions the measurement of the simultaneity of two phenomena, then questions the very notion of Time, and manages to reconcile the principle of relativity and the constant speed of light in Michelson's experiments.

In the footsteps of A. Einstein, I began to dream of a vast physical theory that would unify all forces. The universe is one, the laws of gravitation and electricity are similar. After all, it seems logical to want to unify them in a large theory.

The field theories of A. Einstein who aimed to unify the gravitational force with the electromagnetic forces were interested in the changes of reference frames and the forces of inertia. In particular, A. Einstein was not satisfied with the existence in the Newtonian theory of two types of forces: real forces and pseudo-forces called inertia that are added when moving to a non-Galilean reference frame. To remove this contradiction, he imagined bringing any force closer to a force of inertia.

The ideas of A. Einstein did not concretely lead to a theory of fields combining gravitation and electricity. Moreover, they did not include the nuclear interactions discovered during the 20th century. However, they had a large lineage. They have inspired generations of physicists, induced Gauge theories, the Standard Model of the 1950s and 1970s, the Big Bang and black holes.

After reading the main popular works of A. Einstein and some of his most famous articles, I looked into quantum physics, which I still knew very little about. I went through some particularly pointed academic works, often very mathematical. I admit I didn't understand what it was all about.

I also looked at the origins of the theory. This is often easier to understand, because there is not yet this dress which specifies the model, but which obscures the original ideas of the pioneers. With the help of undergraduate books, I gradually understood the model of the electron of N. Bohr, wondering all the same why these two forces, electrostatic and centrifugal, intervened.

It is mainly thanks to the works of L. de Broglie that I began to glimpse what was quantum physics, or rather wave mechanics. Indeed, L. de Broglie insisted in his works more on the wave aspect than on quanta.

One of the first ideas of L. de Broglie was to say: since the light represented as a wave, can also be represented as a particle the photon, it can be the same for particles of matter like the electron, to be represented then as a wave of matter. The second idea was to explain the quantification of energy by circular standing waves. These ideas of L. de Broglie were for me a real trigger in the understanding of quantum physics.

Summary of the ideas and conjectures developed in this essay

We briefly recall here the ideas developed in this essay, classified in the order in which they occur in the various Memoirs. They are grouped into 11 major ideas, the first two of which are directly inspired by those of A. Einstein. The last 3 are not strictly speaking scientific hypotheses in the sense of Karl Popper (that is to say possibly falsifiable).

The 11 ideas set out here may sometimes seem obscure to the reader. For greater clarity, he may refer to the Memoirs where they are developed (the Memoirs are indicated in parentheses).

Idea 1, take up the idea of a Space-Time continuum and treat Time in physical equations exactly the same way as the three dimensions of Space (Memoir 1)

Note: why is Time complex?

More precisely, it is complex Time it which is exactly of the same nature as a dimension of Space x . Indeed, to calculate the norm of a quadri-vector (or a quadri-vector) instant position, we have:

$$4X^2 = x^2 + y^2 + z^2 + (ict)^2 = x^2 + y^2 + z^2 - (ct)^2$$

Some physicists have sought to eliminate this complex time and to reintroduce a real time into the equations of physics, postulating that it was only a matter of convention. It hardly came to fruition.

Idea 2, make any force correspond to a force of inertia, that is to say to a force that is added when passing from a Galilean reference frame to a non-Galilean reference frame (Memoir 1)

Idea 3, build any force on the model of the Coriolis inertial force $\vec{F}_{cor} = m\vec{v}_r \wedge 2\vec{\Omega}$ (Memoir 2)

During a change of frames of reference (for example passage from a Galilean frame of reference to the frame of a rotating carousel $\vec{\Omega}$), we cancel on the one hand a physical quantity (of type rotation vector $\vec{\Omega}$), we keep on the other hand a physical quantity (of type relative speed \vec{v}_r , momentum $m\vec{v}_r$, electric current...). The force or inertial acceleration that is added when changing reference frames is perpendicular (via a vector product) to both the conserved quantity and the cancelled quantity. This is referred to here as the Coriolis inertial force model. The objective is to build any force such as the electrostatic force, the magnetic force of Lorentz, the gravitational force on the model of the Coriolis inertial.

Note

To be able to apply the vector product, we will most often remain with the simple case of a three-dimensional space. This will contain either 3 dimensions of Space, or 2 dimensions of Space and 1 dimension of Time with a temporal dimension treated identically to those of Space.

Idea 4, bring closer the concept of field and the concept of “generalized” rotation vector cancelled during a change of reference frames (Memoir 2)

We bring the notion of field in physics closer to the notion of “generalized” rotation vector that is cancelled during a change of frames of reference. The notion of rotation vector is taken in a broad sense, because the rotation plane can be spatial or spatiotemporal.

The pulsation field $\vec{\Omega}_{xy}^{1/t}$ and magnetic field $\vec{B}_{xy}^{1/t}$ will be qualified as “generalized” rotation type in a *spatial* plane. The electric fields $\vec{E}_{xt}^{1/y}$ and wave vector field $\vec{k}_{xt}^{1/y}$ will be qualified as “generalized” rotation type in a *spatiotemporal* plane.

Note 1

$\vec{\Omega}_{xy}^{1/t}$ indicates that the rotation plane is x, y and that the rotation vector is oriented along t . We have the index $1/t$ because the unit of $\vec{\Omega}$ is the rad/s .

Note 2

We have the electric field $El_{tx}^{1/y}$ defined from the electromagnetic potential quadrivector A^μ ($\mu = t, x, y, z$), and reoriented to construct it as a rotation vector:

$$El_{tx}^{1/y} = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

The electrostatic field $ES_{tx}^{1/y}$ appears as a special case of the electric field when $\frac{\partial A^x}{\partial t} = 0$:

$$ES_{tx}^{1/y} = -\frac{\partial A^t}{\partial x}$$

Similarly, we have the wave vector field $K_{tx}^{1/y}$ defined from the velocity potential V^x and the Newtonian potential V^t :

$$2c^2 K_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$$

The Newtonian gravitational field $Gr_{tx}^{1/y}$ appears as a special case of the wave vector field when $\frac{\partial V^x}{\partial t} = 0$:

$$Gr_{tx}^{1/y} = -\frac{\partial V^t}{\partial x}$$

Even if the rotation vector (in the broad sense) and the field are associated notions, we will distinguish one from the other by presenting:

- the rotation vector as the movement existing before the change of frames of reference,
- the field as the quantity cancelled after the change of reference frames.

The field will therefore be the rotation vector cancelled after the change of reference frames.

Idea 5, what distinguishes an inertial acceleration from an inertial force? (Memoir 2)

An inertial acceleration (centrifugal or Coriolis type) differs from an inertial force in that:

- when an inertial acceleration must be added during a change of frames of reference, what is cancelled is a "generalized" rotation vector in a *spatial plane* (rotation vector associated with a pulsation field $\vec{\Omega}_{xy}^{1/t}$),
- when an inertial force must be added during a change of frames of reference, what is cancelled is a "generalized" rotation vector in a *spatiotemporal plane* (rotation vector associated with an electric field $\vec{E}_{xt}^{1/y}$ or a wave vector field $\vec{K}_{xt}^{1/y}$).

Idea 6, from Maxwell's source field equations, find a velocity similar to the group velocity $v_g = \frac{\partial \Omega}{\partial K}$, and involving the fields $\vec{B}_{xy}^{1/t}$ and $\vec{E}_{xt}^{1/y}$ (Memoir 3)

In his 1924 thesis, L. de Broglie represents the electron particle as a wave packet with a phase velocity $v_\phi = \frac{\Omega}{K}$ and a group velocity $v_g = \frac{\partial \Omega}{\partial K}$. It assimilates the speed v of the electron particle (classically occurring in Newtonian mechanics) to the group velocity v_g of the electron wave packet.

In this essay, we propose to bring together the fields $\vec{\Omega}$ to \vec{B} (both "generalized" rotation vectors in a spatial plane), and the fields \vec{K} to $\vec{E}l$ (both "generalized" rotation vectors in a spatiotemporal plane). Inspired by the ideas of L. de Broglie, we seek to define a group velocity of a moving electric charge (analogous to the group velocity of a moving mass of Newtonian mechanics), and involving the fields \vec{B} and $\vec{E}l$.

We note that we find an equation of this type using the equations of Maxwell Ampère and Maxwell Gauss.

We rewrite Maxwell Ampère (without displacement electric current, and without electric field) in the form:

$$\mu_0 \vec{J}_{xt}^y = \mu_0 q^t \vec{v}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$$

We rewrite Maxwell Gauss (with a single electric field) in the form:

$$\frac{\vec{q}_{xy}^t}{\varepsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x}$$

By dividing Maxwell Ampère by Maxwell Gauss, we find a formula between a speed (here that of electric charges), the field $B_{xy}^{1/t}$ and the field $E_{xt}^{1/y}$, similar to a group velocity:

$$\mu_0 \varepsilon_0 v_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

We have:

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

We get:

$$v_{xt}^y = \frac{1}{c^2} \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

an equation very similar to the group velocity of a wave packet.

Idea 7, source field equations for gravitation, analogous to those of electromagnetism, involving the fields $\vec{\Omega}_{xy}^{1/t}$ and $\vec{K}_{xt}^{1/y}$ (Memoir 3)

We note that from Maxwell's source field equations involving the fields $\vec{E}_{yt}^{1/x}$ and $\vec{B}_{xy}^{1/t}$ it is possible to find an equation similar to the group velocity. We also note that $\vec{\Omega}_{xy}^{1/t}$ and $\vec{K}_{xt}^{1/y}$ intervene themselves in the expression of the group velocity. In this case, one wonders if there are not source field equations for $\vec{\Omega}_{xy}^{1/t}$ and $\vec{K}_{xt}^{1/y}$, similar to Maxwell's source field equations?

Following the reorientation of the electric field $\vec{E}_{yt}^{1/x}$, we can no longer use the Maxwell Gauss equation as it is. Since we have sought to construct an electric force on the model of the Coriolis inertial force or the Lorentz magnetic force, we note that for the two Maxwell's source field equations (Gauss and Ampère), it is necessary to use the Ampère's theorem. Similarly, source field equations involving $\vec{\Omega}_{xy}^{1/t}$ and $\vec{K}_{xt}^{1/y}$, must use Ampère's theorem. Following an analogy between electromagnetism and gravitation, between Time and Space, we substitute the density of electric charge by the density of mass, the density of electric current by the density of momentum.

Idea 8, relate certain source field equations to a quantum number qualified as a source field, to a mediator particle carrying the source field quantum number, to the displacement of a charged fermion and to an interaction (Memoir 5)

We note that the de Broglie source field equation $\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t} \vec{p}^x = \hbar \vec{K}_{ty}^{1/x}$ is associated with the quantization of the orbital angular momentum σ and the spin angular momentum S .

We have for example the following quantification postulated by N. Bohr in his electron model:

$$\sigma = n\hbar = \frac{np}{k}$$

We then propose to associate to the de Broglie source field equation:

- an orbital and spin angular momentum, which will be incorrectly summarized by spin,
- a photon γ carrying integer spin,
- an electrically charged fermion, carrying a half-integer spin which interacts with the photon,
- reactions between particles: the hyperfine transition or the energy transition, i.e., a spin exchange between the photon and the fermion, a disappearance (or appearance) of the photon, a displacement of the electrically charged fermion and an electric current,
- the electromagnetic interaction.

The objective sought throughout Memoir 5 is to associate with three other source field equations, a source field quantum number, a mediating particle, a charge displacement and an interaction.

Idea 9, distinguish physical Time (with 2 senses and similar to a spatial dimension) from felt Time, by the fact that the second is a characteristic of Life (Memoir 6)

We propose to characterize Life, by the fact that it would advance continuously in one of the 4 dimensions of Space-Time, thus creating a distinction between Time (where Life advances continuously) and the 3 dimensions of the Space. Oriented Time, qualified as biological, would contain the precious principle of causality and allow the constitution of a History.

Idea 10, any a priori physical phenomenon with oriented Time (Biological Time) would in fact be a biological phenomenon (Memoir 6)

The Big Bang, whose theory has an oriented Time, would in fact be a biological phenomenon. It would correspond to the beginning of Life, that is to say the moment when Life had the desire to move forward continuously in one direction.

Idea 11, the speed of light always measured constant whatever the movement of the observer, would in fact be a characteristic of the identical advance in Time of human beings living together (Memoir 6)

Synthesis of the 6 Memoirs

The essay was divided into 6 Memoirs, initially for practical reasons, because the equation editor I was using bugged from too many equations. I therefore had to segment the essay into Memoirs, each Memoir evolving little by little and ultimately being able to be linked to a fundamental question.

The **1st Memoir** focuses on one of the main objectives of the work of A. Einstein, that of an invariance of the Laws of Nature whatever the transformation, even if it means adding new terms in the writing of the Law (for example the addition of forces of inertia in the Newton's fundamental principle of dynamics during a transformation of the change of frames of reference type). During the Memoirs, we recall that the original objective of A. Einstein of an invariance of the Laws of Nature, which initially applied to changes of frames of reference, could be extended, in quantum Physics, to so-called Gauge transformations.

The **2nd Memoir** starts from a paradox of Newtonian mechanics underlined in particular by A. Einstein, the existence a priori disturbing and without explanation of 2 types of forces: "normal" forces and inertial forces often qualified as pseudo forces.

Emphasizing the similarities between gravitation and electromagnetism, between Coriolis force and Lorentz magnetic force, we seek in this Memoir to resolve the paradox by proposing that any "normal" force can be reduced to an inertial force. This force of inertia would be added to Newton's fundamental principle of dynamics (Newton's second law), during a transformation of the "generalized" change of reference frame type, that is to say during the cancellation of a "generalized rotation", concept aimed at extending the notion of rotation from a spatial plane to a "spatiotemporal plane".

In the **3rd Memoir**, we are interested in the source field equations of Maxwell: Maxwell Ampère and Maxwell Gauss, in particular the why of a relationship between something specific: the source and something extended: the field. Noting that the relationship between Maxwell Ampère and Maxwell Gauss gives a sort of group velocity $v_g = \frac{d\Omega}{dK}$, we seek the analog for gravitation of Maxwell's source field equations, replacing the magnetic field by a "pulsation field" Ω and the electric field by a "wave vector field" K .

The initial hypotheses of the building are verified, with a keystone which fits perfectly, when the ratio between Ampere gravitation (in fact the Newton's fundamental principle of dynamics) and Gauss gravitation makes it possible to find $v_g = \frac{d\Omega}{dK}$ at the known condition on the Planck length.

The **4th Memoir** is intended as a chronological presentation of 20th century physics, focusing mainly on particle physics, electromagnetic, weak and strong interactions (strong interactions and quarks are also treated in addition to the beginning of the 5th Memoir). In this Memoir, we show that in the second half of the 20th century, the theories on strong and weak interactions mutually and beneficially influenced each other, much like the theories on gravitation, electrostatics and magnetism had done in the first half of the 19th century. Several focuses are also made on the concept of isospin, which surprisingly plays 3 different roles: that of strong nuclear charge, that of weak nuclear charge and that of a (mathematical) analogue of spin.

The **5th Memoir** is again speculative and is perhaps the most fragile in its assumptions and proposals. It begins with a presentation of the theories on the strong interaction set out in the 1950s and 1960s, it can then be seen as a continuation of the 4th Memoir which ended with a presentation of the electroweak model.

Thereafter, the 5th Memoir tries above all to sketch a synthesis of the different interactions. For this, he suggests a bridge between the source field equations proposed in the 3rd Memoir and certain interactions. The search for a synthesis of the interactions goes through the search for analogies between the interactions, first analogies between strong and weak nuclear interactions at short range,

between gravitational and electromagnetic interactions at infinite range, then analogies between all the interactions.

In the 2nd and 3rd Memoirs, we are interested in the analogies between gravitation and electromagnetism. It is emphasized that if the pioneers were mainly inspired by the analogies between gravitation and electromagnetic (or rather electrostatic), subsequently, when more refined theories appeared, the "continuers" rather insisted on the differences between gravitation and electromagnetism. This has also led some physicists, such as A. Einstein, to point out that certain similarities are only superficial, that when you dig deeper, notable and irreducible differences between gravitation and electromagnetics appear. In the 2nd and 3rd Memoirs, on the other hand, we seek to return to the "spirit" of the pioneers, to show that if we push the analogies further, these differences are reduced.

For the strong and weak interactions, the story is comparable: the pioneers did not even see the existence of 2 types of nuclear interactions at the start, then the theories on the strong and weak interactions were often inspired by one and the other, finally, the "continuators" have above all insisted on their differences. To unify weak and strong interactions, we return in the 5th Memoir to the proposals of the pioneers, and we rely, as they had done, on their similarities.

The 6th **Memoir** is interested in the paradox of a "Physical Time" t , which when changing to its complex form it , is treated in the equations of Physics, in the same way as the 3 dimensions of Space, and of a felt Time (or psychological Time), which appears to us to be very different from Space. This last Memoir is again very speculative, the proposals and arguments put forward do not claim to be strictly scientific in the sense of K. Popper, that is to say falsifiable. For example, it is proposed that the concepts of Big Bang, black hole and entropy would be more related to Biology than to Physics...

Contents

Essential point of the essay: source field equations for gravitation analogous to those of electromagnetism	2
Introduction: why are the laws of gravity and electricity so similar?.....	4
Preface to the reader, what can this essay bring him?	7
Explain to my youngest daughter Mycènes (14 years old) the essay Invariances and Transformations	8
Summary of the ideas and conjectures developed in this essay	13
Synthesis of the 6 Memoirs	17

Memoir 1 Changes of reference frames and transformations of Local Gauge

Summary of the memoir

In this 1st memoir, we will be interested in changes of reference frames and inertial forces, as well as local gauge transformations and interaction momentum energies.

The starting point will be the following question of A. Einstein. Why in classical physics are there two types of forces: "normal" forces and inertial forces (sometimes called pseudo forces)? We will try to answer this question by assimilating any force to an inertial force.

The study of inertial forces will also be an opportunity to study Bohr's model of the electron. Model proposed in 1913 by Niels Bohr and which involves centrifugal inertial force and electrostatic force.

Through the notions of Gauge invariances and transformations introduced in 1918 by Henry Weyl, Gauge invariances that can be compared to a conservation of the Laws of Nature, Gauge transformations that can be compared to changes in reference frames, we will study the wave equations of E. Schrödinger (1925), Wolfgang Pauli (1927) and Paul Dirac (1928). Thus, will be approached the main wave functions of quantum physics of the 1920s.

Chapter I Why make any “normal” force correspond to an inertial force?

Subject of the chapter

In Newtonian mechanics, the laws of Nature are functions of the choice of a first reference frame considered as Galilean. However, this choice appears a priori since a Galilean reference frame is defined as a reference frame in which Newton's 1st law is verified. That is to say in which the principle of inertia is verified: tendency of a body to maintain its speed in the absence of external influence. There is therefore a certain circularity in the choice of a Galilean reference frame.

It is this paradox that A. Einstein underlines in his book *Relativity* (1st edition in 1916) about Newtonian mechanics. He indicates that according to the choice of the first Galilean reference frame, the laws of Nature are very different. Indeed, in the Newton's fundamental principle of dynamics (Newton's 2nd law) $m\vec{a} = \Sigma \vec{F}$, it is necessary to add (or not) inertial forces according to the reference frame where one is positioned.

To escape this contradiction, A. Einstein proposes the astonishing objective of matching any “normal” force to an inertial force. We return here to the main stages of his reflection.

I.1 Historical review of the march of ideas, A. Einstein, H. Weyl, Chen Ning Yang and Robert Mill, transformation, invariance, and addition of “quantities of inertia”

I.1.1 Einstein, the great dream of a unified theory of fields and gravitation

In the years 1910-1920, the great dream of A. Einstein is to unify electric and magnetic forces with gravitational force in a broad field theory. In his search for unification, A. Einstein advances some great ideas, setting major principles.

One of the great ideas of A. Einstein is to bring the notion of Space-Time deformations closer to that of Newtonian gravitational potential V^t . In the theory of General Relativity, A. Einstein describes the deformations of Space-Time using a tensor (a sort of 4×4 dimensions matrix) corresponding to the second derivatives of the gravitational potential V^t . He is then inspired by the Poisson potential source equation (applied to gravitation with ρ the mass density): $\nabla^2 V^t = 4\pi G\rho$ to link the Space-Time deformation tensor and the quadrivector energy momentum. He thus proposes an equation applied to the gravitational field.

We will come back to this equation later, when we look at the Big Bang theory in the 6th memoir.

One of the major principles set by A. Einstein is certainly the principle of relativity: the laws of Nature must be respected (i.e., they must be invariant) regardless of the choice of reference frame. To obtain this principle, A. Einstein seeks to bring any “normal” force closer to an inertial force.

For example, he postulates the gravitational field as locally equivalent to an acceleration. Thus, he can interpret the gravitational field as a "quantity of inertia" that is added when passing from an immobile frame to a frame accelerating compared to the first. We will come back to this in a later paragraph.

Following the principle of Relativity, A. Einstein manages in the theory of General Relativity to describe gravitation. However, he does not consider either electromagnetism or nuclear interactions, the first models of which were developed in the 1930s.

I.1.2 H. Weyl, local gauge transformation and electromagnetism

In the 1920s, the mathematician H. Weyl took up the idea of A. Einstein to bring any "normal" force closer to an inertial force to make the great laws of Nature invariant. He is interested in electromagnetism, in the invariance of Maxwell's equations, and imagines the notion of Gauge transformation which leaves Maxwell's equations invariant.

Following the ideas of H. Weyl, a local Gauge transformation is a local transformation of the geometric properties of Space-Time. It can also be interpreted in the broad sense, as a change of reference frames with the cancellation (or addition) of movements.

We have the partial derivative:

$$f(x + dx) - f(x) = \partial^\mu f(x) dx_\mu$$

If the unit of measurement (the Gauge) varies from one point to another, the partial derivative is corrected by a factor S^μ . We then have for the partial derivative after a Gauge transformation:

$$f(x + dx) - f(x) = (\partial^\mu + S^\mu) f(x) dx_\mu + \theta(dx^2)$$

H. Weyl attempts to match S^μ with the electromagnetic potential quadrivector A^μ ($\mu = t, x, y, z$). In 1929, he found a satisfactory solution to this problem, by observing the equivalence between the classical writing $p^\mu - eA^\mu$ and his quantum writing $i(\partial^\mu + ieA^\mu)$.

$$p^\mu - eA^\mu \Leftrightarrow i(\partial^\mu + ieA^\mu)$$

He then brings the factor S^μ closer to the energy momentum quadrivector potential ieA^μ .

This energy momentum quadrivector potential ieA^μ can be interpreted as a “quantity of inertia” that is added during a local Gauge transformation, in order to make the great laws of Nature invariant.

Note, reminders on generalized electromagnetic potential energy

In electromagnetism, we define a generalized electromagnetic potential energy, also called electromagnetic interaction energy:

$$Ep = q(A^t - (\vec{v}^a \cdot \vec{A}^a))$$

$$(a = x, y, z)$$

From this generalized electromagnetic potential energy, using the Euler-Lagrange equation, it is possible to find the electromagnetic force. This will be detailed in memoir 2. In relativistic quantum electrodynamics, the addition of the energy momentum quadrivector potential ieA^μ during a local Gauge transformation, makes it possible to obtain a Lagrangian including the generalized electromagnetic potential energy.

Thus, in a way, we can interpret the electromagnetic force deriving from a “quantity of inertia” ieA^μ that we add during a local Gauge transformation, in order to leave the great laws of Nature invariant. We will come back to this in this memoir and the following ones.

I.1.3 The local gauge theories of C. N. Yang and R. Mill on nuclear interactions

The ideas of H. Weyl are taken up and adapted in 1954 by C. N. Yang and R. Mill in a local gauge theory which describes the strong nuclear interaction between protons and neutrons.

C. N. Yang and R. Mill use Local Gauge transformations that belong to rotation groups $SU(n)$. In their model, following a local gauge transformation of $SU(2)$, interaction energies appear in the Lagrangian, which make it possible to find the strong nuclear interaction. We will come back to this in the 4th Memoir.

In the 1960s, the ideas of C. N. Yang and R. Mill were incorporated into the Standard Model to describe strong and weak nuclear interactions. In a way, once again, forces or interactions (in this case nuclear) are related to "quantities of inertia" that are added during local gauge transformations, in order to conserve the major laws of nature.

After this reminder on the progress of ideas, let us now return to the initial ideas of A. Einstein.

I.2 Why do there exist privileged reference frames (called Galilean) in Newtonian mechanics?

I.2.1 The principle sought by A. Einstein

“All reference bodies, whatever their state of motion, are equivalent for the description of nature (formulation of the general laws of nature).”

“How is it possible that certain reference bodies (or their states of motion) are distinguished from other reference bodies (or their states of motion)? What is the reason for this preference?”

“The objection is especially important when the state of movement of the reference body is such that it does not need any external action for its maintenance, for example in the case where the reference body performs a rotational movement. uniform.”

These are some sentences of A. Einstein taking up certain ideas of Ernst Mach, extracted from his book *Relativity*.

This is what may please (or displease) in the scientific approach of A. Einstein, an approach not initially based on empiricism, but on the great principles that A. Einstein then goes on to relate to observations and experiences.

In the second part of *Relativity* A. Einstein explains why, following a principle of general relativity, he seeks to bring any “normal” force closer to an inertial force.

In Newtonian mechanics, when passing from a Galilean or inertial reference frame (in uniform rectilinear motion relative to a first Galilean reference frame), to a non-Galilean reference frame, for example in rotation $\vec{\Omega}$ relative to the first Galilean reference frame (change of frames similar to the cancellation of a movement: the rotation vector $\vec{\Omega}$), it is necessary to add inertial forces in the Newton's fundamental principle of dynamics.

These inertial forces may appear artificial compared to "normal" forces such as gravitational, electrical or other forces. Especially when we note that the choice of the first Galilean reference frame, the one in relation to which we define all the other reference frames, is an a priori choice.

After all, why not choose as the first reference frame, the one in rotation? In this case, we could remove the inertial forces that we add when we pass to this reference frame.

The fundamental question is therefore the following: why this difference in the formulation of physical laws because the choice of a first Galilean reference frame is an a priori choice?

Note on the definition of a Galilean reference frame

A Galilean frame is usually defined as a frame in which the principle of inertia is verified. Some physicists also have the habit of defining a Galilean reference frame as a reference frame in which the Newton's fundamental principle of dynamics applies, without having to add inertial forces.

I.2.2 Matching any “normal” force to an inertial force

To preserve the invariance of the general laws of Nature, A. Einstein has a most original idea. He seeks to show that any "normal" force can be matched to an inertial force. Like an inertial force, a "normal" force would be introduced into the Newton's fundamental principle of dynamics, when a change of reference frames is carried out (change corresponding most often to the cancellation of a movement). That is to say, any force, whether inertial or "normal", would be introduced into the Newton's fundamental principle of dynamics, in order *to correct the cancellation of a movement*.

Thus, Einstein compensates for the a priori choice of the first Galilean reference frame and the addition of inertial forces, which results therefrom for non-Galilean reference frames, by the fact that

any force to be introduced into the Newton's fundamental principle of dynamics (whether the referential is Galilean or not) is itself brought closer to an inertial force.

At the same time, A. Einstein cancels the artificial distinction between “normal” forces and inertial forces.

I.2.3 The case of the gravitational field, equality between heavy mass and inertial mass

In his reflection, A. Einstein first studies the gravitational field. He is interested in the equality between heavy mass and inertial mass.

The heavy mass is involved in the expression of the gravitational force: the greater is the heavy mass of a body, the more the body will undergo a gravitational force. Inertial mass is associated with acceleration in Newton's fundamental principle of dynamics: the greater is the inertial mass of a body, the more its acceleration will have a significant resistance against the forces that apply to it.

A priori, these two masses do not designate the same physical notion. However, they are always equalized in physical equations. We can then obtain a gravitational field equal to an acceleration.

In a way, with the a priori choice of a Galilean reference frame:

- describing an accelerating body subjected to a gravitational field,
- including in the Newton's fundamental principle of dynamics no acceleration of inertia,

A. Einstein substitutes an equally a priori choice of a non-Galilean reference frame:

- describing a stationary body (in zero acceleration) and subjected to a gravitational field,
- including in the Newton's fundamental principle of dynamics an acceleration of inertia locally equivalent to a gravitational field.

Whatever the choice of reference frame, the same thing must ultimately be considered: an acceleration or a gravitational field locally equivalent to an acceleration of inertia.

Subsequently, the objective of A. Einstein is to seek the changes of reference frames corresponding to the forces other than gravitational, in particular the forces of the electrical type. The concern is that the previous reasoning is not directly transposable from gravitation to electricity. Indeed, in an electric field, the acceleration of an electrically charged body is not equal to the electric field but also depends on its mass electric charge. There are therefore no changes of trivial reference frames, which make locally correspond electric field and acceleration (or even another movement).

I.3 Analogy Time and Space, can we treat Time in the same way as Space?

The idea of treating the Time dimension identically to the three dimensions of Space is not a priori linked to the notion of force of inertia and change of reference frames. It nevertheless seems interesting to mention it here, because the notion of change from Galilean reference frame to rotating reference frame will be in the next expanded memory by bringing together the notions of Time and Space.

In the writings of A. Einstein on Relativity, this identical treatment of Time and Space is a recurring idea. We observe it for example in the proposal of a continuum of Space-Time or in the deformations of distances in the image of those of durations.

This conception of Time and Space is moreover one of the great upheavals in 20th century physics. Despite the difficulty of intuitively accepting a resemblance between Time and Space and finding reasons for it beyond a simplifying desire for models of the Universe, A. Einstein and many others found that the analogy between the Time and Space held great promise in understanding the Universe.

In a letter from A. Einstein, we note the reading during his young years, of works by H. Poincaré. Indeed, we owe to H. Poincaré the latter the first analyses presented of a resemblance between Space and Time. It was while studying problems of symmetry and translation that H. Poincaré realized that positions and instants could play interchangeable roles in the observation of the movement of a body A relative to an experimenter E.

Let us take the example of a car A in uniform rectilinear motion. If we modify the position of observation of the experimenter E, the car A is seen in another configuration, it is the principle of relativity applied to Space.

If we modify the instants of the experimenter E, for example later instants, the car A is also seen in another configuration, it is the principle of relativity applied to Time.

If we modify the position of E, it is possible to modify the instants of E, earlier or later, so that car A is always observed in the same configuration. Thus, the translation of the instants of E comes to compensate that of the positions of E, and makes it possible to observe an identical movement.

Note: to distinguish the concept of Space x, y, z from a mathematical space with three dimensions and which can include the dimension Time, for example t, x, y we will use in these memoirs a capital letter for the first (Space x, y, z), and a lower case for the second (space t, x, y). We will do the same for Time t , where we will use a capital letter.

I.4 Conclusion of the chapter

In summary, it is to respond to a principle of general relativity, that is to say to an invariance in the formulation of the general laws of Nature, that in order to compensate for the a priori choice of a first Galilean reference frame, we seek here to assimilate any “normal” force to an inertial force to be added during a change of reference frames from Galilean to non-Galilean.

Chapter II Reminders on the inertial forces in Newtonian mechanics

Subject of the chapter

We first discuss the objectives set by A. Einstein for the theories of Special Relativity and General Relativity.

We then remind the forces of inertia used in so-called “general” Newtonian mechanics: centrifugal force, Coriolis force and mass times acceleration. We will focus on their main characteristics in order to find methods that will make it possible to match any “normal” force to an inertial force.

II.1 From the objectives of Special Relativity to those of General Relativity

II.1.1 On the Galilean transformation and on the Lorentz transformation, the case of Special Relativity

The Galilean transformation makes it possible to pass from a Galilean reference frame R^0 (or inertial) to another Galilean reference frame $R^{0'}$ (also inertial). The transition from R^0 to $R^{0'}$ leaves the Space interval invariant:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$

In these 2 Galilean reference frames R^0 and $R^{0'}$, the fundamental principle of dynamics is respected without there being any inertial force to add when changing reference frames.

We give the Galilean transform:

$$\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}$$

The Lorentz transform proposed by H. Lorentz (in 1904) and corrected by Henry Poincaré (in 1905) notably leaves invariant the d'Alembert wave equation $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, Maxwell's equations or the Space-Time interval:

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c\Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c\Delta t'^2$$

Following the ideas of A. Einstein exposed in the theory of special relativity, the Lorentz transform corresponds to the passage in Space-Time from a Galilean (or inertial) frame R^0 to another Galilean (or inertial) frame $R^{0'}$. Whether you are in R^0 or $R^{0'}$, it allows both:

- to measure the same speed of light c in vacuum,
- to leave invariant all the laws of Nature, in particular the fundamental principle of dynamics without there being forces of inertia to add to it.

We have the Lorentz transform:

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \\ \text{with } \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}$$

We note that when $v \ll c$, $\gamma \approx 1$, we find the Galilean transform from the Lorentz transform.

II.1.2 On the objectives set by the theories of special relativity and general relativity

Special relativity stops at the changes of Galilean reference frames from R^0 to $R^{0'}$. The Newton's fundamental principle of dynamics must remain invariant during a change of reference frames, without having to add inertial forces.

A. Einstein sets much more ambitious goals for the theory of general relativity. This one studies the changes of reference frames from Galilean to non-Galilean from R^0 to R' . In so-called "general" Newtonian mechanics, the Newton's fundamental principle of dynamics must remain invariant during a change of reference frames from R^0 to R' , even if it means adding inertial forces.

Note, main difference between Newtonian and Einsteinian mechanics

In Newtonian mechanics, we therefore find these notions of restricted and general by the addition or not of inertial forces in the Newton's fundamental principle of dynamics.

The main difference between Newtonian mechanics and Einsteinian mechanics is therefore not to be found between restricted and general, but in an extension from Space to Space-Time. In so-called "restricted" Newtonian mechanics, it is the Space interval $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$ which is preserved during the transition from R^0 to R^0' , both Galilean references.

In so-called "restricted" Einsteinian mechanics, it is the Space-Time interval $s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c\Delta t^2$ which is preserved when R^0 to R^0' , both also Galilean frames.

II.2 Reminder of some invariants by the Lorentz transform

We remind here some equations invariant by the Lorentz transform, and therefore following the ideas of A. Einstein, invariants by change of Galilean (or inertial) reference frames in the context of Special Relativity.

II.2.1 The Space-Time Interval

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c\Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c\Delta t'^2$$

Note 1 on the concept of quadrivector

On the model of the Space-Time interval, in Special Relativity, quadrivectors are built:

- position (\vec{r}, ct)
- speed $\gamma(\vec{v}, c)$
- acceleration $(\gamma \frac{d}{dt}(\gamma\vec{v}), \frac{d}{dt}(\gamma c))$
- energy momentum $(\vec{p} = \gamma m\vec{v}, \frac{E^t}{c} = \frac{\gamma mc^2}{c})$
- force $\gamma(\vec{f}, \frac{1}{c} \frac{dE^t}{dt})$ or $\gamma(\vec{f}, \frac{\vec{f} \cdot \vec{v}}{c})$
- electromagnetic potential $(\vec{A}, \frac{A^t}{c})$
- electric current density $(\vec{j}, \rho^t c)$
- wave vector $(\vec{k}, \frac{\omega^t}{c}) = \frac{1}{h}(\vec{p}, \frac{E^t}{c})$

whose norm is invariant by the Lorentz transform:

$$\bar{X}^2 = x_1^2 + x_2^2 + x_3^2 - x_4^2 = x_1'^2 + x_2'^2 + x_3'^2 - x_4'^2$$

These quadrivectors are conserved by changing Galilean (or inertial) reference frames in the context of Special Relativity.

Note 2 on a quadrivector electromagnetic field?

The quantities $\vec{E}\vec{l} \cdot \vec{B} = \vec{E}\vec{l}' \cdot \vec{B}'$ and $\frac{1}{c^2} \vec{E}\vec{l} \cdot \vec{E}\vec{l} - \vec{B} \cdot \vec{B} = \frac{1}{c^2} \vec{E}\vec{l}' \cdot \vec{E}\vec{l}' - \vec{B}' \cdot \vec{B}'$ are also invariant by Lorentz transform. We could therefore imagine an electromagnetic field quadrivector of the type:

$$\left(\frac{\vec{E}\vec{l}}{c}, B \right)$$

Nevertheless, the nature of B , here scalar, remains unclear and in Special Relativity we rather define an electromagnetic field tensor (or Maxwell Faraday tensor). $\vec{\vec{F}}_{\mu\nu}$

II.2.2 The d'Alembert wave equation

$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

By Lorentz transform, we also have:

$$\frac{1}{c^2} \frac{\partial^2 u'}{\partial t'^2} = \frac{\partial^2 u'}{\partial x'^2}$$

Note

u can be a scalar as in the case of the d'Alembert wave equation. It can also be more generally an electric field, a magnetic field, a scalar, or vector potential. It then checks the wave equation:

$$\frac{1}{c^2} \frac{\partial^2 F}{\partial t^2} = \Delta F$$

II.2.3 The Newton's fundamental principle of dynamics

We recall the fundamental principle of dynamics, in restricted Newtonian mechanics, with conservation of this principle by the Galileo transform, and therefore valid in all Galilean references within the framework of Newtonian mechanics:

$$\frac{dm\vec{v}}{dt} = \vec{F}$$

We recall the fundamental principle of dynamics, in restricted Einsteinian mechanics (extension of Space to Space-Time and therefore presented in the form of a quadrivector), with conservation of this principle by the Lorentz transform, and therefore valid in all Galilean (or inertial) reference frames:

$$\gamma \left[\frac{d\gamma m_0 \vec{v}}{dt}, \frac{d\gamma m_0 c}{dt} \right] = \gamma \left[\vec{F}, \frac{1}{c} \frac{dE}{dt} \right]$$

with $\gamma = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}}$ and m_0 the resting mass of the body studied by the observer.

This last principle summarizes both the fundamental principle of dynamics in Space: $\frac{d\gamma m_0 \vec{v}}{dt} = \vec{F}$ and the expression of energy in Time: $E = \gamma m_0 c$.

II.3 General information on inertial forces in so-called "general" Newtonian mechanics

II.3.1 Two strategies

In Newtonian mechanics, a physicist has two a priori equivalent strategies for applying the Newton's fundamental principle of dynamics.

Let be the **1st strategy** (case of Newtonian mechanics called "restricted"), he applies this principle in a Galilean reference frame. We put below on the left the terms related to the movement and on the right the terms of type force R^0 :

$$[m(\vec{a})]_{R^0} = \Sigma[\vec{F}]_{R^0}$$

Acceleration \vec{a} can for example decompose into a relative acceleration \vec{a}_r and acceleration of drive (or inertia) \vec{a}_i . We have:

$$[m(\vec{a})]_{R^0} = [m(\vec{a}_r + \vec{a}_i)]_{R^0} = \Sigma[\vec{F}]_{R^0}$$

Let be the **2nd strategy** (case of Newtonian mechanics called "general"), he applies this principle in a non-Galilean reference frame R' . It is then necessary to add inertial forces $[\vec{F}_i]_{R'}$.

For instance, if \vec{a}_r is the remaining acceleration in the reference frame R' , the inertial force to be added will be:

$$[\vec{F}_i]_{R'} = -[m\vec{a}_i]_{R'}$$

The fundamental principle of dynamics is applied in R' :

$$[m\vec{a}_r]_{R'} = [\vec{F}_i]_{R'} + \Sigma[\vec{F}]_{R'}$$

with $[\vec{F}_i]_{R'} = -[m\vec{a}_i]_{R^0}$ and $\Sigma[\vec{F}]_{R'} = \Sigma[\vec{F}]_{R^0}$ (the sum of the forces is considered to remain the same when changing reference frames).

If the body studied is immobile in R' , $\vec{a}_r = \vec{0}$. We apply the fundamental principle of dynamics, with on the left the zero movement and on the right the terms of type force:

$$\vec{0} = [\vec{F}_i]_{R'} + \Sigma[\vec{F}]_{R'}$$

Like two communicating vessels, the movement is cancelled on the left, and is replaced by forces of inertia on the right.

II.3.2 Three types of inertial force in Newtonian mechanics

Three types of inertial forces can be distinguished in so-called "general" Newtonian mechanics.

The first two meet in rotational movements. These are the centrifugal force and the Coriolis force. They are added to the Newton's fundamental principle of dynamics when moving from a Galilean reference frame R^0 to a reference frame R^Ω where we cancel the rotation vector $\vec{\Omega}$.

The third inertial force is of a different nature and is not always interpreted as an inertial force. It is the mass once the acceleration of inertia that is added during a change of reference frames where we cancel the acceleration (or part of the acceleration) and which according to the principle of equivalence of A. Einstein is locally equivalent to a gravitational force.

Remarks

A1. Note that it is implicitly assumed that mass is a quantity that is conserved during changes of reference frames.

R2. For the three forces, we talk about inertial forces, but we can just as easily talk about inertial acceleration. The mass m appears as a quantity apart, because preserved in the changes of references.

II.4 Local equivalence between a gravitational field and an acceleration

II.4.1 Elevator Thought Experiment

In 1907, A. Einstein had in his own words "the happiest idea of all" his "life". Germinates in him, the idea of a local equivalence between an accelerated motion and a gravitational field. A. Einstein calls this equivalence the principle of equivalence, a principle implicitly present in the work of G. Galileo and I. Newton.

In the article published in 1907 *On the principle of relativity and the consequences drawn from it*, A. Einstein developed this principle of equivalence which would lead him a few years later to the theory of General Relativity.

In the book *Relativity*, to explain this principle, A. Einstein takes the example of an elevator immersed in a vacuum and carrying an experimenter E. A man outside the elevator pulls with constant force using a rope attached to the elevator, communicating to it a uniformly accelerated movement.

Experimenter E drops objects of different mass, these take exactly the same time to reach the floor of the elevator (experimenter E repeats the experiment of G. Galileo from the top of the Tower of Pisa). Experimenter E knows the mechanics of I. Newton, and like him, he can deduce the existence of a gravitational field. Indeed, for experimenter E in the elevator, it is impossible to know:

- if he is in a uniform gravitational field,
- or if he undergoes a uniformly accelerated movement (movement that can be cancelled by an adequate change of reference frames).

II.4.2 Equations of the elevator thought experiment

Let be R^0 , the reference frame where E is immobile and which he considers to be Galilean. In R^0 , we apply the Newton's fundamental principle of dynamics for a body subjected to a gravitational force (for example one of the objects that the experimenter drops):

$$m[\vec{a}]_{R^0} = m[\vec{G}r]_{R^0}$$

Now let's show that $\vec{G}r$ corresponds well to the acceleration of inertia $[\vec{a}_i]_{R^a}$ that is added in the fundamental principle of dynamics, when switching to the non-Galilean reference frame R^a where we cancel the acceleration \vec{a} .

We apply in R^a the Newton's fundamental principle of dynamics (on the left terms related to the movement $\vec{0} = m[\vec{a}]_{R^a}$, on the right the terms of type force $m[\vec{G}r']_{R^a} - m[\vec{a}_i]_{R^a}$):

$$\begin{aligned} \vec{0} &= m[\vec{a}]_{R^a} = m[\vec{G}r']_{R^a} - m[\vec{a}_i]_{R^a} \\ \vec{0} &= [\vec{G}r']_{R^a} - [\vec{a}_i]_{R^a} \end{aligned}$$

with $\vec{G}r'$ the transformed of $\vec{G}r$ when passing from R^0 to R^a .

Like two communicating vessels, the movement is cancelled on the left, and is replaced by an inertial force $[\vec{F}_i]_{R^a} = -m[\vec{a}_i]_{R^a}$ on right.

We obtain for the terms on the right:

$$[\vec{G}r']_{R^a} = [\vec{a}_i]_{R^a}$$

It is assumed that the mass of the object and the gravitational field are invariant during the passage from R^0 to R^a . So, we have:

$$[\vec{G}r]_{R^0} = [\vec{G}r']_{R^a} = [\vec{a}_i]_{R^a}$$

In R^a , the gravitational field is therefore locally equivalent to the acceleration of inertia that is added in the Newton's fundamental principle of dynamics, in order to correct the cancellation of a motion.

This principle of local equivalence calls for the following **remarks**.

R1 on the causes of accelerated motion and on the equation $\vec{G}r = \vec{a}$

If a gravitational field always implies an acceleration, the cause of an acceleration is not always a gravitational field, it can be for example an acceleration of centrifugal inertia or an electromagnetic field.

Thus, the equation $\vec{G}r = \vec{a}$ may appear misleading, because if the gravitational field $\vec{G}r$ can always be equated with acceleration \vec{a} , \vec{a} cannot always be equated with $\vec{G}r$.

We realize that this idea, despite its interest, has some weaknesses, including the evacuation of the electromagnetic field.

A. Einstein was obviously aware of this and attempts to reintroduce him into a global field theory concerned him all his life.

R2 on the objective of this memoir and the next

In the Newton's fundamental principle of dynamics, there is a correspondence between:

- the cancellation to the left of a movement,
- the addition to the right of an inertial force.

A. Einstein notes that:

- the cancellation on the left of a movement of uniform acceleration type,
- involves the addition to the right of an inertial force locally equivalent to the gravitational force.

Hence his idea of matching any "normal" force to a inertial force that is added when cancelling a movement.

This is also the objective followed in this memoir and in the next one. It will be proposed to match:

- the cancellation on the left of a "generalized movement",
- the addition to the right of any "normal" force or inertia.

II.5 Examination of centrifugal and Coriolis inertial forces

II.5.1 Mathematical reminders on centrifugal and Coriolis inertial forces

In 1835, Gustave Coriolis published an article entitled *On the equations of relative motion of body systems*. Using an energetic approach, he develops the notion of compound centrifugal forces. The latter will take the name of Coriolis force.

To describe the inertial forces of centrifugal and Coriolis, let us now remind the kinematic approach, an approach today the most commonly used.

Let be a body A on a rotating carousel with rotation vector $\vec{\Omega}$ and possessing on this carousel a relative speed \vec{v}_r .

In the reference frame R^0 (considered as Galilean), we have the speed of the body A:

$$[\vec{v}]_{R^0} = \vec{v}_r + \vec{\Omega} \wedge \vec{OM}$$

In the reference frame R^Ω where we cancel the rotation vector $\vec{\Omega}$ (reference frame R^Ω itself in rotation relative to R^0), we have the speed of the body A:

$$[\vec{v}]_{R^\Omega} = \vec{v}_r$$

Note

In the reference frame $R^{\Omega \times v_r}$, where we cancel at once $\vec{\Omega}$ and \vec{v}_r , the body A has a null speed:

$$[\vec{v}]_{R^{\Omega \times v_r}} = \vec{0}$$

In the reference frame R^0 , we have the acceleration of the body A:

$$[\vec{a}]_{R^0} = \left[\frac{d\vec{v}}{dt} \right]_{R^0} = \left[\frac{d\vec{v}_r}{dt} \right]_{R^0} + \vec{\Omega} \wedge \left[\frac{d\vec{OM}}{dt} \right]_{R^0} + \left[\frac{d\vec{\Omega}}{dt} \right]_{R^0} \wedge \vec{OM}$$

We have:

$$\left[\frac{d\vec{v}_r}{dt} \right]_{R^0} = \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \vec{\Omega} \wedge \vec{v}_r$$

And:

$$\left[\frac{d\vec{O}M}{dt} \right]_{R^0} = [\vec{v}]_{R^0} = \vec{v}_r + \vec{\Omega} \wedge \vec{O}M$$

We obtain the acceleration of the body A in R^0 :

$$\begin{aligned} [\vec{a}]_{R^0} &= \left[\frac{d\vec{v}_r}{dt} \right]_{R^0} + \vec{\Omega} \wedge \vec{v}_r + \vec{\Omega} \wedge (\vec{v}_r + \vec{\Omega} \wedge \vec{O}M) + \left[\frac{d\vec{\Omega}}{dt} \right]_{R^0} \wedge \vec{O}M \\ [\vec{a}]_{R^0} &= \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \left[\frac{d\vec{\Omega}}{dt} \right]_{R^0} \wedge \vec{O}M + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) + 2\vec{\Omega} \wedge \vec{v}_r \end{aligned}$$

Let us place ourselves in the special case of a uniform circular movement $\left[\frac{d\vec{\Omega}}{dt} \right]_{R^0} = 0$, we have:

$$\begin{aligned} [\vec{a}]_{R^0} &= \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) + 2\vec{\Omega} \wedge \vec{v}_r \\ [\vec{a}]_{R^0} &= [\vec{a}_r]_{R^\Omega} + [\vec{a}_e]_{R^0} + [\vec{a}_c]_{R^0} \end{aligned}$$

$\left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} = [\vec{a}_r]_{R^\Omega}$ is the relative acceleration of the body A in the reference frame R^Ω .

$\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) = [\vec{a}_e]_{R^0}$ is the drive acceleration, which multiplied by a mass corresponds to the centrifugal inertial force.

$2\vec{\Omega} \wedge \vec{v}_r = [\vec{a}_c]_{R^0}$ is the Coriolis acceleration (or complementary acceleration), which multiplied by a mass corresponds to the Coriolis inertial force.

We have the following two inertial forces to add in R^Ω to apply the fundamental principle of dynamics:

- the centrifugal inertial force

$$\begin{aligned} \vec{F}_{cent} &= -m\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) \\ \vec{F}_{cent} &= mr\Omega^2\vec{n} \end{aligned}$$

- the Coriolis inertial force

$$\vec{F}_{cor} = -m2\vec{\Omega} \wedge \vec{v}_r = m\vec{v}_r \wedge 2\vec{\Omega} = \vec{p}_r \wedge 2\vec{\Omega}$$

Remarks

R1 Note that the Coriolis acceleration is composed of two distinct terms:

- the derivative of speed $\left[\frac{d\vec{O}M}{dt} \right]_{R^0}$ multipliée par la rotation $\vec{\Omega}$,
- the derivative of $\left[\frac{d\vec{v}_r}{dt} \right]_{R^0} = \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \vec{\Omega} \wedge \vec{v}_r$.

R2 When the rotation vector $\vec{\Omega}$ tends to 0, we find the same derivative of the relative speed:

$$\begin{aligned}\left[\frac{d\vec{v}_r}{dt}\right]_{R^0} &= \left[\frac{d\vec{v}_r}{dt}\right]_{R^\Omega} + \vec{\Omega} \wedge \vec{v}_r \\ \left[\frac{d\vec{v}_r}{dt}\right]_{R^0} &= \left[\frac{d\vec{v}_r}{dt}\right]_{R^\Omega} + 0\end{aligned}$$

II.5.2 Potential energy of centrifugal and Coriolis inertial forces

We remind the elementary work of a force \vec{F} :

$$\delta W = \vec{F} \cdot d\vec{l}$$

The total work along a trajectory C is equal to:

$$W = \int_C \vec{F} \cdot d\vec{l}$$

When the total work of a force is independent of the trajectory followed by the particle, the force is said to be conservative. We define a potential energy Ep (to within a constant):

$$Ep = - \int \vec{F} \cdot d\vec{l}$$

The force is then the gradient (the derivative relative to Space), of the potential energy:

$$\vec{F} = -gr\vec{a}d(Ep)$$

We then have:

$$r\vec{\partial}t\vec{F} = \vec{0}$$

$$\text{because } r\vec{\partial}t(gr\vec{a}d) = \vec{0}$$

Note

Gravitational force and electrostatic force are conservative forces. The Lorentz magnetic force is not a conservative force. Nevertheless, it can be found from a generalized electromagnetic potential: $A^t - \vec{A}^a \cdot \vec{v}$ ($a = x, y, z$) via the Euler-Lagrange equation.

Let's calculate the potential energy of the centrifugal inertial force in the reference frame R^Ω :

$$[\vec{F}_{cent}]_{R^\Omega} = mr\Omega^2\vec{n}$$

$$[Ep(\vec{F}_{cent})]_{R^\Omega} = - \int \vec{F}_{cent} \cdot d\vec{l}$$

$$[Ep(\vec{F}_{cent})]_{R^\Omega} = - \int mr\Omega^2\vec{n} \cdot d\vec{r} = - \int mr\Omega^2 dr$$

$$[Ep(\vec{F}_{cent})]_{R^\Omega} = -\frac{1}{2}mr^2\Omega^2 + cst$$

We have the drive speed in R^0 :

$$v_e = r\Omega$$

We have kinetic energy in R^0 due to the rotation vector Ω :

$$[Ec(\Omega)]_{R^0} = \frac{1}{2}mr^2\Omega^2$$

So, we have (to within a constant):

$$[Ec(\Omega)]_{R^0} = -[Ep(\vec{F}_{cent})]_{R^0}$$

We calculate the work of the Coriolis force in the reference frame R^Ω :

$$[\delta W(\vec{F}_{cor})]_{R^\Omega} = \vec{F}_{cor} \cdot d\vec{l} = 0$$

We have indeed a Coriolis force perpendicular to \vec{v}_r , and therefore to $d\vec{l}$.

We have for the potential energy of the Coriolis force:

$$[Ep(\vec{F}_{cor})]_{R^\Omega} = cst$$

We will usually take a null constant.

Remarks

R1 on the potential term of potential energy

The term potential here seems particularly appropriate. Indeed, the potential energy appears after a change of reference frames where we cancel a movement (rotation vector $\vec{\Omega}$) and therefore kinetic energy. This energy is potentially available and transformable into kinetic energy if the reverse reference frame change is made.

R2 How to understand intuitively (without calculations) that certain forces and accelerations are perpendicular to the velocities from which they derive?

The Coriolis force and the so-called normal acceleration are both perpendicular to velocities.

In the case of a rotational movement of linear velocity $\vec{v} \begin{bmatrix} \cos \omega t \\ \sin \omega t \end{bmatrix}$, we have a derivative $\frac{d\vec{v}}{dt} \begin{bmatrix} -\omega \sin \omega t \\ \omega \cos \omega t \end{bmatrix}$ perpendicular to velocity \vec{v} .

This is because we are interested in rotational movements, that certain forces and accelerations are perpendicular to the speeds from which they derive.

II.6 Conclusion of the chapter

In so-called "general" Newtonian mechanics, we correct the cancellation of part of the movement of the reference body during a change of Galilean to non-Galilean reference frames, by adding in the Newton's fundamental principle of dynamics some forces of inertia.

We want to generalize this to "normal" forces. The concern is that forces and movements are not totally equivalent. That is to say, the passage from one to the other is not a priori so easy, since masses and electric charges intervene.

Except as A. Einstein points out for the gravitational field locally equivalent to an acceleration and therefore to a motion.

In the next memoir, we will focus on matching the cancellation of "generalized movements" (left) to the addition of "normal" forces (right). But in the immediate future, let's study the models of the electron of Bohr, de Broglie and Schrödinger.

Chapter III Bohr, de Broglie and Schrödinger electron models

Subject of the chapter

As part of our study of inertial forces, we are now interested in Bohr, de Broglie and Schrödinger electron models that involve electrostatic force and centrifugal inertial force. It will also be an opportunity to return to the quantum and wave Physics of the 1920s.

III.1 The mysterious spectral lines of hydrogen

Spectroscopy studies the light spectra of a physical phenomenon. During the 19th century, this discipline experienced a spectacular development under the impetus of Gustav Kirchhoff and Robert Wilhelm Bunsen. It is observed that in its normal state, matter does not emit any radiation, but once excited (excitation which consists of an energy input), it can re-emit energy in the form of light radiation.

For example, experimenters fill capillary tubes with hydrogen gas and excite it with a difference in electrical potentials applied to both ends of the tube. They study the spectrum of light re-emitted by hydrogen. To their surprise, they observe spectral lines. That is, the spectrum of light is discontinuous and only a few wavelengths are re-emitted.

The figure below gives part of the light spectrum of hydrogen, with the so-called Balmer series in the visible, the so-called Lyman series in the invisible ultraviolet, and the so-called Paschen series in the invisible ultraviolet.

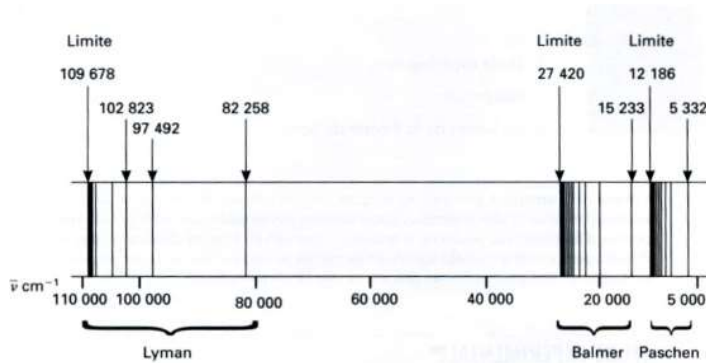


Figure 1: part of the light spectrum of hydrogen gas

Nota

$\bar{\nu}$ is the wavenumber, inverse of the wavelength, with $\bar{\nu} = \frac{1}{\lambda}$. It expresses itself in m^{-1} .

$K = \frac{2\pi}{\lambda}$ is the angular wavenumber. It expresses itself in $rad.m^{-1}$.

In this essay, the term wave vector will be used to designate K .

Following these observations, physicists are looking for an empirical relationship between the characteristics of the different spectral lines. It was Johann Jakob Balmer in 1885 who first established the relationship:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{m^2} \right)$$

with R_H the Rydberg constant of hydrogen, λ the wavelength of the re-emitted light, and m an integer.

Balmer's relationship is then generalized by Johannes Rydberg and Walther Ritz:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

n and m are integers with $n < m$.

III.2 Ernest Rutherford's Planetary Model of the Atom (1911)

In 1909, E. Rutherford, assisted by Hans Geiger and Ernest Marsden, conducted experiments on the bombardment of gold leaf by particles. α (helium nuclei He^{2+}). The 3 experimenters find that most of the particles α pass through the gold leaf. Nevertheless, a few, in a very small proportion, are deflected along very wide angles.

Following these experiments, E. Rutherford proposed in 1911 the Rutherford atom model comparable to a small planetary system. In the center, a dense and positively charged nucleus, at the origin of the deviation following very wide angles of some particles α . Around, especially the vacuum that does not deflect the particles α , as well as negatively charged electrons that orbit on circular or elliptical trajectories.

Rutherford's planetary model has a precursor: the Saturnian model proposed by Hantarō Nagaoka a few years earlier in 1904. These two models, however, clash with the theory of the radiation of the electron in accelerated motion (especially in rotation).

Indeed, according to experimental observations and Maxwell's equations, an electron subjected to acceleration emits energy in the form of electromagnetic waves (this is the same principle as a transmitting radio antenna).

In 1897, Joseph Larmor establishes the following relationship:

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

with P radiated power and a the acceleration of the electric charge.

According to this relationship, the electron orbiting the nucleus must lose energy by radiating electromagnetic waves, and thus eventually crash into the nucleus.

In contradiction with this relationship, one of the fundamental ideas of quantum physics will be to propose that the electron, even in (accelerated) rotational motion, does not radiate energy continuously. It is only during the passage from one rotational movement to another (change of orbits), that the electron radiates an electromagnetic wave. Outside, the electron is said to be in a steady or permanent state.

Note

In quantum physics, the steady state of the electron and the rotational motion are therefore brought together. Let us emphasize here the link with the sentence of A. Einstein previously quoted: "The objection is especially important when the state of movement of the reference body is such that it does not need for its maintenance any external action, for example in the case where the reference body performs a uniform rotational movement."

III.3 Model of the Bohr electron in the hydrogen atom

In 1913, Niels Bohr published an article entitled *On the Constitution of Atoms and Molecules* in which he brought together three a priori different fields of physics. It first takes up Rutherford's planetary model. It then uses the Planck constant h and its link with the kinetic moment proposed by Max Planck in 1900. Finally, he finds the Rydberg constant R_H and explains (in part) the mysterious spectral lines of hydrogen.

III.3.1 Bb Explain the steady state of the electron based on the planetary model and Newtonian gravitation

In his paper, N. Bohr seeks to understand why the electron is in a stable (or permanent to use its term) state. Why in Rutherford's model does not fall on the proton? To answer this, he is inspired like E. Rutherford by the planetary model, as well as Newtonian gravitation.

Let's take the example of the Moon. It does not fall on the Earth because the attractive gravitational force is compensated by the centrifugal effect of the rotational motion of the Moon around the Earth. In the case of the electron model proposed by N. Bohr, the centripetal effect of the attractive electrostatic force of the proton on the electron is compensated by the centrifugal effect due to the rotational motion of the electron around the proton.

The attractive electrostatic force is given by Coulomb's law:

$$F_{Es} = -\frac{e^2}{4\pi\epsilon_0 r^2}$$

In the case of a circular motion of the electron around the proton, we have:

$$\vec{v} = \vec{\Omega} \wedge \vec{r}$$

By deriving, we obtain the acceleration which has a normal component with respect to the speed:

$$\frac{\partial v}{\partial t} = -\frac{v^2}{r}$$

The centrifugal effect of the rotational motion is given by:

$$m \frac{\partial v}{\partial t} = -m \frac{v^2}{r}$$

The application of the fundamental principle of dynamics in the Galilean reference frame R^0 to an electron rotating around the proton and subjected to an electrostatic force, is written:

$$m \frac{\partial v}{\partial t} = F_{Es}$$

We obtain:

$$m \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

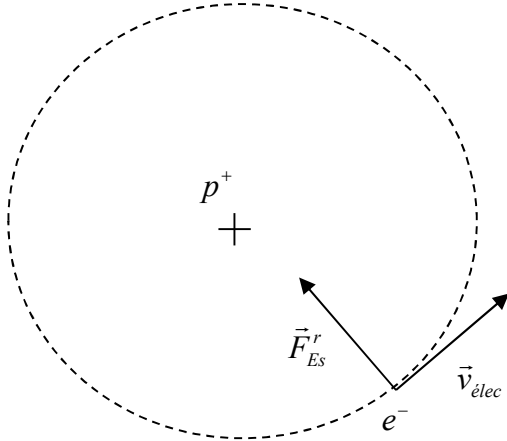


Figure 2: electron "stabilized" by an electrostatic force and by a rotational movement around the proton (the index r for \vec{F}_{Es}^r indicates that the electrostatic force is radial in the model).

III.3.2 Mechanical energy in R^0

We have the electrical potential energy in R^0 :

$$Ep(F_{Es}) = -\frac{e^2}{4\pi\epsilon_0 r}$$

Using $m\frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$, one obtains for kinetic energy in R^0 :

$$Ec = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r} = -\frac{1}{2}Ep(F_{Es})$$

We have mechanical energy E in R^0 :

$$E = Ec + Ep(F_{Es}) = -Ec = -\frac{1}{2}mv^2 = \frac{1}{2}Ep(F_{Es}) = -\frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r}$$

III.3.3 Centrifugal inertial force in R^Ω

In Newtonian theory and Bohr's model, the Moon and the electron are respectively "stabilized" by an attractive force and by their rotational motion. Let us show that they can also be considered immobile and stabilized by an attractive force and a centrifugal inertial force.

We place ourselves in the non-Galilean reference frame R^Ω where the movement of the electron is cancelled. In R^Ω , we always have the same attractive force:

$$F_{Es} = -\frac{e^2}{4\pi\epsilon_0 r^2}$$

We have a centrifugal inertial force:

$$F_{Cent} = m\frac{v^2}{r}$$

In R^Ω , the electron is immobile, we apply the principle of inertia:

$$0 = F_{Es} + F_{Cent}$$

We find the same equation:

$$m \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

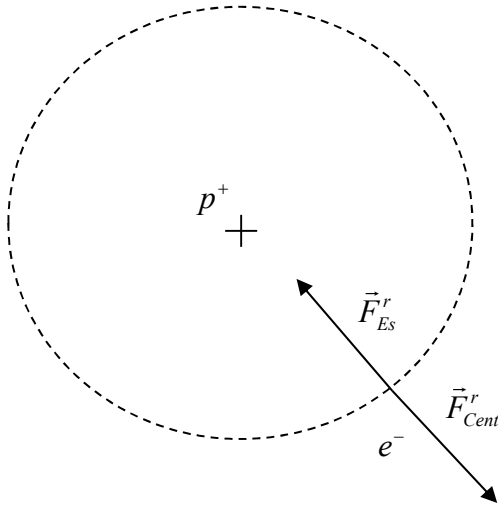


Figure 3: electron "stabilized" by an electrostatic force and by a centrifugal inertial force

III.3.4 Mechanical energy in R^Ω

We have the electrical potential energy in R^Ω :

$$Ep(F_{Es}) = -\frac{e^2}{4\pi\epsilon_0 r}$$

We have the potential energy of the centrifugal inertial force in R^Ω (counted here positively when the force is repulsive):

$$Ep(F_{Cent}) = \frac{1}{2}mv^2$$

We have the same mechanical energy in R^Ω :

$$E = Ep(F_{Es}) + Ep(F_{Cent}) = -\frac{1}{2}mv^2$$

Note 1 on the a priori artificial side of introducing an inertial force

Introducing a centrifugal inertial force may seem artificial. However, this has the advantage of being placed in a reference frame R^Ω more natural for the electron: it is the one where it is motionless and does not radiate.

This will have all its importance in the model of the electron of L. de Broglie where L. de Broglie models the electron by a standing wave (immobile wave in Space).

Note 2, why combine electrostatic force and centrifugal inertial force?

What remains unexplained is why the association in the Bohr electron model of these two forces: electrostatic force and centrifugal inertial force?

We have the same question with the Kepler-Newton planetary model, why the association of these two forces: gravitational force and centrifugal inertia force?

III.3.5 Quantification of kinetic moment

We recall the kinetic moment of a particle with a mass m :

$$\vec{\sigma} = \vec{r} \wedge m\vec{v}$$

In his article, N. Bohr draws on the ideas of M. Planck and A. Einstein on quantifying energy and action S using \hbar . He hypothesizes that kinetic momentum σ_n of the electron revolving around the proton is quantified, and that it is an integer multiple of \hbar :

$$\sigma_n = r_n m v_n = n\hbar$$

The integer n corresponds to the possible orbits of the electron, $n = 1$ corresponds to the lowest and most stable of orbits, $n = 2$ corresponds to an orbit a little higher and a little less stable, and so on.

III.3.6 Electron velocities, Bohr velocity

From the equations $m v_n^2 = \frac{e^2}{4\pi\epsilon_0 r_n}$ and $r_n m v_n = n\hbar$, we deduce the different velocities of the electron, according to its orbit and independently of its mass.

We have:

$$m r_n v_n \times v_n = \frac{e^2}{4\pi\epsilon_0}$$

$$n\hbar \times v_n = \frac{e^2}{4\pi\epsilon_0}$$

We obtain:

$$v_n = \frac{e^2}{4\pi\epsilon_0 n\hbar}$$

For $n = 1$, we have the highest velocity of the electron (also called Bohr velocity):

$$v_1 = v_{Bohr} = \frac{e^2}{4\pi\epsilon_0 \hbar}$$

Note

We often use:

$v_{Bohr} = \frac{e^2}{4\pi\epsilon_0 \hbar} = \alpha_{em} c$ with $\alpha_{em} = \frac{e^2}{4\pi\epsilon_0 \hbar c}$ the fine structure constant, also called the electromagnetic coupling constant.

The fine structure constant $\alpha_{em} \approx \frac{1}{137}$ was proposed in 1916 by Arnold Sommerfeld to explain fine differences between the spectral lines of hydrogen. It relates the speed of light to the speed of the electron in its most stable orbit. We will see in the 4th memoir that the fine structure constant is frequently used in relativistic quantum electrodynamics and in the Standard Model.

III.3.7 Rays of the hydrogen atom, Bohr radius

According to the hypothesis of quantification of the kinetic moment, we have the radius:

$$r_n = \frac{n\hbar}{m v_n}$$

By replacing v_n by $v_n = \frac{e^2}{4\pi\epsilon_0 n\hbar}$, we get:

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{me^2} n^2$$

For $n = 1$, we have the smallest radius. It is called the Bohr radius:

$$r_1 = r_{Bohr} = \frac{4\pi\epsilon_0 \hbar^2}{me^2}$$

It is made to correspond to the radius of the hydrogen atom. Numerically, we have:

$$r_{Bohr} \approx 0,529 \overset{0}{A}$$

III.3.8 Energy levels, finding the constant R_H

From the radius r_n , that is to say of the different possible orbits for the electron, N. Bohr defines energy levels:

$$E_n = -\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 r_n}$$

With:

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{me^2} n^2$$

$$E_n = -\frac{1}{2} \frac{me^4}{(4\pi\epsilon_0 \hbar)^2} \frac{1}{n^2}$$

N. Bohr applies M. Planck's quanta hypothesis to the transition from one energy level to another:

$$E_m - E_n = h\nu = \frac{\hbar 2\pi c}{\lambda}$$

He obtains:

$$\frac{1}{\lambda} = \frac{E_m - E_n}{\hbar 2\pi c} = \frac{me^4}{(4\pi)^3 \epsilon_0^2 (\hbar)^3 c} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

We had the relationship of Balmer Rydberg Ritz:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

N. Bohr gets for the constant R_H :

$$R_H = \frac{me^4}{(4\pi)^3 \epsilon_0^2 (\hbar)^3 c}$$

By calculation, he finds a value very close to the experimental measurement:

$$R_H \approx 109,678 \text{ cm}^{-1}$$

Thanks to its explanatory simplicity and its results confirmed by experience, the success of the Bohr electron model is rapid and considerable. He will inspire many physicists like Arnold Sommerfeld with his constant of fine structure in the search for a relativistic model. However, it is with the ideas of L. de Broglie and E. Schrödinger that the Bohr electron model will find his true extension.

III.4 Ideas of L. de Broglie, the electron at the same time wave and particle

III.4.1 Circular standing waves

During the 1st World War, L. de Broglie worked on antennas installed at the top of the Eiffel Tower. He then became interested in the theory of antennas, as well as the standing waves involved in this theory and comprising a multiple and integer number of nodes.

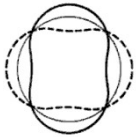
After reading Bohr's paper, L. de Broglie notes strong similarities between the mathematical model of standing waves and the Bohr electron model. A few years later, he modified the Rutherford-Bohr planetary model and imagined a new model for the electron. This model is no longer a point particle, it is both a wave and a particle. We talk about particle wave duality.

In a Galilean reference frame R^0 , the electron wave is considered as a progressive wave, propagating at the speed v_n , as the electron particle of the Bohr model.

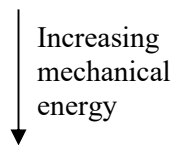
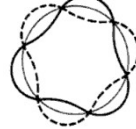
In the electron reference frame R^Q , the electron wave is considered a standing wave, that is to say immobile in Space. L. de Broglie approximates the number n of nodes of the electron standing wave to the number n of the mechanical energy level E_n .

Instead of the orbits of the electron, L. de Broglie imagines circular standing waves. The following figure proposes different possible configurations of the circular standing wave, with a number n of nodes and mechanical energy. E_n , both crescents. For the circular standing wave, we have a number of nodes equal to the own mode n of the standing wave.

$n=4$ 4 nodes



$n=6$ 6 nodes



$n=10$ 10 nodes

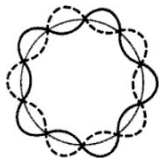


Figure 4: example of the circular standing wave with increase in the number of nodes and mechanical energy

III.4.2 Linking the data of Newtonian mechanics to those of wave mechanics

Following his idea of particle wave, L. de Broglie wished to link the data of classical Newtonian mechanics to those of a new mechanics called wave mechanics, and which he conceived as a

generalization of Newtonian mechanics. Thus, he seeks to relate the speed and momentum of the electron particle to a speed and momentum of the electron progressive wave.

L. de Broglie interprets the electron not quite as a wave, but rather as a packet of waves (of very similar wavelengths) with a group velocity $v_g = \frac{\partial\omega}{\partial k}$ different from its phase velocity $v_\phi = \frac{\omega}{k}$.

To make the link between the particle and the electron wave, L. de Broglie matches the speed v of the electron particle (as defined in Classical Newtonian mechanics) to the group velocity v_g of the electron wave:

$$v = v_g = \frac{\partial\omega}{\partial k}$$

He also matches the momentum p of the electron closer to the momentum of a photon of wavelength λ proposed by A. Einstein:

$$p = \frac{h}{\lambda}$$

According to L. de Broglie, this formula also remains valid for a non-relativistic electron of mass $m_{\acute{e}lec}$ and wavelength λ . That is to say, we have:

$$p = m_{\acute{e}lec}v = \frac{h}{\lambda}$$

III.4.3 Quantification condition

By posing $p = m_{\acute{e}lec}v = \frac{h}{\lambda}$ for an electron, L. de Broglie intuitively sheds light on the Bohr quantification condition:

$$\sigma_n = r_n m_{\acute{e}lec} v_n = n\hbar$$

So, that the circular electron standing wave is not destroyed on its circular trajectory of radius r_n and length $l = 2\pi r_n$ (that is, to avoid destructive interference), the length of this trajectory must be equal to n times the wavelength (with n an integer) in R^Ω :

$$l = 2\pi r_n = n\lambda_n$$

Now, for a progressive electron wave we have in R^0 :

$$p_n = m_{\acute{e}lec}v_n = \frac{h}{\lambda}$$

We obtain:

$$l = 2\pi r_n = n\lambda_n = n \frac{h}{m_{\acute{e}lec}v_n}$$

We find again the Bohr quantification condition:

$$r_n m_{\acute{e}lec} v_n = n\hbar$$

Note

It is reminded that in R^Ω , the electron wave is standing. In R^0 , the electron wave is progressive with velocity $v = v_g = \frac{\partial\omega}{\partial k}$.

That we describe the electron in R^0 or R^Ω , it is considered in a stable or permanent state and does not radiate energy.

III.5 Schrödinger electron wave equation

III.5.1 Historical Preamble

In 1926, E. Schrödinger was inspired by Bohr's model of the electron and the ideas of L. de Broglie. In a few months, he published a series of fundamental articles later grouped in a collection *Collected Papers on Wave Mechanics*. In the 1st article, he proposes the famous equation that bears his name.

The equation is then a formidable intellectual revolution. Thanks to it, we understand many properties of chemical elements and their electrons (energy level, chemical bonds, etc.). It sheds light on the periodic table of chemical elements by Dmitri Mendeleev (1870). It can be considered as the act of fusion between physics and chemistry.

Note on the method used by E. Schrödinger

In his first article from 1926: *Quantification and proper values*, E. Schrödinger obtained the Schrödinger equation at the 3rd page.

To do this, he starts from Hamilton's partial differential equation:

$$H(q, p = \frac{\partial S}{\partial q}) = E$$

With q the position, p the momentum and $S = \int_{t_1}^{t_2} L dt$ the action, integral of the Lagrangian L .

He introduces the wave function ψ in the form:

$$S = k \log(\psi)$$

with k a constant having the dimensions of an action.

He obtains for Hamilton's partial differential equation:

$$H(q, \frac{k}{\psi} \frac{\partial \psi}{\partial q}) = E$$

Using an analogy with a Kepler motion, he ends up after some contortions to his equation:

$$\Delta \psi + \frac{2m}{\hbar^2} (E_m - E_p) \psi = 0$$

With:

$$E_p = -\frac{e^2}{4\pi\epsilon_0 r}$$

Subsequently, he gives to the constant k the value of \hbar .

We are now inspired by the ideas of L. de Broglie to find the Schrödinger equation.

III.5.2 Refractive medium

We remind the d'Alembert wave equation for a monochromatic wave ψ :

$$\Delta\psi - \frac{n^2}{v_{\phi 0}^2} \partial_t^2 \psi = 0$$

with $v_{\phi 0}$ the phase velocity of the wave when the refractive index $n = 1$ (that is, in a vacuum).

Usually, we take $v_{\phi 0} = c$.

We have the phase velocity of the wave in a refractive medium of index $n \neq 1$:

$$v_{\phi} = \frac{v_{\phi 0}}{n}$$

Note

Do not confuse here the refractive index n and n the energy level of the stationary states of the electron.

In the reference frame R^{Ω} where the electron wave is immobile, we have a standing wave function that can be written: $\psi(x, y, z, t) = f(x, y, z)e^{i\Omega t}$ with $f(x, y, z)$ function of Space variables.

We obtain a wave equation in the form:

$$\Delta\psi + \frac{\Omega^2 n^2}{v_{\phi 0}^2} \psi = 0$$

III.5.3 Phase velocity

It is assumed that the electron is similar in R^0 as a progressive wave having a phase velocity:

$$v_{\phi} = \frac{\Omega}{K}$$

The momentum of the electron wave is equal to:

$$p = \frac{h}{\lambda} = \hbar K$$

The energy of the electron wave is equal to:

$$E = \hbar \Omega$$

We obtain for the phase velocity of the electron wave in a refringent medium:

$$v_{\phi} = \frac{v_{\phi 0}}{n} = \frac{\Omega}{K} = \frac{\hbar \Omega}{\hbar K} = \frac{E}{p}$$

III.5.4 Schrödinger wave equation

Following a classical reasoning, we have the relationship between mechanical energy, kinetic energy, and potential energy: $E_m = E_c + E_p$ with $E_p = qA^t$ electrical potential energy.

We have for kinetic energy:

$$E_c = \frac{p^2}{2m_{elec}}$$

We obtain:

$$E_c = E_m - E_p = \frac{p^2}{2m_{elec}}$$

We deduce the momentum of the electron:

$$p = \sqrt{2m_{elec}(E_m - E_p)}$$

We obtain for the phase velocity of the electron wave:

$$v_\phi = \frac{v_{\phi 0}}{n} = \frac{E_m}{p} = \frac{\hbar\Omega}{\sqrt{2m_{elec}(E_m - E_p)}}$$

By putting in the square, we have:

$$\frac{v_{\phi 0}^2}{n^2} = \frac{\hbar^2\Omega^2}{2m_{elec}(E_m - E_p)}$$

We obtain:

$$\frac{\Omega^2 n^2}{v_{\phi 0}^2} = \frac{2m_{elec}}{\hbar^2} (E_m - E_p)$$

We had the wave equation:

$$\Delta\psi + \frac{\Omega^2 n^2}{v_{\phi 0}^2} \psi = 0$$

By substituting $\frac{\Omega^2 n^2}{v_{\phi 0}^2}$ to $\frac{2m_{elec}}{\hbar^2} (E_m - E_p)$, we find the Schrödinger equation:

$$\Delta\psi + \frac{2m_{elec}}{\hbar^2} (E_m - E_p) \psi = 0$$

The choice of Cartesian coordinates is ill-suited to solve this equation. Spherical coordinates are usually used: r, θ, ϕ with $\psi(r, \theta, \phi) = \psi(r)\psi(\theta, \phi)$.

III.5.5 Solving the Schrödinger equation for the radial part $\psi(r)$

It is proposed to solve the Schrödinger equation for the radial part $\psi(r)$ and for the first atomic orbital called 1s (we usually speak of layer K and energy level $n = 1$).

For the electron level $n = 1$, we have the mechanical and potential energies:

$$E_m = -\frac{m_{elec}e^4}{8\varepsilon_0^2(2\pi\hbar)^2}$$

$$E_p = -\frac{e^2}{4\pi\varepsilon_0 r}$$

We have the wave vector K :

$$K = \frac{p}{\hbar} = \frac{m_{elec}v}{\hbar} = \frac{m_{elec}\alpha_{em}c}{\hbar} = \frac{m_{elec}e^2}{4\pi\epsilon_0\hbar^2} = \frac{1}{r_{Bohr}}$$

which is the inverse of the radius of the Bohr atom.

Nota

The Schrödinger radial wave equation corresponds to the wave equation of the Bohr electron model. We find in $K = \frac{1}{r_{Bohr}}$ the Bohr's radius.

Note that the radial part wave equation does not explain atomic orbitals and therefore chemical bonds. In addition, the tangential or angular part ($\psi(\theta, \phi)$) of the wave equation must be used to unify physics and chemistry.

We obtain:

$$\frac{2m_{elec}}{\hbar^2} E_m = \frac{2m_{elec}}{\hbar^2} \frac{m_{elec}e^4}{8\epsilon_0^2(2\pi\hbar)^2} = -K^2$$

$$\frac{2m_{elec}}{\hbar^2} E_p = -\frac{2K}{r}$$

So, we have a differential equation in the form:

$$\Delta\psi + \frac{2m_{elec}}{\hbar^2}(E_m - E_p)\psi = 0$$

$$\Delta\psi - (K^2 - \frac{2K}{r})\psi = 0$$

The differential equation is solved in spherical coordinates for the radial part $\psi(r)$:

$$\nabla_r^2\psi(r) = (K^2 - \frac{2K}{r})\psi(r)$$

A solution is found in the form of:

$$\psi(r) = Ae^{-Kr}$$

Normalization of the wave function involves:

$$\int_0^\infty \psi^2 4\pi r^2 dr = 1$$

the volume element $d\tau$ being equal to $4\pi r^2 dr$.

There is a wave function for the first atomic orbital 1s:

$$\psi(1s) = \sqrt{\frac{K^3}{\pi}} e^{-Kr}$$

We obtain as a solution a wave independent of Time and standing in the reference frame R^Ω . We find the wave vector K . However, it does not play its role as a "propagator" as it would in the case of a progressive wave, because it is not preceded by a complex i .

Note, simplified Schrödinger equation

We can start from a wave equation independent of Time. We have:

$$\Delta\psi + \frac{\Omega^2}{v_\phi^2}\psi = 0$$

If a complex phase velocity is set:

$$v_\phi = \frac{i\Omega}{K}$$

We obtain the differential equation:

$$\Delta\psi - K^2\psi = 0$$

By solving the equation in Cartesian coordinates, we find a solution in the form:

$$\psi(x) = Ae^{-Kx}$$

III.5.6 Group velocity v_g of the electron progressive wave

According to the ideas of L. de Broglie, the electron velocity of the Bohr model corresponds to the group velocity of the electron progressive wave (also called an electron wave packet). We resume here his demonstration which makes it possible to verify this idea.

We place ourselves in the reference frame R^0 where the electron progressive wave propagates at the speed v_g . Let ν the frequency and λ the wavelength of the electron progressive wave.

According to the Rayleigh relation on group speed, we have:

$$\frac{1}{v_g} = \frac{\partial K}{\partial \Omega} = \frac{\partial \frac{1}{\lambda}}{\partial \nu} = \frac{\partial \frac{1}{\lambda}}{\partial \nu} = \frac{\partial \frac{\nu}{v_\phi}}{\partial \nu} = \frac{1}{v_{\phi_0}} \frac{\partial \nu}{\partial \nu}$$

So, we have:

$$\frac{1}{v_g} = \frac{\partial K}{\partial \Omega} = \frac{\partial p}{\partial E_m} = \frac{\partial \sqrt{2m_{elec}(E_m - E_p)}}{\partial E_m} = \frac{m_{elec}}{\sqrt{2m_{elec}(E_m - E_p)}}$$

We derive from it the group speed of the electron wave:

$$v_g = \frac{\sqrt{2m_{elec}(E_m - E_p)}}{m_{elec}}$$

And with:

$$p = \sqrt{2m_{elec}(E_m - E_p)} = m_{elec}v$$

We find a group velocity v_g of the electron wave equal to the (Newtonian) speed v of the electron particle:

$$v_g = \frac{m_{elec}v}{m_{elec}} = v$$

III.5.7 Index n

In the absence of an electrostatic field, we have the phase velocity of the electron progressive wave:

$$v_{\phi 0} = \frac{\hbar\Omega}{\sqrt{-2m_{elec}E_m}}$$

In the presence of an electrostatic field, we have the phase velocity of the electron progressive wave:

$$\frac{v_{\phi 0}}{n} = \frac{\hbar\Omega}{\sqrt{2m_{elec}(E_m - E_p)}}$$

By squared:

$$\frac{v_{\phi 0}^2}{n^2} = \frac{\hbar^2\Omega^2}{2m_{elec}(E_m - E_p)}$$

By eliminating $v_{\phi 0}^2$, we get the index n :

$$n^2 = 1 - \frac{E_p}{E_m} = 1 - \frac{E_p}{\hbar\Omega}$$

Note

The existence of a refringent environment ($n \neq 1$) is here related to the presence of an electrostatic field.

III.5.8 Lagrangian of the Schrödinger equation

We remind the Lagrangian from the Schrödinger equation:

$$L = \bar{\psi}^\dagger \left(i\hbar\partial_t + \frac{\hbar^2(\partial_x - i\frac{e}{\hbar}A^x)^2}{2m_{elec}} - eA^t \right) \psi$$

Applying the Euler-Lagrange equation $\frac{\partial L}{\partial x} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = 0$ to this Lagrangian L , we find again the Schrödinger equation:

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m_{elec}} (\partial_x - i\frac{e}{\hbar}A^x)^2 + eA^t \right) \psi$$

III.5.9 Probabilistic interpretation of the wave function

In 1926, Max Born proposed to interpret the square of the wave function, or the module squared if the wave function is complex, $|\psi|^2$, as the probability density of the presence of the particle.

By integrating on any volume V , we obtain the probability of finding the object in this volume:

$$P = \int_V |\psi(x, y, z)|^2 d\tau$$

We also have the condition of normalization on an infinite volume:

$$P = \int_{\infty} |\psi(x, y, z)|^2 d\tau = 1$$

This interpretation is included in what is now called the Copenhagen interpretation, in reference to the Institute of Physics directed by N. Bohr and located in that city.

There are other interpretations, more or less close (and more or less compatible) with that of Copenhagen. For example, instead of having an electron located in a small volume of Space, the electron is distributed throughout Space, with a density ρ proportional to $|\psi|^2$. We have for the total electric charge of the electron: $e = \int_{\infty} \rho(x, y, z) d\tau$.

We find in a way in these 2 interpretations, the duality wave corpuscle with:

- for the Copenhagen interpretation, a probabilistic electron, localized and therefore more particle,
- and for the second interpretation (the one that could be called de Broglie), an electron distributed throughout Space and therefore more wave.

III.5.10 Conclusion of the chapter

The electron models of Bohr, de Broglie and Schrödinger favours two reference frames, continually passing from one to the other in the reasoning conducted. One R^0 considered Galilean, where the electron is rotating and is interpreted as a progressive wave verifying $p = \hbar K$. The other R^Ω considered non-Galilean, where the electron is immobile and is interpreted as a standing wave. It is in this one that we obtain the functions of standing wave solutions of the Schrödinger equation.

In his writings, L. de Broglie presents wave mechanics as a generalization of Newtonian mechanics. We will now focus on different wave equations that can be interpreted as extensions of the Newton's fundamental principle of dynamics.

Chapter IV Local gauge theories, applications to different wave functions (from Dirac 1928, Schrödinger 1925 to Pauli 1927)

Subject of the chapter

We are interested in the conceptual similarities between:

- Newtonian mechanics dating from the 17th century and allowing to describe gravitation and planetary systems,
- local gauge theories developed especially in the 20th century, and allowing to describe the electromagnetic force, strong and weak nuclear interactions, and particle systems.

This will make it possible to study the wave functions of Dirac, Schrödinger and Pauli proposed in the 1920s, from a local gauge theory angle.

IV.1 General information on Local Gauge theories

The terms invariance of Gauge and transformation of Gauge may seem a priori enigmatic, they are in fact inspired by notions already present in Newtonian mechanics. We propose here to clarify these 2 terms via their similarities with Newtonian mechanics.

IV.1.1 What is invariant?

Invariance is the respect of the fundamental laws of Nature regardless of the change of reference frames or the transformation of local gauge, that is to say the principle of relativity dear to A. Einstein.

In Newtonian mechanics, when cancelling a rotation vector $\vec{\Omega}$ (i.e., during a change of reference frames), centrifugal and Coriolis inertial forces are added, in order to respect the Newton's fundamental principle of dynamics.

In a similar way, in local gauge theories, when cancelling part of the phase of the wave function (i.e., during a local gauge transformation), quadrivectors of interaction momentum energies are added (associated with electromagnetic interaction or strong and weak interactions), in order to respect the wave equation.

It is this respect for the wave equation, regardless of the local Gauge transformation, that is considered the desired Gauge invariant.

Thus, in Newtonian mechanics, the fundamental law of Nature, that is to say the invariant to be respected during a change of references, this is the Newton's fundamental principle of dynamics $m\vec{a} = \Sigma\vec{F}$, even if it means adding inertial forces.

In wave mechanics (or quantum physics), the fundamental law of Nature or the invariant to be respected during a local gauge transformation, this is the wave equation, even if it means adding potential momentum.

Note 1

We will see, however, that we do not manage to generalize this principle as far as we would like. Indeed, depending on the case where we stand, the wave equation to be respected is not exactly the same.

In a vacuum (no mass and no electric charge), the wave equation to be respected is the one proposed by Jean Le Rond d'Alembert in 1746:

$$\Delta\psi - \frac{1}{c^2} \frac{\partial^2\psi}{\partial t^2} = 0$$

In relativistic quantum physics, the wave equation to be respected is that of Klein Gordan proposed in 1926:

$$\Delta\psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

In non-relativistic quantum physics, the wave equation to be respected is that of free Schrödinger proposed in 1925 (absence of electric potential):

$$\Delta\psi - \frac{i2m}{\hbar} \frac{\partial \psi}{\partial t} = 0$$

This last free Schrödinger wave equation reminds the heat equation (or scattering equation: the variation of a concentration in Time is proportional to the relative surplus of this concentration in its infinitesimal environment) proposed by Joseph Fourier in 1807:

$$\Delta u - \frac{1}{\alpha} \frac{\partial u}{\partial t} = 0$$

Note 2

It should be noted that the Maxwell Faraday tensor, as well as Maxwell's equations, are also conserved during a local gauge transformation. Maxwell's tensor Faraday and Maxwell's equations are therefore considered fundamental laws of Nature in local gauge theories.

IV.1.2 What do we transform?

A local gauge transformation consists in modifying (partially cancelling) on the one hand the phase of the wave function, on the other hand the potentials (electromagnetic or other).

We can compare a transformation of Local Gauge to a change of reference frames of Newtonian mechanics where we modify the movement of the reference frame (for example, we cancel a rotation vector $\vec{\Omega}$).

IV.1.3 What do we do to preserve invariance (of the laws of Nature) during transformation?

In Newtonian mechanics, when we make a change of Galilean to non-Galilean reference frames, we add inertial forces in the Newton's fundamental principle of dynamics. These forces can derive from potential energy, as this is the case with centrifugal inertial force.

In local gauge theories, during a local gauge transformation, the derivative of the wave function is transformed into a covariant derivative. We will see that this amounts to adding a potential momentum energy quadrivector, which is associated with an interaction.

These interactions are in a way the counterpart of the inertial forces of Newtonian mechanics. In their nature, they differ since it is not the centrifugal force or the Coriolis force, but the electromagnetic interaction and weak and strong nuclear interactions.

Nota

In relativistic quantum electrodynamics or in the Standard Model, the addition of a potential momentum energy quadrivector ieA^μ ($\mu = t, x, y, z$) (in partial derivatives or wave equations) is like adding an energy of interaction in the Lagrangian. This interaction energy can represent electromagnetic, strong and weak interactions.

In the case of electromagnetic interaction, the interaction energy is precisely equal to the generalized potential energy, which with the help of the Euler-Lagrange equation makes it possible to find the electromagnetic force.

On the other hand, for strong and weak interactions, this is not the case. In fact, for these interactions, the notion of force does not exist, in the sense that these interactions do not intervene in the Newton's fundamental principle of dynamics.

IV.1.4 What is the reference?

In Newtonian mechanics, the basic reference frame is the Galilean reference frame, where the fundamental principle of dynamics applies without having inertial forces to add.

For local gauge theories, vacuum (i.e., a homogeneous, linear and isotropic medium without mass and electric charge) could appear as the reference. The wave equation to be respected would then be that of Alembert.

However, this is not suitable for relativistic quantum physics where the wave equation to be respected is that of Klein Gordan. Compared to that of Alembert, there appears an additional mass term.

We do not know of a local gauge transformation that allows us to pass from the d'Alembert wave equation to that of Klein Gordan. In the latter case, the reference would rather be a "mass" medium, homogeneous, linear and isotropic.

A similar problem arises in non-relativistic quantum physics where the wave equation to be respected is that of free Schrödinger. We do not know of a local gauge transformation that allows us to pass from the Alembert wave equation to the free Schrödinger equation.

IV.2 Some reminders about wave equations

IV.2.1 D'Alembert's wave equation (1746)

We remind the function of a progressive, sinusoidal, plane, and monochromatic wave that propagates in direction of x and t :

$$\psi = \cos(\omega t - \vec{k} \cdot \vec{x} + \phi)$$

with ϕ the phase at the origin, ω the pulsation and \vec{k} the wave vector.

We have in complex notation:

$$\psi = e^{i(\omega t - \vec{k} \cdot \vec{x} + \phi)}$$

The wave function verifies the Alembert equation (wave propagation equation):

$$\frac{1}{v_\phi^2} \partial_t^2 \psi - \Delta \psi = 0$$

with $\Delta = \nabla^2 = \partial_x^2$ the Laplacian operator (second derivative relative to Space), v_ϕ the phase velocity of the wave.

Note on the method of J. le Rond d'Alembert

To find his wave equation $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, J. le Rond d'Alembert seeks to model the string of a violin and is inspired by the Newton's fundamental principle of dynamics $m \frac{\partial^2 x}{\partial t^2} = \Sigma \vec{F}$.

$\frac{\partial^2 u}{\partial t^2}$ represents acceleration, $\frac{\partial^2 u}{\partial x^2}$ represents the sum of the forces and $\frac{1}{c^2}$ represents the term mass (as if energy E_0 was equal to 1 inspired by $E_0 = mc^2$).

IV.2.2 Dispersion relationship

We have the phase velocity, also called dispersion relationship between the pulsation ω and the wave vector k . In a dispersive medium, the index n is a function of the pulsation ω . We have:

$$v_\phi = \frac{\omega}{k} = \frac{c}{n(\omega)}$$

In a non-dispersive medium, the index n is independent of ω , we have:

$$v_\phi = \frac{\omega}{k} = \frac{c}{n}$$

In a vacuum, $n = 1$, we have:

$$v_\phi = \frac{\omega}{k} = c$$

IV.2.3 Momentum and energy operators

Following the ideas of L. de Broglie of a particle wave duality, we pass from a particle characterized by its momentum p^x and its energy $E^t = m^t c^2$, to a wave characterized by its wave vector k^x and its pulsation ω^t through relationships:

$$\begin{aligned} p^x &= \hbar k^x \\ E^t &= m^t c^2 = \hbar \omega^t \end{aligned}$$

For a progressive wave toward the x direction, a easy wave function ψ is for example:

$$\psi = \cos(\omega^t t - \vec{k}^x \vec{x})$$

This wave function can be rewritten from p^x and E^t :

$$\psi = \cos\left(\frac{E^t}{\hbar} t - \frac{\vec{p}^x}{\hbar} \vec{x}\right)$$

In complex notation:

$$\psi = e^{i\left(\frac{E^t}{\hbar} t - \frac{\vec{p}^x}{\hbar} \vec{x}\right)}$$

We have the partial derivative with respect to x of the wave function ψ :

$$\begin{aligned} \partial_x \psi &= -i \frac{p^x}{\hbar} \psi \\ -i \hbar \partial_x \psi &= p^x \psi \end{aligned}$$

Following the previous relation, in Quantum Physics (or Wave Mechanics), we usually define a momentum operator:

$$\hat{p}^x = -i \hbar \partial_x$$

The operator \hat{p}^x "acts" on the wave function and makes it possible to obtain a momentum:

$$\hat{p}^x \psi = -i \hbar \partial_x \psi = p^x \psi$$

We have the partial derivative with respect to t of the wave function ψ :

$$\begin{aligned} \partial_t \psi &= i \frac{E^t}{\hbar} \psi \\ i \hbar \partial_t \psi &= E^t \psi \end{aligned}$$

Following the previous relation, in Quantum Physics, we usually define an energy operator:

$$\hat{E}^t = i \hbar \partial_t$$

The operator \hat{E}^t "acts" on the wave function and makes it possible to obtain an energy:

$$\hat{E}^t \psi = i\hbar \partial_t \psi = E^t \psi$$

Note on the notation of an operator

In Quantum Physics, we also frequently use notation $\hat{\ }^$ to designate an operator. It gives \hat{p} or \hat{E} , for momentum and energy operators. In the rest of the chapter, we will also discuss \hat{S} for spin kinetic moment operator and $\hat{\mu}_S$ for the spin magnetic moment operator.

IV.2.4 Dispersion relations in wave mechanics

For a particle wave, the relationship between energy E and momentum p can be seen as a relationship between ω and k , and be interpreted as a relationship of dispersion.

We have the following dispersion relations.

1/ For a non-relativistic particle wave of mass m :

$$E = \frac{p^2}{2m}$$

2/ For a relativistic particle wave of mass m :

$$E = \sqrt{p^2 c^2 + m^2 c^4}$$

3/ For a particle wave (relativistic or not) of zero mass and zero potential energy:

$$E = pc$$

Note

For relativistic mechanics, we note that the passage of $E = \sqrt{p^2 c^2 + m^2 c^4}$ to $E = pc$ for a particle of zero mass is obvious.

For non-relativistic mechanics, the index n can be used with $n^2 = 1 - \frac{E_p}{E}$ and E_p the potential energy of an electrostatic force. By making tender E_p towards 0, we find $n = 1$ and $v_\phi = \frac{c}{n} = c = \frac{\omega}{k} = \frac{E}{p}$. So, $E = pc$.

IV.2.5 Dispersion relationships, energy mass momentum relationships, and wave equations

In Quantum Physics (or Wave Mechanics), to pass from momentum energy relationships to wave equations, we usually make substitutions:

$$p^x \rightarrow -i\hbar \partial_x = \hat{p}^x$$

$$E^t \rightarrow i\hbar \partial_t = \hat{E}^t$$

Thus from $E^2 = p^2 c^2$, we find the d'Alembert's wave equation:

$$p^2 - \frac{1}{c^2} E^2 = 0$$

$$\Delta \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

Similarly, from relativistic energy $E = \sqrt{p^2 c^2 + m^2 c^4}$, we find the Klein Gordan wave equation:

$$\Delta\psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

Similarly, from kinetic energy $E = \frac{p^2}{2m}$, we find the free Schrödinger wave equation:

$$\Delta\psi - \frac{i2m}{\hbar} \frac{\partial \psi}{\partial t} = 0$$

IV.2.6 Wave vector pulsation quadrivector

In wave mechanics, the energy momentum quadrivector of the particle is connected to a pulsation quadrivector wave vector of the wave.

We have the quadrivector energy momentum:

$$4\vec{p} = \left(\frac{E^t}{c}, \vec{p}^x \right)$$

We have the pulsation quadrivector wave vector:

$$4\vec{k} = \left(\frac{\omega^{1/t}}{c}, \vec{k}^{1/x} \right)$$

We have the relationship between the two quadrivectors:

$$4\vec{k} = \frac{4\vec{p}}{\hbar}$$

These two quadrivectors are characteristics specific to the particle wave.

IV.3 Simple study of a local gauge transformation, case of Time

IV.3.1 Wave function

Let be a wave function in the reference frame R^0 (similar to vacuum):

$$[\psi]_{R^0} = \cos\left(\frac{q}{\hbar}(t - \theta(t))\right)$$

with q the electric charge density and $\theta(t)$ a phase depending on the moments.

Note

The terms of Space are not indicated here.

$\frac{q}{\hbar}$ plays the role of the pulsation ω .

We can see it like this:

$$\omega = \frac{E^t}{\hbar} = \frac{qA_0^t}{\hbar} = \frac{q}{\hbar} \text{ with the electric potential } A_0^t = id.$$

In complex form, this gives:

$$[\psi]_{R^0} = e^{i\frac{q}{\hbar}(t - \theta(t))}$$

We have in the reference frame R^0 :

$$[t]_{R^0} = t$$

$$[\partial_t]_{R^0} = \frac{\partial}{\partial t}$$

We have the derivative of ψ with respect t in R^0 :

$$[\partial_t]_{R^0}\psi = \left(i\frac{q}{\hbar} - i\frac{q}{\hbar}\frac{\partial\theta(t)}{\partial t}\right)\psi$$

Note

The derivative of ψ with respect to t will play the role of the wave equation to be respected during the transformation of Local Gauge.

IV.3.2 Local Gauge Transformation

We perform a transformation of Local Gauge, comparable to a change of reference frames from R^0 to R^{A^t} , where physical quantities are modified, and others are retained.

Electric charge density q is the quantity retained from R^0 to R^{A^t} .

$A^t = \frac{\partial\theta(t)}{\partial t} = \partial_t\theta(t)$ is the quantity cancelled or subtracted from R^0 to R^{A^t} (A^t is of electric potential type).

When transforming Local Gauge from R^0 to R^{A^t} , the following 2 transformations apply.

On the one hand, we perform a translation in Time, which transforms the phase of the wave function:

$$t \rightarrow [t]_{R^{A^t}} = t - \theta(t)$$

$$[\psi]_{R^0} = e^{i\frac{q}{\hbar}(t-\theta(t))} \rightarrow [\psi]_{R^{A^t}} = [\psi]_{R^0} e^{i\frac{q}{\hbar}\theta(t)} = e^{i\frac{q}{\hbar}t}$$

On the other hand, the electrical potential is modified by subtracting $A^t = \partial_t\theta(t)$:

$$[A^t]_{R^0} = 0 \rightarrow [A^t]_{R^{A^t}} = [A^t]_{R^0} - \partial_t\theta(t) = -\partial_t\theta(t)$$

Note 1

We speak here of transformation of Local Gauge because the function $\theta(t)$ is a function of a local variable, the instants.

Note 2

The transformation of electrical potential A^t given above, retains the Maxwell Faraday tensor, as well as Maxwell's equations.

IV.3.3 The invariant: the partial derivative with respect to t

In R^0 , we have the partial derivative with respect to t of the wave function:

$$[\partial_t]_{R^0}[\psi]_{R^0} = \left(i\frac{q}{\hbar} - i\frac{q}{\hbar}A^t\right)[\psi]_{R^0}$$

In a linear transformation, the partial derivative $\partial_t\psi$ turns into $a\partial_t\psi$ in the same way as ψ turns into $a\psi$.

The objective is that during a transformation of Local Gauge from R^0 to R^{A^t} , the partial derivative $[\partial_t]_{R^0}[\psi]_{R^0}$ turns into $[\partial_t]_{R^{A^t}}[\psi]_{R^{A^t}}$ in the same way as $[\psi]_{R^0}$ turns into $[\psi]_{R^{A^t}}$.

If the first partial derivative with respect to Time transforms in the same way, the second partial derivative also, and a wave equation of the kind $-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$ also.

Note

We are interested here only in the partial derivative with respect to Time. We will have an analogous reasoning for the second partial derivative with respect to Space $\frac{\partial^2\psi}{\partial x^2}$.

For this purpose, we define the so-called covariant derivative:

$$[\partial_t]_{R^{A^t}}[\psi]_{R^{A^t}} = ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^{A^t}})[\psi]_{R^{A^t}}$$

We check that the covariant derivative thus defined, verifies our objective:

$$\begin{aligned} [\partial_t]_{R^{A^t}}[\psi]_{R^{A^t}} &= ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^{A^t}})[\psi]_{R^{A^t}} \\ &= ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^{A^t}})[\psi]_{R^0} e^{i\frac{q}{\hbar}\theta(t)} \\ &= [\partial_t]_{R^0}([\psi]_{R^0} e^{i\frac{q}{\hbar}\theta(t)}) - i\frac{q}{\hbar}[A^t]_{R^0} e^{i\frac{q}{\hbar}\theta(t)}[\psi]_{R^0} - i\frac{q}{\hbar} e^{i\frac{q}{\hbar}\theta(t)} \partial_t \theta(t) [\psi]_{R^0} \\ &= e^{i\rho\theta(t)} [\partial_t]_{R^0}([\psi]_{R^0} + i\frac{q}{\hbar} \partial_t \theta(t) e^{i\frac{q}{\hbar}\theta(t)}[\psi]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^0} e^{i\frac{q}{\hbar}\theta(t)}[\psi]_{R^0} - i\frac{q}{\hbar} e^{i\frac{q}{\hbar}\theta(t)} \partial_t \theta(t) [\psi]_{R^0} \\ &= e^{i\frac{q}{\hbar}\theta(t)} [\partial_t]_{R^0}([\psi]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^0} e^{i\frac{q}{\hbar}\theta(t)}[\psi]_{R^0} \\ &= e^{i\frac{q}{\hbar}\theta(t)} ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^0})[\psi]_{R^0} \end{aligned}$$

So, we have:

$$[\partial_t]_{R^{A^t}}[\psi]_{R^{A^t}} = e^{i\frac{q}{\hbar}\theta(t)} ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^0})[\psi]_{R^0}$$

We obtain the desired shape, with the wave function ψ and its partial derivative $\partial_t\psi$ which transforms in the same way by multiplying by $e^{i\frac{q}{\hbar}\theta(t)}$:

$$[\psi]_{R^0} \rightarrow [\psi]_{R^{A^t}} = e^{i\frac{q}{\hbar}\theta(t)}[\psi]_{R^0}$$

$$[\partial_t]_{R^0}[\psi]_{R^0} \rightarrow [\partial_t]_{R^{A^t}}[\psi]_{R^{A^t}} = e^{i\frac{q}{\hbar}\theta(t)} ([\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^0})[\psi]_{R^0}$$

In R^{A^t} , we give in abbreviated form the partial derivative:

$$[\partial_t]_{R^{A^t}} = [\partial_t]_{R^0} - i\frac{q}{\hbar}[A^t]_{R^{A^t}}$$

In **conclusion**, during the passage from R^0 to R^{A^t} , to have a partial derivative that transforms in the same way as the wave function, a term must be added $-i\frac{q}{\hbar}A^t$ comparable to an electrostatic potential energy and from which an electrostatic force can be derived.

IV.4 Simple study of a local gauge transformation, case of Space

IV.4.1 Wave function

Let be a wave function in the reference frame R^0 (similar to vacuum):

$$[\psi]_{R^0} = e^{i\frac{q}{\hbar}(x-\theta(x))}$$

with q the electric charge density and $\theta(x)$ a phase depending on the positions.

Note

The terms of Time are not indicated here.

$\frac{q}{\hbar}$ plays the role of the wave vector K .

We can see it like this:

$$K = \frac{p^x}{\hbar} = \frac{qA_0^x}{\hbar} = \frac{q}{\hbar} \text{ with magnetic potential } A_0^x = id$$

IV.4.2 Local Gauge Transformation

The transformation of Local Gauge is here comparable to a change of reference frames from R^0 to R^{A^x} .

$A^x = \frac{\partial\theta(x)}{\partial x} = \partial_x\theta(x)$ is the quantity cancelled or subtracted from R^0 to R^{A^x} . A^x is of the magnetic vector potential type.

For the covariant derivative, we obtain a relation in Space similar to that of Time:

$$[\partial_x]_{R^{A^x}}[\psi]_{R^{A^x}} = ([\partial_x]_{R^0} - i\frac{q}{\hbar}[A^x]_{R^{A^x}})[\psi]_{R^{A^x}}$$

Note

We can generalize to Space-Time the covariant derivative:

$$[\partial_\mu]_{R^{A^\mu}}[\psi]_{R^{A^\mu}} = ([\partial_\mu]_{R^0} - i\frac{q}{\hbar}[A^\mu]_{R^{A^\mu}})[\psi]_{R^{A^\mu}}$$

with $\mu = t, x, y, z$

To have exactly the same form of the covariant derivative with respect to t and to x , The usual sign of the three components of the magnetic vector potential is reversed.

We have the desired form, that is to say that the partial derivative $[\partial_x]_{R^0}[\psi]_{R^0}$ turns into $[\partial_x]_{R^{A^x}}[\psi]_{R^{A^x}}$ in the same way as $[\psi]_{R^0}$ turns into $[\psi]_{R^{A^x}}$.

$$[\psi]_{R^0} \rightarrow [\psi]_{R^{A^x}} = e^{i\frac{q}{\hbar}\theta(x)}[\psi]_{R^0}$$

$$[\partial_x]_{R^0}[\psi]_{R^0} \rightarrow [\partial_x]_{R^{A^x}}[\psi]_{R^{A^x}} = e^{i\frac{q}{\hbar}\theta(x)}[\partial_x]_{R^0}[\psi]_{R^0}$$

In R^{A^x} , we give in abbreviated format the partial derivative:

$$[\partial_x]_{R^{A^x}} = [\partial_t]_{R^0} - i\frac{q}{\hbar}[A^x]_{R^{A^x}}$$

In **conclusion**, during the passage from R^0 to R^{A^x} , to have a partial derivative that transforms in the same way as the wave function, a term must be added $-i\frac{q}{\hbar}A^x$ comparable to a magnetic potential momentum.

The addition of an electrostatic potential energy and a magnetic potential momentum in the partial derivatives is equivalent to adding a generalized electromagnetic potential energy:

$$Ep = q(A^t - (\vec{v}^a \cdot \vec{A}^a))$$

in the Lagrangian. Using the Euler-Lagrange equation, we can derive the electromagnetic force from the generalized electromagnetic potential energy. We will come back to that in the next memoir.

IV.5 From the D'Alembert wave equation (1746) to the Dirac wave equation (1928) in an electromagnetic field

IV.5.1 General

We are now interested in the Dirac wave equations of a massless particle, then a particle with mass, then a particle with mass in an electromagnetic field.

By putting these equations squared, we find 3 wave equations, first that of Alembert (in a vacuum), then that of Klein Gordan free, then that of Dirac in an electromagnetic field.

We would like the passages from a particle without mass, to a particle with mass, then to a particle with mass in an electromagnetic field, to be interpreted as transformations of Gauge.

We will see that if this is indeed the case for the second passage (from free particle to particle in an electromagnetic field), it is not a priori the case for the first passage (from particle without mass to particle with mass).

IV.5.2 Free Dirac equation of a massless particle in the reference frame R^0 (the vacuum)

We remind the free Dirac equation of a massless particle in a reference frame R^0 (similar to vacuum):

$$(\beta[\partial_{ct}]_{R^0} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^0})\psi = 0$$

By elevating this equation squared, we find the d'Alembert wave equation in R^0 :

$$([\partial_{ct}^2]_{R^0} - [\partial_x^2]_{R^0})\psi = 0$$

Reminder about the matrices used

We remind the 3 Pauli matrices 2×2 used by W. Pauli to introduce spin into the Schrödinger equation (we will come back to this):

$$\begin{aligned} I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \sigma^1 = \sigma^x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^2 = \sigma^y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma^3 = \sigma^z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

We remind the matrices 4×4 β and α^i defined from the 3 Pauli matrices:

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \quad i = 1, 2, 3$$

We also frequently use matrices 4×4 γ^0 et γ^i :

$$\begin{aligned} \gamma^0 = \beta &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \\ \gamma^i = \gamma^0 \alpha^i &= \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \end{aligned}$$

IV.5.3 Free Dirac equation of a relativistic particle with mass (electron type)

In 1928, when P. Dirac proposed his equation, his objective was not to obtain an equation that raised to square, allowed to find the D'Alembert wave equation, but an equation that raised squared, allowed to find the relativistic equation of dispersion of A. Einstein:

$$E = \sqrt{p^2 c^2 + m^2 c^4}$$

That is, if we translate as a function of wave, we have the Klein Gordan wave equation:

$$\Delta\psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

In the context of relativistic wave mechanics of the electron, P. Dirac proposes the wave equation of a particle with mass (electron type), called the free Dirac equation:

$$(\beta[\partial_{ct}]_{R^c} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^c} + i\frac{mc}{\hbar})\psi = 0$$

Nota: the reference frame R^c is always the basic reference frame, but it is no longer a question of vacuum reference frame R^0 because there is a mass m .

It can be written in abbreviated format:

$$(\gamma^\mu [\partial_\mu]_{R^c} + i\frac{mc}{\hbar})\psi = 0$$

The wave function ψ used by P. Dirac is here a spinor with 4 components (we also speak of bispineur with two components each). Like the non-relativistic Pauli wave equation proposed a few months earlier, the Dirac equation includes the notion of spin via the 3 Pauli matrices.

By elevating the free Dirac equation squared, we obtain the free Klein Gordan wave equation:

$$([\partial_{ct}^2]_{R^c} - [\partial_x^2]_{R^c} + \frac{m^2 c^2}{\hbar^2})\psi = 0$$

$$([\partial_\mu^2]_{R^c} + \frac{m^2 c^2}{\hbar^2})\psi = 0$$

$$\text{with } \partial_\mu^2 = \partial_{ct}^2 - \nabla^2, \mu = t, x, y, z$$

Note 1, brief explanation of P. Dirac's method

P. Dirac is looking for a wave equation that is squared, making it possible to find:

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

$$\text{ou } E = \sqrt{p^2 c^2 + m^2 c^4}$$

In 1928, he assumed the existence of coefficients. A, B, C, D that meet this objective and that verify the conditions:

$$\nabla^2 - \frac{1}{c^2} \partial_t^2 = (A\partial_x + B\partial_y + C\partial_z + \frac{i}{c} D\partial_t)(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c} D\partial_t)$$

$$AB + BA = 0$$

$$A^2 = B^2 = \dots = 1$$

$$(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t)\psi = k\psi$$

$$(\nabla^2 - \frac{1}{c^2}\partial_t^2)\psi = k^2\psi$$

He finds for A, B, C, D some matrices 4×4 equal to:

$$A = i\beta\alpha^1$$

$$B = i\beta\alpha^2$$

$$C = i\beta\alpha^3$$

$$D = \beta$$

Note 2 on the passage from d'Alembert to Klein Gordan (i.e., from free Dirac without mass to free Dirac with mass)

To switch from free Dirac without mass to free Dirac with mass, the term $i\frac{mc}{\hbar}$ is added. We would like to equate this addition to a transformation of Local Gauge from R^0 to R^c :

$$\beta[\partial_{ct}]_{R^c} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^c} = \beta[\partial_{ct}]_{R^0} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^0} - i\frac{m}{\hbar}c$$

Unfortunately, it does not work because we do not find in the term $i\frac{m}{\hbar}c$ a potential momentum energy. We perceive here an unclarified point of relativistic quantum theory, because the basic reference frame is not R^0 (the vacuum), but a reference frame R^c which already contains a mass.

IV.5.4 Dirac equation of a relativistic particle with mass (electron type) in an electromagnetic field

We remind the covariant derivative during a local gauge transformation from R^c to R^{A^μ} , with A^μ the cancelled (or subtracted) electromagnetic potential quadrivector, and $\frac{q}{\hbar}$ the quantity retained:

$$[\partial_\mu]_{R^{A^\mu}} = [\partial_\mu]_{R^c} - i\frac{q}{\hbar}A^\mu$$

with $\mu = t, x, y, z$

For a free electron, we have the Dirac equation in R^c :

$$(\beta[\partial_{ct}]_{R^c} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^c} + i\frac{m}{\hbar}c)\psi = 0$$

For an electron in an electromagnetic field, we have the Dirac equation in R^{A^μ} :

$$(\beta([\partial_{ct}]_{R^{A^\mu}} + i\frac{q}{\hbar}A^t) + \beta\vec{\alpha} \circ ([\vec{\partial}_x]_{R^{A^\mu}} + i\frac{q}{\hbar}A^x) + i\frac{m}{\hbar}c)\psi = 0$$

Note on the Lagrangian and the Euler-Lagrange equation

Previously, we highlighted the conceptual links between Newtonian mechanics and local gauge theories used in quantum physics.

Let us now observe that from the Lagrangians L and the Euler-Lagrange equation, we can find both:

- the invariant equation to be respected of Newtonian mechanics: $\dot{p}_i = F_i$ (with $i = x, y, z$),
- the invariant equation to be respected by relativistic quantum physics: $\Delta\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi$.

Historically, the Lagrangian L was introduced in 1788 by Joseph-Louis Lagrange for a new formulation of Newtonian mechanics based on the principle of least action. Since then, Lagrangian has been used in many physical theories, especially in Quantum Physics.

In Newtonian mechanics, we define a Lagrangian $L = E_c - E_p = T - V$ (difference between kinetic energy and potential energy).

The application of the Euler-Lagrange equation to this Lagrangian makes it possible to find the Newton's fundamental principle of dynamics:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$$

We have the generalized amount of movement:

$$\vec{p}_i = \frac{\partial L}{\partial \dot{q}_i}$$

We have the generalized force:

$$F_i = \frac{\partial L}{\partial q_i}$$

We find:

$$\dot{p}_i = F_i$$

In relativistic quantum physics, we define a Lagrangian density:

$$L(q_i, \dot{q}_i, t) \rightarrow L(\varphi, \partial_\mu \varphi, x_\mu)$$

$$\text{With } \mu = t, x, y, z$$

Lagrangian density $L(\varphi, \partial_\mu \varphi, x_\mu)$ also verifies the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial L}{\partial (\partial_\mu \varphi)} \right) - \frac{\partial L}{\partial \varphi} = 0$$

We have the Lagrangian density of a free fermion (by abuse of language, we simply speak of Lagrangian of a free fermion):

$$L_{free} = i\hbar c \bar{\psi} \gamma_\mu \partial^\mu \psi - mc^2 \bar{\psi} \psi$$

Applying to this Lagrangian the Euler-Lagrange equation, we find the Dirac wave equation:

$$(i\gamma_\mu \partial^\mu - \frac{mc}{\hbar})\psi$$

By squared, we find the Equation of Klein Gordan:

$$\Delta\psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

IV.6 Schrödinger wave equations (1925), non-relativistic electron without spin

IV.6.1 General

In non-relativistic quantum physics, the wave equation used as an invariant is the free Schrödinger equation:

$$\Delta\psi - \frac{i2m}{\hbar} \frac{\partial \psi}{\partial t} = 0$$

We will see that by a local gauge transformation, we pass from the free Schrödinger equation to the usual Schrödinger equation.

Note

We are here in the non-relativistic case, because we place ourselves implicitly in a reference frame where the electron particle is immobile in Space and where the electron wave is standing.

IV.6.2 Free Schrödinger wave equation

We start from the dispersion relationship (non-relativistic) connecting the energy to the momentum:

$$E = \frac{p^2}{2m}$$

Using the usual substitutions of quantum mechanics, we find in R^c the free Schrödinger equation:

$$i\hbar[\partial_t]_{R^c} = \frac{1}{2m}(-i\hbar[\partial_x]_{R^c})^2$$

Note

R^c is here also a reference where there remains a mass m . It differs a priori from that seen in the relativistic case.

IV.6.3 Schrödinger wave equation

We perform the transformation of Local Gauge (change of reference frames from R^c to R^{A^μ}):

$$\begin{aligned} [\psi(\vec{x}, t)]_{R^{A^\mu}} &= e^{i\frac{q}{\hbar}\theta(\vec{x}, t)} [\psi(\vec{x}, t)]_{R^0} \\ [A^\mu]_{R^{A^\mu}} &= [A^t]_{R^c} - \partial_\mu \theta(\vec{x}, t) = -\partial_\mu \theta(\vec{x}, t) \end{aligned}$$

As before, covariant derivatives are defined:

$$\begin{aligned} [\partial_t]_{R^{A^\mu}} &= [\partial_t]_{R^c} - i\frac{q}{\hbar}[A^t]_{R^{A^\mu}} \\ [\partial_x]_{R^{A^\mu}} &= [\partial_x]_{R^c} - i\frac{q}{\hbar}[A^x]_{R^{A^\mu}} \end{aligned}$$

In R^{A^μ} , we obtain the wave equation:

$$\begin{aligned} i\hbar([\partial_t]_{R^{A^\mu}} + i\frac{q}{\hbar}[A^t]_{R^{A^\mu}}) &= \frac{1}{2m}(-i\hbar([\partial_x]_{R^c} + i\frac{q}{\hbar}[A^x]_{R^{A^\mu}}))^2 \\ i\hbar[\partial_t]_{R^{A^\mu}} - q[A^t]_{R^{A^\mu}} &= \frac{1}{2m}(-i\hbar[\partial_x]_{R^{A^\mu}} + q[A^x]_{R^{A^\mu}})^2 \end{aligned}$$

We find in R^{A^μ} the Schrödinger wave equation (with the addition of the term $q[A^x]_{R^{A^\mu}}$ which corresponds to a potential magnetic momentum):

$$i\hbar[\partial_t]_{R^{A^\mu}} = \frac{1}{2m}(-i\hbar[\partial_x]_{R^{A^\mu}} + q[A^x]_{R^{A^\mu}})^2 + q[A^t]_{R^{A^\mu}}$$

IV.7 Pauli wave equations (1927), non-relativistic electron with spin

IV.7.1 Pauli non-relativistic wave equation, an electron with a spin in a magnetic field

In 1922, Otto Stern and Walther Gerlach noted the influence of the magnetic field on silver atoms. In 1925, to explain this phenomenon (and others such as the fine doubling of lines in spectroscopy), S. Goudsmit and G. Uhlenbeck proposed that particles are endowed with a kinetic moment on itself and a magnetic moment on itself. They call this new property the spin.

In 1927, W. Pauli modified Schrödinger's electron model and incorporated the notion of spin. In Pauli's non-relativistic wave equation, W. Pauli describes an electron with a spin in an outer magnetic field.

From the three Paul matrices 2×2 $\sigma^1, \sigma^2, \sigma^3$, W. Pauli defines a spin kinetic moment operator of the electron:

$$\vec{S} = \frac{\hbar}{2} \vec{\sigma}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

From this spin kinetic moment operator $\vec{S} = \frac{\hbar}{2} \vec{\sigma}$, W. Pauli defines a spin magnetic moment operator of the electron:

$$\vec{\mu}_S = \frac{e \hbar}{m_e} \frac{\vec{\sigma}}{2}$$

W. Pauli works on matrices 2×2 . He defines for the wave function ψ a spinor with 2 components: ψ_1 and ψ_2 (each component being of the same type as the Schrödinger wave function).

The magnetic moment of spin can be likened to a small magnet. Let be the potential energy of interaction between the magnetic moment of spin of the electron and an exterior magnetic field \vec{B} in which the electron is immersed:

$$E_p = \vec{\mu}_S \cdot \vec{B} = \frac{e \hbar}{m_e} \frac{\vec{\sigma}}{2} \cdot \vec{B}$$

We obtain the Pauli wave equation here written as operators:

$$\left(\frac{1}{2m_e} (\vec{P} + e\vec{A}^x)^2 - eA^t + \frac{e \hbar}{m_e} \frac{\vec{\sigma}}{2} \cdot \vec{B} - E \right) \psi = 0$$

In this Pauli wave equation is:

- on the one hand the energy of a particle without spin immersed in an electromagnetic field:

$$E_1 = \frac{1}{2m_e} (\vec{P} + e\vec{A}^x)^2 - eA^t$$

, i.e., the Schrödinger equation part,

- on the other hand, the interaction energy between the spin magnetic moment and the exterior magnetic field: \vec{B}

$$E_2 = \vec{\mu}_S \cdot \vec{B} = \frac{e \hbar}{m_e} \frac{\vec{\sigma}}{2} \cdot \vec{B}$$

, that is, the part added by W. Pauli.

Note 1 on the spin

It should be noted that the notion of spin is not specifically relativistic. Indeed, it is found in both the Dirac wave equation and the Pauli wave equation. It appears via the constant $\frac{\hbar}{2}$ and Pauli matrices σ_1 , σ_2 , σ_3 which are present in the Dirac wave equation and in the Pauli wave equation.

Note 2 on relativistic and not relativistic

It is difficult to understand why it is necessary to distinguish two cases: one relativistic with the Klein Gordan and Dirac wave equations in an electromagnetic field, the other "classical" or non-relativistic with the Schrödinger and Pauli wave equations.

The non-relativistic case is often presented as a borderline case of the relativistic case. The passage is still non-trivial between Pauli wave equation and Dirac wave equation. Instead of a borderline case of each other, shouldn't we rather see two different cases qualitatively?

Note 3 on the sign of the magnetic vector potential

In this essay, an opposite sign is used for the magnetic vector potential in order to have the same type of construction for the electric field and the magnetic field (we will come back to this in the next memoir). So, we have for the Pauli wave equation:

$$\left(\frac{1}{2m_e}(\vec{P} - e\vec{A}^x)^2 - eA^t + \frac{e}{m_e} \frac{\hbar}{2} \vec{\sigma} \cdot \vec{B} - E\right)\psi = 0$$

IV.7.2 Local Gauge Transformation

Let's go in R^{A^μ} from the Schrödinger wave equation:

$$i\hbar[\partial_t]_{R^{A^\mu}} = \frac{1}{2m} (-i\hbar[\partial_x]_{R^{A^\mu}} + q[A^x]_{R^{A^\mu}})^2 + q[A^t]_{R^{A^\mu}}$$

Let's take for the potential momentum:

$$q[A^x]_{R^{A^\mu}} = -e\vec{A}^x$$

and for potential energy:

$$q[A^t]_{R^{A^\mu}} = -eA^t + \frac{e}{m_e} \frac{\hbar}{2} \vec{\sigma} \cdot \vec{B}$$

We thus find the Pauli wave equation from that of Schrödinger:

$$i\hbar[\partial_t]_{R^{A^\mu}} = \frac{1}{2m} (-i\hbar[\partial_x]_{R^{A^\mu}} - e\vec{A}^x)^2 - eA^t + \frac{e}{m_e} \frac{\hbar}{2} \vec{\sigma} \cdot \vec{B}$$

IV.8 Conclusion of the Memoir

In this 1st memoir, we wanted to emphasize that from Newtonian mechanics and local gauge theories used in quantum physics, emerged a very general principle whose objective was the invariance of laws of Nature during changes of reference frames or during transformations of local gauge.

In the case of Newtonian mechanics, to preserve this invariance, when cancelling a rotation, we add the centrifugal and Coriolis inertial forces. In the case of local gauge theories, to preserve this invariance, when cancelling a phase of the wave function, a quadrivector potential momentum energy associated with an interaction energy is added. In the case of electromagnetic interaction, the interaction energy is precisely equal to the generalized potential energy from which the electromagnetic force derives.

In the next memoir, still with this objective of invariance, we will try to show that when we cancel "generalized rotations", we must add both the centrifugal and Coriolis inertial forces, and the electromagnetic force.

We will also think about a more general theory that would encompass changes of reference frames and transformations of Local Gauge.

Bibliography of the memoir 1

On the theory of relativity, the principle of invariance of the laws of Nature and the gravitational field

- [1] A. Einstein, Relativity. Gauthier-Villars, Paris, 1956 for the French-language edition, 1916 for the 1st German-language edition
- [2] A. Einstein, Physics, philosophy, politics, texts chosen and commented by Françoise Balibar. Seuil, Paris, 2002 (for paperback edition)

On inertial forces in Newtonian mechanics

- [3] H. Lumbroso, problème résolu de mécanique du point et des systèmes de ponts. Dunod, Paris, 1984

On the model of the Bohr electron

- [4] N. Bohr, On the Constitution of Atoms and Molecules. Philosophical Magazine 26, 1913, p.1-24
- [5] R. Didier, Chimie générale. Technique et documentation, Paris, 1984

On the model of the electron de Broglie

- [6] L. de Broglie, thèse de 1924
- [7] L. de Broglie, The wave nature of the electron. Nobel Lecture, 1929
- [8] L. de Broglie, Physique nouvel et quanta. Bibliothèque de philosophie scientifique, Paris, 1937
- [9] L. de Broglie, Eléments de théorie des quanta et de mécanique ondulatoire, Paris, Gauthier-Villars, 1953

On the model of the Schrödinger electron

- [10] Alain Laverne, A propos de l'équation de Schrödinger. chapitre II, Paris
- [11] R. Didier, Chimie générale. Technique et documentation, Paris, 1984
- [12] Erwin Schrödinger, Collected Papers On Wave Mechanics, London, 1928, translated from the German edition (1926)

On the model of the Pauli electron

- [13] Wikipedia, Pauli equation
- [14] Jean Hladik, Michel Chrysos, Pierre-Emmanuel Hladik, Lorenzo Ugo Ancarani, Mécanique quantique Atomes et noyaux Applications technologiques. Dunod, Paris, 1997

On the model of the Dirac electron

- [15] Wikipedia, Dirac equation

On the theories of Local Gauge

- [16] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [17] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007

Table of Contents

Memoir 1 Changes of reference frames and transformations of Local Gauge	21
Chapter I Why make any “normal” force correspond to an inertial force?	22
I.1 Historical review of the march of ideas, A. Einstein, H. Weyl, Chen Ning Yang and Robert Mill, transformation, invariance, and addition of “quantities of inertia”	22
I.2 Why do there exist privileged reference frames (called Galilean) in Newtonian mechanics?	24
I.3 Analogy Time and Space, can we treat Time in the same way as Space?	25
I.4 Conclusion of the chapter	26
Chapter II Reminders on the inertial forces in Newtonian mechanics	27
II.1 From the objectives of Special Relativity to those of General Relativity	27
II.2 Reminder of some invariants by the Lorentz transform	28
II.3 General information on inertial forces in so-called "general" Newtonian mechanics	29
II.4 Local equivalence between a gravitational field and an acceleration	30
II.5 Examination of centrifugal and Coriolis inertial forces	32
II.6 Conclusion of the chapter	35
Chapter III Bohr, de Broglie and Schrödinger electron models	36
III.1 The mysterious spectral lines of hydrogen	36
III.2 Ernest Rutherford's Planetary Model of the Atom (1911)	37
III.3 Model of the Bohr electron in the hydrogen atom	37
III.4 Ideas of L. de Broglie, the electron at the same time wave and particle	43
III.5 Schrödinger electron wave equation	45
Chapter IV Local gauge theories, applications to different wave functions (from Dirac 1928, Schrödinger 1925 to Pauli 1927)	52
IV.1 General information on Local Gauge theories	52
IV.2 Some reminders about wave equations	54
IV.3 Simple study of a local gauge transformation, case of Time	57
IV.4 Simple study of a local gauge transformation, case of Space	60
IV.5 From the D'Alembert wave equation (1746) to the Dirac wave equation (1928) in an electromagnetic field	61
IV.6 Schrödinger wave equations (1925), non-relativistic electron without spin	64
IV.7 Pauli wave equations (1927), non-relativistic electron with spin	66
IV.8 Conclusion of the Memoir	67
Bibliography of the memoir 1	69

Memoir 2 "Generalized" rotation vectors and fields in a spatiotemporal plane

Summary of the memoir

This 2nd memoir will propose more conjunctures than the 1st. We will return to the analogies between gravitation and electrostatics (works of the late 18th, early 19th century), then between fluid mechanics and magnetism (works of the 19th century), finally between mechanics of continuous media and relativistic quantum Electrodynamics (works of the middle of the 20th century).

From these analogies, we will propose the notions of pulsation field $\vec{\Omega}$ and wave vector field \vec{K} . These two fields will apply respectively in a spatial plane and in a spatiotemporal plane. Subsequently, we will try to build any force of classical physics on the model of the Coriolis inertial force, with during a change of reference frames a conserved quantity and a cancelled quantity are gotten. This cancelled quantity is similar to a "generalized" rotation vector (before changing reference frames) or a field (after changing reference frames). This will match the cancellation of "generalized" rotations with the addition of forces in the fundamental principle of dynamics.

This memoir will also be an opportunity to return to the physics of the 19th century, physics that was particularly involved in electricity and magnetism. Among others will be evoked the law of Charles-Augustin Coulomb for electrostatics (1785), the equation of Simeon Denis Poisson (1813) (which A. Einstein was largely inspired for his theory of General Relativity), the work of Michael Faraday on magnetic induction (from 1821), the magnetic force of Pierre-Simon Laplace (from 1820) precursor of the magnetic force of Hendrik Antoon Lorentz (work of the end of the 19th century).), as well as the Gustave Coriolis inertial force (1835).

Chapter I Newtonian gravitational and Electrostatic Coulomb analogies

Purpose of the chapter

We recall the main analogies between gravitation and electrostatics, analogies that have often allowed a mutual enrichment of these two disciplines of physics.

I.1 Newtonian gravitational potential and Coulomb electric potential

I.1.1 Historical reminders on gravitation

The theory of gravitation developed during the 17th century on the basis of astronomical observations. The first notable advances are due to Johannes Kepler who from 1609 to 1618 set out his three laws on the movements of the planets.

The 1st law states that the trajectory of the planets around the Sun is elliptical, with the Sun occupying one of the 2 foci of the ellipse.

The 2nd law states that the areas swept by the vector radius of a planet, are swept in equal durations.

The 3rd law states that the square of the period T of revolution of the planet, varies like the cube of the semi-major axis a of the ellipse:

$$\frac{T^2}{a^3} = \text{constant}$$

In 1687, nearly 80 years after J. Kepler, Isaac Newton published in Latin probably the most famous work of Physics. This is *Philosophiae naturalis principia mathematica*, often referred to simply as *Principia*: the Principles. In the book, based on the physical and mathematical principles laid down by I. Newton, Kepler's 3 laws are found and explained. Truly revolutionary, *Principia* shows that the nature of the universe can be described in a simple way by mathematical equations.

In *Principia*, I. Newton unifies two notions that a priori have nothing to do with each other:

- the elliptical motion of the planets (Kepler's 1st law),
- the fall of bodies under the effect of a gravitational force that decreases with the distance in $\frac{1}{r^2}$ from the source (a mass) according to Newton's universal law of gravitation:

$$\vec{F} = -G \frac{m_1 m_2}{r^2} \vec{u}_r$$

I. Newton brings together movements and forces via his famous principle of dynamics (also called Newton's 2nd law):

$$\vec{p} = \Sigma \vec{F}$$

Note on the unification of I. Newton

In a way, I. Newton unified 2 a priori distinct physical phenomena and made them correspond to the same physical notion called force. He was able to compare these 2 phenomena.

The first phenomenon is the outward repulsion to which a body is subjected during a rotational movement (or more generally an elliptical movement). To quantify this phenomenon, we can place ourselves in the rotating reference frame and use the centrifugal inertial force.

The second phenomenon is the attraction to which two mass bodies are subject to each other. To quantify this phenomenon, I. Newton introduced the gravitational force.

So, we do not compare cabbages and carrots, but 2 phenomena that are of the same physical nature: strength.

Some minds (of which A. Einstein was a part) were still saddened that we compare 2 things that are not exactly of the same nature: a gravitational force and a centrifugal inertial force, the latter qualified by some as a *pseudo force* because added during a change of reference frames.

By proposing in this essay, that any force is reduced to a *pseudo force* added during a change of reference frames, we evacuate the distinction between force and pseudo force. Important conclusions can also be drawn from this on the notion of mass.

I.1.2 Historical reminders on electrostatics

At the end of the 18th, beginning of the 19th, electricity and in particular electrostatics progressed rapidly on the basis of laboratory or field experiments. The pioneers are for example Ewald Georg von Kleist with the so-called Leiden bottle in 1745 (ancestor of the capacitor) or Luigi Galvani with the electric current (concept of animal electricity).

Although electrical observations are frequently made in the laboratory and are not turned towards the stars, a little curiously, theoretical developments of electricity are often carried out in analogy with Newtonian gravitation. New concepts are proposed, such as potential, which enriches both electrostatic theory and gravitational theory.

In the 1780s, using torsional balances, Charles-Augustin Coulomb carried out experiments on electric charges. Like the gravitational force, he measures an electrostatic force that decreases with the distance in $\frac{1}{r^2}$ from the source (an electric charge). Nevertheless, this force is not always attractive: it is repulsive when electric charges are of the same nature and attractive when they are of opposite nature. We have:

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r$$

Also in the 1780s, Pierre-Simon de Laplace developed the notion of potential. He shows that in a vacuum, a potential satisfies the differential equation:

$$\Delta\varphi^t = \nabla^2\varphi^t = \frac{\partial^2\varphi^t}{\partial x^2} + \frac{\partial^2\varphi^t}{\partial y^2} + \frac{\partial^2\varphi^t}{\partial z^2} = 0$$

with Δ the Laplacian.

In 1813, Siméon Denis Poisson continued the work of P-S. Laplace and published a article on the Newtonian scalar potential. Like his predecessors, he relies on analogies between electrostatics and gravitation to develop the notion of a potential that decreases in $\frac{1}{r}$ from a point of origin. For example, we have for the Newtonian gravitational potential:

$$V^t = -G \frac{m_1}{r}$$

and for the Coulomb electric potential:

$$A^t = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

It is also in this article that Poisson's source potential equation is introduced:

$$f = \Delta\varphi^t$$

with f a function representing the source, and φ^t a scalar potential.

In the 1830s, Carl Friedrich Gauss took over the work of S-D. Fish. It is also inspired by the work of Leonhard Euler on fluid mechanics (1750s, field of fluid flow velocities), M. Faraday on magnetism (1830s, magnetic field viewable on oriented iron filings).

With others, C. F. Gauss generalized the fertile notion of field to gravitation and electrostatics. He proposes that a density of mass or electric charge (corresponding to the source), generate respectively a divergent gravitational field and an electric field. He thus gives their modern form to the so-called Gaussian source field relations.

I.1.3 Summary of analogies

We remind in the following table the main analogies between the laws of Newtonian gravitation and Coulomb electrostatics.

	Newtonian gravitation	Electrostatics
Source	Mass m^t	Electric charge q^t
Constant	$-4\pi G$	$\frac{1}{\epsilon_0}$
Potential	$V^t = -G \frac{m_1}{r}$	$A^t = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$
Potential energy	$Ep = m_2 V^t$ $Ep = -G \frac{m_1 m_2}{r}$	$Ep = q_2 A^t$ $Ep = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$
Field	$Gr = -\frac{\partial V^t}{\partial r}$ $\vec{G}r = -G \frac{m_1}{r^2} \vec{u}_r$	$Es = -\frac{\partial A^t}{\partial r}$ $\vec{E}s = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \vec{u}_r$
Force	$\vec{F} = m_2 \vec{G}r$ $\vec{F} = -G \frac{m_1 m_2}{r^2} \vec{u}_r$	$\vec{F} = q_2 \vec{E}s$ $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r$
Potential source relationship by Poisson	$4\pi G \rho_{mas} = \Delta V^t$	$-\frac{\rho_{élec}}{\epsilon_0} = \Delta A^t$
Field source relationship by Gauss	Integral: $\oiint_{\Sigma} \vec{G}r(M) \cdot d^2 \vec{S} = - \iint_V 4\pi G \rho_{mas} dV$ $= -4\pi G M_{int}$ Local: $div \vec{G}r = -4\pi G \rho_{mas}$	Integral: $\oiint_{\Sigma} \vec{E}s(M) \cdot d^2 \vec{S} = \iint_V \frac{\rho_{élec}}{\epsilon_0} dV = \frac{Q_{int}}{\epsilon_0}$ Local: $div \vec{E}s = \frac{\rho_{élec}}{\epsilon_0}$

Note on a potential speed quadrivector

Like the electric potential A^t which is the temporal component of an electromagnetic potential quadrivector: (A^t, A^x, A^y, A^z) , it will be proposed in this memoir that the Newtonian potential V^t is the temporal component of a potential velocities quadrivector: (V^t, V^x, V^y, V^z) . Velocities V^x, V^y, V^z can be interpreted as "speeds in Space", the Newtonian potential V^t could be interpreted as a "speed in Time".

I.2 The notions of energy and power in gravitation and electrostatics

My wife is a physics teacher. To explain the difference between power and energy, she uses a metaphor that has always seemed instructive and pedagogical to me. This metaphor is all the more useful because it also highlights the similarities between gravitation and electrostatics. Indeed, in these two disciplines we find the same two notions: power and energy.

Let's take the example of a 10-storey building served by a staircase and an elevator. A young woman is on the ground floor and has to go up to the 10th floor with her packages. The same energy will be spent by the young woman whether she uses the elevator or the stairs.

On the other hand, the power will be different. With the elevator, the power used will be much greater than with the staircase. Indeed, the duration of climb will be much shorter with the elevator than with the staircase.

We list the different notions that we find both in gravitation and electrostatics.

Number of packets

Masse dm ou charge dq

Packet flow

Mass flow $I_m = \frac{dm}{dt}$ or electric current $I_q = \frac{dq}{dt}$

Height dr of the building

Field

Newtonian gravitational field: Gr

Electrostatic field: Es

Difference in potentials

Difference in Newtonian gravitational potentials

$$dV^t = \vec{G}r \cdot d\vec{r}$$

Difference in electrical potentials (voltage)

$$U = dA^t = \vec{E}s \cdot d\vec{r}$$

Force

Gravitational force:

$$\vec{F}_{Gr} = dm \times \vec{G}r$$

Electrostatic force:

$$\vec{F}_{Es} = dq \times \vec{E}s$$

Power

Gravitational power:

$$P_{Gr} = I_m \times dV^t = \frac{dm}{dt} \times dV^t$$

$$P_{Gr} = \frac{dm}{dt} \times \vec{G}r \cdot d\vec{r} = \vec{F}_{Gr} \cdot \vec{v}$$

$$\text{with } \vec{v} = \frac{d\vec{r}}{dt}$$

Electric power:

$$P_{Es} = I_q \times U = I_q \times dA^t = \frac{dq}{dt} \times dA^t$$

$$P_{Es} = \frac{dq}{dt} \times \vec{E}s \cdot d\vec{r} = \vec{F}_{Es} \cdot \vec{v}$$

Energy

Gravitational energy:

$$dE_{Gr} = P_{Gr} \times dt = dm \times dV^t = dm \times \vec{G}r \cdot d\vec{r}$$

$$dE_{Gr} = \vec{F}_{Gr} \cdot d\vec{r}$$

Electrical energy:

$$dE_{Es} = P_{Es} \times dt = dq \times dA^t = dq \times \vec{E}s \cdot d\vec{r}$$

$$dE_{Es} = \vec{F}_{Es} \cdot d\vec{r}$$

I.3 Conclusion of the chapter

While gravitation and electrostatics have developed in very different experimental terrains, the observation of planets for the former, laboratory or field experiments for the latter, the theories describing them share many common notions, such as those of energy, power, source, field or decreasing potential from source in $\frac{1}{r}$. Electrical and gravitational theories have often influenced each other and allowed mutual enrichment.

The notion of electric current proposed, among others, by L. Galvani, was inspired by the notion of fluid. In the next chapter, we see that magnetism has particularly developed in analogy with fluid mechanics.

Chapter II Analogies fluid mechanics and magnetism, analogies $\vec{\Omega}$ and \vec{B}

Purpose of the chapter

At the beginning of the 19th century, parallel to electrostatics, another area of research is booming, it is magnetism. With the law of Biot and Savart (1820), we find a magnetic potential that decreases in $\frac{1}{r}$ such as the Newtonian gravitational potential and the Coulombian electric potential. However, the similarities between magnetism and gravitation are much less strong than between electrostatics and gravitation. This time, it is mainly through analogies between fluid mechanics and magnetism that this discipline will progress.

In this chapter, we will remind the main analogies between fluid mechanics (as well as mechanics in the broad sense) and magnetism, between the vortex vector $\vec{\Omega}$ or rotation vector $\vec{\Omega}$ and the magnetic field \vec{B} .

We are interested in the rotation vector $\vec{\Omega}$, because its cancellation during a change of reference frames is related to the addition of inertial forces.

We will end with a reminder of the generalized electromagnetic potential energy, which from the Euler-Lagrange equation, makes it possible to find the electromagnetic force.

II.1 Law of Biot and Savart (1820), analogies gravitation, electrostatics and magnetism

In the 1820s, the Danish Hans Christian Ørsted discovered a link between electricity and magnetism through experiments that remained famous. For example, he observes that a wire carrying an electric current is able to move the magnetic needle of a compass. That is, an electric current can be the source of a magnetic field capable of moving the needle.

A few months later, following a series of experiments, Jean-Baptiste Biot and Félix Savart propose the law giving the magnetic field created by a distribution of electric currents. As for the gravitational field and the electric field, the 2 experimenters find a magnetic field that decreases in Space in $\frac{1}{r^2}$ from a source of electrical currents. The law differs from other fields, however, since there is a vector product:

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int_C \frac{Id\vec{l} \wedge \vec{u}_r}{r^2}$$

For a volume distribution of currents, we have:

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \iint_{\tau} \frac{\vec{I}d\tau \wedge \vec{u}_r}{r^2}$$

To be compared with the electrostatic and gravitational fields:

$$\vec{E}_s(r) = \frac{1}{4\pi\epsilon_0} \iint_{\tau} \frac{\rho_{elec}d\tau \times \vec{u}_r}{r^2}$$

$$\vec{G}_r(r) = -G \iint_{\tau} \frac{\rho_{mas}d\tau \times \vec{u}_r}{r^2}$$

We have the following analogies, less numerous than between gravitation and electrostatics:

	Newtonian gravitation	Electrostatics	Magnetism
Source	Mass m	Charge q	Current I
Constant	$-4\pi G$	$\frac{1}{\epsilon_0}$	μ_0
Potential	$V^t = -G \frac{m}{r}$	$A^t = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$	$\vec{A}^{x,y,z} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l}}{r}$
Field	$Gr = -gradV^t$ $\vec{G}r(r)$ $= -G \iint_{\tau} \frac{\rho_{mas} d\tau \times \vec{u}_r}{r^2}$	$Es = -gradA^t$ $\vec{E}s(r)$ $= \frac{1}{4\pi\epsilon_0} \iint_{\tau} \frac{\rho_{elec} d\tau \times \vec{u}_r}{r^2}$	$\vec{B}(r) = r \dot{\vec{A}}^{x,y,z}$ $\vec{B}(r)$ $= \frac{\mu_0}{4\pi} \iint_{\tau} \frac{\vec{I} d\tau \wedge \vec{u}_r}{r^2}$
Poisson, source potential relationship	$4\pi G \rho_{mas} = \Delta V^t$	$-\frac{\rho_{elec}}{\epsilon_0} = \Delta A^t$	$-\mu_0 \vec{j} = \Delta \vec{A}$

Note

By integrating the law of Biot and Savart on any closed loop Γ (which a priori is not an electrical circuit), we find Ampère's theorem which describes the relationship between a magnetic field and an electric current:

$$\oint_{\Gamma} \vec{B}(\vec{r}) \cdot d\vec{r} = \mu_0 I_{int}$$

II.2 Analogies between fluid mechanics and magnetism**II.2.1 Historical reminders**

In the 1st half of the 19th century, research on magnetism made great progress, in particular thanks to the work of Michael Faraday and André-Marie Ampère who were both interested in two different areas of magnetism.

A.-M. Ampère studies the correspondence between the loop of electric current and the magnetic field, which is grouped together in magnetostatics.

M. Faraday studies the correspondence between the variation of the flux of the magnetic field through a circuit and the appearance of an induced electric current in this circuit, which is grouped together in induction phenomena.

In the 1860s, in a vast synthesis, James Clerk Maxwell transcribed the work of M. Faraday and A.-M. Ampère into mathematical equations. For this synthesis, J. C. Maxwell is frequently inspired by fluid mechanics. For example, it matches the magnetic field to the speed of a fluid.

Let us now turn to two analogies between fluid mechanics and magnetism.

The first is between the vortex vector $\vec{\Omega}$ and the current $\mu_0 \vec{j}$, both considered *sources*, as well as between the velocity vector \vec{v} and the magnetic field \vec{B} , both considered *as fields*. We place ourselves in the case of magnetostatics, where generally the sources (currents or magnets) are known, and where the magnetic field \vec{B} generated by these sources is constant in Time.

The second is between the vortex vector $\vec{\Omega}$ and the magnetic field \vec{B} , both considered *as fields*.

In the rest of this memoir, we will rely mainly on the 2nd analogy (between $\vec{\Omega}$ and \vec{B}). The first analogy, however, retains all its interest, because it indicates this ambivalence between a physical quantity (here $\vec{\Omega}$) which can be both *source* and *field*.

II.2.2 Analogy between vortex vector $2\vec{\Omega}$ and current $\mu_0\vec{j}$ both considered as sources, and between \vec{v} and \vec{B} both considered as fields

In 1893, Henri Poincaré published a book on fluid mechanics entitled *Théorie des Tourbillons* (*Tourbillon or vortex or whirlpool theory*). The book is the result of lessons professed in 1891-92. Inspired by the work of J. C. Maxwell, H. Poincaré leads analogies between hydrodynamics (rotational, permanent, and incompressible flow) and magnetostatics. In a related article, he proposes the following analogies.

Let be $\vec{v}(M)$ the local velocity at a point M, of an incompressible perfect fluid characterized by a vortex vector $\vec{\Omega}$. Let be \vec{j} the electric current density vector.

Since the fluid is incompressible, we have $div\vec{v} = 0$.

According to $div\vec{v} = 0$, we can derive $\vec{v}(M)$ from a vector potential \vec{A} with $\vec{v} = r\partial t\vec{A}$.

According to Maxwell-Thomson (or Maxwell-flux), we have: $div\vec{B} = 0$.

We can derive $\vec{B}(M)$ from a vector potential \vec{A} with $\vec{B} = r\partial t\vec{A}$.

The analogies of H. Poincaré are summarized in the table below.

	Hydrodynamics (rotational, permanent, incompressible)	Magnetostatic
Potential	\vec{A}	\vec{A}
Source	$2\vec{\Omega}$	$\mu_0\vec{j}$
Field	$\vec{v}(M)$	$\vec{B}(M)$
Conservation equation	$div\vec{v} = 0$	$div\vec{B} = 0$
Potential field relationship	$\vec{v} = r\partial t\vec{A}$	$\vec{B} = r\partial t\vec{A}$
Potential source relationship	$2\vec{\Omega} = -\Delta\vec{A}$	$\mu_0\vec{j} = -\Delta\vec{A}$
Source field relationship	$2\vec{\Omega} = r\partial t\vec{v}$	$\mu_0\vec{j} = r\partial t\vec{B}$

Note

In the case of a uniform vector $\vec{\Omega}$, we have:

$$\vec{v} = \vec{\Omega} \wedge \vec{OM}$$

$$2\vec{\Omega} = r\partial t\vec{v} = r\partial t(\vec{\Omega} \wedge \vec{OM})$$

In the case of a uniform electric current $\mu_0\vec{j}$, we have:

$$\vec{B} = \frac{1}{2}\mu_0\vec{j} \wedge \vec{OM}$$

$$\mu_0 \vec{j} = r \vec{\partial} t \vec{B} = \frac{1}{2} r \vec{\partial} t (\mu_0 \vec{j} \wedge \vec{O}M)$$

II.2.3 Analogy between vortex vector $\vec{\Omega}$ and magnetic field \vec{B} both considered as fields

The second analogy is to bring together:

- on the one hand the local velocity $\vec{V}(M)$ with magnetic vector potential \vec{A} (we rewrite $\vec{v}(M)$ in capital letters $\vec{V}(M)$ to highlight the link with potential),
- on the other hand the vortex vector $\vec{\Omega} = \frac{1}{2} r \vec{\partial} t \vec{V}$ with the magnetic field $\vec{B} = r \vec{\partial} t \vec{A}$.

These analogies are summarized in the table below.

	Fluid mechanics	Magnetostatic
Potential	\vec{V}	\vec{A}
Source	$div \vec{\Omega} = 0$	$div \vec{B} = 0$
Field	$2\vec{\Omega} = r \vec{\partial} t \vec{V}$ $2\Omega_{xy}^{1/t} = \frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}$	$\vec{B} = r \vec{\partial} t \vec{A}$ $B_{xy}^{1/t} = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$
Conservation equation		$\mu_0 \vec{j} + \Delta \vec{A} = 0$
Potential field relationship	An analogy will be proposed later.	$\mu_0 \vec{j} = r \vec{\partial} t \vec{B} - (\mu_0 \varepsilon_0 \frac{\partial \vec{E}l}{\partial t})$ $\mu_0 j_{yt}^x = \frac{\partial B_{yx}^{1/t}}{\partial y} - (\mu_0 \varepsilon_0 \frac{\partial E_{tx}^{1/y}}{\partial t})$ $\mu_0 j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x} - (\mu_0 \varepsilon_0 \frac{\partial E_{ty}^{1/x}}{\partial t})$ <p>(Maxwell Ampere's relation in a space x, y, t with 3 dimensions)</p>

Note 1

Displacement currents of type $\vec{j}_D = \varepsilon_0 \frac{\partial \vec{E}l}{\partial t}$ are absent in magnetostatics. For this reason, they are indicated in parentheses in the table above.

Note 2

We will come back in a few paragraphs on the notations of the type $\Omega_{xy}^{1/t}$, $B_{xy}^{1/t}$ or $E_{ty}^{1/x}$ used here.

II.2.4 Identification vortex vector $\vec{\Omega}$ and rotation vector $\vec{\Omega}$

In fluid mechanics, a vortex vector is defined $\vec{\Omega} = \frac{1}{2} r \vec{\partial} t \vec{V}$.

When the material points of the fluid share the same circular motion, the vortex vector $\vec{\Omega}$ identifies with the speed of rotation ω of material points. Let's check this for uniform circular motion in a spatial plane x, y.

Let be the vectors position and speed of the material points:

$$\vec{M} \begin{pmatrix} \cos \omega t \\ \sin \omega t \\ z \end{pmatrix}$$

$$\vec{V}(M, t) \begin{pmatrix} -\omega \sin \omega t \\ \omega \cos \omega t \\ 0 \end{pmatrix}$$

with $\omega = \frac{d\theta}{dt}$ the rotational speed of the material points.

Under another writing, we have:

$$\vec{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\vec{V}(M, t) \begin{pmatrix} -x\omega \\ y\omega \\ 0 \end{pmatrix}$$

We calculate the 3 components of the vortex vector $\vec{\Omega} = \frac{1}{2} r \vec{\partial} t \vec{V}$

$$\Omega_z = \Omega_{xy}^{1/z} = \frac{1}{2} \left(\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y} \right) = \frac{1}{2} (\omega - -\omega) = \omega$$

$$\Omega_y = \Omega_{xz}^{1/y} = 0$$

$$\Omega_x = \Omega_{yz}^{1/x} = 0$$

We actually obtain an identification between the vortex vector $\vec{\Omega} = \frac{1}{2} r \vec{\partial} t \vec{V}$ and rotational speed $\omega = \frac{d\theta}{dt}$:

$$\vec{\Omega} = \vec{\Omega}_{xy}^{1/z} = \omega \vec{e}_z$$

Note that $\vec{\Omega}$ is perpendicular to the plane of rotation x, y . In a three-dimensional space, we usually define the vectors vortexes or rotations $\vec{\Omega}$ (as well as kinetic moment) as perpendicular to the plane of rotation.

II.3 Analogies between rotation vector $\vec{\Omega}$ and magnetic field \vec{B}

In the following paragraphs, we remind the main analogies between a rotation vector $\vec{\Omega}$ and a magnetic field \vec{B} .

II.3.1 Larmor relationship

At the end of the 1890s, Joseph Larmor noted that for usually achievable magnetic field values, if the electron is placed on a carousel that rotates at the speed of rotation:

$$\vec{\Omega} = \frac{e}{2m_e} \vec{B}$$

with e the electric charge (in absolute value) of the electron and m_e its mass, then the movement of the electron is as if the electron is not rotating and the magnetic field \vec{B} do not exist. There is compensation between the effects of the magnetic field and the effects of the ride.

From a reference frame change point of view, the Larmor relation can be interpreted:

$$R^{\Omega+B} = R^0$$

with R^0 a Galilean reference frame.

II.3.2 Analogy between potential velocity \vec{V} and magnetic potential \vec{A}

The magnetic field \vec{B} derives from a magnetic potential \vec{A} :

$$\vec{B} = r \partial_t \vec{A}$$

In three dimensions, x, y, t, we have:

$$B_{xy}^{1/t} = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

Similarly, the rotation vector $\vec{\Omega}$ derives from a potential velocity \vec{V} :

$$2\vec{\Omega} = r \partial_t \vec{V}$$

In three dimensions, x, y, t, we have:

$$2\Omega_{xy}^{1/t} = \frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}$$

Note 1

The notation $\Omega_{xy}^{1/t}$ indicates that the rotation is done in the plane x, y. In three dimensions, x, y, t, $\Omega_{xy}^{1/t}$ is Time-oriented and expresses itself in $rad \times s^{-1}$.

By analogy, we pose for $B_{xy}^{1/t}$ the same notation, even if the unit of the magnetic field is the tesla and not the s^{-1} .

Note 2

We will come back later on this notion of potential velocity \vec{V} . The idea to remember is that a potential velocity corresponds to a velocity cancelled after a change of reference frames.

II.3.3 Analogy between the velocity potential of a uniform circular motion and the magnetic potential of a uniform magnetic field

We have the relationship between the linear velocity (here in the form of a potential velocity \vec{V}^s) and the rotation vector $\vec{\Omega}^{1/t}$ of a uniform circular movement and the radius \vec{r} (r, s, t orthogonal three-dimensional coordinate system):

$$\vec{V}^s = \vec{\Omega}^{1/t} \wedge \vec{r}$$

Similarly, we have the relationship between the magnetic vector potential, the uniform magnetic field and the radius \vec{r} :

$$\vec{A}^s = \frac{1}{2} \vec{B}^{1/t} \wedge \vec{r}$$

II.3.4 Reminders on the orbital magnetic moment of a magnetic dipole

In magnetism, we define the orbital magnetic moment $\vec{\mu}$ a magnetic dipole, which can be interpreted as a small magnet.

In the case of a rotating charged particle q , we have the orbital magnetic moment (perpendicular to the plane of rotation of the charged particle):

$$\vec{\mu} = \frac{1}{2} \vec{r} \wedge q \vec{v}$$

In integral form, on a closed outline C , we get if \vec{r} always perpendicular to $q \vec{v}$:

$$\vec{\mu} = \left(\oint_C \frac{1}{2} r q \vec{v} \cdot d\vec{l} \right) \vec{n}$$

with \vec{n} unit vector perpendicular to the plane of rotation.

We also define the orbital magnetic moment $\vec{\mu}$ a magnetic dipole in the form of:

$$\vec{\mu} = \iint_S I d\vec{s} = I \vec{S}$$

with I the electric current

\vec{S} and $d\vec{s}$ perpendicular to surfaces S and ds ,

$\vec{n} ds = d\vec{s}$ unit vector perpendicular to surface ds .

Note 1

We want to show the identity of the 2 definitions of the orbital magnetic moment $\vec{\mu}$.

$$\oint_C \frac{1}{2} r q \vec{v} \cdot d\vec{l} = \iint_S I ds ?$$

If C is a closed outline with perimeter $2\pi r$ and S a disk with surface πr^2 , we obtain the relationships:

$$\frac{1}{2} r q \vec{v} \cdot \oint_C d\vec{l} = I \iint_S ds ?$$

$$\frac{1}{2} r q v \times 2\pi r = I \times \pi r^2 ?$$

We have the electric current:

$$qv = I$$

We find:

$$\vec{\mu} = \left(\oint_C \frac{1}{2} r q \vec{v} \cdot d\vec{l} \right) \vec{n} = \iint_S I d\vec{s} = I \vec{S}$$

Note 2

The passage from $\oint_C \frac{1}{2} r q \vec{v} \cdot d\vec{l}$ to $\iint_S I ds$ is gotten via Stockes' theorem.

In local form, we have:

$$\frac{1}{2} r \partial t (r q \vec{v}) = I \vec{n}$$

According to Stockes' theorem we have:

$$\oint_C \frac{1}{2} r q \vec{v} \cdot d\vec{l} = \iint_S \frac{1}{2} r \vec{\partial} t (r q \vec{v}) \cdot d\vec{s} = \iint_S I ds$$

II.3.5 Analogy between orbital kinetic moment and orbital magnetic moment

We remind the orbital kinetic moment of a particle of mass m :

$$\vec{\sigma} = \vec{r} \wedge m\vec{v}$$

We have the relationship between

- the orbital magnetic moment $\vec{\mu}$ of a particle with charge q , mass m
- and orbital kinetic moment $\vec{\sigma}$ of this particle (a sign $-$ is introduced):

$$\vec{\mu} = -\frac{q}{2m}\vec{\sigma}$$

We often use $\vec{\mu} = \gamma\vec{\sigma}$ with $\gamma = -\frac{q}{2m}$ the gyromagnetic ratio.

We have the relationship between the orbital magnetic moment $\vec{\mu}_{Bohr}$ of an electron with charge $-e$, mass m_e and its orbital kinetic moment $\vec{\sigma}_e$:

$$\vec{\mu}_{Bohr} = \frac{e}{2m_e}\vec{\sigma}_e$$

(so-called Bohr-Procopiu relationship with $\vec{\mu}_{Bohr}$ called the Bohr magneton).

According to the Bohr electron model, kinetic moment $\vec{\sigma}_e$ is quantified:

$$\vec{\sigma}_e = n\hbar\vec{z}$$

We obtain the magnetic moment of the electron in its most stable state ($n = 1$):

$$\vec{\mu}_{Bohr} = \frac{e\hbar}{2m_e}\vec{z}$$

Note

We have a comparable analogy between the kinetic moment of spin \vec{S} and the magnetic moment of spin $\vec{\mu}_S$:

$$\vec{\mu}_S = g\frac{q}{2m}\vec{S}$$

with spin kinetic moment $\vec{S} = \pm\frac{1}{2}\hbar\vec{z}$ and g the Landé factor.

For the electron, we have the Landé factor $g \approx -2$.

II.3.6 Potential energy of a magnetic dipole as a function of \vec{B} and $\vec{\Omega}$

We have the potential energy of a magnetic dipole $\vec{\mu}$ in an external magnetic field \vec{B} :

$$Ep = -\vec{\mu} \cdot \vec{B}$$

Note

We can also interpret Ep as an interaction energy between a magnetic dipole and an exterior magnetic field \vec{B} .

In memoirs 4 and 5, when we look at elementary particles, the magnetic dipole will be brought closer to the fermions and the exterior magnetic field to the photons.

We have the Larmor relation for the electron:

$$\vec{B} = \frac{2m_e}{e} \vec{\Omega}$$

We have the Bohr-Procopiu magneton for the electron:

$$\vec{\mu} = \frac{e}{2m_e} \vec{\sigma}_e$$

We then have the potential energy of an electron magnetic dipole in an exterior magnetic field \vec{B} :

$$Ep = -\frac{e}{2m_e} \vec{\sigma}_e \cdot \frac{2m_e}{e} \vec{\Omega}$$

$$Ep = -\vec{\sigma}_e \cdot \vec{\Omega}$$

With $\vec{\sigma}_e = \hbar \vec{z}$, we obtain:

$$Ep = -\hbar \Omega$$

Note 1

We find a relationship close to the energy of a photon proposed by A. Einstein:

$$E = h\nu = \frac{h}{2\pi} 2\pi\nu = \hbar\Omega$$

At the beginning of his thesis, L. de Broglie generalized Einstein's relationship applied to photons, to mass particles such as the electron. For this, he brings together two famous equations proposed by A. Einstein. He equalizes the energies of a standing wave with pulsation $\Omega_0 = 2\pi\nu_0$ and that of corpuscle at rest with mass m_0 :

$$E_0 = h\nu_0 = \hbar\Omega_0 = m_0c^2$$

We will come back to these equations in memoir 3 on the source field equations.

Note 2

Both the photon and the electron have a spin kinetic moment \vec{S} . On the other hand, the electron is the only one to possess a charge and a mass, and therefore a magnetic moment of spin $\vec{\mu}_S = g \frac{q}{2m} \vec{S}$. If a photon is not subject to the effects of a magnetic field \vec{B} , an electron considered as a small magnet of magnetic moment with a spin $\vec{\mu}_S$, is oriented according to \vec{B} for a minimization of the moment with force $\vec{M} = \vec{\mu}_S \wedge \vec{B}$.

II.3.7 Analogy between the Coriolis force and the Lorentz magnetic force

We have the Coriolis force:

$$\vec{F}_{Cor} = m\vec{v} \wedge 2\vec{\Omega}$$

and the Lorentz magnetic force:

$$\vec{F}_{Lor} = q\vec{v} \wedge \vec{B}$$

Both are perpendicular:

- to a momentum $\vec{p} = m\vec{v}$ or to a current $\vec{j} = q\vec{v}$,
- to a rotation vector $\vec{\Omega}$ or to a magnetic field \vec{B} .

II.3.8 Potential energy and potential momentum

For gravitation and electrostatics, we define a potential energy that derived (with respect to Space) makes it possible to find respectively the gravitational force of Newton and the electrostatic force of Coulomb.

$$\vec{F}_{Gr} = -\frac{\partial E p_{Gr}^t(\vec{r}, \vec{v}, t)}{\partial \vec{r}} = -\frac{\partial m^t V^t}{\partial \vec{r}}$$

$$\vec{F}_{Es} = -\frac{\partial E p_{Es}^t(\vec{r}, \vec{v}, t)}{\partial \vec{r}} = -\frac{\partial q^t A^t}{\partial \vec{r}}$$

Note on Energy $E p^t$

Energy $E p^t$ is here considered as a time-oriented vector component.

In magnetism, the Lorentz magnetic force does not work. Its potential energy is constant, and the magnetic force cannot be found by deriving it.

It is the same in mechanics with the Coriolis force, analogous to the Lorentz magnetic force, which does not work. Its potential energy is constant, and the Coriolis force cannot be found by deriving it.

Nevertheless, in Electromagnetism, we define a generalized potential that makes it possible to find via the Euler Lagrange equation the Lorentz electromagnetic force (including the Lorentz magnetic force). This will be detailed in the next paragraph.

In addition, from the magnetic vector potential A^μ ($\mu = x, y, z$) and from the electrical charge q^t , we define a magnetic potential momentum. We can do the same for Newtonian mechanics where we define a gravitational potential momentum from the velocity potential V^μ ($\mu = x, y, z$) and from the mass m^t .

We have the following table:	Gravitation	Electromagnetism
Potential energy	$E p_{Gr}^t = m^t V^t$	$E p_{Es}^t = q^t A^t$ (electrostatic)
Potential momentum in the x direction	$p^x = m^t V^x$	$p^x = q^t A^x$
Potential momentum in the y direction	$p^y = m^t V^y$	$p^y = q^t A^y$
Potential momentum in the z direction	$p^z = m^t V^z$	$p^z = q^t A^z$

Note: it can be seen that q^t and m^t are both Time-oriented. We will come back to it extensively.

II.4 Reminders on generalized electromagnetic potential energy

II.4.1 Generalized potential energy of the Lorentz electromagnetic force

A magnetic dipole $\vec{\mu}$ has a magnetic potential energy $Ep = -\vec{\mu} \cdot \vec{B}$ when it is placed in a magnetic field \vec{B} .

Nevertheless, as mentioned in the previous paragraph, the Lorentz magnetic force and the Coriolis inertial force do not work. The potential energies involved are therefore constant. These two forces cannot be derived (with respect to Space) from a potential energy.

However, there is a generalized electromagnetic potential energy in the sense of the Euler-Lagrange equations:

$$Ep = q(A^t - (\vec{v}^\mu \cdot \vec{A}^\mu))$$

$$\text{with } \mu = x, y, z$$

Note: for generalized electromagnetic potential energy, we also abusively speak of generalized electromagnetic potential. But it is indeed an energy because we include the electric charge q .

From this generalized potential, it is possible to find the Lorentz electromagnetic force.

We will see the demonstration in the next paragraph.

II.4.2 Demonstration

In the case of a non-conservative force, called generalized because it derives from a generalized potential energy Ep , we have the relationship:

$$\vec{F} = \frac{d}{dt} \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{v}} - \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{r}}$$

We try to show that from $Ep = q(A^t - (\vec{v}^\mu \cdot \vec{A}^\mu))$, we find again the Lorentz electromagnetic force:

$$\vec{F}_{Lor} = q(\vec{E}l + \vec{v} \wedge \vec{B})$$

According to the equations of Maxwell Thomson (zero divergence of the magnetic flux) and Maxwell Faraday, we have:

$$\vec{B} = \vec{\nabla} \wedge \vec{A}$$

$$\vec{\nabla} \wedge \vec{E}l = -\frac{\partial \vec{B}}{\partial t}$$

Therefore:

$$\vec{\nabla} \wedge \vec{E}l = -\frac{\partial \vec{\nabla} \wedge \vec{A}}{\partial t} = \vec{\nabla} \wedge \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\vec{\nabla} \wedge \left(\vec{E}l + \frac{\partial \vec{A}}{\partial t}\right) = 0$$

$$\vec{E}l + \frac{\partial \vec{A}}{\partial t} = \vec{\nabla} A^t$$

$$\vec{F}_{Lor} = q(-\vec{\nabla}A^t - \frac{\partial \vec{A}}{\partial t} + \vec{v} \wedge (\vec{\nabla} \wedge \vec{A}))$$

And according to Willard Gibbs' formula:

$$\vec{v} \wedge (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla}(\vec{v} \cdot \vec{A}) - (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A}$$

So, we have for the Lorentz electromagnetic force:

$$\vec{F}_{Lor} = q(-\vec{\nabla}A^t - \frac{\partial \vec{A}}{\partial t} \vec{\nabla}(\vec{v} \cdot \vec{A}) - (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A})$$

$$\vec{F}_{Lor} = -q(\frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A}) + q(-\vec{\nabla}A^t + \vec{\nabla}(\vec{v} \cdot \vec{A}))$$

$$\vec{F}_{Lor} = -q(\frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A}) + q\vec{\nabla}(-A^t + (\vec{v} \cdot \vec{A}))$$

$$\vec{F}_{Lor} = -q(\frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A}) - \frac{\partial q(A^t - (\vec{v} \cdot \vec{A}))}{\partial \vec{r}}$$

By introducing: $Ep = q(A^t - (\vec{v} \cdot \vec{A}))$, we obtain:

$$\vec{F}_{Lor} = -q(\frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{A}) - \frac{\partial Ep}{\partial \vec{r}}$$

We have:

$$\frac{\partial Ep}{\partial \vec{v}} = -q\vec{A}$$

Therefore:

$$\frac{d}{dt} \frac{\partial Ep}{\partial \vec{v}} = -q \frac{d\vec{A}}{dt}$$

With:

$$d\vec{A} = \frac{\partial \vec{A}}{\partial t} dt + \frac{\partial \vec{A}}{\partial x} dx + \frac{\partial \vec{A}}{\partial y} dy + \frac{\partial \vec{A}}{\partial z} dz$$

$$\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \frac{\partial \vec{A}}{\partial x} \dot{x} + \frac{\partial \vec{A}}{\partial y} \dot{y} + \frac{\partial \vec{A}}{\partial z} \dot{z}$$

So, we have:

$$\frac{d}{dt} \frac{\partial Ep}{\partial \vec{v}} = -q \frac{\partial \vec{A}}{\partial t} - q(\frac{\partial \vec{A}}{\partial x} \dot{x} + \frac{\partial \vec{A}}{\partial y} \dot{y} + \frac{\partial \vec{A}}{\partial z} \dot{z})$$

We can notice in passing:

$$\begin{aligned} \frac{\partial \vec{A}}{\partial x} \dot{x} + \frac{\partial \vec{A}}{\partial y} \dot{y} + \frac{\partial \vec{A}}{\partial z} \dot{z} &= \begin{pmatrix} \dot{x} \frac{\partial A_x}{\partial x} + \dot{y} \frac{\partial A_x}{\partial y} + \dot{z} \frac{\partial A_x}{\partial z} \\ \dot{x} \frac{\partial A_y}{\partial x} + \dot{y} \frac{\partial A_y}{\partial y} + \dot{z} \frac{\partial A_y}{\partial z} \\ \dot{x} \frac{\partial A_z}{\partial x} + \dot{y} \frac{\partial A_z}{\partial y} + \dot{z} \frac{\partial A_z}{\partial z} \end{pmatrix} = \left(\dot{x} \frac{\partial}{\partial x} + \dot{y} \frac{\partial}{\partial y} + \dot{z} \frac{\partial}{\partial z} \right) \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \\ &= \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \end{aligned}$$

Therefore:

$$\begin{aligned} \frac{d\vec{A}}{dt} &= \frac{\partial \vec{A}}{\partial t} + \left(\frac{\partial \vec{A}}{\partial x} \dot{x} + \frac{\partial \vec{A}}{\partial y} \dot{y} + \frac{\partial \vec{A}}{\partial z} \dot{z} \right) = \frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A} \\ \frac{d}{dt} \frac{\partial Ep}{\partial \vec{v}} &= -q \frac{d\vec{A}}{dt} = -q \left(\frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \cdot \vec{A} \right) \end{aligned}$$

Therefore:

$$\vec{F}_{Lor} = \frac{d}{dt} \frac{\partial q(A^t - (\vec{v} \cdot \vec{A}))}{\partial \vec{v}} - \frac{\partial q(A^t - (\vec{v} \cdot \vec{A}))}{\partial \vec{r}}$$

satisfies the relationship seen above.

$$\vec{F}_{Lor} = \frac{d}{dt} \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{v}} - \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{r}}$$

Note 1

We remind the Lagrangian of the Lorentz electromagnetic force:

$$L = \frac{1}{2} m \vec{v}^2 - q(A^t - (\vec{v} \cdot \vec{A})) = Ec - Ep$$

Note 2

We remind the general Euler-Lagrange equation for a generalized force that does not derive from a potential:

$$\vec{F} = \frac{d}{dt} \frac{\partial Ec(\vec{r}, \vec{v}, t)}{\partial \vec{v}} - \frac{\partial Ec(\vec{r}, \vec{v}, t)}{\partial \vec{r}}$$

with Ec kinetic energy.

We find the Euler-Lagrange equation for the electromagnetic force by equalizing the two terms, one with potential energy, the other with kinetic energy:

$$\vec{F}_{Lor} = \frac{d}{dt} \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{v}} - \frac{\partial Ep(\vec{r}, \vec{v}, t)}{\partial \vec{r}}$$

$$\vec{F}_{Lor} = \frac{d}{dt} \frac{\partial Ec(\vec{r}, \vec{v}, t)}{\partial \vec{v}} - \frac{\partial Ec(\vec{r}, \vec{v}, t)}{\partial \vec{r}}$$

$$\frac{d}{dt} \frac{\partial Ec - Ep}{\partial \vec{v}} - \frac{\partial Ec - Ep}{\partial \vec{r}} = 0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \vec{v}} - \frac{\partial L}{\partial \vec{r}} = 0$$

II.5 Conclusion of the chapter

In this chapter, it was pointed out that Electromagnetism had developed in analogy with fluid mechanics. We have also listed the many analogies that exist between the rotation vector $\vec{\Omega}$ (or vortex vector $\vec{\omega}$) and the magnetic field \vec{B} .

The analogies were made in the 3 dimensions of Space x, y, z . In the next chapter, we will extend these analogies to the 4th temporal dimension t by treating Time as a dimension of Space.

Chapter III Analogies between the mechanics of continuous media and relativistic quantum electrodynamics

Purpose of the chapter

We study the analogies between:

- continuous media mechanics (CMM), extension of fluid mechanics to all types of continuous media,
- relativistic quantum electrodynamics.

Relativistic quantum Electrodynamics is a theory developed in the mid-20th century. It aims to reconcile the Electromagnetism of the 19th century with the relativistic quantum physics of the 1920s and 1930s. It uses a Lagrangian formalism.

In some respects, relativistic quantum Electrodynamics may appear as a theory using tools analogous to those of the mechanics of continuous media, with extension of the 3 dimensions of Space to the 4 dimensions of Space-time.

III.1 Historical presentation of relativistic quantum Electrodynamics (QED)

III.1.1 Symbiosis between Electromagnetism, Special Relativity and Quantum Physics

In the previous chapter, it was pointed out that the Electromagnetism of the 19th had progressed thanks to analogies with fluid dynamics.

At the beginning of the 20th century, A. Einstein developed the theory of Special Relativity, then the theory of General Relativity. The General Relativity presents itself as a generalization of Newtonian mechanics and describes the Newtonian gravitational field. For its mathematical tools, General Relativity takes up those of the mechanics of continuous media, an extension of fluid mechanics to other media.

Relativistic quantum Electrodynamics knows its main results in the years 1930-1950, in particular with the work of Paul Dirac, then with those of Richard Feynman (adapting in his thesis of 1942, the principle of least action to quantum mechanics).

We can consider relativistic quantum Electrodynamics as a direct extension of 19th century electromagnetism, with the addition of Planck's constant \hbar .

We can then interpret relativistic quantum Electrodynamics as a symbiosis between Electromagnetism, Quantum Physics and Special Relativity. Like Special Relativity, Relativistic quantum Electrodynamics includes the Lorentz transform and is based on a 4-dimensional Space-Time. It also goes far in the analogies between Time and Space, since Time is often presented with two senses, as a dimension of Space.

For this reason, in relativistic quantum electrodynamics, we have a focus on the principle of least action which implies a final cause or at least, which makes it possible to free oneself from the principle of causality.

III.1.2 Dirac relativistic wave equation as the fundamental law to be respected, absence of constant G

Relativistic quantum Electrodynamics stems mainly from the work of Paul Dirac in the 1920s. Thus, it is the relativistic wave equation of Dirac that serves as a fundamental law to be respected during the transformations of local gauge and not the non-relativistic wave equations of Schrödinger and Pauli.

Like quantum physics of the 1920s and 1930s, relativistic quantum Electrodynamics largely takes up the tools of analytical mechanics of Joseph-Louis Lagrange, Carl Gustav Jakob Jacobi and William

Rowan Hamilton. (late 18th century, early 19th century), analytical mechanics that historically had been presented as a reinterpretation of Newtonian mechanics, and which subsequently found many other applications.

Nevertheless, as a paradox, relativistic quantum Electrodynamics does not reconcile the Planck constant \hbar and the Newton constant G . That is, it does not incorporate into its equations neither Newtonian gravitation (in particular the gravitational force). $\vec{F} = -G \frac{m_1 m_2}{r^2} \vec{u}_r$, nor Einsteinian General Relativity (these two theories using the constant G). In contrast, Like General Relativity, relativistic quantum Electrodynamics takes up the tools of the mechanics of continuous media, in particular the 4-dimensional tensor tool.

These are the analogies between the tools of the mechanics of continuous media and the tools of relativistic quantum Electrodynamics, which we will study in this chapter.

Note

For relativistic quantum electrodynamics, the abbreviation QED is frequently used as quantum electrodynamics.

III.2 Reminders on the continuous media mechanics (CMM)

The continuous media mechanics studies the deformations of a continuous medium, often a fluid, sometimes a solid. For this reason, it is often interpreted as an extension of fluid mechanics. The continuous medium is characterized by the movements of material points in the three dimensions of Space.

III.2.1 Descriptions of Lagrange and Euler

We distinguish two descriptions for the movement of a material point, that of Joseph-Louis Lagrange and that of Leonhard Euler (2 famous mathematicians of the 18th century again gathered in a physical model). These two descriptions are briefly reminded here.

Description of Lagrange

We follow in its movement a fluid particle p that is at the point M and at the moment t .

We define for this particle p :

- a particle velocity (we talk about particulate velocity):

$$\vec{v}_p(t) = \frac{d\vec{r}_p}{dt}$$

- an acceleration of the particle:

$$\vec{a}_p(t) = \frac{d\vec{v}_p}{dt} = \frac{d^2\vec{r}_p}{dt^2}$$

This description is similar to that of Newtonian mechanics. It is not very used in CMM because it requires knowledge about each particle of the fluid. The following description, description of Euler, is more commonly used.

Description of Euler

We place ourselves in a fixed point of observation M and we measure at this point M , at the moments t , the local velocity of the particles $\vec{v}(M, t)$, function of independent variables: positions of M and t .

The set of local velocities, or velocity field $\vec{v}(M, t)$, describes the movement of the fluid.

For a fluid particle p that is at the observation point M at the moments t , we can confuse the local velocity and the velocity of the particle:

$$\vec{v}_p(t) = \vec{v}(M, t)$$

By contrast, the acceleration is different. We give the relationship between the acceleration of a particle and the local velocity:

$$\vec{a}_p(t) = \frac{d\vec{v}_p}{dt} = \frac{d\vec{v}(M, t)}{dt} + (\vec{v}(M, t) \cdot \text{grad})\vec{v}(M, t)$$

with $\frac{d\vec{v}(M, t)}{dt}$ the local acceleration and $(\vec{v}(M, t) \cdot \text{grad})\vec{v}(M, t)$ the convective acceleration.

We have:

$$(\vec{v}(M, t) \cdot \text{grad})\vec{v}(M, t) = \text{grad}\left(\frac{v^2}{2}\right) + \text{rot}\vec{v} \wedge \vec{v} = \text{grad}\left(\frac{v^2}{2}\right) + 2\vec{\Omega} \wedge \vec{v}$$

with $\text{rot}\vec{v} = 2\vec{\Omega}$

In convective acceleration, we recognize:

- $\text{grad}\left(\frac{v^2}{2}\right)$ which corresponds to the centrifugal inertial acceleration,
- $2\vec{\Omega} \wedge \vec{v} = \text{rot}\vec{v} \wedge \vec{v}$ which corresponds to the Coriolis inertial acceleration.

Note 1

We can interpret the passage from one to the other of these 2 descriptions, as a change of reference frames. The frame of reference according to Euler's description, follows the particle, and therefore cancels part of its motion. It requires the addition of inertial accelerations equal to:

$$(\vec{v}(M, t) \cdot \text{grad})\vec{v}(M, t) = \text{grad}\left(\frac{v^2}{2}\right) + 2\vec{\Omega} \wedge \vec{v}$$

Note 2

Following an analogy with the magnetic vector potential $\vec{A}(M, t)$, we define in this memoir a potential velocity vector $\vec{V}(M, t)$ corresponding to the movement cancelled during a change of reference frames. The potential term is adequate, because it is a movement that does not exist in the reference frame where it is cancelled, but which is potentially recoverable if the change of reference frame is reversed.

To highlight the resemblance between this potential velocity vector $\vec{V}(M, t)$ and the velocity field $\vec{v}(M, t)$ of the CMM, we will both write them in capital letters $\vec{V}(M, t)$.

III.2.2 Rotation rate tensor

In CMM, we define a tensor $\bar{\bar{L}}$ 3×3 gradient of local velocities $\vec{V}(M, t)$. It makes it possible to characterize the movements of the continuous medium.

The components of this tensor $\bar{\bar{L}}$ are $\text{Grad}(\vec{V}(M, t))$. We obtain:

$$L_{ij} = \frac{\partial V_i}{\partial x_j}$$

In the general case, the tensor $\bar{\bar{L}}$ is neither symmetrical nor antisymmetric. It is decomposed into:

- a symmetric tensor $\bar{\bar{D}} = \frac{1}{2}(\text{Grad}\vec{V}(M, t) + \text{Grad}\overleftarrow{V}(M, t))$ called a deformation rate tensor (or tensor of deformation speeds),
- an antisymmetric tensor $\bar{\bar{\Omega}} = \frac{1}{2}(\text{Grad}\vec{V}(M, t) - \text{Grad}\overleftarrow{V}(M, t))$ with a zero diagonal, and called a rotation rate tensor (or tensor of rotational speeds).

Note

Decomposition into deformation rates and rotation rates can be interpreted as a decomposition in variation of the norm of a vector and variation of its direction.

A material point in the continuous medium can deform and undergo **shears** that are characterized by:

$$D_{ij} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right)$$

A material point can also undergo **rotations** that are characterized by:

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial V_j}{\partial x_i} - \frac{\partial V_i}{\partial x_j} \right)$$

In the spatial plane x, y, we have the rotation (oriented in the direction of z):

$$\Omega_{xy}^z = \frac{1}{2} \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right)$$

We have the rotation rate tensor $\bar{\bar{\Omega}}$:

$$\bar{\bar{\Omega}} = \begin{bmatrix} 0 & \Omega_{xy}^z & \Omega_{xz}^y \\ -\Omega_{xy}^z & 0 & \Omega_{yz}^x \\ -\Omega_{xz}^y & -\Omega_{yz}^x & 0 \end{bmatrix}$$

The following diagram illustrates in two dimensions of Space, the deformations that a continuous medium can undergo:

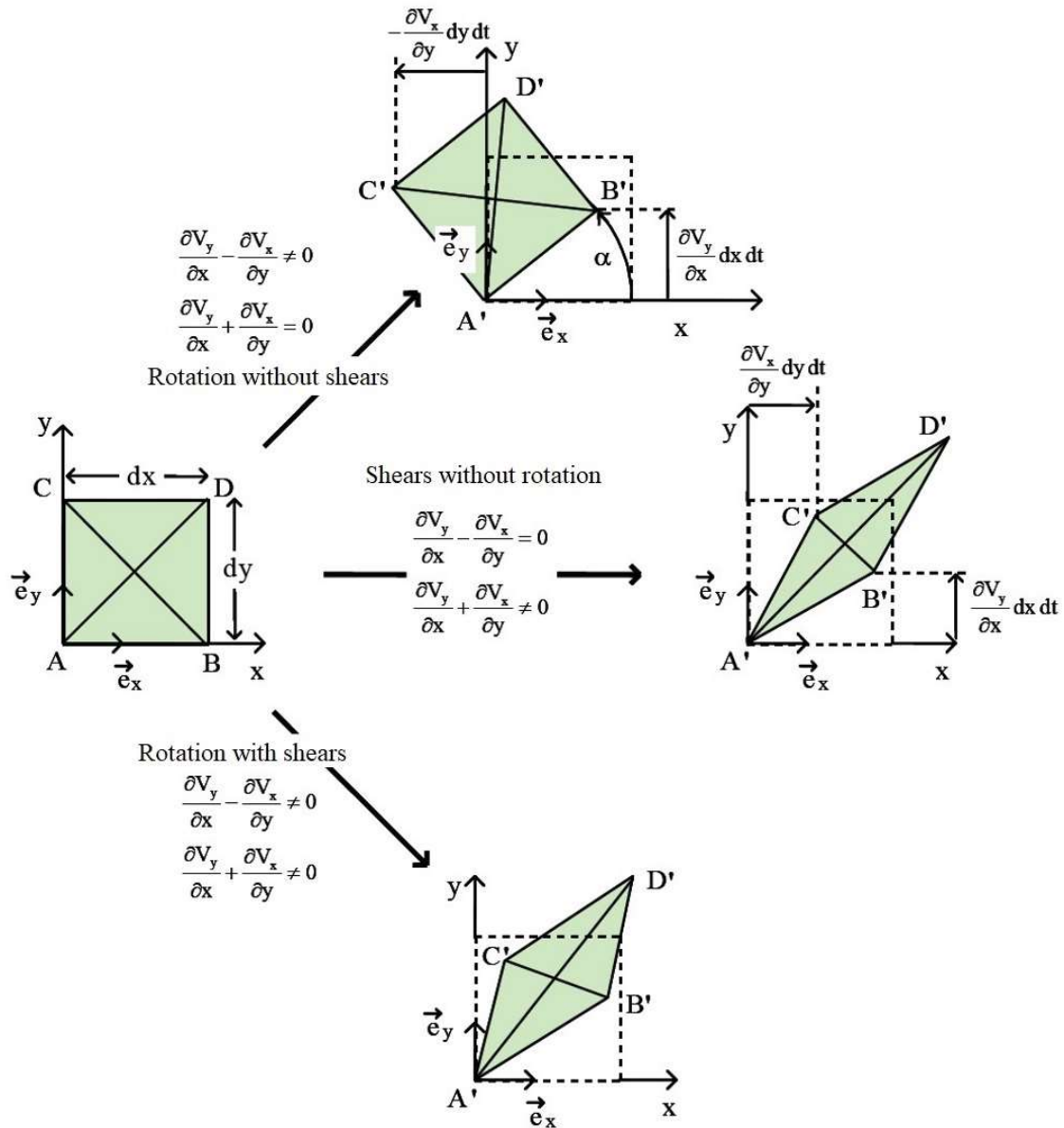


Figure 1: shears and rotations, from *Fluid Mechanics* by Jean-François Sini

III.3 Reminders on Maxwell Faraday's tensor, analogies between the tools of CMM and relativistic quantum Electrodynamics

III.3.1 Maxwell Faraday's tensor

Like CMM and General Relativity, relativistic quantum Electrodynamics uses the tensor tool. The most frequently used tensor is Maxwell Faraday's, noted here $\bar{\bar{F}}_{\mu\nu}$.

$\bar{\bar{F}}_{\mu\nu}$ is an antisymmetric tensor, describing the electromagnetic field. Despite its name inspired by two famous British physicists of the 19th century, it was proposed around the 1950s.

There are several forms for $\bar{\bar{F}}_{\mu\nu}$. One is presented below:

$$\bar{\bar{F}}_{\mu\nu} = \begin{bmatrix} 0 & -\frac{El_x}{c} & -\frac{El_y}{c} & -\frac{El_z}{c} \\ \frac{El_x}{c} & 0 & -B^z & B^y \\ \frac{El_y}{c} & B^z & 0 & -B^x \\ \frac{El_z}{c} & -B^y & B^x & 0 \end{bmatrix}$$

The components of the tensor $\bar{\bar{F}}_{\mu\nu}$ are defined from the electromagnetic quadrivector.

$$F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu = \frac{\partial A^\nu}{\partial \mu} - \frac{\partial A^\mu}{\partial \nu}$$

μ, ν correspond to t, x, y, z .

We propose to redefine the components of electric and magnetic fields on the model of tensor components $\bar{\bar{F}}_{\mu\nu}$ (it will be argued in the next chapter). We then have:

$$El_{tx} = F_{tx} = \partial_t A^x - \partial_x A^t = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

$$B_{xy} = F_{xy} = \partial_x A^y - \partial_y A^x = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

We obtain for the tensor:

$$\bar{\bar{F}}_{\mu\nu} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ \frac{El_{xt}}{c} & 0 & B_{xy} & B_{xz} \\ \frac{El_{yt}}{c} & B_{yx} & 0 & B_{yz} \\ \frac{El_{zt}}{c} & B_{zx} & B_{zy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ -\frac{El_{tx}}{c} & 0 & B_{xy} & B_{xz} \\ -\frac{El_{ty}}{c} & -B_{xy} & 0 & B_{yz} \\ -\frac{El_{tz}}{c} & -B_{xz} & -B_{yz} & 0 \end{bmatrix}$$

III.3.2 Lagrangian of a fermion in QED

In relativistic quantum Electrodynamics, we define the Lagrangian (or Lagrangian density) of a fermion (for example an electron) in an electromagnetic field (associated with a photon):

$$L = i\hbar c \bar{\psi} \gamma_\mu (\partial^\mu + iq c A^\mu) \psi - mc^2 \bar{\psi} \psi$$

$$L = \bar{\psi} (i\hbar c \gamma_\mu \partial^\mu - mc^2) \psi - qc \bar{\psi} \gamma_\mu \psi A^\mu$$

Note 1

In particle physics, fermions are the particles that undergo the field, as opposed to the bosons that generate the field. Fermions have a half-integer spin and respond to the Fermi-Dirac statistic. Bosons have an integer spin and respond to the Bose-Einstein statistic. For electromagnetic interaction, the boson is actually a photon. We will come back to this extensively in future memoirs.

Note 2

We have the quadrivector charge density electric currents:

$$j^\mu = (\rho^t, j^x, j^y, j^z)$$

$$\text{with } (\mu = t, x, y, z)$$

We have the generalized electromagnetic potential energy:

$$Ep = q^t(A^t - (\vec{v} \cdot \vec{A})) = j^\mu A_\mu$$

In QED, we define j^μ by:

$$j^\mu = qc\bar{\psi}\gamma_\mu\psi$$

We find in the Lagrangian of a fermion, the generalized electromagnetic potential energy:

$$qc\bar{\psi}\gamma_\mu\psi A^\mu = j^\mu A_\mu$$

By applying Euler-Lagrange $\frac{\partial L}{\partial x} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) = 0$ to the Lagrangian, we find the Dirac wave equation in an electromagnetic field:

$$(i\hbar c\gamma_\mu(\partial^\mu + iqcA^\mu) - mc^2)\psi = 0$$

From the Maxwell Faraday tensor, we define a Lagrangian that allows to include the creation and disappearance of photons:

$$L = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$$

By associating the two Lagrangians, we obtain the Lagrangian of relativistic quantum Electrodynamics:

$$L = \bar{\psi}(i\hbar c\gamma_\mu\partial^\mu - mc^2)\psi - qc\bar{\psi}\gamma_\mu\psi A^\mu - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$$

$$L = \bar{\psi}(i\hbar c\gamma_\mu\partial^\mu - mc^2)\psi - j^\mu A_\mu - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$$

III.3.3 Find Maxwell's source field equations from a Lagrangian

Let's define a Lagrangian simply from the generalized electromagnetic potential energy and Maxwell Faraday tensor:

$$L = -j^\mu A_\mu - \frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu}$$

$$L = -j^\mu A_\mu - \frac{1}{4\mu_0}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$L = -\frac{1}{2\mu_0}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\nu A_\mu \partial^\mu A^\nu) - j^\mu A_\mu$$

By applying Euler-Lagrange $\frac{\partial L}{\partial A_\nu} - \partial_\mu(\frac{\partial L}{\partial(\partial_\mu A_\nu)}) = 0$ to this Lagrangian, we find the two Maxwell's source field equations (Ampère and Gauss) in the form source tensor:

$$\mu_0 j^\mu = \partial_\mu F^{\mu\nu}$$

Note 1

According to the initial conventions taken on the constants μ_0 and ε_0 , we can also find the following source tensor relation:

$$\frac{j^\mu}{\varepsilon_0} = \partial_\mu F^{\mu\nu}$$

Note 2

Through these tensor source relationships, we note the construction symmetry for the 2 source field equations of Maxwell Ampère and Maxwell Gauss, symmetry not very visible in the equations proposed by Oliver Heaviside and Willard Gibbs at the end of the 19th century, but explicitly apparent here.

In memoir 3, we will also propose the same mode of construction for the 2 Maxwell's source field equations.

III.3.4 Analogies between CMM tools and those of the QED

Let us now highlight the analogies between:

- the tensor $\bar{\bar{\Omega}}_{\mu\nu}$ ($\mu, \nu = x, y, z$) of rotation rates in the three dimensions of Space,
- the Maxwell Faraday tensor $\bar{\bar{F}}_{\mu\nu}$ ($\mu, \nu = t, x, y, z$) in the four dimensions of Space-Time.

Both are antisymmetric and constructed from intersecting differences in potential derivatives.

$$\bar{\bar{\Omega}}_{\mu\nu} = \begin{bmatrix} 0 & \Omega_{xy}^z & \Omega_{xz}^y \\ -\Omega_{xy}^z & 0 & \Omega_{yz}^x \\ -\Omega_{xz}^y & -\Omega_{yz}^x & 0 \end{bmatrix}$$

$$\bar{\bar{F}}_{\mu\nu} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ -\frac{El_{tx}}{c} & 0 & B_{xy} & B_{xz} \\ -\frac{El_{ty}}{c} & -B_{xy} & 0 & B_{yz} \\ -\frac{El_{tz}}{c} & -B_{xz} & -B_{yz} & 0 \end{bmatrix}$$

$$\Omega_{xy}^z = \frac{1}{2} \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right)$$

$$El_{tx} = F_{tx} = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

$$B_{xy} = F_{xy} = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

Two observations are made:

- the tools of the QED are similar to an extension of 3 to 4 dimensions of the tools of the CMM with a consideration of the Time dimension,
- if we bring the rotation vector Ω closer to the magnetic field B , it is the electric field El which appears as the extension to the Time dimension.

Note 1

Note that to describe the weak and strong interactions, symmetric tensors similar to maxwell Faraday's are used in the Standard Model:

$$\bar{\bar{F}}_{\mu\nu}^a = \begin{bmatrix} 0 & El_x^a & El_y^a & El_z^a \\ -El_x^a & 0 & -B_z^a & B_y^a \\ -El_y^a & B_z^a & 0 & -B_x^a \\ -El_z^a & -B_y^a & B_x^a & 0 \end{bmatrix}$$

For strong interaction, with a parameter a ranging from 1 to 8, strong fields are defined El^a and B^a analogous to electric and magnetic fields.

For weak interaction, with a parameter a ranging from 1 to 3, weak fields are also defined El^a and B^a analogous to electric and magnetic fields.

Note 2

We can wonder about the antisymmetric character of all these tensors, especially since as we have seen in CMM, the tensor $\bar{\bar{L}}$ is in the general case neither symmetrical nor antisymmetric.

Only the rotation rate tensor $\bar{\bar{\Omega}}$ built from $\bar{\bar{L}}$ is symmetrical.

Some models suggest that the anti-symmetry of the tensors of relativistic quantum electrodynamics and the Standard Model, is related to the conservation of charges and mass.

The question remains open.

III.4 Conclusion of the chapter

Like General Relativity, relativistic quantum Electrodynamics uses 4-dimensional tensors of Space-Time, tools previously developed in the mechanics of continuous media in 3 dimensions of Space.

The Maxwell-Faraday electromagnetic tensor $\bar{\bar{F}}_{\mu\nu}$ explicitly reveals the formal similarities between the magnetic field $B_{xy} = F_{xy}$ and the electric field $El_{tx} = F_{tx}$. The electric field is built on exactly the same mode as the magnetic field, with an extension to the Time dimension.

In the next chapter, we will use this idea to define the electric field as a "generalized" rotation vector in a spatiotemporal plane.

Chapter IV "Generalized" rotation vectors and fields

Purpose of the chapter

We propose here to explain the notions:

- of "generalized" rotation vector taken in a broad sense (in a spatial plane and then in a spatiotemporal plane) that is cancelled during a change of reference frames,
- of the field obtained after the change of reference frames and cancellation of the "generalized" rotation vector.

We will define the notions of wave vector field $\vec{K}_{xt}^{1/y}$ and pulsation field $\vec{\Omega}_{xy}^{1/t}$. On the model of the electric field $\vec{E}l_{xt}^{1/y}$ extension to the Time dimension of the magnetic field $\vec{B}_{xy}^{1/t}$, the wave vector field $\vec{K}_{xt}^{1/y}$ will be the extension to the Time dimension of the pulsation field $\vec{\Omega}_{xy}^{1/t}$.

IV.1 Analogies between magnetic fields \vec{B} , electric field $\vec{E}l$ and rotation vector $\vec{\Omega}$

IV.1.1 Analogies between magnetic fields \vec{B} and electric field $\vec{E}l$

In the Electromagnetism of the 19th century, we conventionally derive the electric field $\vec{E}l$ and the magnetic field \vec{B} from an electric scalar potential A^t and from a magnetic vector potential \vec{A} .

We have the following equations where electric field and magnetic field appear formally different:

$$\vec{E}l = -grad(A^t) - \frac{\partial \vec{A}^\mu}{\partial t}$$

$$(\mu = x, y, z)$$

$$\vec{B} = r\partial t \vec{A}^\mu$$

We have for example for the component B_z of the magnetic field \vec{B} :

$$B_z = \frac{\partial A^x}{\partial y} - \frac{\partial A^y}{\partial x}$$

During the 1930s, relativistic quantum Electrodynamics developed, influenced by the theory of Relativity. In particular, it takes up the notion of space-time quadrivector, and introduces an electromagnetic potential quadrivector that will be noted here $4\vec{A}$ or \vec{A}^μ ($\mu = t, x, y, z$):

$$4\vec{A} = \begin{bmatrix} A^t \\ A^{x'} \\ A^{y'} \\ A^{z'} \end{bmatrix} = \begin{bmatrix} A^t \\ -A^x \\ -A^y \\ -A^z \end{bmatrix}$$

with A^t the electric scalar potential and \vec{A} the magnetic vector potential (we put here an apostrophe $A^{x'}$ to distinguish the old from the new terms of the magnetic vector potential, but thereafter we will not put more).

Note on the electrical potential defined as a time component

We can wonder about the deep reasons for defining an electromagnetic quadrivector with an electric potential in time component and a magnetic potential in spatial components.

The only answer to this, it is true unsatisfactory, is to see that it works perfectly in the calculations.

As already mentioned in the previous chapter, this electromagnetic potential quadrivector $4\vec{A}$ highlights much more than the equations of the 19th century the similarities between fields \vec{B} and field $\vec{E}l$.

We have for example for the component El_x :

$$El_x = -\frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x} = \frac{\partial A^{x'}}{\partial t} - \frac{\partial A^t}{\partial x}$$

And for the component B_z :

$$-B_z = -\frac{\partial A^y}{\partial x} + \frac{\partial A^x}{\partial y} = \frac{\partial A^{y'}}{\partial x} - \frac{\partial A^{x'}}{\partial y}$$

To bring the fields $\vec{E}l$ and \vec{B} closer together, we modify their indices, and we now take those of the Maxwell Faraday tensor. For example, we have for El_x and B_z :

$$El_{tx} = El_x = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

$$B_{xy} = -B_z = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

We obtain thus a field $\vec{E}l$ and a field \vec{B} which are constructed identically (except for the index) using the electromagnetic potential quadrivector \vec{A}^μ ($\mu = t, x, y, z$).

IV.1.2 Orient the fields $\vec{E}l$ and \vec{B} in analogy with a rotation vector $\vec{\Omega}$

In mechanics, the rotation vector $\vec{\Omega}_{xy}$ with respect to a spatial plan x, y is oriented perpendicular to this plane. It is therefore classically oriented in the direction of z in a three-dimensional space x, y, z.

Here we keep the idea of treating Time in the same way as Space. In a three-dimensional space (2 for Space and 1 for Time), the dimension t is perpendicular to the spatial plane x, y. So, we have a rotation vector $\Omega_{xy}^{1/t}$ oriented according to Time (and in line with its units that are in $rad \times s^{-1}$, hence here the $1/t$).

The same notation is used for the fields $B_{xy}^{1/t}$ et $El_{tx}^{1/y}$ which will be respectively associated with a "generalized" rotation in a spatial plane x, y and oriented in the direction of t, to a "generalized" rotation in a spatiotemporal plane t, x and oriented in the direction of y.

We have the following notations that we will keep from now on:

$$2\Omega_{xy}^{1/t} = \frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}$$

$$B_{xy}^{1/t} = -B_z = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

$$El_{tx}^{1/y} = El_x = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

Following these analogies, an important consequence is the *reorientation* of the electric field $El_{tx}^{1/y} = El_x$. Indeed, it is usually considered that the electric field $El_{tx}^{1/y} = El_x$ is oriented in the direction of x. To continue the analogy to the end, it must be reoriented in the direction of y. The interest is to define the field $El_{tx}^{1/y}$ as a true "generalized" rotation vector in a spatiotemporal plane. The inappropriate thing is to have to use most of the classic formulas where the electric field intervenes.

If the electric field is redirected, the magnetic field is not $B_{xy}^{1/t} = -B_z$ which retains its usual orientation in the dimension perpendicular to x, y. Whether this dimension is z in a space x, y, z, or t in a space x, y, t, it remains the dimension perpendicular to the plane x, y.

Note 1

In a 4-dimensional space, the problem arises again since two dimensions can be perpendicular to a plane. In this essay, we will most often stick to the simple case of 3 dimensions.

Note 2

If the electric field is reoriented, we will see that this will not be the case for the electric force, nor for other so-called classical forces.

IV.1.3 Definition of a potential velocity quadrivector

We have previously highlighted the similarities between electrostatics and gravitation, between electrical potential A^t and Newtonian potential V^t .

On the model of the electromagnetic potential quadrivector $4\vec{A} = \begin{bmatrix} A^t \\ A^x \\ A^y \\ A^z \end{bmatrix}$, we define a potential velocity quadrivector from the linear velocities that we cancel during a change of reference frames and the Newtonian potential:

$$4\vec{V} = \begin{bmatrix} V^t \\ V^x \\ V^y \\ V^z \end{bmatrix}$$

Note

Note that in Special Relativity, the velocity quadrivector is defined as:

$$4\vec{v} = \begin{bmatrix} \gamma c \\ \gamma v^x \\ \gamma v^y \\ \gamma v^z \end{bmatrix}$$

with $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

So, it's not the same physical notion.

IV.2 Vector generalized rotations and fields in spatial or spatiotemporal planes

We propose to define the fields $\vec{B}_{xy}^{1/t}$, $\vec{E}_{xt}^{1/y}$, $\vec{\Omega}_{xy}^{1/t}$, $\vec{K}_{xt}^{1/y}$ such as generalized rotation vectors in spatial and spatiotemporal planes that are cancelled during a change of reference frames. We place ourselves here in a space with three dimensions orthogonal between them (two dimensions x, y for Space, and one dimension t for Time).

We will distinguish a little artificially the notion of field deriving from a potential from that of generalized rotation vector in a spatial or spatiotemporal plane, by the fact that:

- the generalized rotation vector is the physical quantity present before the change of reference frames,
- the field is the physical quantity obtained after the cancellation of the "generalized" rotation vector when changing reference frames and potentially available if the reverse reference frame change is performed.

IV.2.1 Pulse field $\vec{\Omega}_{xy}^{1/t}$

We define a pulsation field corresponding to the rotation vector in a spatial plane $\vec{\Omega}_{xy}^{1/t}$:

$$2\Omega_{xy}^{1/t} = \frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y}$$

This is defined from a velocity vector potential (with components V^x, V^y), corresponding to the linear velocities cancelled when changing reference frames.

IV.2.2 Magnetic field $\vec{B}_{xy}^{1/t}$

We define a usual magnetic field in a vacuum, derived from the magnetic potential A^x, A^y :

$$B_{xy}^{1/t} = \frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y}$$

Like the pulsation field, the magnetic field appears as a generalized rotation vector in a spatial plane. The velocity potentials V^x, V^y and magnetic potentials A^x, A^y are both components defined in Space.

Note on the Larmor relation

We have highlighted the analogies between velocity vector potential V^x, V^y and magnetic vector potential A^x, A^y . By contrast, the root causes that require distinguishing these velocity potential and magnetic potential will remain unexplained here.

It is noted that using the Larmor relation between $\vec{\Omega}_{xy}^{1/t}$ à $\vec{B}_{xy}^{1/t}$, one can find a simple relationship between velocity potential and magnetic potential.

We have:

$$\vec{\Omega}_{xy}^{1/t} = \frac{e}{2m_e} \vec{B}_{xy}^{1/t}$$

with $-e$ the charge of the electron and m_e the mass of the electron.

We have for the potentials:

$$\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y} = \frac{e}{m_e} \left(\frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y} \right)$$

If we equalize one by one, we find:

$$V^x = \frac{e}{m_e} A^x$$

$$V^y = \frac{e}{m_e} A^y$$

We have the equality between potential momentums:

$$m_e V^x = eA^x$$

$$m_e V^y = eA^y$$

IV.2.3 Electric field $\vec{E}l_{tx}^{1/y}$

In the same way, we define an electric field $\vec{E}l_{tx}^{1/y}$ deriving from an electric potential A^t and from a magnetic potential A^x :

$$El_{tx}^{1/y} = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

IV.2.4 Wave vector field $\vec{K}_{tx}^{1/y}$

The magnetic field $\vec{B}_{xy}^{1/t}$ and the electric field $\vec{E}l_{tx}^{1/y}$ both derive from the electromagnetic quadrivector potential. The first corresponds to a generalized rotation vector in a spatial plane, the second corresponds to a generalized rotation vector in a spatiotemporal plane.

The pulsation field $\vec{\Omega}_{xy}^{1/t}$ corresponds to a vector rotation in a spatial plane and derives from the velocity vector potential.

We are looking for the counterpart of $\vec{\Omega}_{xy}^{1/t}$, generalized rotation vector in a spatiotemporal plane and derived from the velocity quadrivector potential.

In wave theory, we have the analogies between Time and Space for the following quantities:

Time	Space
Period T in s	Wavelength λ in m
Frequency $\nu = \frac{1}{T}$ in Hz or s^{-1}	Wavenumber $\sigma = \frac{1}{\lambda}$ in m^{-1}
Pulse $\omega = 2\pi\nu = \frac{2\pi}{T}$ in $rad \times s^{-1}$	Wave vector $k = 2\pi\sigma = \frac{2\pi}{\lambda}$ in $rad \times m^{-1}$

The analogue in the Space of the pulsation $\omega = 2\pi\nu$, it is the wave vector $k = 2\pi\sigma$.

The idea is to define a wave vector field $\vec{K}_{tx}^{1/y}$ equal to:

$$2c^2 K_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$$

$2c^2 \vec{K}_{tx}^{1/y}$ is in $m \times s^{-2}$ (as an acceleration).

Note 1

The Newtonian potential V^t must be in $m^2 \times s^{-2}$ in order to $\frac{\partial V^t}{\partial x}$ is in $m \times s^{-2}$.

Note 2

We add here c^2 in the expression of the wave vector field $2c^2\vec{K}_{tx}^{1/y}$ for the sake of homogeneity. In a medium other than vacuum (where the phase velocity of the wave v_ϕ is no longer equal to c), it should be possible to generalize with:

$$2v_\phi^2 K_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$$

IV.2.5 Newtonian gravitational field and Einsteinian gravitational field

We propose to decompose the wave vector field $2c^2 K_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$ in two terms.

One, the Newtonian gravitational field Gr_{tx}^y which appears as a special case of the wave vector field when $\frac{\partial V^x}{\partial t} = 0$, $Gr_{tx}^y = -\frac{\partial V^t}{\partial x}$.

The other, an acceleration field $a_{ein_{xt}}^y = -\frac{\partial V^x}{\partial t}$ that is added during an implicit change of reference frames. It is referred to here as an Einsteinian acceleration or an Einsteinian gravitational field in reference to the principle proposed by A. Einstein of equivalence between acceleration and gravitational field. This acceleration occurs classically in the fundamental principle of Newton's dynamics:

$$m\vec{a}_{ei} = \Sigma\vec{F}$$

We have the relationship between the wave vector field $\vec{K}_{xt}^{1/y}$, the Newtonian gravitational field and the Einsteinian gravitational field:

$$2c^2\vec{K}_{tx}^{1/y} = \vec{G}_{tx}^{1/y} - \vec{a}_{ein_{xt}}^y$$

Note 1

To be checked if it can be generalized in this form in refractive index media $n \neq 1$:

$$2v_\phi^2\vec{K}_{tx}^{1/y} = \vec{G}_{tx}^{1/y} - \vec{a}_{ein_{xt}}^y?$$

Note 2

Let us observe that like $\vec{K}_{tx}^{1/y}$, $\vec{G}_{tx}^{1/y}$ and $\vec{a}_{ein_{xt}}^y$ can both be considered generalized rotation vectors in a spatiotemporal plane t, x and oriented according to y .

IV.2.6 Tensor $\vec{\vec{D}}_{\mu\nu}$ of wave vector field and pulsation field

We remind the Maxwell Faraday tensor.

$$\vec{\vec{F}}_{\mu\nu} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ -\frac{El_{tx}}{c} & 0 & B_{xy} & B_{xz} \\ -\frac{El_{ty}}{c} & -B_{xy} & 0 & B_{yz} \\ -\frac{El_{tz}}{c} & -B_{xz} & -B_{yz} & 0 \end{bmatrix}$$

By analogy between \vec{K} and $\vec{E}l$, then between $\vec{\Omega}$ and \vec{B} , we define the pulsation wave vector tensor in the 4 dimensions of Space-Time:

$$\bar{\bar{D}}_{\mu\nu} = \begin{bmatrix} 0 & cK_{tx} & cK_{ty} & cK_{tz} \\ -cK_{tx} & 0 & \Omega_{xy} & \Omega_{xz} \\ -cK_{ty} & -\Omega_{xy} & 0 & \Omega_{yz} \\ -cK_{tz} & -\Omega_{xz} & -\Omega_{yz} & 0 \end{bmatrix}$$

We have it in the following form if we divide all the terms by c :

$$\bar{\bar{D}}_{\mu\nu} = \begin{bmatrix} 0 & K_{tx} & K_{ty} & K_{tz} \\ -K_{tx} & 0 & \frac{\Omega_{xy}}{c} & \frac{\Omega_{xz}}{c} \\ -K_{ty} & -\frac{\Omega_{xy}}{c} & 0 & \frac{\Omega_{yz}}{c} \\ -K_{tz} & -\frac{\Omega_{xz}}{c} & -\frac{\Omega_{yz}}{c} & 0 \end{bmatrix}$$

IV.3 Conclusion of the chapter

In this chapter, we defined "generalized movements" of the type generalized rotation vectors in a spatial or spatiotemporal plane, rotation vectors that become potential quantities, that is to say fields deriving from potentials, when they are cancelled during a change of reference frames.

The real objective that we will address in the next chapter is to match any cancellation of a generalized movement during a change of reference frames to a so-called amount of inertia (such as force, acceleration, potential energy, etc.), which is added to preserve the invariance of the main laws of Nature. In the next chapter, the main law of Nature to be preserved will be the fundamental principle of Newton's dynamics.

Chapter V Build any force on the model of the Coriolis inertial force and the Lorentz magnetic force

Purpose of the chapter

We now seek to construct any so-called classical force (we exclude strong and weak nuclear interactions) on the model of the Coriolis inertial force, a force that is added in the fundamental principle of Newton's dynamics when we cancel the rotation vector $\vec{\Omega}$.

For this we will use fields and generalized rotation vectors in spatial and spatiotemporal planes, which was defined in the previous chapter.

V.1 Objective and angles of attack

V.1.1 Objectives and angles of attack of General Relativity

In the theory of General Relativity, A. Einstein set as his main objective the respect of the principle of generalized relativity, that is to say an invariance of the laws of Nature regardless of the change of reference frames.

For this, A. Einstein seeks to preserve the space-time interval $ds^2 = g_{ij}dx^i dx^j$ regardless of the change of reference frame. He describes the notion of field as deformations of Space-Time and draws inspiration from the source potential equation of Poisson gravitation to propose the so-called Einstein equation of General Relativity. We will come back to this in the last Memoir, when we talk about the Big Bang theory.

In this chapter, we take up the objective of A. Einstein of General Relativity, an invariance of the laws of Nature regardless of the change of reference frames. However, we limit ourselves to changes of reference frames of the type cancellation of a generalized rotation vector. In addition, we will use another angle of attack based more on the so-called classical forces, on the Electromagnetism of the 19th century and on the Quantum Electrodynamics of the 20th.

V.1.2 Angle of attack, modelled on the inertial Coriolis force

In the 1st Memoir, we mentioned the reasons for bringing any "normal" force closer to an inertial force:

- first, a conceptual difficulty in accepting that there are privileged so-called inertial or Galilean reference frames. The reference frames in rotational movement pose in particular problem because they also "work" in isolation. One thinks in particular of the Bohr electron model, where the rotating electron in a stationary state, does not radiate energy,
- secondly, a difficulty in accepting the existence of two types of forces: inertial forces that are added during a change of reference frames and "normal" forces that would be somehow pre-existing.

We also highlighted the analogies between:

- the generalized electromagnetic potential energy that must be added (in local gauge theories, in partial derivatives of the wave function ψ) when cancelling part of the wave function phase ψ (corresponding to a transformation of local gauge),
- the inertial forces to be added (in the fundamental principle of Newton's dynamics) when cancelling the rotation vector $\vec{\Omega}$ (corresponding to a change of reference frames),

in order to have an invariant for the main laws of Nature, for wave equations or for the fundamental principle of Newton's dynamics.

In this 2nd Memoir, we have highlighted the similarities between the Coriolis force $\vec{F}_{Cor} = m\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$ and the Lorentz magnetic force $\vec{F}_{Lor} = q\vec{v}^y \wedge \vec{B}_{xy}^{1/t}$, forces defined from a momentum or electric current and a field (pulsation field or magnetic field), and perpendicular to these quantities.

We also highlighted the similarities between pulsation field $\vec{\Omega}_{xy}^{1/t}$, wave vector field $\vec{K}_{xt}^{1/y}$, magnetic field $\vec{B}_{xy}^{1/t}$ and electric field $\vec{E}_{xt}^{1/y}$.

Following these analogies and similarities, the idea is to build the electric force, the magnetic force and the gravitational force on the model of the Coriolis inertial force, taking up or expanding the following 3 characteristics:

- a quantity cancelled during a change of reference frames, a quantity to be matched to a generalized rotation vector in a spatial or spatiotemporal plane. The cancelled quantity is $\Omega_{xy}^{1/t}$, $K_{xt}^{1/y}$, $B_{xy}^{1/t}$, $E_{xt}^{1/y}$. The change of reference frames is performed from a reference state R^0 to a reference frame R^Ω , R^K , R^B , R^{El} where one cancels one of the 4 generalized rotation vectors,
- a quantity retained during the change of reference frames (which will take the form of a speed, a momentum, a current, an electric charge or a mass),
- a force to be added in the fundamental principle of Newton's dynamics, force both perpendicular to the conserved quantity and the cancelled quantity.

Note 1 on the electric field and electric force

Usually, the electric force and the electric field have the same direction. Reorienting the electric field $\vec{E}_{xt}^{1/y}$ from x to y (such as a rotation vector), achieves an electric force that is perpendicular to both the electric charge q^t and to the field $\vec{E}_{xt}^{1/y}$. The electric force thus defined will retain the same orientation as the usual one. We will come back to that.

Note 2, restriction with vector product

On the model of the Coriolis force, all the forces studied here will be built from a vector product, a notion proposed by Willard Gibbs at the end of the 19th century.

The vector product is defined in three dimensions of a space, this implies an important restriction: to place itself only in three-dimensional spaces, for example a space with two dimensions of Space and one dimension of Time.

Note 3 on contact forces

It will be admitted in this Memoir that the so-called contact forces (friction, etc.) that intervene in the fundamental principle of Newton's dynamics can always be reduced to gravitational and/ or electric forces and that therefore the study of the latter forces is sufficient.

V.2 Quantities retained and quantities cancelled when changing reference frame, speed, mass, and electric charge

V.2.1 Findings on quantities retained and cancelled when changing reference frame

When looking at a Coriolis acceleration $\vec{a}_{cor_{yt}}^x = 2\vec{\Omega}_{xy}^{1/t} \wedge \vec{v}_r^y$, we find that when changing reference frame from R^0 to R^Ω , there is a quantity cancelled the field $\vec{\Omega}_{xy}^{1/t}$ and a quantity retained specific to the body studied: \vec{v}_r^y the relative speed of the body studied.

By analogy between the Coriolis force and magnetism, we propose that it may be the same for the electromotor field $\vec{E}m_{yt}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$. When changing the reference frame from R^0 to R^B . There is a quantity cancelled the generalized rotation vector $\vec{B}_{xy}^{1/t}$ and a conserved quantity specific to the body studied: \vec{v}^y the speed of the electric charge.

If we take the table potential momentums and potential energies, each potential energy and potential momentum can be broken down into:

- a quantity retained (lowercase) when changing reference frames, the electric charge q^t and the mass m^t ,
- a cancelled quantity (in uppercase)) when changing reference frames, electromagnetic potential quadrivectors A^μ and speed potential quadrivectors V^μ (from which the fields derive).

	Electromagnetism	Gravitation
Potential energy Ep^t	$Ep^t = q^t A^t$	$Ep^t = m^t V^t$
Potential momentum in the direction of x	$p^x = q^t A^x$	$p^x = m^t V^x$
Potential momentum in the direction of y	$p^y = q^t A^y$	$p^y = m^t V^y$

It is noted that the quantities retained are specific to the particle and that the quantities cancelled are external to it.

V.2.2 Conserved quantities in physics, Noether's theorem

In 1918, Emilie Noether proposed a famous theorem that bears her name, and which speaks of quantities conserved during transformations. Let us now say a few words.

Let be a Lagrangian L with symmetry. This means that L is not modified, when applied a family of transformations parameterized by a variable s , sending a position q to a new position $q(s)$.

This is expressed by the nullity of the Lagrangian derivative with respect to s :

$$\frac{\partial L}{\partial s} = 0$$

Noether's theorem states that there is a quantity C conserved during the transformation parameterized by s and which is equal to:

$$C = \frac{\partial L}{\partial \dot{q}(s)} \frac{\partial q(s)}{\partial s} = p \frac{\partial q(s)}{\partial s}$$

Note that as for the Lagrangian L , we have for C :

$$\frac{\partial C}{\partial s} = 0$$

For example, when L is invariant by a translation into Space $\frac{\partial L}{\partial q} = 0$, the variable s is then the position q . We obtain as a conserved quantity:

$$C = \frac{\partial L}{\partial \dot{q}} \frac{\partial q}{\partial q} = p \frac{\partial q}{\partial q} = p$$

The momentum is therefore the quantity conserved when the Lagrangian is invariant by translation in Space.

Similarly, it is shown that the quantity conserved is:

- the energy (i.e., Hamiltonian) H when the Lagrangian is invariant by translation in Time,
- the kinetic moment when the Lagrangian is invariant by rotation,
- an electric charge current density quadrivector when the Lagrangian is invariant by phase change of the wave function, that is, by global gauge transformation.

Let's look at this last point in the next paragraph.

V.2.3 Application of Noether's theorem: conservation of the electric charge current density quadrivector when the Lagrangian is invariant by global gauge transformation

Let's take the simple example of a global gauge transformation that applies to the wave function:

$$\psi(x) \rightarrow e^{i\alpha}\psi(x)$$

α can take any real value,
 x here refers to a local variable, i.e., a moment or position.

Note

We differentiate between global gauge transformation and local gauge transformation with α independent of the time or position variables for the first and $\alpha(x)$ function of a time or position variable for the second.

We have the Lagrangian of a free electron described by the wave function ψ :

$$L = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - \frac{mc}{\hbar})\psi$$

Note that this Lagrangian is invariant by $\psi(x) \rightarrow e^{i\alpha}\psi(x)$.

According to Noether's theorem, there is a quantity conserved during the transformation.

We have for the electric charge density:

$$j^t = \frac{\partial L}{\partial \dot{\psi}(\alpha)} \frac{\partial \psi(\alpha)}{\partial \alpha}$$

For the electric current density:

$$j^{\mu} = \frac{\partial L}{\partial (\vec{\nabla}\psi(\alpha))} \frac{\partial \psi(\alpha)}{\partial \alpha}$$

$$(\mu = x, y, z)$$

By developing these two equations, we obtain the electric charge current density quadrivector defined in relativistic quantum Electrodynamics:

$$j^{\mu} = -e\bar{\psi}\gamma^{\mu}\psi$$

$$(\mu = t, x, y, z)$$

V.3 Coriolis inertial acceleration and induced electromotor field, both obtained from the cancellation of a generalized rotation vector in a spatial plane

V.3.1 What distinguishes an inertial acceleration from an inertial force?

When changing reference frames, we cancel (or modify) a movement. In this case, we wonder why we add in the fundamental dynamic principle an inertial force and not an inertial acceleration, acceleration which is indeed much more similar to a movement?

The notion of force is widely used in physics. It is strange, or at least less intuitive than the notion of acceleration. In the fundamental principle of dynamics, it is not quantities of movements that intervene on either side of the relationship. On one side (on the right), we have a vector sum of forces. On the other side (on the left), we have a variation of movement (or acceleration) once a mass (whose presence we cannot intuitively understand).

The notions of acceleration and inertial acceleration are much more intuitive. When a body is in rectilinear motion, it can have tangential acceleration. When this body enters rotation, it acquires normal acceleration in addition to tangential acceleration. If we place ourselves in the reference frame R^Ω where we cancel the rotation vector $\vec{\Omega}$, we must add a centrifugal inertial acceleration which is precisely the normal acceleration. All this is very well demonstrated mathematically and can intuitively be understood.

By contrast, the notion of inertial force (because of the consideration of mass) is not demonstrated mathematically.

To explain what distinguishes the notion of inertial acceleration from the notion of inertial force, the following idea is proposed.

What if *the inertial acceleration* were added in all general laws of Nature invariant by change of reference frames, when we cancel during a change of reference frames *a generalized rotation vector* in a *spatial plane*?

What if *the inertial force* were added in all general laws of Nature invariant by change of reference frames, when we cancel during a change of reference frames *a generalized rotation vector* in a *spatiotemporal plane*?

The Coriolis inertial acceleration is perpendicular to a velocity \vec{v}_r and a rotation vector (the first term retained and the second term cancelled when changing reference frames). For a force, it would be almost the same. Nevertheless, the force would be distinguished from an inertial acceleration by the cancellation not of a generalized rotation vector in a *spatial plane*, but in a *spatiotemporal plane*.

Thus, the force would be perpendicular to something analogous to the velocity. \vec{v}_r and to a generalized rotation vector in a *spatiotemporal plane*.

It is this idea that we will evaluate in the next paragraphs.

V.3.2 Generalized rotation vectors in a spatial plane

Let be the pulsation field $\vec{\Omega}_{xy}^{1/t}$ and magnetic field $\vec{B}_{xy}^{1/t}$ both correspond to a generalized rotation vector in a spatial plane x, y:

$$2\vec{\Omega}_{xy}^{1/t} = \left(\frac{\partial V^y}{\partial x} - \frac{\partial V^x}{\partial y} \right)_{xy}^{1/t}$$

$$\vec{B}_{xy}^{1/t} = \left(\frac{\partial A^y}{\partial x} - \frac{\partial A^x}{\partial y} \right)_{xy}^{1/t}$$

V.3.3 Coriolis inertial acceleration

By cancelling the rotation vector $\vec{\Omega}_{xy}^{1/t}$, that is, by going from R^0 to R^Ω :

- we add the centrifugal inertial acceleration (with the drive speed $\vec{v}_e^y = \vec{\Omega}_{xy}^{1/t} \wedge \vec{x}$ which is also cancelled when changing reference frames):

$$\vec{a}_{cent\ ty}^x = -\vec{v}_e^y \wedge \vec{\Omega}_{xy}^{1/t} = -(\vec{\Omega}_{xy}^{1/t} \wedge \vec{x}) \wedge \vec{\Omega}_{xy}^{1/t}$$

- we add the Coriolis inertial acceleration with the relative velocity \vec{v}_r^y as the quantity retained:

$$\vec{a}_{cor\ ty}^x = -\vec{v}_r^y \wedge 2\vec{\Omega}_{xy}^{1/t}$$

Note

x holds here the role of the normal (or radial) coordinate and y the role of tangential coordinate.

We have the following figure for the Coriolis inertial acceleration:

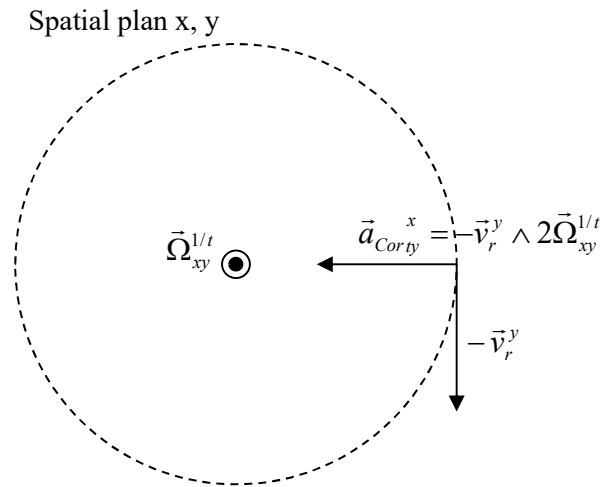


Figure 2: Coriolis inertial acceleration

Note

We have in R^0 , the absolute speed:

$$\vec{v}_a^y = \vec{v}_e^y + \vec{v}_r^y$$

V.3.4 Induced electromotor field

The magnetic field $\vec{B}_{xy}^{1/t}$ corresponds to a generalized rotation in a spatial plane. According to our initial idea, if we cancel the generalized rotation vector $\vec{B}_{xy}^{1/t}$ when changing reference frame from R^0 to R^B , we expect to get something to add that is more like an acceleration than a force.

This is precisely what we see, since we obtain an induced electromotor field $\vec{E}m$ when passing from R^0 to R^B :

$$\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$$

with \vec{v}^y the speed of the electric charges, quantity conserved during the passage from R^0 to R^B

We have the following figure:

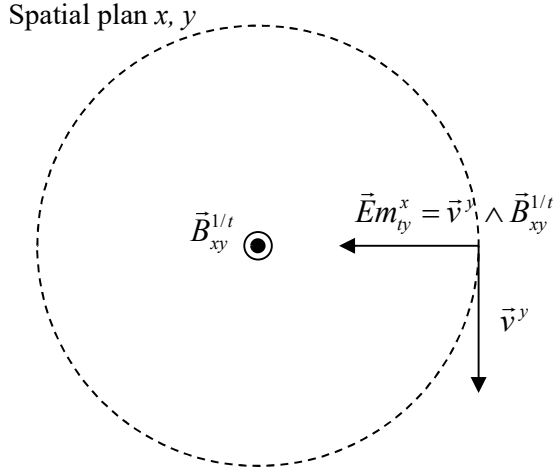


Figure 3: induced electromotor field

Note 1

$\vec{E}m_{ty}^x$ usually has the same direction as the electric field $El_{tx}^y = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$. However, here, since we redirect the electric field El_{tx}^y , they no longer have the same direction.

Note 2

Since we reorient the electric field El_{tx}^y , we can no longer use Maxwell Faraday's equation as it is:

$$r\vec{\partial}t\vec{E}l = -\frac{\partial\vec{B}}{\partial t}$$

On the other hand, since the electromotor field Em is not oriented, we can use it in Maxwell Faraday's equation:

$$r\vec{\partial}t\vec{E}m = -\frac{\partial\vec{B}}{\partial t}$$

We can also apply Lenz Faraday's law to it. We therefore have an electromotive induced force:

$$e = \int \vec{E}m \cdot d\vec{l} = -\frac{d\Phi_c}{dt}$$

Φ_c refers to the total magnetic flux cut by the electrical circuit when it is moved. We will come back to these phenomena of induction in the next Memoir.

V.4 Forces obtained from the cancellation of a generalized rotation vector in a spatiotemporal plane

V.4.1 Generalized rotation vectors and fields in a spatiotemporal plane

We have the electric field:

$$\vec{E}l_{tx}^{1/y} = \frac{\partial A^x}{\partial t} - \frac{\partial A^t}{\partial x}$$

We have the wave vector field:

$$2c^2\vec{K}_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$$

V.4.2 Electric force and Coulomb electrostatic force

For the electric force, the quantity conserved during the change of reference frames from R^0 to R^{El} is the electric charge q^t .

As we want to build the (classical) forces on the model of the Coriolis inertial acceleration, we need an electric charge of the vector type. That is, in addition to its norm, it needs a direction and a sens. In physics (quantum, relativistic, etc.), charge density is frequently defined as the temporal component of an electric charge current density quadrivector. This makes it possible to orient the electric charge density according to Time.

We build an electric force on the model of the Coriolis force, q^t the quantity retained, $\vec{E}l_{tx}^{1/y}$ the generalized rotation vector in a spatiotemporal plane cancelled during the passage from R^0 to R^{El} . We obtain:

$$\vec{F}_{Elty}^x = \vec{q}^t \wedge \vec{E}l_{tx}^{1/y}$$

We have the following figure:

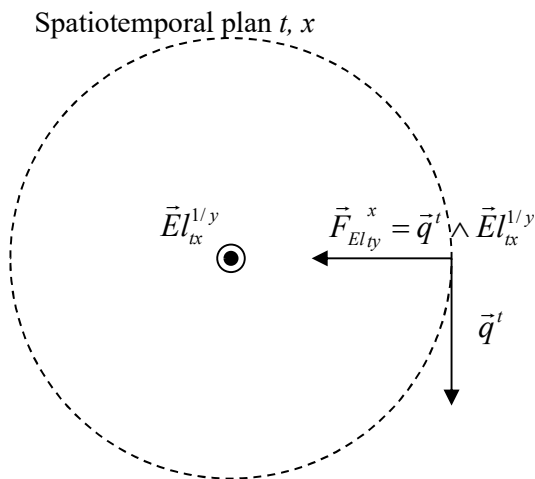


Figure 4: electric force

Note 1

We note that the electric force thus obtained has the same direction as the electric force as it is usually defined.

Note 2

For an electromagnetic wave, we speak of horizontal or vertical polarization when the electric field is horizontal or vertical. A vertical or horizontal antenna is then used to transmit or receive this type of wave.

In the case of a reorientation of the electric field (which then has the same direction as the wave vector \vec{k} of the electromagnetic wave), the polarization of the electromagnetic wave corresponds to the direction of the electric force and no longer to that of the electric field.

Note 3

The term electric force is used quite infrequently, since always coupled with the magnetic force (except in the case of the electrostatic force), it gives the electromagnetic force.

When $\frac{\partial A^x}{\partial t} = 0$, magnetic phenomena are absent, and we simply have the electrostatic field:

$$\vec{E}_{S_{tx}}^{1/y} = -\frac{\partial A^t}{\partial x}$$

We also obtain the Coulomb electrostatic force (special case of the electric force):

$$\vec{F}_{Coul_{ty}}^x = \vec{q}^t \wedge \vec{E}_{S_{tx}}^{1/y}$$

Note, intuitive analogy

In a uniform circular motion, we have the relationship between the rotation angular velocity $\Omega^{1/t}$ and rotation linear speed V^θ : $V^\theta = r\Omega^{1/t}$ (V^θ being considered here as a speed potential).

To find $\Omega^{1/t}$, one can derive V^θ with respect to r .

$$\Omega^{1/t} = \frac{\partial V^\theta}{\partial r} = \frac{\partial r\Omega^{1/t}}{\partial r} = \Omega^{1/t}$$

In electrostatics, we have:

$$E_{S_{tx}}^{1/y} = -\frac{\partial A^t}{\partial x}$$

If we continue the analogy, the electric potential A^t plays the role of a "generalized rotation linear velocity" in a cancelled spatiotemporal plane when moving from R^0 to R^{El} .

V.4.3 Gravitational force

We have seen that the wave vector field $\vec{K}_{tx}^{1/y}$ corresponding to the "generalized" rotation vector in a spatiotemporal plane, is the counterpart of the pulsation field $\vec{\Omega}_{xy}^{1/t}$ corresponding to the rotation vector in a spatial plane.

Following a strict analogy with the electric force, the idea is to build a gravitational force with:

- a wave vector field $\vec{K}_{tx}^{1/y}$, whose corresponding "generalized" rotation vector cancels out when changing reference frame from R^0 to R^K ,
- a conserved quantity that will no longer be an electric charge \vec{q}^t but a mass directed in Time \vec{m}^t .

A difficulty is therefore to accept the vectorization of the mass \vec{m}^t directed in Time. It is true that mass is very rarely vectorized in classical mechanics. In relativistic physics, we sometimes distinguish between the notions of longitudinal mass and transverse mass.

Following a strict analogy with the electric force, we have the gravitational force (which includes the so-called Newtonian gravitation and so-called Einsteinian gravitation):

$$\vec{F}_{K_{ty}}^x = \vec{m}^t \wedge 2c^2 \vec{K}_{tx}^{1/y}$$

with:

$$2c^2 \vec{K}_{tx}^{1/y} = \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x}$$

We have:

$$2c^2 \vec{K}_{tx}^{1/y} = \vec{G}r_{tx}^{1/y} - \vec{a}_{Ein_{xt}}^y$$

with:

$$a_{ein_{xt}}^y = -\frac{\partial V^x}{\partial t}$$

$$Gr_{tx}^{1/y} = -\frac{\partial V^t}{\partial x}$$

We have the following figure:

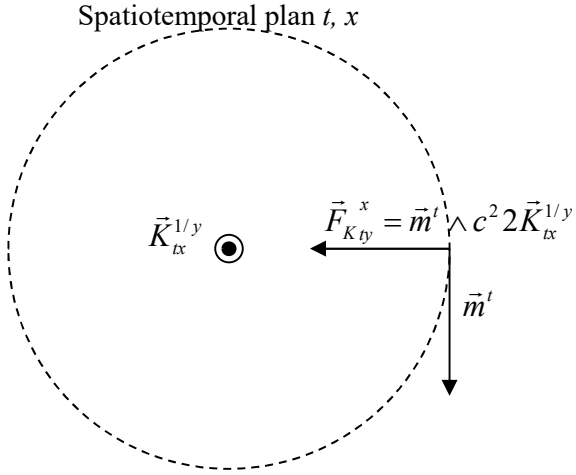


Figure 5: *gravitational force*

In the case of Newtonian gravitation, we have:

$$\vec{G}r_{tx}^{1/y} = 2c^2 \vec{K}_{tx}^{1/y} = \left(-\frac{\partial V^t}{\partial x}\right)_{tx}^y$$

with:

$$\frac{\partial V^x}{\partial t} = 0$$

We have the classical Newtonian gravitational force that is added in the reference frame R^{Gr} where we cancel $\vec{G}r_{tx}^{1/y}$:

$$\vec{F}_{Gr_{ty}}^x = \vec{m}^t \wedge \vec{G}r_{tx}^{1/y} = -\vec{m}^t \wedge \left(\frac{\partial V^t}{\partial x}\right)_{tx}^y$$

In the case of Einsteinian gravitation, we have:

$$\vec{a}_{Ein_{xt}}^y = -2c^2 \vec{K}_{tx}^{1/y} = \left(-\frac{\partial V^x}{\partial t}\right)_{xt}^y$$

with:

$$\frac{\partial V^t}{\partial x} = 0$$

We have the Einsteinian gravitational force that is added in the reference frame R^a where we cancel $\vec{a}_{ein_{xt}}^y$:

$$\vec{F}_{Einty}^x = \vec{m}^t \wedge \vec{a}_{Eintxt}^y = -\vec{m}^t \wedge \left(\frac{\partial V^x}{\partial t} \right)_y$$

It is pointed out that the Einsteinian gravitational force \vec{F}_{Einty}^x is oriented in the same direction x as the speed \vec{v}^x .

Note 1, infinite wavelength

In the event that we have:

$$m \vec{a}_{Eintxt}^y = m \vec{G} r_{tx}^{1/y}$$

We obtain:

$$\begin{aligned} -\frac{\partial V^x}{\partial t} &= -\frac{\partial V^t}{\partial x} \\ 2c^2 K_{tx}^{1/y} &= \frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x} = 0 \end{aligned}$$

The wavelength $\lambda^y = \frac{2\pi}{K_{tx}^{1/y}}$ is therefore infinite.

Note 2, analogue of the Larmor relation for generalized rotation vectors in a spatiotemporal plane

We recall the Larmor relation which translates identical effects of a pulsation field $\vec{\Omega}_{xy}^{1/t}$ and a magnetic field $\vec{B}_{xy}^{1/t}$ (i.e., 2 fields in a spatial plane), when the mass and electric charge values are correctly chosen (here m_e electron mass and $-e$ electron charge):

$$2m_e \vec{\Omega}_{xy}^{1/t} = e \vec{B}_{xy}^{1/t}$$

We look for an analogous relation for rotation vectors in a spatiotemporal plane.

We apply the fundamental principle of Newton's dynamics for an electron in both a Newtonian gravitational field and an electric field:

$$\begin{aligned} m_e \vec{a}_{Eintxt}^y &= m_e \vec{G} r_{tx}^{1/y} - e \vec{E} l_{tx}^{1/y} \\ m_e \vec{a}_{Eintxt}^y - m_e \vec{G} r_{tx}^{1/y} &= -e \vec{E} l_{tx}^{1/y} \end{aligned}$$

Moving to velocity potentials:

$$\begin{aligned} m_e \left(-\frac{\partial V^x}{\partial t} + \frac{\partial V^t}{\partial x} \right) &= -e \vec{E} l_{tx}^{1/y} \\ m_e \left(\frac{\partial V^x}{\partial t} - \frac{\partial V^t}{\partial x} \right) &= e \vec{E} l_{tx}^{1/y} \end{aligned}$$

We obtain:

$$2m_e c^2 \vec{K}_{tx}^{1/y} = e \vec{E} l_{tx}^{1/y}$$

We find a relation analogous to the Larmor relation for generalized rotation vectors in a spatiotemporal plane. $2m_e c^2 \vec{K}_{tx}^{1/y}$ and $e \vec{E}_{tx}^{1/y}$ have the same effects here that can be compensated.

V.4.4 What distinguishes an inertial acceleration from an inertial force?

An inertial acceleration (centrifugal or Coriolis type) is distinguished from an inertial force by the fact that:

- when an inertial acceleration must be added during a change of reference frames, what is cancelled is a "generalized" rotation vector in a *spatial plane* (rotation vector associated with a pulsation field $\vec{\Omega}_{xy}^{1/t}$),
- when an inertial force must be added during a change of reference frames, what is cancelled is a "generalized" rotation vector in a *spatiotemporal plane* (rotation vector associated with an electric field $\vec{E}_{xt}^{1/y}$ or a wave vector field $\vec{K}_{xt}^{1/y}$).

A force is oriented in Space and an acceleration is oriented in Time. An inertial acceleration is in a way the analogue, in Time, of the inertial force of inertia.

V.4.5 Definitions of an electric-type force and a gravitational-type force

We call electric force, any force whose quantity conserved during the change of reference frames is an electric charge \vec{q}^t .

The quantity cancelled when changing frames of reference is a generalized rotation vector in a spatiotemporal plane (corresponding to the field $\vec{E}_{xt}^{1/y}$), we have an electric force $\vec{F}_{Elty}^x = \vec{q}^t \wedge \vec{E}_{tx}^{1/y}$ oriented in Space (e.g. in the direction x).

We call gravitational force, any force whose quantity conserved during the change of reference frames is a mass \vec{m}^t .

The quantity cancelled when changing frames of reference is a generalized rotation vector in a spatiotemporal plane (corresponding to the field $\vec{K}_{tx}^{1/y}$), we have a gravitational force $\vec{F}_{Kty}^x = \vec{m}^t \wedge c^2 2 \vec{K}_{tx}^{1/y}$ oriented in Space (e.g. in the direction x).

V.5 Transition from Coriolis inertial acceleration and electromotor field to forces

In the preceding paragraphs, it was proposed:

- on the one hand, that an inertial acceleration or an electromotor field are added when we cancel a generalized rotation in a spatial plane,
- on the other hand, that a force is added when we cancel a generalized rotation in a spatiotemporal plane.

The goal now is to see how to move from an inertial acceleration (or an electromotor field) to a gravitational or electric force.

V.5.1 Transition from centrifugal, Coriolis, Einsteinian inertial accelerations to centrifugal, Coriolis, Einsteinian gravitation inertial forces

The Einsteinian gravitational field $(\vec{a}_{Ein})_{xt}^y = (-\frac{\partial v^x}{\partial t})_{xt}^y$ is of the acceleration type such as centrifugal inertial accelerations $(\vec{a}_{Cent})_{ty}^x$ and as Coriolis inertial accelerations $(\vec{a}_{Cor})_{ty}^x$.

In addition, $(\vec{a}_{Ein})_{xt}^y$, $(\vec{a}_{Cent})_{ty}^x$, $(\vec{a}_{Cor})_{ty}^x$ are of the generalized rotation type in a spatiotemporal plane t, y or t, x.

So, according to these 2 points, we can expect, when we cancel one of these 3 accelerations during a change of reference frames (and that we keep a mass \vec{m}^t), to have to add a gravitational-type inertial force.

This is exactly what Newtonian mechanics predicts, since when we cancel one of these accelerations, we must add a centrifugal inertial force. $(\vec{F}_{Cent})_{tx}^y$, a Coriolis inertial force $(\vec{F}_{Cor})_{ty}^x$ or an Einsteinian gravitational force $(\vec{F}_{Ein})_{ty}^x$.

Transition from a Coriolis inertial acceleration to a Coriolis inertial force

Let be the Coriolis inertial acceleration $(\vec{a}_{Cor})_{ty}^x$ with \vec{v}^y the quantity retained when the reference frame is changed, and $\vec{\Omega}_{xy}^{1/t}$ the cancelled rotation vector:

$$(\vec{a}_{Cor})_{ty}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$$

Note that the Coriolis acceleration $(\vec{a}_{Cor})_{ty}^x$ can be thought of as a generalized rotation vector in a spatiotemporal plane (however, we will have to reorient this acceleration to get what we want).

If we cancel $(\vec{a}_{Cor})_{ty}^x$ when changing reference frames, and the quantity retained is a mass \vec{m}^t , we obtain the Coriolis gravitational inertial force oriented according to x (always built on the same model):

$$(\vec{F}_{Cor})_{ty}^x = \vec{m}^t \wedge \left(\frac{\partial V^x}{\partial t}\right)_{tx}^y$$

Even if the precise reason remains to be clarified, it is noted that the Coriolis acceleration must be reoriented in order to become the generalized rotation vector in a spatiotemporal plane $\left(\frac{\partial V^x}{\partial t}\right)_{tx}^y$, that is cancelled when reference frames are changed.

We have the passage from the Coriolis acceleration (following x) to the generalized rotation vector in a spatiotemporal plane t, x:

$$(\vec{a}_{Cor})_{ty}^x \rightarrow \left(\frac{\partial V^x}{\partial t}\right)_{tx}^y$$

We have the two equivalent expressions to obtain the Coriolis inertial force (it's a gravitational-type force):

$$(\vec{F}_{Cor})_{ty}^x = \vec{m}^t \wedge \left(\frac{\partial V^x}{\partial t}\right)_{tx}^y$$

$$(\vec{F}_{Cor})_{ty}^x = m^t (\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x = -m^t (\vec{a}_{Cor})_{ty}^x$$

In the first, $\left(\frac{\partial V^x}{\partial t}\right)_{tx}^y$ is a generalized rotation vector in a spatiotemporal plane. In the second, $(\vec{a}_{Cor})_{ty}^x = (\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x$ is an inertial acceleration.

Note

Let us observe that the vector product applies only once, either in the expression of the Coriolis acceleration, or in the expression of the Coriolis force.

V.5.2 From the electromotor field $\vec{E}m_{ty}^x$ to the Lorentz magnetic force $(\vec{F}_{Lor})_{ty}^x$

Transition from an electromotor field to a Lorentz magnetic force

When changing reference frames that cancels the magnetic field $\vec{B}_{xy}^{1/t}$ and who keeps \vec{v}^y (the speed of the electrical circuit), we must add an electromotor field of the type:

$$\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$$

Note that the electromotor field $\vec{E}m_{ty}^x$ is constructed in the same way as Coriolis acceleration $(\vec{a}_{Cor})_{ty}^x$. The analogy will continue when we switch from the electromotor field to the Lorentz magnetic force.

If we cancel $\vec{E}m_{ty}^x$ when changing reference frame, and the amount retained is an electric charge \vec{q}^t , we obtain an electric force oriented according to x (always built on the same model):

$$(\vec{F}_{Lor})_{ty}^x = \vec{q}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y$$

Even if the precise reason remains to be clarified, it is noted that the electromotor field must be reoriented in order to become the generalized rotation vector in a spatiotemporal plane corresponding to the induced electric field $\vec{E}i_{tx}^y = \left(\frac{\partial A^x}{\partial t}\right)_{tx}^y$, that is cancelled when reference frames are changed.

We have the passage of an electromotor field $\vec{E}m_{ty}^x$ to a generalized rotation vector in a spatiotemporal plane corresponding to the induced electric field $\vec{E}i_{tx}^y$:

$$\vec{E}m_{ty}^x \rightarrow \vec{E}i_{tx}^y = \left(\frac{\partial A^x}{\partial t}\right)_{tx}^y$$

We have the two equivalent expressions to obtain the Lorentz magnetic force (of electric type):

$$\begin{aligned} (\vec{F}_{Lor})_{ty}^x &= \vec{q}^t \wedge \vec{E}i_{tx}^y = \vec{q}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y \\ (\vec{F}_{Lor})_{ty}^x &= (q^t \vec{v}^y \wedge \vec{B}_{xy}^{1/t})_{ty}^x = q^t (\vec{E}m)_{ty}^x \end{aligned}$$

In the first, $\vec{E}i_{tx}^y = \left(\frac{\partial A^x}{\partial t}\right)_{tx}^y$ is a rotation vector in a spatiotemporal plane. In the second, $\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$ is an electromotor field.

The electromotor field $\vec{E}m_{ty}^x$ has the same orientation as the Lorentz magnetic force $(\vec{F}_{Lor})_{ty}^x$. As with gravitation, the vector product applies only once.

Note on acceleration and electromotor field

From the above, it is possible to better understand why in a Newtonian gravitational field the acceleration of a body does not depend on its mass, while in an electrostatic field the acceleration of a body depends on its mass charge.

Indeed, the analogue of gravitation, for electricity, it is not a spatiotemporal vector of the type an acceleration, but a spatiotemporal vector of the type an induced field.

Thus, the analogue of: “In a Newtonian gravitational field, the acceleration of a body does not depend on its mass”, it is: “in a Coulombian electrostatic field, the induced field of a body does not depend on its electric charge”.

V.5.3 Axial vectors (or pseudo-vectors) and true vectors (or polar vectors)

When a quantity is completely defined by an intensity, an axis, and a direction of rotation around that axis, as in the case of a rotation vector $\vec{D}_{xy}^{1/t}$, this quantity is said to be an axial vector or a pseudo-vector. It is sometimes written surmounted by a curvilinear arrow: $\vec{\curvearrowright}$, indicating that this quantity requires knowledge of a direction of rotation.

According to the ideas developed here, the notion of generalized rotation vector in a spatial plane is identical to that of axial vector. The notation $\vec{\Omega}^{1/t}$ or $\vec{B}^{1/t}$ is therefore here equivalent to $\vec{\Omega}$ or \vec{B} .

For "classical" vectors, of the type a velocity \vec{v} or a force \vec{F} , sometimes referred to as true vectors or polar vectors.

The components of an axial vector are transformed by changing coordinate systems, like those of a polar vector. However, when moving from a direct coordinate system to an inverse coordinate system, the components of an axial vector do not change signs, unlike those of a polar vector.

True (or polar) vector

A true vector \vec{v} of components (v^x, v^y, v^z) in the coordinate system $(O, \vec{e}^x, \vec{e}^y, \vec{e}^z)$ is represented in the inverse coordinate system $(O, -\vec{e}^x, -\vec{e}^y, -\vec{e}^z)$ by a vector of components $(-v^x, -v^y, -v^z)$.

The velocity vector \vec{v} , the momentum vector $\vec{p} = m\vec{v}$, the force vector \vec{F} are examples of true vectors. They are all in the plane of rotation, hence the qualifier of polar.

Axial vector of a generalized rotation in a spatial plane

An axial vector $\vec{A}^{1/t}$ or pseudo-vector of components (a^x, a^y, a^z) in the coordinate system $(O, \vec{e}^x, \vec{e}^y, \vec{e}^z)$ is represented in the inverse coordinate system $(O, -\vec{e}^x, -\vec{e}^y, -\vec{e}^z)$ by the same components (a^x, a^y, a^z) .

The rotation vector $\vec{\Omega}_{xy}^{1/t}$, the magnetic field $\vec{B}_{xy}^{1/t}$, the kinetic moment at a point O: $\vec{L}_{Oxy}^{1/t} = \vec{OM} \wedge m\vec{v}$, the moment of a force at point O: $\vec{M}_{Oxy}^{1/t} = \vec{OM} \wedge \vec{F}$ are examples of axial vectors. They are all perpendicular to the spatial plane of rotation, hence the qualifier axial.

We can notice:

- that the vector product of two true vectors behaves like an axial vector, for example $\vec{L}_{Oxy}^{1/t} = \vec{OM} \wedge m\vec{v}$,
- that the vector product of a polar vector and an axial vector behaves like an axial vector, for example $\vec{F} = q\vec{v} \wedge \vec{B}_{xy}^{1/t}$.

Axial vector of a generalized rotation in a spatiotemporal plane

Usually, the electric field $\vec{E}l$ is presented as a true vector. It is proposed in this Memoir to interpret it as an axial vector of a generalized rotation in a spatiotemporal plane. We thus have the electric force, true vector, vector product of a true vector \vec{q}^t and an axial vector $\vec{E}l_{xt}^{1/y}$:

$$\vec{F}^x = \vec{q}^t \wedge \vec{E}l_{xt}^{1/y}$$

Like the electric field, the wave vector field $\vec{K}_{tx}^{1/y}$ is also an axial vector of a generalized rotation in a spatiotemporal plane.

An acceleration vector \vec{a}^x , a force vector \vec{F}^x , an electromotor field vector $\vec{E}m^x$ are all 3 true vectors, vector products of a true vector and an axial vector of a generalized rotation.

V.6 General table (quantities cancelled and retained when changing reference frames)

To conclude the chapter, the table below summarizes what has just been stated:

	Retained quantity: velocity	Retained quantity: velocity
--	------------------------------------	------------------------------------

<p>Cancelled quantity: rotation vector in a <i>spatial plane</i></p>	<p><i>Coriolis</i> Cancelled quantity $\Omega_{xy}^{1/t}$ Retained quantity \vec{v}^y (relative velocity) We must add a Coriolis inertial acceleration $\vec{a}_{Cor\ ty}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$</p>	<p><i>Electromotor</i> Cancelled quantity $B_{xy}^{1/t}$ Retained quantity \vec{v}^y (velocity of the electrical circuit). We must add an electromotor field $\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$</p>
	<p>Retained quantity: mass</p>	<p>Retained quantity: electric charge</p>
<p>Cancelled quantity: rotation vector in a <i>spatiotemporal plane</i></p>	<p><i>Gravitation</i> Cancelled quantity $K_{tx}^{1/y}$ Retained quantity \vec{m}^t It is necessary to add a gravitational-type force $\vec{F}_{K\ ty}^x = \vec{m}^t \wedge 2c^2 \vec{K}_{tx}^{1/y}$</p>	<p><i>Electromagnetism</i> Cancelled quantity $El_{tx}^{1/y}$ Retained quantity \vec{q}^t It is necessary to add an electric-type force $\vec{F}_{El\ ty}^x = \vec{q}^t \wedge \vec{E}l_{tx}^{1/y}$</p>

Chapter VI Thoughts on R^0 and new interpretation for the mass?

Purpose of the chapter

To conclude this Memoir, we conduct some final thoughts on the reference frame R^0 , as well as on a new physical interpretation to be given to the mass, in view of what has been proposed in the previous chapters.

VI.1 Some thoughts on the reference frame R^0

VI.1.1 On a general definition for R^0

If we take up the definition of a Galilean reference frame, as a reference frame in which the fundamental principle of Newton's dynamics applies, without having inertial forces to add, this definition can be extended to R^0 and define R^0 , as a reference frame in which the fundamental principle of Newton's dynamics applies, without having forces to add, since all forces are inertial forces:

$$\left[\frac{d\vec{p}}{dt} = \Sigma \vec{F}_{inertie} = 0 \right]_{R^0}$$

Thus, R^0 is the reference frame in which all the movements of spatial and spatiotemporal rotations are carried out and in which no (inertial) force exists. No generalized rotation has been undone in this reference frame R^0 .

Once again, the definition is a bit circular and remains theoretical. What is the true meaning of R^0 ? What does it correspond to physically?

Note, on the Galilean reference frame

We can interpret the Galilean reference frame as an intermediate reference frame where the generalized rotation vectors $\vec{B}_{xy}^{1/t}$, $\vec{E}l_{tx}^{1/y}$, $\vec{G}r_{tx}^{1/y} = -\frac{\partial v^t}{\partial x}$ have been cancelled (and where the corresponding fields have been added $\vec{B}_{xy}^{1/t}$, $\vec{E}l_{tx}^{1/y}$, $\vec{G}r_{tx}^{1/y}$, as well as their corresponding forces in the fundamental principle of Newton's dynamics) and where the generalized rotation vectors $\Omega_{xy}^{1/t}$, $\frac{\partial v^x}{\partial t}$ have not been cancelled.

VI.1.2 On Foucault's pendulum and Mach's principle

Let us take the case of the Coriolis force again and dwell on the Foucault pendulum experiment.

The rotation of the plane of oscillation of the Foucault pendulum makes it possible to prove that the Earth rotates on itself. This rotation of the plane of oscillation is explained by the Coriolis force. What remains difficult to grasp is the physical understanding of this reference frame R^0 in relation to which the Earth rotates?

According to Mach's principle, this reference frame R^0 could be represented by all the masses, as well as the movements of the Universe. This is a satisfactory idea for the mind, but Mach's principle remains a conjecture that is difficult to prove.

Be that as it may, by generalizing the case of the Coriolis force to all forces, we have on the one hand aggravated the question of the physical meaning of the reference frame R^0 , since it is now posed to all forces.

On the other hand, it can also be seen as a simplification. Indeed, when the question of the reference frame R^0 will be solved for the Coriolis force, it will also be solved for all forces.

VI.1.3 What would be the speed of rotation of the Sun on itself according to a Foucault pendulum placed on its surface?

For the Earth, the rotation vector Ω which is used in the Coriolis force or in the calculation of the oscillation period of the Foucault pendulum, is quasi similar with the speed of rotation of the Earth on itself relative to the Sun (the corresponding period $T = \frac{1}{\Omega}$ is called the sidereal day).

A priori, this is only a (good) approximation, because the notion of Ω used in the Coriolis force and requiring a Galilean reference frame differs from the notion of rotational speed of the Earth on itself and taking as a reference the Sun (non-Galilean reference a priori).

An interesting experiment would be to measure the period of oscillation of a Foucault pendulum around the Sun and see what speed of rotation on itself would be obtained for the Sun? What would then be the reference for the Sun?

Note on the non-equivalence of reference frames

According to Foucault's pendulum, Earth and Sun are not equivalent references in the quest for R^0 . The Sun seems a much better approximation of R^0 than the Earth, but obviously the Sun is not R^0 . The unresolved question is: what is R^0 ?

VI.1.4 Understand what is R^0 , same question for Coriolis force or weight

The Earth rotates on itself with Ω , but in relation to what? That is to say, in relation to which reference frame R^0 , the Earth is spinning? The Sun is already a good approximation of R^0 , but as we have seen, it is only an approximation.

In the reference frame R^Ω , where the rotation of the Earth has been cancelled, when a body retains a velocity \vec{v}^y , we must add a Coriolis inertial acceleration of the type $\vec{a}_{Cor_{ty}}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$.

Similarly, in a gravitational field $\vec{G}r_{tx}^{1/y}$, we have like a "rotation" in a spatiotemporal plane; but in relation to what is this "spatiotemporal rotation" carried out? That is to say, in relation to which reference frame R^0 , we "turn" in the spatiotemp plane. The issue is also not resolved.

In the reference frame R^{Gr} , where the "spatiotemporal rotation" was cancelled $\vec{G}r_{tx}^{1/y}$, when a body retains a mass \vec{m}^t , it is necessary to add an inertial force of the type $\vec{P}_{ty}^x = \vec{m}^t \wedge \vec{G}r_{tx}^{1/y}$, which is more commonly called body weight.

In conclusion, the question of understanding the nature of R^0 , arises for both the Coriolis force and the weight.

VI.2 A new interpretation for the mass \vec{m}^t ?

Some see aging as the analogue in Time of velocity v in Space. We will propose in this paragraph that this analogue, for physics, is not aging, but mass. \vec{m}^t .

VI.2.1 An analogue in Time of velocity v in Space?

In the previous chapters, we proposed the parallel between:

- the Coriolis acceleration $\vec{a}_{Cor_{ty}}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$ in Space,
- the weight $\vec{P}_{ty}^x = \vec{m}^t \wedge \vec{G}r_{tx}^{1/y}$ (or more generally $\vec{F}_{K_{ty}}^x = \vec{m}^t \wedge 2c^2\vec{K}_{tx}^{1/y}$) in Time.

In the case of Coriolis acceleration, \vec{v}^y is the quantity conserved when a change of reference frames cancelling the rotation vector in a spatial plane is made: $\vec{\Omega}_{xy}^{1/t}$.

In the case of weight, the mass \vec{m}^t (and by extension matter) appears as the analogue in time of velocity \vec{v}^y in Space. Indeed, \vec{m}^t is the quantity conserved when a change of reference frames is made cancelling the "generalized" rotation vector in a spatiotemporal plane: $\vec{G}l_{tx}^{1/y}$.

In physics, we define velocities $\vec{v}^{x,y,z}$ in Space, but not "velocities in Time". The mass \vec{m}^t would be interpreted precisely as this "velocity" in Time?

If so, the mass \vec{m}^t of a body would be the measure of its "velocity" in Time. By extension, everything that is material and endowed with a mass \vec{m}^t would have a "velocity" in Time.

Note 1 on gravitational potential

Like the mass \vec{m}^t , the gravitational potential V^t is also the analogue in Time of a velocity in Space. By contrast, it's about velocity potential $V^{x,y,z}$ that is cancelled when reference frames are changed (not the reference frame that is retained).

Note 2 on mass energy (energy at rest in Space)

The mass energy $E^t = m^t c^2$, which a physical concept similar to mass, would also be interpreted as a "velocity" in Time.

Note 3 on analogue of m^t

According to the above, the analogue in the Space of \vec{m}^t in Time, it's not the momentum $\vec{p}^{x,y,z}$, but the velocity $\vec{v}^{x,y,z}$. If we multiply frequently \vec{v} by m^t , it is to get \vec{p} , a physical quantity that can be compared in particular with energy E^t in the equations.

Similarly, the analogue in Space of the gravitational force in Time is not the Coriolis inertial force, but the Coriolis inertial acceleration. If one frequently multiplies an inertial acceleration by m^t , it is to obtain an inertial force and therefore a physical quantity that can be compared with other forces in the equations.

Note 4 on extensivity and intensity in Space and Time

The mass ∂m^t of a volume ∂Vol is a so-called extensive quantity in Space: this mass ∂m^t can add up in Space.

The small distance ∂x traveled by a system for a small period of time ∂t is a so-called intensive quantity in Space: this distance travelled ∂x is the same at any point in the Space of the isolated system studied.

If we swap Space and Time, ∂m^t and ∂x , we obtain:

The distance ∂x travelled by a system for a period of Time ∂t is a so-called extensive quantity in Time: this distance travelled ∂x can add up in Time.

The mass ∂m^t of a volume ∂Vol is a so-called time-intensive quantity: this mass ∂m^t is the same at all times of the Time of the isolated system studied.

Note 5 on density of mass as the analogue in Time of velocity in Space

You can also see the mass volumetric density masse (or volumetric mass) $\rho^t = \frac{\partial m^t}{\partial Vol}$ as the analogue in the Time of velocity $v^x = \frac{\partial x}{\partial t}$ in Space. Both are also intensive quantities in Space, defined locally.

Note 6 on electric charge

The electric charging \vec{q}^t is also close to the notion of "speed" in Time. We have the parallels:

$$\begin{aligned}\vec{E}m_{ty}^x &= \vec{v}^y \wedge \vec{B}_{xy}^{1/t} \\ \vec{F}_{Elty}^x &= \vec{q}^t \wedge \vec{E}_{tx}^{1/y}\end{aligned}$$

This observation is also part of a broader question: what brings together, but ultimately distinguishes a mass from an electric charge?

VI.2.2 A cinematic in Space-Time?

Following a relativistic approach to movements, a non-zero mass body \vec{m}^t would have a "velocity" in Time different from the observer who measures its mass. Similarly, a zero-mass body \vec{m}^t would have a "velocity" in Time identical to the observer.

More generally, matter and motion could be described by a kinematics extended to Space-Time. Both would be interpreted as movements in Space-Time.

Many academic books on the mechanics of point, fluids or solids are composed of:

- a first kinematic chapter that describes the movements in Space,
- a second kinetic chapter that introduces the notions of mass and momentum.

It is proposed here, in a way, to group these two chapters into a single kinematic chapter that would describe the movements in Space-Time.

Note 1 on the enigma of the absence of negative mass

Unlike electric charges (and other charges such as strong, weak nuclear ...), there is no negative mass. This remains an enigma and has the consequence that the gravitational force is always attractive.

In the absence of negative mass, there is not a priori for "velocity" in Time any notion of sense as for speed in Space.

It should be noted, however, that the study interval $[-c, +c]$ of a speed $\vec{v}^{x,y,z}$ in Space is restricted as the interval $[0, +\infty[$ of a mass \vec{m}^t .

This difference in restriction in the two study intervals: $[-c, +c]$ and $[0, +\infty[$, could it simply be due to a difference in views in the way speed and mass are measured?

Some physicists such as John Wheeler or Richard Feynman have also proposed to interpret the signs of electric charges, as advances in one direction of Time or in the other direction.

Note 2 on relativistic mass γm^t

In Special Relativity, we introduce the notion of relativistic mass γm^t , with:

- m^t the resting mass of a body with respect to an observer,
- γm^t its mass when this body has a velocity v with respect to the observer.

For example, photons, if they were at rest in Space from an observer, would have zero mass m^t , and therefore a zero "velocity" in Time.

Note 3 on "Light Time"

In the Memoir 6 more speculative than the previous ones, we will propose the notion of Light Time, with as a first meaning: a Light Time that would be that of Light and photons, and with as a second meaning: a Light Time that would also be ours, human beings, who will measure a zero mass $m^t = 0$, that is, a zero "speed" in the Time of Light, if the Light were at rest in Space with respect to us.

VI.3 Conclusion of the Memoir, on the choice of the inertial reference framework

In this 2nd Memoir, we have taken up the initial principle set by A. Einstein in the theory of General Relativity: an invariance of the laws of Nature regardless of the change of reference frames.

Nevertheless, we have presented a different approach from that proposed by A. Einstein in General Relativity, where he relies on a conservation of the Space-Time interval:

$$ds^2 = g_{ij} dx^i dx^j$$

regardless of the change of reference frames.

Here, we focused on the classical forces. We wanted to erase the differences between "normal" force and inertial force by building any classical force on the model of the Coriolis inertial force.

The main idea was as follows: to choose a reference frame describing a body:

- in generalized rotational motion in a spatial and/or spatiotemporal plane,
- subjected to no force, inertial acceleration, or electromotor field,

we have the possible change in a reference frame describing the body:

- motionless in Space-Time, except its own speed \vec{v}^x and its own mass \vec{m}^t , the generalized rotation vectors in spatial or spatiotemporal planes that have been cancelled when changing reference frames,
- subjected to forces, inertial accelerations and/or electromotor fields.

In the next Memoir, we will propose using the fields $K_{tx}^{1/y}$ and $\Omega_{xy}^{1/t}$ defined in this Memoir, some source field equations fields analogous to Maxwell's equations and which apply to gravitation.

Bibliography of the Memoir 2

On Newtonian gravitation and the Coriolis force

- [1] H. Lumbroso, *Problèmes résolus de mécanique du point et des systèmes de points*. Dunod, Paris, 1984.

On electrostatics

- [2] H. Lumbroso, *Problèmes résolus d'électrostatique et dynamique des particules chargées*. Dunod, Paris, 1984.

On magnetism

- [3] A. Einstein, L. Infeld, *The Evolution of Physics: The Growth of Ideas from Early Concepts to Relativity and Quanta*, Cambridge University Press, 1938
[4] H. Lumbroso, *Electromagnétisme, problèmes résolus*. McGraw-Hill, Paris, 1985.

On relativistic quantum Electrodynamics

- [5] R. Feynman, *The principle of least action in Quantum Mechanics*, thèse, Princeton University, 1942
[6] Wikipedia, Electromagnetic tensor
[7] Wikipedia, Lagrangian
[8] Wikipedia, Maxwell's equations

On fluid mechanics and the mechanics of continuous media

- [9] H. Lumbroso, *Problèmes résolus mécanique des fluides*. Dunod, Paris, 1989.
[10] Jean-François Sini, *Mécanique des fluides*. Centrale Nantes, 2008.

On the mechanics of solids

- [11] C. Grossetête et P. Olive, *Mécanique des systèmes et du solide*. Ellipses, Paris, 1999.

Table of Contents

Memoir 2	"Generalized" rotation vectors and fields in a spatiotemporal plane.....	73
Chapter I	Newtonian gravitational and Electrostatic Coulomb analogies	74
I.1	Newtonian gravitational potential and Coulomb electric potential.....	74
I.2	The notions of energy and power in gravitation and electrostatics	76
I.3	Conclusion of the chapter.....	78
Chapter II	Analogies fluid mechanics and magnetism, analogies Ω and B.....	79
II.1	Law of Biot and Savart (1820), analogies gravitation, electrostatics and magnetism.....	79
II.2	Analogies between fluid mechanics and magnetism.....	80
II.3	Analogies between rotation vector Ω and magnetic field B	83
II.4	Reminders on generalized electromagnetic potential energy	89
II.5	Conclusion of the chapter.....	92
Chapter III	Analogies between the mechanics of continuous media and relativistic quantum electrodynamics	93
III.1	Historical presentation of relativistic quantum Electrodynamics (QED).....	93
III.2	Reminders on the continuous media mechanics (CMM)	94
III.3	Reminders on Maxwell Faraday's tensor, analogies between the tools of CMM and relativistic quantum Electrodynamics.....	97
III.4	Conclusion of the chapter.....	101
Chapter IV	"Generalized" rotation vectors and fields.....	102
IV.1	Analogies between magnetic fields B , electric field E and rotation vector Ω	102
IV.2	Vector generalized rotations and fields in spatial or spatiotemporal planes	104
IV.3	Conclusion of the chapter.....	108
Chapter V	Build any force on the model of the Coriolis inertial force and the Lorentz magnetic force	109
V.1	Objective and angles of attack.....	109
V.2	Quantities retained and quantities cancelled when changing reference frame, speed, mass, and electric charge	110
V.3	Coriolis inertial acceleration and induced electromotor field, both obtained from the cancellation of a generalized rotation vector in a spatial plane.....	113
V.4	Forces obtained from the cancellation of a generalized rotation vector in a spatiotemporal plane	115
V.5	Transition from Coriolis inertial acceleration and electromotor field to forces	120
V.6	General table (quantities cancelled and retained when changing reference frames).....	123
Chapter VI	Thoughts on $R0$ and new interpretation for the mass?	125
VI.1	Some thoughts on the reference frame $R0$	125
VI.2	A new interpretation for the mass mt ?	126
VI.3	Conclusion of the Memoir, on the choice of the inertial reference framework.....	128
Bibliography of the Memoir 2.....		130

Memoir 3 Maxwell's source field equations to gravitational equations

Summary of the Memoir

In this 3rd Memoir, we are interested in Maxwell's source field equations, equations involving magnetic and electric fields, electric charge and electric currents, and applying to electromagnetism. Following an analogy electricity gravitation, we will propose source equations fields involving the pulsation and wave vector fields, the mass and the momentum, and applying to gravitation. At the end of the Memoir, we will also propose for gravitation the analogue of Maxwell Faraday's homogeneous equation.

This Memoir is also an opportunity to discuss some important discoveries in physics. We return to the work of André-Marie Ampère, in particular the observation that a magnet and a current loop produce identical effects (work carried out from 1820), then to Maxwell's equations proposed by James Clerk Maxwell in 1865 and transcribed using partial derivatives in 1884 by Oliver Heaviside and Willard Gibbs.

We also talk about Louis de Broglie's idea (1924) of an electron that is both wave and particle. In passing, we deal with a little Newtonian mechanics (Isaac Newton's *Principia* published in 1687), Hamiltonian mechanics (William Rowan Hamilton around 1833), Einsteinian mechanics (with $E = mc^2$ proposed by Albert Einstein in 1905 as part of Special Relativity). There is also mention of the Compton scattering (1922). We also discuss the gravitational waves suggested by A. Einstein in 1916 as part of General Relativity and confirmed experimentally in 2015 by researchers at LIGO (Laser Interferometer Gravitational-Wave Observatory).

Chapter I Original idea of memory, velocity of electric charge and historical reminders

Purpose of the chapter

We evoke the original idea of this Memoir: a velocity of the electric charge comparable to a group velocity. We then perform a historical reminder on Maxwell's equations, then on gravitational waves.

I.1 Original idea of the Memoir, a velocity of the electric charge comparable to a group velocity?

In 1924, L. De Broglie defends a thesis in which he proposes that the electron corpuscle is also similar to an electron wave. According to him, the velocity of the electron corpuscle orbiting the nucleus of the atom (in the Bohr electron model), corresponds to the group velocity $v_g = \frac{\partial \Omega}{\partial K}$ of the electron wave. Thus, with great audacity for his time, he associates the notion of velocity developed in Newtonian mechanics with that of group velocity developed in wave mechanics.

In this Memoir, we will see that by making the ratio of the two Maxwell source field equations (that of Gauss and that of Ampère), we can find a physical quantity, ratio of the electrical current density: $j = \rho v$ (unit: $\frac{A}{m^2}$ or $\frac{C}{m^2 \times s}$) and the electrical charge volumetric density (or electrical charge bulk density): ρ (unit: $\frac{C}{m^3}$), involving magnetic fields B and electric field El :

$$\mu_0 \varepsilon_0 \frac{j}{\rho} = \mu_0 \varepsilon_0 v = \frac{\partial B}{\partial El}$$

This last equation is formally similar to that of group velocity, especially if we bring the magnetic field B closer to a pulsation field Ω , and the electric field El closer to a wave vector field K .

Based on these findings, the idea developed in the next chapters is that, like Maxwell source field equations, involving B and El , and applying to electromagnetism, there are source field equations involving the pulsation field Ω and the wave vector field K , and applying to gravitation.

The sources of these equations will no longer be charge volumetric density ρ and electrical current density j , but mass volumetric density ρ (unit: $\frac{Kg}{m^3}$), as well as masses m and momentums p . By making the ratio of these source field equations applied to gravitation, it must be possible to regain velocity group $v_g = \frac{\partial \Omega}{\partial K}$ of the wave of a mass particle.

One of the questions will be to determine the constants that intervene in the source field equations of gravitation. In Maxwell's source field equations, we use constants μ_0 et ε_0 , related by the relationship:

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Note that these constants appear in $\mu_0 \varepsilon_0 v = \frac{\partial B}{\partial El}$, while there is no constant for group velocity $v_g = \frac{\partial \Omega}{\partial K}$. At a minimum, it is expected that the gravitational G constant will be found in the source field equations of gravitation.

I.2 Historical reminders, Maxwell source field equations

During the 1860s, J. C. Maxwell published a voluminous treatise of more than 1,000 pages on electricity and magnetism. In this treatise entitled *A Treatise on Electricity and Magnetism*, the physicist is inspired by the work of Michael Faraday on the magnetic field, as well as those of William

Thomson (Lord Kelvin) or André-Marie Ampère. He proposes the mathematization of this work, inspired by the tools of fluid mechanics.

J. C. Maxwell opposes the Newtonian conception of forces with remote action. It takes up Mr. Faraday's conception of a continuous medium supporting the transformations of a field through Space, transformations of a field that thus makes it possible to propagate the force.

In his treatise, J. C. Maxwell proposes eight equations that are now grouped together as Maxwell's equations. In 1884, these eight equations were transcribed by O. Heaviside and W. Gibbs into four equations and rewritten using partial derivatives.

We are interested here in the first two, the so-called source field and designated by Maxwell Gauss:

$$\frac{\rho}{\varepsilon_0} = \text{div}\vec{E}l$$

$$\mu_0\vec{J} = \text{rot}\vec{B} - \mu_0\varepsilon_0 \frac{\partial\vec{E}l}{\partial t}$$

As the name suggests, these two equations connect electric and magnetic fields to sources of electrical charge density and electrical current density. They differ from Maxwell Thomson's homogeneous equations:

$$\text{div}\vec{B} = 0$$

and Maxwell Faraday:

$$\text{rot}\vec{E}l = -\frac{\partial\vec{B}}{\partial t}$$

which they involve only electric field and magnetic field.

Before J. C. Maxwell's treatise, the maxwell Gauss and Maxwell Ampère equations had already been proposed in different forms. The main innovation of J. C. Maxwell is to add in the Maxwell Ampere equation a displacement current:

$$\vec{J}_D = \varepsilon_0 \frac{\partial\vec{E}l}{\partial t}$$

which on the model of the conservation of the mass, allows the conservation of the electric charge.

The principles of conservation of mass or electric charge are thus translated into the same equation:

$$\frac{\partial\rho}{\partial t} + \text{div}(\rho\vec{v}) = 0$$

with ρ the volumetric density of mass or electric charge and \vec{v} the velocity of particles of mass or electric charge.

I.3 Drawing inspiration from gravitational waves

In 1916, as part of General Relativity, A. Einstein proposed the notion of gravitational waves, which he interpreted as an oscillation of the curvature of Space-Time. It also relies on an analogy with electromagnetic waves.

According to the principle of the transmitting antenna, an electric charge in accelerated motion in an antenna (for example, a sinusoidal alternating electric current in an antenna) radiates an electromagnetic wave propagating at the speed of light in a vacuum. Similarly, a mass in accelerated motion could radiate a gravitational wave propagating at the speed of light in a vacuum.

According to the principle of the receiving antenna, an electromagnetic wave generates an acceleration of the electric charges present in an antenna (for example an alternating current). Similarly, a gravitational wave would generate an acceleration of the masses present in an antenna.

Let's study the case of a radio antenna called half-wave with length $l = \frac{\lambda}{2}$ traversed by a sinusoidal electric current:

$$I(z, t) = I_0 e^{i\omega t}$$

The antenna radiates an electromagnetic field. We give below the electric field part in polar coordinates (r, θ) :

$$\vec{E}l_\theta(M, t) \approx \frac{iI_0}{2\pi\epsilon_0 cr} \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} e^{i(\omega t - k r)} \vec{u}_\theta$$

The half-wave radio antenna and the electric field can be symbolized by the following figure:

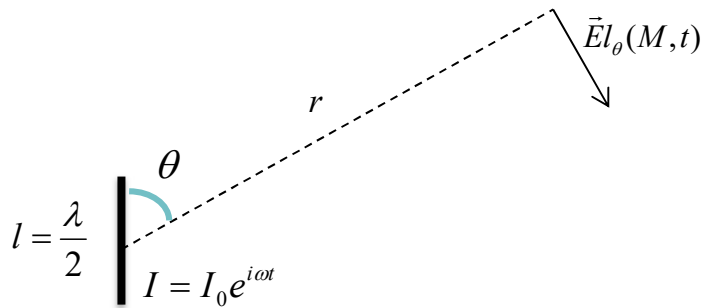


Figure 1: half-wave radio antenna

The principle of the transmitting and receiving antenna derives in particular from Maxwell Ampere's source field equation:

$$\mu_0 \vec{j} = \text{rot} \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}l}{\partial t}$$

This equation translates that a current (a moving electric charge) produces a magnetic field \vec{B} and an electric field $\vec{E}l$. Conversely, these two fields behave in a vacuum like an electromagnetic wave, which generates an electric current in an antenna.

Note, transition from Maxwell's equations to d'Alembert's wave equation

From Maxwell Faraday, considering Maxwell Gauss and Maxwell Ampère, we find

$$\Delta \vec{E}l - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}l = \vec{\nabla} \left(\frac{\rho}{\epsilon_0} \right)$$

with

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

In a vacuum, we obtain:

$$\Delta \vec{E}l - \frac{1}{c^2} \frac{\partial^2 \vec{E}l}{\partial t^2} = 0$$

From Maxwell Ampère, considering Maxwell Thomson and Maxwell Faraday, we find:

$$\Delta \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{B} = \vec{\nabla} \times (\mu_0 \vec{j})$$

In a vacuum, we obtain:

$$\Delta \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

These 2 equations indicate that electric and magnetic fields follow the d'Alembert wave equation in a vacuum.

In 2015, the hypothesis of A. Einstein of a gravitational wave appears to be confirmed. Detectors called LIGO (Laser Interferometer Gravitational-Wave Observatory), one located in Louisiana, the other in the state of Washington record the expected signals. Without going into the details of the complex operation of these detectors, they are based on a principle of receiving antennas, with masses set in motion accelerated during the presence of gravitational waves.

Since there are source field equations in electromagnetism (Maxwell Ampère and Maxwell Gauss) describing the link between the antennas (i.e., the source: charge in accelerated motion) and the electromagnetic wave (i.e., the fields \vec{E} et \vec{B} spreading at speed c in a vacuum), we conjecture for gravitation analogous source field equations, describing the link between antennas (i.e., the source: mass in accelerated motion) and the gravitational wave.

In the 2nd Memoir, we highlighted the analogies between the fields \vec{B} , \vec{E} , $\vec{\Omega}$, \vec{K} . We would therefore like source field equations applying to gravitation and involving field $\vec{\Omega}$ and field \vec{K} . It is these source field equations that we will research and propose in this 3rd memory.

Note

We conjecture here gravitational waves constructed from fields $\vec{\Omega}$ and \vec{K} rather than simply from the Newtonian gravitational field $Gr = -\frac{\partial V^t}{\partial r}$ as General Relativity does. We will come back to this delicate point at the end of this Memoir, and in the 6th Memoir when we will evoke the Poisson's source potential equation:

$$4\pi G \rho_{mas} = \Delta V^t$$

an equation on which General Relativity is based.

I.4 Conclusion of the chapter

Following an analogy gravitation electromagnetism, we conjecture the existence of source field equation applying to gravitation, and involving fields $\vec{\Omega}$ et \vec{K} .

This conjecture comes in particular from a velocity of the electric charge comparable to a group velocity and obtained from Maxwell's source field equations. It is also inspired by gravitational waves.

In the next chapter, we will return to the notions of phase velocity and group velocity.

Chapter II Reminders phase velocity, group velocity, extensions of relativistic mechanics and Newtonian mechanics to wave mechanics, elements of Special Relativity

Purpose of the chapter

L. de Broglie's particle wave approach was first justified because it made it possible to intuitively explain certain aspects of relativistic mechanics. Yet, as a paradox, the ideas of L. de Broglie first led to the Schrödinger wave equation which is not relativistic.

In this chapter, we will explain the notions of phase velocity and group velocity that are at the heart of L. de Broglie's "wave" approach. We will also see how the "wave" ideas of L. de Broglie can intuitively illuminate certain points of relativistic mechanics, as well as Newtonian mechanics.

II.1 Reminders on phase velocity and group velocity

II.1.1 Phase velocity

The phase velocity $v_\phi = \frac{dx}{dt}$ of a progressive wave is the velocity at which the phase of the wave $\phi = \omega t - kx$ spreads into Space.

Let be a progressive monochromatic wave defined by the wave function:

$$\psi(x, t) = \psi_0 \cos(\omega t - kx)$$

The phase plan is the set of points with the same phase value ϕ . The phase plan is located in x at the moment t , and in $x + dx$ at the moment $t + dt$.

So, we have:

$$\phi = \omega t - kx$$

$$\phi = \omega(t + dt) - k(x + dx)$$

By subtraction, we obtain:

$$0 = \omega dt - k dx$$

So, we have a phase velocity ratio of the pulsation ω and the wave vector k :

$$v_\phi = \frac{dx}{dt} = \frac{\omega}{k}$$

Nota

In a refractive medium (medium where we observe a refraction of light, that is to say a deviation of the light ray), we have the relationship (with n the refractive index of the medium):

$$n = \frac{c}{v_\phi} < 1$$

II.1.2 Group velocity

In the 1880s, in order to better understand wave phenomena, physicists Louis Georges Gouy and John William Rayleigh proposed to distinguish a phase velocity and a group velocity. Group velocity is thought of as the speed of a wave packet of very close pulsations (or periods), as well as a very close wave vector (or wavelengths).

Let us take the simple case of a wave packet consisting of the superposition of two waves of close pulsations ω_1 and ω_2 , of close wave vectors k_1 and k_2 , with amplitude equal to the unit:

$$\psi(x, t) = \cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$$

$$\psi(x, t) = 2 \cos\left(\frac{\omega_1 + \omega_2}{2} t - \frac{k_1 + k_2}{2} x\right) \cos\left(\frac{\omega_2 - \omega_1}{2} t - \frac{k_2 - k_1}{2} x\right)$$

The wave packet obtained is the product of 2 terms:

- the first is a monochromatic phase velocity wave $v_\phi = \frac{\omega_1 + \omega_2}{k_1 + k_2}$ corresponding to a weighted average of the pulsations of the two waves by their respective wave vectors,
- the second is a monochromatic wave of phase velocity $v_\phi = \frac{\omega_2 - \omega_1}{k_2 - k_1}$. It acts as an amplitude modulator of the first term.

There is a phenomenon of beating. A high-frequency sinusoid with characteristics close to those of the two pulsations $\omega_1 + \omega_2$ is modulated in amplitude by a sinusoid of lower pulsation $\omega_2 - \omega_1$. This one, of low frequency, forms a kind of envelope around the other.

For values close to the two pulsations and the two wave vectors, the group velocity of the wave packet is approximately equal to the velocity of the wave. $v_g = \frac{\omega_2 - \omega_1}{k_2 - k_1}$. By making tender $\omega_2 - \omega_1$ and $k_2 - k_1$ towards 0, we obtain the group velocity:

$$v_g = \frac{d\omega}{dk}$$

Note

For an electromagnetic wave, the phase velocity and the group velocity are related by the relationship (valid for low frequencies only):

$$v_g v_\phi = \frac{c^2}{n^2}$$

II.2 Extensions of relativistic mechanics and Newtonian mechanics to wave mechanics

II.2.1 General information on relativistic mechanics

Relativistic mechanics developed at the beginning of the 20th century, thanks in particular to the work of A. Einstein in the theory of Relativity. In many ways, it resembles classical Newtonian mechanics of which it is often considered a generalization. We find, for example, the notions of energy and momentum. Nevertheless, it differs singularly from it in certain respects.

For example, a coefficient $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ marking the deformations of durations and distances is introduced into relativistic mechanics.

Thus, in classical mechanics, we have the momentum: $p = mv$ and in relativistic mechanics, the momentum: $p = \gamma mv$.

Similarly, in classical mechanics, we have the energy momentum relationship:

$$E = \frac{p^2}{2m} + Ep$$

and in relativistic mechanics, we have the energy momentum relationship:

$$E = \frac{c^2}{v} p = \gamma mc^2$$

It is sometimes difficult to intuitively understand the reasons for these differences. We will now see that the wave approach of L. de Broglie makes it possible to intuitively understand certain equations of relativistic mechanics.

II.2.2 Ideas of L. de Broglie to intuitively understand the Lorentz transform

We recall here the ideas of L. de Broglie developed at the beginning of his thesis (1924), which make it possible to understand intuitively the Lorentz transform. As a paradox, these ideas imbued with relativism lead a few years later to the Schrödinger equation which has nothing relativistic (but which has everything waving).

Let be a standing wave of wave function:

$$\psi = \cos \omega_0 t'$$

In his thesis, L. de Broglie schematizes this wave function by a clock.

Let be an observer B motionless with respect to this wave function and let be an observer A mobile with respect to B , advancing at uniform rectilinear velocity v_g (this is a Newtonian mechanical velocity in the sense of describing point bodies, but we write v_g because L. de Broglie then makes it correspond to a group velocity).

The question asked by L. de Broglie is how A will the wave (or the clock) see?

According to L. de Broglie, A sees it as a progressive (and no longer stationary or standing) wave propagating at phase velocity $v_\phi = \frac{\omega}{k}$ and with wave function:

$$\psi = \cos \omega \left(t - \frac{x}{v_\phi} \right)$$

Following the relativistic ideas of A. Einstein, we have the relationship between the energy E of a relativistic body moving at velocity v_g and the energy E_0 of the same body if it is considered motionless:

$$E = \gamma E_0 \text{ with } \gamma = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} > 1$$

We have $E = \hbar \omega$ with ω the pulsation of the progressive wave, $E_0 = \hbar \omega_0$ with ω_0 the pulsation of the standing wave.

We obtain:

$$\omega = \gamma \omega_0$$

with $\gamma > 1$. The progressive wave therefore has a higher pulsation and frequency than the standing wave.

We have for the wave function of the progressive wave:

$$\psi = \cos \gamma \omega_0 \left(t - \frac{x}{v_\phi} \right)$$

We have the relationship between the group velocity and the phase velocity:

$$v_g v_\phi = c^2$$

with $n = 1$ since we are in a vacuum.

If we replace v_ϕ by v_g in the wave function, we obtain:

$$\psi = \cos \gamma \omega_0 \left(t - \frac{v_g}{c^2} x \right)$$

If we equalize the phases of the two standing and progressive wave functions, we obtain:

$$\phi = \omega_0 t' = \gamma \omega_0 \left(t - \frac{v_g}{c^2} x \right)$$

By eliminating the pulsation on both sides ω_0 , we find the Lorentz transform between the moments t' of the observer B and t of the observer A :

$$t' = \gamma \left(t - \frac{v_g}{c^2} x \right)$$

Following the ideas of L. de Broglie, the relativistic effect is interpreted as the variation of the period T (and the frequency $f = \frac{\omega}{2\pi}$) of a wave as a function of the phase velocity of the wave $v_\phi = \frac{c^2}{v_g}$, velocity measured relative to an observer. In other words, when a standing wave moves relative to an observer and becomes progressive with a phase velocity $v_\phi = \frac{c^2}{v_g}$, the observer measures a variation in the period T and in the frequency f of the wave relative to the period T_0 and to the frequency f_0 of the standing wave.

Note 1

By analogy between Space and Time, by substituting ct by x , we have the Lorentz transform for the positions:

$$x' = \gamma(x - v_g t)$$

Note 2

What is not explained intuitively here is why we have the relationship between energies:

$$E = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} E_0, \text{ that is, the origin of } \gamma = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}}$$

We will look at this in a future paragraph.

In summary, L. de Broglie questions the variations in the frequency of a wave, or more generally the frequency variations of any periodic phenomenon, when it becomes mobile.

Let be a periodic phenomenon, motionless in relation to an observer, with a pulsation ω_0 . When this periodic phenomenon becomes mobile, with a uniform rectilinear velocity v_g relative to the observer, does this phenomenon retain the same pulsation ω_0 for the observer?

Special Relativity answers that no. When a periodic phenomenon of initial pulsation ω_0 becomes mobile, with a uniform rectilinear velocity v_g relative to the observer, the observer measures for the periodic phenomenon a new pulsation ω always greater than ω_0 and equal to:

$$\omega = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \omega_0$$

Note on mass

According to relations $E = \hbar\omega = mc^2 = \gamma E_0 = \gamma \hbar\omega_0 = \gamma m_0 c^2$, it is noted that the faster a body moves, the more its mass $m = \gamma m_0$ increases.

II.2.3 Phenomenon of duration expansion (or slowing down of clocks)

The term Time dilation refers to an effect of Special Relativity, according to which the time interval between two events measured in any Galilean (or inertial) reference frame R is always greater than the time interval measured in the Galilean reference frame R^0 (in relative motion with respect to R), where these two events have the same spatial position (but do not take place at the same time).

In Special Relativity, Time is not the same according to the reference frame. To characterize these differences, clocks are used, each of which is specific to a reference frame. To an observer, the clock of a moving reference frame seems slowed down compared to the clock of a stationary (or a standing) reference frame. We are talking about slowing down the clocks.

To find the phenomenon of dilation of durations, we can use the Lorentz transform:

$$t' = \gamma \left(t - \frac{v_g}{c^2} x \right)$$

We have the Time interval between 2 events:

$$\Delta t' = \gamma \left(\Delta t - \frac{v_g}{c^2} \Delta x \right)$$

In the case of a fixed body in R^0 , with the 2 events separated from Δt_0 having the same spatial position, we have: $\Delta x_0 = 0$. We obtain:

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \Delta t_0$$

$$\Delta t \geq \Delta t_0$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \geq 1$$

Note, length contractions

We have the analogous phenomenon (and inverse in the result) for lengths. Following the Lorentz transform:

$$\Delta l' = \gamma (\Delta l - v_g \Delta t)$$

When the 2 measures allowing to obtain Δl_0 are made in R^0 at the same moment, we have: $\Delta t_0 = 0$. We obtain:

$$\Delta l = 1 - \frac{v_g^2}{c^2} \Delta l_0$$

$$\Delta l \leq \Delta l_0$$

II.2.4 Group velocity, check that γ works

We are now looking for the relationship between group velocity v_g and γ .

The group velocity of the particle wave must check:

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial E}{\partial p} = \frac{\partial E}{\partial \frac{E v_g}{c^2}}$$

We obtain:

$$\frac{c}{v_g} = \frac{\partial E \frac{v_g}{c}}{\partial E}$$

The energy E is proportional to γ and to a constant (in Special Relativity, we take the energy of an motionless mass $E_0 = m_0 c^2$, and we have $E = \gamma m_0 c^2 = \gamma E_0 = \gamma \hbar \omega_0$).

So, we have the differential equation to solve to find the relationship between v_g and γ :

$$\frac{c}{v_g} = \frac{\partial \frac{v_g}{c} \gamma \hbar \omega_0}{\partial \gamma \hbar \omega_0} = \frac{\partial \frac{v_g}{c} \gamma}{\partial \gamma}$$

Note that if we propose:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}}$$

That is, if v_g and γ verify the relationship:

$$(\gamma)^2 - \left(\gamma \frac{v_g}{c}\right)^2 = 1$$

$$\left(\gamma \frac{v_g}{c}\right)^2 = (\gamma)^2 - 1$$

The differential equation is well solved:

$$\frac{\partial \frac{v_g}{c} \gamma}{c \partial \gamma} = \frac{\partial \sqrt{(\gamma)^2 - 1}}{c \partial \gamma} = \frac{2\gamma}{2c \sqrt{(\gamma)^2 - 1}} = \frac{1}{v_g}$$

The value γ (dilation of durations) is therefore imposed by the phase velocity, the group velocity, the relationship $v_g v_\phi = c^2$, the relationships $E_0 = \hbar \omega_0 = m_0 c^2$ and $p = \hbar k$.

Note 1

If we pose $E = \gamma E_0 = \gamma m_0 c^2$, from the relationship $(\gamma)^2 - \left(\gamma \frac{v_g}{c}\right)^2 = 1$, multiplying by $m_0^2 c^4$, we find again:

$$(\gamma)^2 m_0^2 c^4 - \left(\gamma \frac{v_g}{c}\right)^2 m_0^2 c^4 = m_0^2 c^4$$

$$E^2 - p^2 c^2 = m_0^2 c^4$$

Note 2

In the theory of Relativity, the energy momentum quadrivector (or four-momentum) is defined by:

$$4p^\mu = (p^t c = \gamma m_0 c^2, p^x = \gamma m_0 v^x, p^y = \gamma m_0 v^y, p^z = \gamma m_0 v^z)$$

$$4p_0^\mu = (p_0^t c = m_0 c^2, p_0^x = m_0 v^x, p_0^y = m_0 v^y, p_0^z = m_0 v^z) \text{ with } \gamma = 1$$

We have:

$$E = p^t c = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v_g^2}{c^2}}}$$

If we develop in series at the limit $\frac{v_g}{c} \ll 1$, we obtain:

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^2} + \dots$$

If $v = 0$, we find again $E_0 = m_0 c^2$.

$\frac{1}{2} m_0 v^2$ corresponds to kinetic energy.

The term $\frac{3}{8} m_0 \frac{v^4}{c^2}$, as well as the following, can be seen as a relativistic correction to kinetic energy.

A relativistic mass is frequently defined m with:

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v_g^2}{c^2}}}$$

II.2.5 Dispersion relation in the case of classical Newtonian mechanics

In the context of the Schrödinger equation, we saw in Memoir 1 that wave mechanics could also be considered as an extension (or even a generalization) of Newtonian classical mechanics. Here we make some reminders on group and phase velocities in mechanics applied to Newtonian mechanics.

Group velocity

We have the energy of classical Newtonian mechanics (non-relativistic):

$$E = \hbar\Omega = \frac{p^2}{2m} + Ep$$

with Ep a potential energy.

We obtain the momentum of classical Newtonian mechanics:

$$p = mv = \sqrt{2m(\hbar\Omega - Ep)}$$

According to de Broglie, we have:

$$p = \hbar K$$

We obtain:

$$\frac{\partial \hbar K}{\partial \hbar\Omega} = \frac{\partial \sqrt{2m(\hbar\Omega - Ep)}}{\partial \hbar\Omega} = \frac{m}{\sqrt{2m(\hbar\Omega - Ep)}} = \frac{m}{p} = \frac{1}{v_g}$$

We find again the group velocity:

$$v_g = \frac{\partial \Omega}{\partial K}$$

Phase velocity

For the phase velocity, we have the dispersion relation of Newtonian wave mechanics:

$$v_\phi = \frac{\Omega}{K} = \frac{E}{p} = \frac{E}{\sqrt{2m(E - Ep)}}$$

We can define a refractive index n with v_ϕ^0 the phase velocity of the wave in the absence of potential:

$$\frac{v_\phi}{n} = v_\phi^0 = \frac{E}{\sqrt{2mE}}$$

We obtain for the index n by eliminating v_ϕ in the last two expressions:

$$n^2 = 1 - \frac{Ep}{E}$$

We obtain for the phase velocity:

$$v_\phi = \sqrt{1 - \frac{Ep}{E}} \frac{E}{\sqrt{2mE}} = n \frac{E}{\sqrt{2mE}}$$

When n tends towards 1 (Ep tends towards 0), we find again the dispersion relationship between the momentum and the energy for a wave in a vacuum:

$$v_\phi = v_\phi^0 = v_g^0 = \frac{E}{\sqrt{2mE}} = \frac{E}{p}$$

II.2.6 Phase velocity, finding the dispersion relationship between energy and momentum in the case of relativistic mechanics

For a photon, we have a phase velocity equal to:

$$v_\phi = \frac{\omega}{k} \text{ with } v_\phi = v_g = c$$

If we pose $E = \hbar\omega$ $p = \hbar k$, we have the dispersion relation:

$$v_\phi = \frac{E}{p}$$

$$E = pv_\phi = pc$$

For any particle wave, we always have the dispersion relation:

$$E = pv_\phi \text{ (by contrast } v_\phi \neq v_g \neq c)$$

We have:

$$v_g v_\phi = c^2$$

Note: In non-relativistic theories, we do not use $v_g v_\phi = c^2$ to have the relationship between momentum and energy.

We obtain the dispersion relationship between the momentum and the energy of relativistic mechanics:

$$E = \frac{c^2}{v_g} p$$

When v_g tends towards c , we find the dispersion relationship of a wave in a vacuum:

$$E = pc$$

Nota 1

In Special Relativity, we have: $E^2 = \frac{c^2}{v_g^2} p^2 = p^2 c^2 + m_0^2 c^4$. We also find the dispersion relationship of a wave in a vacuum: $E = pc$, when the mass of the particle tends to 0.

Note 2

The dispersion relationship $E = p v_\phi^0 = pc$ is a borderline case:

- in relativistic mechanics, when the mass of the particle tends to 0,
- in classical Newtonian mechanics, when the potential energy (of the particle in a field) tends to 0.

II.3 Some reminders about the relativistic Doppler-Fizeau effect

II.3.1 Interpretation of the relativistic Doppler-Fizeau effect

The interpretation of the relativistic effect may be reminiscent of the Doppler-Fizeau effect. Indeed, for these two effects, we have a variation in the frequency as a function of the velocity v_g between the body emitting the signal and the body receiving it. We will remind in this paragraph and the next one that the cumulation of the Doppler-Fizeau effect and the dilation effect of the durations corresponds exactly to the relativistic effect (Lorentz transform).

Let's first consider the case of a transmitter that moves at velocity v_g and a receiver that is fixed. Following the "classic" Doppler effect, we have the relationship between the frequency f_{rec} received by the fixed receiver and frequency f_{trans} issued by the mobile transmitter:

$$f_{rec} = \frac{1}{1 + \frac{v_g}{c}} f_{trans}$$

Depending on the expansion effect of the durations, the durations of the mobile transmitter measured by the fixed receiver are increased by a factor $\frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}}$. The frequencies of the mobile transmitter

measured by the fixed receiver are therefore reduced by a factor $\sqrt{1 - \frac{v_g^2}{c^2}}$. So, we have:

$$f_{rec0} = \sqrt{1 - \frac{v_g^2}{c^2}} f_{trans}$$

By cumulating the 2 effects, we have the relativistic Doppler-Fizeau effect:

$$f_{rec0} = \frac{1}{1 + \frac{v_g}{c}} \times \sqrt{1 - \frac{v_g^2}{c^2}} f_{trans} = \frac{\sqrt{\left(1 - \frac{v_g}{c}\right)\left(1 + \frac{v_g}{c}\right)}}{1 + \frac{v_g}{c}} f_{trans}$$

$$f_{rec0} = \sqrt{\frac{1 - \frac{v_g}{c}}{1 + \frac{v_g}{c}}} f_{trans}$$

Next consider the case of a receiver that moves at velocity v_g and a transmitter that is fixed. Following the Doppler effect, we have the relationship between the frequency f_{rec} received by the mobile receiver and frequency f_{trans} issued by the fixed transmitter:

$$f_{rec} = \left(1 - \frac{v_g}{c}\right) f_{trans}$$

Depending on the expansion effect of the durations, the durations of the fixed transmitter measured by the mobile receiver are reduced by a factor $\sqrt{1 - \frac{v_g^2}{c^2}}$. The frequencies of the mobile transmitter measured by the fixed receiver are therefore increased by a factor $\frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}}$. So, we have:

$$f_{rec} = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} f_{trans}$$

By cumulating the 2 effects, we have the relativistic Doppler-Fizeau effect:

$$f_{rec} = \frac{\left(1 - \frac{v_g}{c}\right)}{\sqrt{1 - \frac{v_g^2}{c^2}}} f_{trans0} = \frac{\left(1 - \frac{v_g}{c}\right)}{\sqrt{\left(1 - \frac{v_g}{c}\right)\left(1 + \frac{v_g}{c}\right)}} f_{trans0}$$

$$f_{rec} = \sqrt{\frac{1 - \frac{v_g}{c}}{1 + \frac{v_g}{c}}} f_{trans0}$$

We get the same formula as before. The relativistic Doppler-Fizeau effect is perfectly symmetrical and depends only on the relative speed between transmitter and receiver.

II.3.2 Relativistic Doppler effect via the Lorentz transform

We can also find the relativistic Doppler effect from the Lorentz transform.

Let be a monochromatic source, fixed in a reference frame R associated with a coordinate system $Oxyzt$, and which emits a flat light wave, with frequency f , in the direction Ou of the plan xy , such as $(Ox, Ou) = \theta$. An observer A is associated with a reference frame R' , this reference frame associated with a coordinate system $Ox'y'z't'$ and in uniform rectilinear translation of velocity v_g^x in the direction x relative to the reference frame R . The emitted signal (i.e., the plane light wave) makes an angle θ in R and an angle θ' in R' .

The diagram below summarizes this statement:

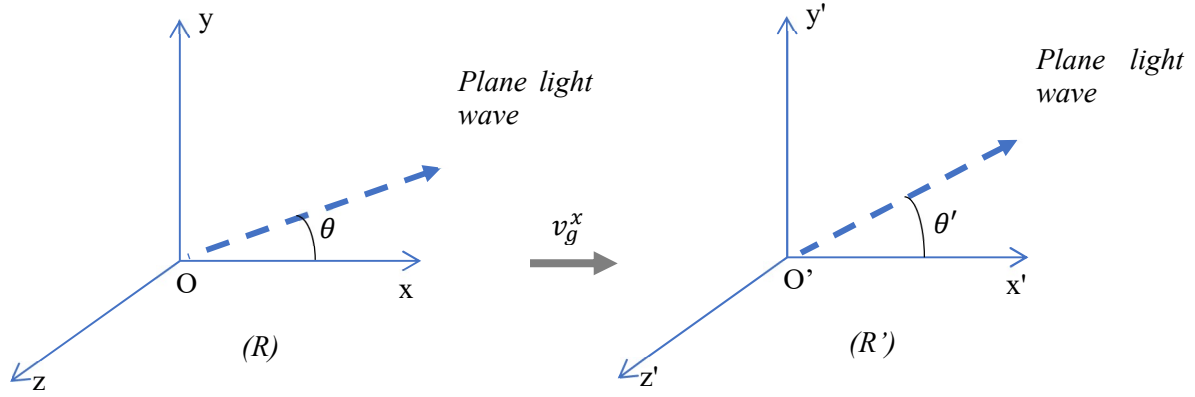


Figure 2: Plane light wave

We have the 4 components of the wave quadrivector in R :

$$\begin{aligned} k_x &= \frac{2\pi f}{c} \cos\theta \\ k_y &= \frac{2\pi f}{c} \sin\theta \\ k_z &= 0 \\ \frac{\omega}{c} &= \frac{2\pi f}{c} \end{aligned}$$

We have the 4 components of the wave quadrivector in R' :

$$\begin{aligned} k'_x &= \frac{2\pi f'}{c} \cos\theta' \\ k'_y &= \frac{2\pi f'}{c} \sin\theta' \\ k'_z &= 0 = k_z \\ \frac{\omega'}{c} &= \frac{2\pi f'}{c} \end{aligned}$$

We have the Lorentz transform for the time component:

$$\begin{aligned} \frac{\omega'}{c} &= \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \left(\frac{\omega}{c} - \frac{v_g}{c} k_x \right) \\ \frac{2\pi f'}{c} &= \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \left(\frac{2\pi f}{c} - \frac{v_g}{c} \frac{2\pi f}{c} \cos\theta \right) \end{aligned}$$

We deduce the relationship between the frequency f_{trans} issued in R and frequency f'_{rec} received by observer A motionless in R' :

$$\begin{aligned} f' &= \gamma \left(1 - \frac{v_g}{c} \cos\theta \right) f \\ f'_{rec} &= \gamma \left(1 - \frac{v_g}{c} \cos\theta \right) f_{trans} \end{aligned}$$

We have the Lorentz transform of the following component x :

$$\begin{aligned} k'_x &= \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \left(k_x - \frac{v_g}{c} \frac{\omega}{c} \right) \\ \frac{2\pi f'}{c} \cos\theta' &= \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \left(\frac{2\pi f}{c} \cos\theta - \frac{v_g}{c} \frac{2\pi f}{c} \right) \theta' = \frac{f}{f'} \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} \left(\cos\theta - \frac{v_g}{c} \right) \end{aligned}$$

$$\cos\theta' = \frac{\cos\theta - \frac{v_g}{c}}{1 - \frac{v_g}{c} \cos\theta}$$

We have the Lorentz transform of the following component y :

$$\begin{aligned} k'_y &= k_y \\ \frac{2\pi f'}{c} \sin\theta' &= \frac{2\pi f}{c} \sin\theta \end{aligned}$$

For the observer A with velocity v_g^x relative to the source, there is usually a change in the frequency (and therefore the wavelength) and the direction of the wave.

When $\cos\theta = 1$, i.e., when the light wave is emitted following x and has the same direction as v_g^x , we find again:

$$\begin{aligned} f'_{rec} &= \frac{\left(1 - \frac{v_g}{c}\right)}{\sqrt{1 - \frac{v_g^2}{c^2}}} f_{trans} = \frac{\left(1 - \frac{v_g}{c}\right)}{\sqrt{\left(1 - \frac{v_g}{c}\right)\left(1 + \frac{v_g}{c}\right)}} f_{trans} \\ f'_{rec} &= \sqrt{\frac{1 - \frac{v_g}{c}}{1 + \frac{v_g}{c}}} f_{trans} \end{aligned}$$

The relativistic effect is therefore the combination of the classical Doppler effect and the phenomenon of slowing down the clocks of Special Relativity.

When $\cos\theta = \frac{\pi}{2}$, the wave propagates according to y perpendicular to v_g^x , we have:

$$f'_{rec} = \frac{1}{\sqrt{1 - \frac{v_g^2}{c^2}}} f_{em}$$

II.4 Some reminders of special relativistic electromagnetism

II.4.1 Invariance of Maxwell's equations by the Lorentz transform

It is reminded that Maxwell's equations are invariant by Lorentz transform and therefore by change of Galilean reference frames in the context of Special Relativity. On a dans R and R' , 2 reference frames in rectilinear motion uniform with respect to each other:

$$\text{Maxwell Gauss: } \frac{\rho}{\varepsilon_0} = \text{div}\vec{E}l \text{ in } R \text{ et } \frac{\rho}{\varepsilon_0} = \text{div}\vec{E}l' \text{ in } R'$$

$$\text{Maxwell Ampère: } \mu_0 \vec{j} = \text{rot}\vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}l}{\partial t} \text{ in } R \text{ and } \mu_0 \vec{j}' = \text{rot}\vec{B}' - \mu_0 \varepsilon_0 \frac{\partial \vec{E}l'}{\partial t} \text{ in } R'$$

$$\text{Maxwell Thomson: } \text{div}\vec{B} = 0 \text{ in } R \text{ and } \text{div}\vec{B}' = 0 \text{ in } R'$$

$$\text{Maxwell Faraday: } \text{rot}\vec{E}l = -\frac{\partial \vec{B}}{\partial t} \text{ in } R \text{ and } \text{rot}\vec{E}l' = -\frac{\partial \vec{B}'}{\partial t} \text{ in } R'$$

Note on Maxwell Ampère

Maxwell Ampère is invariant by the Lorentz transform if Maxwell Gauss is verified.

Note on mass m_0 and electric charge q

In Special Relativity, it is postulated that mass m_0 and electric charge q (as well as the electric charge density ρ) are relativistic invariants.

II.4.2 Electromagnetic field quadrivector?

Remember that the quantities $\vec{E}\vec{l}\cdot\vec{B} = \vec{E}\vec{l}'\cdot\vec{B}'$ et $\frac{1}{c^2}\vec{E}\vec{l}\cdot\vec{E}\vec{l} - \vec{B}\cdot\vec{B} = \frac{1}{c^2}\vec{E}\vec{l}'\cdot\vec{E}\vec{l}' - \vec{B}'\cdot\vec{B}'$ are invariant by Lorentz transform. We could therefore imagine an electromagnetic field quadrivector of the type:

$$\left(\frac{\vec{E}\vec{l}}{c}, B\right)$$

Nevertheless, the nature of B , here scalar, remains fuzzy and in Special Relativity we rather define an electromagnetic field tensor or Maxwell Faraday tensor $\vec{F}_{\mu\nu}$.

II.4.3 Transformation of electromagnetic fields during a passage from R to R'

Let be a change of Galilean reference frame from R to R' , with R' animated by a uniform rectilinear velocity v_g^x in the direction x . We remind the transforms of the electromagnetic field to pass from the reference frame R to R' :

$$\begin{aligned} El_x' &= El_x \\ El_y' &= \gamma(El_y - v_g^x B_z) \\ El_z' &= \gamma(El_z + v_g^x B_y) \\ B_x' &= B_x \\ B_y' &= \gamma\left(B_y + \frac{v_g^x}{c^2} El_z\right) \\ B_z' &= \gamma\left(B_z - \frac{v_g^x}{c^2} El_y\right) \end{aligned}$$

Note 1

Note that these are not quadrivectors, because the transformation is not exactly that of Lorentz.

Note 2

We find these transforms, via the Lorentz transform of the Lorentz force, which is a force quadrivector. For El , we place ourselves in the special case where the electric charge q is immobile in R . For B , we place in the special case where the electric charge q moves according to z .

II.4.4 Lorentz transformation of fields K and Ω

In a space $x y t=z$ (plane light wave emitted in a plane xy), we have the Lorentz transforms for the pulsation and wave vector, here for the fields K and Ω :

$$\begin{aligned} K_{zt}^{y'} &= K_{zt}^y \\ K_{yt}^{x'} &= \gamma\left(K_{yt}^x - \frac{u}{c^2} \Omega_{xy}^z\right) \\ \Omega_{xy}^{z'} &= \gamma\left(\Omega_{yx}^z - u K_{yt}^x\right) \end{aligned}$$

With reorientation of the electromagnetic field, we obtain the values of the electromagnetic field:

$$\begin{aligned} El_{zt}^{y'} &= El_{zt}^y \\ El_{yt}^{x'} &= \gamma(El_{yt}^x - u B_{xy}^z) \\ B_{xy}^{z'} &= \gamma\left(B_{yx}^z - \frac{u}{c^2} El_{yt}^x\right) \end{aligned}$$

Analogous forms are obtained for the pulsation field, wave vector field and for the reoriented electromagnetic field. When the electromagnetic field is reoriented, it transforms by a Lorentz transform and can then be considered a quadrivector.

II.5 Conclusion of the chapter

We have explained the notions of group velocity and phase velocity, which according to the ideas of L. de Broglie make it possible to interpret wave mechanics as an extension (or even a generalization) of both classical Newtonian mechanics and relativistic mechanics.

After studying the phase and group velocities of wave mechanics, we will look at the possible links between phase velocity, group velocity and Maxwell source field equations.

Chapter III Maxwell's source field equations and group velocity

Purpose of the chapter

Several times, we have mentioned the idea of L. de Broglie to bring the velocity of an electron particle moving around the nucleus (velocity used for example in the Bohr electron model) closer to the group velocity of an electron wave.

The purpose of this chapter is to see if we can also compare the velocity of a moving electric charge (i.e., an electric current) to the group velocity of a wave.

Using the source field equations of Maxwell Ampère and Maxwell Gauss (involving charge and electric currents), we will see that it is possible to find an equation involving the velocity of an electric charge and formally resembling that of the group velocity.

III.1 Reminder on Maxwell's sources field equations

Maxwell Ampère

Maxwell Ampère's equation describes the magnetic field and electric field generated by a density of electric currents (moving electric charges).

Maxwell Ampère is reminded in integral form:

$$\mu_0 \iint_S \vec{j} \cdot d\vec{S} = \oint_C \vec{B} \cdot d\vec{l} - \mu_0 \varepsilon_0 \iint_S \frac{\partial \vec{E}l}{\partial t} \cdot d\vec{S}$$

Maxwell Ampère is reminded in local form:

$$\begin{aligned} \mu_0 \vec{j} &= \text{rot} \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}l}{\partial t} \\ \mu_0 (\vec{j} + \vec{j}_D) &= \text{rot} \vec{B} \end{aligned}$$

with $\vec{j}_D = \varepsilon_0 \frac{\partial \vec{E}l}{\partial t}$ the displacement current introduced by J. C. Maxwell.

Maxwell Gauss

Maxwell Gauss's equation describes the electric field generated by a density of electric charges.

Maxwell Gauss is reminded in integral form:

$$\frac{\Sigma Q_{int}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \iint_V \rho d\tau = \iint_S \vec{E}l \cdot d\vec{S}$$

with ρ the density of electric charges.

Maxwell Gauss is reminded in local form:

$$\frac{\rho}{\varepsilon_0} = \text{div} \vec{E}l$$

Gauss gravitation

There are strong analogies between Maxwell Gauss and Gauss gravitation, giving the gravitational field generated by a mass density. We remind here Gauss gravitation in integral form:

$$-4\pi G(\Sigma M) = -4\pi G \iint_V \rho_m d\tau = \iint_S \vec{G}r \cdot d\vec{S}$$

With ρ_m the mass volumetric density.

Gauss gravitation is reminded in local form:

$$-4\pi G\rho_m = \text{div}\vec{G}r$$

III.2 Rewriting Maxwell's sources field equations in a 3-dimensional Space-Time

We place ourselves in a space oriented with three dimensions x, y, t.

III.2.1 Maxwell Ampère

We have Maxwell Ampère in local form:

$$\mu_0\vec{j} = r\partial t\vec{B} - \mu_0\varepsilon_0 \frac{\partial\vec{E}l}{\partial t}$$

Moving to partial derivatives, we have with an electric current density in the y direction:

$$\mu_0j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x} - \mu_0\varepsilon_0 \frac{\partial El_{ty}^{1/x}}{\partial t}$$

$$\mu_0j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x} - \frac{\partial El_{ty}^{1/x}}{c^2\partial t}$$

Note

The electric field has been reoriented $El_y = El_{ty}^{1/x}$ in the manner of the magnetic field (i.e., in the manner of a generalized rotation vector), which makes it possible to have in the expression of Maxwell Ampère the same formalism for the electric field and the magnetic field.

This gives with an electric current in the x direction:

$$\mu_0j_{yt}^x = \frac{\partial B_{yx}^{1/t}}{\partial y} - \mu_0\varepsilon_0 \frac{\partial El_{tx}^{1/y}}{\partial t}$$

$$\mu_0j_{yt}^x = \frac{\partial B_{yx}^{1/t}}{\partial y} - \frac{\partial El_{tx}^{1/y}}{c^2\partial t}$$

III.2.2 Maxwell Gauss

Maxwell Gauss is reminded locally:

$$\text{div}\vec{E}l = \frac{\rho}{\varepsilon_0}$$

The electric field is reoriented on the model of the magnetic field, Maxwell Gauss is transformed into an analogue of Maxwell Ampère.

We have with a volume density of electric charge ρ_{xy}^t following t:

$$\frac{\rho_{xy}^t}{\varepsilon_0} = \frac{\partial El_{xt}^{1/y}}{\partial x} - \frac{\partial El_{yt}^{1/x}}{\partial y}$$

Note

In the previous Memoir, during the study of the Maxwell Faraday tensor $\bar{\bar{F}}_{\mu\nu}$ and from the rewriting of Maxwell's source field equations with this tensor, we have already seen that we can construct these equations under the same mode.

III.3 From Maxwell's sources field equations to group velocity of a particle wave**III.3.1 Maxwell's sources field equations**

Let us now seek, from the equations of Maxwell Gauss and Maxwell Ampère, to find the equation of group velocity.

We place ourselves in the case that:

- the electric field El is permanent in Time and depends on only one variable in space x . It therefore depends neither on t nor on y .
- the magnetic field B depends only on x (no electric current following x).

According to **Maxwell Ampère**, we have:

$$\mu_0 j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial El_{ty}^{1/x}}{\partial t}$$

(no current following x)

The field $El_{ty}^{1/x}$ is permanent.

So, we have:

$$\mu_0 j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x}$$

According to **Maxwell Gauss**, we have:

$$\frac{\rho_{xy}^t}{\varepsilon_0} = \frac{\partial El_{xt}^{1/y}}{\partial x} - \frac{\partial El_{yt}^{1/x}}{\partial y}$$

The field El does not depend on y .

So, we have:

$$\frac{\rho_{xy}^t}{\varepsilon_0} = \frac{\partial El_{xt}^{1/y}}{\partial x}$$

III.3.2 Link between group velocity and Maxwell's source field equations

Dividing **Maxwell Ampère** by **Maxwell Gauss**, we find:

$$\mu_0 \varepsilon_0 \frac{j_{xt}^y}{\rho_{xy}^t} = \frac{\partial B_{xy}^{1/t}}{\partial x} \times \frac{\partial x}{\partial El_{xt}^{1/y}}$$

We have:

$$j_{xt}^y = \rho_{xy}^t v^y$$

with v^y the velocity of the electric charge.

We then have:

$$\mu_0 \varepsilon_0 v^y = \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

With $\mu_0 \varepsilon_0 = \frac{1}{c^2}$, we obtain:

$$v^y = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

We find a relationship that resembles that of group velocity:

$$v_g = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$$

According to the Larmor relation (which contains two generalized rotation vectors in a spatial plane), we have:

$$B_{xy}^{1/t} = \frac{2m_{elec}}{e} \Omega_{xy}^{1/t}$$

From the fundamental principle of Newton's dynamics, we saw that we had a similar relation that contains two generalized pawns in a spatiotemporal plane:

$$E_{xt}^{1/y} = \frac{2m_{elec}c^2}{e} K_{xt}^{1/y}$$

Note, vacuum check

Let's check the following relationship in a vacuum:

$$\frac{B}{El} = \frac{\Omega}{c^2 K}$$

In a vacuum, we have:

$$c = \frac{\Omega}{K}$$

From $\frac{B}{El} = \frac{\Omega}{c^2 K}$, we obtain:

$$\frac{B}{El} = \frac{1}{c}$$

This verifies the relationship between the field norms \vec{B} and $\vec{E}l$ in a vacuum.

By substituting the magnetic and electric fields by the pulsation field and wave vector field, we obtain:

$$v^y = c^2 \frac{\partial \frac{2m_{elec}}{e} \Omega_{xy}^{1/t}}{\partial \frac{2m_{elec}c^2}{e} K_{xt}^{1/y}}$$

$$v^y = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$$

We obtain a velocity of the electric charge identical to the expression of a group velocity.

III.4 Conclusion of the chapter

From Maxwell's source field equations, it is possible to find a velocity of the electric charge $v^y = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$ similar to a group velocity $v_g = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$.

Further to this finding:

- since there are source field equations associating the fields B and El , equations that make it possible to find a group velocity,
- in addition, since the fields Ω and K are involved in the definition of group velocity,

let's see if it is possible to obtain source field equations applied to gravitation, involving fields Ω and K , and allowing to find a group velocity.

Chapter IV Ampère and Gauss theorems, source field equations for gravitation

Purpose of the chapter

In this chapter we propose analogues for gravitation, Maxwell's source field equations for electromagnetism.

We first give a graphical representation of Ampère's theorem. This makes it possible to explain Maxwell's source field equations, and then to propose different source field equations applying to gravitation.

We place ourselves here in a space x, y, t oriented.

IV.1 Space-oriented sources: electric current density, Maxwell Ampere equation

IV.1.1 Magnetic field

We have according to Ampere's theorem (magnetic part) in a 3-dimensional space x, y, l :

$$\mu_0 \iint_{S_{xl}} \vec{j}^y \cdot d\vec{S}_{xl}^y = \oint_{\Gamma^l} \vec{B}^{1/l}(M) \cdot d\vec{l}$$

S_{xl} is a surface in the spatial plane x, l

\vec{S}_{xl}^y perpendicular to the surface S_{xl}

Γ^l is a contour following l

\vec{j}^y the density of the oriented in the y direction and to be integrated on a surface S_{xl}

Note on electric current densities

The volumetric current density \vec{j} is expressed in Ampere per square meter: $\frac{A}{m^2}$.

We also define a surface current density \vec{j}_S , which is expressed in $\frac{A}{m}$

The current I , linear density, is expressed in Ampere.

In the case of spatial symmetries, we have the equivalence relations for current densities:

$$\vec{j}^x dVol = \vec{j}_S^x dS^x = \vec{I}^x dx$$

In the case of magnetism, for the 3rd dimension perpendicular to the plane of rotation x, y , it is assumed to be able to reason similarly on a spatial dimension l or on a temporal dimension t .

So, we take $\vec{B}^{1/t} = \vec{B}^{1/l}$ directed following Time.

We also take Γ^t directed following Time.

Finally, we can have: $d\vec{l} = cd\vec{t}$.

We have according to Ampère's theorem (magnetic part) in a 3-dimensional space x, y, t :

$$\mu_0 \iint_{S_{xt}} \vec{j}_{xt}^y \cdot d\vec{S}_{xt}^y = \oint_{\Gamma^t} \vec{B}_{xy}^{1/t}(M) \cdot cd\vec{t}$$

S_{xt} is a surface in the spatiotemporal plane x, t

\vec{S}_{xt}^y is perpendicular to the spatiotemporal surface S_{xt}

Γ^t is a contour directed following Time, included in the surface S_{xt}

\vec{j}^y the volumetric current of electric charges density, directed following y and to be integrated on a surface S_{xt}

Graphically, Ampère's theorem translates as:

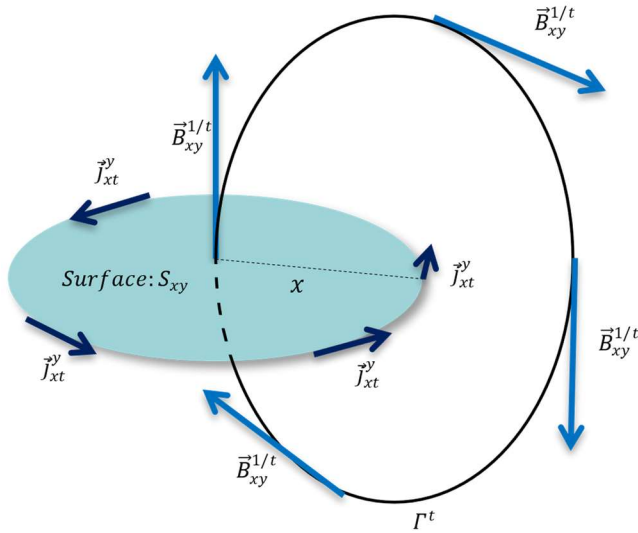


Figure 3: Maxwell Ampère, magnetic field $\vec{B}_{xy}^{1/t}$ and current volumetric density \vec{J}_{xt}^y

By applying Stokes' theorem to:

$$\mu_0 \iint_{S_{xt}} \vec{J}_{xt}^y \cdot d\vec{S}_{xt}^y = \oint_{\Gamma^t} \vec{B}_{xy}^{1/t}(M) \cdot c d\vec{t}$$

We have:

$$\mu_0 \iint_{S_{xt}} \vec{J}_{xt}^y \cdot d\vec{S}_{xt}^y = - \iint_{S_{xt}} \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x} d\vec{S}_{xt}^y$$

We obtain the local shape to be integrated on a surface S_{xt} :

$$\mu_0 \vec{J}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$$

Note

We take l as the 3rd dimension of Space, but we can just as easily choose z as the notation. We find the magnetic field oriented according to z as it is conventionally defined: $\vec{B}^{1/z}$.

IV.1.2 Electric field (displacement currents)

We have according to Ampère's theorem (displacement currents part):

$$\mu_0 \iint_{S_{tx}} \vec{J}_{xt}^y \cdot d\vec{S}_{tx}^y = -\mu_0 \epsilon_0 \oint_{\Gamma^x} \vec{E} l_{ty}^{1/x} \cdot d\vec{x}$$

S_{tx} is a surface in the spatiotemporal plane x, t

\vec{S}_{tx}^y perpendicular to the spatiotemporal surface S_{tx}

Γ^x contour following x , included in the surface S_{tx}

\vec{J}^y the volumetric current density, directed following y and to be integrated on a surface S_{tx}

We have graphically:

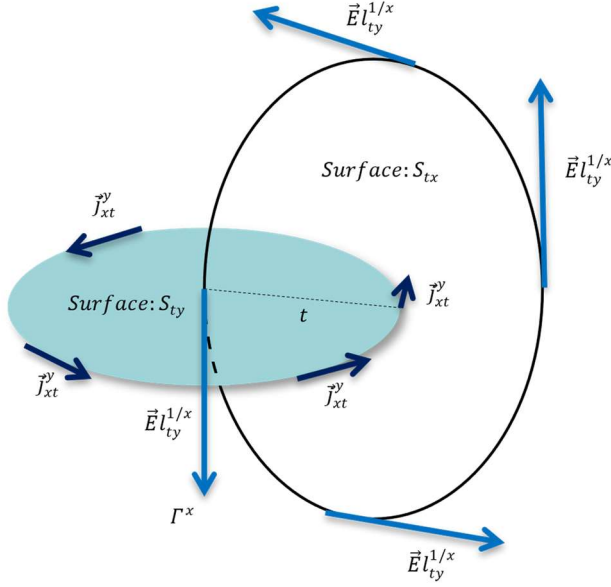


Figure 4: Maxwell displacement currents, field $\vec{E}_l^{1/x}$ and current volumetric density \vec{J}_{xt}^y

In local form, we have:

$$\mu_0 \vec{J}_{xt}^y = -\mu_0 \varepsilon_0 \frac{\partial \vec{E}_l^{1/x}}{\partial t}$$

If we combine the two Maxwell Ampère equations, we have:

$$\mu_0 \vec{J}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}_l^{1/x}}{\partial t}$$

IV.2 Time-oriented sources: electric charge density, Maxwell Gauss equation

IV.2.1 Electric field 1

According to Ampère and Gauss's theorems, we have:

$$\frac{1}{\varepsilon_0} \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma^y} \vec{E}_l^{1/y} \cdot d\vec{y}$$

S_{xy} is a surface in the spatial plane x, y

\vec{S}_{xy}^t perpendicular to the surface S_{xy}

Γ^y contour following y, included in the surface S_{xy}

$\vec{\rho}_{xy}^t$ the volumetric electric charge density directed following t and to be integrated on a surface S_{xy}

Note 1 on electric charge densities

The volumetric electric charge density ρ^t is expressed in Coulomb per cubic meter: $\frac{C}{m^3}$.

We also usually define a surface electric charge density which is expressed in $\frac{C}{m^2}$, and a linear electric charge density in $\frac{C}{m}$.

The electric charge q is expressed in C.

In the case of spatial symmetries, we have the equivalence relations for the current density and electric charge density:

$$\vec{j}^x dVol = \vec{j}_S^x dS = \vec{i}^x dx$$

$$\vec{\rho}^t dVol = \vec{\sigma}^t dS = \vec{\lambda}^t dl = d\vec{q}^t$$

Note 2 on Ampère and Gauss's theorems

With the reorientation of the electric field on the model of the magnetic field, the usual Maxwell Gauss relation turns into an analogue of Ampère's theorem.

Thus, we can see the theorem used above as a Gauss theorem, analogous to Ampère by a permutation of t by y .

In the rest of this Memoir, we will rather speak of Gauss for time-oriented sources and Ampere for space-oriented sources, knowing that all the source field equations proposed here are fundamentally based on the same Ampere's theorem.

We have graphically:

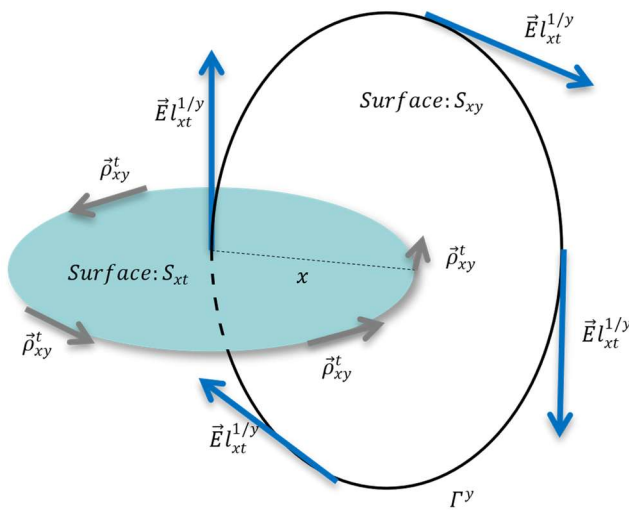


Figure 5: Maxwell Gauss 1, field $\vec{E}l_{xt}^{1/y}$ and volumetric electric charge density $\vec{\rho}_{xy}^t$

In local form, we have:

$$\frac{\vec{\rho}_{xy}^t}{\epsilon_0} = \frac{\partial \vec{E}l_{xt}^{1/y}}{\partial x}$$

IV.2.2 Electric field 2

According to Ampère and Gauss's theorems, we have:

$$\frac{1}{\epsilon_0} \iint_{S_{yx}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{yx}^t = - \oint_{\Gamma^x} \vec{E}l_{yt}^{1/x} \cdot d\vec{x}$$

S_{yx} is a surface in the spatial plane x, y

\vec{S}_{xy}^t perpendicular to the spatial surface S_{yx}

Γ^x contour following x , included in the surface S_{yx}

$\vec{\rho}_{xy}^t$ the volumetric electric charge density directed following t and to be integrated on a surface S_{xy}

We have graphically:

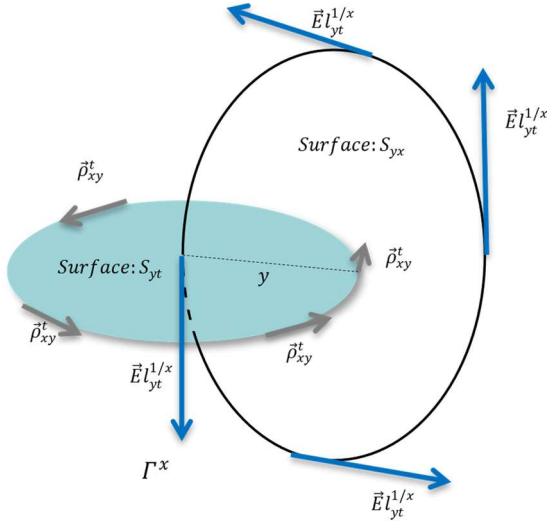


Figure 6: Maxwell Gauss 2, field $\vec{E}l_{yt}^{1/x}$ and volumetric electric charge density $\vec{\rho}_{xy}^t$

In local form, we have:

$$\frac{\vec{\rho}_{xy}^t}{\varepsilon_0} = -\frac{\partial \vec{E}l_{yt}^{1/x}}{\partial y}$$

If we combine the two Maxwell Gauss equations, we have:

$$\frac{\vec{\rho}_{xy}^t}{\varepsilon_0} = \frac{\partial \vec{E}l_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{E}l_{yt}^{1/x}}{\partial y}$$

IV.3 Analogies electromagnetism and gravitation, momentum-type sources oriented in Space, Ampere's theorem

IV.3.1 Analogues of charge, current densities, and a focus about constants

In analogy to the electromagnetism source source field equations, we want to construct source source field equations for fields $\vec{\Omega}$ and \vec{K} .

There are two problems:

- what are the analogues of electric charge density and electric current density?
- which constants are used for the gravitational source field equations. Is it the same (permittivities ε_0 and permeability μ_0 used in electromagnetism, gravitational constant G) or do we have to find others?

IV.3.2 Current density and momentum density

In electromagnetism, we integrate the volumetric electric current density $\vec{j}_{xt}^y = \vec{j}_{xl}^y$ on a spatial plane l , with the idea that the temporal dimension t and the spatial dimension l can correspond: $l = ct$.

For gravitation, temporal dimension and spatial dimension cannot a priori correspond. The analogue of the volumetric electric current density \vec{j}_{xt}^y must be integrated on a spatiotemporal plane x, t . The analogue of \vec{j}_{xt}^y is therefore not simply a volumetric momentum density \vec{p} .

It is proposed here that the analogue of the linear electric current density \vec{l}_{xt}^y (usually called electric current I) is for gravitation, a linear momentum density that will have to be integrated on t . It then

corresponds to the derivative with respect to Time of the momentum \vec{p} , i.e., $(\frac{d\vec{p}}{dt})_{tx}^y$ or to \vec{p}_{tx}^y to use the notation of I. Newton (or to $(\sum \vec{F})_{xt}^y$ if we apply the fundamental principle of Newton's dynamics).

To then switch from linear density to volumetric density, the same method is used for the current I or for \vec{p} : we divide by m^2 .

To differentiate respectively linear density and volumetric density, the notations are used \vec{p}^y and \vec{p}^y . We obtain the following table, with indication of the units for each concept used:

	Linear density	Volumetric density
Current of electric charges	$\vec{I}^y dy$	$\vec{j}^y dVol$
Unit	A	$\frac{A}{m^2}$
Current of masses (derived from the momentum)	$\vec{p}^y dy$	$\vec{p}^y dVol$
Unit	$\frac{Kg.m}{s^2}$	$\frac{Kg}{m.s^2}$

We define Vol , a volume on 2 dimensions of Space and 1 of Time.

In case of symmetry of 2 dimensions of Space-Time, we have the equivalence relations for the electric current density and for the momentum derived with respect to Time density:

$$\begin{aligned} \vec{j}^y dVol &= \vec{I}^y dy \\ \vec{p}^y dVol &= \vec{p}^y dy \end{aligned}$$

$\vec{p}^y d\vec{y}$ corresponds to the notion of elementary work δW .

We end this paragraph with the following summary table:

	Momentum linear density, that is, the derivative of the momentum with respect to t	Momentum volumetric density on x, y, t
Current of masses (derived from the momentum)	$(\frac{d\vec{p}^x}{dt})_{tx}^y = \vec{p}_{tx}^y$	$(\frac{d\vec{p}^x}{dt})_{tx}^y = \vec{p}_{tx}^y$
Elementary work δW	$(\frac{d\vec{p}^x}{dt})_{tx}^y dy = \vec{p}_{tx}^y d\vec{y}$	$(\frac{d\vec{p}^x}{dt})_{tx}^y dVol = \vec{p}_{tx}^y dVol$

IV.3.3 The constant to use

In the following paragraphs, we will see that if we introduce Planck's constant \hbar (or rather $-\hbar$), the 4 source field equations obtained from sources oriented in Space, correspond to 3 equations already known. A for the 4th equation, a priori unknown, it will allow to find the wavelength of Compton:

$$\lambda_c = 2\pi \frac{\hbar}{mc}$$

IV.3.4 Source: linear momentum density and pulsation field $\vec{\Omega}_{xy}^{1/t}$

According to Ampère's theorem we have in a space x, y, t, with k a constant in $\frac{Kg}{s}$ to be determined:

$$\iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} k \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$$

$\frac{Kg}{m \cdot s^2} \cdot m^2$	$\frac{Kg}{s} \cdot \frac{m}{s}$
------------------------------------	----------------------------------

S_{xt} is a surface in the spatiotemporal plane x, t

\vec{S}_{xt}^y perpendicular to the spatiotemporal surface S_{xt}

Γ^t contour following t , included in the surface S_{xt}

$\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$ the volumetric momentum density (or the volumetric current of masses density) directed following y and to be integrated on a surface S_{xt}

By applying Stockes' theorem to:

$$\frac{1}{k} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$$

We have:

$$\frac{1}{k} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \iint_{S_{xt}} \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} d\vec{S}_{xt}^y$$

We obtain the local form to be integrated on a surface S_{xt} :

$$\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial k \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

We observe that we obtain an equation very close to the fundamental principle of Newton's dynamics, if we replace:

- on the one hand the volumetric momentum density $\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$ by the linear momentum density $\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$
- on the other hand, the constant k by Planck's constant \hbar .

We then have:

$$\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

If we go back this last equation, we have in the 1st column of the table below:

$\frac{1}{\hbar} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \iint_{S_{xt}} \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} d\vec{S}_{xt}^y$	$\frac{1}{k} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \iint_{S_{xt}} \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} d\vec{S}_{xt}^y$
$\frac{1}{\hbar} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$	$\frac{1}{k} \iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$
$\frac{1}{\hbar} \iint_{S_{xt}} \vec{p}_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$	$\frac{1}{k} \iint_{S_{xt}} \vec{p}_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$

We look for the value of the constant k as a function of \hbar which allows to pass from one to the other of the 2 equations above:

$$\begin{aligned} \frac{1}{k} \vec{p}_{xt}^y \cdot d\vec{S}_{xt}^y &= \frac{1}{\hbar} \vec{p}_{xt}^y \cdot d\vec{S}_{xt}^y \\ \frac{1}{k} \vec{p}_{xt}^y dx dt &= \frac{1}{\hbar} \vec{p}_{xt}^y dx dt \end{aligned}$$

However, in the case of a symmetry of 2 dimensions of Space-Time, we have the equivalence:

$$\vec{p}_{xt}^y dVol = \vec{p}_{xt}^y dy$$

$$\vec{p}_{xt}^y dx dy dct = \vec{p}_{xt}^y dy$$

We take the equation again:

$$\frac{1}{k} \vec{p}_{xt}^y dx dct = \frac{1}{\hbar} \vec{p}_{xt}^y dx dct$$

We multiply each term by dy :

$$\frac{1}{k} \vec{p}_{xt}^y dx dy dct = \frac{1}{\hbar} \vec{p}_{xt}^y dx dy dct$$

We replace $\vec{p}_{xt}^y dx dy dct$ by $\vec{p}_{xt}^y dy$:

$$\frac{1}{k} \vec{p}_{xt}^y dy = \frac{1}{\hbar} \vec{p}_{xt}^y dy dx dct$$

We obtain the searched relationship between \hbar and k :

	$\hbar = k \times dx dct$	$\hbar dy = k \times dVol$
Unit	$\frac{Kg \cdot m^2}{s} = \frac{Kg}{s} \cdot m^2$	$\frac{Kg \cdot m^2}{s} \cdot m = \frac{Kg}{s} \cdot m^3$

By choosing the constant k correctly, we find on the one hand Ampère's theorem applied to the momentum linear density \vec{p}_{tx}^y and to the pulsation field $\vec{\Omega}_{xy}^{1/t}$, and on the other hand the fundamental principle of Newton's dynamics:

$$\iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \hbar \vec{\Omega}_{xy}^{1/t} \cdot d\vec{c}t$$

$\frac{Kg \cdot m}{s^2} \cdot m^2$	$\frac{Kg \cdot m^2}{s} \cdot \frac{m}{s}$
------------------------------------	--

$$\iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \iint_{S_{xt}} \frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x} d\vec{S}_{xt}^y$$

We have graphically:

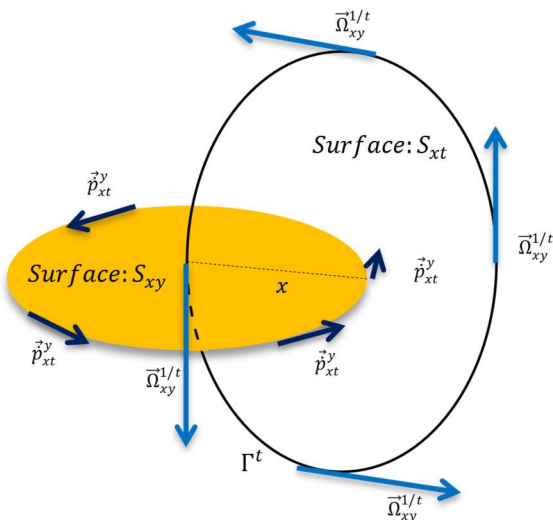


Figure 7: source \vec{p}_{xt}^y and field $\vec{\Omega}_{xy}^{1/t}$

Why we find the fundamental principle of Newton's dynamics:

Using the relation of Max Planck and A. Einstein between an energy and a pulsation $E = \hbar\Omega$ ($E = hv$), we get:

$$\left(\frac{d\vec{p}^x}{dt}\right)_{xt}^y = -\frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x} = -\frac{\partial E}{\partial x}$$

We find the fundamental principle of Newton's dynamics with forces that derive with respect to space of a (potential) energy.

This is also William Rowan Hamilton's 2nd canonical equation:

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

with the Hamiltonian H matching with energy $E = \hbar\Omega$.

We will call:

$$\left(\frac{d\vec{p}^x}{dt}\right)_{xt}^y = -\frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

the **Newton's** source field equation.

Note 1, on Special Relativity

Newton's source field equation is here verified in the context of Einsteinian Special Relativity (invariance by the Lorentz transform). We have, with m_0 the resting mass and with $\vec{\Omega}_{0xy}^{1/t}$ the resting pulsation field, the relationship:

$$\left(\frac{d\gamma m_0 \vec{v}^x}{dt}\right)_{xt}^y = -\frac{\partial \hbar \gamma \vec{\Omega}_{0xy}^{1/t}}{\partial x}$$

We obtain:

$$\left(\frac{dm_0 \vec{v}^x}{dt}\right)_{xt}^y = -\frac{\partial \hbar \vec{\Omega}_{0xy}^{1/t}}{\partial x}$$

We find above the fundamental principle of dynamics verified in the framework of Newtonian "special or restricted" mechanics (invariance by the Galileo transform).

Note 2, on rotation Ω

In the relationship $E = \hbar\Omega$ ($E = hv$), We replace the notion of pulsation ω by the notion of rotation Ω , Implicitly, the frequency or wave phenomenon of the particle can be reduced to a rotation and a pulsation field Ω . The mystery remains to understand what exactly this rotation would be: what revolves and around what?

Note 3, on frictional forces

To recover the fundamental principle of Newton's dynamics, we admit here that all force derives from potential energy. We must therefore disregard, for example, frictional forces that do not derive from potential energy.

We can propose an analogue of the law of Biot and Savart, in the context of "Pulsatiostatics" with $\vec{\Omega}_{xy}^{1/t}$ time independent:

	Magnetostatics	« Pulsatiostatics »
Law of Biot and Savart	$\vec{B}_{xy}^{1/t} = \frac{\mu_0}{4\pi} \int_C \frac{I_{xt}^y d\vec{y} \wedge \vec{x}}{x^3}$	$\vec{\Omega}_{xy}^{1/t} = \frac{-1}{4\pi\hbar} \int_C \frac{\left(\frac{dp^x}{dt}\right)_{xt}^y d\vec{y} \wedge \vec{x}}{x^3}$

One can also propose an analogue of Ampère's	Magnetostatics	« Pulsatiostatics »
--	----------------	---------------------

theorem in local form:		
Ampere's theorem in local form	$\mu_0 \vec{j} = \text{rot} \vec{B}$	$\frac{1}{\hbar} \vec{p} = \text{rot} \vec{\Omega}$

Note on the tensor of momentum flux density

For those familiar with fluid dynamics or General Relativity, $\hbar \vec{\Omega}_{xy}^{1/t}$ has a physical significance very similar to that of the tensor of momentum flux density, noted here Π_{ik} .

In a volume V , we have the relationship between the momentum density and the tensor of momentum flux density:

$$\frac{\partial}{\partial t} \iiint \rho v_i dV = - \iiint \frac{\partial \Pi_{ik}}{\partial x_k} dV$$

According to the Green–Ostrogradski theorem, we have:

$$\frac{\partial}{\partial t} \iiint \rho v_i dV = - \oiint \vec{\Pi}_{ik} \cdot d\vec{S}$$

On a surface S , we also have:

$$\frac{\partial}{\partial t} \iint \rho v_i dS = - \iint \frac{\partial \Pi_{ik}}{\partial x_k} dS$$

According to Stokes' theorem, we have:

$$\frac{\partial}{\partial t} \iint \rho v_i dS = - \oint \vec{\Pi}_{ik} \cdot d\vec{l}$$

Compare to:

$$\iint_{S_{xt}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{xt}^y = - \oint_{\Gamma^t} \hbar \vec{\Omega}_{xy}^{1/t} \cdot d\vec{c}^t$$

IV.3.5 Source: linear momentum density and wave vector field $\vec{K}_{ty}^{1/x}$

According to Ampère's theorem we have in a space x, y, t , with the linear momentum density as the source $\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$ and as a field $\vec{K}_{ty}^{1/x}$:

$$\frac{1}{\hbar} \iint_{S_{tx}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{tx}^y = \oint_{\Gamma^x} \vec{K}_{ty}^{1/x} \cdot d\vec{x}$$

S_{tx} is a surface in the spatiotemporal plane x, t

\vec{S}_{tx}^y perpendicular to the spatiotemporal surface S_{tx}

Γ^x contour following x , included in the surface S_{tx}

$\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$ the linear momentum density directed following y and to be integrated on a surface S_{tx}

Note

The remarks made in the previous paragraph for volumetric and linear momentum densities also hold here. It is assumed that we always have a two-dimensional symmetry of Space-Time. We therefore directly propose Ampère's theorem from the linear momentum density $\left(\frac{d\vec{p}^x}{dt} \right)_{tx}^y$ and the constant \hbar .

We have graphically:

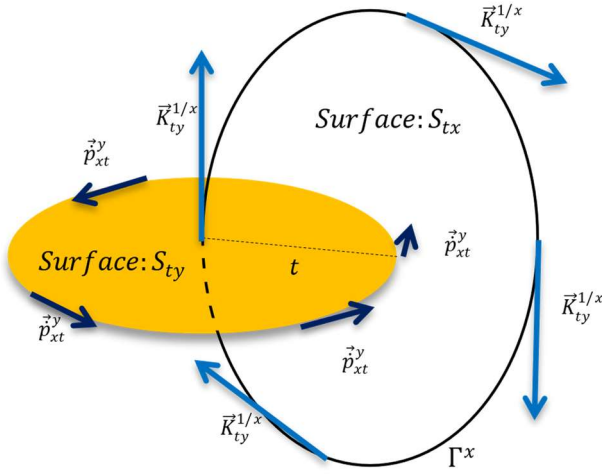


Figure 8: source \vec{p}_{xt}^y and field $\vec{K}_{ty}^{1/x}$

By applying Stockes' theorem to:

$$\frac{1}{\hbar} \iint_{S_{tx}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{tx}^y = \oint_{\Gamma^x} \vec{K}_{ty}^{1/x} \cdot d\vec{x}$$

We get:

$$\frac{1}{\hbar} \iint_{S_{tx}} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \cdot d\vec{S}_{tx}^y = \iint_{S_{tx}} \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t} \cdot d\vec{S}_{tx}^y$$

We have the local form to integrate on a surface S_{tx} :

$$\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

If the derivative with respect to Time is equal to the partial derivative with respect to Time, we find the relation of Einstein and de Broglie relating the momentum to the wavelength:

$$\vec{p}_{ty}^x = \hbar \vec{K}_{ty}^{1/x}$$

that is:

$$p = \frac{h}{\lambda}$$

$$\gamma m_0 v = \frac{h}{\lambda}$$

with $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and v the velocity of the body studied.

We will call:

$$\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

the **de Broglie's** source field equation (Einstein's name will be used for a future source field equation where the famous equation comes into play $E_0 = mc^2$).

If we combine the two source field equations involving the momentum, we have:

$$-\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

Note

Following the ideas proposed here, we observe that the constant \hbar is not directly related to electromagnetism but to gravitation.

Historical note

The relationship $p = \frac{h\nu}{c} = \frac{h}{\lambda}$ was first proposed by A. Einstein to give the momentum of a photon from its wavelength. We have the relations for a photon of zero mass: $E = h\nu = \frac{hc}{\lambda}$ and $E = pc$. We thus find $p = \frac{h}{\lambda}$.

In 1924, L. de Broglie proposed to extend this relationship $p = \frac{h}{\lambda}$ to all particles, especially mass particles such as the electron. According to L. de Broglie's hypothesis, particles are associated with waves. The association between the momentum p of a particle and the wavelength λ of the wave is done precisely through the relationship $p = \frac{h}{\lambda}$.

IV.4 Analogies electromagnetism and gravitation, mass-type sources oriented in Space, Ampere's theorem

We propose in this paragraph an analogy, which unlike that of the previous paragraph, does not correspond exactly to Maxwell's equations. This analogy nevertheless seems to be of interest since it contains equations known from quantum physics.

IV.4.1 A second analogue for gravitation at the electric current linear density?

Previously, it was pointed out that we could pass from the equations of Ampère to Gauss by permuting moment and position.

Note that if we swap x and t in momentum linear density $\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y$, we find a quantity also oriented in Space and which, moreover, includes a term of mass with the replacement of \vec{p}^x by \vec{m}^t . We obtain:

$$\left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y$$

In the following paragraphs, we propose to construct and test source field equations created from momentum and mass densities oriented in Space, as are current densities in the case of Ampere's theorem.

We summarize in the table below, the sources oriented in Space that we will test:

	Impulsion	Mass
Momentum or mass linear densities, both oriented in Space	$\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \vec{p}_{tx}^y$	$\left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y$
Unit	$\frac{Kg \cdot m}{s^2}$	$\frac{Kg}{m}$

IV.4.2 Source: linear mass density and pulsation field $\vec{\Omega}_{xy}^{1/t}$

According to Ampère's theorem in a space x, y, t:

$$\frac{c^2}{\hbar} \iint_{S_{xt}} \left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y \cdot d\vec{S}_{xt}^y = \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$$

S_{xt} is a surface in the spatiotemporal plane x, t

\vec{S}_{xt}^y perpendicular to the spatiotemporal surface S_{xt}

Γ^t contour following t , included in the surface S_{xt}

$\left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y$ the linear mass density directed following y and to be integrated on a surface S_{xt}

We have graphically:

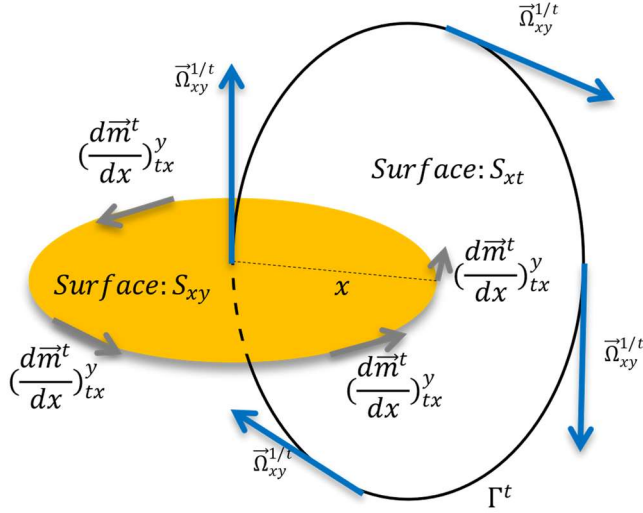


Figure 9: source $\left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y$ and field $\vec{\Omega}_{xy}^{1/t}$

By applying Stockes' theorem to:

$$\frac{c^2}{\hbar} \iint_{S_{xt}} \left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y \cdot d\vec{S}_{xt}^y = \oint_{\Gamma^t} \vec{\Omega}_{xy}^{1/t} \cdot d\vec{t}$$

We have:

$$\frac{c^2}{\hbar} \iint_{S_{xt}} \left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y \cdot d\vec{S}_{xt}^y = \iint_{S_{xt}} \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} \cdot d\vec{S}_{xt}^y$$

We obtain the local form to be integrated on a surface S_{xt} :

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

Considering that the derivative with respect to x is equal to the partial derivative with respect to x , we have:

$$\vec{m}^t c^2 = \hbar \vec{\Omega}_{xy}^{1/t}$$

We find the famous relations of A. Einstein and M. Planck giving the mass energy E_0 of a motionless particle of mass m_0 and the energy of a body considered a standing (or stationery) wave of frequency ν_0 :

$$E_0 = m_0 c^2 = h \nu_0$$

If the particle becomes mobile with velocity v , then it has a relative mass:

$$m = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0 = \gamma m_0$$

Similarly, if the wave is no longer standing (or stationery) and becomes progressive (or travelling), it then has a frequency:

$$\nu = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \nu_0 = \gamma \nu_0$$

We then have for the energy E of the moving particle or the progressive (or travelling) wave:

$$E = \gamma E_0 = \gamma m_0 c^2 = mc^2 = h \gamma \nu_0 = h \nu = \sqrt{p^2 c^4 - m_0^2 c^4}$$

We have:

$$\begin{aligned} mc^2 &= h \nu \\ mc^2 &= \hbar \omega \end{aligned}$$

We will call:

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

the **Einstein** source field equation.

Note

The Einstein source field equation is here verified in the context of Einsteinian Special Relativity (invariance by the Lorentz transform). We have with m_0 the mass at rest and with $\vec{\Omega}_{0xy}^{1/t}$ the pulsation field at rest, the relationship:

$$\begin{aligned} \frac{c^2}{\hbar} \left(\frac{d\gamma m_0^t}{dx} \right)_{tx}^y &= \frac{\partial \gamma \vec{\Omega}_{0xy}^{1/t}}{\partial x} \\ m_0 c^2 &= \hbar \omega_0 = E_0 \end{aligned}$$

IV.4.3 Source: linear mass density and wave vector field $\vec{K}_{ty}^{1/x}$

According to Ampère's theorem in a space x, y, t :

$$\frac{c^2}{\hbar} \iint_{S_{tx}} \left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y \cdot d\vec{S}_{tx}^y = - \oint_{\Gamma^x} \vec{K}_{ty}^{1/x} \cdot d\vec{x}$$

S_{tx} is a surface in the spatiotemporal plane x, t

\vec{S}_{tx}^y perpendicular to the spatiotemporal surface S_{tx}

Γ^x contour following x , included in the surface S_{tx}

$\left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y$ the linear mass density directed following y and to be integrated on a surface S_{tx}

We have graphically:

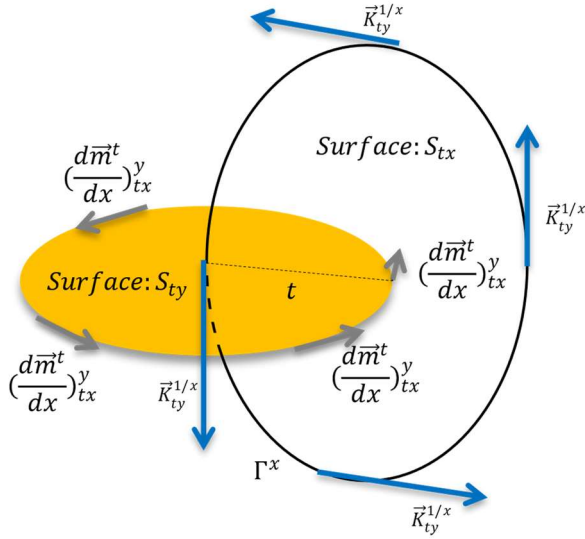


Figure 10: source $(\frac{d\vec{m}^t}{dx})_{tx}^y$ and field $\vec{K}_{ty}^{1/x}$

By applying Stokes' theorem to:

$$\frac{c^2}{\hbar} \iint_{S_{tx}} (\frac{d\vec{m}^t}{dx})_{tx}^y \cdot d\vec{S}_{tx}^y = - \oint_{\Gamma^x} \vec{K}_{ty}^{1/x} \cdot d\vec{x}$$

We get:

$$\frac{c^2}{\hbar} \iint_{S_{tx}} (\frac{d\vec{m}^t}{dx})_{tx}^y \cdot d\vec{S}_{tx}^y = - \iint_{S_{tx}} \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t} \cdot d\vec{S}_{tx}^y$$

We have the local form to integrate on a surface S_{tx} :

$$\frac{c^2}{\hbar} (\frac{d\vec{m}^t}{dx})_{tx}^y = - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

That we can write in a simpler way:

$$\frac{dmc^2}{dx} = - \frac{\partial \hbar K^x}{\partial t}$$

If we assimilate partial derivative and derivative, we have:

$$\frac{c^2}{\hbar} \frac{dm}{dx} = \frac{dK}{dt}$$

$$c^2 dm = \frac{dx}{dt} d\hbar K$$

We place ourselves in the case of:

$$\frac{dx}{dt} = c$$

$$cdm = d\hbar K$$

By integrating to within a constant, we have:

$$cm = \hbar K$$

We find again the Compton wavelength:

$$\lambda_c = \frac{2\pi}{K} = 2\pi \frac{\hbar}{mc} = \frac{h}{mc}$$

We will call:

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y = - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

the **Compton** source field equation.

We also have:

$$\frac{c^2}{\hbar} \left(\frac{d\gamma \vec{m}_0^t}{dx} \right)_{tx}^y = - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

with \vec{m}_0^t the mass of the particle if it were motionless (or stationary),

with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and v the velocity of the body studied,

and from the moment we have: $\vec{K}_{ty}^{1/x}$, the wave is progressive (or travelling) according to $p = \frac{h}{\lambda}$.

If we combine the two source field equations involving mass, we have:

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

Note 1, on the source field equations used by Newtonian mechanics and relativistic mechanics

Newtonian classical mechanics and its extension to the wave mechanics (non-relativistic) of Bohr, de Broglie and Schrödinger use the de Broglie source field equation: $p = \hbar k$ and the Newton source field equation: $\frac{dp}{dt} = - \frac{\partial \hbar \Omega}{\partial r} = - \frac{\partial E}{\partial r}$.

Special Relativity and its extension of Dirac's relativistic wave mechanics (Relativistic quantum Electrodynamics) use the de Broglie source field equation: $p = \hbar k$ and the Einstein source field equation: $E_0 = mc^2 = \hbar \Omega_0$.

Note 2, Newton and Compton, Einstein and de Broglie, analogy between Space and Time

There is a formal analogy between:

- the Newton source field equation $\left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x}$ and the Compton source field equation $\left(\frac{d\vec{m}^t c^2}{dx} \right)_{tx}^y = - \frac{\partial \hbar \vec{K}_{ty}^{1/x}}{\partial t}$,
- the de Broglie source field equation $\vec{p}^x = \hbar \vec{K}_{ty}^{1/x}$ and the Einstein source field equation $\vec{m}^t c^2 = \hbar \vec{\Omega}_{xy}^{1/t}$.

Indeed, we pass from one to the other by swapping x and ct , i.e., Space and Time. For example, by swapping x and ct , the momentum \vec{p}^x becomes mass energy $E_0^t = \vec{p}^t c = \vec{m}^t c^2$ (with here $\gamma = 1$).

IV.5 Reminders on the Compton wavelength

IV.5.1 Compton scattering

In 1923, Arthur Compton observed what is now called the Compton effect or Compton scattering. It is an experimental confirmation of light considered particle, that is to say formed of photons.

During Compton scattering, an incident photon hits a free particle (e.g., an electron or proton). A scattered photon is emitted with a slightly higher wavelength than the incident photon (i.e., a slightly lower energy).

The difference in wavelength between the incident photon and the scattered photon is proportional to a constant value λ_c named Compton wavelength. We have the following relationship:

$$\Delta\lambda = \lambda_c(1 - \cos \theta)$$

$$\text{with } \lambda_c = \frac{h}{mc}$$

where:

$\Delta\lambda$ is the shift between the wavelengths of the incident photon and the scattered photon,

λ_c is the Compton wavelength,

θ is the scattering angle of the scattered photon,

m mass of the free particle (e.g., electron or proton).

We can compare the constant λ_c at a quantum wavelength. Unlike the de Broglie wavelength $\lambda = \frac{h}{p}$, the Compton wavelength does not correspond to an observable wavelength in a propagation, it is a priori only an auxiliary calculation.

IV.5.2 Schema of a scattering

We schematize a Compton scattering between a photon and an electron initially at rest. The electron and photon are scattered respectively with angles φ and θ .

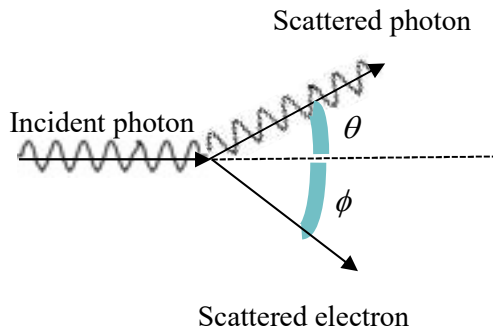


Figure 11: Compton scattering

The table below shows the Compton wavelength of the electron, proton, and neutron. It is noted that λ_c is very close for proton and neutron, these 2 particles therefore have very close masses.

Particle	Symbol	Valeur
Electron	$\lambda_{c,e}$	$\approx 2,426 \times 10^{-12}m$
Proton	$\lambda_{c,p}$	$\approx 1,321 \times 10^{-15}m$
Neutron	$\lambda_{c,n}$	$\approx 1,320 \times 10^{-15}m$

Note on the study of the infinitely small

In particle colliders, it is often said that the more energy you increase, the more small distances you probe.

This can be intuitively understood via a wave approach. The more we increase energy $E = h\nu$ and thus the frequency ν , the more we decrease the wavelength λ and the more we probe small distances.

This can also be understood through the relationship $E = mc^2 = c\hbar K$. The more we increase energy $E = mc^2$, the more we increase K inverse of a distance and more we probe small distances.

IV.6 Hamiltonian mechanics reminders

IV.6.1 Hamilton's canonical equations

In 1833, William Rowan Hamilton was inspired by the work of Joseph-Louis Lagrange and proposed a reformulation of Newtonian classical mechanics. Instead of the fundamental principle of dynamics, he imagines 2 equations of identical form and with first derivatives.

We remind here some principles of Hamiltonian mechanics, which was also frequently used by pioneers of quantum and wave mechanics such as L. de Broglie, E. Schrödinger or W. Heisenberg.

Let q be a position variable, p a momentum variable (called also conjugate momentum) and H the energy of the system studied (also called Hamiltonian). We have the two Hamilton's canonical equations:

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

IV.6.2 Link with the gravitational source field equations

We have already pointed out that the **2nd canonical equation** $\dot{p} = -\frac{\partial H}{\partial q}$ corresponds to the Newton source field equation with $H = \hbar\Omega$.

Note

It is customary to say that the Hamiltonian H represents energy. Nevertheless, it is a "wave" energy. $H = \hbar\Omega$ of type field, a priori possibly distinct from mass energy $E_0 = p_0^t = m_0^t c^2$ of type source.

To find the **1st canonical equation** $\dot{q} = \frac{\partial H}{\partial p}$, the definition of group velocity and the de Broglie source field equation must be used.

We have:

$$v_g = \frac{\partial \Omega}{\partial K} = \frac{\partial \hbar\Omega}{\partial \hbar K}$$

By definition, we have:

$$H = \hbar\Omega$$

According to L. de Broglie:

$$p = \hbar K_{unit}$$

In addition:

$$v_g = \dot{q}$$

We find:

$$\dot{q} = \frac{\partial H}{\partial p}$$

For example, if we have a non-relativistic mechanical energy of the form:

$$H = \hbar\Omega = \frac{p^2}{2m} + Ep(q)$$

Using Hamilton's 1st canonical equation (i.e., the de Broglie source field equation and the definition of group velocity), we find the usual definition of momentum:

$$\begin{aligned}\dot{q} &= \frac{\partial H}{\partial p} = \frac{p}{m} \\ p &= m\dot{q}\end{aligned}$$

IV.7 Analogies between electromagnetism and gravitation, Time-oriented sources, Gauss's theorem

IV.7.1 Maxwell Gauss electrostatic and Gauss gravitation

In this paragraph, we will focus on the analogies between the source field equations: Maxwell Gauss and Gauss gravitation (or Newton's Law of Universal Gravitation). These analogies have already been discussed at length in this essay. Since their discoveries, they have also been the subject of numerous studies by physicists. Here we briefly recall the main analogies between Coulomb electrostatics and Newtonian gravitation.

	Coulomb electrostatics	Newtonian gravitation
Charge density, mass density	ρ^t	ρ^t
Constant	$\frac{1}{\epsilon_0}$	$-4\pi G$
Field (fields not redirected)	$Es = -\frac{\partial A^t}{\partial r}$ $\vec{E}s = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \vec{u}_r$	$Gr = -\frac{\partial V^t}{\partial r}$ $\vec{G}r = -G \frac{m_1}{r^2} \vec{u}_r$
Gauss (fields not redirected)	$\text{div}\vec{E}s = \frac{\rho_{elec}}{\epsilon_0}$	$\text{div}\vec{G}r = -4\pi G \rho_{mass}$
Curl (fields not redirected)	$r\vec{\partial}t\vec{E}s = \vec{0}$	$r\vec{\partial}t\vec{G}r = \vec{0}$

IV.7.2 Gauss gravitation 1

Following a strict analogy with Maxwell Gauss equation proposed previously for electromagnetism and the electric field, we have for gravitation and the field $\vec{K}_{xt}^{1/y}$:

$$-4\pi G \iint_{S_{xy}} \vec{p}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma^y} 2c^2 \vec{K}_{xt}^{1/y} \cdot d\vec{y}$$

S_{xy} is a surface in the spatiotemporal plane x, y

\vec{S}_{xy}^t perpendicular to the spatial surface S_{xy}

Γ^y contour following y , included in the surface S_{xy}

$\vec{\rho}_{xy}^t$ the mass volumetric density directed following t and to be integrated on a surface S_{xy}

Note on units for mass densities

The mass volumetric density ρ^t is expressed in kilograms per cubic metre: $\frac{Kg}{m^3}$.

We also define a mass surface density which is expressed in $\frac{Kg}{m^2}$, and a mass linear density in $\frac{Kg}{m}$.

The mass m is expressed in Kg.

In the case of spatial symmetries, we have equivalence relations for mass densities:

$$\vec{\rho}^t dVol = \vec{\sigma}^t dS = \vec{\lambda}^t dl = d\vec{m}^t$$

We have the relationship between the wave vector field and the gravitational field:

$$2c^2 \vec{K}_{xt}^{1/y} = \frac{\partial V^t}{\partial x} - \frac{\partial V^x}{\partial t}$$

$$2c^2 \vec{K}_{xt}^{1/y} = \vec{G}_{xt}^{1/y} - \frac{\partial V^x}{\partial t}$$

Note

According to the notations used in this Memoir, we have:

$$\vec{G}_{xt}^{1/y} = \frac{\partial V^t}{\partial x}$$

$$\vec{G}_{tx}^{1/y} = -\frac{\partial V^t}{\partial x}$$

We obtain the source field relation with the gravitational field:

$$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma^y} \left(\vec{G}_{xt}^{1/y} - \frac{\partial V^x}{\partial t} \right) \cdot d\vec{y}$$

We remind the Gauss gravitation equation which should be obtained from the "usual" relationship:

$$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma^y} \vec{G}_{xt}^{1/y} \cdot d\vec{y}$$

Note that in Gauss gravitation "usual" equation, there is no term $-\frac{\partial V^x}{\partial t}$, corresponding to acceleration $-\frac{\partial V^x}{\partial t}$ cancelled when changing reference frames from R^0 to R^K (cancelled acceleration referred to as Einsteinian acceleration in previous Memoirs).

For Gauss gravitation "usual" equation, we will speak of "gravitostatic", by analogy with electrostatics, even if it should perhaps not be seen as an identical meaning. We propose here 3 hypotheses to this difference between the Gauss gravitation "usual" equation and the Gauss gravitation equation proposed here, 3 hypotheses that are also compatible with each other.

First hypothesis

Gravitostatic can correspond like electrostatics in case there is no term $-\frac{\partial V^x}{\partial t}$ or $-\frac{\partial A^x}{\partial t}$, that is, the case where there is no acceleration cancelled when changing reference frames.

We then have:

$$2c^2 \vec{K}_{xt}^{1/y} = \vec{G}_{xt}^{1/y} - \frac{\partial V^x}{\partial t} = \vec{G}_{xt}^{1/y} - 0$$

$$\vec{E}l_{xt}^{1/y} = \vec{E}s_{xt}^{1/y} - \frac{\partial A^x}{\partial t} = \vec{E}s_{xt}^{1/y} - 0$$

Second hypothesis

The cancelled acceleration does not appear in Gauss gravitation "usual" equation, because it is implicitly present in the acceleration used. For example, when applying the fundamental principle of Newton's dynamics, we can have:

$$\begin{aligned} m\vec{a} &= m\vec{G}r + \Sigma\vec{F} \\ m(\vec{a}_1 + \vec{a}_2) &= m\vec{G}r + \Sigma\vec{F} \end{aligned}$$

with \vec{a}_2 that is matched to $\frac{\partial V^x}{\partial t}$,

with $\vec{G}r$ that is matched to $\vec{G}r_{xt}^{1/y}$,

and with $\Sigma\vec{F}$ the sum of the non-gravitational forces.

We get:

$$\begin{aligned} m\vec{a}_1 &= m(\vec{G}r - \vec{a}_2) + \Sigma\vec{F} \\ m\vec{a}_1 &= m2c^2\vec{K} + \Sigma\vec{F} \end{aligned}$$

In this case, it is noted that the choice of the initial reference frame R^0 , is not the same, since it is also necessary to add \vec{a}_2 .

Third hypothesis

Cancellation of acceleration is not considered $\frac{\partial V^x}{\partial t}$ in Newtonian theory, nor indeed in General Relativity. Should we then see a weakness of these 2 theories and a track to explain dark matter other than an addition of additional mass?

In a future chapter, seeking to transpose Maxwell Faraday and the phenomena of induction of electromagnetism to gravitation, we will focus on the so-called Neuman case, where we define an electromotive field:

$$\vec{E}m = \frac{\partial \vec{A}^x}{\partial t}$$

This electromotor field explains (with also the case of Lorentz) the phenomena of magnetic induction. It will be suggested that this is a possible explanation for the non-compliance with the Keplerian expectation for galaxies.

End of the 3 hypotheses

We summarize in this table the equations of Gauss gravitation, distinguishing the gravitational case and the gravitostatic one:

Gravitation	Gravitostatic
$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma y} 2c^2\vec{K}_{xt}^{1/y} \cdot d\vec{y}$	$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t = \oint_{\Gamma y} \vec{G}r_{xt}^{1/y} \cdot d\vec{y}$
$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t$ $= -\oint_{\Gamma x} (\vec{G}r_{xt}^{1/y} - \frac{\partial V^x}{\partial t}) \cdot d\vec{x}$	

We have graphically:

Gravitation	Gravitostatic
-------------	---------------

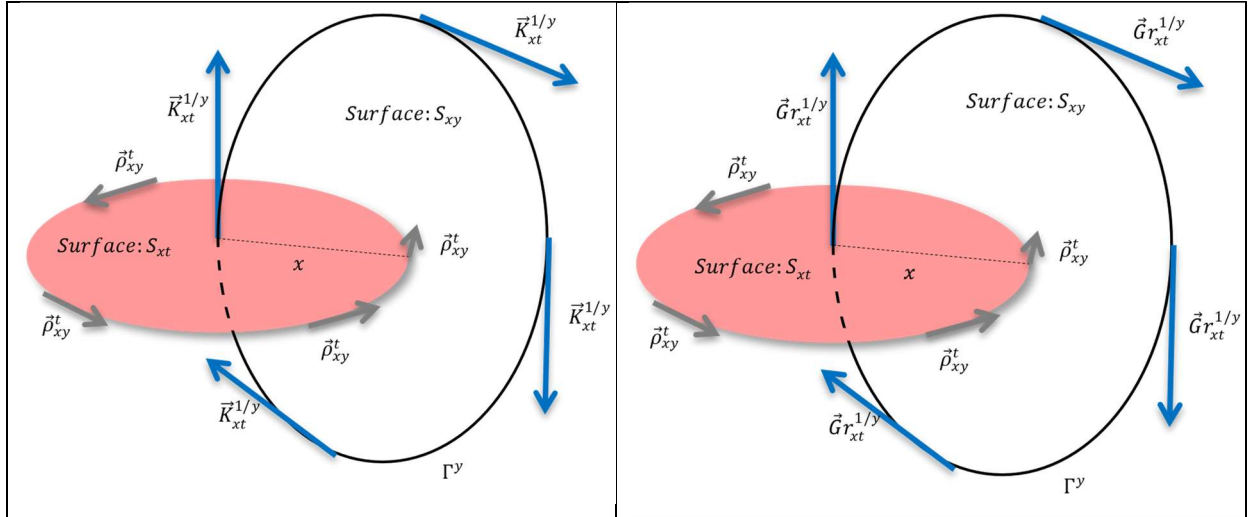


Figure 12: Gauss gravitation 1, fields $\vec{K}_{xt}^{1/y}$, $\vec{G}_{r_{xt}}^{1/y}$ and mass volumetric density $\vec{\rho}_{xy}^t$

By applying Stockes' theorem to:

$$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy} = \oint_{\Gamma^y} 2c^2 \vec{K}_{xt}^{1/y} \cdot d\vec{y}$$

We get:

$$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy} = - \iint_{S_{xt}} \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x} d\vec{S}_{xy}$$

We have the local form to integrate on a surface S_{xy} :

Gravitation	Gravitostatic
$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x}$	$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial \vec{G}_{r_{xt}}^{1/y}}{\partial x}$

IV.7.3 Gauss gravitation 2

Still following a strict analogy with the proposed Maxwell Gauss equation, we have:

Gravitation	Gravitostatic
$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy} = - \oint_{\Gamma^x} 2c^2 \vec{K}_{yt}^{1/x} \cdot d\vec{x}$	$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy} = - \oint_{\Gamma^x} \vec{G}_{r_{yt}}^{1/x} \cdot d\vec{x}$
$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy} = - \oint_{\Gamma^x} (\vec{G}_{r_{yt}}^{1/x} - \frac{\partial V^y}{\partial t}) \cdot d\vec{x}$	

S_{yx} is a surface in the spatial plane x, y

\vec{S}_{xy}^t perpendicular to the space surface S_{yx}

Γ^x contour following x, included in the surface S_{yx}

$\vec{\rho}_{xy}^t$ the mass volumetric density directed following t and to be integrated on a surface S_{xy}

We have the relationship between the wave vector field and the gravitational field:

$$2c^2 \vec{K}_{yt}^{1/x} = \frac{\partial V^t}{\partial y} - \frac{\partial V^y}{\partial t}$$

$$2c^2 \vec{K}_{yt}^{1/x} = \vec{G}r_{yt}^{1/x} - \frac{\partial V^y}{\partial t}$$

We have graphically:

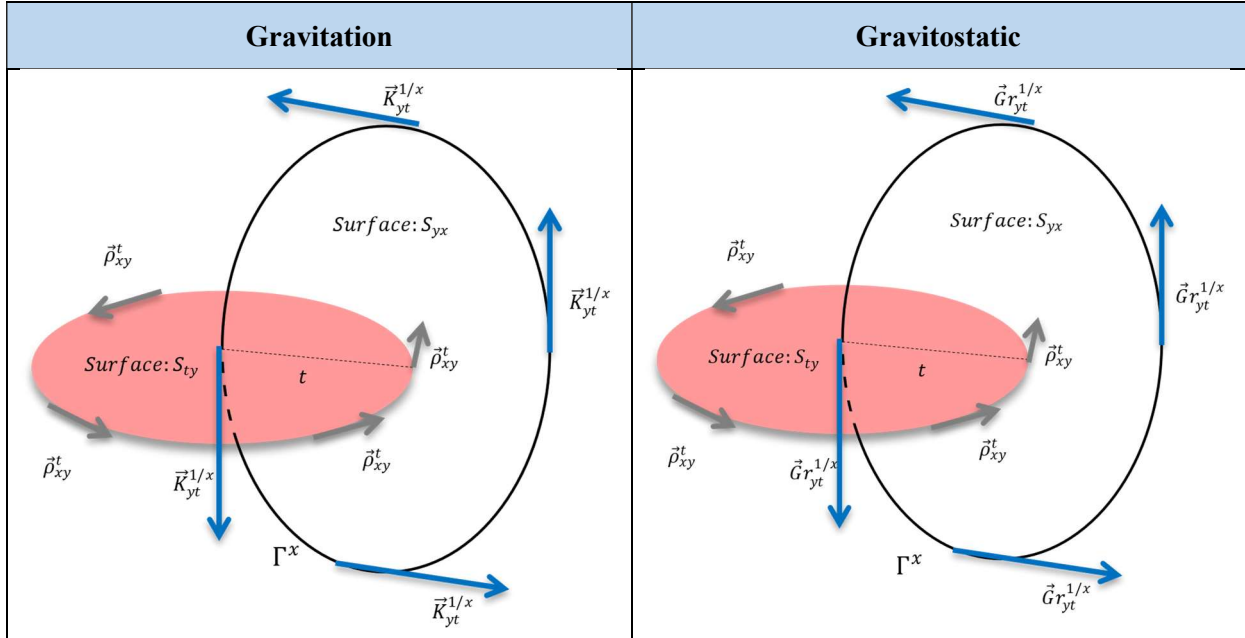


Figure 13: Gauss gravitation 2, fields $\vec{K}_{yt}^{1/x}$, $\vec{G}r_{yt}^{1/x}$ and mass volumetric density $\vec{\rho}_{xy}^t$

In local form, we have:

Gravitation	Gravitostatic
$-4\pi G \vec{\rho}_{xy}^t = -\frac{\partial 2c^2 \vec{K}_{yt}^{1/x}}{\partial y}$	$-4\pi G \vec{\rho}_{xy}^t = -\frac{\partial \vec{G}r_{yt}^{1/x}}{\partial y}$

If we combine the two Gauss gravitation equations, we have:

Gravitation	Gravitostatic
$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial 2c^2 \vec{K}_{yt}^{1/x}}{\partial y}$	$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial \vec{G}r_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{G}r_{yt}^{1/x}}{\partial y}$
$-4\pi G \vec{\rho}_{xy}^t = \left(\frac{\partial \vec{G}r_{xt}^{1/y}}{\partial x} - \frac{\partial^2 V^x}{\partial x \partial t} \right) - \left(\frac{\partial \vec{G}r_{yt}^{1/x}}{\partial y} - \frac{\partial^2 V^y}{\partial y \partial t} \right)$	
$-4\pi G \vec{\rho}_{xy}^t = \left(\frac{\partial \vec{G}r_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{G}r_{yt}^{1/x}}{\partial y} \right) - \left(\frac{\partial^2 V^x}{\partial x \partial t} - \frac{\partial^2 V^y}{\partial y \partial t} \right)$	

Note 1, clarify the source field equations

Let us explain in words the Maxwell source field equations and those just proposed.

Maxwell Gauss source field

$$\frac{\vec{\rho}_{xy}^t}{\varepsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{E}_{yt}^{1/x}}{\partial y}$$

A volumetric density of electric charge (motionless or stationary in Space) generates an electric field variable in Space.

Maxwell Ampère source field

$$\mu_0 \vec{j}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$$

A volumetric density of electric current (electric charge moving in Space, for example a rotation movement) generates a magnetic field variable in Space.

Maxwell displacement current source field

$$\vec{j}_{xt}^y = - \frac{\partial \vec{E}_{ty}^{1/x}}{\partial t}$$

A volumetric density of electric current (electric charge moving in Space, for example a rotation movement) generates an electric field variable in Time.

Newton source field

$$\left(\frac{d\vec{p}^x}{dt}\right)_{tx}^y = - \frac{\partial \hbar \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

$$\frac{dp}{dt} = - \frac{\partial \hbar \Omega^t}{\partial x}$$

A linear momentum density (accelerated moving mass in space) generates a pulsation field variable in Space.

By reexplaining the fundamental principle of Newton's dynamics according to a source field approach, we emphasize the analogy between this principle and Ampère's theorem:

- Fundamental principle: an accelerated moving mass generates a pulsation field (or potential energy) that varies in Space.
- Ampere's theorem: a moving electric charge generates a magnetic field that varies in Space.

Keep in mind that in the fundamental principle of Newton's dynamics, mass is not necessarily constant and that by analogy, this should apply to electric charge in Ampère's theorem.

De Broglie source field

$$\left(\frac{d\vec{p}^x}{dt}\right)_{tx}^y = \frac{\partial \hbar \vec{K}_{ty}^{1/x}}{\partial t}$$

$$p = \hbar K$$

A linear momentum density (mass in motion accelerated in Space) generates a wave vector field variable in Time.

Einstein source field

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx}\right)_{xt}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

$$E = mc^2 = \hbar \Omega$$

A linear density of mass (motionless or stationary in Space) generates a pulsation field variable in Space.

Compton source field

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx}\right)_{xt}^y = - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

$$\frac{dmc^2}{dx} = -\frac{\partial \hbar K^{1/x}}{\partial t}$$

A linear density of mass (motionless or stationary in Space) generates a wave vector field variable in Time.

Gauss gravitation source field

$$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^y}{\partial x} - \frac{\partial 2c^2 \vec{K}_{yt}^x}{\partial y}$$

A mass volumetric density (motionless or stationary in Space) generates a wave vector field variable in Space.

Note 2

For gravitational source field equations (space-oriented sources, equations with constant \hbar) the flows of sources occur through spatiotemporal surfaces.

For Maxwell Ampere source field equations (space-oriented sources, the magnetic part equation with constant μ_0 and the displacement current with constant ε_0), the flows of sources also occur through spatiotemporal surfaces.

For Maxwell Gauss source field equations (time-oriented sources, with constant ε_0) and the 2 equations of Gauss gravitation (sources oriented in Time, with constant $-4\pi G$), the flows of sources occur through spatial surfaces.

IV.8 Tensor approach

IV.8.1 Maxwell Faraday electromagnetic tensor $\vec{\vec{F}}_{\mu\nu}$

We remind Maxwell Faraday's electromagnetic tensor:

$$\vec{\vec{F}}_{\mu\nu} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ \frac{El_{xt}}{c} & 0 & B_{xy} & B_{xz} \\ \frac{El_{yt}}{c} & B_{yx} & 0 & B_{yz} \\ \frac{El_{zt}}{c} & B_{zx} & B_{zy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ -\frac{El_{tx}}{c} & 0 & B_{xy} & B_{xz} \\ -\frac{El_{ty}}{c} & -B_{xy} & 0 & B_{yz} \\ -\frac{El_{tz}}{c} & -B_{xz} & -B_{yz} & 0 \end{bmatrix}$$

Maxwell source field equations can be put in the compact form below which translates their form identity in a Space-Time: ct, x, y, z

$$\partial_\mu F_{\mu\nu} = \mu_0 j^\mu$$

$$j^\mu = (j^t, j^x, j^y, j^z)$$

IV.8.2 Gravitation tensor $\vec{\vec{D}}_{\mu\nu}$

We have the source field equations of gravitation:

$$-\frac{1}{\hbar c} \left(\frac{d\vec{p}^x}{dt} \right)_{xt} = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{c \partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial ct}$$

$$\frac{-2\pi G}{c^2} \vec{\rho}_{xy}^t = \frac{\partial \vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{K}_{yt}^{1/x}}{\partial y}$$

We define the gravitation tensor $\bar{\bar{D}}_{\mu\nu}$:

$$\bar{\bar{D}}_{\mu\nu} = \begin{bmatrix} 0 & K_{tx} & K_{ty} & K_{tz} \\ -K_{tx} & 0 & \frac{\Omega_{xy}}{c} & \frac{\Omega_{xz}}{c} \\ -K_{ty} & -\frac{\Omega_{xy}}{c} & 0 & \frac{\Omega_{yz}}{c} \\ -K_{tz} & -\frac{\Omega_{xz}}{c} & -\frac{\Omega_{yz}}{c} & 0 \end{bmatrix}$$

We define the quadrivector energy momentum density:

$$p^\mu = \left(-\frac{2\pi G}{c^2} \vec{p}_{xy}^t, \frac{-1}{\hbar c} \vec{p}_{yt}^x, \frac{-1}{\hbar c} \vec{p}_{zt}^y, \frac{-1}{\hbar c} \vec{p}_{xt}^z \right)$$

Nota

This quadrivector energy momentum density here has a different meaning from the quadrivector energy momentum of Special Relativity equal to $p^\mu = (p^t c = \gamma c^2 m^t, p^x = \gamma m^t v^x, p^y, p^z)$.

Note that for energy, it is here a volumetric density while for momentum, it is here a linear density. To see if this can pose a problem of homogeneity in the definition of this tensor?

We obtain the compact form that emphasizes the identity of form of the gravitation source field equations in a Space-Time ct, x, y, z :

$$\partial_\mu D_{\mu\nu} = p^\mu$$

Note

Considering the Time dimension as identical to a Space dimension, constructing all the fields in the same mode, therefore makes it possible to have source field equations that are singularly compact and simple. This works for electromagnetism as well as gravitation.

IV.9 Conclusion of the chapter

Following an analogy between gravitational electromagnetism, several source field equations for gravitation have been proposed using the theorems of Ampere and Gauss.

As expected, we found the Gauss gravitation equation, although slightly modified compared to Newton's law of universal gravitation. More surprisingly, we also found the fundamental principle of Newton's dynamics, as well as the de Broglie wavelength momentum relation for Maxwell Ampère's analogues.

At the beginning of this Memoir, it was pointed out that it was possible to find a group velocity of an "electrically charged" particle wave from Maxwell's equations. In the next chapter, we will test whether it is also possible to find a group velocity of a "mass" particle wave from the source field equations obtained for gravitation.

Chapter V Conditions for obtaining group velocity

Purpose of the chapter

From the source field equations of gravitation obtained in the previous chapter, we look for which conditions must respect certain constants, in order to find the expected group velocities.

V.1 Uniting constants \hbar and G in the same model?

V.1.1 Reminder on the type-group velocity obtained from Maxwell's equations

Previously, in the context of electromagnetism, we divided one by the other the following source field equations:

$$\mu_0 j_{xt}^y = \frac{\partial B_{xy}^{1/t}}{\partial x}$$

$$\frac{\rho_{xy}^t}{\varepsilon_0} = \frac{\partial E_{xt}^{1/y}}{\partial x}$$

By taking $j_{xt}^y = \rho_{xy}^t v^y$, we had:

$$\mu_0 \varepsilon_0 v^y = \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

With the condition between constants $\mu_0 \varepsilon_0 = \frac{1}{c^2}$, we obtained for the velocity of an electric charge:

$$v^y = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E_{xt}^{1/y}}$$

a relationship similar to that of the group velocity:

$$v_g = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$$

Thus, so that the "classical" velocity of an electric charge (i.e., an electrically charged particle) is well comparable to the group velocity of a wave, we used on one hand a relationship between electric current density j_{xt}^y and electric charge density ρ_{xy}^t , with $j_{xt}^y = \rho_{xy}^t v^y$, on the other hand a condition relating to constants $\frac{1}{\varepsilon_0}$ and μ_0 , with $\mu_0 \varepsilon_0 = \frac{1}{c^2}$.

For the "classical" velocity of a mass (i.e., a mass particle) to be equal to the group velocity of a wave, we will now seek to have on the one hand a relationship between the momentum density \vec{p}_{xt}^y and mass density ρ_{xy}^t , on the other hand a condition relating to constants G and \hbar .

V.1.2 Search for a relationship between momentum volumetric density \vec{p}_{xt}^y and mass volumetric density ρ_{xy}^t

We have the table below giving the different momentum mass densities:

	Momentum linear density	Momentum volumetric density	Mass volumetric density	Momentum volumetric density expressed as a function of mass volumetric density and velocity
	\vec{p}_{xt}^y	\vec{P}_{xt}^y	ρ_{xy}^t	$\rho_{xy}^t dy (\frac{d\vec{v}^x}{dt})_{tx}^y$
Units	$\frac{Kg.m}{s^2}$	$\frac{Kg}{m.s^2}$	$\frac{Kg}{m^3}$	$\frac{Kg}{m^3} \times m \times \frac{m}{s^2}$

In order to have consistency at the unit level, we propose the relationship between the momentum volumetric density, the mass volumetric density and the velocity of a mass:

$$\vec{P}_{xt}^y = (\frac{d\vec{p}^x}{dt})_{xt}^y = \rho_{xy}^t dy (\frac{d\vec{v}^x}{dt})_{xt}^y$$

Based on the following analogies:	Electromagnétisme	Constantes	Gravitation	Constantes
Ampère Space-oriented sources	$\mu_0 \vec{j}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$	μ_0	$\frac{1}{\hbar} \vec{p}_{xt}^y = -\frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$ $\frac{1}{k} \vec{P}_{xt}^y = -\frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$	$-\frac{1}{\hbar}$ $-\frac{1}{k}$
Gauss Time-oriented sources	$\vec{\rho}_{xy}^t = \frac{\partial \vec{E}l_{xt}^{1/y}}{\partial x}$	$\frac{1}{\epsilon_0}$	$-\frac{2\pi G}{c^2} \vec{\rho}_{xy}^t = \frac{\partial \vec{K}l_{xt}^{1/y}}{\partial x}$	$-\frac{2\pi G}{c^2}$
Relationship between space-oriented and time-oriented sources	$j_{xt}^y = \rho_{xy}^t v^y$		$\vec{P}_{xt}^y = (\frac{d\vec{p}^x}{dt})_{xt}^y$ $= \rho_{xy}^t dy (\frac{d\vec{v}^x}{dt})_{xt}^y$	
Relationship between constants		$\frac{1}{c^2} = \mu_0 \epsilon_0$?

We would like to examine, in the case of gravitation, by dividing the gravitational source field relation of the 1st line by that of the 2nd line, whether it is possible to find a group velocity, as well as a condition between the constants used, in particular between \hbar and G .

Another way of saying, the physical meaning of $\mu_0 \epsilon_0$. It is the inverse of the speed of light squared, is there a physical meaning to be found on the gravitational side for constants \hbar and G ?

In summary, what is the condition between the constants \hbar and G , so that the group velocity is equal to:

$$v_g = v^x = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$$

V.1.3 Uniting constants \hbar and G in the same model?

We will use the following source field equations that we will divide one by the other:

	Source equations fields	Units
Ampère gravitation	$\frac{1}{k} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$ $\frac{1}{k} \rho_{xy}^t dy \left(\frac{d\vec{v}^x}{dt} \right)_{xt}^y = - \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$	$\frac{s \text{ Kg}}{\text{Kg m} \cdot \text{s}^2} = \frac{1}{\text{m} \cdot \text{s}}$ $\frac{s \text{ m Kg m}}{\text{Kg m}^3 \text{ s}^2} = \frac{1}{\text{m} \cdot \text{s}}$
Gauss gravitation	$-\frac{2\pi G}{c^2} \vec{\rho}_{xy}^t = \frac{\partial \vec{K}_{xt}^{1/y}}{\partial x}$	$\frac{\text{m}^3 \text{ Kg}}{\text{Kg} \cdot \text{s}^2 \text{ m}^3} = \frac{1}{\text{s}^2}$

We want to:

$$v_g = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} \times \frac{\partial x}{\partial \vec{K}_{xt}^{1/y}} = \frac{1}{k} \rho_{xy}^t dy \left(\frac{d\vec{v}^x}{dt} \right)_{xt}^y \times \frac{c^2}{2\pi G \rho_{xy}^t} ?$$

$$v_g = \frac{1}{k} \times \frac{dy dv^x}{dct} \times \frac{c^3}{2\pi G} ?$$

With $\hbar = k \times dx dct$, we want to:

$$v_g = \frac{dx dct}{\hbar} \times \frac{dy dv^x}{dct} \times \frac{c^3}{2\pi G} ?$$

$$v_g = \frac{dx dy}{\hbar} \times \frac{c^3}{2\pi G} dv^x ?$$

So, that the group velocity v_g of a wave is equal to the "classical" velocity dv^x of a mass particle, it is necessary to:

$$\frac{dx dy}{\hbar} \times \frac{c^3}{2\pi G} = 1 ?$$

Note on introduced infinitesimals

In the term momentum volumetric density $\vec{p}_{xt}^y = \rho_{xy}^t dy \left(\frac{d\vec{v}^x}{dt} \right)_{tx}^y$, we introduce the 2 infinitesimals: dy and $d\vec{v}^x$. We then find ourselves at the end of the calculations with one infinitesimal too many: dv^x , whereas we would only like v^x .

It remains to be seen whether we should not rather introduce a term such as $\rho_{xy}^t \frac{dy v^x}{dt}$, with if v^x is constant as a function of Time, obtaining: $\rho_{xy}^t v^x \frac{dy}{dt}$ and therefore the desired term?

In the case of a symmetry of Space, we pose $dx = dy$. It is therefore necessary to:

$$(dx)^2 = \frac{2\pi G \hbar}{c^3}$$

$m^2 = \frac{\text{m}^3}{\text{Kg} \cdot \text{s}^2} \cdot \frac{\text{Kg} \cdot \text{m}^2 \cdot \text{s}^3}{\text{s}} \cdot \frac{\text{s}^3}{\text{m}^3}$	Units
--	--------------

If we correct, and take as elementary length: $dy = 2\pi dx$ (this may seem more relevant, because we have rather a cylindrical symmetry on x, y, t), we get:

$$2\pi(dx)^2 = \frac{2\pi G \hbar}{c^3}$$

We find the Planck length:

$$dx = l_p = \sqrt{\frac{G \hbar}{c^3}}$$

In **conclusion**, so that the "classic" velocity v^x of a mass particle is well equal to the group velocity $v_g = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$ of a wave, we have the condition between the constants \hbar and G , on the infinitesimal length:

$$l_p = \sqrt{\frac{G\hbar}{c^3}}$$

$$c = \sqrt{\frac{G\hbar}{cl_p^2}}$$

Search for a relationship between \hbar and k

We posed in the previous chapter:

$$\hbar = dx dct \times k$$

In the case of symmetry between dct and dy , we have:

$$\hbar = dx dy \times k$$

$$\hbar = 2\pi(dx)^2 \times k$$

We obtain the relation:

$$\hbar = 2\pi l_p^2 \times k$$

Note on the direction of group velocity

We find a group velocity for a mass oriented according to x (instead of y for an electric charge). The case of mass is a priori more satisfactory, because perpendicular to both $\vec{\Omega}_{xy}^{1/t}$ and $\vec{K}_{xt}^{1/y}$. To see where is the error at the level of v^y for the electric current? We have:

$$v_g = v^x = \frac{\partial \Omega_{xy}^{1/t}}{\partial K_{xt}^{1/y}}$$

$$v^y = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$$

Thus, instead of asking:

$$j_{xt}^y = \rho_{xy}^t v^y$$

Should we not rather ask:

$$\vec{j}_{xt}^y = \vec{\rho}_{xy}^t \wedge \vec{v}^x ?$$

We would then have for the group velocity:

$$v^x = c^2 \frac{\partial B_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$$

V.1.4 Justification for wave vector expression

In Memoir 2, following an analogy with the pulsation field, we posed for the wave vector field (in vacuum):

$$2c^2 K_{xt}^{1/y} = \frac{\partial V^t}{\partial x} - \frac{\partial V^x}{\partial t}$$

$$2c^2 K_{xt}^{1/y} = G r_{xt}^{1/y} - \frac{\partial V^x}{\partial t}$$

The constant c^2 was introduced for the sake of homogeneity, and without any real justification.

The fact of proposing source field equations of gravitation involving the pulsation field and wave vector field, then to find from these source field equations, the expression of the group velocity $v_g = \frac{\partial \Omega^{1/t}}{\partial K^{1/y}}$ provides justification for the expression $2c^2 K_{xt}^{1/y} = \frac{\partial v^t}{\partial x} - \frac{\partial v^x}{\partial t}$ and for the introduction of c^2 .

We can also see this group velocity as the keystone (or the stumbling block) and the expected justification for a number of hypotheses made in Memoirs 1, 2, 3.

V.1.5 Planck mass and Planck electric charge

From the smallest wavelength $\lambda_p = 2\pi l_p$ ($dy = 2\pi dx$), constructed as the perimeter of a radius circle $dx = l_p$:

$$\lambda_p = 2\pi \sqrt{\frac{G\hbar}{c^3}}$$

and the Compton wavelength relation:

$$m_p c = \frac{2\pi\hbar}{\lambda_p}$$

we obtain the Planck mass:

$$m_p c = \hbar \sqrt{\frac{c^3}{G\hbar}}$$

$$m_p = \sqrt{\frac{\hbar c}{G}}$$

We also frequently introduce the Planck duration constructed from $t_p = \frac{l_p}{c}$:

$$t_p = \sqrt{\frac{G\hbar}{c^5}}$$

We thus obtain minimum bounds for length, mass, duration..., similar notions, but still different from the quantification of a physical quantity, which would then take a discrete series of values.

We touch on quantum gravity and a quantum of gravitational interaction, but are we really there? We will return to this delicate question at the end of the 5th Memoir.

We can also find l_p and m_p by equalizing mass energy E_0 and potential energy E_p , then using the Compton wavelength. By analogy between gravitation and electrostatics, a Planck electric charge is obtained by substituting gravitational potential energy with electrostatic energy (or by substituting directly on the term $m_p = \sqrt{\frac{\hbar c}{G}}$ the constant G by the constant $\frac{1}{4\pi\epsilon_0}$). The table below summarizes the 2 approaches for gravitation and electrostatics.

	Gravitation	Electrostatique
Constant	G	$\frac{1}{4\pi\epsilon_0}$

Compton wavelength	$m_p = \frac{\hbar}{cl_p}$	$m_p = \frac{\hbar}{cl_p}$
Potential energy between 2 masses or 2 Planck charges	$E_p = \frac{Gm_p^2}{l_p}$	$E_p = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$
Mass energy	$E_0 = m_p c^2$	$E_0 = m_p c^2$
	$E_0 = E_p$ $m_p c^2 = \frac{Gm_p^2}{l_p}$ Using the Compton wavelength: $\frac{\hbar}{cl_p} c^2 = \frac{Gm_p^2}{l_p}$ We get:	$E_0 = E_p$ $m_p c^2 = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$ Using the Compton wavelength: $\frac{\hbar}{cl_p} c^2 = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$ We get:
Planck mass or charge	$m_p = \sqrt{\frac{\hbar c}{G}}$	$q_p = \sqrt{4\pi\epsilon_0 \hbar c}$

Note on entropy S of a black hole

For the record, we recall the Bekenstein-Hawking relationship, proposed in the 1970s by Stephen Hawking and Jacob Bekenstein, giving entropy S of a black hole as a function of its surface A and Planck length l_p :

$$S = k_B \frac{A}{4l_p^2} = k_B \ln W$$

with k_B the Boltzmann constant,

and $W = \exp\left(\frac{A}{4l_p^2}\right)$ the number of configurations.

We will come back to this more broadly in the 6th Memoir.

V.2 Uniting constants \hbar and ϵ_0 in the same model?**V.2.1 Reminders on the fine structure constant α and its interpretation from Planck's charge**

Historically, the fine structure constant α was introduced by A. Sommerfeld in 1916 in a relativistic model decrying the electron and inspired by the non-relativistic model of the Bohr atom. The constant α was first interpreted as the ratio between the speed of the electron in Bohr's model and the speed of light in vacuum .

The constant α was named in reference to the fine structure, which describes the duplication of the spectral lines of a particle, and of which the relativistic model of Sommerfeld proposed a first explanation. Subsequently, the spin model provided a better understanding of these duplications.

We remind here this constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

The constant α being dimensionless, its very existence implies the existence of an underlying mechanism fixing its value. We will suggest in the next paragraph a way forward to this mechanism.

The constant α can also be interpreted as the square of the ratio between the elementary charge and the Planck charge:

$$\alpha = \frac{e^2}{q_p^2}$$

Note that the model of the Bohr atom, the constant α , Planck electric charge and many other physical notions include both constants ε_0 and \hbar . The first constant ε_0 is present in the source field equation Gauss electromagnetic, the second constant \hbar is present in Ampere gravitation. We will therefore look for a model that unites these 2 source field equations and thus these 2 constants ε_0 and \hbar .

V.2.2 Uniting the constants \hbar and ε_0 in the same model?

Previously, Ampere electromagnetism was first divided by Gauss electromagnetism, then Ampere gravitation by Gauss gravitation to obtain group velocities or similar. Let us now see what happens if we divide Ampere gravitation by Gauss electromagnetism, that is to say if we "mix" gravitation and electricity.

We use the following source field equations that are divided one by the other:

	Source equations fields	Units
Ampère Gravitation	$\frac{1}{k} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$ $\frac{1}{k} \rho_{xy}^t dy \left(\frac{d\vec{v}^x}{dt} \right)_{xt}^y = - \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$	$\frac{s \text{ Kg}}{\text{Kg m. s}^2} = \frac{1}{\text{m. s}}$ $\frac{s \text{ m Kg m}}{\text{Kg m}^3 \text{ s}^2} = \frac{1}{\text{m. s}}$
Gauss Electromagnetism	$\frac{\vec{\rho}_{exy}^t}{\varepsilon_0} = \frac{\partial \vec{E}_{l_{xt}}^{1/y}}{\partial x}$	

With $\vec{\rho}_{exy}^t$, the electric charge volumetric density, to be distinguished from the mass volumetric density.

By analogy with $v^y = \frac{c^2 \partial B_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$, we introduce in the desired ratio the term c^2 . We have (to within a sign -):

$$\frac{\partial 2\vec{\Omega}_{xy}^{1/t}}{\partial x} \times \frac{c^2 \partial x}{\partial \vec{E}_{l_{xt}}^{1/y}} = \frac{2}{k} \rho_{xy}^t dy \left(\frac{d\vec{v}^x}{dt} \right)_{xt}^y \times \frac{\varepsilon_0 c^2}{\rho_{exy}^t}$$

With $\hbar = k \times dx dct$, we have:

$$\frac{c^2 \partial 2\Omega_{xy}^{1/t}}{\partial E l_{xt}^{1/y}} = \frac{dx dct}{\hbar} \times \frac{dy dv^x}{dct} \times 2\varepsilon_0 c^3 \times \frac{\rho_{xy}^t}{\rho_{exy}^t}$$

With $dy = 2\pi dx$, we have:

$$\frac{c^2 \partial 2\Omega_{xy}^{1/t}}{\partial E l_{xt}^{1/y}} = dx^2 \times \frac{4\pi \varepsilon_0 c^3}{\hbar} \times \frac{\rho_{xy}^t}{\rho_{exy}^t} \times dv^x$$

We set aside the ratio of charge densities $\frac{\rho_{xy}^t}{\rho_{exy}^t}$ and we propose the condition to have a group velocity of

the type $\frac{c^2 \partial 2\Omega_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$:

$$dx^2 \times \frac{4\pi\epsilon_0 c^3}{\hbar} = 1$$

We obtain the condition on an infinitesimal quantity (not exactly homogeneous at a distance, because of the term $\frac{\rho_{xy}^t}{\rho_{exy}^t}$ set aside):

$$le_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$$

The analogies are given in the following table:

	Gauss gravitation Ampère gravitation	Gauss electrostatic Ampère gravitation
Constant used in Gauss	$-G$	$\frac{1}{4\pi\epsilon_0}$
Constant used in Ampere	$-\hbar$	$-\hbar$
Conditions on infinitesimal quantities	$l_p = \sqrt{\frac{G\hbar}{c^3}}$	$le_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$
Compton wavelength	$m_p = \frac{\hbar}{cl_p}$	$m_p = \frac{\hbar}{cl_p}$
Potential energy between 2 Planck masses or 2 Planck charges	$E_p = \frac{Gm_p^2}{l_p}$	$E_p = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$ We take l_p and not le_p , because we will equalize E_p with E_0
Mass energy	$E_0 = m_p c^2$	$E_0 = m_p c^2$
	$E_0 = E_p$ $m_p c^2 = \frac{Gm_p^2}{l_p}$ Using the Compton wavelength: $\frac{\hbar}{cl_p} c^2 = \frac{Gm_p^2}{l_p}$ We get:	$E_0 = E_p$ $m_p c^2 = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$ Using the Compton wavelength: $\frac{\hbar}{cl_p} c^2 = \frac{1}{4\pi\epsilon_0} \frac{(q_p)^2}{l_p}$ We get:
Planck mass or Planck charge	$m_p = \sqrt{\frac{\hbar c}{G}}$	$q_p = \sqrt{4\pi\epsilon_0 \hbar c}$
	$\frac{Gm_p^2}{l_p} = m_p c^2$	$\frac{q_p^2}{4\pi\epsilon_0 le_p} = q_p c^2$
Planck mass or Planck charge from Planck lengths	$m_p = \frac{c^2 l_p}{G}$	$q_p = 4\pi\epsilon_0 c^2 le_p$
		If we pose $le_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$, we

		actually find: $q_p = \sqrt{4\pi\epsilon_0\hbar c}$
--	--	--

In the case of the Bohr atom, we have for an electron of mass m_e and speed v_e , on the Bohr orbit of radius r_B :

$$m_e v_e^2 = m_e (\alpha c)^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_B}$$

Following de Broglie's ideas:

$$m_e v_e = \frac{\hbar}{r_B}$$

$$\frac{\hbar}{r_B} \alpha c = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_B}$$

We get:

$$e = \sqrt{\alpha 4\pi\epsilon_0 \hbar c}$$

To obtain a group velocity of type $\frac{c^2 \partial^2 \Omega_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}$, the condition must be checked:

$$l e_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$$

By analogy between gravitation and electrostatics, we define the Planck charge q_p function of $l e_p$:

$$q_p = 4\pi\epsilon_0 c^2 l e_p$$

By replacing with $l e_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$, We have the condition to respect on q_p , to get the group velocity:

$$q_p = 4\pi\epsilon_0 c^2 \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$$

$$q_p = \sqrt{4\pi\epsilon_0 \hbar c}$$

The fine-structure constant is defined as the ratio of the elementary electric charge and the Planck electric charge:

$$\alpha = \frac{e^2}{q_p^2}$$

We therefore have the condition to respect on the fine structure constant, to obtain the group velocity

$$\frac{c^2 \partial^2 \Omega_{xy}^{1/t}}{\partial E l_{xt}^{1/y}}:$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

V.2.3 Planetary models

To finish this "mixing" gravitation and electricity, we propose the table below which highlights the analogies between the Kepler-Newton planetary model and that of Bohr. The differences are explained in the case of the Bohr model by a "crossing" of the gravitational and electromagnetism source field equations.

	Kepler-Newton planetary model The planet is "stabilized" by the gravitational force generated by the Sun and by the centrifugal inertial force	Planetary model of the Bohr electron The electron is "stabilized" by the electrostatic force generated by the proton and by the centrifugal inertial force
Source field equations of Ampère-type	$-\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$	$-\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$
Constants	$-\hbar$	$-\hbar$
Source field equations of Gauss-type	$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial 2c^2 \vec{K}_{yt}^{1/x}}{\partial y}$	$\frac{\vec{\rho}_{exy}^t}{\epsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{E}_{yt}^{1/x}}{\partial y}$
Constants	$-G$	$\frac{1}{4\pi\epsilon_0}$
Condition on an infinitesimal quantity for obtaining a group velocity	$l_p = \sqrt{\frac{G\hbar}{c^3}}$	$le_p = \sqrt{\frac{\hbar}{4\pi\epsilon_0 c^3}}$
Infinitesimal mass and infinitesimal electric charge	$m_p = \sqrt{\frac{\hbar c}{G}}$ $m_p = \frac{c^2 l_p}{G}$	$q_p = \sqrt{4\pi\epsilon_0 \hbar c}$ $q_p = 4\pi\epsilon_0 c^2 le_p$
Study scale	Planetary system	Atom

It should be noted that the planetary model of the Bohr electron does not involve magnetism and the constant μ_0 , Hence its sometimes somewhat "old-fashioned" appearance.

V.3 Uniting the constants μ_0 and G in the same model?

In the previous paragraph, we looked for a model including constants ϵ_0 and \hbar from the source field equations of electromagnetism and gravitation. Note that always from these source field equations, it is also possible to "mix" the constants μ_0 and G .

However, unlike ϵ_0 and \hbar , There is a priori no known physical notion that includes both μ_0 and G , that is, magnetism and gravitation.

However, let's see what happens if we divide Ampere electromagnetism by Gauss gravitation.

	Source field equations	Units
Ampère electromagnetism	$\mu_0 \vec{j}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$ $\vec{j}_{xt}^y = \rho_{exy}^t \vec{v}_{xt}^y$ $\mu_0 \rho_{exy}^t \vec{v}_{xt}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$	
Gauss gravitation	$-\frac{4\pi G}{c^2} \vec{\rho}_{xy}^t = \frac{\partial 2\vec{K}_{xt}^{1/y}}{\partial x}$	$\frac{m^3 \text{ Kg}}{\text{Kg} \cdot s^2 m^3} = \frac{1}{s^2}$

We have (to within the sign -):

$$\frac{\partial B_{xy}^{1/t}}{\partial x} \times \frac{\partial x}{\partial 2K_{xt}^{1/y}} = \mu_0 \rho_{exy}^t v_{xt}^y \times \frac{c^2}{4\pi G \rho_{xy}^t}$$

$$\frac{\partial B_{xy}^{1/t}}{\partial 2K_{xt}^{1/y}} = \frac{\mu_0 c^2}{4\pi G} \times \frac{\rho_{exy}^t}{\rho_{xy}^t} v_{xt}^y$$

If we put aside $\frac{\rho_{exy}^t}{\rho_{xy}^t}$, we have the condition for obtaining a group velocity:

$$\frac{\mu_0 c^2}{4\pi G} = 1$$

This results in a condition that cannot be verified. To explain why, we cannot "mix" here gravitation and magnetism?

V.4 Conclusion of the chapter

As hoped at the beginning of this Memoir, in order to find a "classical" velocity of a mass particle equal to a group velocity of a wave, by dividing the source field equation of Ampere gravitation by that of Gauss gravitation, we obtain a condition between \hbar and G , a condition giving an infinitesimal length identical to the Planck length.

Similarly, we obtain a condition between \hbar and ϵ_0 . On the other hand, it was not possible to obtain a condition between μ_0 and G .

Thanks to the condition between \hbar and G , quantum physics and Newtonian gravity are unified in the same model. For this, it was hypothesized, in the previous Memoir, that the gravitational field $\vec{G}r$ is a constituent of the wave vector field \vec{K} (the other constituent being the acceleration cancelled by changing reference frames). This hypothesis is supported by obtaining the group velocity.

In this and the previous chapter, there has been a lot of talk about source field equations. Nevertheless, there has been little debate about the underlying reasons for these relationships between sources and fields. Why and how can a source located in Space-Time generate a field extended to Space-Time? It is these questions that we will focus on in the next chapter.

Chapter VI Can we intuitively understand Ampère's theorem? Study of 2 tracks

Purpose of the chapter

We examine 2 tracks to intuitively understand what can connect a local source to the derivative of a field that extends to Space-Time.

VI.1 What is demonstrated and what is not

In the previous chapter, source field equations for gravitation were proposed in analogy with those of electromagnetism. However, for all these source field equations, there is a priori no demonstration.

Indeed, in vector analysis, what we prove from Stokes' theorem is the link between a vector along a closed contour and the rotational flux of this vector through a surface S (i.e., the passage from one to two dimensions):

$$\oint_C \vec{B} \cdot d\vec{l} = \iint_S \text{rot} \vec{B} \cdot d\vec{S}$$

By contrast, what is not proven is the link between a field generally extended in Space-Time and a source generally located in Space-Time.

So, why $\mu_0 \vec{j} = \text{rot} \vec{B}$? A local relation that yields Ampère's theorem in integral form:

$$\mu_0 \iint_S \vec{j} \cdot d\vec{S} = \oint_C \vec{B} \cdot d\vec{l} = \iint_S \text{rot} \vec{B} \cdot d\vec{S}$$

We will not make here a mathematical proof of $\mu_0 \vec{j} = \text{rot} \vec{B}$. We will simply propose two tracks that could lead to a better understanding of the different source field equations and Ampère's theorem.

Note 1

There is a proof of Ampère's theorem from the law of Biot and Savart:

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int_C \frac{Id\vec{l} \wedge \vec{r}}{r^3}$$

, but then you have to prove that law.

Note 2

We have the same problem of demonstration with the Maxwell Gauss source field equation:

$$\frac{\rho}{\varepsilon_0} = \text{div} \vec{E}$$

, also called Gauss's theorem. As we treat here identically Maxwell Gauss, Maxwell Ampère and the source field equations of electromagnetism and gravitation, to demonstrate one of these equations is to demonstrate the others.

VI.2 First track: 2 analogies between fluid mechanics and magnetism

We remind the **first** analogy between fluid mechanics and magnetism, analogy mainly used in this essay:

	Fluid mechanics	Magnetism
Field	$2\vec{\Omega}$	\vec{B}

Vector potential	\vec{V}	\vec{A}
Condition of field conservation	$div\vec{\Omega} = 0$	$div\vec{B} = 0$
Potential field relationship	$2\vec{\Omega} = rot\vec{V}$	$\vec{B} = rot\vec{A}$

We have also mentioned a **second** analogy between fluid mechanics and magnetostatics, an analogy proposed by H. Poincaré in one of his lectures. Compared to the first analogy, the rotation vector (or vortex vector) $\vec{\omega}$ corresponds to the current density $\mu_0\vec{j}$, the magnetic field \vec{B} corresponds to the velocity field \vec{V} .

We have the following table:

	Fluid mechanics (rotational, permanent, incompressible)	Magnetostatics
Source	$\vec{\omega}(M)$ Vortex vector	$\vec{j}(M)$ Current density
Constant	2	μ_0
Field	$\vec{V}(M)$	$\vec{B}(M)$
Potential vecteur	\vec{A}	\vec{A}
Condition of field conservation	$div\vec{V} = 0$	$div\vec{B} = 0$
Potential field relationship	$\vec{V} = rot\vec{A}$	$\vec{B} = rot\vec{A}$
Condition of source conservation	$div\vec{\omega} = 0$	$div\mu_0\vec{j} = 0$
Source field relationship	$2\vec{\omega} = rot\vec{V}$	$\mu_0\vec{j} = rot\vec{B}$
Potential source relationship	$\Delta\vec{A} + 2\vec{\omega} = 0$	$\Delta\vec{A} + \mu_0\vec{j} = 0$
Ampère's theorem	$\iint_S 2\vec{\omega}(M) \cdot d\vec{S} = \oint_{\Gamma} \vec{V}(M) \cdot d\vec{l}$	$\iint_S \mu_0\vec{j}(M) \cdot d\vec{S} = \oint_{\Gamma} \vec{B}(M) \cdot d\vec{l}$

Note 1

It should be noted that $\vec{\omega}$ and $\vec{\Omega}$ are two distinct notations for the same rotation vector.

It is proposed here to use:

- $\vec{\omega}$ when the rotation vector is interpreted as a source or a rotation of a body on itself (spin rotation), both located in Space (even if, as we will see, the source and the body rotating on itself do not have the same limits in Space),
- $\vec{\Omega}$ when the rotation vector is interpreted as a field or an orbital rotation, both extended in Space.

We will come back to this in the next paragraph, when we study the second track.

Note 2

Let us observe that in fluid mechanics, the vector $\vec{\Omega}$ (or $\vec{\omega}$) is not generally oriented according to Time, but according to a dimension of Space. It is therefore noted $\vec{\omega}$ and not $\vec{\omega}^t$.

In the case of a source $\vec{\omega}$ uniform, we have the velocity field:

$$\vec{V} = \vec{\omega} \wedge \vec{OM} = \vec{\Omega} \wedge \vec{OM}$$

$$2\vec{\omega} = r\vec{\partial}t\vec{V} = r\vec{\partial}t(\vec{\omega} \wedge \vec{OM})$$

In the case of a source $\mu_0\vec{j}$ uniform, we have the magnetic field:

$$\vec{B} = \frac{1}{2}\mu_0\vec{j} \wedge \vec{OM}$$

$$\mu_0\vec{j} = r\vec{\partial}t\vec{B} = \frac{1}{2}r\vec{\partial}t(\mu_0\vec{j} \wedge \vec{OM})$$

It is noted that \vec{B} takes the form of a magnetic moment.

The comparison of the two analogies is interesting. In the first, the rotation vector $\vec{\Omega}$ plays the role of a field. In the second, the rotation vector $\vec{\omega}$ plays the role of a source. We thus have the same physical quantity which alternately plays the role of a field or a source, hence a possible passage from one to the other.

The same applies to \vec{B} , which can be analysed as the source of the “field” \vec{A} .

We summarize the 2 analogies in the following table:

	Fluid mechanics (rotational, permanent, incompressible)	Magnetostatics (with in particular zero displacement currents)	Magnetism
Source	$\vec{\omega}(M)$	$\vec{j}(M)$	$\vec{B}(M)$
Constant	2	μ_0	1
Field	$\vec{V}(M)$	$\vec{B}(M)$	$\vec{A}(M)$
Condition of source conservation	$div2\vec{\omega}=0$	$div\mu_0\vec{j}=0$	$div\vec{B}=0$
Source field relationship	$2\vec{\omega} = r\vec{\partial}t\vec{V}$	$\mu_0\vec{j} = rot\vec{B}$	$\vec{B} = r\vec{\partial}t\vec{A}$
	If the source $\vec{\omega}$ is uniform in a volume <i>Vol</i> of Space, the field \vec{V} is calculated in this volume <i>Vol</i> : $\vec{V} = \vec{\omega} \wedge \vec{OM}$	If the source $\mu_0\vec{j}$ is uniform in a volume <i>Vol</i> of Space, the field \vec{B} is calculated in this volume <i>Vol</i> : $\vec{B} = \frac{1}{2}(\mu_0\vec{j} \wedge \vec{OM})$	If \vec{B} is uniform in a volume <i>Vol</i> of Space, \vec{A} is calculated in this volume <i>Vol</i> : $\vec{A} = \frac{1}{2}(\vec{B} \wedge \vec{OM})$
Ampère's theorem	$\iint_S 2\vec{\omega} \cdot d\vec{S} = \oint_R \vec{V} \cdot d\vec{l}$	$\iint_S \mu_0\vec{j} \cdot d\vec{S} = \oint_R \vec{B} \cdot d\vec{l}$	$\iint_S \vec{B} \cdot d\vec{S} = \oint_R \vec{A} \cdot d\vec{l}$
Law of Biot and Savart	If the source $\vec{\omega}$ is located in <i>Vol</i> a finite volume of Space, the field \vec{V} is calculated at any point in Space according to the	If the source $\mu_0\vec{j}$ is located in <i>Vol</i> a finite volume of Space, the field \vec{B} is calculated at any point of Space according	If \vec{B} is located in <i>Vol</i> a finite volume of Space, \vec{A} is calculated at any point in Space according to the

	analogue of Biot and Savart's law: $\vec{V}(OM) = \frac{1}{2\pi} \iiint_{Vol} \frac{\vec{\omega} d\tau \wedge \vec{OM}}{OM^3}$	to the law of Biot and Savart: $\vec{B}(OM) = \frac{\mu_0}{4\pi} \iiint_{Vol} \frac{\vec{j} d\tau \wedge \vec{OM}}{OM^3}$	analogue of Biot and Savart's law: $\vec{A}(OM) = \frac{1}{4\pi} \iiint_{Vol} \frac{\vec{B} d\tau \wedge \vec{OM}}{OM^3}$
--	---	--	--

In variable regime, we can try the following analogy:

	Mécanique des fluides	Electromagnétisme
Source field relationship		$\mu_0 \vec{j} = r \vec{\partial} t \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E} l}{\partial t}$
Source field relationship	$2(\vec{\omega} + \vec{\omega}_D) = rot \vec{V}$	$\mu_0 (\vec{j} + \vec{j}_D) = rot \vec{B}$
		$\vec{j}_D = \epsilon_0 \frac{\partial \vec{E} l}{\partial t}$
Condition of source conservation	$div \vec{\omega} + \frac{\partial \omega^t}{\partial t} = 0$	$div \vec{j} + \frac{\partial q^t}{\partial t} = 0$

See then the physical meanings of $\vec{\omega}_D$ and ω^t ?

VI.3 Second track: change of points of view, study of two bodies of the Earth-Sun type

The second track repeats the old trick of A. Einstein: see if we can explain two phenomena of different appearance but in reality equal (or even identical), by different points of observation.

Note, search for relationships between something localized in Space and something extended in Space

The Ampère and Gauss source field relations relate a source located in Space and an extended field in Space. The track followed here is therefore to look for phenomena that relate something localized in Space with something extensive.

Two phenomena are cited here for the record:

- The first is an orbital rotation that is extended in Space and a spin rotation (rotation on oneself) that is localized in Space. The two can be related by a change of reference frames (it is this phenomenon that is examined in this second track).
- The second is the particle wave duality, with a wave that is extended into Space and a corpuscle that is localized.

VI.3.1 Earth Sun System

Let be a system with two bodies for example the Earth and the Sun.

Let be R^{Solis} the solar reference system

In R^{Solis} , the Sun is motionless, and the Earth has, as a first approximation, two movements:

- an orbital rotational motion of angular velocity $[\Omega_{Terra}]_{R^{Solis}}$.

We have:

$$[\Omega_{Terra}]_{R^{Solis}} = \frac{2\pi}{T_{Terra}}$$

with $T_{Terra} \approx 365 \text{ days}$ to be converted to seconds to express the angular velocity in $rad \times s^{-1}$.

- a rotational movement on itself (spin rotation) of angular velocity $[\omega_{Terra}]_{R^{Solis}}$.

We have:

$$[\omega_{Terra}]_{R^{Solis}} = \frac{2\pi}{t_{Terra}}$$

with $t_{Terra} \approx 24hours$.

Let be R^{Terra} the terrestrial reference frame

In R^{Terra} , the Earth is motionless, and the Sun has, as a first approximation, two movements:

- an orbital rotational motion of angular velocity $[\Omega_{Solis}]_{R^{Terra}}$.

We have:

$$[\Omega_{Solis}]_{R^{Terra}} = \frac{2\pi}{T_{Solis}}$$

with $T_{Solis} \approx 24hours$.

- a rotational movement on itself (spin) of angular velocity $[\omega_{Solis}]_{R^{Terra}}$.

We have:

$$[\omega_{Solis}]_{R^{Terra}} = \frac{2\pi}{t_{Solis}}$$

with $t_{Solis} \approx 365days$.

The following figure illustrates:

- when in R^{Solis} , the Earth rotates in orbit in one direction,
- when in R^{Terra} , the Sun rotates in the other direction.

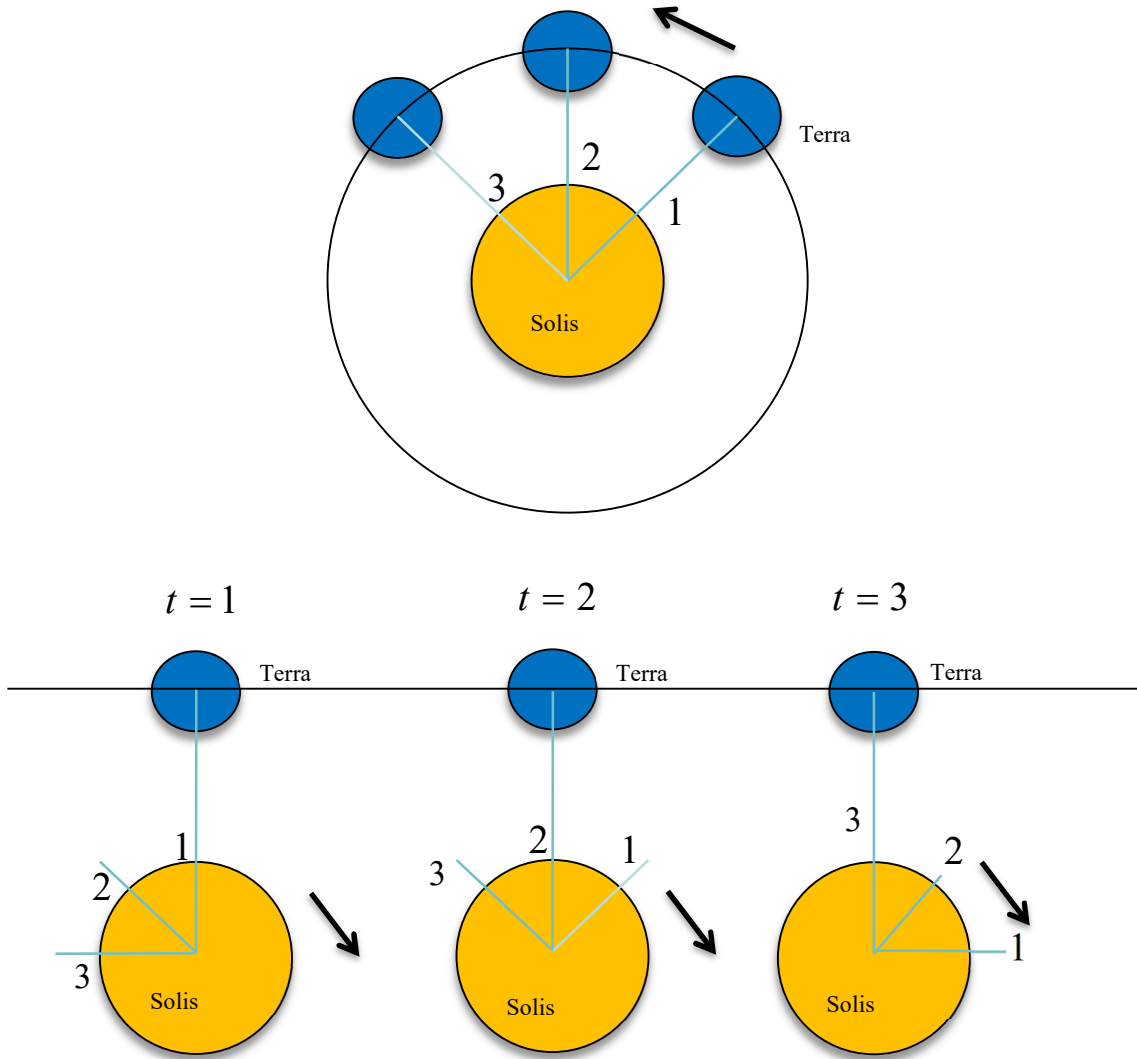


Figure 14: Earth Sun system

In a two-body Earth-Sun system, we therefore have the relations:

$$[\Omega_{Terra}]_{R^{Solis}} = -[\omega_{Solis}]_{R^{Terra}}$$

$$[\omega_{Terra}]_{R^{Solis}} = -[\Omega_{Solis}]_{R^{Terra}}$$

To switch from R^{Solis} to R^{Terra} , it is necessary to cancel the movements of the Earth: $[\Omega_{Terra}]_{R^{Solis}}$ and $[\omega_{Terra}]_{R^{Solis}}$. Thus, we obtain a reference that is immobile.

We have the change of reference frames:

$$R^{Terra} = R^{Solis + \omega_{Terra} + \Omega_{Terra}}$$

with ω_{Terra} and Ω_{Terra} movements cancelled when passing from R^{Solis} to R^{Terra}

With $[\omega_{Terra}]_{R^{Solis}} = -[\Omega_{Solis}]_{R^{Terra}}$, we get:

$$R^{Terra} = R^{Sol + Solis + \Omega_{Terra}}$$

$$R^{Terra + \Omega_{Solis}} = R^{Solis + \Omega_{Terra}}$$

We see that this works for both the Earth and the Sun. This is desirable since there is no reason a priori to favour one celestial body more than another.

It is noted that the relationship $[\omega_{Terra}]_{R^{Solis}} = -[\Omega_{Solis}]_{R^{Terra}}$ can remind a source field relation, with $[\omega_{Terra}]_{R^{Solis}}$ the rotation on itself that would play something local and $[\Omega_{Solis}]_{R^{Terra}}$ the orbital rotation that would play something extensive.

VI.3.2 Combination of the 2 tracks

If we combine this second track, change of points of view, with the table of analogies between Fluid Mechanics and Magnetostatics proposed in the first track, we get:

	Fluid mechanics	Magnetostatics
Source	$\vec{\omega}(M)$	$\vec{j}(M)$
Constant	2	μ_0
Field	$\vec{V}(M)$	$\vec{B}(M)$
Condition of source conservation	$div 2\vec{\omega}=0$	$div \mu_0 \vec{j}=0$
Source field relationship	$2\vec{\omega} = r \partial_t \vec{V}$	$\mu_0 \vec{j} = rot \vec{B}$
Ampère's theorem	$\iint_S 2\vec{\omega} \cdot d\vec{S} = \oint_{\Gamma} \vec{V} \cdot d\vec{l}$	$\iint_S \mu_0 \vec{j} \cdot d\vec{S} = \oint_{\Gamma} \vec{B} \cdot d\vec{l}$

- with $-\vec{\omega}(M)$ the spin rotation of the body studied (localized in O), measured by the other bodies (located in M), that is to say in the reference frame of these other bodies.
- and with $\vec{V}(M)$ the speed of the other bodies at the distance OM , in the reference frame of the body studied (located in O).

VI.4 Analogies between Gauss's theorem and Earth-Sun or Earth-Star systems

VI.4.1 Intuitively understand the transition from local to extended

We always try to understand in an intuitive way the Ampère's theorem, and the one we propose as its analogue (with permutation of moments and positions), the Gauss's theorem.

We always pursue the track of a relationship between a source (localized) and a field (extended), corresponding to a change of views between a spin rotation (localized) and an orbital rotation (extended).

We will now focus on a qualitative comparison between Gauss's theorem and Earth Sun or Earth Star systems.

VI.4.2 Study of spherical or cylindrical symmetry sources

Spherical symmetry (3 dimensions)

Let be the case of a finite source, with spherical symmetry of radius R in Space, of volumetric density ρ^t (of electric charge or mass) uniformly distributed (e.g., a planet of uniformly distributed mass volumetric density).

Applying Gauss's theorem, we have:

- first a field (electrostatic $Es(r)$ or gravitational $Gr(r)$ field), inside the source (electric or mass source) $r \leq R$, which increases in $\frac{K\rho^t}{3} \times r$ with K the constant function of the nature of the field,
- then an (electrostatic or gravitational) field outside the source $r \geq R$, which decreases in $\frac{K\rho^t R^3}{3} \times \frac{1}{r^2}$. We also have a constant flow of the field through spherical surfaces of $4\pi r^2$, and a zero limit of the field at infinity.

Cylindrical symmetry (2 dimensions +1)

Let be the case of a source with cylindrical symmetry of radius R and infinite height $h \rightarrow +\infty$ (In a way, the passage of the previous case from 3 to 2 dimensions), of the same uniformly distributed volumetric density ρ^t (electric charge or mass).

Applying Gauss's theorem, we have:

- first a field (electrostatic $Es(r)$ or gravitational $Gr(r)$ field), inside the source (electric or mass source) $r \leq R$, which increases in $\frac{K\rho^t}{2} \times r$,
- then an (electrostatic or gravitational) field outside the source $r \geq R$, which decreases in $\frac{K\rho^t R^2}{2} \times \frac{1}{r}$. We also have a constant flow of the field through cylindrical surfaces of $2\pi r \times h$, and a zero limit of the field at infinity.

We summarize in the following table these 2 cases, as well as the relationships and analogies that can be used in the following pages:

	Fluid mechanics (rotational, permanent, incompressible)	Magnetostatics	Electrostatics	Gravitation
Source	$\vec{\omega}^{x,y,z}(M)$ Vortex vector	$\vec{j}^{x,y,z}(M)$ Current volumetric density	$\rho^t(M)$ Electrical volumetric charge density	$\rho^t(M)$ Mass volumetric density
Constant	2	μ_0	$\frac{1}{\epsilon_0}$	$-4\pi G$
Field (with cylindrical source)	$\vec{V}_{rz}^\theta(M)$	$\vec{B}_{rz}^\theta(M)$	$\vec{E}s_{rt}^\theta(M)$	$\vec{G}r_{rt}^\theta(M)$
Study space	x, y, z	x, y, z	x, y, t	x, y, t
Source conservation	$\text{div}\vec{\omega}=0$	$\text{div}\mu_0\vec{j}=0$	$\frac{\partial\rho^t}{\partial t}=0$	$\frac{\partial\rho^t}{\partial t}=0$
Source field relationship	$2\vec{\omega} = r\partial t\vec{V}$	$\mu_0\vec{j} = \text{rot}\vec{B}$	$\frac{\vec{\rho}^t}{\epsilon_0} = \frac{\partial\vec{E}s_{xt}^y}{\partial x}$	$-4\pi G\vec{\rho}^t = \frac{\partial\vec{G}r_{xt}^y}{\partial x}$
Ampère's theorem	$2 \iint_{S_{xy}} \vec{\omega}_{xy}^z \cdot d\vec{S}_{xy}^t$ $= \oint_{\Gamma y} \vec{V}_{xt}^y \cdot d\vec{y}$	$\mu_0 \iint_{S_{xy}^z} \vec{j}_{xy}^z \cdot d\vec{S}_{xy}^t$ $= \oint_{\Gamma y} \vec{B}_{xt}^y \cdot d\vec{y}$	$\frac{1}{\epsilon_0} \iint_{S_{xy}} \vec{\rho}_{xy}^t \cdot d\vec{S}_{xy}^t$ $= \oint_{\Gamma y} \vec{E}s_{xt}^y \cdot d\vec{y}$	$-4\pi G \iint_{S_{xy}} \vec{\rho}_{xy}^t$ $\cdot d\vec{S}_{xy}^t = \oint_{\Gamma y} \vec{G}r_{xt}^y \cdot d\vec{y}$
Gauss's theorem (non-redirectioned fields)			$\frac{1}{\epsilon_0} \iiint_V \rho^t dV$ $= \oiint_\Sigma \vec{E}s \cdot d^2\vec{S}$	$-4\pi G \iiint_V \rho^t dV$ $= \oiint_\Sigma \vec{G}r \cdot d^2\vec{S}$
Law of Biot and Savart	$\vec{V}(OM)$ $= \frac{1}{2\pi} \iiint_{Vol} \frac{\vec{\omega}d\tau \wedge \vec{O}}{OM^3}$	$\vec{B}(OM)$ $= \frac{\mu_0}{4\pi} \iiint_{Vol} \frac{\vec{j}d\tau \wedge \vec{O}M}{OM^3}$	$\vec{E}s(OM)$ $= \frac{1}{4\pi\epsilon_0} \iiint_{Vol} \frac{\vec{\rho}^td\tau \wedge \vec{O}}{OM^3}$	$\vec{G}r(OM)$ $= -G \iiint_{Vol} \frac{\vec{\rho}^td\tau \wedge \vec{O}M}{OM^3}$

Cylindrically symmetrically symmetric source of radius R and infinite height $r \leq R$	$V_{rz}^\theta(r) = \omega r$	$B_{rz}^\theta(r) = \mu_0 \frac{j r}{2}$ $B_{rz}^\theta(r) = \mu_0 \frac{I r}{2\pi R^2}$ Avec: $j = \frac{I}{\pi R^2}$	$ES_{rt}^\theta(r) = \frac{1}{\epsilon_0} \frac{\rho^t r}{2}$	$Gr_{rt}^\theta(r) = -4\pi G \frac{\rho^t r}{2}$
Cylindrically symmetrically symmetric source of radius R and infinite height $r \geq R$	$V_{rz}^\theta(r) = \frac{\omega R^2}{r}$	$B_{rz}^\theta(r) = \mu_0 \frac{j R^2}{2r}$	$ES_{rt}^\theta(r) = \frac{1}{\epsilon_0} \frac{\rho^t R^2}{r}$	$Gr_{rt}^\theta(r) = -4\pi G \frac{\rho^t R^2}{r}$
Spherically symmetrically symmetric source of radius R $r \leq R$			$ES_{rt}^{\theta\varphi}(r) = \frac{1}{\epsilon_0} \frac{\rho^t r}{3}$	$Gr_{rt}^{\theta\varphi}(r) = -4\pi G \frac{\rho^t r}{3}$
Spherically symmetrically symmetric source of radius R $r \geq R$			$ES_{rt}^{\theta\varphi}(r) = \frac{1}{\epsilon_0} \frac{\rho^t R^3}{3r^2}$	$Gr_{rt}^{\theta\varphi}(r) = -4\pi G \frac{\rho^t R^3}{3r^2}$

VI.4.3 Field within the source of uniformly distributed density, increasing in r

Let's take the example of a simplified Earth-Sun system, where the Earth no longer has an orbital rotation around the Sun and simply a spin rotation $-\vec{\omega}$ of 24 hours.

From Earth, an observer sees the Sun rotate with an orbital rotation $\vec{\Omega}$ of 24 hours. If other bodies, for example stars, are fixed with respect to the Sun, the terrestrial observer also sees them rotate with an orbital rotation. of 24 hours.

Note that this example corresponds *precisely* to the case of a source, with spherical or cylindrical symmetry, radius R in Space, with uniformly distributed mass volumetric density ρ^t , with $r \leq R$.

Indeed, we have:

- on the one hand, a uniformly distributed source $\vec{\omega}$, for example in a Space volume $Vol = \frac{4}{3}\pi R^3$ or $Vol = \pi R^2 \times h$. The source $\vec{\omega}$ is **uniformly distributed** within this volume Vol , because all the stars in this volume see the Earth rotate on itself with the **same spin rotation** $-\vec{\omega}$,
- on the other hand, a field \vec{V} which increases in r inside the source $\vec{\omega}$ uniformly distributed over the volume Vol .

Fluid mechanics (rotational, permanent, incompressible)	
Source field relationship	$2\vec{\omega} = r\vec{\partial}_t\vec{V}$
Condition	$div\vec{\omega} = 0$
	If the source $\vec{\omega}$ is uniformly distributed across a Space volume Vol , the field \vec{V} is calculated in this volume Vol :

	$\vec{V} = \vec{\omega} \wedge \vec{OM}$
--	--

- with $-\vec{\omega}(M)$ the spin rotation of the Earth (located in O), measured by the stars (located in M), that is to say in the reference frame of these stars.
- and with $\vec{V}(M)$ the speed of stars at distance OM , in the reference frame of the Earth.

It should be noted that the spatial location of the source $\vec{\omega}(M)$ does not correspond to the Earth, but to the sphere of $Vol = \frac{4}{3}\pi R^3$ (or to the cylinder of $Vol = \pi R^2 \times h$), within which the stars (of position M) see the Earth turn (of position O) with the same spin rotation $-\vec{\omega}(M)$. Even if the source is localized, it is a priori more extensive than the body studied (here the Earth).

Conclusion of the paragraph

More generally, let us remember that the source $\vec{\omega}(M)$ **uniformly distributed** in a volume Vol corresponds to a **common reference frame**, within which all studied bodies of position M , measure for the reference body located in O (for example an observer O), the same spin rotation $-\vec{\omega}(M)$.

For his part, the observer O measures for all studied bodies within Vol , a velocity field $\vec{V}(M)$ responding to the source field equation $2\vec{\omega}(M) = r\partial t\vec{V}(M)$ (we even have $\vec{V} = \vec{\omega} \wedge \vec{OM}$).

VI.4.4 Field outside the source, with a zero limit to infinity

We now wish to continue the analogy with Gauss's theorem, to get out of the source $\vec{\omega}$ of volume Vol and have a field \vec{V} who:

- on the one hand, decreases in Space by moving away from the source,
- on the other hand, has a zero limit to infinity.

Let's take the example of a simplified Earth-Sun system, where the Earth no longer has an orbital rotation around the Sun and simply a spin rotation $-\vec{\omega}$ of 24 hours. Previously, it had been proposed that the other bodies in the Universe, called stars, are fixed with respect to the Sun, and therefore also see the Earth with a spin rotation. $-\vec{\omega}$ of 24 hours.

Now propose that stars are not fixed with respect to the Sun (more realistic case). Nevertheless, let us remain in a simplified case where the Earth, in relation to these stars, has no orbital rotation, but simply a spin rotation.

At a very distant distance from the Earth (tending towards infinity), what will be for these stars the spin rotations $-\vec{\omega}_1, \dots, -\vec{\omega}_m, \dots, -\vec{\omega}_n, \dots$ of the Earth? They are likely to be all possibilities. The spin rotation $-\vec{\omega}_m$ will for example be in a sense and $-\vec{\omega}_n$ in another sense. For the terrestrial observer, these stars will have a zero average orbital rotation $\vec{\Omega}$ and therefore a zero average speed (and also a field) $\vec{V} = \vec{\Omega} \wedge \vec{r}$.

We have the somewhat paradoxical idea, but intuitively understandable, that the terrestrial reference frame (that is to say, the reference frame where the Earth is fixed) is both in itself and at infinity.

Qualitatively, we are therefore similar here to Gauss's theorem, case of a source with spherical or cylindrical symmetry of radius R in Space, of uniformly distributed volumetric density ρ^t din the source, with $r \geq R$, and a field \vec{V} which is infinitely zero.

VI.4.5 Field outside source, decay

Let now be a Star very far from the Sun, which sees the Sun rotate with a spin rotation of $-\vec{\omega}_{E-S}$. The Sun itself sees the Earth rotate with a spin rotation of $-\vec{\omega}_{S-T}$.

If the distance Earth to Sun is negligible compared to the distance Sun Star and if the spin rotations are all in the same plane, the Star sees the Earth rotate with a spin rotation of $-\vec{\omega}_{E-T} = -\vec{\omega}_{E-S} - \vec{\omega}_{S-T}$.

In the case that $-\vec{\omega}_{E-S}$ is opposite sense to $-\vec{\omega}_{S-T}$, the terrestrial observer sees the Star rotate with an orbital rotation $\vec{\Omega}_{E-T}$ lower than that of the Sun $\vec{\Omega}_{S-T}$.

Moving away from Earth, it is likely that stars see Earth with spin rotations. $-\vec{\omega}_1, \dots, -\vec{\omega}_n$ more and more divergent.

For the terrestrial observer, these stars have an average orbital rotation $\vec{\Omega}$ which is decreasing.

To qualitatively find Gauss's theorem, it is necessary for the terrestrial observer, that the stars have an average orbital rotation $\vec{\Omega}$ which decreases in $\frac{1}{r^3}$ (or $\frac{1}{r^2}$ in 2 dimensions of Space) and a field $\vec{V} = \vec{\Omega} \wedge \vec{r}$ which decreases in $\frac{1}{r^2}$ (or $\frac{1}{r}$ in 2 dimensions of Space).

In other words, what are the characteristics to attribute to a Space, to find these laws of decay of the field?

VI.5 Conclusion of the chapter

We proposed 2 tracks that could lead to a better understanding of source field equations and theorems of Ampère and Gauss. One insists on the interchangeability between source and field, the other on a difference in points of view to move from local to extent.

By combining the 2 tracks, it was found that a vortex vector source $\vec{\omega}(M)$ **uniformly distributed** in a volume Vol , corresponds to a **common reference frame** of volume Vol , within which all the bodies studied of position M , measure for the reference body located in O (for example an observer O), the same spin rotation $-\vec{\omega}(M)$.

For its part, the observer O measures for all the studied bodies within Vol , a velocity field $\vec{V}(M)$ responding to the source field equation $2\vec{\omega}(M) = r\vec{\partial}t\vec{V}(M)$.

The target (not met here) is to broaden the demonstration of the relationship $2\vec{\omega}(M) = r\vec{\partial}t\vec{V}(M)$ to any sources (and then to the types of reference frames to be matched with those sources) and to the bodies M outside the source.

In the next chapter, we will focus on homogeneous field equations, i.e., equations involving only fields.

Note on the principle of least action

As mentioned in Memoir 1, Joseph-Louis Lagrange proposed in 1788 to demonstrate the fundamental principle of Newton's dynamics from the principle of least action. This could also be a way to demonstrate source field equations.

Chapter VII Gravitationally analogous to Maxwell Faraday's equation?

Purpose of the chapter

Previously, analogues for gravitation to Maxwell's source field equations were proposed. Now let's study what could be the gravitational analogue of Maxwell Faraday's field field equation.

VII.1 On the phenomena of electrical induction

VII.1.1 Historical reminders

Mr. Faraday's main experiments took place in the 1830s. As A.-M. Ampere, M. Faraday is inspired by the experiences of H. C. Ørsted. Nevertheless, he works on another branch of magnetism, that of electromagnetic induction phenomena. A.-M. Ampère studied how a direct electric current (the *source*) can generate a magnetic field like a magnet. By contrast, Faraday studied how a magnetic field can induce an electric current in a circuit, (in relation to an electric *field*).

Initially, Mr. Faraday studied continuous currents in time. He made rapid progress when he extended his work to variable currents in time, which could generate variable magnetic fields in time.

Its fundamental idea is that a variation in time of the flow of a magnetic field through an electric circuit generates in this circuit the appearance of an electromotor field (or an *e.m.f.* electromotive force of induction) and therefore of an induced electric current.

Mr. Faraday's work is at the origin of the electric generator (dynamo) and the electric motor. They allow the conversion of mechanical energy into electrical energy, and vice versa.

For example, a metal bar sliding on two rails generates a variation in Time of the magnetic field flux through the electrical circuit consisting of the two rails, the movable bar and a fixed bar. This induces the appearance in the circuit of an induction electromotive force *e.m.f.* and an induced electric current.

VII.1.2 Faraday's Law

We recall Faraday's law, a mathematical translation of his ideas:

$$e.m.f. = - \frac{d\Phi}{dt}$$

The *e.m.f.* is the electromotive force that sets in motion the electrical charges of the circuit. Contrary to its name, it is not a force since it is homogeneous at a voltage (i.e., at a difference in electrical potentials) and is expressed in volts. If it is multiplied by an electric charge q^t , it can be likened to the work of an electric force.

Note on the law of moderation

We often talk about Lenz Faraday's law of moderation, enunciated by Heinrich Lenz in 1834 following the work of Mr. Faraday. According to this law, the direction of the current induced in the circuit is such that by its effects, it opposes the variation of the magnetic flux which gave rise to it.

In the case of a closed circuit, purely resistive, of total resistance R , we have the current:

$$i = \frac{|e.m.f.|}{R} = \frac{d\Phi}{Rdt}$$

To find the direction, sense of the current, Lenz Faraday's law of moderation is used. If we take the example of a metal bar sliding on two rails, the current $i = qv$ passing through the movable bar

creates a Lorentz magnetic force $\vec{F}_{Lor} = q\vec{v} \wedge \vec{B}$. The direction, sense of the current is determined by the direction, sense of the magnetic force, which must oppose the force moving the movable bar (for example, the weight of the movable bar).

VII.1.3 Neuman's case and Lorentz's case

In the phenomena of magnetic induction, we usually distinguish two cases:

- **the case of Neuman**, a fixed electrical circuit in a variable in Time field $\vec{B}_{xy}^{1/t}$,
- **the case of Lorentz**, displacement of the electrical circuit in a stationary field $\vec{B}_{xy}^{1/t}$ (magnetic field independent of Time).

In the case of Neuman, we have:

$$e.m.f. = \oint_{circuit} \frac{\partial A^x}{\partial t} \cdot d\vec{x} = -\frac{d\Phi}{dt}$$

Φ is the magnetic flux through the circuit.

In the case of Lorentz, we have:

$$e.m.f. = \oint_{circuit} (\vec{v}^y \wedge \vec{B}_{xy}^t)_{ty}^x \cdot d\vec{x} = -\frac{d\Phi_c}{dt}$$

Φ_c is the total magnetic flux cut by the circuit as it moves.

If the closed filiform circuit undergoes deformation without there being a discontinuity of velocity (without switching), one has $d\Phi_c = d\Phi$ since the flow of \vec{B}_{xy}^t is conservative. The *e.m.f.* induced then has the same expression in both cases:

$$e.m.f. = -\frac{d\Phi}{dt}$$

In **the case of Lorentz**, we take into account an electromotor field:

$$\vec{E}m_{ty}^x = (\vec{v}^y \wedge \vec{B}_{xy}^t)_{ty}^x$$

with \vec{v}^y the velocity of movement of the electrical circuit.

We have the electric force defined from this electromotor field:

$$\vec{F}_{El_{ty}}^x = q^t (\vec{E}m)_{ty}^x = (q^t \vec{v}^y \wedge \vec{B}_{xy}^t)_{ty}^x$$

In **the case of Neuman**, we take into account an electric field of induction:

$$\vec{E}i_{tx}^y = \left(\frac{\partial A^x}{\partial t}\right)_{tx}^y$$

translation of the variation of the magnetic field in Time.

As we reorient in this essay the electric induction field $\vec{E}i$, and not the electromotor field $\vec{E}m$ (which retains the same direction as the electric force \vec{F}_{El}), $\vec{E}i$ and $\vec{E}m$ do not have the same direction.

In the case of Neuman, we define an electromotive field:

$$\vec{E}m_{ty}^x = (\vec{u}^t \wedge \vec{E}i_{tx}^y)_{ty}^x = \vec{u}^t \wedge \left(\frac{\partial A^x}{\partial t}\right)_{tx}^y$$

with \vec{u}^t unit vector directed in Time.

We have the electric force defined from the electromotor field $\vec{E}m$ or from the electric field of induction $\vec{E}i$:

$$\vec{F}_{El_{ty}}^x = q^t (\vec{E}m)_{ty}^x = \vec{q}^t \wedge \vec{E}i_{tx}^y = \vec{q}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y$$

Reunion of the two cases Lorentz and Neuman

If we combine the two cases, we have an electromotive field:

$$\vec{E}m_{ty}^x = (\vec{v}^y \wedge \vec{B}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y)_{ty}^x$$

We have the electric force defined from this electromotor field:

$$\vec{F}_{El_{ty}}^x = q^t (\vec{E}m)_{ty}^x = q^t (\vec{v}^y \wedge \vec{B}_{xy}^{1/t} + \vec{u}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y)_{ty}^x$$

VII.1.4 Maxwell Faraday equation

We remind Maxwell Faraday's equation in integral form:

$$f.e.m. = \oint_{\Gamma^x} \vec{E}m_{ty}^x \cdot d\vec{x} = \iint_{S_{xy}} \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t} \cdot d\vec{S}_{xy}^t = -\frac{d\Phi}{dt}$$

with Γ^x a closed contour and S_{xy} a surface.

Maxwell Faraday is recalled locally:

$$rot(\vec{E}m_{ty}^x) = \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t}$$

Note

Classically we have $rot(\vec{E}l) = \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t}$. But as we reorient the electric field $\vec{E}l$ and not the electromotor field $\vec{E}m$, we use in Maxwell Faraday (in local form) the electromotor field (which retains the same direction as the electric force).

Previously, we proposed analogues for gravitation to Maxwell's electromagnetic source field equations. Now let's see what could be the analogue for gravitation to Maxwell Faraday's field field equation. To do this, let's take a closer look at this equation.

VII.2 Maxwell Faraday's equation transcribed from electromagnetism to gravitation

VII.2.1 From an integrated electromotor field on a closed contour to a potential difference

In Maxwell Faraday in integral form, two parts can be distinguished.

A first part that makes the link between the induced *e. m. f.* and the electromotive field $\vec{E}m_{yt}^x$, that is:

$$e.m.f. = \oint_{\Gamma^x} \vec{E}m_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{v}^y \wedge \vec{B}_{xy}^{1/t} + \vec{u}^t \wedge \left(\frac{\partial \vec{A}^x}{\partial t}\right)_{tx}^y)_{ty}^x \cdot d\vec{x}$$

A second part that makes the link between the electromotor field $\vec{E}m_{yt}^x$ and the magnetic part, with the variation of the magnetic flux, i.e.:

$$\oint_{\Gamma^x} \vec{E} m_{yt}^x \cdot d\vec{x} = \iint_{S_{xy}} \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t} \cdot d\vec{S}_{xy}^t = -\frac{d\Phi}{dt}$$

Note 1

Integration on a closed contour Γ^x reflects the transition from an electromotor field to a difference in electric potentials.

If we multiply by a electric charge q^t , the integration on a closed contour Γ^x translates the transition from an electric force to an electrical energy.

Note 2

Note that a Coulomb electrostatic field of the type $(\vec{u}^t \wedge \vec{E}_{S_{tx}^{1/y}}^x)_{ty}^x$ cannot induce a *e. m. f.* in a closed contour. We have:

$$f. e. m. = \oint_{\Gamma^x} (\vec{u}^t \wedge \vec{E}_{S_{tx}^{1/y}}^x)_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{u}^t \wedge \frac{\partial \vec{A}^t}{\partial x})_{ty}^x \cdot d\vec{x} = \vec{u}^t \wedge (\vec{A}^t(M) - \vec{A}^t(M)) = 0$$

Only the terms $(\vec{v}^y \wedge \vec{B}_{xy}^{1/t})_{ty}^x$ and $(\vec{u}^t \wedge (\frac{\partial \vec{A}^x}{\partial t})_{tx}^y)_{ty}^x$ can induce *e. m. f.* in a closed contour.

For an electric field $\vec{E}_{tx}^{1/y} = \frac{\partial \vec{A}^x}{\partial t} - \frac{\partial \vec{A}^t}{\partial x}$, we have:

$$f. e. m. = \oint_{\Gamma^x} (\vec{u}^t \wedge \vec{E}_{tx}^{1/y})_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{u}^t \wedge (\frac{\partial \vec{A}^x}{\partial t} - \frac{\partial \vec{A}^t}{\partial x}))_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{u}^t \wedge \frac{\partial \vec{A}^x}{\partial t})_{ty}^x \cdot d\vec{x}$$

The electric potential \vec{A}^t does not intervene, only the magnetic potential is taken into account \vec{A}^x .

VII.2.2 Analogy of electromagnetism and gravitation

We have the Coriolis acceleration:

$$\vec{a}_{Cor_{ty}}^x = -(\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x$$

Similarly, we have the Einsteinian acceleration (acceleration cancelled during a change of reference frames):

$$\vec{a}_{Ein_{ty}}^x = (\vec{u}^t \wedge \frac{\partial \vec{V}^x}{\partial t})_{ty}^x$$

with \vec{u}^t unit vector directed in Time.

Note on $\frac{\partial \vec{V}^x}{\partial t}$

In the Gauss gravitational source field equation, we add an additional term: $\frac{\partial \vec{V}^x}{\partial t}$ compared to the "usual" Gauss gravitation equation. We have introduced this additional term $\frac{\partial \vec{V}^x}{\partial t}$ as an acceleration cancelled when changing reference frame from R to R^K . It is noted that it is also analogous, for electromagnetism, to the electric field of induction $\vec{E}_{tx}^y = (\frac{\partial \vec{A}^x}{\partial t})_{tx}^y$.

We have Maxwell Faraday in integral form:

$$\oint_{\Gamma^x} \vec{E} m_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} ((\vec{v}^y \wedge \vec{B}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \frac{\partial \vec{A}^x}{\partial t})_{ty}^x) \cdot d\vec{x} = \iint_{S_{xy}} \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t} \cdot d\vec{S}_{xy}^t = \frac{-d\Phi_B}{dt}$$

with Φ_B the magnetic field flux.

Its analogue for gravitation is:

$$\begin{aligned} \oint_{\Gamma^x} (-\vec{a}_{Cor_{ty}}^x + \vec{a}_{Ein_{ty}}^x) \cdot d\vec{x} &= \oint_{\Gamma^x} ((\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \frac{\partial \vec{V}^x}{\partial t})_{ty}^x) \cdot d\vec{x} = \iint_{S_{xy}} \frac{\partial 2\vec{\Omega}_{xy}^{1/t}}{\partial t} \cdot d\vec{S}_{xy}^t \\ &= \frac{2d\Phi_\Omega}{dt} \end{aligned}$$

with Φ_Ω the pulsation field flow.

Note 1

There is no sign here – here in $\frac{2d\Phi_\Omega}{dt}$. We'll see why in the next paragraph.

Note 2

By integrating on a closed contour Γ^x , we have the passage from an acceleration to a difference of Newtonian potentials:

$$\Delta V^t = \oint_{\Gamma^x} (-\vec{a}_{Cor_{ty}}^x + \vec{a}_{Ein_{ty}}^x) \cdot d\vec{x} = \oint_{\Gamma^x} ((\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \frac{\partial \vec{V}^x}{\partial t})_{ty}^x) \cdot d\vec{x}$$

If we multiply the above equation by a mass m^t , Integration on a closed contour Γ^x translates the transition from a gravitational force to a gravitational energy.

Note 3

Note that the Newtonian gravitational force $\vec{F}_{Gr_{ty}}^x = \vec{m}^t \wedge (\frac{\partial V^t}{\partial x})_{tx}^y$ cannot produce a difference in Newtonian potentials in a closed contour. We have:

$$\Delta V^t = \oint_{\Gamma^x} (\vec{u}^t \wedge \frac{\partial \vec{V}^t}{\partial x})_{ty}^x \cdot d\vec{x} = \vec{u}^t \wedge (V^t(M) - V^t(M)) = 0$$

Only the terms $(\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x$ and $(\vec{u}^t \wedge \frac{\partial \vec{V}^x}{\partial t})_{ty}^x$ can produce a difference in Newtonian potentials in a closed contour.

For a wave vector field $\vec{K}_{tx}^{1/y} = \frac{\partial \vec{V}^x}{\partial t} - \frac{\partial \vec{V}^t}{\partial x}$, we have:

$$\Delta V^t = \oint_{\Gamma^x} (\vec{u}^t \wedge \vec{K}_{tx}^{1/y})_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{u}^t \wedge (\frac{\partial \vec{V}^x}{\partial t} - \frac{\partial \vec{V}^t}{\partial x}))_{ty}^x \cdot d\vec{x} = \oint_{\Gamma^x} (\vec{u}^t \wedge \frac{\partial \vec{V}^x}{\partial t})_{ty}^x \cdot d\vec{x}$$

Newtonian potential V^t does not intervene, only the velocity potential V^x is taken into account.

VII.2.3 Changes in reference frames, understand Faraday gravitation

We recognize in $\vec{a}_{Ein_{ty}}^x - \vec{a}_{Cor_{ty}}^x$ the terms of acceleration when changing the reference frames from R^0 to R^Ω .

We recall the decomposition of an acceleration:

$$[\vec{a}]_{R^0} = \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{O}M + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) + 2\vec{\Omega} \wedge \vec{v}_r$$

with $\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$ the centrifugal inertial acceleration and $2\vec{\Omega} \wedge \vec{v}_r$ the Coriolis acceleration.

The Einsteinian acceleration corresponds to the acceleration cancelled when changing reference frames from R^0 to R^Ω , i.e., the difference between the initial acceleration $[\vec{a}]_{R^0}$ and the remaining acceleration $\left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega}$.

We have:

$$\vec{a}_{Ein} = [\vec{a}]_{R^0} - \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega}$$

Without explaining it physically, we note that we must also consider the centrifugal inertial acceleration $\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$ as a remaining acceleration. We get:

$$\vec{a}_{Ein} = [\vec{a}]_{R^0} - \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} - \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$$

Note

It is a little easier to understand why there is no explicit equivalent to centrifugal inertia acceleration $\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$ in electromagnetism. This equivalent must be implicitly included in the electric field of induction $\vec{E}_{tx}^y = \left(\frac{\partial A^x}{\partial t} \right)_{tx}^y$, as it is here included in the Einsteinian acceleration.

So, we have:

$$\vec{a}_{Ein_{ty}}^x = \vec{a}_{cor_{ty}}^x + \frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{O}M$$

$$\vec{a}_{Ein_{ty}}^x - \vec{a}_{cor_{ty}}^x = \frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{O}M$$

We have the velocity potential vector defined from the pulsation field:

$$\vec{V} = \vec{\Omega}^t \wedge \vec{O}M$$

We have:

$$2\vec{\Omega} = r\vec{\partial}t\vec{V} = r\vec{\partial}t(\vec{\Omega}^t \wedge \vec{O}M)$$

Deriving with respect to time:

$$\frac{\partial 2\vec{\Omega}^t}{\partial t} = \frac{\partial r\vec{\partial}t(\vec{\Omega}^t \wedge \vec{O}M)}{\partial t}$$

$$\frac{\partial 2\vec{\Omega}^t}{\partial t} = r\vec{\partial}t\left(\frac{\partial \vec{\Omega}^t}{\partial t} \wedge \vec{O}M\right)$$

Based on Stokes' theorem along a closed path Γ^x delimiting a surface S_{xy} , we have:

$$\oint_{\Gamma^x} \left(\frac{\partial \vec{\Omega}^t}{\partial t} \wedge \vec{O}M \right) \cdot d\vec{x} = \iint_{S_{xy}} r \vec{\partial}t \left(\frac{\partial \vec{\Omega}^t}{\partial t} \wedge \vec{O}M \right) \cdot d\vec{S}_{xy}^t = \iint_{S_{xy}} \frac{\partial 2\vec{\Omega}_{xy}^t}{\partial t} \cdot d\vec{S}_{xy}^t = \frac{2d\Phi_\Omega}{dt}$$

We therefore find:

$$\Delta V^t = \oint_{\Gamma^x} (\vec{a}_{Einty}^x - \vec{a}_{Corty}^x) \cdot d\vec{x} = \oint_{\Gamma^x} \left(\frac{\partial \vec{\Omega}^t}{\partial t} \wedge \vec{O}M \right) \cdot d\vec{x} = \iint_{S_{xy}} \frac{\partial 2\vec{\Omega}_{xy}^t}{\partial t} \cdot d\vec{S}_{xy}^t = \frac{2d\Phi_\Omega}{dt}$$

We have Maxwell Faraday's analogue for gravitation:

$$\Delta V^t = \oint_{\Gamma^x} (\vec{a}_{Einty}^x - \vec{a}_{Corty}^x) \cdot d\vec{x} = \frac{2d\Phi_\Omega}{dt}$$

This last equation will be called Faraday gravitation.

Note 1

For a good analogy between gravitation and electromagnetism, it is also necessary that $\vec{\Omega}_{xy}^t$ is conservative flow as \vec{B}_{xy}^t .

Note 2, look for examples of Faraday gravitation: where can there be a Newtonian potential greater than expected?

Maxwell Faraday can be presented as the generation of a difference in electric potentials from a variation in the flux of the magnetic field. Maxwell Faraday can be interpreted as obtaining an additional Coulomb electric potential, without having to go through electrostatic Maxwell Gauss.

Following an analogy between gravitation and electromagnetism, let's look for examples where Faraday gravitation could intervene, that is to say cases where we would have a Newtonian potential higher than expected if we simply used Gauss gravitation?

Note 3, "luminous mass" and "dynamic mass", the enigma of dark matter

In 1933, astronomer Fritz Zwicky studied a small group of 7 galaxies in the Berenice's Hair cluster. These galaxies are comparable to vast vortices composed of spirals or arms of stars that revolve around a particularly dense center of stars.

For each galaxy, Zwicky measured the distances of stars from the galactic center, their rotation periods around the galactic center, as well as their average velocities. From a formula inspired by Kepler's 3rd law $\frac{T^2}{a^3} = \frac{4\pi^2\lambda^2}{GM}$ with $\lambda = 1 + \frac{m}{M}$, with M the estimated mass of the galactic center and m the mass of the rotating star ($m \ll M$), the astronomer deduces the "dynamic mass" of the Galaxy.

F. Zwicky also measured brightness L emitted by the 7 galaxies, and through the relation of proportionality $L \propto M^3$, he obtained for each galaxy a "luminous mass".

The astronomer then notes that the speed of stars in the arms of galaxies is much higher than one would expect. Indeed, the dynamic mass is 400 times greater than the luminous mass.

In the 1970s, the American astronomer Vera Rubin took over the work of F. Zwicky using observation instruments with greater precision. She calculated the rotation speed of galaxy arms as a function of the distance to the galactic center. The rotation speed of a spiral galaxy is supposed to decrease away from the center, following a Keplerian decay.

However, like F. Zwicky's observations, V. Rubin noted that the stars on the periphery rotate far too fast. For example, for the Andromeda galaxy, the velocities remain virtually constant as one moves away from the center.

To explain these velocities in peripheries much higher than the Keplerian decay, it is necessary to postulate a Newtonian potential higher than expected. Indeed, a higher rotational speed implies a higher centrifugal inertial force, and therefore a higher Newtonian gravitational attractive force to balance the centrifugal inertial force, and therefore a higher Newtonian potential.

If we use "usual" Gauss gravitation (without the term $\frac{\partial \vec{v}^x}{\partial t}$), an additional Newtonian potential involves additional mass to generate this Newtonian potential.

Thus, following the work of F. Zwicky and V. Rubin, it is proposed the hypothesis of a dark matter, a halo of non-visible matter surrounding galaxies, a halo that would represent up to 90% of the mass of the galaxy. This dark matter would generate the additional Newtonian potential, which would explain the velocities of stars higher than expected Keplerian.

Another way to obtain an additional Newtonian potential would be to use Faraday gravitation, and in particular the term $\frac{\partial \vec{v}^x}{\partial t}$ which is also proposed to be added to Gauss gravitation. We can see the rotation vector (or pulsation field) $\vec{\Omega}$ of a galaxy as the analogue of the magnetic field \vec{B} , the arms of stars of a galaxy as the analogue of an electrical circuit delimiting a closed surface.

In this case, the variation in the shape of the arms of stars would produce a variation in the flux of the pulsation field. $\vec{\Omega}$ through the closed surface bounded by the arms and would induce the appearance of an additional difference in Newtonian potentials on the periphery of the galaxy (as there appears an additional difference of Coulombian electric potentials in electrical circuits during induction phenomena). Track ahead.

Note 4 on the proposed modification of Gauss gravitation

It is proposed in this essay to modify Gauss gravitation slightly, i.e., to modify the classical equation

$-4\pi G \vec{\rho}_{xy}^t = \frac{\partial \vec{G}_{xt}^{1/y}}{\partial x} - \frac{\partial \vec{G}_{yt}^{1/x}}{\partial y}$ to $-4\pi G \vec{\rho}_{xy}^t = \frac{\partial 2c^2 \vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial 2c^2 \vec{K}_{yt}^{1/x}}{\partial y}$, this amounts to introducing terms such as $\frac{\partial \vec{v}^x}{\partial t}$ et $\frac{\partial \vec{v}^y}{\partial t}$.

These terms that are added in Gauss gravitation are the analogues for electromagnetic induction of the Neuman case where an electromotive field appears: $\vec{E}m_{xt}^{1/y} = \frac{\partial \vec{A}^x}{\partial t}$, $\vec{E}m_{yt}^{1/x} = \frac{\partial \vec{A}^y}{\partial t}$ and an electromotive induction force appears: $e = \int \vec{E}m \cdot d\vec{l} = -\frac{d\Phi}{dt}$, with Φ the magnetic field flux.

VII.3 Tensor approach, Maxwell Faraday and Maxwell Thomson

VII.3.1 Maxwell Faraday electromagnetic tensor $\vec{\vec{F}}_{\mu\nu}$

We remind Maxwell Faraday's electromagnetic tensor:

$$\vec{\vec{F}}_{\mu\nu} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ \frac{El_{xt}}{c} & 0 & B_{xy} & B_{xz} \\ \frac{El_{yt}}{c} & B_{yx} & 0 & B_{yz} \\ \frac{El_{zt}}{c} & B_{zx} & B_{zy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{El_{tx}}{c} & \frac{El_{ty}}{c} & \frac{El_{tz}}{c} \\ -\frac{El_{tx}}{c} & 0 & B_{xy} & B_{xz} \\ -\frac{El_{ty}}{c} & -B_{xy} & 0 & B_{yz} \\ -\frac{El_{tz}}{c} & -B_{xz} & -B_{yz} & 0 \end{bmatrix}$$

We remind Maxwell Faraday and Maxwell Thomson given from the tensor $\vec{\vec{F}}_{\mu\nu}$:

$$\partial_\gamma F_{\alpha\beta} + \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} = 0$$

In space x, y, z , we have:

$$\frac{\partial F_{xy}}{\partial z} + \frac{\partial F_{yz}}{\partial x} + \frac{\partial F_{zx}}{\partial y} = 0$$

$$\frac{\partial B_{xy}}{\partial z} + \frac{\partial B_{yz}}{\partial x} + \frac{\partial B_{zx}}{\partial y} = 0$$

This corresponds to:

$$\text{div} \vec{B} = 0$$

In space x, y, ct , we have:

$$\frac{\partial F_{xy}}{\partial ct} + \frac{\partial F_{yt}}{\partial x} + \frac{\partial F_{tx}}{\partial y} = 0$$

$$\frac{\partial B_{xy}}{\partial ct} + \frac{\partial El_{yt}}{c \partial x} + \frac{\partial El_{tx}}{c \partial y} = 0$$

$$\frac{\partial B_{xy}}{\partial ct} + \frac{\partial El_{tx}}{c \partial y} - \frac{\partial El_{ty}}{c \partial x} = 0$$

This corresponds to:

$$\frac{\partial \vec{B}}{\partial t} + \text{rot} \vec{E}l = 0$$

$$\text{rot} \vec{E}l = \begin{pmatrix} \frac{\partial El_{tx}}{\partial y} \\ \frac{\partial El_{ty}}{\partial x} \\ 0 \end{pmatrix}$$

In plane x, y

VII.3.2 Gravitational tensor $\bar{\bar{D}}_{\mu\nu}$

We remind the gravitational tensor:

$$\bar{\bar{D}}_{\mu\nu} = \begin{bmatrix} 0 & K_{tx} & K_{ty} & K_{tz} \\ -K_{tx} & 0 & \frac{\Omega_{xy}}{c} & \frac{\Omega_{xz}}{c} \\ -K_{ty} & -\frac{\Omega_{xy}}{c} & 0 & \frac{\Omega_{yz}}{c} \\ -K_{tz} & -\frac{\Omega_{xz}}{c} & -\frac{\Omega_{yz}}{c} & 0 \end{bmatrix}$$

Following a strict analogy between the 2 tensors $\bar{\bar{F}}_{\mu\nu}$ and $\bar{\bar{D}}_{\mu\nu}$, we pose for the analogue of Maxwell Faraday and Maxwell Thomson:

$$\partial_\gamma D_{\alpha\beta} + \partial_\alpha D_{\beta\gamma} + \partial_\beta D_{\gamma\alpha} = 0$$

In space x, y, ct , we have:

$$\partial_{ct} D_{xy} + \partial_x D_{yt} + \partial_y D_{tx} = 0$$

$$\partial_{ct} \frac{\Omega_{xy}}{c} - \partial_x K_{ty} + \partial_y K_{tx} = 0$$

In the spaces x, z, ct and y, z, ct , we have:

$$\partial_{ct} \frac{\Omega_{xz}}{c} - \partial_x K_{tz} + \partial_z K_{tx} = 0$$

$$\partial_{ct} \frac{\Omega_{yz}}{c} - \partial_y K_{tz} + \partial_z K_{ty} = 0$$

We thus find Faraday gravitation.

In space x, y, z :

$$\partial_z D_{xy} + \partial_x D_{yz} + \partial_y D_{zx} = 0$$

$$\partial_z \Omega_{xy} + \partial_x \Omega_{yz} + \partial_y \Omega_{zx} = 0$$

$$\text{div} \vec{\Omega} = 0$$

We find the pulsation field with conservative flow. For a pulsation field, there can be no monopole.

VII.4 Conclusion of the chapter

In electricity, a variation in the flux of the magnetic field \vec{B} involves the appearance of a difference in electrical potentials in a closed contour. Similarly, for gravitation, if we follow a strict analogy with electricity, a variation in the flux of the pulsation field $\vec{\Omega}$ involves the appearance of a difference in Newtonian potentials in a closed contour.

To see if this can explain, in the Galaxies, the observed velocities of stars that do not respect the Keplerian expectation?

Chapter VIII Wave character of electric, magnetic, wave vector and pulsation fields

Purpose of the chapter

In previous chapters, analogues of Maxwell's equations for gravitation have been proposed. As was pointed out at the beginning of this Memoir, from Maxwell's equations, it is possible to find wave equations for electric and magnetic fields, and thus to propose electromagnetic waves that propagate at the speed c of light in vacuum.

This had also been one of the great contributions of J. C. Maxwell to bring together theoretically, in the 1860s, electromagnetism and optics. Almost two decades later, in 1888, Heinrich Hertz discovered electromagnetic waves in the air.

From the analogues of Maxwell's equations obtained previously, and always following a strict analogy with electromagnetism, we will propose in this chapter wave equations for the wave vector and pulsation fields. We will then obtain gravitational waves that also propagate at speed c in vacuum.

VIII.1 Electric field and wave vector field

Take the rotational of Maxwell Faraday equation, taking into account Maxwell Gauss and Maxwell Ampere, we obtain for the electric field $\vec{E}l_{xt}^{1/y}$, and by analogy for the wave vector field $\vec{K}_{xt}^{1/y}$, the wave equations in the table below:

	Electromagnetism	Gravitation
Wave equation of spatiotemporal fields in vacuum, absence of charge, current, mass and momentum densities	$\frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial t^2} = 0$	$\frac{\partial^2 \vec{K}_{xt}^{1/y}}{\partial x^2} - \frac{l_p^2 c}{\hbar G} \frac{\partial^2 \vec{K}_{xt}^{1/y}}{\partial t^2} = 0$
Constants	ε_0	$-\frac{1}{2\pi G}$
Constants	μ_0	$-\frac{2\pi l_p^2 c}{\hbar}$
Conditions on constants	$\mu_0 \varepsilon_0 = \frac{1}{c^2}$	$\frac{l_p^2 c}{\hbar G} = \frac{1}{c^2}$
Wave equation of spatiotemporal fields in vacuum	$\frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial x^2} - \frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial ct^2} = 0$	$\frac{\partial^2 \vec{K}_{xt}^{1/y}}{\partial x^2} - \frac{\partial^2 \vec{K}_{xt}^{1/y}}{\partial ct^2} = 0$
Wave equation of spatiotemporal fields in the presence of charge, current, mass and momentum densities	$\frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial x^2} - \frac{\partial^2 \vec{E}l_{xt}^{1/y}}{\partial ct^2} = \frac{1}{\varepsilon_0} \frac{\partial \vec{\rho}_{xy}^t}{\partial x} + \mu_0 \frac{\partial \vec{j}_{xt}^y}{\partial t}$	$\frac{\partial^2 c^2 \vec{K}_{xt}^{1/y}}{\partial x^2} - \frac{\partial^2 c^2 \vec{K}_{xt}^{1/y}}{\partial ct^2} = -2\pi G \frac{\partial \vec{\rho}_{xy}^t}{\partial x} - \frac{2\pi l_p^2 c}{\hbar} \frac{\partial \vec{p}_{xt}^y}{\partial t}$ $\frac{\partial^2 c^2 \vec{K}_{xt}^{1/y}}{\partial x^2} - \frac{\partial^2 c^2 \vec{K}_{xt}^{1/y}}{\partial ct^2} = -2\pi G \frac{\partial \vec{\rho}_{xy}^t}{\partial x} - \frac{c}{\hbar} \frac{\partial \vec{p}_{xt}^y}{\partial t}$

Wave equation of potentials	$\frac{\partial^2 A^t}{\partial x^2} - \frac{\partial^2 A^t}{\partial ct^2} = -\frac{\rho^t}{\varepsilon_0}$	$\frac{\partial^2 V^t}{\partial x^2} - \frac{\partial^2 V^t}{\partial ct^2} = 4\pi G \rho^t$
------------------------------------	--	--

Unit verification:

	$\frac{\partial^2 c^2 \vec{K}_{xt}^{1/y}}{\partial x^2}$	$\frac{2\pi l_p^2 c \partial \vec{p}_{xt}^y}{\hbar \partial t}$
Units	$\frac{1}{ms^2}$	$\frac{m^2 \times Kg \times s \times m \times m}{m^2 \times Kg \times s^2 \times m^3 \times s} = \frac{1}{ms^2}$

Nota

To obtain the wave equation of electromagnetic potentials, we must respect the condition of the Lorenz gauge:

$$\vec{\nabla} \cdot \vec{A}^{x,y,z} + \mu_0 \varepsilon_0 \frac{\partial A^t}{\partial t} = 0$$

VIII.2 Magnetic fields and pulsation

Take the rotational equation of Maxwell Ampère, taking into account Maxwell Thomson and Maxwell Faraday, we obtain for the magnetic field $\vec{B}_{xy}^{1/t}$, and by analogy for the pulsation field $\vec{\Omega}_{xy}^{1/t}$, the wave equations in the table below:

	Electromagnetism	Gravitation
Wave equation of spatial fields in vacuum, absence of charge, current, mass and momentum densities	$\frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial t^2} = 0$	$\frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{l_p^2 c}{G \hbar} \frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial t^2} = 0$
Constants	ε_0	$-\frac{1}{2\pi G}$
Constants	μ_0	$-\frac{2\pi l_p^2 c}{\hbar}$
Conditions on constants	$\mu_0 \varepsilon_0 = \frac{1}{c^2}$ $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	$\frac{l_p^2 c}{\hbar G} = \frac{1}{c^2}$ $c = \sqrt{\frac{\hbar G}{l_p^2 c}}$
Wave equation of spatial fields in vacuum	$\frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial ct^2} = 0$	$\frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial ct^2} = 0$
Wave equation of spatial fields in the presence of charge, current, mass and momentum densities	$\frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{B}_{xy}^{1/t}}{\partial ct^2} = \frac{\mu_0 \partial \vec{j}_{xt}^y}{\partial x}$	$\frac{\partial^2 c \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 c \vec{\Omega}_{xy}^{1/t}}{\partial ct^2}$ $= -\frac{2\pi l_p^2 c \partial \vec{p}_{xt}^y}{\hbar \partial x}$

		$\frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial ct^2} = -\frac{1}{\hbar} \frac{\partial \vec{p}_{xt}^y}{\partial x}$
--	--	--

Note on Klein Gordan's wave equation

From the wave equation:

$$\frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial ct^2} = -\frac{1}{\hbar} \frac{\partial \vec{p}_{xt}^y}{\partial x}$$

If we replace the source $\vec{p}_{xt}^y = \left(\frac{d\vec{p}^x}{dt}\right)_{xt}^y$ by the source $-\left(\frac{d\vec{m}^t}{dx}\right)_{xt}^y c^2 = \left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y c^2$ (that is, by a permutation of x and t), we obtain (further deriving the source by a partial derivative $\left(\frac{\partial \vec{m}^t}{\partial x}\right)_{tx}^y c^2$):

$$\frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial x^2} - \frac{\partial^2 \vec{\Omega}_{xy}^{1/t}}{\partial ct^2} = -\frac{1}{\hbar} \frac{\partial^2 \vec{m}^t c^2}{\partial x^2}$$

By replacing $\vec{\Omega}_{xy}^{1/t}$ by ψ any wave function:

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial ct^2} = \frac{1}{\hbar} \frac{\partial^2 m^t c^2}{\partial x^2} \psi$$

By expanding to the 3 dimensions of Space:

$$\Delta \psi - \frac{\partial^2 \psi}{\partial ct^2} = \frac{1}{\hbar} \Delta m^t c^2 \psi$$

We obtain a wave equation that is reminiscent of Klein Gordan's wave equation:

$$\Delta \psi - \frac{\partial^2 \psi}{\partial ct^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

VIII.3 Examples of solving wave equations

For an electric field $El_{xt}^{1/y}$, we have as a solution of the wave equation in vacuum, for example the following wave function:

$$El_{xt}^{1/y}(t, y) = El' \cos(\omega t - ky)$$

with $\frac{\omega}{k} = c$.

In this example, the electromagnetic wave propagates in the direction y.

Note 1 on longitudinal and transverse waves

We speak of a longitudinal wave when the deformation of the medium is in the same direction as the propagation of the wave. Sound is a good example.

We speak of a transverse wave when the deformation of the medium is in a direction perpendicular to the propagation of the wave. A wave or an electromagnetic wave (as it is usually presented), are good examples.

In this essay, by reorienting in a space x, y, t, the electric field $El_{xt}^{1/y}$ from direction x to direction y, one obtains for the electric field part, a longitudinal wave. Indeed, the deformations of the electric field are then in the direction of the propagation of the wave.

The electric force remains oriented according to x. We therefore preserve, for the electric force part, a transverse wave.

As for the magnetic field $B_{xy}^{1/t}$ which deforms following t , one can intuition for the magnetic field part, a wave that "propagates in Time" in the same direction as the deformations of the magnetic field, that is to say a wave "longitudinal in Time".

Note 2 on polarized waves

In the case of a polarized wave (types of rectilinear polarization, circular polarization, vertical rectilinear polarization, horizontal rectilinear polarization...), the type of polarization is usually indicated by the directions of the electric field. As it is proposed to reorient it, the polarization type is then indicated by the directions of the electric force to which an electric charge would be subjected.

By analogy between electromagnetism and gravitation, for a pulsation field $\Omega_{xy}^{1/t}$, we have as a solution of the wave equation in vacuum, for example the following wave function:

$$\Omega_{xy}^{1/t}(t) = \Omega' \cos(\omega t - ky)$$

with $\frac{\omega}{k} = c$.

This example can be interpreted as a "virtual carrousel", propagating in the y direction of Space, and possessing a velocity of rotation $\Omega_{xy}^{1/t}(t, y)$ varying sinusoidally according to the moments t and the positions y occupied by this "virtual carrousel".

If this "virtual carrousel" is stationary with respect to an observer, it responds to a standing wave function:

$$\Omega_{xy}^{1/t}(t) = \Omega_0 \cos(\omega t)$$

Physically, this "virtual carrousel" can be interpreted as motionless in Space relative to the observer, and possessing a velocity of rotation $\Omega_{xy}^{1/t}(t)$ sinusoidally varying over time.

Note on physical quantities both field or potential and wave function

In this chapter, we have studied physical quantities of the field or potential type, which under certain conditions, are also wave functions. In the next Memoir, we will study the Yukawa potential (describing nuclear interactions), which is also both potential and wave function.

VIII.4 Conclusion of the chapter

The different equations obtained show that the variations in Space and Time of a moving (accelerated) electric charge, generate an electromagnetic wave composed of an electric field and a magnetic field, a wave propagating at speed c in a vacuum.

Similarly, variations in Space and Time of an accelerated moving mass generate a gravitational wave composed of a wave vector field and a pulsation field, a wave propagating at speed c in a vacuum.

Light characterized by its velocity c in vacuum, is therefore a form of electromagnetic radiation, and also gravitational radiation.

Conversely, these electromagnetic and gravitational waves (that is, in the visible part of the spectrum, light) can generate in "antennas" accelerated movements of electric charges and masses, i.e., electric currents and mass currents.

Chapter IX Why 2 types of electric charges and only 1 of mass? Example of a simplified tidal model

Purpose of the chapter

We try to understand the differences that remain between electromagnetism and gravitation, in particular the existence in electromagnetism of two natures of electric charges, positive or negative, and the existence in gravitation of a single nature of mass, always positive. In other words, the existence in electricity of two natures of forces, attractive and repulsive, and the existence in gravitation of a single nature of force, always attractive.

To understand these differences between gravitation and electromagnetism, we will take as an example a static and simplified model of the tides.

IX.1 Analogies between electromagnetism and gravitation

IX.1.1 Why 2 types of charge and only 1 of mass?

In the introduction of this essay, we have pointed out the similarities between gravitation and electromagnetism. We also noted some differences. In Memoirs 2 and 3, we noted that most of these differences fade away, if the analogy is taken a little further than usual.

For example, in an electric field, the acceleration of a body is a function of its electric mass charge, whereas in a gravitational field, the acceleration of this body does not depend on its mass. If we push the analogy a little further, as we did in Memoir 2, in electromagnetism, the analogue of acceleration is not acceleration, but the induced field (or electromotor field). We can then say: in an electric field, the induced field of a body does not depend on its electric charge.

Similarly, in this Memoir 3, Maxwell's equations have been proposed for the gravitation of analogues. These analogues have allowed us to find wave equations for gravitation similar to those of electromagnetism.

Nevertheless, despite all these analogies, there remains an important difference for which no explanation has been provided. There are two natures of electric charge and one nature of mass. What for?

IX.1.2 The electron that goes back in Time

As often in this essay, we try to understand if the differences between electricity and gravitation, cannot be explained simply by changes of reference frames and by different movements in Space-Time?

In a way, we use the old "trick" of A. Einstein, and we take up the old lens of John Archibald Wheeler and Richard Feynman. In the 1940s, these two physicists had proposed to explain the difference in nature between the electron and the positron by a different movement in Time: the electron advances in Time for example in the same sense as we human observers, the positron advances in Time in the opposite sense to us.

This interpretation of the + and – charges provides a convincing and intuitive explanation for the annihilation between an electron and a proton. Indeed, this annihilation is interpreted as an electron making a U-turn in Time and turning into a positron. Positron and electron are therefore here the two faces of the same particle, differentiated by a reversal of their progress in Time (we will come back to this in the 6th memory).

Note

In the 1950s, J. A. Wheeler, continuing the work of A. Einstein on General Relativity, worked on a theory called Geometrodynamics. This theory was an attempt to describe Space-Time in terms of geometry and possibly motions, without recourse to the notion of matter (we also followed a similar

objective, when in the 2nd memory, it was proposed to interpret mass as an analogue in Time of velocity in Space).

Geometrodynamics is also related to super-substantialism, a doctrine according to which Space-Time is the only physical substance of the Universe. All physical properties then consist of properties, points or regions of Space-Time.

To return to our initial objective: to understand why 2 natures of charge and 1 only nature of mass, note that the idea of an electron that goes back in time, can not a priori be transposed to that of a mass that goes back in time. Indeed, there is no negative mass. The analogy of electromagnetism and gravitation does not work a priori here.

Nevertheless, in the next paragraphs, we will look at whether, pursuing more the analogy between gravitation and electromagnetism, it would not be possible to make it compatible with the idea of J. A. Wheeler and R. Feynman: that of an electron going back in Time?

First, let's do some reminders about dipole moments.

IX.2 Reminders on dipole moments and dipoles

IX.2.1 Dipole moment of an electrostatic dipole

An electrostatic dipole is defined by a zero-sum distribution of electric charges, with a barycentre of positive charges not coinciding with that of negative charges. The simplest dipole is a pair of charges of opposite sign, distant by a length $2a$. We have the following figure:

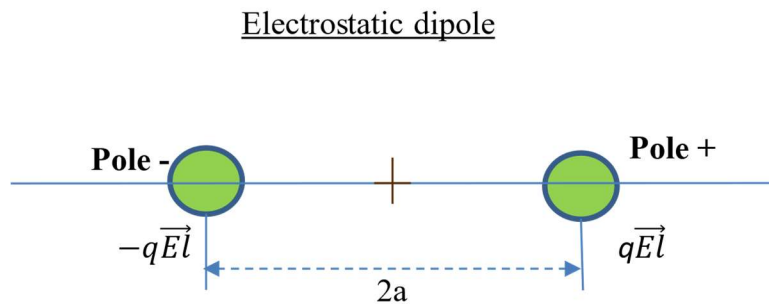


Figure 15: simplest electrostatic dipole

We usually define a dipole moment $\vec{\mu}$ of an electrostatic dipole, oriented from charge $-q$ toward charge $+q$ by the formula:

$$\vec{\mu} = 2q\vec{a}$$

To calculate the electrostatic field generated by this dipole, we frequently place ourselves in the case where $r = OM \gg a$ (with O the center of the dipole). We obtain simple radial and orthoradial components for the electrostatic field:

$$El_r = -\frac{\partial A^t}{\partial r} = \frac{1}{4\pi\epsilon_0} \frac{2\mu \cos\theta}{r^3}$$

$$El_\theta = -\frac{1}{r} \frac{\partial A^t}{\partial r} = \frac{1}{4\pi\epsilon_0} \frac{\mu \sin\theta}{r^3}$$

with $\theta = (\vec{\mu}, \overrightarrow{OM})$.

It is noted that for an electrostatic dipole, there is a faster decrease in the electrostatic field, in $\frac{1}{r^3}$, instead of $\frac{1}{r^2}$ for a point charge.

Electrostatic field lines answer the equation:

$$r = K \sin^2 \theta$$

We obtain field lines:

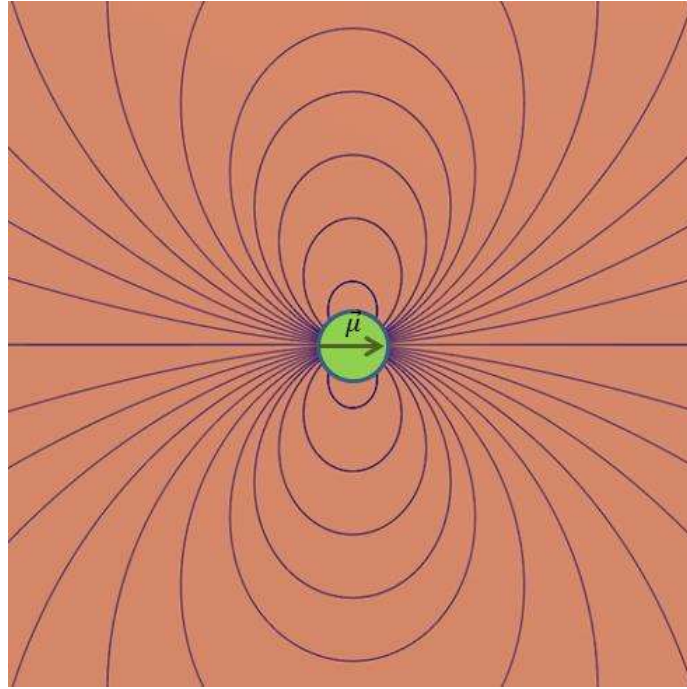


Figure 16: Field lines for electrostatic or magnetic dipoles

Note on electric field reorientation

Since it is proposed to redirect the electric field (but not the electric force), these field lines correspond to the directions of an electric force, to which a point electric charge would be subjected.

We have the potential energy of a dipole $\vec{\mu}$ placed in an external electric field \vec{E} :

$$E_p = -\vec{\mu} \cdot \vec{E}$$

We have the torque exerted on the dipole $\vec{\mu}$ by an external electric field \vec{E} :

$$\vec{\Gamma} = \vec{\mu} \wedge \vec{E}$$

IX.2.2 Dipole moment of a magnetic dipole

Under magnetostatic conditions, we usually define the magnetic moment (orbital) $\vec{\mu}$ of a magnetic dipole in the form of:

$$\vec{\mu} = \iint_S I d\vec{s} = I\vec{S}$$

with I the electric current,

\vec{S} and $d\vec{s}$ perpendicular to surfaces S and ds ,

$\vec{n}ds = d\vec{s}$ unit vector perpendicular to the surface ds .

To calculate the magnetic field generated by this magnetic dipole, we frequently place ourselves in the case where $r = OM \gg a$ (with O the center of the magnetic dipole). This makes it possible to obtain simple radial and orthoradial components for the magnetic field:

$$B_r = \frac{\mu_0}{4\pi} \frac{2\mu \cos\theta}{r^3}$$

$$B_\theta = \frac{\mu_0}{4\pi} \frac{\mu \sin\theta}{r^3}$$

with $\theta = (\vec{\mu}, \overrightarrow{OM})$.

Magnetic field lines also answer the equation:

$$r = K \sin^2 \theta$$

IX.2.3 Dipole moment of a kinetic dipole

We have the following analogies between the dipole moments of an electrostatic, magnetic and kinetic dipole:

	Electrostatics Electrostatic dipole	Magnetostatics Magnetic dipole	"Pulsatiostatic" or "Kinetostatic" Kinetic dipole
Source field equation	$\frac{\vec{\rho}_{xy}^t}{\epsilon_0} = \frac{\partial \vec{E}_{xt}^{1/y}}{\partial x}$	$\mu_0 \vec{j}_{xy}^y = \frac{\partial \vec{B}_{xy}^{1/t}}{\partial x}$	$-\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt}\right)_y = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$
Independence of Time	Static electric charges	$\frac{\partial \vec{B}_{xy}^{1/t}}{\partial t} = \vec{0}$	$\frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial t} = \vec{0}$
Linear density	$\vec{\lambda}^t$	\vec{j}^y	\vec{p}^y
Dipole moment $\vec{\mu}$	$\vec{\mu} = 2q\vec{a}$ Electrostatic dipole moment	$\vec{\mu} = I\vec{S} = \frac{1}{2}\vec{r} \wedge q\vec{v}$ Magnetic dipole moment	$\vec{\mu} = \frac{1}{2}\vec{r} \wedge m\vec{v}$ Kinetic dipole moment
Potential energy of the dipole in an exterior field (fields not reoriented)	$E_p = -\vec{\mu} \cdot \vec{E}$	$E_p = -\vec{\mu} \cdot \vec{B}$	$E_p = -\vec{\mu} \cdot 2\vec{\Omega}$
Torque exerted on the dipole by an exterior field (fields not reoriented)	$\vec{\Gamma} = \vec{\mu} \wedge \vec{E}$	$\vec{\Gamma} = \vec{\mu} \wedge \vec{B}$	$\vec{\Gamma} = \vec{\mu} \wedge 2\vec{\Omega}$

Since potential energies E_p (in absolute terms) tend to be maximized and couples $\vec{\Gamma}$ (en module) tend to be minimized, the different dipole moments $\vec{\mu}$ tend to orient themselves along field lines.

We symbolize on the following diagrams, the electrostatic, magnetic, kinetic and "gravitostatic" dipoles, by 2 different sources of meanings, close in Space and separated by $2a$, and a zero sum of these 2 sources as soon as we deviate a little in Space.

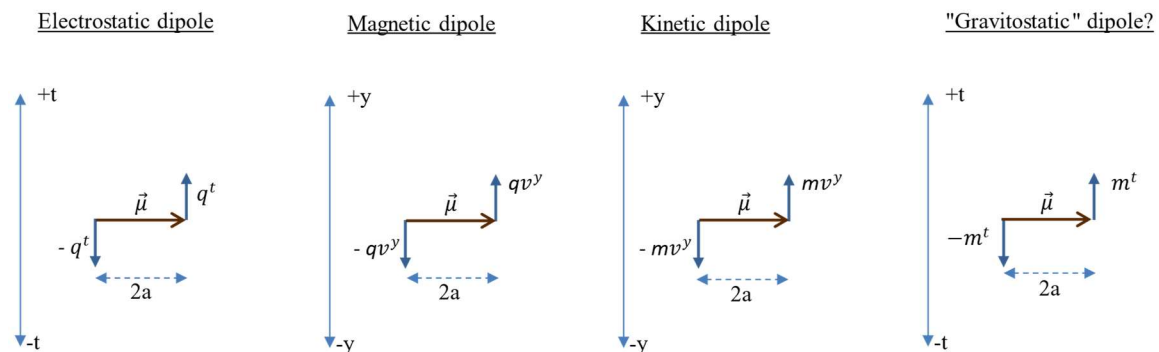


Figure 17: diagrams highlighting the analogies between the 4 dipoles

In the absence of 2 types of mass, positive and negative, there is no "gravitostatic" dipole.

In the next few paragraphs, we will focus on the tides. We will see that we observe a phenomenon with similarities with a "gravitostatic" dipole. Indeed, under the effect of an external gravitational field, we observe the polarization of a static mass, that is to say the appearance of 2 differentiated poles.

IX.3 Static model I of the tides, existence of a "gravitostatic" dipole?

IX.3.1 Simplified models of the tides

We propose in this part and in the following two, some simplified models of the tides observed on the Earth, based on the combination of 2 phenomena:

- the first phenomenon is the deformation of the Earth (appearance of 2 bulges or bulges on both sides of the Earth), deformation due in particular to the gravitational force generated by the Moon (and also by the Sun). We will talk about static tide. This phenomenon will be studied in this and the following section.
- the second phenomenon is the displacement of these 2 bulges following the movements of the Earth relative to the Moon (and the Sun). We will talk about a dynamic tide. We will stop on the simplest and most impactful movement: that of the rotation on itself of the Earth relative to the moon. It already makes it possible to understand variations in sea level during a rotation period.

We will use 2 planets:

- a planet T (as Terra in Latin) composed of a hard core, not or little deformable (in brown), surrounded by a deformable layer (in blue) and can symbolize an immense ocean.
- a planet L (as Luna in Latin) formed of a single and deformable material (in grey).

To explain the 2 bulges, we will always place ourselves in static cases, that is to say an Earth and a Moon immobile with respect to each other. Here we will discuss 2 static models often used to explain static tides.

These 2 static models achieve the same result, the explanation of 2 bulges totally symmetrical with respect to the cutting plane of the Earth (plane passing through its center and perpendicular to the Earth-Moon axis). Nevertheless, they have subtle differences that are interesting to study.

Note

The term planet is used for the *Moon*, even if it is a *moon*.

IX.3.2 Study of a system with 2 deformable and static masses

Electrostatics studies the phenomena created by static electric charges for the observer. In this Memoir, by analogy, we speak of "gravitostatic" for static masses. In this chapter, these masses will also be deformable for the observer.

We assume here that the 2 planets T and L are static relative to each other. The distance between their 2 centres remains constant. Under the effects of both, these 2 planets (which alone in the Universe would be spherical in shape) deform. For example, under the effect of the gravitational force of planet L on planet T, 2 bulges form on planet T.

The formation of these 2 bulges is explained by a gravitational force of planet L on planet T which decreases with distance, and therefore by masses of planet T subjected to gravitational forces of different values.

At the center of planet T, we pose a mass m subjected to gravitational force:

$$\vec{P} = m\vec{G}r$$

In the 1st bulge of planet T, the masses (closer to planet L) are subjected to a greater gravitational force \vec{P} than at the center (the difference is noted by the gravitational field $\vec{\Lambda}$):

$$\vec{P} = m(\vec{G}r + \vec{\Lambda})$$

These masses will be more attracted (than the other masses of planet T) and will therefore tend to move closer to planet L. It appears on planet T a 1st bulge, just facing planet L.

In the 2nd bulge of planet T, the masses (farther from planet L) are subjected to a gravitational force \vec{P} less important than at the center (the difference is denoted by the gravitational field $-\vec{\Lambda}$):

$$\vec{P} = m(\vec{G}r - \vec{\Lambda})$$

These masses will be less attracted (than the other masses of planet T) and will therefore tend to move away from planet L. It appears on planet T a 2nd bulge, on the other side of the planet, symmetrical to the 1st bulge, and whose existence is due to an opposite reason.

Planet L is deformed by the gravitational force of planet T, and 2 bulges also appear. We have the following diagram:

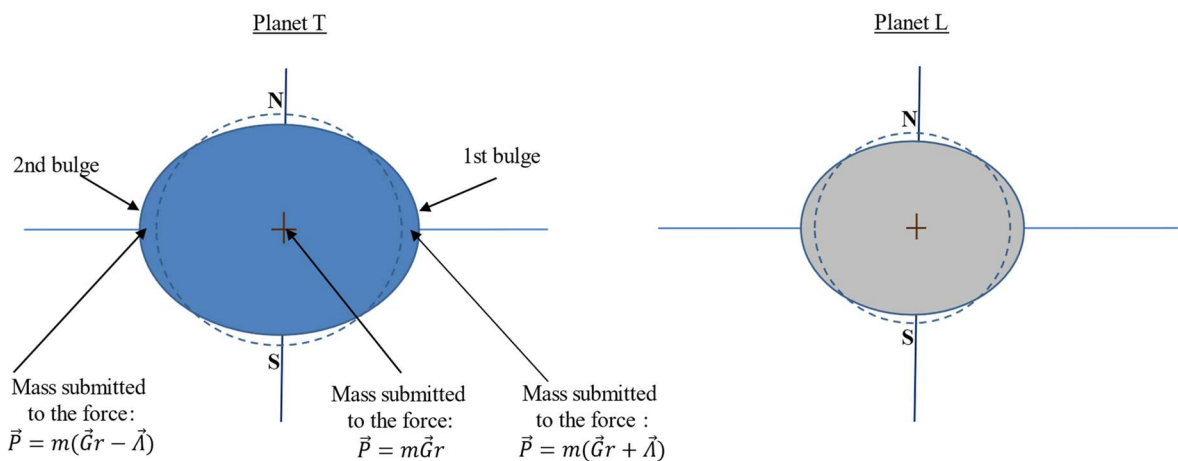


Figure 18: planets T and L deforming under the effects of each other

Rating on scales

Neither the planets nor the bulges are here to scale.

Note on centrifugal forces

If these 2 planets were subject only to gravitational forces, they would attract and end up crashing against each other. It is therefore implicitly assumed here that other forces intervene and hold their centres, so that they are static with respect to each other.

Note on flattenings at Earth's North and South Poles

For the Earth, the flattenings observed at the North Pole and the South Pole are not (or very little) due to gravitational force. They are essentially caused by the centrifugal force generated by the rotation of the Earth on itself.

On the following diagrams, we propose for planets T and L, an elevation, frontal and transverse sections. Note that for reasons of symmetry, the frontal and transverse sections are in fact identical for each planet.

There is also a slight flattening on the sides which compensates, at the material level, the 2 bulges.

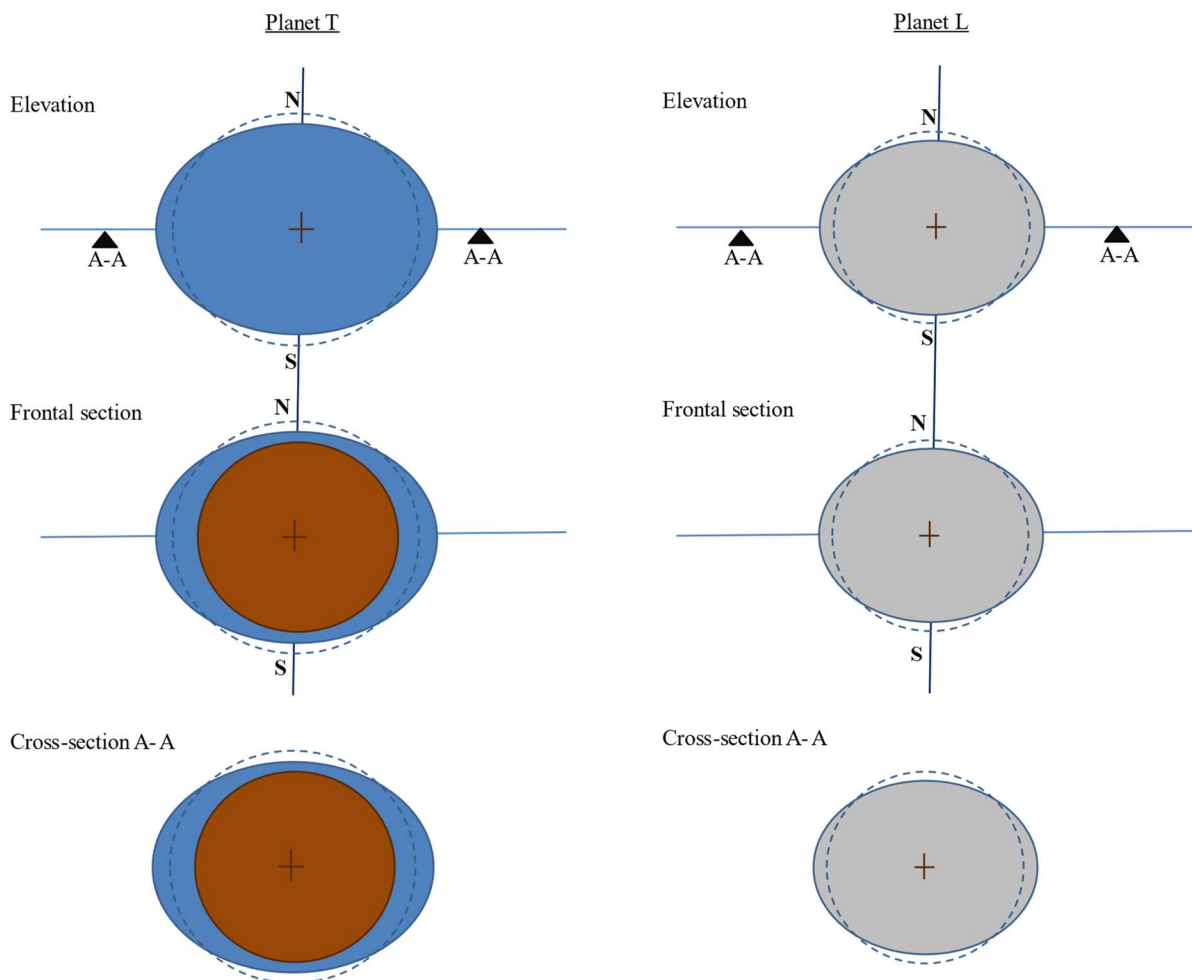


Figure 19: planets T and L, elevations, and sections

IX.3.3 "Gravitostatic" dipole

It is observed that the 2 bulges, which are created on planet T in the case of the "gravitostatic", under the effect of the gravitational force of planet L, are similar to a phenomenon of polarization of planet T. There is indeed appearance of a pole + and a pole -, with differentiation of these 2 poles.

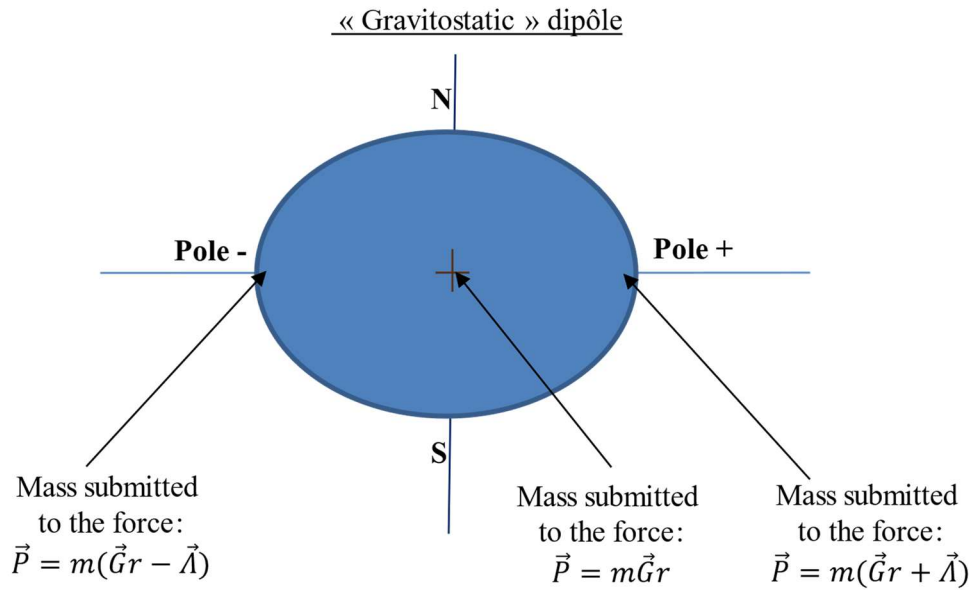


Figure 20: "Gravitostatic" dipole

In the case where we take as a reference for weight:

$$\vec{P} = m\vec{G}r = \vec{0}$$

We find as a positive mass at the + pole, and as a negative mass at the - pole. In the following diagram, we highlight the analogies between "gravitostatic" dipole and electrostatic dipole.

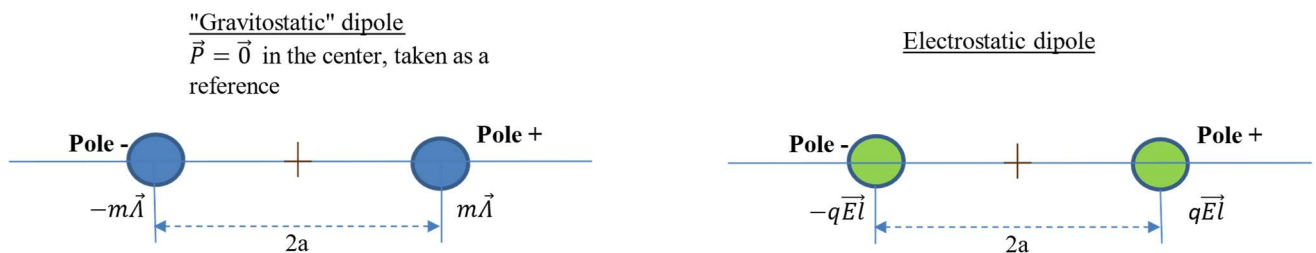


Figure 21: analogies between dipoles

IX.4 Static model II of tides, existence of attractive and repulsive forces for gravitation?

IX.4.1 Study of a system with 2 deformable and static masses with respect to each other, system rotating on itself with respect to a Galilean reference frame

It is now assumed that the Earth-Moon system rotates relative to a Galilean reference frame, with the center of mass of the Earth Moon system (i.e., the mass-weighted barycenter of the Earth Moon system) as the center of rotation.

Compared to Model I, Model II may seem a bit more complicated. It requires the introduction of an additional movement, that of the rotation on itself of the Earth Moon system. Nevertheless, it has a significant advantage: it is sufficient in itself to explain that the Earth and the Moon are static with respect to each other.

Indeed, in Model I, we have only one attractive force. To prevent the Earth and the Moon from colliding (since they attract each other), it is necessary to assume the existence of other forces or other movements that stabilize the system.

To perform experiments on this system, it would be necessary to use as 2 long stakes supporting respectively the Earth and the Moon and then study the deformations of each planet, deformations induced by the other planet.

In Model II, thanks to rotational motion, a centrifugal repulsive force is obtained, which balances the gravitational attractive force. The system is stabilized, static, internally. In particular, the distance between the centers of the Earth and the Moon remains constant. There is no need to call for force or other movement.

Gravitational attractive force

For the first bulge of the Earth (the one facing the Moon), it is explained as in Model I by the gravitational attractive force of the Moon on the Earth. By this force alone, we obtain a first bulge a priori a little larger than in model I.

We give the expression of the gravitational force at a point M of the Earth:

$$\vec{F}_{Gr}(M) = m \frac{Gm_L}{r^2} \vec{u}_{ML}$$

with:

r distance between a point M of the Earth and the center L of the Moon,

m mass of the volume studied at point M,

m_L mass of the Moon,

\vec{u}_{ML} unit vector oriented from point M to center L of the Moon.

We have the following deformations (which are not at full scale):

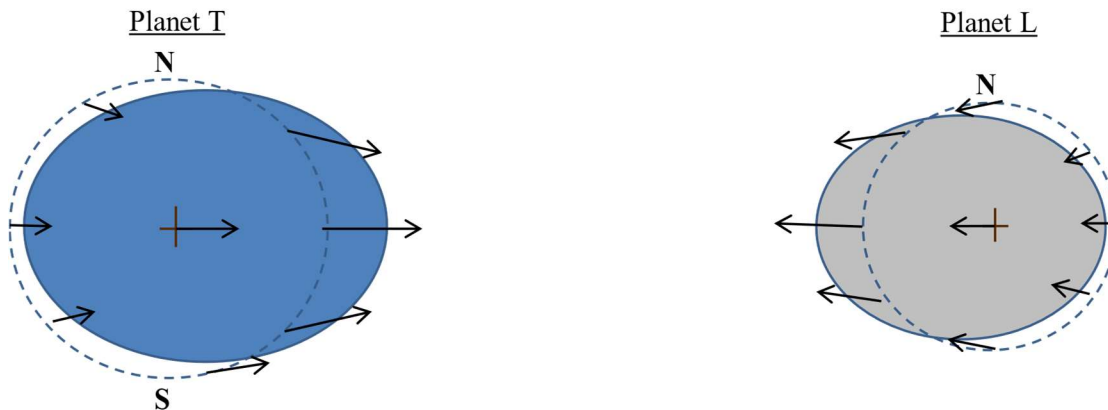


Figure 22: Gravitational attractive force that creates a first bulge

Centrifugal force (repulsive)

To explain the 2nd bulge, it is assumed that the Earth Moon system has a rotation on itself with respect to a Galilean reference frame, with W the center of mass of the Earth Moon system also defined as the center of this rotation. We have the speed of rotation:

$$\Omega_M = \frac{2\pi}{T_M}$$

with $T_M = 27.3 \text{ days}$ (quite close to a month or menstrual cycle)

It should be noted that we always remain in the case of statics, since the Earth and the Moon are always static with respect to each other.

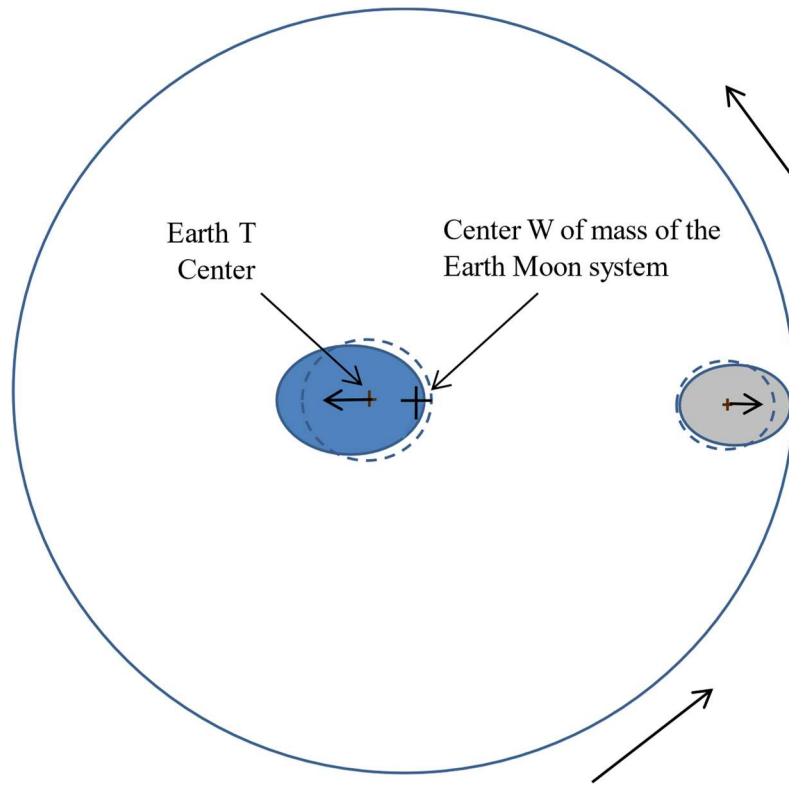


Figure 23: the Earth-Moon system on a carousel of W center and rotational speed $\Omega_M = \frac{2\pi}{T_M}$

Note

In reality, the Moon is much smaller than the Earth, and the center of mass of the Earth Moon system is as in the figure above, located inside the Earth.

By assumption, the centres of the Earth and the Moon are static relative to each other. In the reference frame R^{Ω_M} , they are therefore static or immobile. In this reference frame R^{Ω_M} , we apply the fundamental principle of Newton's dynamics, for a mass m located at the point T center of the Earth:

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} = \vec{0}$$

Nota

It is often noted R^* the reference frame of the center of mass W of the system studied (here the Earth and the Moon). It should be noted that in the example studied, R^* is considered like a Galilean reference frame. We go from R^* to R^{Ω_M} by cancelling the rotational speed Ω_M and then adding the centrifugal inertial force $\vec{F}_{cen}(T)$ when applying the fundamental principle of Newton's dynamics in R^{Ω_M} .

We therefore have, including the centrifugal force (inertial force):

$$\begin{aligned} \vec{F}_{cen}(T) + \vec{F}_{Gr}(T) &= \vec{0} \\ \vec{F}_{cen}(T) &= -\vec{F}_{Gr}(T) \end{aligned}$$

For a mass m localized in T, we have:

$$\begin{aligned} \vec{F}_{cen}(T) &= m\vec{a}_{cen}(T) = -m\Delta_{TW}\Omega_M^2\vec{u}_{TL} \\ \vec{F}_{Gr}(T) &= m\vec{G}r(T) = m\frac{Gm_L}{\Delta_{TL}^2}\vec{u}_{TL} \end{aligned}$$

So, we have:

$$\begin{aligned}\vec{F}_{cen}(T) &= m\vec{a}_{cen}(T) = -m\vec{G}r(T) \\ \vec{F}_{cen}(T) &= -m\Delta_{TW}\Omega_M^2\vec{u}_{TL} = -m\frac{Gm_L}{\Delta_{TL}^2}\vec{u}_{TL}\end{aligned}$$

with the following notations:

m mass of the volume studied,

T center of the Earth,

L center of the Moon,

W center of mass of the Earth-Moon system,

Δ_{TL} distance between the center of the Earth and that of the Moon,

Δ_{TW} distance between the centre of the Earth and the centre of mass W,

Δ_{LW} distance between the center of the Moon and the center of mass W,

m_L mass of the Moon,

m_T mass of the Earth,

Ω_M speed of rotation on itself (or spin speed) of the Earth Moon system,

T_M period of rotation on itself of the Earth Moon system in a Galilean reference frame,

\vec{u}_{TL} unit vector in the direction of Earth Moon,

\vec{u}_{LT} unit vector in the direction of Moon Earth.

We have analogous equations on the Moon side:

$$\begin{aligned}\vec{F}_{cen}(L) &= m\vec{a}_{cen}(L) = m\vec{G}r(L) \\ \vec{F}_{cen}(L) &= -m\Delta_{LW}\Omega_M^2\vec{u}_{LT} = m\frac{Gm_T}{\Delta_{LW}^2}\vec{u}_{LT} \\ \vec{a}_{cen}(L) &= -\Delta_{LW}\Omega_M^2\vec{u}_{LT} \\ \vec{G}r(L) &= \frac{Gm_T}{\Delta_{LW}^2}\vec{u}_{LT}\end{aligned}$$

Why is centrifugal force the same at every point on Earth?

The centrifugal force at a point M is a function of the distance between M and the center of rotation. However, in this case, not all points on the Earth have of course the same distance from the center of mass W.

In fact, in the case of a solid rotation, not all points on the Earth revolve around W. They revolve around other points. On the other hand, they all perform a rotation of radius Δ_{TW} and period T_M . Thus, they are all subjected to the same centrifugal force. The following figure helps to better understand.

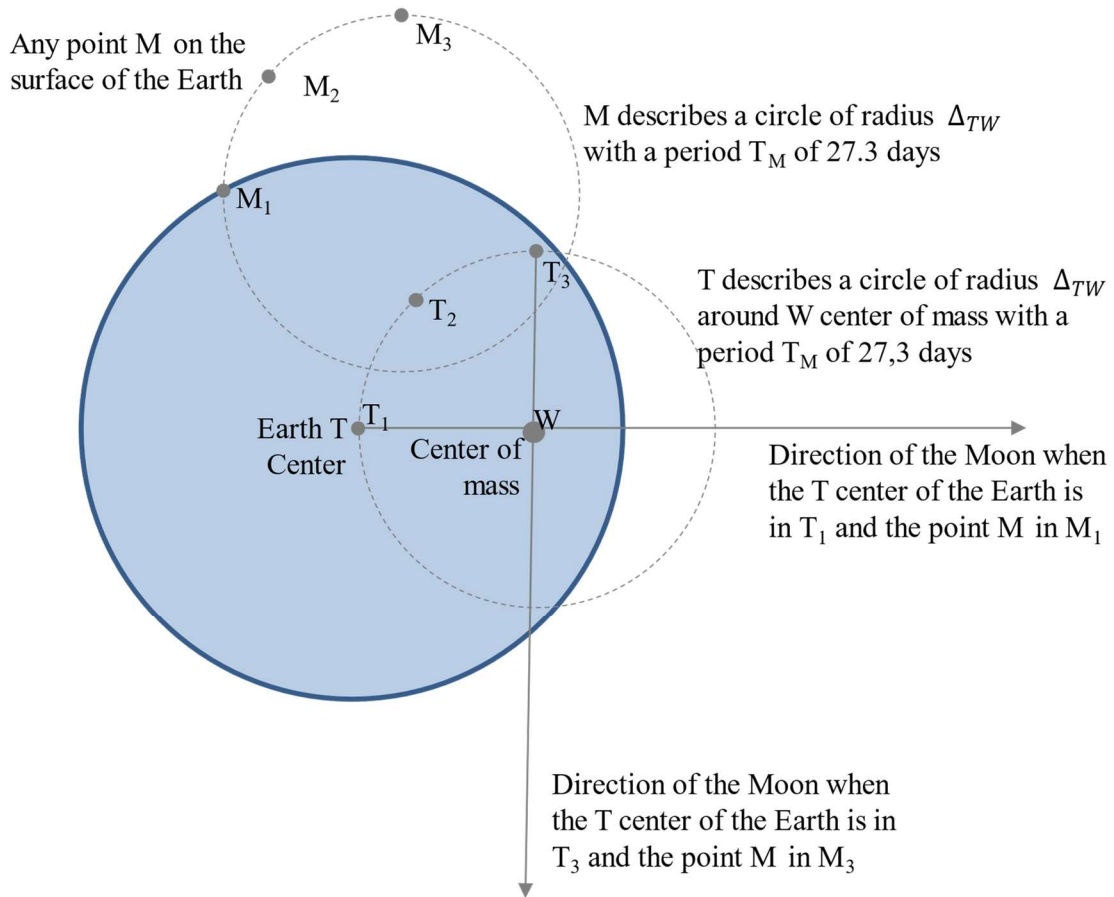


Figure 24: solid rotation of the Earth

Note that the vectors $\overrightarrow{T_1M_1}, \overrightarrow{T_2M_2}, \overrightarrow{T_3M_3}, \dots$, have the same direction, the same sense and the same norm in the course of Time. They are said to be equipollents. The trajectories of all points on the Earth form circles of the same radius. We are for the Earth (and also for the Moon) in the case of a circular translational motion.

We therefore have for any point M of the Earth:

$$\begin{aligned} \vec{F}_{cen}(T) &= m_T \vec{a}_{cen}(T) = \vec{F}_{cen}(M) = m_T \vec{a}_{cen}(M) \\ \vec{a}_{cen}(T) &= \vec{a}_{cen}(M) = -\Delta_{TW} \Omega_M^2 \vec{u}_{TL} \end{aligned}$$

We have:

$$\vec{F}_{cen}(T) = -m \frac{G m_L}{\Delta_{TL}^2} \vec{u}_{TL}$$

Therefore:

$$\vec{F}_{cen}(M) = -m \Delta_{TW} \Omega_M^2 \vec{u}_{TL} = -m \frac{G m_L}{\Delta_{TL}^2} \vec{u}_{TL}$$

Note

The orbits of the different points of the Earth are actually slightly elliptical, but to simplify the model, we will consider them here as circular. This centrifugal force should not be confused with that due to the rotation of the Earth on itself (spin) which increases with the distance from the point considered to the axis of rotation.

We schematize the deformations of the Earth and the Moon due to the rotation of the Earth Moon system:



Figure 25: centrifugal repulsive force that creates a second bulge

Tidal force, vector sum of gravitational attractive force and centrifugal repulsive force

For a mass m localized in M and subjected to the tidal force, we have the following figure:

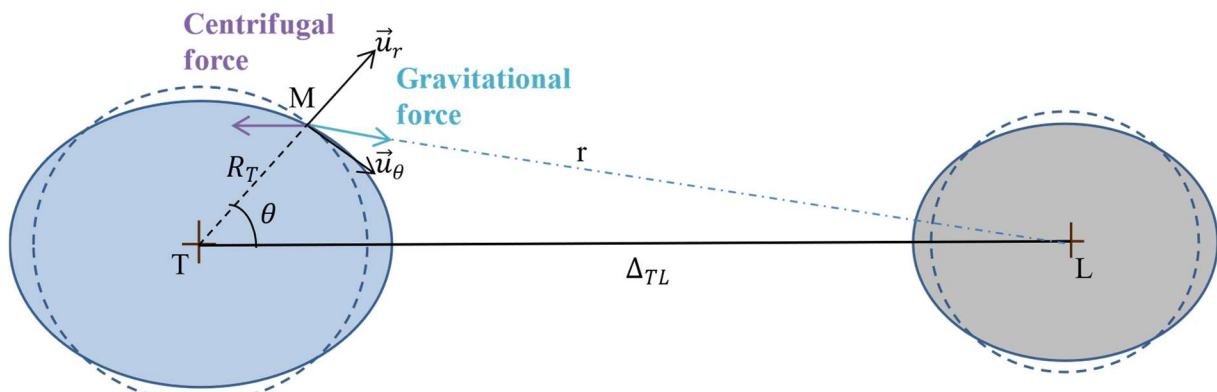


Figure 26: Attractive and repulsive force superposition

We recall the 2 forces to which is subjected this mass m located in M :

$$\vec{F}_{cen}(M) = -m \frac{Gm_L}{\Delta_{TL}^2} \vec{u}_{TL}$$

$$\vec{F}_{Gr}(M) = m \frac{Gm_L}{r^2} \vec{u}_{ML}$$

When we add the effects of gravitation and the effects of centrifugal force, we obtain the tidal force:

$$\vec{F}_{Marée}(M) = \vec{F}_{Gr}(M) + \vec{F}_{cen}(M)$$

$$\vec{F}_{Marée}(M) = m \left(\frac{Gm_L}{r^2} \vec{u}_{ML} - \frac{Gm_L}{\Delta_{TL}^2} \vec{u}_{TL} \right)$$

with:

m mass of the volume studied,

Δ_{TL} the distance between the centre of the Earth and that of the Moon,

r the distance between a point M of the Earth and the centre L of the Moon,

R_T the radius of the Earth,

m_L the mass of the Moon.

Then by making some simplifications, with $\Delta_{TL} \gg R_T$, one obtains by projecting on the axes of the point M considered:

$$\vec{F}(M) \approx mg \frac{m_L}{m_T} \left(\frac{R_T}{\Delta_{TL}}\right)^3 ((3\cos^2\theta - 1)\vec{u}_r + \frac{3}{2}\sin 2\theta \vec{u}_\theta)$$

$$\|\vec{F}(M)\| \approx mg \frac{m_L}{m_T} \left(\frac{R_T}{\Delta_{TL}}\right)^3 \sqrt{3\cos^2\theta + 1}$$

with:

the constant $g = \frac{Gm_T}{R_T^2}$ acceleration of gravity on the Earth's surface,

θ the latitude of the point M considered.

We have the deformations of the 2 planets with indicated the attractive and repulsive forces:

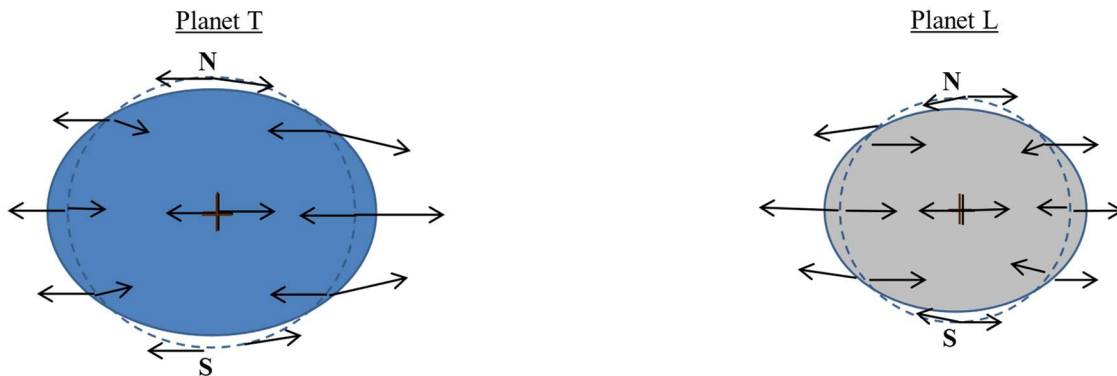


Figure 27: attractive and repulsive forces

We have the deformations with indicated the tidal forces:



Figure 28: tidal forces

Why are the 2 bulges identical when the causes are different?

The gravitational attractive force distorts the Earth. Nevertheless, these deformations are symmetrical with respect to a plane perpendicular to the Earth-Moon axis, plane slightly offset towards the Moon with respect to the center T of the Earth.

The centrifugal repulsive force has the effect, in a way, of shifting this plane and passing it through the center T of the Earth. Thus, the 2 bulges are symmetrical with respect to this plane passing through T and perpendicular to the Earth-Moon axis.

IX.4.2 Bridging the gap between "gravitostatic" and electrostatics?

In the static model II, we note the presence of two forces, one repulsive and the other attractive, as in the case of electrostatics.

Let us retain from the study of this model II, that any system with 2 static bodies with respect to each other, and rotating on itself with respect to a Galilean reference frame, can be interpreted by an

observer located in this Galilean reference frame, as presenting a repulsive force between these 2 bodies.

When an observer notices a repulsion between 2 electric charges, he usually explains this repulsion by an identical sign of the 2 charges (charges + or charges -). Could he finally explain this repulsion by a 2-body system having a rotation on itself, and generating a centrifugal force greater than the attractive force between the 2 bodies?

Similarly, when an observer notices an attraction between 2 electric charges, he usually explains this attraction by an opposite sign of the 2 charges (charges + and charges -). Could he finally explain this attraction by a 2-body system having a rotation on itself, and generating a centrifugal force lower than the attractive force between the 2 bodies?

These interpretations would bring together electrostatic and "gravitostatic", with bodies (or charges) of the same nature. These bodies would simply be distinguished by their respective movements and explain the existence of attractive and repulsive forces.

IX.4.3 Explain 2 types of electric charge simply by movements?

Model of the Bohr hydrogen atom, proton and electron

In the Bohr model of the hydrogen atom, the proton is generally considered fixed with respect to a Galilean reference frame, and the electron as rotating around the proton. The electron is "stabilized" by an attractive electric force and a repulsive centrifugal force depending on the speed of the electron in the Galilean reference frame.

Note that in this Bohr model, we are not in the case of electrostatics, since the 2 charges + and -, are mobile with respect to each other.

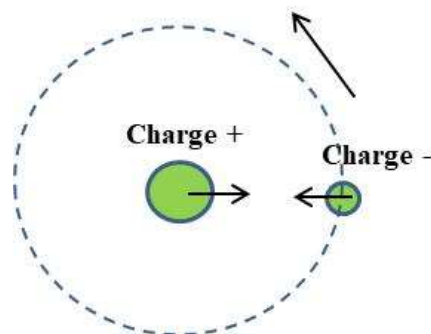


Figure 29: Bohr model of the hydrogen atom

Model with 2 electric charges of the same nature, static in relation to each other, rotating on a carrousel (charges in circular translational motion)

In this model, we are again in the case of electrostatics. When the carrousel turns in one sense, for example clockwise, the 2 charges + repel each other under the effect of centrifugal force. When the carrousel turns in the other sense, the 2 charges - also repel each other under the effect of centrifugal force. The difference in nature between the + and - charges is explained here by the sense of rotation of the carrousel.

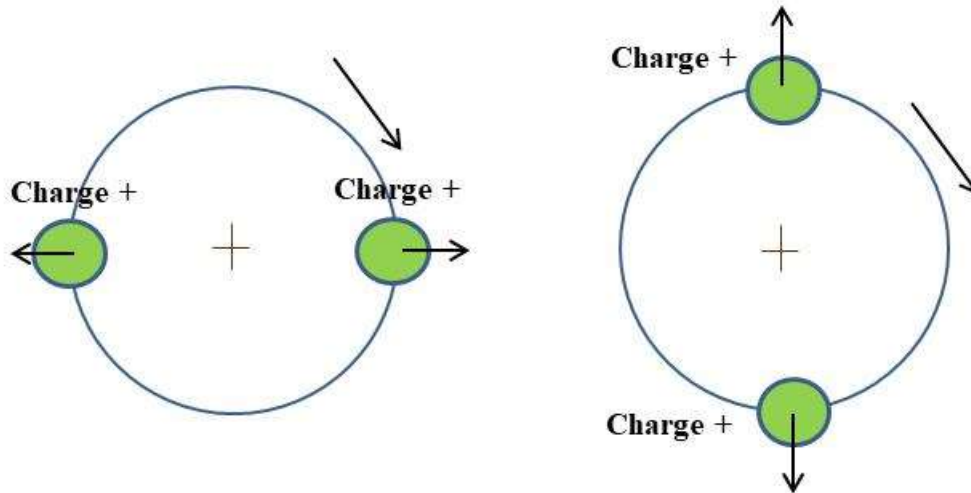


Figure 30: 2 electric charges + on a carousel

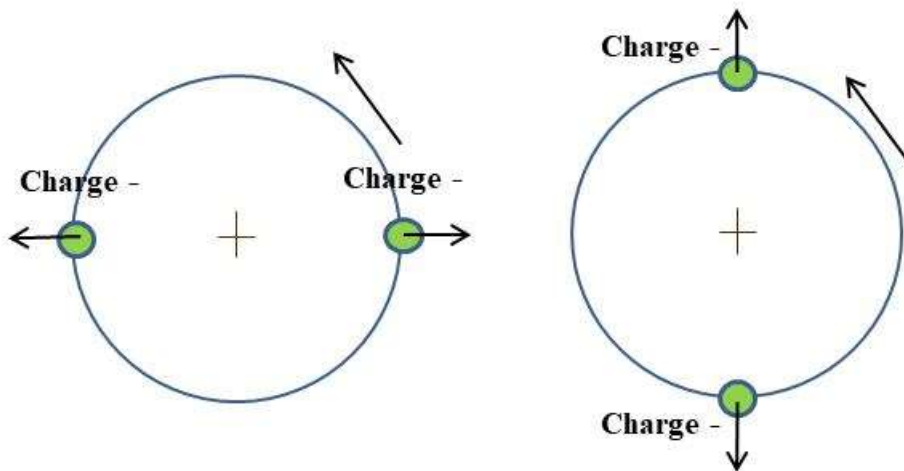


Figure 31: 2 electric charges - on a carousel

Note 1 on the sense of rotation reference

Here we assume that the carousel is clockwise for electric charges +, and in the "trigo" or counterclockwise sense for electric charges -. It is a random choice, obviously without justification. Moreover, if an observer looking "over" these 2 carrousel, then looks at them "below", he will see them turn in the other sense.

The reference of the sense of rotation of these carousel is therefore not defined in relation to an external observer, but in relation to the charges between them., with the idea that a carousel formed of 2 static charges + turns in one sense, and that a carousel formed of 2 static charges - turns in the other sense.

According to these ideas, if we superimpose a + charge and a charge -, we obtain a carousel of low speed of rotation on itself compared to a Galilean reference frame, and therefore 2 charges that attract each other. To stabilize the system, it is necessary to use other forces or movements, such as a rotation of the electron around the proton (case of the Bohr atom).

Note 2 on a circular translation

We talk about carousel and rotation. Nevertheless, as for the Earth and the Moon in Model II, it would in fact be a circular translation for the points of the bodies studied.

Note 3, the analogue of the carrousel in electromagnetism is the magnetic field

A mass on a merry-go-round, its analogue in electromagnetism, is an electric charge in a magnetic field.

IX.4.4 The electron that goes back in Time

We take up in this paragraph the idea of J. A. Wheeler and R. Feynman of an electron going back in Time, an electron that would be a positron. We mix this idea with that of the carrousel with 2 static bodies in relation to each other. We have the following figure:

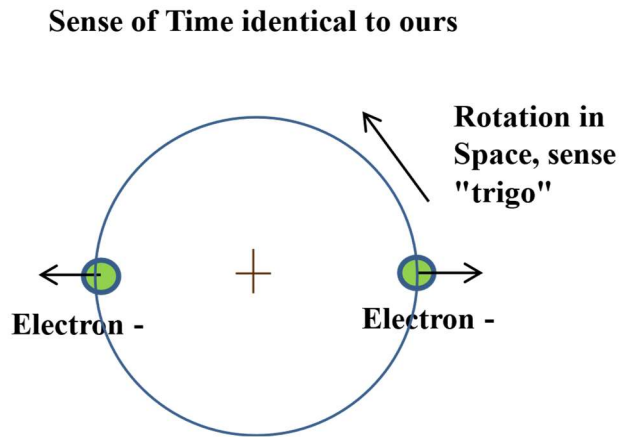


Figure 32: 2 charges - on a carrousel turning in the sense "trigo", advancing in the sense of Time identical to ours

When we go back in time, we modify the previous figure, with following the idea of J. A. Wheeler and R. Feynman, an electron that transforms into a positron. Note that during this time revival, the characteristics of the 2-body system are preserved, since the carrousel always turns in the same trigo sense. We have the following figure:

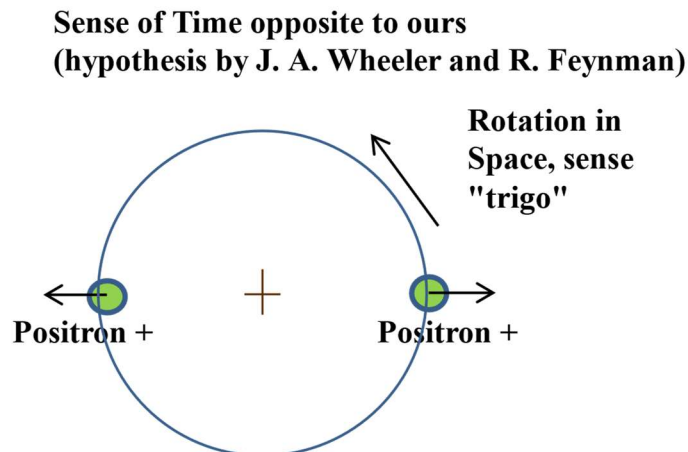


Figure 33: 2 charges + on a carrousel turning in the sense "trigo", advancing in the sense of Time opposite to ours

In our Time, an observer sees this carrousel rotate clockwise sense, and therefore in the opposite sense to that of the carrousel of 2 electrons of charge -. He can therefore conclude the existence of 2 positrons of charge +. We have the following figure:

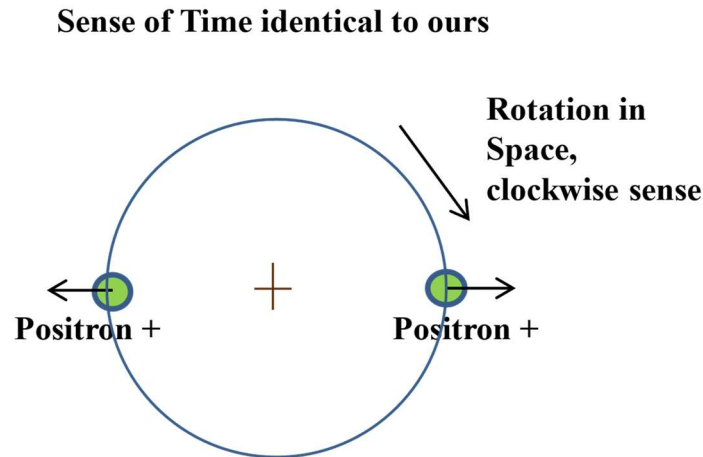


Figure 34: 2 charges + on a carousel turning clockwise sense, in the sense of Time identical to ours

IX.4.5 Decomposition of an acceleration

We remind the decomposition of an acceleration:

$$[\vec{a}]_{R^0} = \left[\frac{d\vec{v}_r}{dt} \right]_{R^\Omega} + \frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{O}M + \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M) + 2\vec{\Omega} \wedge \vec{v}_r$$

In the 2nd Memoir, it was proposed that the Coriolis force $m\vec{v}_r \wedge 2\vec{\Omega}$ is for gravitation the analogue of the Lorentz force $q\vec{v}_r \wedge \vec{B}$ of electromagnetism. In the previous chapter, it was proposed that the drive acceleration $\frac{\partial \vec{\Omega}}{\partial t} \wedge \vec{O}M$ is the analogue of the phenomena of induction of M. Faraday.

By contrast, as pointed out in the same chapter, there is no analogue in electromagnetism to centrifugal inertial acceleration $\vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$. To see if finally, there would not be an analogue in the phenomena of repulsion observed between 2 electric charges of the same nature?

IX.5 Tides in motion

This section does not deal with analogies between "gravitostatic" and electrostatic. It simply has the advantage of proposing a dynamic tidal model, explaining in a simplified way the variations in the level of the ocean during a day.

IX.5.1 The formation of continents

We slightly modify the previous patterns, adding to planet T, 3 non-deformable islands: X, Y and Z. Island X is positioned on the North Pole of Planet T. Islands Y and Z are positioned on the equator of planet T, with island Y facing planet L, and island Z on the side, relative to L.

At the bottom left of the figure, we symbolize the constant levels of the ocean, functions of the position of the 3 islands in relation to planet L.

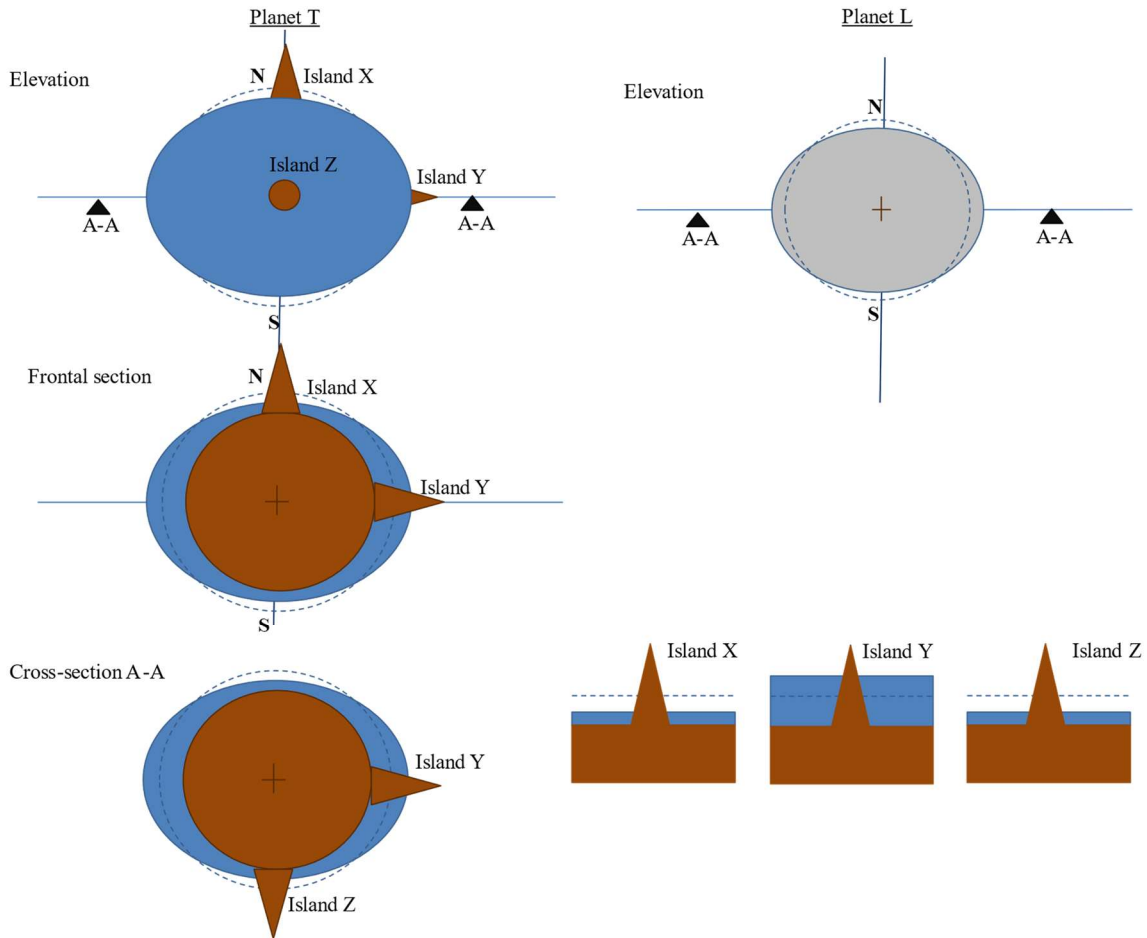


Figure 35: Planet T with 3 islands

IX.5.2 Study of moving masses

We now wish to observe for the 3 islands tidal phenomena, that is to say periodic variations in the level of the ocean.

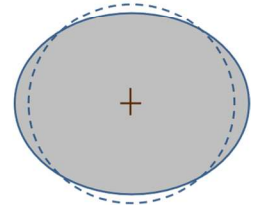
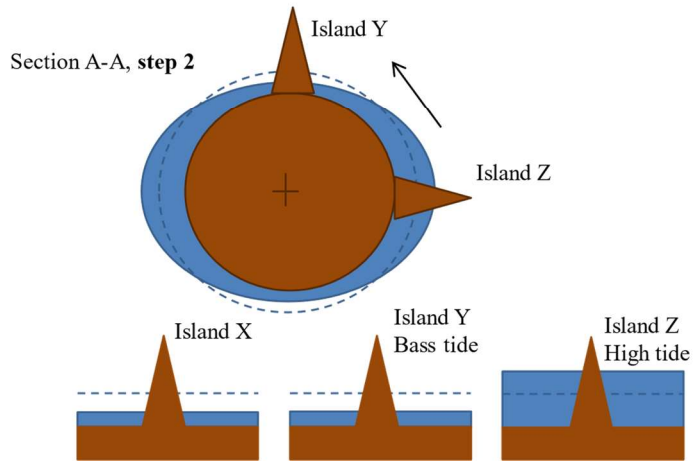
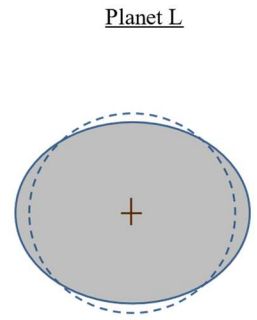
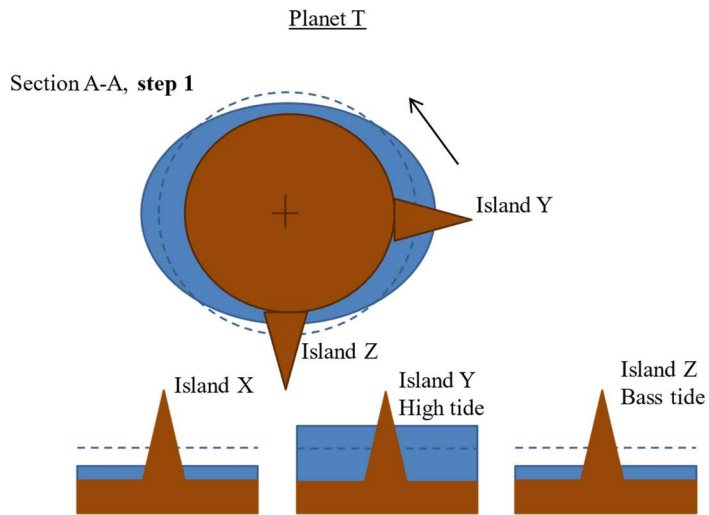
In the case of the Earth, the Moon and the Sun, the movements are numerous and complex, hence many periodic phenomena (high tides, etc.). We stop here at a simplified case, with:

- a planet L always static (in model I in a Galilean reference frame, in model II on the carrousel of the Earth Moon system),
- a planet T rotating on itself with respect to L, according to a period T_J of approximately 24h50.

T_J is the period of rotation on itself (spin) of the Earth in the lunar reference frame (duration close to one Earth day). In this lunar reference frame, we have the speed of rotation of the Earth on itself (spin):

$$\Omega_J = \frac{2\pi}{T_J}$$

For Island X, located at the North Pole of planet T, there is no variation in ocean level. On the other hand, for islands Y and Z, located on the equator, there are 2 cycles of high tide, low tide during the period of 24:50. We give on the following diagrams a division into 4 stages of these tides.



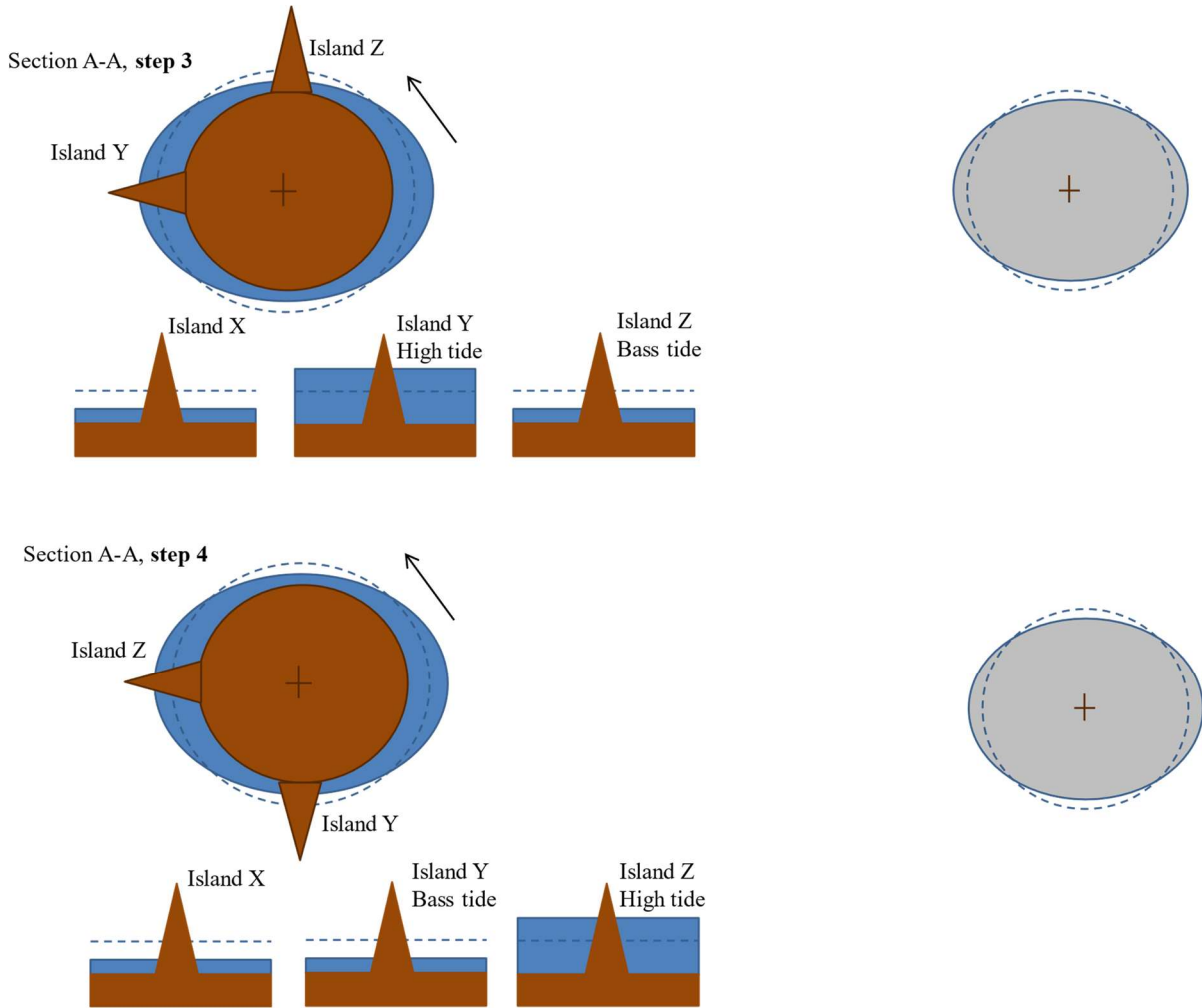


Figure 36: 4 steps of rotation of planet T, with variation of the ocean level of islands Y and Z

In the following diagram, we give for island Y, the sinusoidal variations of the ocean level over a period of 24h50.

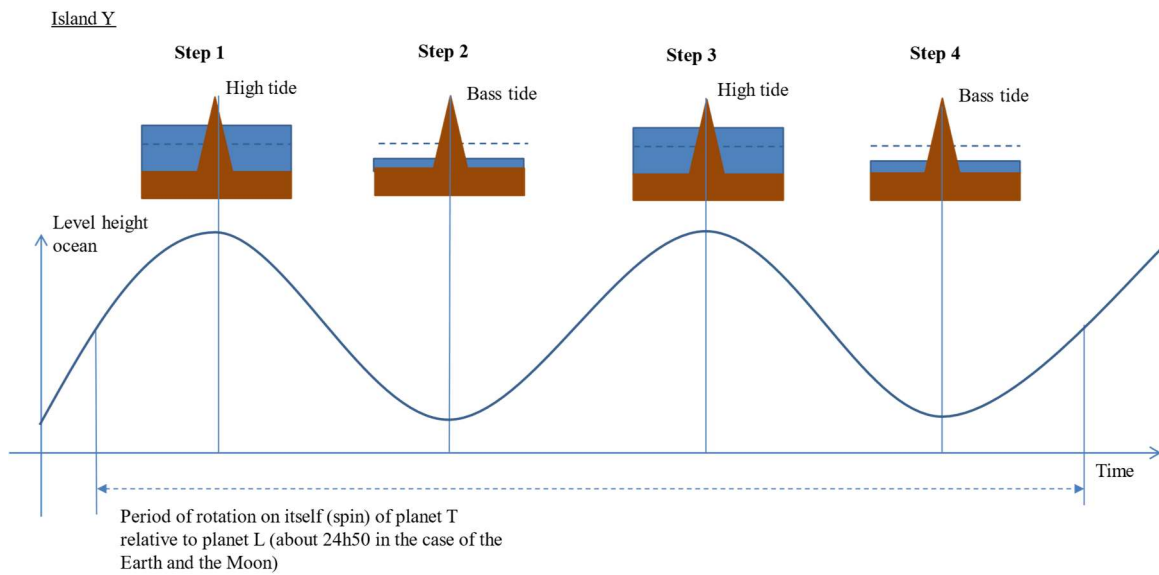


Figure 37: Sinusoidal variations in ocean level

IX.6 Complements on accelerations and inertial forces, summary of their analogues on the one hand between Gravitation and Electromagnetism, on the other hand between Space and Time

IX.6.1 Analogues of Coriolis inertial acceleration

	Gravitation	Analogue for electromagnetism
Space	Coriolis inertial acceleration $\vec{a}_{Cor_{ty}}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$	Electromotor field $\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$
Analogue for Time	Gravitational force $\vec{F}_{K_{ty}}^x = \vec{m}^t \wedge 2c^2 \vec{K}_{tx}^{1/y}$	Electric force $\vec{F}_{El_{ty}}^x = \vec{q}^t \wedge \vec{E}_{tx}^{1/y}$

IX.6.2 Analogues of the acceleration of drive inertia due to the variation in rotation velocity $\vec{\Omega}_{xy}^{1/t}$

	Gravitation	Analogue pour l'électromagnétisme
Space	Drive inertial acceleration due to variation in rotation velocity $\vec{\Omega}_{xy}^{1/t}$ $\vec{a}_{i_{ty}}^x = \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial t} \wedge \vec{O}M$ $\frac{\partial \vec{V}^x}{\partial t}$ $rot(\vec{a}_{i_{ty}}^x) = \frac{\partial 2\vec{\Omega}_{xy}^{1/t}}{\partial t}$	Neumann case of induction $\vec{E}m_{ty}^x = -\frac{\partial \vec{B}}{2\partial t} \wedge \vec{O}M$ $\frac{\partial \vec{A}^x}{\partial t}$ $rot(\vec{E}m_{ty}^x) = \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t}$
Analogue for Time	$\frac{\partial \vec{V}^t}{\partial x}$ Gravitational field	$\frac{\partial \vec{A}^t}{\partial x}$ Electrostatic field

IX.6.3 Analogues of centrifugal drive inertia acceleration

	Gravitation	Analogue for electromagnetism
Space	Centrifugal drive inertia acceleration $\vec{a}_{cent} = \vec{\Omega} \wedge (\vec{\Omega} \wedge \vec{O}M)$	Repellent electric field?
Analogue pour le Temps	?	?

IX.7 Conclusion

In this chapter, we focused on the analogies and differences between electrostatics and "gravitostatic", testing whether it is possible to erase certain differences (such as 2 natures of charge and 1 single nature of mass) by a kinetic approach in Space-Time.

In these "tests", we took simplified and static tidal models as examples. This was an opportunity first to look at a hypothetical "gravitostatic" dipole, then to try an explanation of the repulsive electric force by movements in Space-Time.

According to this last "test", the + and - charges would in fact be identical, and it would be their movements in Space-Time that would distinguish them. Thus, there would be only one nature of electric charge, as there is only one nature of mass (this mass itself being interpreted as a "velocity" in Time, according to the ideas proposed in Memoir 2).

However, it must be admitted, we have remained far from a quantitative explanation. Some tracks have been sketched, but there are still many unexplained elements, such as the ratio always constant and very far from 1, between the mass of a proton and that of an electron.

There are also other important differences between electromagnetism and gravitation, including the quantization by a photon particle representing the electromagnetic field, and the absence of an analogous particle for the gravitational field. We will return in future Memoirs to this question of quantification and particles representing fields.

Chapter X Form and conclusion of the Memoir

X.1 Summary of the main equations (in a space x, y, t)

In this Memoir, many analogies between electromagnetism and gravitation have been conducted. This allowed us to propose new source field equations for gravitation. Often, and surprisingly, these analogies were not easily spotted, precisely because they had not been taken far enough.

For example, it has been proposed that the analogue of Maxwell Gauss $\frac{\rho}{\epsilon_0} = \text{div}\vec{E}l$, is for gravitation, as expected, Gauss gravitation $-4\pi G\rho = \text{div}\vec{G}r$, is Newton's law of universal gravitation, albeit slightly modified.

Similarly, it has been proposed that the analogue of Maxwell Ampère $\mu_0\vec{J} = \text{rot}\vec{B}$, is for gravitation, the fundamental principle of Newton's dynamics $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ (written here as Hamilton's 2nd canonical equation).

Similarly, it has been proposed that the analogue of Maxwell Ampère drive current $\mu_0\vec{J} = \mu_0\epsilon_0\frac{\partial\vec{E}l}{\partial t}$, is for gravitation, the Einstein and de Broglie's relation $p = \frac{h}{\lambda}$.

Finally, it has been proposed that the analogue of Maxwell Faraday $\text{rot}\vec{E}l = -\frac{\partial\vec{B}}{\partial t}$, is for gravitation, the rotation velocity $\vec{\Omega}$ of a carrousel that varies over time, and the necessary addition of an inertial acceleration: $\vec{a}_i = \frac{\partial\vec{\Omega}}{\partial t} \wedge \vec{O}M$.

These main analogies are recalled in the following form.

Source field equations:

	Electromagnetism	Gravitation
Time-oriented sources (Gauss)	$\frac{\vec{\rho}_{xy}^t}{\epsilon_0} = \frac{\partial\vec{E}l_{xt}^{1/y}}{\partial x} - \frac{\partial\vec{E}l_{yt}^{1/x}}{\partial y}$ Electrostatics: $\frac{\vec{\rho}_{xy}^t}{\epsilon_0} = \frac{\partial\vec{E}s_{xt}^{1/y}}{\partial x} - \frac{\partial\vec{E}s_{yt}^{1/x}}{\partial y}$	$-4\pi G\vec{\rho}_{xy}^t = \frac{\partial 2c^2\vec{K}_{xt}^{1/y}}{\partial x} - \frac{\partial 2c^2\vec{K}_{yt}^{1/x}}{\partial y}$ Gravitostatic: $-4\pi G\vec{\rho}_{xy}^t = \frac{\partial\vec{G}r_{xt}^{1/y}}{\partial x} - \frac{\partial\vec{G}r_{yt}^{1/x}}{\partial y}$
Space-oriented sources (Ampère)	$\mu_0\vec{J}_{xt}^y = \frac{\partial\vec{B}_{xy}^{1/t}}{\partial x} - \mu_0\epsilon_0\frac{\partial\vec{E}l_{ty}^{1/x}}{\partial t}$	$-\frac{1}{\hbar}\left(\frac{d\vec{p}^x}{dt}\right)_{xt}^y = \frac{\partial\vec{\Omega}_{xy}^{1/t}}{\partial x} - \frac{\partial\vec{K}_{ty}^{1/x}}{\partial t}$ $-\frac{c^2}{\hbar}\left(\frac{d\vec{m}^t}{dx}\right)_{tx}^y = \frac{\partial\vec{K}_{ty}^{1/x}}{\partial t} - \frac{\partial\vec{\Omega}_{xy}^{1/t}}{\partial x}$

Note

These source field equations are verified in restricted Einsteinian mechanics, i.e., they are invariant by the Lorentz transform.

Maxwell Faraday electromagnetism homogeneous field equations:

$$\oint_{\Gamma^x} ((\vec{v}^y \wedge \vec{B}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \frac{\partial \vec{A}^x}{\partial t})_{ty}^x) \cdot d\vec{x} = \iint_{S_{xy}} \frac{-\partial \vec{B}_{xy}^{1/t}}{\partial t} \cdot d\vec{S}_{xy}^t = -\frac{d\Phi_B}{dt}$$

$$\oint_{\Gamma^x} \vec{E} m_{ty}^x \cdot d\vec{x} = -\frac{d\Phi_B}{dt}$$

Faraday gravitation homogeneous field equations:

$$\oint_{\Gamma^x} ((\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t})_{ty}^x + (\vec{u}^t \wedge \frac{\partial \vec{v}^x}{\partial t})_{ty}^x) \cdot d\vec{x} = \iint_{S_{xy}} \frac{\partial 2\vec{\Omega}_{xy}^t}{\partial t} \cdot d\vec{S}_{xy}^t = \frac{2d\Phi_\Omega}{dt}$$

$$\oint_{\Gamma^x} (-\vec{a}_{Cor}^x + \vec{a}_{Ein}^x) \cdot d\vec{x} = \frac{2d\Phi_\Omega}{dt}$$

X.2 Conclusion of the Memoir

Maxwell's equations proposed in the 19th century may appear to have a complex form. However, we see that if we push the analogy between Time and Space, between magnetic field and electric field, between Maxwell Gauss and Maxwell Ampère, we find much simpler forms, both for source field equations and for homogeneous field equations, both for electromagnetism and for gravitation.

In addition, in order to find velocities of electric charges or masses equal to group velocities, by dividing the proposed source field equations, we find on the one hand a known condition between the constants ε_0 and μ_0 , on the other hand a condition between \hbar and G giving an infinitesimal length identical to the Planck length.

Nevertheless, there remains an unresolved question: this is the underlying reason for the equalities between sources and fields in Space-Time. This question is eminently important, because these source field equations appear to be the most fundamental of physics.

We will come back in the 5th Memoir on these source field equations. But first, in the 4th Memoir, we will be interested in nuclear interactions and physics of the 20th century.

Bibliography of Memoir 3

On Newtonian gravitation

- [1] H. Lumbroso, Problèmes résolus de mécanique du point et des systèmes de points. Dunod, Paris, 1984

On electrostatics

- [2] H. Lumbroso, Problèmes résolus d'électrostatique et dynamique des particules chargées. Dunod, Paris, 1984

On magnetism

- [3] H. Lumbroso, Electromagnétisme, problèmes résolus. McGraw-Hill, Paris, 1985.

On Maxwell's equations and antennas

- [4] Wikipedia, Équations de Maxwell
[5] H. Lumbroso, Problèmes résolus sur les ondes électromagnétiques. Dunod, Paris, 1991.

On group velocity

- [6] Wikipedia, vitesse d'une onde
[7] Wikipedia, wave velocity

On the wave approach of classical and relativistic mechanics

- [8] L. de Broglie, thèse de 1924, Paris
[9] L. de Broglie, Eléments de théorie des quanta et de mécanique ondulatoire, Gauthier-Villars, Paris, 1953

On the momentum flux density tensor

- [10] Wikipedia, Tenseur de densité du flux d'impulsion

On Planck dimensions:

- [11] Vincent Isoz, site internet sur la physique et les mathématiques
<http://www.sciences.ch/htmlfr/physatomique/physatommecquantond02.php#dimensionsplanck>

On dark matter:

- [12] Wikipedia, Dark matter

On the tides:

- [13] Guillaume Pages, Année universitaire 2018/2019 Mémoire de Master 2 LES MARÉES
[14] Wikipedia, Marée, <https://fr.wikipedia.org/wiki/Mar%C3%A9e>
[15] <http://hmf.enseciht.fr/travaux/CD9598/travaux/optsee/hym/nome35/maree/2.htm>

Table of Contents

Memoir 3	Maxwell's source field equations to gravitational equations.....	133
Chapter I	Original idea of memory, velocity of electric charge and historical reminders	134
I.1	Original idea of the Memoir, a velocity of the electric charge comparable to a group velocity?.....	134
I.2	Historical reminders, Maxwell source field equations.....	134
I.3	Drawing inspiration from gravitational waves.....	135
I.4	Conclusion of the chapter.....	137
Chapter II	Reminders phase velocity, group velocity, extensions of relativistic mechanics and Newtonian mechanics to wave mechanics, elements of Special Relativity	138
II.1	Reminders on phase velocity and group velocity.....	138
II.2	Extensions of relativistic mechanics and Newtonian mechanics to wave mechanics.....	139
II.3	Some reminders about the relativistic Doppler-Fizeau effect.....	146
II.4	Some reminders of special relativistic electromagnetism.....	149
II.5	Conclusion of the chapter.....	151
Chapter III	Maxwell's source field equations and group velocity.....	152
III.1	Reminder on Maxwell's sources field equations.....	152
III.2	Rewriting Maxwell's sources field equations in a 3-dimensional Space-Time.....	153
III.3	From Maxwell's sources field equations to group velocity of a particle wave.....	154
III.4	Conclusion of the chapter.....	156
Chapter IV	Ampère and Gauss theorems, source field equations for gravitation.....	157
IV.1	Space-oriented sources: electric current density, Maxwell Ampere equation.....	157
IV.2	Time-oriented sources: electric charge density, Maxwell Gauss equation.....	159
IV.3	Analogies electromagnetism and gravitation, momentum-type sources oriented in Space, Ampere's theorem.....	161
IV.4	Analogies electromagnetism and gravitation, mass-type sources oriented in Space, Ampere's theorem.....	168
IV.5	Reminders on the Compton wavelength.....	173
IV.6	Hamiltonian mechanics reminders.....	174
IV.7	Analogies between electromagnetism and gravitation, Time-oriented sources, Gauss's theorem.....	175
IV.8	Tensor approach.....	181
IV.9	Conclusion of the chapter.....	182
Chapter V	Conditions for obtaining group velocity.....	183
V.1	Uniting constants \hbar and \mathbf{G} in the same model?.....	183
V.2	Uniting constants \hbar and ϵ_0 in the same model?.....	188
V.3	Uniting the constants μ_0 and \mathbf{G} in the same model?.....	192
V.4	Conclusion of the chapter.....	193
Chapter VI	Can we intuitively understand Ampère's theorem? Study of 2 tracks.....	194
VI.1	What is demonstrated and what is not.....	194
VI.2	First track: 2 analogies between fluid mechanics and magnetism.....	194
VI.3	Second track: change of points of view, study of two bodies of the Earth-Sun type.....	197
VI.4	Analogies between Gauss's theorem and Earth-Sun or Earth-Star systems.....	200
VI.5	Conclusion of the chapter.....	204
Chapter VII	Gravitationally analogous to Maxwell Faraday's equation?.....	205

VII.1	On the phenomena of electrical induction.....	205
VII.2	Maxwell Faraday's equation transcribed from electromagnetism to gravitation.....	207
VII.3	Tensor approach, Maxwell Faraday and Maxwell Thomson	212
VII.4	Conclusion of the chapter.....	214
Chapter VIII Wave character of electric, magnetic, wave vector and pulsation fields ...		215
VIII.1	Electric field and wave vector field.....	215
VIII.2	Magnetic fields and pulsation.....	216
VIII.3	Examples of solving wave equations	217
VIII.4	Conclusion of the chapter.....	218
Chapter IX Why 2 types of electric charges and only 1 of mass? Example of a simplified tidal model		219
IX.1	Analogies between electromagnetism and gravitation.....	219
IX.2	Reminders on dipole moments and dipoles.....	220
IX.3	Static model I of the tides, existence of a "gravitostatic" dipole?	223
IX.4	Static model II of tides, existence of attractive and repulsive forces for gravitation?.....	226
IX.5	Tides in motion.....	236
IX.6	Complements on accelerations and inertial forces, summary of their analogues on the one hand between Gravitation and Electromagnetism, on the other hand between Space and Time.	240
IX.7	Conclusion.....	240
Chapter X Form and conclusion of the Memoir.....		242
X.1	Summary of the main equations (in a space x, y, t).....	242
X.2	Conclusion of the Memoir.....	243
Bibliography of Memoir 3.....		244

Memoir 4 Elements on nuclear interactions

Summary

This 4th Memoir focuses on what happens in the nucleus of the atom, that is to say nuclear interactions. It is mainly a historical account of the main models on this subject, from the 1910s to the 1970s.

We will first mention the scattering of Ernest Rutherford (1911), the work of W. Heisenberg on isospin (1932), the theory of E. Fermi on the weak interaction (1933), Hideki Yukawa's model for nuclear interactions (1935). This model involves a photon-type mediating particle, but with mass (the particle was named mesotron by H. Yukawa, it was later called the meson).

We will then talk about the work of Chen Ning Yang and Robert Mills (1954), decrying the strong interaction, from Gauge transformations belonging to the group $SU(2)_I$. We will focus on the Glashow-Weinberg-Salam electroweak model (1960s), bringing together the electromagnetic interaction based on the group $U(1)_Q$, the weak interaction based on $U(1)_T$ and a 3rd interaction based on $U(1)_{Y_W}$. Finally, we will briefly mention the BEH mechanism (mechanism proposed independently in 1964 by R. Brout and F. Englert, as well as by P. Higgs) to assign a mass to particles.

Chapter I Ernest Rutherford's scattering (1911) and particle collisions

Purpose of the chapter

We are interested in Rutherford scattering, a precursor of experiments of collisions between particles and involving the electromagnetic force. We will approach collisions between particles according to a classical approach, then according to a relativistic quantum approach.

I.1 The discovery of radioactivity by Henri Becquerel (1886)

Usually, the first research on the nucleus is dated by the discovery in 1896 of radioactivity. Studying uranium salts away from sunlight, Henri Becquerel observed that these salts emit radiation. Subsequent experiments conducted by H. Becquerel himself, as well as by Marie Skłodowska-Curie and Pierre Curie concluded that there were three types of radiations called α , β and γ (following the first 3 letters of the Greek alphabet).

The discovery aroused strong interest among physicists. Many people studied uranium salts and the nature of their radiation. Quickly, it was realized that the radiations α are positively charged, that the radiations β are negatively charged, and that the radiations γ are neutral. In addition, it was found that radiations α are much more massive than radiations β . Gradually, it was understood that radiations α consist of positively charged helium nuclei, that radiations β consist of negatively charged electrons, and that radiations γ are an electromagnetic wave.

During the 1930s and 1940s, physicists came to distinguish between two types of nuclear interactions. They linked radiation β to the so-called weak interaction and the decay of the nucleus. They associated the so-called strong interaction with the cohesion of the nucleus, i.e., what holds the nucleons together.

Note on the scope of weak and strong interactions

Weak and strong nuclear interactions are characterized by a very low scope (at the level of the nucleus of the atom). In this, they are opposed to gravitational and electromagnetic forces that have an infinite scope.

I.2 Rutherford's experiments (1909-1911)

From 1909 to 1911, E. Rutherford and his two collaborators Hans Geiger and Ernest Marsden used radiations of particles α (helium nuclei) to cause collisions between particles. We speak of incident particles before the collision and scattered particles after the collision, hence the term Rutherford scattering.

These experiences are of great importance for several reasons. First, they can be presented as the precursors of collisions between particles in accelerators. Then, they allow E. Rutherford to propose a new model for the atom.

Let's explain one of these experiences. Using a beam of particles α , E. Rutherford and his two collaborators bombard a very thin layer of gold. Most of the particle beam passes through the gold layer and reaches its target, i.e., propagates in a straight line. However, some particles α , in a very small proportion (of the order of 0.01%), are deflected with a very wide angle.

To explain these results, E. Rutherford imagines a planetary model of the atom, which a few years later will inspire N. Bohr in his explanation of the luminous lines emitted by excited atoms (see Memoir 1).

In this planetary model, the atom is mainly composed of vacuum, which explains why most particles α can get through. In the centre, there is an electrically positively charged nucleus.

By effect of the electrostatic Coulomb force, this nucleus deflects in a small proportion and with a very large angle some particles α . Finally, around the nucleus, there are electrons of negative charge in rotation.

I.3 Rutherford scattering in classical mechanics

Let us study Rutherford scattering in the context of classical mechanics (non-quantum and non-relativistic), such as E. Rutherford describes it himself.

Let be incident particles of positive charge q (helium nuclei) in motion that collide with positive charged particles Q (the nuclei of the atoms of the gold leaf). Using the Coulomb electrostatic force, we deduce the effective cross-section σ of Rutherford scattering:

$$\frac{d\sigma}{d\Omega} = \left(\frac{qQ}{16\pi\epsilon_0 E_0} \right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

with:

$E_0 = \frac{1}{2}mv^2$ the initial energy of the incident particle on the scattering center (immobile particles of the gold leaf),

θ angle of the scattered particle,

σ the effective cross-section of incident particles,

Ω the solid angle of the scattered particles.

According to the diagram below, if an incident particle passes through the corona $d\sigma$, the scattered particle passes through the corona $d\Omega$. The scattering center is for example an atom nucleus of gold leaf.

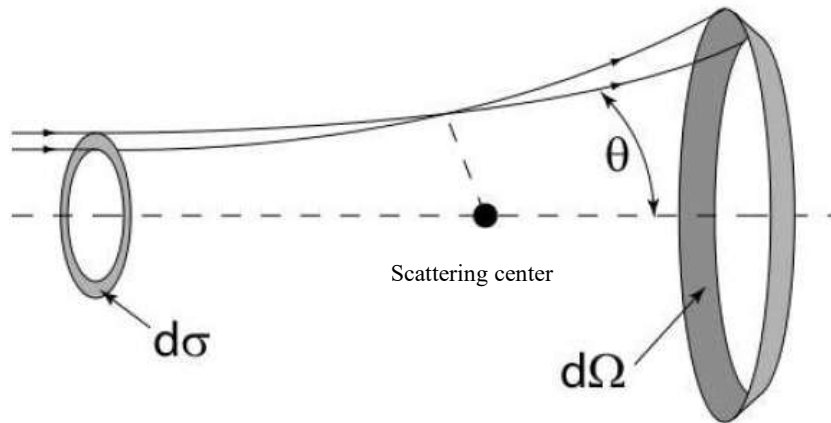


Figure 1: Rutherford scattering (source Wikipedia)

For electron or proton particles of electric charge e , we usually express the effective section σ using α_{em} . We have:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha_{em}\hbar c}{4E_0} \right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

Note

$$\alpha_{em} = \frac{e^2}{\hbar c 4\pi\epsilon_0} \approx \frac{1}{137} \text{ (fine-structure constant or electromagnetic coupling constant).}$$

I.4 Rutherford scattering in the context of relativistic quantum electrodynamics

I.4.1 Preview on mediating particles

Let us give a simplified version of Rutherford scattering in the context of relativistic Quantum Electrodynamics (QED). We take up Richard Feynman's ideas on collisions between particles, ideas developed in the years 1940-1950.

We will come back to this later, but note that one of the main ideas of relativistic quantum electrodynamics is that an incident particle 1 (for example, an electron 1) does not react directly with a particle 2 of the scattering center (for example, another electron 2) via the Coulomb electrostatic force. There is a mediating particle called boson that reacts on the one hand with the incident electron 1, on the other hand with the electron 2 of the scattering center. In the case of electromagnetic interaction, the mediating particle is the photon.

I.4.2 Probability amplitude of a propagator and vertices

We remind Klein Gordon's relativistic wave equation:

$$((p^\mu)^2 c^2 - m^2 c^4)\psi(0) = 0$$

with $(p^\mu) = (E^t, p^x, p^y, p^z)$ the quadrivector energy momentum of Special Relativity.

In relativistic quantum electrodynamics, the Klein Gordon wave function is interpreted as the boson wave equation (Mediating particle of interactions).

From this equation, R. Feynman defines a propagator, associated with the boson that propagates with a quadrivector energy momentum p^μ and a mass m :

$$Pr o p a = \frac{i}{(p^\mu)^2 c^2 - m^2 c^4}$$

$$Pr o p a = \frac{i}{(p^\mu)^2 - m^2}$$

with the constant c chosen as equal to 1.

R. Feynman also defines a probability amplitude of vertices 1 and 2 describing the emission absorption of the boson by particles 1 and 2. This probability amplitude depends directly on the coupling constant with the particles 1 and 2 (for example proportional to charges with coupling g_1 or g_2).

We have:

$$Vertice = g_1$$

$$Vertice = g_2$$

We obtain the probability amplitude M during the collision between the particles 1 and 2:

$$M \propto g_1 \cdot \frac{i}{(p^\mu)^2 - m^2} \cdot g_2$$

We have the probability:

$$Probability \propto \left| \frac{g_1 g_2}{(p^\mu)^2 - m^2} \right|^2$$

This interpretation makes it possible to develop a simple graphical method and to calculate the probability of certain processes. This graphical method is called Feynman diagrams.

We have below the Feynman diagram corresponding to the collision between 2 moving particles and the intervention of an intermediate boson.

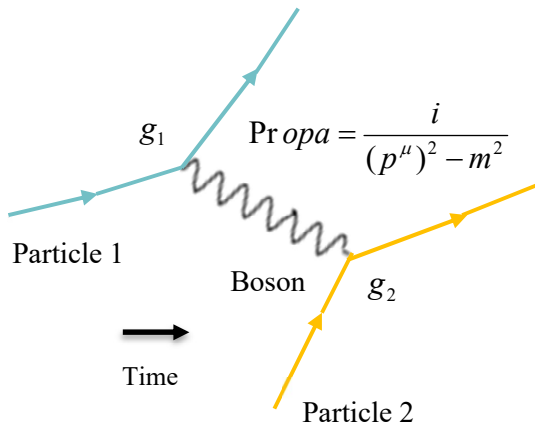


Figure 2: collision between two particles 1 and 2, intermediate boson

Note

The arrows of a Feynman diagram indicate the sense of the particle's momentum and not the sense of the reaction (which is indicated by Time). The particles have an arrow oriented in the sense of Time. Antiparticles have an arrow oriented in the opposite sense of Time.

In QED, fermions (particles of matter) have a straight line. Photons (light-mediating particles) have a wavy line.

I.4.3 Example of electromagnetic interaction

For electromagnetic interaction, the mediating particle is the photon γ of zero mass with $Propa = \frac{i}{(p^\mu)^2}$ and coupling charge $g = e$. So, we have:

$$Probability \propto \left| \frac{e^2}{(p^\mu)^2} \right|^2 = \frac{e^4}{(p^\mu)^4} \propto \frac{\alpha_{em}^2}{(p^\mu)^4}$$

We give the Feynman diagram for the Rutherford scattering between two moving electrons.

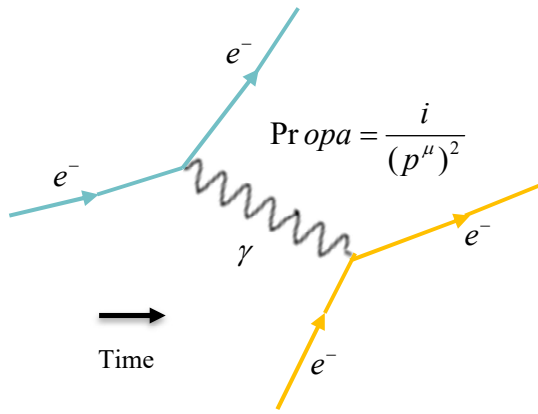


Figure 3: collision between two electrons and intermediate photon γ

I.4.4 Rediscover Rutherford scattering in the classic setting

We give the cross section for the collision of charged particles, during Rutherford scattering, in the context of relativistic electrodynamics:

$$\frac{d\sigma}{d\Omega} \propto \frac{\alpha_{em}^2}{(p^\mu)^4} E_0^2$$

(p^μ) is the amount of momentum energy transferred to the scattering center by the incident particle during the scattering process. It is also equal to the amount of momentum energy transported by the photon mediating particle.

E_0 is the initial energy of the incident particle.

We have the relationship between the initial energy E_0 , the momentum energy transferred (p^μ) , and angle θ of the scattered particle:

$$(p^\mu)^2 = 2E_0^2(1 - \cos \theta)$$

$$(p^\mu)^2 = 4E_0^2 \sin^2\left(\frac{\theta}{2}\right)$$

From $\frac{d\sigma}{d\Omega} \propto \frac{\alpha_{em}^2}{(p^\mu)^4} E_0^2$ and using $(p^\mu)^2 = 4E_0^2 \sin^2\left(\frac{\theta}{2}\right)$, we find the expression of Rutherford scattering in the context of the classical approach:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha_{em} \hbar c}{4E_0}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

I.5 Conclusion of the chapter

Rutherford scattering is the precursor to particle collisions. E. Rutherford gave it a classic description. This scattering can also be more finely described by relativistic quantum electrodynamics and Feynman diagrams.

These collisions make it possible to study incident and scattered particles, mediating particles, as well as the interactions involved. For example, they provide information on certain characteristic properties of particles. Among these properties is the spin or isospin that we will deal with in the next chapter.

Chapter II The spin of Uhlenbeck and Goudsmit (1925) and the isospin of Heisenberg (1932)

Purpose of the chapter

The spin S is a physical notion proposed by G. Uhlenbeck and S. Goudsmit in 1925, to describe electromagnetic interaction. The isospin I is a notion proposed by W. Heisenberg in 1932, by mathematical analogy with spin, to describe nuclear interactions. We remind here some analogies between the spin which reverses during a hyperfine transition of the electron, and the isospin which reverses during a decay β^- when you go from a neutron to a proton.

II.1 Le spin S by G. Uhlenbeck and S. Goudsmit (1925)

II.1.1 Change of states and modification of the orbital angular momentum of an electron

In 1913, the physicist N. Bohr managed to explain the luminous lines emitted by an excited hydrogen atom, by proposing the model of an electron rotating around the proton. In this model, the electron can only occupy certain possible orbits (or possible orbital angular momentums). N. Bohr matches these orbits to electron states and energy levels. The passage from one state to another of the electron corresponds to the emission of a luminous line by the hydrogen atom.

Nevertheless, in the Bohr model of the electron, there are still some unexplained phenomena such as a doubling of the light lines. There is not a single line, but actually two lines very close in wavelength called doublets.

II.1.2 Change of states and modification of the spin S of an electron, hyperfine transition

In 1922, Otto Stern and Walther Gerlach passed a beam of silver atoms through a magnetic field. While conventionally, silver atoms should not undergo the effect of the magnetic field, the 2 experimenters observe a separation of the beam in two.

To explain these facts (doubling of the lines, separation of the beam in two and others such as the anomalous Zeeman effect), George Uhlenbeck and Samuel Goudsmit proposed in 1925 that particles such as the electron have their own magnetism (this must also be the case of silver atoms in the experiment of Stern and Gerlach).

In a way, the electron is not only comparable to a small electric charge, but also to a small magnet. The model represents the electron as a small charged and massive sphere, rotating on itself and possessing a magnetic moment of spin, connected to a spin angular momentum taking values of the type: $+\frac{\hbar}{2}$, $-\frac{\hbar}{2}$ (hence the 2 beams for silver atoms of the Stern and Gerlach experiment).

The doublets of light lines very close in wavelength to each other, are explained by an inversion of the spin of the electron, passing for example from $+\frac{\hbar}{2}$ to $-\frac{\hbar}{2}$, inversion of the spin called hyperfine transition.

Even if the electron has one of its physical characteristics modified (in this case spin), it is indeed the same particle in two different states.

The spin hypothesis is particularly fruitful. As we saw in Memoir 1, it is then incorporated into many models such as the non-relativistic Pauli wave equation or the Dirac relativistic wave equation. Spin also inspired W. Heisenberg to develop the notion of isospin., which a priori has nothing to do physically with spin, but has strong formal and mathematical analogies.

II.1.3 Mathematical treatment of total angular momentum and spin

In quantum mechanics, we define operators on the wave function ψ , transcriptions of notions from classical mechanics to quantum mechanics. We thus have the position and momentum operators on a wave function ψ defined by:

$$\hat{x}\psi = x \cdot \psi$$

$$\hat{p}^x\psi = -i\hbar \frac{\partial\psi}{\partial x}$$

In classical mechanics, the oriented angular momentum following z, has the expression:

$$L^z = xp^y - yp^x$$

In quantum mechanics, the orbital angular momentum operator is defined on the wave function ψ :

$$\hat{L}^z = \hat{x}\hat{p}^y - \hat{y}\hat{p}^x$$

The total angular momentum operator noted \hat{J} is the vector sum of the orbital angular momentum operator noted \hat{L} and the (intrinsic) spin angular momentum operator noted \hat{S} :

$$\hat{J} = \hat{L} + \hat{S}$$

We usually define \hat{J} using \hat{J}^3 the angular momentum along the z-axis and its norm $(\hat{J})^2 = (\hat{J}^1)^2 + (\hat{J}^2)^2 + (\hat{J}^3)^2$.

We look for eigenstates (eigenvectors) common to $(\hat{J})^2$ and \hat{J}^3 , as well as their eigenvalues. Mathematically, it is shown that $|j, m\rangle$ is the desired eigenstate, $\hbar^2 j(j+1)$ the eigenvalue of $(\hat{J})^2$, $\hbar m$ the eigenvalue of \hat{J}^3 .

We get:

$$(\hat{J})^2|j, m\rangle = \hbar^2 j(j+1)|j, m\rangle$$

$$\hat{J}^3|j, m\rangle = \hbar m|j, m\rangle$$

with j integer or half integer,
with $-j \leq m \leq +j$ per jump of one unit.

The mathematical treatment is exactly the same for an orbital angular momentum L or a spin angular momentum S . In the case of S , we usually define angular momentum operators \hat{S}^3 and $(\hat{S})^2 = (\hat{S}^1)^2 + (\hat{S}^2)^2 + (\hat{S}^3)^2$. We have the eigenstates common to $(\hat{S})^2$ and \hat{S}^3 , as well as eigenvalues:

$$(\hat{S})^2|s, m_s\rangle = \hbar^2 s(s+1)|s, m_s\rangle$$

$$\hat{S}^3|s, m_s\rangle = \hbar m_s|s, m_s\rangle$$

with s integer or half integer,
with $-s \leq m_s \leq +s$ per jump of one unit.

II.1.4 Building a composite body formed by 2 particles with spin $\frac{1}{2}$

By binding 2 particles bearing spin $s = \frac{1}{2}$ and $s' = \frac{1}{2}$ in an orbital angular momentum state $L = 0$, we can construct 2 total spin states $S = 1$ and $S = 0$. The total spin state $S = 1$ in turn breaks down into 3 states, with $S^3 = \pm 1$ and $S^3 = 0$.

Below, we take as example 2 electrons e and e' with spin s^3 and $s^{3'}$.

$$|S = 1, S^3 = 1\rangle = \left|s = \frac{1}{2}, s^3 = \frac{1}{2}; s' = \frac{1}{2}, s^{3'} = \frac{1}{2}\right\rangle = |e_R; e_R'\rangle$$

$$\begin{aligned} |S = 1, S^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left|s = \frac{1}{2}, s^3 = \frac{1}{2}; s' = \frac{1}{2}, s^{3'} = -\frac{1}{2}\right\rangle + \left|s = \frac{1}{2}, s^3 = -\frac{1}{2}; s' = \frac{1}{2}, s^{3'} = \frac{1}{2}\right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|e_R; e_L'\rangle + |e_L; e_R'\rangle) \end{aligned}$$

$$|S = 1, S^3 = -1\rangle = \left|s = \frac{1}{2}, s^3 = -\frac{1}{2}; s' = \frac{1}{2}, s^{3'} = -\frac{1}{2}\right\rangle = |e_L; e_L'\rangle$$

$$\begin{aligned} |S = 0, S^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left|s = \frac{1}{2}, s^3 = \frac{1}{2}; s' = \frac{1}{2}, s^{3'} = -\frac{1}{2}\right\rangle - \left|s = \frac{1}{2}, s^3 = -\frac{1}{2}; s' = \frac{1}{2}, s^{3'} = \frac{1}{2}\right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|e_R; e_L'\rangle - |e_L; e_R'\rangle) \end{aligned}$$

II.2 Heisenberg isospin (1932) defined as a strong nuclear charge of the neutron and proton

II.2.1 Analogies of proton and neutron behaviour

The isospin I is a concept introduced by Werner Heisenberg in 1932 to explain certain facts involved in nuclear interactions.

At the end of the 1920s, it was noticed that vis-à-vis a certain nuclear interaction (those we call today the strong interaction), the proton and the neutron behave in the same way.

Like electromagnetism, where the intensity of the interaction is the same in absolute value between 2 electrons, between 2 protons or between 1 electron and 1 proton, the intensity of the strong nuclear interaction is roughly the same in absolute value between 2 protons, between 2 neutrons or between 1 proton and 1 neutron.

It is also noted that proton and neutron have a very close mass. Finally, it is known that the neutron can decay into a proton during decay. β^- where an electron is emitted. This reaction involves another nuclear interaction that is now called the weak interaction.

To explain these different facts, W. Heisenberg imagined that the proton and the neutron are two sides of the same particle called the nucleon.

First, he assigned to the proton and neutron a strong opposite nuclear charge, which he named isospin. This strong opposite nuclear charge explains the attraction between proton and neutron and therefore the cohesion of the atomic nucleus.

Then, he transposed the change of spin state of the electron via the hyperfine transition, to a change of state of the isospin of the particles of the nucleus (protons and neutrons) via decay. β^- . Neutron and proton are nucleons of opposite isospins ($+\frac{\hbar}{2}$ for the proton, and $-\frac{\hbar}{2}$ for the neutron), the transition from one to the other is explained by an inversion of their isospin.

Note 1

For spin and isospin values, rather than saying $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$, it is also used very frequently $+\frac{1}{2}$ or $-\frac{1}{2}$.

Note 2

By convention, in quantum physics, it is usually chosen that it is the 3rd component of spin, noted here S^3 , that perpendicular to the plane x, y and oriented according to z, which is equal to $\pm\frac{\hbar}{2}$ and which reverses during the hyperfine transition.

Similarly, it is chosen that it is the 3rd component of isospin, noted here I^3 , which is equal to $\pm \frac{\hbar}{2}$ and which reverses during decay β^- .

Note 3

Like the proton and the electron which respectively have an electric charge Q of $+1$ and -1 , it will be seen in a future chapter on the strong interaction, that the Yang-Mills theory assigns to the neutron and the proton a strong nuclear charge identified with isospin, and respectively equal to $-\frac{1}{2}$ and $+\frac{1}{2}$.

Note 4

In the 1950s and 1960s, the notion of isospin is taken up in the Quark Model, with the notion of flavor (for quarks of 1st generation) which corresponds to that of isospin. An up flavored quark has an isospin equal to $+\frac{1}{2}$ and a down flavor quark has an isospin equal to $-\frac{1}{2}$. We'll come back to that.

Note 5

At the time of W. Heisenberg, there was only one notion of isospin noted here I . Today, we can distinguish 3 distinct notions:

- the weak isospin or weak charge noted T ,
- the isospin I (which is reversed when one passes from a neutron to a proton),
- the strong nuclear charge that will be noted in this essay Co as color.

I and Co are sometimes confused as in the Yang-Mills theory on strong interactions. The strong charge Co is then considered equal to isospin I (we speak of strong isospin and strong charge for I).

Weak isospin or weak charge T is a notion proposed by S. Glashow to describe the weak interaction, in strict analogy with the Yang-Mills theory describing the strong interaction, where the strong charge is considered equal to the isospin I .

In that case, I and T are both considered to be the charge of a nuclear interaction, respectively strong and weak. They also share the same mathematical formalism, since they are associated with the gauge transformations of the group $SU(2)$.

We will return to all these concepts at length in this Memoir and in the next.

II.2.2 Mediating particle

To explain the change of state of the particle (electron or nucleon), W. Heisenberg proposes the existence of a mediating particle, carrying the quantum quantity spin or isospin, and which allows the conservation of the quantum quantity in reactions between particles. The mediating particle is emitted or absorbed by the particle when the latter changes state.

Thus, just as a light particle such as the electron, can by changing its spin state emit or absorb an ultra-light particle that is the photon, so a heavy particle that is the nucleon could by changing its isospin state emit light particles that are electrons or positrons.

This is what we observe in decay β^- , when the neutron turns into a proton and an electron is emitted.

Like isospin, the particle mediating hypothesis will be particularly fruitful. Nevertheless, in the case of isospin change, the electron and positron will not in fact be the mediating particle sought. It will soon be seen that other mediating particles will be proposed for nuclear interactions, in particular by Hideki Yukawa in 1935.

II.2.3 Mathematical treatment of isospin I

We have the same mathematical treatment for spin and isospin. Isospin operators are usually defined \hat{f}^3 and $(\hat{f})^2 = (\hat{f}^1)^2 + (\hat{f}^2)^2 + (\hat{f}^3)^2$.

We have the eigenstates common to $(\hat{I})^2$ and \hat{I}^3 , as well as eigenvalues:

$$(\hat{I})^2|i, m_i\rangle = \hbar^2 i(i+1)|i, m_i\rangle$$

$$\hat{I}^3|i, m_i\rangle = \hbar m_i|i, m_i\rangle$$

with i integer or half integer,
with $-s \leq m_i \leq +s$ per jump of one unit.

By binding 2 particles carrying an isospin $i = \frac{1}{2}$, we can construct 4 states of total isospin $I = 1$ and $I = 0$, $I^3 = \pm 1$. Here we take as an example a proton and an isospin neutron i^3 opposites (as well as an antiproton and an antineutron).

For matter, we have the proton neutron doublet $\begin{pmatrix} n \\ p \end{pmatrix}$. For antimatter, we have the doublet $\begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$.

We get:

$$|I = 1, I^3 = 1\rangle = \left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle = |p; -\bar{n}\rangle$$

$$\begin{aligned} |I = 1, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle + \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|p; \bar{p}\rangle + |n; -\bar{n}\rangle) \end{aligned}$$

$$|I = 1, I^3 = -1\rangle = \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle = |n; \bar{p}\rangle$$

$$\begin{aligned} |I = 0, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle - \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|p; \bar{p}\rangle - |n; -\bar{n}\rangle) \end{aligned}$$

II.3 Analogies between spin S reversal and isospin I reversal

II.3.1 Spin S reversal during a hyperfine transition of the electron

The hyperfine transition of the hydrogen atom is the emission by the hydrogen atom of invisible light, which has a wavelength of 21cm in the case where the electron returns to its most stable state (lowest orbit or Bohr radius). This corresponds to the passage from an energy level to a new level very close to the previous one, hence the term hyperfine.

As mentioned above, G. Uhlenbeck and S. Goudsmit proposed that this hyperfine transition corresponds to the passage of a movement where electron and proton rotate on themselves in the same sense, to a movement where electron and proton rotate on themselves in opposite senses (we speak of a spin passage of the proton and the electron from parallel to antiparallel).

To take a comparison with the Earth-Sun system, a hyperfine transition would be equivalent to reversing the spin of the Earth or that of the Sun. For example, if we reverse the spin of the Earth, for Earthlings, the Sun no longer rises in the east, but in the west. If we reverse the spin of the Sun, for Earthlings, the Sun always rises in the east, but its rotation on itself in one year (relative to the Earth) is done in the other sense.

We can write the hyperfine transition (with global conservation of the spin S^3 during the reaction and conservation of the electric charge Q by the electron):

$$e^-(S^3 = \frac{\hbar}{2}, Q = -1) \rightarrow e^-(S^3 = -\frac{\hbar}{2}, Q = -1) + \gamma(S^3 = \hbar, Q = 0)$$

Nota

In the case of the hydrogen atom, the most stable case (least excited atom for spin) corresponds to electron and proton of antiparallel spins.

We have the following figure which symbolizes the passage from parallel to anti-parallel with emission of a photon of energy $\Delta E = 10^{-6} eV$.

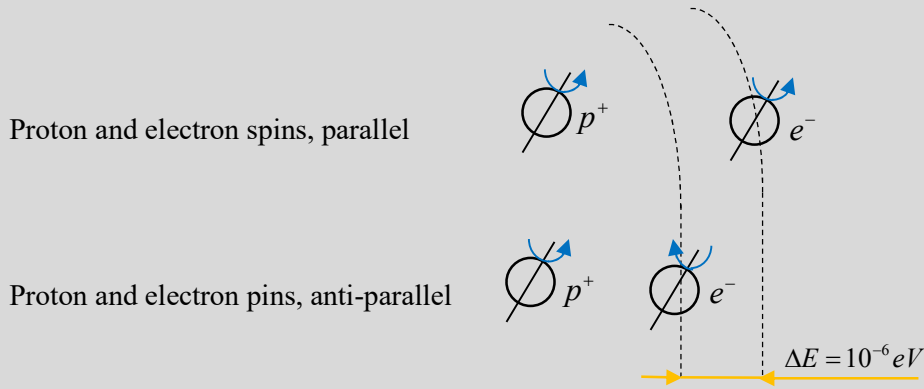


Figure 4: Hyperfine transition of the electron

II.3.2 Isospin I inversion of the nucleon during a decay β^-

By analogy with the hyperfine transition that reverses the spin S of the electron, we assume a reaction that reverses isospin I of the nucleon. For example, we have the passage from a neutron to a proton observed during a decay β^- .

$$n(I^3 = -\frac{\hbar}{2}, Q = 0) \rightarrow p(I^3 = \frac{\hbar}{2}, Q = 1) + B(I^3 = -\hbar, Q = -1)$$

By analogy with the photon γ who carries a spin $S^3 = \hbar$, here we assume the existence of a boson B who carries an isospin $I^3 = -\hbar$ and which allows the conservation of I^3 .

Note

During the passage from the neutron to the proton, the electric charge Q is not conserved at nucleon level.

In the next Memoir, it will be proposed that the analogue of electric charge Q when reversing spin S , this is the weak hypercharge Y_W (concept introduced in the Electroweak Model) when reversing isospin I .

As the electric charge Q is conserved by the electron during a hyperfine transition, the weak hypercharge Y_W is conserved by the nucleon (left-handed) when this nucleon passes from neutron to proton:

$$n(I^3 = -\frac{\hbar}{2}, Y_W = 1) \rightarrow p(I^3 = \frac{\hbar}{2}, Y_W = 1) + B(I^3 = -\hbar, Y_W = 0)$$

II.4 Mediating particle and the notion of interaction

II.4.1 Classical mechanics and electromagnetism (before the 1920s), do not distinguish the particle that undergoes the field from the one that generates it

In mechanics and classical electromagnetism, a body (mass or electrically charged) entering a field, is subjected to a force from this field.

As a possible ambivalence, the body (mass or charged) can itself create a field (gravitational or electromagnetic), to which other bodies will be subjected. We then speak of sources (electric charge, electric current, magnet or mass) for the body generating the field.

During the years 1920-1930, a new approach appeared with the idea of distinguishing particles undergoing the field and particles generating the field (or carrying the field).

The particles undergoing the field are called fermions because they follow the Fermi-Dirac statistic. They have a half-integer spin. Often, they are persistent particles of matter, in the sense that they do not appear or disappear frequently.

They participate in scattering reactions: collision with another fermion and then transformation of certain characteristics such as spin, momentum, etc. They also participate in annihilation reactions, for example between an electron and a positron. They follow the exclusion principle proposed by W. Pauli in 1926: all fermions in the same system cannot simultaneously share the same quantum state.

The particles generating the field are the mediating particles mentioned above. They are called bosons because they follow the Bose-Einstein statistic. They have an entire spin.

These mediating particles are often ephemeral, in the sense that they can appear or disappear easily. Among them are photons, particles of light. Bosons do not respect the Pauli exclusion principle and can therefore, in the same system, simultaneously share the same quantum state.

Note on Supersymmetry

With this new conception of a mediating particle (or boson) representing the field, we lose the idea of interchangeability between fermion particles that can undergo a field and boson particles that can create a field.

In the 1960s, Russian researchers developed Supersymmetry models with the idea of matching each fermion particle, a boson particle, in order to find this interchangeability between particle that creates the field and particle that undergoes it.

The particles predicted by supersymmetry models were ultimately not discovered. Despite the theoretical interest they had, these models have not been confirmed.

II.4.2 The notion of interaction

The notion of force dear to classical mechanics is gradually replaced by the notion of interaction. This can be seen as a more all-encompassing approach than that of force. It also distinguishes between particles of matter undergoing the field and mediating particles generating it.

Particles undergoing the field and carrying a half integer spin S^3 or a half integer isospin I^3 , cannot react directly to each other. They interact with mediating particles carrying integer spin or integer isospin, which act as intermediaries.

During the hyperfine transition, it is the electromagnetic interaction that intervenes. The photon γ is presented as the mediating particle of the interaction (as well as the representative of the electromagnetic field).

When decay β^- , we will see at the end of this Memoir that the Electroweak Model involves 3 interactions: electromagnetic interaction, weak interaction and a last one called hyper interaction.

II.5 Conclusion

In this chapter, the notion of isospin I introduced by W. Heisenberg in 1932 was mentioned.

The isospin I first has analogies with electric charge Q , in the sense that I is presented as the strong nuclear charge with opposite values for the proton and neutron, thus explaining their attraction in the nucleus.

The isospin I also has analogies with spin S both mathematical and physical during reactions between particles. The spin S reverses during a hyperfine transition transforming an electron into an electron of opposite spin, with the two electrons being presented as two faces of the same particle. The isospin I reverses when switching from a neutron to a proton, with the two nucleons being presented as two sides of the same particle.

In the next chapter, we will focus on decay β^\pm and the first weak model on weak interaction, that of Enrico Fermi in 1933. In particular, we will discuss the idea of electromagnetic currents charge density quadrivector $j_{e.m.}^\mu$ (noted here j_Q^μ because carrying an electric charge Q) and weak currents charge density quadrivector j_{weak}^μ .

Chapter III Fermi's theory about weak interaction (1933)

Purpose of the chapter

We summarize in broad outline the Fermi theory on the weak interaction, a theory based on an analogy with electromagnetism and on a universality of the phenomena of decay of particles.

On the model of the electromagnetic current charge density quadrivector j_Q^μ formed with electron and positron decaying into photon-mediating particles γ , Fermi theory defines a weak current charge density quadrivector j_{weak}^μ also decaying into particles.

III.1 Analogy between particle decays

III.1.1 Fermi's idea: universality of decay phenomena

In 1933, E. Fermi proposed a first theory on weak interaction and decay β^- . His idea is to make an analogy between:

- the decay of an electron and a positron into a photon, a decay that involves electromagnetic interaction, the photon in turn materializing as a positron and an electron,
- the decay β^- , decay that involves the weak interaction, with the neutron decaying into a proton, an electron and an antineutrino.

Thus, E. Fermi tries to show the universality of several phenomena of decay of particles.

Note

W. Heisenberg had studied analogies between hyperfine transition and decay β^- to develop his notion of isospin. For his part, E. Fermi studies analogies between electron positron annihilation and decay β^- to develop a general notion of current charge density quadrivector.

Note that hyperfine transition and positron electron annihilation both generate a photon and involve electromagnetic interaction. We will come back to this later.

III.1.2 Dirac's idea: current charge density quadrivector, electron and positron couple

A few years before E. Fermi, P. Dirac proposes that the electron e (wave function ψ) and positron \bar{e} (wave function $\bar{\psi}$) couple forms an electromagnetic currents charge density quadrivector $j_Q^\mu(e^-, \bar{e}^+) = Q\bar{\psi}\gamma^\mu\psi$ (with $\mu = x, y, z, t$ or $\mu = 0, 1, 2, 3$ and Q the number of elementary electric charges e).

We have the interaction energy between the currents charge density quadrivector j_Q^μ and electromagnetic potential A^μ :

$$E = e j_Q^\mu A^\mu$$

The electron positron couple also forms an electromagnetic current charge density quadrivector $j_Q^{\mu'}(\bar{e}^{+'}, e^{-'})$. If the disintegration (or decay) and reintegration is punctual (very fast), the current $j_Q^\mu(e^-, \bar{e}^+)$ interacts directly with the current $j_Q^{\mu'}(\bar{e}^{+'}, e^{-'})$. We do not then have intervention of a mediating particle such as the photon.

Note 1

For brevity, we will often speak abusively of current $j_Q^\mu(e^-, \bar{e}^+)$, whereas it is actually a current charge density quadrivector $j_Q^\mu(e^-, \bar{e}^+)$ ($\mu = t, x, y, z$). The term $j_Q^t(e^-, \bar{e}^+) = Q\bar{\psi}\gamma^0\psi$ represents the density of electric charge. The terms $j_Q^{x,y,z}(e^-, \bar{e}^+) = Q\bar{\psi}\gamma^{1,2,3}\psi$ represent the densities of electric currents.

For electromagnetic interaction, the interaction energy $E = ej_Q^\mu A^\mu$ corresponds to the electromagnetic generalized potential energy.

Note 2

In the energy of interaction $E = ej_Q^\mu A^\mu$, the current term ej_Q^μ represents the electron and positron, the term potential quadrivector A^μ represents the photon γ .

Note 2

We remind the Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We remind the matrixs γ :

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \text{ with } i = 1,2,3$$

We define the conjugate spinor:

$$\bar{\psi} = \psi^\dagger \gamma^0$$

with ψ^\dagger adjoint (or adjugate) matrix, transposed from the conjugate.

We have the reaction during the interaction between the two currents $j_Q^\mu(e^-, \bar{e}^+)$ and $j_Q^{\mu'}(\bar{e}^{-'}, e^{+'})$:

$$e^- + \bar{e}^+ \rightarrow \bar{e}^{-'} + e^{+'}$$

This corresponds to the following diagram:

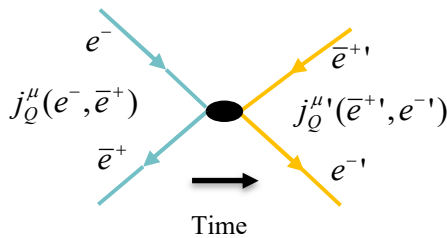


Figure 5: Positron electron annihilation, then positron electron materialization

Note on electric current and electric-type force

According to the photoelectric effect of A. Einstein and electron model of N. Bohr's, we can interpret the term interaction energy $E = ej_Q^\mu A^\mu$ as the effect of a photon represented by the potential quadrivector A^μ on an electron carrying electric charge eQ and in "stable" orbit around the core.

According to N. Bohr, the photon by interacting with the electron, generates a change in orbital angular momentum (or spin according to G. Uhlenbeck and S. Goudsmit) of the electron, a change of orbit and a displacement of the electron. We have a displacement of charge eQ and therefore an electric current j_Q^μ . We find the idea that an electric-type force has the effect of a displacement of electric charge.

Note that in electromagnetic interaction, 2 quantum numbers are used:

- First, the spin S which is exchanged between the photon and the electron and whose exchange generates a change of orbit and a displacement of the electron,

- Next, the electrical charge Q characteristic specific to the electron, set in motion when the electron changes orbit.

III.2 Fermi theory

III.2.1 Weak current

In 1933, E. Fermi took up P. Dirac's idea of the positron electron couple forming an electromagnetic current. He imagined that the proton neutron and electron anti-neutrino pairs can also form a so-called weak current. The neutron proton weak current $j_{weak}^\mu(n, p^+)$ interacts with another antineutrino electron weak current $j_{weak}^\mu(\bar{\nu}_e, e^-)$.

Note, foretaste on weak isospin T (also called weak charge)

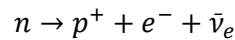
Weak isospin T is a quantum number proposed in the 1960s by S. Glashow as part of the Electroweak Model.

Weak isospin T sharing similarities with spin S and isospin I . Like them, T is group-based $SU(2)$. The 3rd component of weak isospin is frequently used: T^3 .

There are also analogies between T^3 and Q , since these are quantities transported respectively by weak currents and electromagnetic currents. It is often referred to as T^3 as the number of elementary weak charges, by analogy with Q which is the number of elementary electric charges.

We will see at the end of this Memoir that there are several types of weak currents. j_{weak}^μ . The current $j_{T^3}^\mu$ is one of them.

When decay β^- , we have the reaction between the two weak currents $j_{weak}^\mu(n, p^+)$ and $j_{weak}^\mu(\bar{\nu}_e, e^-)$:



This corresponds to the following diagram:

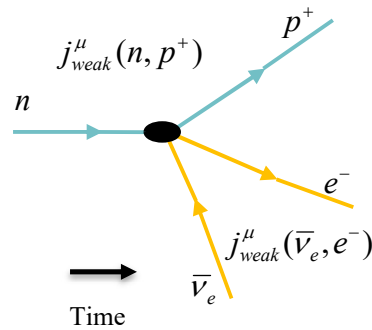


Figure 6: decay β^-

III.2.2 Taking into account a mediating particle for the positron electron current

For electromagnetic interaction, we have a probability amplitude M between two currents $j_Q^\mu(e^-, \bar{e}^+)$ and $j_Q^{\mu'}(\bar{e}^-, e^+)$ equal to:

$$M = \frac{e^2}{(p^\mu)^2} j_Q^\mu(e^-, \bar{e}^+) j_Q^{\mu'}(\bar{e}^-, e^+)$$

with p^μ ($\mu = t, x, y, z$) the momentum energy quadrivector carried during the interaction between the two currents j_Q^μ et $j_Q^{\mu'}$, i.e., the energy momentum quadrivector carried by the photon mediating particle γ .

This corresponds to the following diagram:

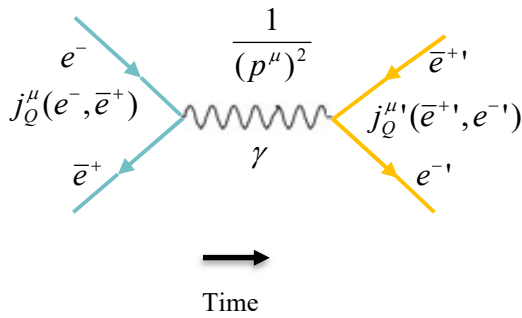


Figure 7: annihilation electron positron, photon, then materialization electron positron

Note

In the relationship $M = \frac{e^2}{(p^\mu)^2} j_\mu j^{\mu'}$, we actually have a contravariant-covariant summons.

III.2.3 Probability amplitude

At the time of E. Fermi, the idea of the mediating particle is in its infancy and E. Fermi assumes that the weak interaction is punctual.

For the reaction $n \rightarrow p + e^- + \bar{\nu}_e$, E. Fermi constructs a probability amplitude M for the interaction between the two weak currents:

$$M = \frac{8G_F}{\sqrt{2}} j_{weak}^\mu(n, p^+) \cdot j_{weak}^\mu(\bar{\nu}_e, e^-)$$

E. Fermi also notes that the probability amplitude is proportional to a constant G_F playing for weak interaction the role of $\frac{e^2}{(p^\mu)^2}$. G_F is now called Fermi constant. E. Fermi finds as a value for this constant:

$$G_F = 1,166 \times 10^{-5} GeV^{-2}$$

III.3 Foretaste of Glashow-Weinberg-Salam electroweak model and of quark model

The Glashow-Weinberg-Salam electroweak model and the Quark Model both use the notion of weak Fermi current, which will be particularly fruitful. Here we give a foretaste of these two theories.

III.3.1 Foretaste of the Glashow-Weinberg-Salam electroweak model

If the weak interaction is no longer considered punctual as in the Fermi model, we have a diagram involving mediating particles (as used in the Glashow-Weinberg-Salam electroweak model that will be studied at the end of this Memoir):

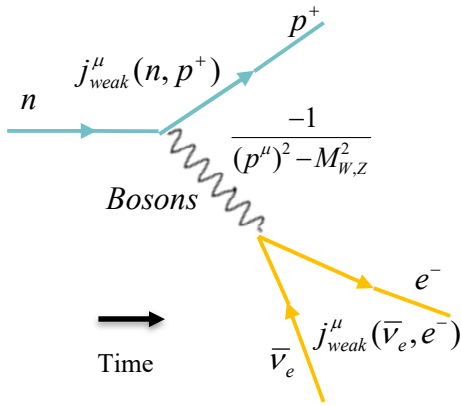


Figure 8: decay β^- with mediating particles

We have a probability amplitude M equal to:

$$M = \frac{-g_T^2}{(p^\mu)^2 - M_{W,Z}^2} j_{weak}^\mu(n, p^+) \cdot j_{weak}^\mu(\bar{\nu}_e, e^-)$$

g_T is the weak elementary charge related to the weak interaction (the analogue of the electric elementary charge e).

$M_{W,Z}$ is the mass of weak bosons W^\pm and Z^0 , mediating particles of the Electroweak Model.

Note

At the end of this Memoir, it will be seen that $g_T T^3$ for the weak interaction is the analogue of eQ for electromagnetic interaction.

The idea is to approach the energy momentum quadrivector (p^μ) toward 0, to find the Fermi model:

$$\frac{-g_T^2}{(p^\mu)^2 - M_W^2} \xrightarrow{(p^\mu) \rightarrow 0} \frac{g_T^2}{M_W^2} = \frac{8G_F}{\sqrt{2}}$$

According to Fermi's theory, coupling is introduced:

$$\frac{g_T^2}{M_W^2} = \frac{8G_F}{\sqrt{2}}$$

We will see that this relationship makes it possible to find the mass of weak bosons. W^\pm .

Note: why is it assumed that weak bosons W^\pm and Z^0 have a mass unlike the photon γ ?

It is assumed that weak bosons W^\pm and Z^0 have mass, because the weak interaction has a very small range (or scope) compared to electromagnetic and gravitational interactions (which have infinite range).

Following the ideas of H. Yukawa proposed in 1935, we approach the Compton wavelength $\lambda_c = \frac{h}{mc}$ to the range of the interaction, with m the mass of the particle mediating the interaction. For weak interactions, since the range (and therefore λ_c) is very small, it is assumed that the mediating particles W^\pm and Z^0 have a relatively large mass.

This will be explained in the next chapter on the Yukawa mesotron.

III.3.2 Foretaste of the Quark Model

In the Quark Model proposed by M. Gell-Mann and G. Zweig in the 1960s, the neutron is formed by two down quarks and one up quark. The proton is formed by two up quarks and one down quark. The notions of up and down represent the flavor of the quark and are distinguished by isospin I^3 . An up-flavor quark has an isospin $I^3 = \frac{1}{2}$, a down flavor quark has an isospin $I^3 = -\frac{1}{2}$.

The Quark Model seeks to model the strong interaction. In this context, the notions of flavor and isospin I can be brought closer to the strong nuclear charge.

By analogy with the strong interaction, isospin I^3 and the strong nuclear charge, the Electroweak Model defined for the weak interaction, a weak isospin, or a weak nuclear charge T^3 , which takes for the up and down quarks the same values.

We have:

$$T^3 = I^3 = \frac{1}{2} \text{ for the up quark } u.$$

$$T^3 = I^3 = -\frac{1}{2} \text{ for the down quark } d.$$

Note

This is true only for so-called left-handed up and down quarks. We will explain in a future chapter the notions of helicity and right-handed and left-handed particles.

In the Quark Model, leptons (electron and neutrino) have no isospin I . Indeed, the latter is considered a strong charge, and leptons are not subject to strong interaction.

On the other hand, leptons are subject to the weak interaction and the Electroweak Model assigns to the electron and the neutrino (left-handed only), a weak charge corresponding to that of the down quark and the up quark.

We have:

$$T^3 = \frac{1}{2} \text{ for the neutrino } \nu_e \text{ (} T^3 = -\frac{1}{2} \text{ for the antineutrino } \bar{\nu}_e \text{).}$$

$$T^3 = -\frac{1}{2} \text{ for electron } e^-.$$

We can rewrite the decay β^- with an up quark that transforms into a down quark:

$$d(T^3 = -\frac{1}{2}, Q = -\frac{1}{3}) \rightarrow u(T^3 = \frac{1}{2}, Q = \frac{2}{3}) + e^-(T^3 = -\frac{1}{2}, Q = -1) + \bar{\nu}_e(T^3 = -\frac{1}{2}, Q = 0)$$

The weak boson W^- is the mediating particle involved in the reaction. The boson W^- carries an electric charge $Q = -1$. Currents $j_{T^-}^\mu(d, \bar{u})$ and $j_{T^-}^\mu(\bar{\nu}_e, e^-)$ are the weak currents interacting with the boson W^- (we will come back to this when we deal with the Electroweak Model).

We have the corresponding Feynman diagram:

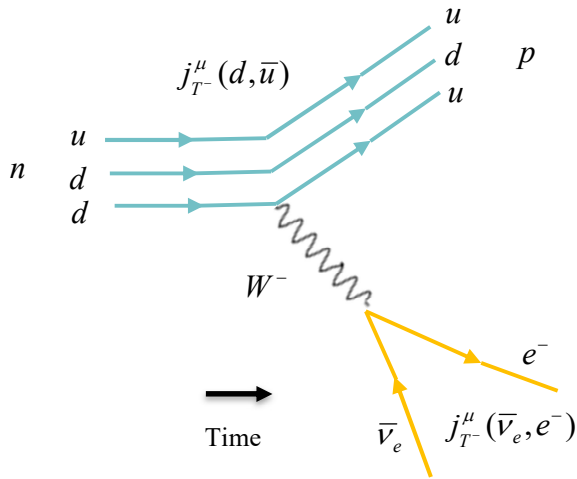


Figure 9: decay β^- according to the Quark Model

Note on the neutrino

The neutrino was proposed by W. Pauli in 1930 to explain the apparent non-conservation of angular momentum during a decay. β^- . W. Pauli initially used the term neutron. To distinguish it from the neutron present in the nucleus, it was renamed a neutrino by physicists Edoardo Amaldi and Enrico Fermi.

Like the proton neutron doublet (or up quark and down quarks), the electron and neutrino form a doublet of opposite weak charges (and different electric charges).

$$e^-(T^3 = -\frac{1}{2}, Q = -1)$$

$$\nu_e(T^3 = \frac{1}{2}, Q = 0)$$

However, unlike both nucleons (and two quarks), electron and neutrino do not have a close mass. The electron has a priori a mass much higher than that of the neutrino.

III.4 Fermi process and Gamow Teller process

III.4.1 Decay β^- , Fermi process and Gamow Teller process

For decay β^- , there are two types of weak interaction or two types of processes:

- vector interaction V, with the weak boson W^- that does not carry spin, and which decays into two particles of opposite half-integer spin. It is called the Fermi process.
- axial interaction A, with the weak boson W^- who carries a spin 1 or -1, and which decays into two particles of the same half-integer spin. It is called the Gamow Teller process.

III.4.2 Fermi process, vector interaction

We can write schematically the Fermi process, with a boson W^- that does not carry spin:

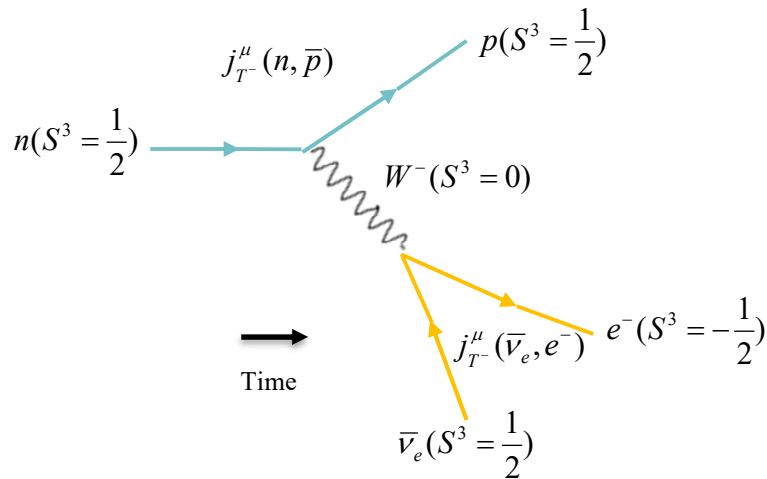


Figure 10: Fermi process

III.4.3 Gamow Teller process, axial interaction

We can write schematically the process of Gamow Teller, with a boson W^- with an integer spin:

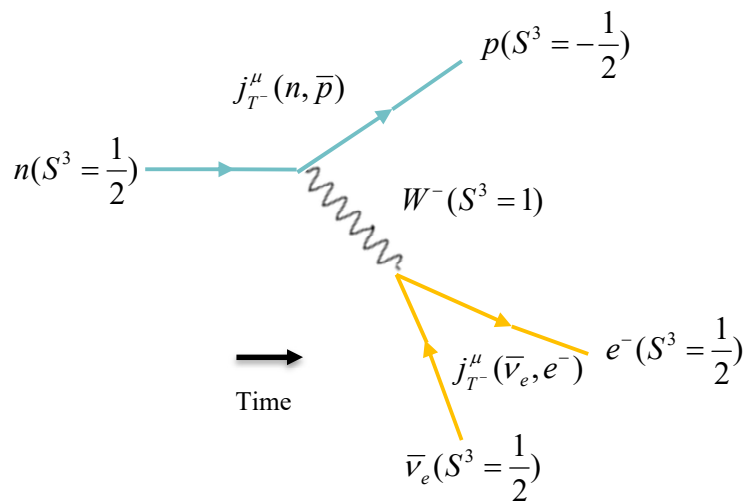


Figure 11: Gamow Teller process

Nota 1

W^- is indeed a boson, because in the 2 cases it has an integer spin.

Nota 2

For the 2 diagrams, the spin values S^3 are given as examples. Other values can be carried by the particles.

III.5 Conclusion of the chapter

P. Dirac had the idea of an electromagnetic current composed of a positron electron couple that decays into a photon, then materializes into an electron positron couple.

Inspired by this idea and following a generalizing approach to decay phenomena, E. Fermi proposes a weak current formed by a proton neutron couple that decays, then materializes into an antineutrino electron couple.

Fermi's weak interaction theory does not include the notion of mediating particles. In the next chapter, we will focus on the theory of the Yukawa mesotron which proposes a mass mediating particle to explain the small range of nuclear interactions.

Chapter IV The Mesotron of Yukawa (1935)

Purpose of the chapter

We explain here the theory of Hideki Yukawa, which in order to explain the small range of nuclear interactions, proposes a mass-mediating particle: the mesotron. Inspired by the Schrödinger wave equation that describes the electron, H. Yukawa proposes a wave equation that describes the mesotron. He also proposes to confuse this wave equation with the so-called Yukawa potential generated by the mesotron. At the end of the chapter, we will evoke the pion mesons (or pi mesons), particles discovered in the years 1940-1950 and in some way experimental confirmation of the Yukawa mesotron.

IV.1 Hideki Yukawa's theory to describe nuclear interactions, the mesotron or meson

In 1935, H. Yukawa developed a successful theory to describe nuclear interactions within the nucleus between nucleons (for example between a proton and a neutron).

To explain that these nuclear interactions have a limited scope R_N (approximately $R_N = 10^{-15}m$, which corresponds to the radius of the nucleus of the hydrogen atom or a nucleon), H. Yukawa assumes the existence of a mass mediating particle: the mass mesotron m_{mes} , particle later called the meson.

H. Yukawa proposes to bring the Compton wavelength λ_C closer to the limited range of the nuclear interaction:

$$\frac{\lambda_C}{2\pi} = \frac{1}{K_C} = 2R_N$$

Using the Compton relation:

$$m_{mes}c = \frac{h}{2\pi \lambda_C} = \frac{h}{\lambda_C} = \hbar K_C$$

H. Yukawa deduces the mass of the mediating particle, the mesotron or meson:

$$m_{mes}c = \frac{\hbar}{2R_N} = \frac{h}{\lambda_C}$$

Digital application

$$m_{mes}c^2 = \frac{1,973 \times 10^2 eV \cdot nm}{2 \times 10^{-15}} \approx 100 MeV$$

Following the idea of H. Yukawa, we can thus relate the scope R_N from an interaction to the mass of its mediating particle m :

$$R_N = \frac{\hbar}{2mc}$$

Note 1 on Heisenberg's uncertainty principle

To find the mass of the mediating particle, we can also use Einstein's source field equation $mc^2 = \hbar\Omega$ and Heisenberg's uncertainty principle, instead of Compton's relation.

We have:

$$\Delta E \Delta t \approx \frac{\hbar}{2}$$

We suppose $\Delta E = m_{mes}c^2 = \hbar\Omega$ (i.e., Einstein's source field equation).

We arrive at:

$$\Delta t \approx \frac{\hbar}{2\Delta E} = \frac{\hbar}{2m_{mes}c^2}$$

We suppose $R_N \approx c\Delta t$, assuming that the speed of the mesotron is close to that of light. We find:

$$R_N \approx \frac{\hbar}{2m_{mes}c}$$

Note 2 on the radius of the hydrogen atom

Remind that:

- the radius of the nucleus of the hydrogen atom (i.e., a proton) corresponds to the range of the nuclear interaction R_N ,
- the radius of the hydrogen atom corresponds in the Bohr model to the orbit of the electron (stabilized by the electrostatic force and the centrifugal inertia force) in its most stable energy level.

IV.2 Analogy Schrödinger wave equation and Yukawa wave equation, case of an electron and a mesotron

In the 1st Memoir, taking up the ideas of L. de Broglie, a method was proposed to find the Schrödinger equation that applies to an electron. We propose here a similar method to find the Klein Gordon wave equation independent of Time, wave equation that applies to a mesotron according to H. Yukawa.

We first remind in a few lines the method for the electron wave. We then approach the mesotron wave.

IV.2.1 Electron wave method

We start from the following wave equation:

$$\nabla^2\psi + \frac{\Omega^2}{v_\phi^2}\psi = 0$$

Following the ideas of L. Broglie, it is stated that the electron is similar to a phase velocity wave:

$$v_\phi = \frac{\Omega}{K}$$

We obtain for the wave equation:

$$\nabla^2\psi + K^2\psi = 0$$

According to de Broglie's source field equation, we have the relation of the momentum with the wave vector:

$$p = \hbar K$$

According to Einstein's source field equation, we have the relation of energy with pulsation:

$$E = mc^2 = \hbar\Omega$$

Speed v_ϕ of the electron wave is then equal to:

$$v_\phi = \frac{\Omega}{K} = \frac{E}{p}$$

We have the relationship between the energies: $E = E_c + V$ (with V potential energy).

$$E_c = E - V = \frac{p^2}{2m_{elec}}$$

The momentum can be deduced from this:

$$p = \sqrt{2m_{elec}(E - V)}$$

We have the wave vector:

$$K = \frac{\sqrt{2m_{elec}(E - V)}}{\hbar}$$

$$K^2 = \frac{2m_{elec}(E - V)}{\hbar^2}$$

We find the Schrödinger equation:

$$\nabla^2\psi + \frac{2m_{elec}}{\hbar^2}(E - V)\psi = 0$$

IV.2.2 Mesotron wave method

We start from the wave equation:

$$\nabla^2\psi + \frac{\Omega^2}{v_\phi^2}\psi = 0$$

It is assumed that the meson is similar to a phase velocity wave (in complex):

$$v_\phi = \frac{\Omega}{iK}$$

We obtain the wave equation:

$$\nabla^2\psi - K^2\psi = 0$$

We use the Compton relation:

$$m_{mes}c = \hbar K$$

We obtain the Klein Gordon wave equation independent of time that applies to a mesotron:

$$\nabla^2\psi - \left(\frac{m_{mes}c}{\hbar}\right)^2\psi = 0$$

IV.3 The potential of Yukawa

IV.3.1 Nuclear field potential

H. Yukawa uses the Klein Gordon wave equation independent of time:

$$\nabla^2 \psi - \left(\frac{m_{mes}c}{\hbar}\right)^2 \psi = 0$$

and interprets the wave function ψ as a spherical symmetry potential.

H. Yukawa confuses thus:

- the wave function ψ representing the mesotron particle wave,
- the potential of the nuclear field generated by the mesotron mediating particle.

H. Yukawa rewrites the time-independent Klein Gordon wave equation in spherical coordinates:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi(r)}{dr} \right) = \left(\frac{m_{mes}c}{\hbar}\right)^2 \psi(r)$$

Note

The wave function $\psi(r)$ is considered a potential, so it is the part cancelled during a change of reference frames.

From the wave function $\psi(r)$, we can define a nuclear field (also called mesonic field because created by the meson) equal to $\frac{\partial \psi(r)}{\partial r}$.

By solving the differential equation, we obtain a spherically symmetric Yukawa potential (presented here as a potential energy, because we have the term g^2):

$$\psi(r) = -g^2 \frac{e^{-\left(\frac{m_{mes}c}{\hbar}r\right)}}{r} = -g^2 \frac{e^{-\left(\frac{2\pi}{\lambda_C}r\right)}}{r}$$

One can interpret g^2 as a coupling constant between the meson and a nucleon, g as an elementary charge of nuclear interaction.

Reminder on the Compton wavelength

$$\frac{m_{mes}c}{\hbar} = \frac{2\pi}{\lambda_C} = K_C$$

IV.3.2 Screened potentials

If we make the mass of the mesotron m_{mes} tend toward 0, we obtain a Coulombian or Newtonian potential which decreases in $\frac{1}{r}$ dans l'Espace et défini à une constante multiplicative près.

We have the electrostatic potential energy generated by an electric charge q on another electric charge $-q$:

$$qA^t(r) = \frac{-g^2}{r} = \frac{-q^2}{4\pi\epsilon_0 r}$$

We have the gravitational potential energy generated by a mass m on another mass m :

$$mV^t(r) = \frac{-g^2}{r} = \frac{-Gm^2}{r}$$

The Yukawa potential is sometimes presented as a screened Coulombian or Newtonian potential.

Note 1, screened magnetic vector potential and superconductivity phenomenon

The Yukawa potential can also be likened to a screened magnetic vector potential \vec{A}^a $a = x, y, z$.

According to the theorem of Biot and Savart, we have a decay of the magnetic field in $\frac{1}{r^2}$ and the vector potential in $\frac{1}{r}$.

We have the Klein Gordon equation independent of time:

$$\nabla^2 \psi - \left(\frac{mc}{\hbar}\right)^2 \psi = 0$$

By replacing the wave function ψ by a screened magnetic vector potential, we obtain:

$$\nabla^2 \vec{A}^a - \left(\frac{mc}{\hbar}\right)^2 \vec{A}^a = 0$$

We replace \vec{A}^a by a magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}^a$, we get:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} = -\left(\frac{mc}{\hbar}\right)^2 \vec{A}^a$$

If we add a rotational on each side, we have:

$$\begin{aligned} \nabla^2 \vec{B} &= -\vec{\nabla} \times \left(\left(\frac{mc}{\hbar}\right)^2 \vec{A}^a\right) \\ \nabla^2 \vec{B} &= -\left(\frac{mc}{\hbar}\right)^2 \vec{\nabla} \times \vec{A}^a = -\left(\frac{mc}{\hbar}\right)^2 \vec{B} \\ \nabla^2 \vec{B} &= -\left(\frac{mc}{\hbar}\right)^2 \vec{B} \end{aligned}$$

We solve this differential equation, and we find:

$$\vec{B} = \vec{B}_0 \exp^{-\frac{mc}{\hbar}x} = \vec{B}_0 \exp^{-\frac{2\pi}{\lambda}x}$$

We obtain a magnetic field \vec{B} which is expelled from the material from a wavelength:

$$\lambda = \frac{h}{mc}$$

This corresponds to the phenomenon of superconductivity where the range of the magnetic field \vec{B} is highly reduced in a superconducting material.

According to the model described here, the photon particle mediating the electromagnetic interaction acquires a mass m . This mass of the photon tends towards infinity $m \rightarrow \infty$, when the magnetic field \vec{B} is increasingly expelled from the material, i.e., when $\lambda \rightarrow 0$.

We will return to this when we discuss the BEH mechanism, which is inspired by the ideas of H. Yukawa and the phenomenon of superconductivity, explains the small range of nuclear interactions and makes it possible to assign a mass to particles.

Note 2, link between mass and orbit

In his model of the electron, N. Bohr assimilates the radius of the orbit of the electron r_{Bohr} to the inverse of the wave vector:

$$r_{Bohr} = \frac{1}{K}$$

Using Broglie's source field relation $p = \hbar K$, we find the radius of the orbit of the electron according to its momentum:

$$r_{Bohr} = \frac{\hbar}{p}$$

It is noted that the higher the momentum of the particle, the smaller the orbit. This relationship $r = \frac{\hbar}{p}$ may be a priori valid for massless particles such as the photon.

In his model of the mesotron, H. Yukawa assimilates the scope R_N of the nuclear interaction to the inverse of the wave vector (with a factor $\frac{1}{2}$):

$$R_N = \frac{1}{2K}$$

Using the Compton relation $mc = \hbar K$, we find the scope R_N of the interaction carried by the particle as a function of its mass:

$$R_N = \frac{\hbar}{2mc}$$

As Bohr does, we can also equate the inverse of the wave vector to an orbit of radius r of the particle:

$$r = \frac{1}{K}$$

Using the Compton relation $mc = \hbar K$, we find the radius of the orbit of the particle as a function of its mass:

$$r = \frac{\hbar}{mc} = 2R_N$$

It is noted that the higher the mass of the particle, the smaller the orbit.

Different masses of particles are given.

Mesotron mass: $m_{mes}c^2 \approx 100MeV$

Pion meson mass: $m_{mes}c^2 \approx 139,6MeV$

Proton mass: $m_{pro}c^2 \approx 938,3MeV$

Boson mass Z^0 : $m_Zc^2 \approx 91,2GeV = 91\,200MeV$

Boson mass W^\pm : $m_Wc^2 \approx 80,4GeV = 80\,400MeV$

The Yukawa mesotron has a mass about 10 times smaller than the proton and an orbit of radius about 10 times larger than the proton.

The boson Z^0 has a mass about 100 times larger than the proton and an orbit about 100 times smaller than the proton.

IV.4 On pions as mediating particles of the strong interaction, foretaste on Yang-Mills theory, the Quark Model, and the Yukawa interaction

IV.4.1 Discovery of the pion meson (1947)

In 1936, a charge particle μ^- , with a mass close to that predicted by H. Yukawa is discovered in cosmic rays. It is called the mu meson. However, it is quickly realized that this particle cannot participate in nuclear reactions. It is actually a kind of large electron that is renamed the muon μ^- (we will come back to this when we study lepton generations.).

In 1947, thanks to collisions between particles, a new particle was discovered that finally seemed to play a role in nuclear interactions. The particle is called the pion meson.

Meanwhile, Yukawa's theory is refined and the existence of three types of mesons is postulated. (+, – and neutral). Neutral, because for a certain nuclear interaction (the one later called strong), in some cases there seems to be no difference between proton and neutron. A proton can change state into a neutron (ditto for a neutron changing state into a proton). For this type of change of state from proton to proton (or neutron to neutron), the mediating particle must be neutral since it does not carry electric charge.

Yukawa's theory agrees with the experiment since we discover 3 types of pions, 2 electrically charged: les pions π^+ and π^- , and 1 electrically neutral: the pion π^0 .

IV.4.2 Pion mesons as mediating particles of strong interaction, Yang-Mills theory

In the 1950s, the theory of H. Yukawa has as an extension a theory of the strong interaction called here of Yang-Mills. The pions π^+ , π^- , π^0 are presented as the mediating particles of the interaction and isospin I is designated both as the strong charge of the interaction and as the quantum quantity carried by the pions.

Note

The Yang-Mills theory describing the strong interaction is mainly based on Gauge transformations. We will come back to this in a future chapter.

This theory of strong interaction and that of electromagnetic interaction both have mediating particles. However, there is an important difference between the 2. In electromagnetic interaction, photons do not carry electric charge: these are spins that they exchange with other particles. In the strong interaction, the pions carry strong charge (or isospin) and they exchange them with the other particles.

The pions π^+ , π^- , π^0 have the following spins S and isospins I :

Pions	(S, S^3)	(I, I^3)
π^+	(0,0)	(1,1)
π^-	(0,0)	(1, -1)
π^0	(0,0)	(1,0)

We give for example some isospin exchange reactions I via the pions, reactions that ensure the cohesion of the nucleus between nucleons.

$$p^+(I^3 = \frac{1}{2}) \rightarrow n(I^3 = -\frac{1}{2}) + \pi^+(I^3 = 1)$$

$$n(I^3 = -\frac{1}{2}) + \pi^+(I^3 = 1) \rightarrow p^+(I^3 = \frac{1}{2})$$

$$n(I^3 = -\frac{1}{2}) \rightarrow p^+(I^3 = \frac{1}{2}) + \pi^-(I^3 = -1)$$

$$p^+(I^3 = \frac{1}{2}) + \pi^-(I^3 = -1) \rightarrow n(I^3 = -\frac{1}{2})$$

$$p^+(I^3 = \frac{1}{2}) \rightarrow p^+(I^3 = \frac{1}{2}) + \pi^0(I^3 = 0)$$

$$n(I^3 = -\frac{1}{2}) \rightarrow n(I^3 = -\frac{1}{2}) + \pi^0(I^3 = 0)$$

An analogous reaction is given, for electromagnetic interaction (hyperfine transition), with spin exchange S^3 and as a mediating particle the photon γ .

$$e_R(S^3 = \frac{1}{2}) \rightarrow e_L(S^3 = -\frac{1}{2}) + \gamma(S^3 = 1)$$

Note

Despite the analogy, let us again emphasize this profound difference. For the strong interaction, it is the strong charge or I^3 which is exchanged between the nucleon and the pion mediating particle. For electromagnetic interaction, this is spin S^3 (and not the electric charge Q) which is exchanged between the electron and the photon mediating particle.

IV.4.3 Foretaste of the Model of quarks, nucleons and mesons described as composite particles

Following the ideas of H. Yukawa, we can describe the strong interaction by the exchange of pion mesons between nucleons. This idea is found in the Quark Model developed in the 1960s, with an extension to other mesons as mediating particles of the strong interaction. In addition to up flavor u and down flavor d corresponding respectively to $I^3 = +\frac{1}{2}$ and $I^3 = -\frac{1}{2}$, the Quark Model proposes strange flavor s and charm flavor c . Like isospin I in Yang-Mills theory, the flavors u , d , s , c are identified with the strong charge.

Note

There are also 2 additional flavors of quarks, t , b , which were subsequently proposed.

In the Quark Model, additional flavors s and c make it possible to describe new mesons and nucleons that are grouped under the term baryons. Baryons and mesons are described as composite particles formed from quarks u , d , s , c (which then become the elementary components of the model).

The Quark Model distinguishes between two main types of particles:

- the baryons formed of 3 quarks (as well as antibaryons formed of 3 antiquarks) and whose best-known representatives are proton and neutron nucleons. These are the particles subject to strong interaction,
- the mesons, particles composed of an even number of quarks and antiquarks. These are the mediating particles of the strong interaction, carrying a strong charge identified with the 4 flavors.

Note

Other particles were later proposed, such as pentaquarks formed by 5 quarks.

IV.4.4 Decomposition of mesons into quarks, quantum numbers

Below is a list of some mesons, with their decomposition into quarks and their quantum numbers (source Wikipedia, Mesons for masses and lifetimes).

		(S, S^3)	(I, I^3)	Ba	St	C	Masses Mev/c^2	Lifetimes s
$\pi^+ u\bar{d}$		(0,0)	(1, +1)	0	0	0	139,6	$2,60 \times 10^{-8}$
$\pi^- \bar{u}d$		(0,0)	(1, -1)	0	0	0	139,6	$2,60 \times 10^{-8}$
$\pi^0(u\bar{u} - d\bar{d})/\sqrt{2}$		(0,0)	(1,0)	0	0	0	139,6	$2,60 \times 10^{-8}$
$K^+ u\bar{s}$		(0,0)		0	1		493,7	$1,24 \times 10^{-8}$
$K^- \bar{u}s$		(0,0)		0	-1		493,7	$1,24 \times 10^{-8}$
$K^0 d\bar{s}$		(0,0)		0	1		497,7	
$D^+ c\bar{d}$		(0,0)		0	0	1	1 869,4	$10,6 \times 10^{-13}$
$D^- \bar{c}d$		(0,0)		0	0	-1	1 869,4	$10,6 \times 10^{-13}$
$D^0 c\bar{u}$		(0,0)		0	0	1	1 864,6	$4,2 \times 10^{-13}$
$D_s^{++} c\bar{s}$		(0,0)		0	1	1	1 969	$4,7 \times 10^{-13}$
$D_s^{--} \bar{s}\bar{c}$		(0,0)		0	-1	-1	1 969	$4,7 \times 10^{-13}$

Note 1

St is the quantum number of strangeness. C is the quantum number of charm.

Note 2

Let's look at the analogy between pion mesons π^+ , π^- , π^0 for isospin I^3 , and kaon mesons K^+ , K^- , K^0 for strangeness St .

Note 3

Mesons are bosons and therefore have whole spins.

Note 4

An assembly of 2 half-integer spin fermions is always an integer spin boson. Hence the recurring idea proposed by many physicists, that like mesons formed of 2 elementary particles (in this case 1 quark and 1 antiquark), any boson can actually be formed of 2 fermions.

IV.4.5 Pions in the Quark Model

If we assemble quarks u and d (and their antiquarks) according to their respective isospin I (as was done previously for the proton and neutron), we obtain 4 states that correspond to the 3 pions π^+ , π^- , π^0 . The 4th state has a zero isospin I and therefore does not correspond to a particle.

We thus differentiate the three pions π^+ , π^- , π^0 :

$$|I = 1, I^3 = 1\rangle = \left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle = |u; -\bar{d}\rangle = |\pi^+\rangle$$

$$\begin{aligned} |I = 1, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle + \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|u; \bar{u}\rangle + |d; -\bar{d}\rangle) = |\pi^0\rangle \end{aligned}$$

$$|I = 1, I^3 = -1\rangle = \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle = |d; \bar{u}\rangle = |\pi^-\rangle$$

$$\begin{aligned} |I = 0, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle - \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|u; \bar{u}\rangle - |d; \bar{d}\rangle) \end{aligned}$$

Note, analogy between pions π^+ , π^- , π^0 and weak bosons W^+ , W^- , $W^0 = Z^0$

In Yukawa's strong interaction theory, pions π^+ , π^- , π^0 carry a strong charge equal to isospin I . Pions π^+ , π^- are called scale operators. By reacting with fermions, they can increase or decrease the strong charge with one I^3 of the fermion.

In the chapter dealing with the electroweak model, we will see that it is defined in analogy to pions π^+ , π^- , π^0 , 3 weak bosons W^+ , W^- , $W^0 = Z^0$ that carry a weak charge T and which are mediating particles of the weak interaction. Weak bosons W^+ , W^- are also of the scale operator type. By reacting with fermions, they can increase or decrease the weak charge with one T^3 of the fermion.

IV.4.6 Elements on the angular momentum of mesons, scalar meson and pseudoscalar meson

We define the total angular momentum \vec{J} of a meson:

$$\vec{J} = \vec{L} + \vec{S}$$

with \vec{L} the orbital angular momentum of a quark relative to the antiquark,

with \vec{S} the sum of the spins of the quark and the antiquark.

Let be the parity operator \hat{P} , the operator that transforms $\vec{x} \rightarrow -\vec{x}$.

For a particle, we define a quantum number of parity P (or intrinsic parity), following the relationship:

$$\hat{P}|s, m_s\rangle = P|s, m_s\rangle$$

with $P = \pm 1$,

s integer or half integer,

$-s \leq m_s \leq +s$ per jump of one unit.

For mesons, the parity quantum number P provides information on orbital angular momentum \vec{L} following the relationship:

$$P = (-1)^{L+1}$$

A so-called pseudo-scalar meson possesses $S = 0$ and $J^P = 0^-$ (that is $P = -1$). This is the case, for example, with pions.

A so-called scalar meson possesses $S = 0$ and $J^P = 0^+$ (that is $P = +1$).

IV.4.7 Elements about the Yukawa interaction

The so-called Yukawa interaction was developed in the 1950s and 1960s following a Lagrangian formalism, in order to describe the strong interaction between nucleons (then called Dirac fields or spinorial fields) and meson mediating particles, for example pions (called pseudo-scalar fields). This interaction is not proposed by H. Yukawa, but it is attributed its name, because it is partly inspired by his idea of mass mediating particle of nuclear interactions.

Note on the Higgs boson

The Yukawa interaction is also used in the Electroweak Model, to couple quarks and leptons (then called Dirac fields) with the Higgs boson (then called scalar field), in order to assign a mass to quarks and leptons. We will come back to this in the chapter dealing with the Electroweak Model.

We remind here the main Lagrangians used in the interaction of Yukawa.

For a pseudoscalar meson represented by the wave function φ (for example, a pion-type mediating particle, $S = 0$ and $J^P = 0^-$), we have the Lagrangian of the Yukawa interaction with the meson which interacts with a fermion represented by the wave function ψ :

$$L_{Yukawa}(\varphi, \psi) = -g\bar{\psi}i\gamma^5\varphi\psi$$

with $\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and g the elementary charge of the Yukawa interaction.

For a scalar meson ($S = 0$ and $J^P = 0^+$), we have the Lagrangian of Yukawa's interaction:

$$L_{Yukawa}(\varphi, \psi) = -g\bar{\psi}\varphi\psi$$

To obtain the total Lagrangian $L_{Total}(\varphi, \psi)$, we must also take into account the Dirac Lagrangian $L_{Fermion}(\psi)$ of the fermion:

$$L_{Fermion}(\psi) = i\hbar c\bar{\psi}\gamma^\mu(D_\mu)\psi - mc^2\bar{\psi}\psi$$

and the Lagrangian of the meson $L_{meson}(\varphi)$:

$$L_{meson}(\varphi) = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$$

$V(\varphi)$ is a term of self-interaction. It is given in the form: $V(\varphi) = \frac{1}{2}\mu^2\varphi^2$ with μ the mass of the meson. It is also given in a more developed form $V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \lambda\varphi^4$, with λ a constant that avoids discrepancies.

Nota

To better understand the origin of $L_{meson}(\varphi) = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$, it is necessary to be interested in the theory of the scalar field (and more generally in the theory of relativistic quantum fields).

Generally, a scalar field is required to be relativistic (i.e., invariant with respect to Lorentz transformations), to be as simple as possible, that it does not contain derivatives with respect to Time of order greater than one.

We obtain the total Lagrangian:

$$L_{Total}(\varphi, \psi) = L_{meson}(\varphi) + L_{Fermion}(\psi) + L_{Yukawa}(\varphi, \psi)$$

IV.5 Conclusion of the chapter

Yukawa's theory introduces the notion of mass mediating particle and explains the limited scope of nuclear interactions.

It proposes a "screened" potential depending on the mass of the mediating particle. When we tend this mass towards 0, we find the shape of the potentials of fields of infinite range, such as the Newtonian gravitational field or the Coulomb electrostatic field.

In the 1950s, the theory of H. Yukawa has as an extension a theory of strong interaction, with pions as a mediating particle, carrying a strong charge equal to isospin I .

In the next chapters, we will focus on the description of interactions between particles from Gauge theories. This description developed particularly from the 1950s following the work of Chen Ning Yang and Robert Mills, it formed the major theoretical corpus of the Standard Model. In preliminary, we will make some mathematical reminders on rotation groups $SU(n)$.

Chapter V Mathematical reminders about rotation groups $SU(n)$, physical applications to gauge transformations

Purpose of the chapter

Gauge theories require a lot of mathematical background. In this chapter, we will make some mathematical reminders on rotation groups $SU(n)$ used to describe Gauge transformations. We will also be interested in quantum numbers (in particular charges) involved in reactions between particles and in Gauge transformations.

V.1 General information on gauge theories and quantum numbers

V.1.1 Gauge invariances and gauge transformations

The expressions gauge invariances and gauge transformations were introduced in 1918 by H. Weyl. He was inspired by the ideas of A. Einstein of an invariance of the laws of Nature whatever the transformation, in this case a Gauge transformation to be compared to a change of frames of reference.

In the first Memoir of this essay, it was pointed out that Gauge theories are based, like Newtonian mechanics, on:

- the concepts of cancelled quantities and quantities retained during a local gauge transformation (or during a change of reference frames),
- "inertial" terms to be added in the wave equation or in the fundamental principle of Newton's dynamics, in order to make these 2 laws invariant ("inertial" terms of the potential energy momentum type or the inertial force type).

Nota

In Gauge theories, terms such as momentum energy potential are added in the wave equation and interaction energy in the Lagrangian.

V.1.2 Historical reminders, march of ideas

The Gauge theory formulated by H. Weyl in the 1920s, describes the electromagnetic interaction, the Gauge transformations belong to the group of rotations $U(1)_Q$, with Q the electric charge.

In 1954, to explain the strong interaction, Chen Ning Yang and Robert Mills introduced a non-abelian Gauge theory where Gauge transformations belong to rotation groups $SU(2)_I$, with isospin I presented as the strong charge.

In the late 1950s, early 1960s, through the Quark Model, Gell-Mann perfected the Yang-Mills theory and proposed a strong interaction theory based on the group $SU(3)_{Flavor}$. The strong charge is based on isospin I and a new quantum number called strangeness St . The whole thing is called flavors.

In the early 1960s. S. Glashow transcribes the Yang-Mills theory, from the strong interaction to the weak interaction, always based on $SU(2)$. By analogy with strong isospin I seen as the strong charge, S. Glashow introduces the notion of weak isospin or weak charge T , with $SU(2)_T$. In the late 1960s, the theory was completed by Steven Weinberg and Abdus Salam in the so-called Electroweak Model.

In 1973, to explain the strong interaction, H. David Politzer, Frank Wilczek and David Gross introduced quantum chromodynamics, a theory based on $SU(3)_{Color}$, with the notion of color charge presented as the strong charge.

We group together these different theories describing electromagnetic interactions, strong and weak, based on groups of rotations $SU(n)$, and essentially developed in the 1950s-1970s, in what is called the Standard Model.

It is to these groups of rotations $SU(n)$, which we will focus on in this chapter. We will first do some mathematical reminders, and then we will see how this physically applies to particles.

Reminder, difference between global and local gauge transformation

We speak of a global gauge transformation on a wave function ψ :

$$\psi \rightarrow \psi' = e^{i\alpha q} \psi$$

with α a constant and q a quantum quantity (e.g., an electric charge).

We speak of local gauge transformation on a wave function ψ when $\alpha(x)$ is a function of local variables x , positions and/or moments:

$$\psi \rightarrow \psi' = e^{i\alpha(x)q} \psi$$

Global and local gauge transforms are both rotations belonging to groups $SU(n)$. Usually, we speak of global version gauge transformations and local version gauge transformations.

V.1.3 2 types of quantum numbers

In particle physics, we note the existence of quantized quantities, to which we associate quantum numbers. We can distinguish 2 types of quantum numbers.

First type: charge-like quantum numbers (noted here X in a general sense) that can generate interaction fields. In the Standard Model, there are 4 of them: electric charge Q , weak charge T , weak hypercharge Y_W , strong charge (or color charge) Co .

In this chapter, we will see that these charge-type quantum numbers are the generators of global gauge transformations (rotations of the group $SU(n)$), that they are retained during a Global Gauge transformation. They can be associated with velocities and obtain common charge density quadrivectors that are also conserved by global gauge transformation.

For example, we have electromagnetic current j_Q^μ , the weak current j_T^μ , the weak hypercharge current $j_{Y_W}^\mu$, the strong current j_{Co}^μ . We will come back to the first 3 currents when we study the Electroweak Model.

On the model of the inertial forces that are added during a change of reference frames, in order to preserve the invariance of the fundamental principle of Newton's dynamics, we add in the Lagrangian interaction energy terms composed of these currents j_Q^μ , j_T^μ , $j_{Y_W}^\mu$ and potential quadrivectors, during a local gauge transform, in order to preserve the invariance of the wave equations.

Second type: quantum numbers that are involved in reactions between particles and that frequently reverse or change. For example, we find the spin S , the isospin I , the baryonic number Ba , strangeness St , etc. These quantum numbers do not generate a priori interaction fields. For example, if an electrical charge Q generates an electric field, there is no spin field or baryonic field.

For a particle and its associated wave function, one can during a reaction between particles, modify these quantum numbers. The particle then transforms into its conjoined particle. For example, for isospin I , the reaction makes it possible to pass from a neutron to a proton. For spin S , the so-called hyperfine transition reaction makes it possible to pass from a right electron to a left electron.

Generally, during reactions between particles, the quantum quantity retains the same total value (at the beginning and end of the reaction). For spin, we actually have a global conservation of total angular momentum.

Nota

The isospin I is here classified in the 2nd type and close to spin S . Following the original ideas of W. Heisenberg and the Yangs-Mills gauge theory, isospin I is presented as the strong charge of the strong interaction. It is therefore classified in the 1st type. The same goes for strangeness St , which in the Quark Model, is presented with the other flavors associated with I , as the strong charge of strong interaction. An explanation for this will be proposed in the next Memoir. It will be suggested that the strong charge (noted here Co) is a concept to be distinguished from I or St , but which is nevertheless a function of these quantum numbers I and St .

V.2 Mathematical reminders about rotation groups $SU(n)$

V.2.1 Elements of mathematics on rotation groups

The Standard Model is based on gauge transformations belonging to the rotation groups of dimension vectors n in complex spaces. These rotation groups are called $SU(n)$, S as special to say that the determinant of the matrix describing the rotation is equal to 1, U as unitary and n the dimension of the vector concerned by the rotation.

We classify these groups $SU(n)$ in the Lie groups named after S. Lie, a Norwegian mathematician who originated these mathematical notations and who published his papers in the 1870s. These groups are usually noncommutative.

Unit matrices satisfy the condition:

$$U^*U = UU^* = I_n$$

The adjoint matrix of U is noted U^* (or U^\dagger in physics, and more specifically in quantum physics).

Nota

An adjoint matrix M of a matrix with complex coefficients is the transposed matrix of the conjugate matrix of M . For example:

$$\begin{pmatrix} 1+i & 13 \\ 3-2i & i \end{pmatrix}^* = \begin{pmatrix} 1-i & 3+2i \\ 13 & -i \end{pmatrix}$$

The group $SO(n)$ is the group of rotations of vectors with dimension n in real spaces. S as special to say that the determinant of the matrix is equal to 1. O as orthogonal and who verifies the condition ${}^tAA = I_n$ with tA the transposed matrix.

In the following paragraphs, some examples of rotation groups are given.

V.2.2 Group rotations $U(1)$

A complex vector V with dimension 1 turns into a vector V' by angle rotation θ :

$$V' = e^{i\theta}V$$

The rotation matrix is $(e^{i\theta})$.

V.2.3 Group rotations $SO(2)$

A real vector V with dimension 2 turns into a vector V' by the rotation matrix:

$$\begin{pmatrix} V^{x'} \\ V^{y'} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} V^x \\ V^y \end{pmatrix}$$

There is a correspondence between rotations $U(1)$ and $SO(2)$ (This is called isomorphism).
We have:

$$V^\pm = \frac{1}{\sqrt{2}}(V^x \pm iV^y)$$

We can rewrite a rotation of $SO(2)$:

$$\begin{pmatrix} V^{+'} \\ V^{-'} \end{pmatrix} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} V^+ \\ V^- \end{pmatrix}$$

$$V^{+'} = e^{i\theta} V^+$$

$$V^{-'} = e^{-i\theta} V^-$$

We have a general form that is part of $U(1)$, with α which is a real:

$$W' = e^{-i\alpha\theta} W$$

V.2.4 Group rotations $SO(3)$

A real vector V with dimension 3 turns into a vector V' by the following rotation matrices corresponding to rotations about the axes x, y and z:

$$R^x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix}$$

$$R^y(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$

$$R^z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$R^x(\theta)$ is an angle rotation θ in a spatial plan y, z around the axis x. $R^y(\theta)$ in a spatial plan x, z around the axis y. $R^z(\theta)$ in a spatial plan x, y around the axis z.

We orient all rotations according to the rule of the right hand.

Note

There is also a correspondence between the rotations of $SO(3)$ and those of $SU(2)$ (this is also called isomorphism).

V.3 Mathematical reminders about generators of group $SU(n)$

V.3.1 General

The generators of a group make it possible to generate by linear combinations all the elements of a group. The number of generators in a group $SU(n)$ is equal to $n^2 - 1$.

Any rotation matrix U of a group $SU(n)$ can be written as the linear combination (at the exponential level) of $n^2 - 1$ generators. We have:

$$U = e^{iH} = e^{i\alpha^a T^a} = e^{i(\alpha^1 T^1 + \alpha^2 T^2 + \alpha^3 T^3 + \dots)}$$

with:

$$a = 1, 2, 3, \dots, n^2 - 1,$$

α^a actual parameters,

T^a matrix n^2 with complex coefficients called generators of the group $SU(n)$.

Note

Here we have matrix exponentials.

V.3.2 Generators of group $SU(2)$

The group $SU(2)$ has 3 generators, for example the halves of the 3 Pauli matrices:

$$\frac{\sigma^1}{2} = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \frac{\sigma^2}{2} = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \frac{\sigma^3}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Note

It is reminded that the Pauli matrices were introduced by W. Pauli to take into account the effects of a magnetic field on an electron. The electron is modelled as a small magnet with spin angular momentum and spin magnetic moment. Taking into account this notion of spin in the Schrödinger wave equation, gives the Pauli wave equation.

In Dirac's relativistic wave equation, the notion of spin is directly incorporated.

A matrix U of $SU(2)$ can be defined as a linear combination of these 3 generators:

$$U(\alpha^1, \alpha^2, \alpha^3) = e^{\frac{i}{2}(\alpha^1 \sigma^1 + \alpha^2 \sigma^2 + \alpha^3 \sigma^3)}$$

Let X be a doublet that turns into X' by rotation $U(\alpha^1, \alpha^2, \alpha^3)$ of $SU(2)$.

So, we have:

$$X \rightarrow X' = U(\alpha^1, \alpha^2, \alpha^3)X$$

$$X \rightarrow X' = e^{\frac{i}{2}(\alpha^1 \sigma^1 + \alpha^2 \sigma^2 + \alpha^3 \sigma^3)}X$$

For a rotation $R^x(\theta)$ with angle θ around the axis x , it is shown that it can be defined from the generator σ^1 :

$$R^x(\theta) = e^{\frac{i}{2}\theta\sigma^1}$$

$$X \rightarrow X' = e^{\frac{i}{2}\theta\sigma^1}X$$

Around the axis y , we have the same from the generator σ^2 :

$$R^y(\theta) = e^{\frac{i}{2}\theta\sigma^2}$$

$$X \rightarrow X' = e^{\frac{i}{2}\theta\sigma^2}X$$

Around the axis z , we have the same from the generator σ^3 :

$$R^z(\theta) = e^{\frac{i}{2}\theta\sigma^3} = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

$$X \rightarrow X' = e^{\frac{i}{2}\theta\sigma^3} X$$

For example, if $\theta = 2\pi$, We have:

$$e^{\frac{i}{2}2\pi\sigma^3} = \begin{pmatrix} e^{-i\frac{2\pi}{2}} & 0 \\ 0 & e^{i\frac{2\pi}{2}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

V.3.3 Generators of group $SU(3)$

In the Quark Model and in Quantum Chromodynamics (both describing the strong interaction and included in the Standard Model), we have triples of particles of dimension 3. The Gauge transformations on these particle triples belong to the group $SU(3)$.

Note, transition from flavors to colors

A little strangely, during the 1960s, we passed:

- from a triplet based on flavors up, down, and strange proposed by M. Gell-Mann in the Quark Model (with a strong charge based on flavors),
- to a triplet based on the colors red, green, and blue in quantum chromodynamics (with a strong charge based on colors).

The group $SU(3)$ has $3^2 - 1 = 8$ generators, e.g., the 8 Gell-Mann matrices, $\lambda^1 \dots \lambda^8$, formed from Pauli matrices:

$$\begin{aligned} \frac{\lambda^1}{2} &= \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{\lambda^2}{2} = \frac{1}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{\lambda^3}{2} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \frac{\lambda^4}{2} &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \frac{\lambda^5}{2} = \frac{1}{2} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \\ \frac{\lambda^6}{2} &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \frac{\lambda^7}{2} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \frac{\lambda^8}{2} = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned}$$

Any transformation of a triplet of particles can be written as a linear combination of the 8 Gell-Mann matrices.

A matrix U of $SU(3)$ can be defined as a linear combination of these 8 generators:

$$U(\alpha^1, \alpha^2, \dots, \alpha^8) = e^{i\frac{1}{2}(\alpha^1\lambda^1 + \alpha^2\lambda^2 + \dots + \alpha^8\lambda^8)}$$

Let X be a triplet of particles that transforms into X' by rotation $U(\alpha^1, \alpha^2, \dots, \alpha^8)$ of $SU(3)$. So, we have:

$$X \rightarrow X' = U(\alpha^1, \alpha^2, \dots, \alpha^8)X$$

$$X \rightarrow X' = e^{i\frac{1}{2}(\alpha^1\lambda^1 + \alpha^2\lambda^2 + \dots + \alpha^8\lambda^8)} X$$

V.3.4 Elements on the structure constant of Lie groups

In group theory, we define the commutator of a couple x, y :

$$[x, y] = xyx^{-1}y^{-1}$$

x and y commute if and only if $[x, y] = 1$.

For three generators of a Lie group, we have the relation:

$$[T_i, T_j] = if_{ijk}T_k$$

This relation defines the algebra of the Lie group. Quantities f_{ijk} called structure constants, are constant parameters that characterize each group.

For $SU(2)$, noncommutative group, the commutation relation is written:

$$[T_i, T_j] = i\varepsilon_{ijk}T_k$$

where the structure constant f_{ijk} is equal to the tensor ε_{ijk} of Levi-Civita.

$\varepsilon_{ijk} = +1$ if (ijk) is an even permutation of (123) ,
 $\varepsilon_{ijk} = -1$ if (ijk) is an odd permutation of (123) ,
 $\varepsilon_{ijk} = 0$ otherwise.

V.4 Physical application, charge defined as a generator of group $SU(n)$, multiplets of particles and wave functions**V.4.1 General**

In the Standard Model, a charge X^a ($a = 1, 2, 3, \dots, n^2 - 1$) is defined as the generator of a group $SU(n)$.

Particles are grouped by multiplet of particles. The number n of $SU(n)$ indicates the number of particles in the multiplet. It also indicates the number of possible states and the number of wave functions associated with the multiplet of particles.

Nota

We still retain the fundamental idea of L. de Broglie to associate a particle with a wave (or wave function). ψ

V.4.2 Electric charge Q , generator of group $U(1)_Q$

In electromagnetic interaction, electric charge Q is the generator of the group $U(1)_Q$.

We have $n = 1$ and thus a multiplet formed of a single particle. A particle with an electric charge Q has only one possible state. If we reason in terms of wave function ψ and probability, there is only one wave function ψ corresponding to the probability amplitude of observing the electrically charged particle.

V.4.3 Strong charge: isospin I , generator of group $U(2)_I$

In the Yang-Mills theory describing the strong interaction, the strong charge I is the generator of the group $U(2)_I$.

We have $n = 2$ and thus a multiplet formed of 2 particles, for example 2 quarks u and d with strong charges $I^3 = +\frac{1}{2}$ and $I^3 = -\frac{1}{2}$ (that is to say, up and down flavors).

We define two wave functions associated with the strong charge I : ψ_u and ψ_d corresponding to the probability amplitude of observing the particle doublet. We have the doublet of wave functions: $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$.

V.4.4 Strong charge of flavors, generator of group $U(3)_{flavors}$

In the initial Quark Model describing the strong interaction, the strong charge is the generator of the group $U(3)_{flavors}$. Compared to the Yang-Mills theory, we have one more flavor that is strange or s .

We have $n = 3$ and thus a multiplet formed by 3 particles, for example 3 quarks u , d and s with strong charges, with flavors up, down, and strange.

We define three wave functions each corresponding to the probability amplitudes of observing the triplet of particles. We have the wavefunction triplet: $\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$.

Note

If we add a charm or c flavor, we have a group $U(4)_{flavors}$ and a quadruplet of wave function: $\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_c \\ \psi_s \end{pmatrix}$.

V.4.5 Weak charge T , generator of group $U(2)_T$

In the Electroweak Model describing the weak interaction, the weak charge T is the generator of the group $U(2)_T$.

We have $n = 2$ and thus a multiplet formed by 2 particles, for instance 2 quarks, one of weak charge $T^3 = +\frac{1}{2}$ (i.e., flavor quarks u , c , t), the other weak charge $T^3 = -\frac{1}{2}$ (i.e., flavor quarks d , s , b).

We define two wave functions associated with the weak charge T : $\psi_{u,c,t}$ and $\psi_{d,s,b}$, corresponding to the probability amplitude of observing the doublet of particles of weak charges $T^3 = +\frac{1}{2}$ and $T^3 = -\frac{1}{2}$. We have the doublet of wave functions: $\psi = \begin{pmatrix} \psi_{u,c,t} \\ \psi_{d,s,b} \end{pmatrix}$.

Note 1

If we stop at the 1st generation of quarks, that is to say the first 2 flavors u and d . We have the doublet of wave functions: $\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$.

Note 2

In the Electroweak Model, this only applies to left-handed quarks (and right-handed antiquarks), that have a non-zero weak charge T . Right-handed quarks (and left-handed antiquarks) have a zero weak charge T and are therefore not subject to weak interaction. We will come back to this in a future chapter.

V.4.6 Strong charge of colors, generator of group $U(3)_{\text{colors}}$

In quantum chromodynamics describing the strong interaction, the strong charge is the generator of the group $U(3)_{\text{colors}}$.

We have $n = 3$ and thus a multiplet formed by 3 quarks, for example 3 quarks of strong charges of color R, G, B (red, green, blue).

We define three wave functions each corresponding to the probability amplitudes of observing the triplet of color particles R, G, B . We have the wavefunction triplet: $\psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$.

Note

The group $SU(3)_{\text{flavors}}$ was developed by M. Gell-Mann prior to $SU(3)_{\text{colors}}$. For this reason, its name is found in the Gell-Mann matrices used in quantum chromodynamics.

That the strong charge is generating $SU(3)_{\text{flavors}}$ or $SU(3)_{\text{colors}}$, the mathematical model is the same for groups.

V.5 Physical application, Global Gauge transformations and conservation of the charges

V.5.1 General

The rotations of group $SU(n)$ allow you to define Global Gauge transformations on the multiplets of wave functions associated with the multiplexes of particles. For example, we have the global gauge

transform on a multiplet of wave functions $\begin{pmatrix} \psi_1 \\ \dots \\ \psi_n \end{pmatrix}$ defined from the matrix U of the group $SU(n)$:

$$\begin{pmatrix} \psi_1 \\ \dots \\ \psi_n \end{pmatrix} \rightarrow \begin{pmatrix} \psi_1 \\ \dots \\ \psi_n \end{pmatrix}' = U \begin{pmatrix} \psi_1 \\ \dots \\ \psi_n \end{pmatrix}$$

V.5.2 Case of $U(1)_Q$, electric charge Q

If U belongs to $U(1)_Q$, then ψ is a singlet of wave functions and we have:

$$U = e^{i\alpha Q}$$

$$\psi \rightarrow \psi' = e^{i\alpha Q} \psi$$

We have the global gauge transformation approximated by the infinitesimal transformation:

$$\psi \rightarrow \psi' = (1 + i\alpha Q)\psi$$

If $\psi \rightarrow \psi' = (1 + i\alpha Q)\psi$ is a symmetry of the system, then the electric charge Q is preserved by the Global Gauge transformation.

Explanation

According to Noether's theorem, any transformation that leaves the equations of motion invariant or in other words, that commutes with the Hamiltonian H of the system (energy of the system), we can associate a conserved quantity.

If the infinitesimal transformation $\psi \rightarrow \psi' = (1 + i\alpha Q)\psi$ leaves the motion invariant, so we have the commutation with the Hamiltonian:

$$[H, (1 + i\alpha Q)] = 0$$

We get:

$$[H, Q] = 0$$

The electric charge Q is here the quantity conserved by the infinitesimal transformation.

Global Gauge transformation example

To pass from the electron of electric charge -1 to the antielectron of electric charge $+1$, we do a rotation of $\pi = \alpha Q$ in the space of electric charges:

$$\bar{e}^+(+1) = e^{i\pi} e^-(-1)$$

$e^{i\pi}$ belongs to the group $U(1)_Q$.

V.5.3 Case of $SU(2)_T$, weak charge T (weak isospin)

If U belongs to $SU(2)_T$, then $\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$ is a doublet of wave functions and we have:

$$U = e^{\frac{i}{2}(\alpha^1 \sigma^1 + \alpha^2 \sigma^2 + \alpha^3 \sigma^3)}$$

$$\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \rightarrow \begin{pmatrix} \psi_{u'} \\ \psi_{d'} \end{pmatrix} = e^{\frac{i}{2}(\alpha^1 \sigma^1 + \alpha^2 \sigma^2 + \alpha^3 \sigma^3)} \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

Note

We are studying here only the first generation of quarks.

The weak charges $T^a = \frac{\sigma^a}{2}$ ($a = 1, 2, 3$) are considered to be the generators of $SU(2)_T$. So, we have 3 weak charges.

If U is a symmetry of the system then the matrices $\frac{\sigma^a}{2}$ are retained, i.e., weak charges $T^a = \frac{\sigma^a}{2}$ are retained.

Example of Global Gauge transformation

Let be the doublet of particles of the nucleus (hadrons) of opposite weak charges, a quark u_L and a quark d_L (the particles are both left-handed, as only left-handed particles are subject to the weak interaction.):

$$X = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \begin{pmatrix} T^3 = \frac{1}{2} \\ T^3 = -\frac{1}{2} \end{pmatrix}$$

Similarly, let be the doublet of particles around the nucleus (leptons) of opposite weak charges, a left-handed electron e_L^- and a left-handed neutrino ν_L :

$$Y = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} = \begin{pmatrix} T^3 = \frac{1}{2} \\ T^3 = -\frac{1}{2} \end{pmatrix}$$

In the space of weak charges T , to switch from the quark doublet $X = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ to the quark doublet $X' = \begin{pmatrix} u_L' \\ d_L' \end{pmatrix}$, we perform a rotation $R^z(\theta) = e^{\frac{i}{2}\alpha^3\sigma^3}$ of $SU(2)_T$ with angle $\alpha^3 = 2\pi$ and $\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

We have:

$$X' = e^{\frac{i}{2}2\pi\sigma^3} X = e^{\frac{i}{2}2\pi\sigma^3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$e^{\frac{i}{2}2\pi\sigma^3} = \begin{pmatrix} e^{-i\frac{2\pi}{2}} & 0 \\ 0 & e^{i\frac{2\pi}{2}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

So, we have:

$$X' = \begin{pmatrix} u_L' \\ d_L' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} d_L \\ u_L \end{pmatrix}$$

We have an identical treatment with the lepton doublet Y :

$$Y' = \begin{pmatrix} \nu_L' \\ e_L' \end{pmatrix} = e^{\frac{i}{2}2\pi\sigma^3} Y = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

Note 1

A rotation about the z-axis with $\alpha^3 = 2\pi$, reverses the weak charge T . A rotation about the z-axis with $\alpha^3 = 4\pi$, leaves invariant weak charge T . We have the same observation for spin and isospin I which share with the weak charge the same mathematical formalism. We will come back to this at the end of this chapter.

Note 2

What we just explained in the previous example about left-handed quarks, left-handed leptons, and the weak charge T , also applies to quarks (both left-handed and right-handed) and isospin I , when the latter is considered a strong charge (this is the case in the Yang-Mills theory that we will study in a future chapter). The high charge is then conserved by gauge transformation of the group $SU(2)_I$. This does not apply to leptons that are not subject to strong interaction.

V.5.4 Case of $SU(3)_{Colors}$, strong charge of colors

If U belongs to $SU(3)_{Colors}$, then $\begin{pmatrix} \psi_R \\ \psi_V \\ \psi_B \end{pmatrix}$ is a triplet of wave functions and we have:

$$U(\alpha^1, \alpha^2, \dots, \alpha^8) = e^{\frac{i}{2}(\alpha^1\lambda^1 + \alpha^2\lambda^2 + \dots + \alpha^8\lambda^8)}$$

$$\begin{pmatrix} \psi_R \\ \psi_V \\ \psi_B \end{pmatrix} \rightarrow \begin{pmatrix} \psi_R' \\ \psi_V' \\ \psi_B' \end{pmatrix} = e^{i\frac{1}{2}(\alpha_1\lambda_1 + \alpha_2\lambda_2 + \dots + \alpha_8\lambda_8)} \begin{pmatrix} \psi_R \\ \psi_V \\ \psi_B \end{pmatrix}$$

If U is a symmetry of the system then the matrices $\frac{\lambda^a}{2}$ are retained, that is, strong charges of color are preserved by Global Gauge transformation.

Note on the strong charge of flavors

This also applies to $SU(3)_{flavors}$, since we have the same mathematical formalism as to groups $SU(3)$. Strong charges of flavor are also preserved by Global Gauge transformation.

V.6 Physical application, currents charge quadrivector conserved during a global gauge transform, Noether's theorem

V.6.1 Electromagnetic currents

In Memoir 2, it was mentioned that using Noether's theorem, one obtains the conservation of an electromagnetic currents charge quadrivector during a global gauge transformation. This is succinctly reminded.

We have the Global Gauge transformation belonging to the group $U(1)_Q$:

$$\psi(x) \rightarrow e^{i\alpha Q} \psi(x)$$

α can take any real value.

We have the Lagrangian of a free electron described by the wave function ψ :

$$L = \bar{\psi}(i\gamma_\mu \partial^\mu - \frac{mc}{\hbar})\psi$$

This Lagrangian is invariant by the global gauge transformation $\psi(x) \rightarrow e^{i\alpha Q} \psi(x)$.

According to Noether's theorem, there is a quantity conserved in the global gauge transformation.

The density of electric charge is the quantity conserved with $\mu = t$:

$$j_Q^t = \frac{\partial L}{\partial \dot{\psi}(\alpha)} \frac{\partial \psi(\alpha)}{\partial \alpha}$$

The density of electric currents is the quantity conserved with $\mu = x, y, z$:

$$j_Q^\mu = \frac{\partial L}{\partial (\vec{\nabla} \psi(\alpha))} \frac{\partial \psi(\alpha)}{\partial \alpha}$$

By developing these two equations, we obtain as conserved quantity the electromagnetic currents charge density quadrivector:

$$j_Q^\mu = -eQ\bar{\psi}\gamma^\mu\psi$$

$$\text{with } \mu = t, x, y, z$$

with e the elementary electric charge and Q the number of elementary electric charges.

We also have the conservation electromagnetic currents charge:

$$\partial_\mu j_Q^\mu = 0$$

with $\mu = t, x, y, z$

V.6.2 Currents carrying a charge X^a

What we have just stated about electromagnetic currents and the group $U(1)_Q$, can generalize to groups $SU(n)_X$, with X^a the charges who are generators of the group $SU(n)$. We then have $n^2 - 1$ currents $j_{X^a}^\mu$ carrying a charge X^a that check conservation of currents charge:

$$\partial_\mu j_{X^a}^\mu = 0$$

with $a = 1, \dots, n^2 - 1$ and $\mu = t, x, y, z$

For instance, in the case of strong interaction of the group $SU(2)_I$ and in the case of weak interaction of the group $SU(2)_T$, we speak respectively of strong currents $j_{I^a}^\mu$ and weak currents $j_{T^a}^\mu$. We have 3 strong currents: $j_{I^1}^\mu, j_{I^2}^\mu, j_{I^3}^\mu$ and 3 weak currents: $j_{T^1}^\mu, j_{T^2}^\mu, j_{T^3}^\mu$.

We will return to this type of weak currents in the chapter dealing with the Glashow-Weinberg-Salam electroweak model.

V.7 Physical application, transformations of spin S and isospin I based on $SU(2)$

In the previous paragraphs, we have mainly mentioned quantum numbers of the charge type, generating the rotations of $SU(n)$.

Note that for quantum numbers of the other type, such as spin S and isospin I , we also have transformations of $SU(2)$. These transformations can reverse these quantum numbers and describe some of the changes observed during a hyperfine transition or beta decay. We will see that the mathematical treatment is identical for spin and isospin.

V.7.1 Hyperfine transition, inversion of spin S^3

Let be a doublet of electrons of opposite spins, of the same momentum et d'hélicités opposées $X =$

$$\begin{pmatrix} e_R^- \\ e_L^- \end{pmatrix} = \begin{pmatrix} S^3 = \frac{1}{2} \\ S^3 = -\frac{1}{2} \end{pmatrix} \text{ (a right-handed electron and a left-handed electron).}$$

We can symbolize a hyperfine transition that reverses the spin states of the two electrons by a rotation $R^z(\alpha = 2\pi)$ around z axis:

$$X' = \begin{pmatrix} e_R'^- \\ e_L'^- \end{pmatrix} = e^{\frac{i}{2}2\pi\sigma^3} X = e^{\frac{i}{2}2\pi\sigma^3} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

We have:

$$e^{\frac{i}{2}2\pi\sigma^3} = \begin{pmatrix} e^{-i\frac{2\pi}{2}} & 0 \\ 0 & e^{i\frac{2\pi}{2}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

So, we have:

$$X' = \begin{pmatrix} e_{R'}^- \\ e_{L'}^- \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} e_L^- \\ e_R^- \end{pmatrix}$$

The electron doublet X' has the spins inverted with respect to the electron doublet X .

More generally, any transformation of a spin doublet can be written as a linear combination of the 3 Pauli matrices. We will see in the following example that it is the same for isospin doublets.

V.7.2 Beta decay, inversion of isospin I^3

Let be the doublet of particles of the nucleus (hadrons) of opposite isospins, a quark u and a quark d :

$$X = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} I^3 = \frac{1}{2} \\ I^3 = -\frac{1}{2} \end{pmatrix}$$

We can symbolize a beta decay that reverses the isospin states of a quark u and of a quark d by rotation $R^Z(\alpha = 2\pi)$ around z axis:

Note

Instead of having a doublet $\begin{pmatrix} u \\ d \end{pmatrix}$, we can also reason on a proton neutron doublet:

$$X = \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} I^3 = \frac{1}{2} \\ I^3 = -\frac{1}{2} \end{pmatrix}$$

In isospin space, to pass from the quark doublet $X = \begin{pmatrix} u \\ d \end{pmatrix}$ to the doublet $X' = \begin{pmatrix} u' \\ d' \end{pmatrix}$, we perform a rotation $R^Z(\alpha) = e^{\frac{i}{2}\alpha\sigma^3}$ of $SU(2)$ with angle $\alpha = 2\pi$ and with $\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Like spin, we have:

$$X' = \begin{pmatrix} u' \\ d' \end{pmatrix} = e^{\frac{i}{2}2\pi\sigma^3} X = e^{\frac{i}{2}2\pi\sigma^3} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$e^{\frac{i}{2}2\pi\sigma^3} = \begin{pmatrix} e^{-i\frac{2\pi}{2}} & 0 \\ 0 & e^{i\frac{2\pi}{2}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

So, we have:

$$X' = \begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} d \\ u \end{pmatrix}$$

Note: foretaste on the hyper interaction and the boson B

In the case of hyperfine transition, the spin number S^3 is retained via the radiated photon γ which carries the difference in spins between right-handed and left-handed electrons.

In the case of beta decay, we will propose in the next Memoir that the isospin number I^3 is conserved via the radiated boson B , which carries the difference in isospins between the neutron and the proton.

Like the photon γ (carrying spins S^3) which is the mediating particle of the electromagnetic interaction of electric charge Q , we will propose that the boson B (carrying isospins I^3) is the particle mediating the hyper interaction of weak hypercharge Y_W .

V.8 Conclusion of the chapter

In this chapter, we first made some mathematical reminders on rotation groups $SU(n)_X$ used by the Standard Model. We then mentioned as physical applications, charges X^a defined in the Standard Model, and generators of these rotation groups $SU(n)_X$.

These charges X^a are preserved by Global Gauge transformation. They are each associated with a current $j_{X^a}^\mu$ which, according to Noether's theorem, is also conserved by a global gauge transformation.

After these mathematical reminders and these physical applications, we will return in more detail to the 1st Gauge theory using a charge X^a with more than one dimension: the Yang-Mills gauge theory describing the strong interaction and having isospin as a strong charge.

Chapter VI Yang-Mills local gauge theory on strong interaction (1954)

Purpose of the chapter

We remind in general the theory of local gauge of Yang-Mills which describes the strong interaction. In the 1st Memoir, we compared Weyl gauge transformations to cancellations (or modifications) of rotation vectors in two dimensions of a real space (or a dimension of a complex space).

We can see the local Yang-Mills gauge theory as the generalization of one to two complex dimensions of Weyl's gauge theory. Thus, Yang-Mills Local Gauge transformations can be compared to cancellations (or modifications) of rotation vectors in two dimensions of a complex space.

VI.1 Preamble

In 1954, inspired by the ideas of A. Einstein and H. Weyl, Chen Ning Yang and Robert Mills published a local gauge theory describing the strong interaction.

The Yang-Mills local gauge theory is based on the same principle as H. Weyl's local gauge theory describing electromagnetism. The objective is to obtain the invariance of the great laws of Nature, in this case:

- Dirac's relativistic wave equation $(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar})\psi = 0$ with $\mu = t, x, y, z$,
- Lagrangian $L = i\hbar c \bar{\psi} \gamma^\mu (\partial_\mu) \psi - mc^2 \bar{\psi} \psi$ which makes it possible to find, using it in the Euler-Lagrange equation, this relativistic wave equation.

The Yang-Mills theory is essentially distinguished from that of Weyl by the number of dimensions used, with 1 complex dimension in Weyl and 2 complex dimensions in Yang-Mills. For example, in Weyl, the electric charge Q has 1 component, in Yang-Mills, the strong charge identified with isospin I^a ($a = 1, 2, 3$) has 2 components. In Weyl, the wave function is a singlet, in Yang-Mills the wave function is a doublet. In Weyl, local gauge transformations on the wave function belong to group of rotations $U(1)_Q$, at Yang-Mills, local gauge transformations on the wave function belong to group of rotations $SU(2)_I$.

The transition from 1 to 2 dimensions has important consequences, because while $U(1)$ is a commutative group, $SU(2)$ is not. This introduces additional terms into the gauge transform of the potential quadrivector and into the Yang-Mills Lagrangian. This will be discussed in the following paragraphs.

Like Yukawa's theory of nuclear interactions, Yang-Mills theory uses mediating particles called pion mesons, which are represented by potential quadrivectors (also called gauge fields).

The main handicap of Yang-Mills is that unlike the Yukawa mesotron, the mediating particles must be massless to obtain the invariance of the laws of Nature (in particular the invariance of the Lagrangian by local gauge transformation). Thus, the very short scope of nuclear interactions is not explained at Yang-Mills.

The Yang-Mills theory is of great historical importance. In the 2 decades following its publication, its ideas were widely reflected in the Standard Model, which was inspired by them to describe both strong and weak interaction.

At the end of the next chapter, we will see briefly how in the Standard Model, a mass is assigned to mediating particles via the BEH (or Higgs) mechanism, and how the small scope of nuclear interactions is thus explained.

In the next paragraphs, we will remind the main characteristics of a Yang-Mills local gauge theory, an essential prerequisite for better understanding the Standard Model.

VI.2 Local gauge transformation, analogy electromagnetic interaction and strong nuclear interaction

VI.2.1 Local gauge transformation in the case of electromagnetic interaction

In the case of electromagnetic interaction, we have the local gauge transformation of the wave function:

$$\psi \rightarrow \psi' = \exp(ieQ\alpha(x))\psi$$

with $\alpha(x)$ function of local variable x (positions and/or moments).

The local gauge transformation belongs to the group of rotations:

$$\exp(ieQ\alpha(x)) \in U(1)_Q$$

Q is the number of elementary electric charges. It is an integer number since the electric charge is quantified as a multiple of elementary electric charges e . We can consider Q as a matrix 1×1 .

We have the local gauge transform of the electromagnetic potential quadrivector:

$$A^\mu \rightarrow A^{\mu'} = A^\mu + \partial_\mu \alpha(x)$$

$$\text{with } \mu = t, x, y, z$$

In the local gauge theory of electromagnetism, the electromagnetic potential quadrivector is associated with a photon γ , mediating particle of electromagnetic interaction.

VI.2.2 Covariant derivative of electromagnetic interaction

In the 1st Memoir, we saw that to obtain the invariance of the relativistic Dirac wave equation, it is necessary to define a covariant derivative:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + iqQA^\mu$$

with $\mu = t, x, y, z$ and q the elementary charge.

We have the relativistic Dirac wave equation in the reference frame R^A where we carried out the transformation of Local Gauge:

$$(i\gamma^\mu D_\mu - \frac{mc}{\hbar})\psi = 0$$

Note

In the case of an electron, $q = -e$, we then have:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieQA^\mu$$

$$(i\gamma^\mu (\partial_\mu - ieQA^\mu) - \frac{mc}{\hbar})\psi = 0$$

VI.2.3 Local Gauge Transformation of the wave function in the case of strong interaction

In the case of the strong interaction, according to the Yang-Mills theory, we have the local gauge transform of a doublet of wave functions:

$$\psi \rightarrow \psi' = \exp(ig_I I^a \cdot \alpha^a(x))\psi$$

$$(a = 1, 2, 3)$$

with $\alpha^a(x)$ function of the local variable x (moments, positions).

ψ and ψ' are here doublets of wave functions.

g_I is the elementary strong charge in analogy with the elementary electric charge e .

I^a is the number of elementary strong charges (or more simply the strong charge). In Yang-Mills, the strong charge is identified with isospin.

I^a ($a = 1,2,3$) are the 3 components of isospin. They are matrices 2×2 frequently defined from Pauli matrices: $I^a = \frac{\sigma^a}{2}$, with σ^a the 3 Pauli matrices.

The local gauge transformation belongs to the group of rotations:

$$\exp(ig_I I^a \cdot \alpha^a(x)) \in SU(2)$$

$$(a = 1,2,3)$$

Note 1

In the next chapter describing the electroweak model and the weak interaction, we will see that the proposed model is largely inspired by that of Yang-Mills. Since the interaction described is weak, we will use the weak charge T , defined by S. Glashow in analogy with the strong charge I of Yang-Mills.

In the table below, we summarize the analogies between electric charges, strong and weak charges:

	Electromagnetic interaction Weyl theory	Strong interaction Yang-Mills theory	Weak interaction Electroweak model
Number of charges conserved per Global Gauge transformation	Q (Electric charge)	I^a (Strong charge) $a = 1,2,3$	T^a (Weak charge) $a = 1,2,3$
Elementary charge	$g_Q = e$	g_I	g_T
Total charge	eQ	$g_I I^a$	$g_T T^a$

Note 2

As already mentioned, isospin I is also to be compared to spin S . Both are reversed during reactions between particles, decay β for isospin I and hyperfine transition for spin S .

In the next Memoir, it will be proposed to distinguish the strong charge noted here Co (like a strong charge of color) from the isospin I .

However, in the case of nucleons and quarks of 1st generation, the strong charge Co remains precisely equal (probably with a very good approximation) to isospin I . We then understand better why the Yang-Mills theory takes isospin I as the strong charge.

VI.2.4 Local gauge transformation of potential quadrivectors in the case of the strong interaction

In the Yang-Mills theory, we do not have a single potential quadrivector A^μ , but three noted: $A^{1\mu}$, $A^{2\mu}$, $A^{3\mu}$ who each interacts with a strong current j_a^μ ($a = 1,2,3$) carrying a strong charge respectively I^1 , I^2 , I^3 .

We have the local gauge transform of the 3 potential quadrivectors (we give it here for $A^{1\mu}$):

$$A^{1\mu} \rightarrow A^{1\mu'} = A^{1\mu} + \partial_\mu \alpha^1(x) + g_I f_{123} \alpha^2(x) A^{3\mu}$$

with f_{123} the group structure constant. For $SU(2)$, f_{123} are the components of the tensor ε^{123} of Levi-Civita.

Note 1

For a group of rotations $SU(n)$, the number of potential quadrivectors is equal to $n^2 - 1$, therefore identical to the number of generators in the group $SU(n)$, to the number of charges X^a and to the number of currents $j_{X^a}^\mu$.

Note 2

In Yang-Mills, the 3 potential quadrivectors of the strong interaction correspond to the mediating particles of the interaction, i.e., three mesons A^1, A^2, A^3 .

To pass from mesons A^1, A^2, A^3 of the theory, to the electrically charged pions π^+, π^- that we observe, we pose the relationships between the potential quadrivectors of mesons:

$$\pi^{+\mu} = \frac{1}{\sqrt{2}}(A^{1\mu} + iA^{2\mu})$$

$$\pi^{-\mu} = \frac{1}{\sqrt{2}}(A^{1\mu} - iA^{2\mu})$$

Unlike the photon γ that does not carry an electrical charge, the pions π^+, π^- carry a strong charge (in this case isospin I^3). The pions π^+, π^- are of the scale operator type, they can increase or decrease by a I^3 , the strong charge of the particle with which they interact.

VI.2.5 Covariant derivative of strong interaction

To obtain the invariance of the Dirac relativistic wave equation, Yang-Mills theory defines a covariant derivative:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_I I^a \cdot A^{a\mu}$$

$$(a = 1,2,3)$$

$g_I I^a$ are the quantities retained during the local gauge transformation.

$A_1^\mu, A_2^\mu, A_3^\mu$: are the three potential quadrivectors cancelled during the local gauge transformation.

We have the relativistic Dirac wave equation in the reference frame R^A where the local gauge transformation is performed:

$$(i\gamma^\mu D_\mu - \frac{mc}{\hbar})\psi = 0$$

$$(i\gamma^\mu (\partial_\mu + ig_I I^a \cdot A^{a\mu}) - \frac{mc}{\hbar})\psi = 0$$

$$(a = 1,2,3)$$

Note

To see if we can associate in a more encompassing theory:

- rotations of $SU(2)$ (or $SU(n)$ if we generalize) corresponding to the Yang-Mills Local Gauge transformations,
- the generalized rotation vectors in a spatial or spatiotemporal plane that we proposed in the 2nd Memoir, and which are cancelled during a change of reference frames.

These were rotations in a real two-dimensional space or in a complex one-dimensional space. With $SU(n)$, We switch to rotations in n complex dimensions.

VI.3 Reminders on the Lagrangians used in electromagnetism and relativistic quantum electrodynamics

VI.3.1 Lagrangian of d'Alembert

We remind the Lagrangian of d'Alembert:

$$[L]_{R^0} = i\bar{\psi}\gamma^\mu(\partial_\mu)\psi$$

with $\mu = 0,1,2,3$ and $\mu = ct, x, y, z$.

The reference frame of the Lagrangian is the vacuum R^0 .

Reminder

The conjugate of the wave function is defined by:

$$\bar{\psi} = \psi^\dagger\gamma^0$$

Applying the Euler Lagrange equation: $\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\psi)}\right) - \frac{\partial L}{\partial\psi} = 0$ or $\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\bar{\psi})}\right) - \frac{\partial L}{\partial\bar{\psi}} = 0$ on the Lagrangian of d'Alembert, then squared, we find the wave equation of d'Alembert:

$$\frac{\partial^2\psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

In compact form:

$$\partial_\mu^2\psi = 0$$

with $\mu = ct, x, y, z$

VI.3.2 Lagrangian of free Dirac

We remind the Lagrangian of free Dirac:

$$[L]_{R^c} = i\hbar c\bar{\psi}\gamma^\mu(\partial_\mu)\psi - mc^2\bar{\psi}\psi$$

The reference frame R^c of the Lagrangian of free Dirac is no longer the vacuum since there are masses.

Applying the Euler Lagrange equation: $\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\psi)}\right) - \frac{\partial L}{\partial\psi} = 0$ or $\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\bar{\psi})}\right) - \frac{\partial L}{\partial\bar{\psi}} = 0$ on the Lagrangian of free Dirac, we find the free Dirac equation in the reference frame R^c :

$$(\beta[\partial_{ct}]_{R^c} + \beta\vec{\alpha} \circ [\vec{\partial}_x]_{R^c} + i\frac{mc}{\hbar})\psi = 0$$

In compact form:

$$(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar})\psi = 0$$

Then squared, we find the Klein Gordan wave equation in R^c :

$$([\partial_{ct}^2]_{R^c} - [\partial_x^2]_{R^c} + \frac{m^2 c^2}{\hbar^2})\psi = 0$$

VI.3.3 Dirac Lagrangian in an electromagnetic field

We remind the Lagrangian of Dirac in an electromagnetic field:

$$[L]_{R^A} = i\hbar c \bar{\psi} \gamma^\mu (D_\mu) \psi - mc^2 \bar{\psi} \psi$$

with R^A the reference frame where the electromagnetic potential quadrivector is cancelled A^μ , with $\mu = 0, 1, 2, 3$ or $\mu = ct, x, y, z$.

We have the covariant derivative (in the case of an electron):

$$D_\mu = \partial_\mu - ieA^\mu$$

We obtain the Dirac Lagrangian in an electromagnetic field:

$$[L]_{R^A} = \bar{\psi} (i\hbar c \gamma^\mu \partial_\mu - mc^2) \psi + \hbar c e \bar{\psi} \gamma^\mu \psi A^\mu$$

$-e\bar{\psi} \gamma^\mu \psi A^\mu$ is the electromagnetic generalized potential energy, also called electromagnetic interaction energy. It is the interaction term between the particle (fermion) of currents charge quadrivector $j_Q^\mu = -e\bar{\psi} \gamma^\mu \psi$ and the electromagnetic potential quadrivector A^μ (representing the photon γ).

We have the electric charge density:

$$j_Q^t = -e\bar{\psi} \gamma^0 \psi = -e\psi^\dagger \psi$$

We find the electrostatic potential energy:

$$Ep^t = -eA^t$$

We have the electric currents:

$$j_Q^\mu = -e\bar{\psi} \gamma^\mu \psi$$

$$\text{with } \mu = x, y, z$$

We find the magnetic generalized potential energy:

$$Ep^t = j_Q^\mu A^\mu = -e\vec{v}^\mu \cdot \vec{A}^\mu$$

$$\text{with } \mu = x, y, z$$

We find the electromagnetic generalized potential energy (but with different signs, because the signs of the magnetic vector potential have been reversed.):

$$Ep^t = -e(A^t - (\vec{v}^\mu \cdot \vec{A}^\mu))$$

$$\text{with } \mu = x, y, z$$

Note on magnetic potential energy

Remind that the magnetic potential energy is zero, the magnetic force of Lorentz does not work. This is because there is a magnetic potential momentum $e\vec{A}^\mu$ ($\mu = x, y, z$) that we have this *magnetic generalized* potential energy.

Note

To switch from the Lagrangian $[L]_{R^c}$ to the Lagrangian $[L]_{R^A}$, we perform the change of reference frames or transformation of local gauge:

$$\psi \rightarrow \psi' = \exp(ieQ\alpha(x))\psi$$

$$A^\mu \rightarrow A^{\mu'} = A^\mu + \partial_\mu\alpha(x)$$

$$\partial_\mu(\exp(ieQ\alpha(x))\psi) = -i(\partial_\mu\alpha(x))$$

$$[L]_{R^c} \rightarrow [L]_{R^A} = [L]_{R^c} + (\partial_\mu\alpha(x))\bar{\psi}\gamma^\mu\psi$$

In relativistic quantum electrodynamics, there is also a Lagrangian describing the dynamics of the photon:

$$L_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$ is the Maxwell Faraday tensor, the electromagnetic field tensor, or the photon field tensor.

We obtain the total Lagrangian in the reference frame R^A where the electromagnetic potential quadrivector is cancelled A^μ :

$$[L]_{R^A} = \bar{\psi}(i\hbar c\gamma^\mu\partial_\mu - mc^2)\psi + \hbar c e\bar{\psi}\gamma^\mu\psi A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

VI.4 Lagrangians involved in the strong interaction

VI.4.1 Lagrangian of d'Alembert

In the Yang-Mills theory, the Lagrangians used for the strong nuclear interaction are built on the same model as the electromagnetic interaction.

In the reference frame R^0 (i.e., vacuum), we have a Lagrangian for a doublet of particles (of the nucleus) corresponding to the d'Alembert wave equation:

$$[L]_{R^0} = i\bar{\psi}\gamma_\mu(\partial^\mu)\psi$$

Note: ψ above is a doublet of wave functions.

VI.4.2 Lagrangian of free Dirac

In the reference frame R^c (with masses), we have the Lagrangian corresponding to Free Dirac:

$$[L]_{R^c} = i\hbar c\bar{\psi}\gamma_\mu(\partial^\mu)\psi - mc^2\bar{\psi}\psi$$

VI.4.3 Lagrangian of Dirac in a strong nuclear field

In the reference frame R^A where potential quadrivectors are cancelled $A^{a\mu}$, we replace ∂^μ by D^μ so that the wave equation is always respected (Gauge invariance principle). The Yang-Mills theory gives the Lagrangian for particles of the nucleus in a strong nuclear field:

$$[L]_{R^A} = i\hbar c\bar{\psi}\gamma^\mu(D_\mu)\psi - mc^2\bar{\psi}\psi$$

We have the covariant derivative:

$$D_\mu = \partial_\mu + ig_I I^a \cdot A^{a\mu}$$

$$(a = 1,2,3)$$

We get:

$$[L]_{RA} = i\hbar c \bar{\psi} \gamma^\mu (\partial_\mu + ig_I I^a \cdot A^{a\mu}) \psi - mc^2 \bar{\psi} \psi$$

$$[L]_{RA} = \bar{\psi} (i\hbar c \gamma^\mu \partial_\mu - mc^2) \psi - \hbar c g_I I^a \bar{\psi} \gamma^\mu \psi \cdot A^{a\mu}$$

$-\hbar c g_I I^a \bar{\psi} \gamma^\mu \psi \cdot A^{a\mu}$ is the term of strong interaction between particles (fermion) of strong currents charge density quadrivectors $j_{I^a}^\mu = g_I I^a \bar{\psi} \gamma^\mu \psi$ and strong potential quadrivectors $A^{a\mu}$ (representing the meson mediating particles of the strong interaction).

There is also a Lagrangian to describe the dynamics of gauge mesons:

$$L_{\text{jauge}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

We have three tensors $F_{\mu\nu}^1, F_{\mu\nu}^2, F_{\mu\nu}^3$ analogues of the electromagnetic tensor $F_{\mu\nu}$ of Maxwell-Faraday.

We give the expression of the tensor $F_{\mu\nu}^3$:

$$F_{\mu\nu}^3 = \partial_\mu A^{3\nu} - \partial_\nu A^{3\mu} + g_I \varepsilon^{123} A^{1\mu} A^{2\nu}$$

ε^{123} is the group structure constant $SU(2)$. For $SU(2)$, these are the components of the tensor ε^{123} of Levi-Civita.

Note 1

The group $SU(2)$ of isospin I is noncommutative, the combination $g_I \varepsilon^{123} A^{1\mu} A^{2\nu}$ involves interactions between mesons A^1, A^2, A^3 with exchange of quantities carried by mesons.

Note 2

Gauge invariance requires that mesons A^1, A^2, A^3 are massless. However, to explain the small range of nuclear interactions, according to the ideas of H. Yukawa, it is necessary that the mediating particles have a mass.

This implies that the Lagrangian includes terms of mass of the form: $L_{\text{mass}} = m_{\text{mes}}^2 A^\mu A_\mu$ with m_{mes} the mass of mesons.

But this Lagrangian L_{mass} is not local gauge invariant. To overcome this problem, we will study in a future chapter the BEH mechanism used by the Electroweak Model.

VI.5 Chapter conclusion, summary on Global and Local Gauge Transformations

During a Global Gauge transformation of $SU(n)$, we have the charges X^a (such as electric charge, strong charge of isospin, strong charge of flavor, weak charge of the Electroweak Model, strong charge of color, etc.) that are retained. We also have the currents charge density quadrivectors (electromagnetic, strong, weak) that are retained and that satisfy the retention equation: $\partial_\mu j_X^\mu = 0$ with X^a the generating charge of the group $SU(n)_X$, $a = 1, \dots, n^2 - 1$ and $\mu = t, x, y, z$.

During a local Gauge transformation of $SU(n)$, we obtain according to a principle of Invariance of the great laws of Nature, covariant derivatives, and interaction terms to be added in the Lagrangian. We have $n^2 - 1$ potential quadrivectors (or gauge fields) that are cancelled during the transformation of Local Gauge. To these $n^2 - 1$ potential quadrivectors, we associate $n^2 - 1$ mediating particles. Those $n^2 - 1$ mediating particles of potential quadrivectors $A^{a\mu}$ react via interaction terms with fermions carrying charges X^a associated with currents $j_{X^a}^\mu$. If $n \geq 2$, the $n^2 - 1$ mediating particles react with each other.

In the next chapter, we will continue to focus on the work of Chen Ning Yang. This time, we will focus on the work on weak interaction and the violation of parity that we encounter there.

Chapter VII Lee Yang Wu's parity violation (1957) and the negative helicity of Golhaber's neutrino (1958)

Purpose of the chapter

Like particles of zero electric charge that are not subject to electromagnetic interaction, like leptons of zero strong charge that are not subject to the strong interaction, we will focus here on particles of zero weak charges that are not subject to the weak interaction.

Symptoms of zero weak charge are manifested in the parity violation encountered in the weak interaction. A violation of parity that implies that certain particles distinguish right from left.

Helicity is defined as the projection of the spin of a particle on its direction of propagation. We study the absence of neutrino of positive helicity (or right-handed neutrino), as well as the non-participation of certain particles in the weak interaction according to their helicity and therefore, having a zero weak charge.

VII.1 Reminders on helicity

VII.1.1 Definition

We call helicity H of a particle, the projection of the spin of this particle on its direction of propagation:

$$H = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|}$$

with \vec{S} the spin (generally e following) and \vec{p} the momentum.

The following figure schematizes for an electron and an antineutrino, the momentum \vec{p} by a thin arrow and spin \vec{S} by a thick arrow.

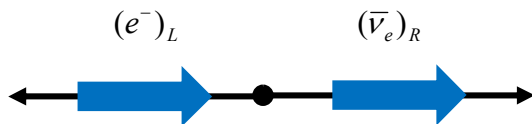


Figure 12: helicity of an electron and an antineutrino

In the figure above, we have the following helicities for the electron and antineutrino:

$$H_{e^-} = -\frac{1}{2}$$

$$\text{with } S_{e^-} = \frac{1}{2}$$

$$H_{\bar{\nu}_e} = \frac{1}{2}$$

$$\text{with } S_{\bar{\nu}_e} = \frac{1}{2}$$

If the helicity is positive, i.e., if the spin has the same direction as the momentum, we speak of a right-handed particle (right or R). If the helicity is negative, i.e., if the spin has a direction opposite to the momentum, we speak of a left-handed particle (left or L). We have on the figure above a left-handed electron $(e^-)_L$ and a right-handed antineutrino $(\bar{\nu}_e)_R$.

VII.1.2 Hyperfine transition example

The notion of helicity makes it possible to distinguish the two faces, at the spin level, of the same particle.

For example, if a right-handed proton p_R has a spin (actually the 3rd component of spin) $S^3 = +\frac{1}{2}$ a left-handed proton p_L with the same momentum has a spin $S^3 = -\frac{1}{2}$. Similarly, if a right-handed electron $(e^-)_R$ has a spin $S^3 = +\frac{1}{2}$ a left-handed electron $(e^-)_L$ with the same momentum has a spin $S^3 = -\frac{1}{2}$.

Generally, during a hyperfine transition, the electron retains the same momentum. Spin and helicity reverse. For example, we go from a right-handed electron to a left-handed electron. We have the reaction:

$$(e^-)_R(S^3 = \frac{1}{2}) \rightarrow (e^-)_L(S^3 = -\frac{1}{2}) + \gamma(S^3 = 1)$$

VII.2 Historical reminders on parity violation and negative helicity of the neutrino

VII.2.1 Parity violation in weak interactions

In the early 1950s, there was an anomaly in the weak interaction decay of two so-called strange mesons θ^+ and τ^+ (today both known as positive kaon K^+) into pions π .

In 1956, to explain this anomaly, Tsung-Dao Lee and Chen Ning Yang predicted a violation of parity in the weak interaction.

Reminder about the parity operator \hat{P}

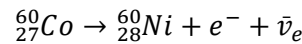
The parity operator \hat{P} reverses the Space coordinates:

$$x \rightarrow x' = -x$$

The orbital angular momentum L and spin angular momentum S are not affected by the parity operation. Energy E and momentum p are reversed at the sign level. We have the following transformations by \hat{P} :

$$\begin{aligned} L &\rightarrow L' = L \\ S &\rightarrow S' = S \\ p &\rightarrow p' = -p \\ E &\rightarrow E' = -E \end{aligned}$$

In 1957, Chien-Shiung Wu's cobalt-60 experiments investigated the possible violation of parity. When decay β^- of cobalt-60, we have the following reaction:



Graphically, this corresponds to the following diagram, with momentum \vec{p} always symbolized by a thin arrow and spin \vec{S} symbolized by a thick arrow. It is noted that the antineutrino $\bar{\nu}_e$ has positive helicity.

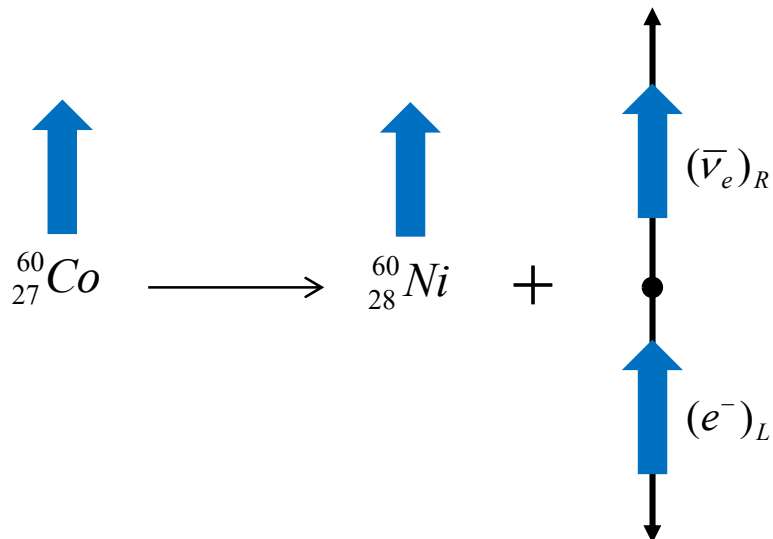


Figure 13: observed schema of decay of the cobalt ${}^{60}_{27}\text{Co}$

We give the symmetric schema obtained by parity operation \hat{P} , with reversed roles for electron and antineutrino: electron e^- is emitted forward, antineutrino $\bar{\nu}_e$ is emitted backwards. In this case, the antineutrino $\bar{\nu}_e$ should have negative helicity. This schema is not observed since there is a maximum violation of parity.

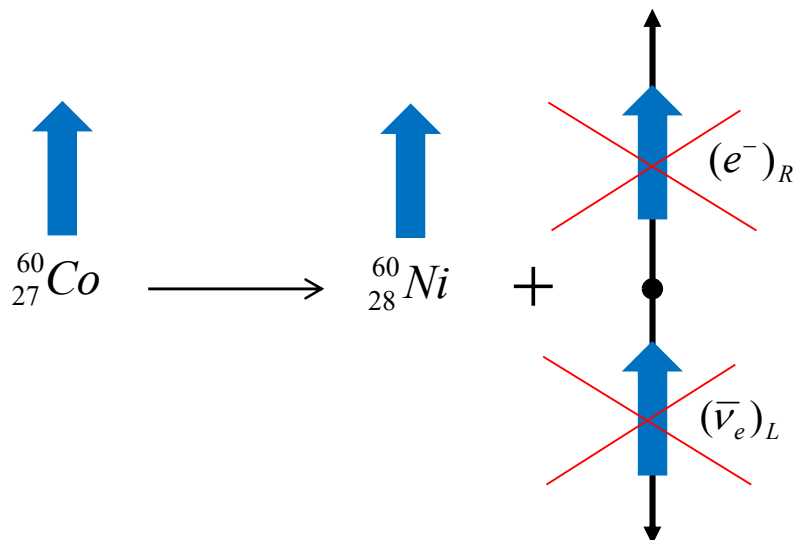


Figure 14: schema obtained by parity operation \hat{P} on the previous schema, schema not observed

The experiments of Chien-Shiung Wu on cobalt 60 therefore conclude at a maximum of the violation of parity (we will return to this experiment in the 6th Memoir).

VII.2.2 Negative helicity of the neutrino and positive helicity of the antineutrino

Inspired by these facts, in 1957 and 1958, Maurice Goldhaber, Lee Grodzins and Andrew Sunyar set up an experiment to measure the helicity of the neutrino. The principle of the experiment is based on the transfer of the helicity of the neutrino to a more easily detectable particle: the photon.

The trio noted that the helicity of the neutrino is always negative (neutrino always left-handed), with a degree of accuracy however quite low: of the order of 10%. Similarly, it was found that the helicity of the antineutrino is always positive (antineutrino always right-handed).

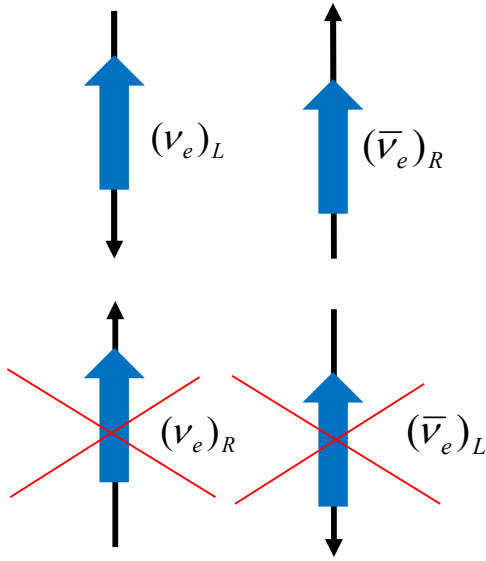


Figure 15: observed neutrino and antineutrino, not observed neutrino and antineutrino

Note on antimatter

Note that during the passage from matter to antimatter, there is reversal of the momentum, but not of the spin. The helicity is therefore reversed. The antiparticle of a right-handed electron $(e^-)_R$ is a left-handed antielectron $(\bar{e}^+)_L$. They both have the same spin, for example $S^3 = \frac{1}{2}$, an opposite momentum and helicity.

Similarly, the antiparticle of a left-handed neutrino $(\nu_e)_L$ is the right-handed antineutrino $(\bar{\nu}_e)_R$.

VII.2.3 Helicity and weak charge T

Goldhaber Grodzins Sanyara's experiment reveals that there are only left-handed neutrinos (and no right-handed neutrinos). On the other hand, there are protons or electrons both right-handed and left-handed.

Nevertheless, it is experimentally observed that only left-handed particles (and right-handed antiparticles) are involved in the weak interaction. Right-handed particles (and left-handed particles) are not subject to the weak interaction.

To explain this, right-handed particles are attributed a zero weak charge. (T, T^3) .

For example, for the right-handed proton $(p)_R$, the right-handed neutron $(n)_R$ or the right-handed electron $(e^-)_R$, we have $(T, T^3) = (0, 0)$.

Similarly, for left-handed antiproton $(\bar{p})_L$, left-handed antineutron $(\bar{n})_L$ or left-handed antielectron $(\bar{e})_L$, we have $(T, T^3) = (0, 0)$.

Note

The weak charge T is a function of the helicity and therefore the spin of the particle. Indeed, depending on their helicity, some particles do not participate in the weak interaction and have a zero weak charge T nulle.

VII.3 Quantum numbers of particles

In the tables below, we give the electric charge Q , the spin (S, S^3) , the helicity H and the weak charge (T, T^3) for some particles and antiparticles.

It is considered that the particles all have a momentum of the same direction and sense, that the antiparticles all have a momentum of the same direction and opposite sense.

VII.3.1 Left-handed leptons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(e^-)_L$	-1	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, -\frac{1}{2})$
$(\nu_e)_L$	0	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, +\frac{1}{2})$

VII.3.2 Right-handed leptons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(e^-)_R$	-1	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{2}$	(0,0)
$(\nu_e)_R$ (Not observed)	0	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{2}$	(0,0)

VII.3.3 Left-handed anti-leptons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(\bar{e}^+)_L$	+1	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{2}$	(0,0)
$(\bar{\nu}_e)_L$ (Not observed)	0	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{2}$	(0,0)

VII.3.4 Right-handed anti-leptons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(\bar{e}^+)_R$	+1	$(\frac{1}{2}, +\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, +\frac{1}{2})$
$(\bar{\nu}_e)_R$	0	$(\frac{1}{2}, +\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, -\frac{1}{2})$

VII.3.5 Left-handed nucleons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(p)_L(uud)_L$	+1	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, +\frac{1}{2})$
$(n)_L(udd)_L$	0	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, -\frac{1}{2})$

VII.3.6 Right-handed nucleons

Particle	Electric charge	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
----------	-----------------	--------------------	-----------------	---------------------------

	Q			
$(p)_R(uud)_R$	+1	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{2}$	(0,0)
$(n)_R(udd)_R$	0	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{2}$	(0,0)

VII.3.7 Left-handed anti-nucleons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(\bar{p})_L(\bar{u}\bar{u}\bar{d})_L$	-1	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{2}$	(0,0)
$(\bar{n})_L(\bar{u}\bar{u}\bar{d})_L$	0	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{2}$	(0,0)

VII.3.8 Right-handed anti-nucleons

Particle	Electric charge Q	Spin (S, S^3)	Helicity H	Weak charge (T, T^3)
$(\bar{p})_R(\bar{u}\bar{u}\bar{d})_R$	-1	$(\frac{1}{2}, +\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, -\frac{1}{2})$
$(\bar{n})_R(\bar{u}\bar{u}\bar{d})_R$	0	$(\frac{1}{2}, +\frac{1}{2})$	$-\frac{1}{2}$	$(\frac{1}{2}, +\frac{1}{2})$

VII.3.9 Particle charge following the interaction

In the table below, we order the particles according to their participation in the interactions described by the Standard Model. It is noted that right-handed neutrinos do not participate in any interaction and have all their charges zero. They have never been detected and a priori do not exist.

Does not participate in the interaction if zero charge	Electromagnetic interaction	Weak interaction	Strong interaction
Left-handed proton $(p)_L$	X	+X	+X
Left-handed neutron $(n)_L$		-X	-X
Right-handed proton $(p)_R$	X		+X
Right-handed neutron $(n)_R$			-X
Left-handed electron $(e^-)_L$	-X	-X	
Left-handed neutrino $(\nu_e)_L$		+X	
Right-handed electron $(e^-)_R$	-X		
Right-handed neutrino $(\nu_e)_R$ (not observed)			

Note

If, as in the Quark Model, it is assumed that the neutron is a composite particle formed by electrically charged quarks, then the neutron is also subject to electromagnetic interaction.

VII.4 Chapter conclusion

The weak interaction has a weak charge T analogous to the electric charge Q of the electromagnetic interaction.

For some particles, depending on their spin and helicity, this weak charge T is null. These particles do not intervene in the weak interaction, or even do not exist a priori like left-handed antineutrinos. This is the cause of the parity violation in weak interactions.

In the next chapter, we focus on the Electroweak Model, which to describe the weak interaction, takes the mathematical Yangs-Mills model developed for the strong interaction, and adapts it to particles subject or not to the weak interaction according to their helicity.

Chapter VIII Glashow-Weinberg-Salam electroweak model (1960s)

Purpose of the chapter

We present the Glashow-Weinberg-Salam electroweak model strongly inspired by the Yang-Mills gauge theory, transcribed from the strong interaction to the weak interaction. The electroweak model brings together in the same theory the electromagnetic interaction, the weak interaction, and an interaction with a weak hypercharge. Y_W . Using the Higgs mechanism, mass is assigned to the mediating bosons of the weak interaction and thus explains the very small range (or scope) of this interaction.

VIII.1 History of the Electroweak Model

The Glashow-Weinberg-Salam electroweak model is developed during the 1960s. It is inspired by the Yang-Mills gauge theory that we saw earlier. In the early 1960s, Sheldon Glashow proposed a first model combining electromagnetic interaction and weak interaction based on a gauge theory of the type $U(1)_Q \times U(1)_{Y_W} \times SU(2)_T$.

However, this model only concerns leptons (electron, neutrinos, etc., i.e., particles outside the nucleus not subject to the strong interaction). The decay of the neutron into proton is not considered. In addition, the mediating bosons of the weak interaction have no mass (like the photon). Implied, the weak interaction has a very large range, which is not experimentally the case.

In the late 1960s, Steven Weinberg and Abdus Salam had the idea of using the Brout-Englert-Higgs mechanism (BEH) to give mass to the mediating bosons of the weak interaction and thus explain the low range. Hadrons (i.e., particles of the nucleus formed by quarks subjected to the strong interaction) are this time integrated into the Electroweak Model. The BEH mechanism also makes it possible to assign mass to leptons and quarks.

In the following paragraphs, the main characteristics of the Electroweak Model will be presented. The presentation is based on an article by S. Glashow and Howard Georgi in *Physics Today* in September 1980.

S. Glashow starts from an analogy between the Coulomb electrostatic force and the weak interaction to define coupling constants. We will come to this in a few paragraphs. First, let's do a reminder about the quantum numbers of the particles used in the Electroweak Model.

VIII.2 Quantum numbers of the Electroweak Model, analogies between strong and weak interactions

VIII.2.1 General

Previously, it was seen in the Yang-Mills theory describing the strong interaction that isospin I is presented as the strong charge. By analogy between strong and weak interactions, we will now see that S. Glashow defines an isospin or weak charge T . Similarly, he defines a weak hypercharge Y_W in analogy with strong hypercharge Y that is found in the Quark Model.

VIII.2.2 Strong hypercharge Y and isospin I^3 , Gell-Mann Nishijima's relationship

Based on experimental data on collisions between particles involving the strong interaction, K. Nishijima (1953) and M. Gell-Mann (1956) proposed a formula linking the charge Q to strong hypercharge Y and to isospin I^3 , for particles of the same strong hypercharge Y rgrouped in a multiplet.

For each particle of the multiplet, we have the Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Y}{2}$$

In addition, we have:

$$Y = B + St$$

with B the baryonic number and St the strangeness number of the particle.

We get:

$$Q = I^3 + \frac{B + St}{2}$$

The particles of the multiplet usually have opposite isospins. Thus, on average for the multiplet, the isospins cancel each other out:

$$\text{average}(I^3) = 0$$

We have the strong hypercharge of each particle equal to that characterizing the multiplet. So, we have an average:

$$\text{average}(Y) = Y$$

According to $Q = I^3 + \frac{Y}{2}$, we then have:

$$\text{average}(Q) = \text{average}(I^3) + \text{average}\left(\frac{Y}{2}\right) = \frac{Y}{2}$$

We have the strong hypercharge of each particle equal to that characterizing the multiplet, equal to twice the average of the charges of the particles of the multiplet.

Example of a doublet

For a neutron proton doublet, we have the values:

Particle	Electric charge Q	Strong hypercharge Y	Isospin identified with strong charge I^3
p	1	1	$+\frac{1}{2}$
n	0	1	$-\frac{1}{2}$

For each particle of the doublet, we have according to the Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Y}{2} = \pm \frac{1}{2} + 1$$

On average, we have:

$$\text{average}(Q) = \frac{Y}{2} = \frac{1}{2}$$

VIII.2.3 Weak hypercharge Y_W and weak charge T^3

In the early 1960s, Sheldon Glashow was looking for a local gauge theory that could unify electromagnetic and weak interactions. By analogy with the Gell-Mann–Nishijima relation, he proposed a relationship between the electric charge, a weak isospin (or weak charge) T^3 and weak hypercharge Y_W . For each particle of a multiplet, we have the Glashow relation:

$$Q = T^3 + \frac{Y_W}{2}$$

On average for the multiplet, we have:

$$average(Q) = \frac{Y_W}{2}$$

Since leptons are subject to weak interaction, S. Glashow assigns them a weak charge T^3 identical to that of the corresponding nucleon. On the other hand, he differentiates left-handed and right-handed particles since the latter are not subject to weak interaction.

Examples of left-handed doublets

For a left-handed lepton doublet (electron $(e^-)_L$, neutrino $(\nu_e)_L$), we have the values:

Particle	Electric charge Q	Weak hypercharge Y_W	Weak charge T^3
$(e^-)_L$	-1	-1	$-\frac{1}{2}$
$(\nu_e)_L$	0	-1	$+\frac{1}{2}$

On average, we have:

$$average(Q) = \frac{Y_W}{2} = -\frac{1}{2}$$

Although Glashow's initial model does not contain hadrons, they are later included in a form analogous to leptons. For a left-handed neutron proton doublet, we have the values:

Particle	Electric charge Q	Strong hypercharge Y	Isospin I^3	Weak hypercharge Y_W	Weak charge T^3
$(p)_L$	1	1	$+\frac{1}{2}$	1	$+\frac{1}{2}$
$(n)_L$	0	1	$-\frac{1}{2}$	1	$-\frac{1}{2}$

On average, we have:

$$moy(Q) = \frac{Y_W}{2} = \frac{1}{2}$$

Example of a right-handed singlet

We define groups of particles containing a single particle. In this case, the single particle has a zero weak charge T^3 and is not subject to weak interaction.

For example, we have the electron of right helicity $(e^-)_R$:

Particle	Electric charge Q	Weak hypercharge Y_W	Weak charge T^3
$(e^-)_R$	-1	-2	0
$(\nu_e)_R$ not observed			

VIII.2.4 Quantum numbers of the Electroweak Model

The table below summarizes the quantum numbers of the particles involved in the Electroweak Model. Ba is the baryonic number, L_e is the leptonic number.

		Charges				
	Particle generations	(S, S^3)	Ba L_e	Q	(T, T^3)	$\frac{Y_W}{2} = Q - T^3$
Left-handed quarks	u_L, c_L, t_L	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	$+\frac{2}{3}$	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{3}$
	d_L, s_L, b_L	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
Right-handed quarks	u_R, c_R, t_R	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{3}$	$+\frac{2}{3}$	(0,0)	$+\frac{4}{3}$
	d_R, s_R, b_R	$(\frac{1}{2}, +\frac{1}{2})$	$+\frac{1}{3}$	$-\frac{1}{3}$	(0,0)	$-\frac{2}{3}$
Left-handed leptons	e_L^-, μ_L^-, τ_L^-	$(\frac{1}{2}, -\frac{1}{2})$	-1	-1	$(\frac{1}{2}, -\frac{1}{2})$	-1
	$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	$(\frac{1}{2}, -\frac{1}{2})$	-1	0	$(\frac{1}{2}, +\frac{1}{2})$	-1
Right-handed leptons	e_R^-, μ_R^-, τ_R^-	$(\frac{1}{2}, +\frac{1}{2})$	-1	-1	(0,0)	-2
	$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ not observed			0	(0,0)	0

Note that:

- the 3 generations of particles (quarks and leptons) have the same number of charges (this would no longer be the case if we included the strong charge in the table above),
- the weak charge (T, T^3) and the weak hypercharge Y_W are functions of helicity and therefore spin.

Like the strong interaction that differentiates hadrons and leptons depending on whether or not they are subject to the strong interaction, the weak interaction differentiates left-handed and right-handed particles depending on whether or not they are subject to the weak interaction.

VIII.3 Coupling constants

VIII.3.1 Analogy with electrostatics

To define the coupling constants involved in the electroweak model, S. Glashow starts from the Coulomb electrostatic force:

$$F_{el} = \frac{QQ'e^2}{4\pi\epsilon_0 r^2}$$

Q and Q' are integers that indicate the number of elementary electric charges.

The electrostatic force is rewritten from the electromagnetic coupling constant α_{em} :

$$\frac{F_{el}}{\hbar c} = \alpha_{em} \frac{QQ'}{r^2}$$

VIII.3.2 Coupling constants α_Y and α_T

On the model of α_{em} , the Electroweak Model defines two additional coupling constants.

We remind the coupling constant of the electromagnetic interaction with an elementary electric charge $e = g_Q$:

$$\alpha_{em} = \alpha_Q = \frac{g_Q^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

On this model, S. Glashow defines a coupling constant α_Y and an elementary charge g_Y associated with weak hypercharge Y_W :

$$\alpha_Y = \frac{g_Y^2}{4\pi\epsilon_0\hbar c}$$

He also defines a coupling constant α_T and an elementary charge g_T associated with weak charge T :

$$\alpha_T = \frac{g_T^2}{4\pi\epsilon_0\hbar c}$$

Note

Like $\alpha_{em} = \alpha_Q$, coupling constants α_Y and α_T are dimensionless.

Numerically, we have the following values (for energy equivalent to $M_Z \approx 90\text{GeV}$):

$$\alpha_{em} = \alpha_Q = \frac{g_Q^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

$$\alpha_T = \frac{g_T^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{30}$$

$$\alpha_Y = \frac{g_Y^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{100}$$

Note

The following alternative notations are also used for coupling constants α_T and α_Y :

$$\alpha_Y = \alpha_1 = \alpha'$$

$$\alpha_T = \alpha_2 = \alpha$$

$$g_Y = g_1 = g'$$

$$g_T = g_2 = g$$

Here we prefer to use the indices Q , Y and T to mark the link between the coupling constant and the quantum number of charges (electric, hyper weak or weak).

VIII.4 Local Gauge transformations and covariant derivatives

VIII.4.1 Group $SU(2)_T$

As before, S. Glashow is inspired by the Yang-Mills theory describing the strong interaction, transcribed for the weak interaction.

He proposes that the weak charge T is the generator of a group of rotations $SU(2)_T$ with $\exp(ig_T T^a \cdot \alpha^a(x)) \in SU(2)_T$.

He also proposes a local gauge transformation on a doublet of wave functions ψ_L , associated with left-handed particles:

$$\psi_L \rightarrow \psi_L' = \exp(ig_T T^a \cdot \alpha^a(x))\psi_L$$

$$(a = 1,2,3)$$

Note

As in Yang-Mills, we usually take a weak charge $T^a = \frac{\sigma^a}{2}$ ($a = 1,2,3$) with σ^a the 3 Pauli matrices.

He defines 3 weak potential quadrivectors $W^{1\mu}$, $W^{2\mu}$, $W^{3\mu}$, corresponding to three massive bosons W^1 , W^2 , W^3 particles mediating the weak interaction.

We have the local gauge transformation of the 3 potential quadrivectors:

$$W^{1\mu} \rightarrow W^{1\mu'} = W^{1\mu} + \partial_\mu \alpha^1(x) + g_T f_{123} \alpha^2(x) W^{3\mu}$$

with $f_{123} = \varepsilon_{123}$ the group structure constant.

VIII.4.2 Group $U(1)_{Y_W}$

For weak hypercharge Y_W to one dimension (such as the electric charge Q), S. Glashow proposes that it be the generator of a group of rotations $U(1)_{Y_W}$ with $\exp(ig_Y Y_W \cdot \alpha(x)) \in U(1)_{Y_W}$.

He also offers a local gauge transformation on a wave function ψ :

$$\psi \rightarrow \psi' = \exp(ig_Y Y_W \cdot \alpha(x))\psi$$

S. Glashow defines a potential quadrivector B^μ corresponding to a boson B . We have the local gauge transformation of the potential quadrivector:

$$B^\mu \rightarrow B^{\mu'} = B^\mu + \partial_\mu \alpha(x)$$

VIII.4.3 Hypercharge $\frac{Y_W}{2}$, covariant derivative and hyper interaction

For a multiplet of particles, we have on average:

$$\text{average}(Q) = \frac{Y_W}{2}$$

Since the charge Q is retained during a global gauge transformation, $\text{average}(Q)$ must also be retained, and therefore also $\frac{Y_W}{2}$.

For electromagnetic interaction, after a local gauge transformation, we have a covariant derivative of the form:

$$D^\mu = \partial^\mu + ieQ \cdot A^\mu$$

For weak hypercharge Y_W , after a local gauge transformation, a covariant derivative of the form is posed:

$$D^\mu = \partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu + \dots$$

We put three small points because we do not yet consider in the covariant derivative of the term weak interaction.

We may feel like we are defining a new interaction based on the particle mediating the boson B and on the weak hypercharge Y_W . It is proposed here that this is indeed the case. We will talk in this Memoir and in the next Memoir of hyper interaction.

VIII.4.4 Weak charge T^3 , covariant derivative and weak interaction

For each particle of a multiplet, we have the Glashow relation between the 3 charges:

$$Q = T^3 + \frac{Y_W}{2}$$

Since the electric charge Q et the weak hypercharge Y_W are quantities retained during a Global Gauge transformation, weak charge T^3 is also.

We can therefore define a Gauge quadrivector $W^{3\mu}$ that is cancelled during a Local Gauge transformation and is associated with the quantity retained T^3 .

We generalize to potential quadrivectors $W^{a\mu}$ and weak charges T^a , with $a = 1,2,3$.

For left-handed fermions subject to weak interaction, the Electroweak Model poses a covariant derivative, after local Gauge transformation:

$$D^\mu = \partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu + ig_T T^a \cdot W^{a\mu}$$

with $a = 1,2,3$

$g_Y \frac{Y_W}{2}$ and $g_T T^a$ are the quantities kept during a Global Gauge transformation.

$W^{1\mu}$, $W^{2\mu}$ and $W^{3\mu}$ are the three quadrivectors cancelled during the local gauge transformation that we match the mediating particles of the weak interaction, that is, to the three weak bosons W^1 , W^2 , W^3 .

B^μ is a cancelled quadrivector that is matched to the boson B .

Note

For right-handed particles not subject to weak interaction, a covariant derivative is posed:

$$D^\mu = \partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu.$$

VIII.5 Study of Lagrangians involved in the Electroweak Model

In order not to weigh down the expressions, we choose for the constants $\hbar = 1$ and $c = 1$.

VIII.5.1 Lagrangian of d'Alembert

The Lagrangians used for the weak interaction are built on the model of those of the electromagnetic interaction and those of the Yang-Mills interaction.

In the reference frame R^0 (the vacuum), we have a Lagrangian for the left-handed electron corresponding to the d'Alembert wave equation:

$$[L]_{R^0} = i\bar{\psi}_L \gamma_\mu (\partial^\mu) \psi_L$$

Note

Antiparticle associated with the wave function $\bar{\psi}_L$ is actually a right-handed antiparticle. It is subject to weak interaction.

VIII.5.2 Lagrangian of Dirac libre

In the reference frame R^c (with masses), we have the Lagrangian for the left-handed electron corresponding to free Dirac:

$$[L]_{R^c} = i\bar{\psi}_L \gamma_\mu (\partial^\mu) \psi_L - m\bar{\psi}_L \psi_L$$

VIII.5.3 Lagrangian of Dirac in a weak field

In the reference frame R^{B+W} where we cancel potential quadrivectors B^μ and $W^{a\mu}$, we replace ∂^μ by D^μ so that the wave equation is always respected (Gauge invariance principle), the electroweak model gives the Lagrangian for left-handed fermions:

$$[L]_{R^{B+W}} = i\bar{\psi}_L \gamma_\mu (\partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu + ig_T T^a \cdot W^{a\mu}) \psi_L - m\bar{\psi}_L \psi_L$$

The Electroweak Model gives the Lagrangian for right-handed fermions:

$$[L]_{R^{B+W}} = i\bar{\psi}_R \gamma_\mu (\partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu) \psi_R - m\bar{\psi}_R \psi_R$$

Like Yang-Mills, there is also a Lagrangian to describe the dynamics of gauge bosons:

$$L_{gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu}$$

(Summation on $a = 1,2,3$)

We have for the tensor $B_{\mu\nu}$:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

with $B_{\mu\nu}$ of the same type as the Maxwell Faraday tensor $F_{\mu\nu}$.

We have for the 3 tensors $W_{\mu\nu}^1, W_{\mu\nu}^2, W_{\mu\nu}^3$:

$$W_{\mu\nu}^3 = \partial_\mu W_\nu^{3\mu} - \partial_\nu W_\mu^{3\mu} + g_T \varepsilon^{123} W^{1\mu} W^{2\nu}$$

Note 1

The group $SU(2)_T$ of weak isospin is noncommutative. As at Yang-Mills, the combination $g_T \varepsilon^{123} W^{1\mu} W^{2\nu}$ involves interactions between weak bosons W^1, W^2, W^3 .

Note 2

As with Yang-Mills, local gauge invariance requires that all four bosons B and W^1, W^2, W^3 are massless. However, to explain the small range of nuclear interactions, it's necessary that bosons W^1, W^2, W^3 , and that observed bosons $W^+, W^-, W^0 = Z^0$ have a mass.

W^+ and W^- are the two bosons carrying an electric charge and involved in decays β^+ and β^- .

This implies that the Lagrangian includes terms of mass of the form:

$$L_{mass} = M_W^2 W^{+\mu} W_\mu^+ + M_W^2 W^{-\mu} W_\mu^- + M_Z^2 Z^{0\mu} Z_\mu^0$$

But this Lagrangian L_{mass} is not Gauge invariant. To overcome this problem, we will see later that the idea of the Electroweak Model is to use the BEH mechanism.

VIII.6 Weak bosons W^+, W^- and Z^0

VIII.6.1 Weak bosons W^+ and W^- electrically charged

As mentioned, decays β^+ and β^- are known since the end of the 19th century with the discovery by H. Becquerel of radioactivity. Following the idea of mediating particles, in the late 1940s, there was a conjecture for decays β^+ and β^- , of electrically charged mediating particles, that interact with the weak currents proposed by E. Fermi.

Intervening respectively in the decays β^+ and β^- , these mediating particles will later be called weak bosons W^+ and W^- . According to H. Yukawa, these weak bosons must have a mass.

VIII.6.2 The neutral weak boson Z^0

In the early 1950s, experimenters noticed the existence of reactions involving weak interactions, but whose exchanges between the particles of the reaction did not seem to involve any electrical charge. (unlike decays β^+ and β^-). For example, muon neutrinos ν_μ appear to react with matter, and then give muon neutrinos ν_μ with slightly different energy.

In 1958, following a certain analogy with the pion meson π^0 electrically neutral mediating particle of the strong interaction, Sydney Bludman puts forward the idea of an electrically neutral weak current that interacts with a weak boson, both mass and electrically neutral, later called the boson Z^0 .

The photon γ and the boson Z^0 have fairly similar characteristics. Both are electrically neutral, that is, they do not carry electrical charges.

Note

There is also the notation W^0 for the boson Z^0 .

VIII.6.3 Weak boson quadrivectors W^+ and W^-

For his Electroweak Model, S. Glashow seeks to obtain from bosons W^1 and W^2 , bosons W^+ and W^- , which respectively carry an electric charge $+$ and $-$. Since these electrically charged weak bosons can increase or decrease the weak charge of a T^3 of the fermions with which they interact, S. Glashow defines them as scale operators.

We give the potential quadrivectors of the weak electrically charged bosons:

$$W^{+\mu} = \frac{1}{\sqrt{2}} (W^{1\mu} + iW^{2\mu})$$

$$W^{-\mu} = \frac{1}{\sqrt{2}}(W^{1\mu} - iW^{2\mu})$$

We give the weak charges carried by the weak currents (i.e., the fermions) with which the electrically charged weak bosons interact:

$$T^+ = T^1 + iT^2$$

$$T^- = T^1 - iT^2$$

If we construct the weak charges T^+ and T^- from the Pauli matrices, we get:

$$T^+ = T^1 + iT^2 = \frac{1}{2} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$T^- = T^1 - iT^2 = \frac{1}{2} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Note 1, reminder on scale operators

Let us briefly remind the theory of scale operators originally used for quantum angular momentum J or for spin S . We also talk about creation and annihilation operators. We define angular momentum scale operators J^\pm from operators J^1 and J^2 :

$$J^+ = J^1 + iJ^2$$

$$J^- = J^1 - iJ^2$$

J^+ and J^- are scale operators for the eigenvalue $\hbar m$ of the operator J^3 .

$|m\rangle$ is a common eigenvector for operators $(J)^2$ and J^3 . We have for the operator J^3 :

$$J^3 |m\rangle = \hbar m |m\rangle$$

Let j be the maximum value of m .

Operators J^\pm allow you to increase or decrease the eigenvalues of the operator J^3 :

$$J^- |m\rangle = \hbar \sqrt{j(j+1) - m(m-1)} |m-1\rangle$$

$$J^+ |m\rangle = \hbar \sqrt{j(j+1) - m(m+1)} |m+1\rangle$$

The theory can be used for spin S , isospin I or for weak charge T , all of which have formal analogies to quantum angular momentum J .

T^+ and T^- are scale operators for the eigenvalue $m\hbar$ of the operator T^3 .

$|m\rangle$ is an eigenvector common to $(T)^2$ and T^3 . We have for the operator T^3 :

$$T^3 |m\rangle = m\hbar |m\rangle$$

Let j be the maximum value of m .

We have:

$$T^- |m\rangle = \hbar \sqrt{j(j+1) - m(m-1)} |m-1\rangle$$

$$T^+|m\rangle = \hbar\sqrt{j(j+1) - m(m+1)}|m+1\rangle$$

Operators T^+ and T^- can be associated with bosons W^+ and W^- that increase or decrease the weak charge T^3 of the particle. These bosons W^+ and W^- therefore carry weak charges T^3 and are electrically charged according to the Glashow relation $Q = T^3 + \frac{Y_W}{2}$.

Note that the weak boson W^3 associated with T^3 is not a scale operator for T^3 . W^3 therefore, cannot increase or decrease the weak charge T^3 of a fermion. It does not carry weak charge T^3 .

Note 2 on the top index of mediating particles

Mediating particles are usually indicated not by the quantum quantity they carry, but by the charge carried by the fermions (weak currents) with which they interact.

For example, bosons W^+ and W^- interact with weak-charge fermions T^+ and T^- . bosons W^+ and W^- carry as weak charge T^3 and $-T^3$.

Bosons W^1, W^2, W^3 interact with weak charge fermions T^1, T^2, T^3 .

Note 3, analogy between pions (or meson pi) π^+, π^-, π^0 and weak bosons $W^+, W^-, W^0 = Z^0$

If we only consider the 1st generation of left-handed quarks, following a strict analogy with pions π^+, π^-, π^0 , we should have for weak bosons $W^+, W^-, W^0 = Z^0$:

$$|T = 1, T^3 = 1\rangle = \left|t = \frac{1}{2}, t^3 = \frac{1}{2}; t' = \frac{1}{2}, t^{3'} = \frac{1}{2}\right\rangle = |u_L; -\bar{d}_L\rangle = |W^+\rangle$$

$$\begin{aligned} |T = 1, T^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left|t = \frac{1}{2}, t^3 = \frac{1}{2}; t' = \frac{1}{2}, t^{3'} = -\frac{1}{2}\right\rangle + \left|t = \frac{1}{2}, t^3 = -\frac{1}{2}; t' = \frac{1}{2}, t^{3'} = \frac{1}{2}\right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|u_L; \bar{u}_L\rangle + |d_L; -\bar{d}_L\rangle) = |Z^0\rangle \end{aligned}$$

$$|T = 1, T^3 = -1\rangle = \left|t = \frac{1}{2}, t^3 = -\frac{1}{2}; t' = \frac{1}{2}, t^{3'} = -\frac{1}{2}\right\rangle = |d_L; \bar{u}_L\rangle = |W^-\rangle$$

$$\begin{aligned} |T = 0, T^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left|t = \frac{1}{2}, t^3 = \frac{1}{2}; t' = \frac{1}{2}, t^{3'} = -\frac{1}{2}\right\rangle - \left|t = \frac{1}{2}, t^3 = -\frac{1}{2}; t' = \frac{1}{2}, t^{3'} = \frac{1}{2}\right\rangle \right) \\ &= \frac{1}{\sqrt{2}} (|u_L; \bar{u}_L\rangle - |d_L; \bar{d}_L\rangle) \end{aligned}$$

If we want to take into account the 2nd generation of quarks, we should a priori replace d_L by $d_L' = -d_L \cos \theta_c + s_L \sin \theta_c$, with θ_c the Cabibbo angle (for the model of N. Cabibbo, see Memoir 5).

Note 4 on weak hypercharge

According to the relationship $Q = T^3 + \frac{Y_W}{2}$, It is noted that bosons $W^+, W^-, W^0 = Z^0$ all have zero weak hypercharge.

VIII.7 Weak currents carrying a weak charge T

VIII.7.1 Definition

We remind the electromagnetic currents charge density quadrivector (we distinguish here right-handed and left-handed particles, even if there is no need for electromagnetic interaction):

$$j_Q^\mu = Q(\bar{\psi}_R \gamma^\mu \psi_R + \bar{\psi}_L \gamma^\mu \psi_L)$$

with $\mu = t, x, y, z$

In analogy with this current and following the ideas of E. Fermi, the Electroweak Model defines 3 weak currents, each of which carries a weak charge T^a and interacts with a weak boson W^a ($a = 1, 2, 3$):

$$j_{T^a}^\mu = T^a \bar{\psi}_L \gamma^\mu \psi_L$$

with $\mu = t, x, y, z$

Note

The weak current only applies to left-handed fermions, since they are the only ones subject to the weak interaction.

From $j_{T^1}^\mu$ and $j_{T^2}^\mu$, we define weak currents carrying a charge T^\pm , who each interacts with a boson W^+ and W^- :

$$j_{T^+}^\mu = j_{T^1}^\mu + i j_{T^2}^\mu$$

$$j_{T^-}^\mu = j_{T^1}^\mu - i j_{T^2}^\mu$$

Note

So, we have:

- 2 bosons W^+ and W^- that interact with weak currents $j_{T^+}^\mu$ and $j_{T^-}^\mu$, and which carry the weak charge T^3 ,
- 1 boson Z^0 that interacts with weak current $j_{T^0}^\mu$.

VIII.7.2 Examples of electromagnetic currents, examples of electrically neutral and electrically charged weak currents

Case of 1st generation leptons

If we simply consider the leptons of 1st generation, we have an electromagnetic current:

$$j_Q^\mu = Q \bar{e} \gamma^\mu e$$

Note

The wave function ψ of the particle is here symbolized directly by the particle.

We have a neutral weak current formed by a doublet of left-handed particles:

$$j_{T^3}^\mu = T^3 \overline{(v_e \ e)}_L \gamma^\mu \begin{pmatrix} v_e \\ e \end{pmatrix}_L$$

$\overline{v_{eL}}$ is the antiparticle of v_{eL} , $\overline{v_{eL}} = \bar{v}_{eR}$ is of right helicity.

The Electroweak Model gives electrically charged weak currents $j_{T^\pm}^\mu$, carrying weak charges T^- and T^+ , Left-handed particle compounds, interacting with bosons W^- and W^+ :

$$j_{T^-}^\mu = \overline{(e_L)}\gamma^\mu(v_{eL})$$

$$j_{T^+}^\mu = \overline{(v_{eL})}\gamma^\mu(e_L)$$

Case of 1st and 2nd generation quarks

If we consider the quarks of 1st and 2nd generation, we have an electromagnetic current:

$$j_Q^\mu = Q\bar{u}\gamma^\mu u + Q\bar{d}\gamma^\mu d + Q\bar{c}\gamma^\mu c + Q\bar{s}\gamma^\mu s$$

$$j_Q^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{2}{3}\bar{c}\gamma^\mu c - \frac{1}{3}\bar{s}\gamma^\mu s$$

We have a neutral weak current:

$$j_{T^3}^\mu = T^3 \overline{(u \ d)}_L \gamma^\mu \begin{pmatrix} u \\ d \end{pmatrix}_L + T^3 \overline{(c \ s)}_L \gamma^\mu \begin{pmatrix} c \\ s \end{pmatrix}_L$$

\bar{u}_L is here the antiparticle of u_L , $\bar{u}_L = \bar{u}_R$ is of right helicity.

The Electroweak Model gives electrically charged weak currents $j_{T^\pm}^\mu$:

$$j_{T^+}^\mu = \overline{(u \ c)}_L \gamma^\mu \begin{pmatrix} d \\ s \end{pmatrix}_L$$

$$j_{T^-}^\mu = \overline{(d \ s)}_L \gamma^\mu \begin{pmatrix} u \\ c \end{pmatrix}_L$$

Note

We will come back to these electrically charged weak currents in the next Memoir.

We do not consider here angle of Cabibbo θ_c (concerning quarks) and angle of Pontecorvo θ_p (concerning leptons). This will also be discussed in the next Memoir.

VIII.8 Study of interaction energies

VIII.8.1 Weak interaction energy

We remind the Lagrangian of left-handed particles proposed by the Electroweak Model:

$$[L]_{R^{B+W}} = i\bar{\psi}_L \gamma_\mu \left(\partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu + ig_T T^a \cdot W^{a\mu} \right) \psi_L - m\bar{\psi}_L \psi_L$$

$$a = 1,2,3$$

We have the weak interaction energy between the currents $j_{T^a}^\mu = T^a \bar{\psi}_L \gamma^\mu \psi_L$ and potentials $W^{a\mu}$:

$$[E]_{RW} = -ig_T T^a \bar{\psi}_L \gamma_\mu \psi_L \cdot W^{a\mu}$$

$$(a = 1,2,3)$$

$$[E]_{RW} = -ig_T j_{T^a}^\mu \cdot W^{a\mu}$$

By replacing with the charged quantities, we obtain:

$$[E]_{RW} = -ig_T \left[\frac{1}{\sqrt{2}} (j_{T^+}^\mu \cdot W^{+\mu} + j_{T^-}^\mu \cdot W^{-\mu}) + j_{T^3}^\mu \cdot W^{3\mu} \right]$$

Note

It should be noted that in this weak charge interaction energy, only left-handed particles are involved.

VIII.8.2 Hyper currents carrying a weak hypercharge Y_W

In analogy with the electromagnetic current, we define a hyper current carrying a weak hypercharge Y_W :

$$j_{Y_W}^\mu = Y_{WL} \bar{\psi}_L \gamma^\mu \psi_L + Y_{WR} \bar{\psi}_R \gamma^\mu \psi_R$$

Note

Remember that left-handed and right-handed particles do not have the same weak hypercharge. Y_W . For example, for a quark u , we have a hyper current:

$$j_{Y_W}^\mu = \frac{1}{3} \bar{u}_L \gamma^\mu u_L + \frac{4}{3} \bar{u}_R \gamma^\mu u_R$$

Since we have the relationship between the charges transported by the currents and conserved by global gauge transformation:

$$\frac{Y_W}{2} = Q - T^3$$

We have the relationship between the currents conserved by global gauge transformation:

$$\frac{j_{Y_W}^\mu}{2} = j_Q^\mu - j_{T^3}^\mu$$

VIII.8.3 Hyper interaction energy

We have the hyper interaction energy between hyper current $j_{Y_W}^\mu$ and potential B^μ :

$$[E]_{RB} = -i \frac{g_Y}{2} (Y_{WL} \bar{\psi}_L \gamma^\mu \psi_L + Y_{WR} \bar{\psi}_R \gamma^\mu \psi_R) \cdot B^\mu = -ig_Y \frac{j_{Y_W}^\mu}{2} \cdot B^\mu$$

$$[E]_{RB} = -ig_Y \frac{j_{Y_W}^\mu}{2} \cdot B^\mu = -ig_Y (j_Q^\mu - j_{T^3}^\mu) \cdot B^\mu$$

If we look at the interaction energies:

- $[E]_{RW} = -ig_T \left[\frac{1}{\sqrt{2}} (j_{T^+}^\mu \cdot W^{+\mu} + j_{T^-}^\mu \cdot W^{-\mu}) + j_{T^3}^\mu \cdot W^{3\mu} \right]$,
- $[E]_{RB} = -ig_Y \frac{j_B^\mu}{2} \cdot B^\mu = -ig_Y (j_Q^\mu - j_{T^3}^\mu) \cdot B^\mu$,

we recognize in:

- the term $-ig_T \left[\frac{1}{\sqrt{2}} (j_{T^+}^\mu \cdot W^{+\mu} + j_{T^-}^\mu \cdot W^{-\mu}) \right]$ the contribution of electrically charged weak currents,
- the terms $-ig_T j_{T^3}^\mu \cdot W^{3\mu}$ et $ig_Y (j_Q^\mu - j_{T^3}^\mu) \cdot B$ the contribution of electrically "neutral" weak currents, respectively with weak charge T^3 and weak hypercharge Y_W .

However, we do not distinguish at first glance what we detect experimentally:

- an electrically neutral weak current $j_{T^0}^\mu$ that interacts with a boson Z^0 of potential $Z^{0\mu}$,
- an electromagnetic current j_Q^μ that interacts with a photon γ of potential A^μ .

For this reason, the Electroweak Model introduces the so-called Weinberg angle.

VIII.8.4 Angle of Weinberg

The Electroweak Model considers that the potentials $W^{3\mu}$, B^μ are linear combinations of potentials $Z^{0\mu}$, A^μ . Following the ideas of S. Glashow, the Electroweak Model defines a matrix of mixing between potentials $W^{3\mu}$, B^μ et les potentiels $Z^{0\mu}$, A^μ .

We have the mixing matrix:

$$\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z^{0\mu} \\ A^\mu \end{pmatrix}$$

θ_W is commonly referred to as the Weinberg angle.

Note, reminder on associations between potential quadrivectors and bosons

The potential quadrivector $W^{3\mu}$ is associated with the boson W^3 .

The potential quadrivector B^μ is associated with the boson B .

The potential quadrivector $Z^{0\mu}$ is associated with the boson Z^0 .

The potential quadrivector A^μ is associated with the photon γ .

The first 2 bosons, W^3 and B , have not been observed experimentally. The next 2, Z^0 and γ , are observed.

Experimentally, we measure:

$$\sin^2 \theta_W \approx 0,231$$

$$\theta_W \approx 28,7^\circ$$

Nota

Intuitively, the Weinberg angle θ_W can be interpreted as the manifestation of the violation of parity by the boson Z^0 . Z^0 reacts by favouring left-handed particles of negative helicity.

By substituting the expressions of $W^{3\mu}$ and B^μ by the expressions of Z^μ and A^μ , we obtain for the interaction energies, contribution of electrically "neutral" currents of weak charge T^3 and weak hypercharge Y_W :

$$\begin{aligned} [E]_{R^{B+W}} &= -i[(g_T \sin \theta_W - g_{Y_W} \cos \theta_W)j_{T^3}^\mu + (g_Y \cos \theta_W)j_Q^\mu] \cdot A^\mu \\ &\quad -i[(g_T \cos \theta_W - g_Y \sin \theta_W)j_{T^3}^\mu - (g_Y \sin \theta_W)j_Q^\mu] \cdot Z^{0\mu} \end{aligned}$$

VIII.8.5 Electromagnetic interaction energy

We have the electromagnetic interaction energy:

$$[E_{em}]_{R^{B+W}} = -i[(g_T \sin \theta_W - g_Y \cos \theta_W)j_{T^3}^\mu + (g_Y \cos \theta_W)j_Q^\mu] \cdot A^\mu$$

The Electroweak Model seeks to find the electromagnetic interaction energy usually defined:

$$[E_{em}]_{RA} = -iej_Q^\mu \cdot A^\mu$$

It is found if we pose the 2 relations between the 3 elementary charges $e = g_Q, g_Y, g_T$:

$$e = g_Y \cos \theta_W$$

$$e = g_T \sin \theta_W$$

We replace the elementary charges in the interaction energies (contribution of "neutral" currents), and we obtain:

$$[E]_{R^{B+W}} = -iej_Q^\mu \cdot A^\mu - i \frac{g_T}{\cos \theta_W} (j_{T^3}^\mu - \sin^2 \theta_W j_Q^\mu) \cdot Z^{0\mu}$$

VIII.8.6 Relationships between coupling constants, elementary charges, and Weinberg angle

We remind the different relationships between coupling constants, elementary charges and the Weinberg angle proposed by the Electroweak Model:

$$\frac{g_Y}{g_T} = \tan \theta_W$$

$$\frac{1}{g_Y^2} + \frac{1}{g_T^2} = \frac{1}{e^2} = \frac{1}{g_Q^2}$$

$$\frac{1}{\alpha_Y} + \frac{1}{\alpha_T} = \frac{1}{\alpha_Q}$$

$$\alpha_Q = \alpha_B \cos^2 \theta_W$$

$$\alpha_Q = \alpha_{T^3} \sin^2 \theta_W$$

$$\frac{g_Y^2 + g_T^2}{g_Y^2 g_T^2} = \frac{1}{e^2}$$

$$\frac{g_Y g_T}{\sqrt{g_T^2 + g_Y^2}} = e = g_Q$$

$$\cos \theta_W = \frac{e}{g_Y} = \frac{g_T}{\sqrt{g_Y^2 + g_T^2}}$$

$$\sin \theta_W = \frac{e}{g_T} = \frac{g_Y}{\sqrt{g_Y^2 + g_T^2}}$$

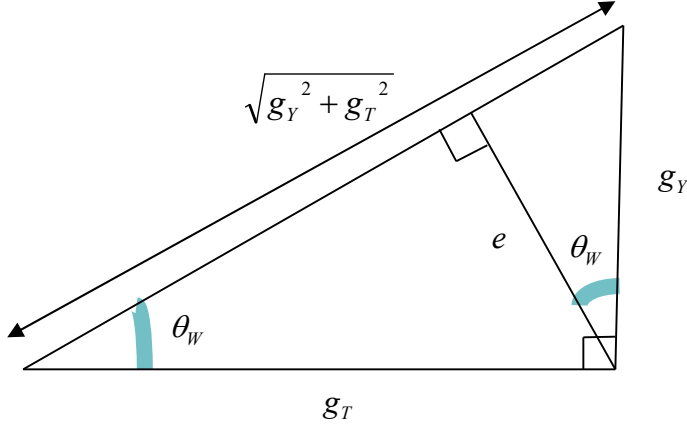


Figure 16: elementary charges and Weinberg angle

VIII.8.7 Neutral weak current carrying weak charge T^0 interacting with the boson Z^0

We define a neutral weak current carrying a weak charge T^0 interacting with the boson Z^0 :

$$j_{T^0}^\mu = j_{T^3}^\mu - \sin^2 \theta_W j_Q^\mu$$

$$j_{T^0}^\mu = \bar{\psi}_L \gamma^\mu (T^3 - \sin^2 \theta_W Q) \psi_L - \bar{\psi}_R \gamma^\mu \sin^2 \theta_W Q \psi_R$$

We obtain in terms of interaction energy:

$$[E]_{R^{B+W}} = -ie j_Q^\mu A^\mu - i \frac{g_T}{\cos \theta_W} j_{T^0}^\mu Z^{0\mu}$$

We have the elementary charge g_{T^0} and the coupling constant α_{T^0} :

$$g_{T^0} = \frac{g_T}{\cos \theta_W} = \frac{e}{\sin \theta_W \cos \theta_W}$$

$$\alpha_{T^0} = \frac{\alpha_T}{\cos^2 \theta_W} = \frac{\alpha_Q}{\sin^2 \theta_W \cos^2 \theta_W}$$

Note

For leptons of 1st generation, we have neutral weak currents interacting with the boson Z^0 :

$$j_{T^0}^\mu = (T^3 - \sin^2 \theta_W Q) \bar{e}_L \gamma^\mu e_L - \sin^2 \theta_W Q \bar{e}_R \gamma^\mu e_R$$

$$j_{T^0}^\mu = (T^3 + \sin^2 \theta_W) \bar{e}_L \gamma^\mu e_L + \sin^2 \theta_W \bar{e}_R \gamma^\mu e_R$$

$$j_{T^0}^\mu = T^3 \bar{\nu}_L \gamma^\mu \nu_L$$

For quarks of 1st generation, we have neutral weak currents interacting with the boson Z^0 :

$$j_{T^0}^\mu = (T^3 - \sin^2 \theta_W \frac{2}{3}) \bar{u}_L \gamma^\mu u_L + (T^3 + \sin^2 \theta_W \frac{1}{3}) \bar{d}_L \gamma^\mu d_L$$

$$- \sin^2 \theta_W \frac{2}{3} \bar{u}_R \gamma^\mu u_R - \sin^2 \theta_W \frac{1}{3} \bar{d}_R \gamma^\mu d_R$$

VIII.8.8 Covariant derivative rewritten according to potentials $W^{+\mu}$, $W^{-\mu}$, A^μ and $Z^{0\mu}$

We have the covariant derivative:

$$D^\mu = \partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu + ig_T T^a \cdot W^{a\mu}$$

with $a = 1, 2, 3$

It is rewritten according to potentials $W^{+\mu}$, $W^{-\mu}$, A^μ and $Z^{0\mu}$:

$$D^\mu = \partial^\mu + i\sqrt{2}g_T(T^+ \cdot W^{+\mu} + T^- \cdot W^{-\mu}) + ieQ \cdot A^\mu + ig_{T^0}(T^3 - \sin^2 \theta_W Q) \cdot Z^{0\mu}$$

VIII.9 Brout Englert Higgs mechanism (BEH), assign mass to bosons and fermions

We now remind in broad outline the BEH mechanism which makes it possible to assign a mass to the three massive bosons W^+ , W^- , and Z^0 , and to the fermions.

VIII.9.1 Superconducting materials

The BEH mechanism is inspired by Vitaly Ginzburg's - Lev Davidovich Landau theory of superconducting materials, where the magnetic field \vec{B} is expelled from inside the material. In this case, the magnetic field \vec{B} no longer has an infinite range, since it is zero inside the superconducting material.

During the study of the Yukawa mesotron, this theory has already been mentioned. It is assumed that the mediating particle associated with the electromagnetic field, in this case a photon γ , acquires mass in the superconducting material. We have for the magnetic field:

$$\vec{B} = \vec{B}_0 \exp\left(-\frac{m_\gamma c}{\hbar} x\right) = \vec{B}_0 \exp\left(-\frac{2\pi}{\lambda} x\right)$$

with m_γ the mass of the mediating particle γ associated with the short-range magnetic field in the superconducting material.

We have the relationship between m_γ and wavelength λ which represents the range of the magnetic field inside the superconducting material:

$$\frac{m_\gamma c}{\hbar} = \frac{2\pi}{\lambda}$$

When the range λ of the magnetic field \vec{B} decreases to 0, the mass of γ tends towards infinity.

It is J. Goldstone, on a suggestion of Y. Nambu, who had the idea of transposing the Ginzburg-Landau theory in superconductivity, to the Gauge theories.

VIII.9.2 Elements on the BEH mechanism

To transpose the model of a superconducting material to all of Space-Time, we postulate the existence of a scalar field present everywhere, even in a vacuum.

The idea is that the fundamental reference frame (the one that can be called the quantum vacuum) has a scalar field with a wave function not zero, but equal to a doublet:

$$\phi_{vacuum} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \frac{\mu}{\lambda} \end{pmatrix}$$

with μ and λ independent constants (nothing to do with the wavelength for λ).

Note: thus, in the Electroweak Model, the fundamental reference frame R^0 or quantum vacuum, has dysmetry, quantitatively towards the distances of 10^{-1} cm, this in order to assign a mass to the gauge boson W^\pm, Z^0 .

Some will only see it as a "pirouette" because the fundamental question is obviously, what exactly is R^0 ?

We remind the Lagrangian of a scalar meson of wave function ϕ used in the Yukawa interaction:

$$L_{meson}(\phi) = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi)$$

Let φ be the doublet of wave functions, representing the scalar field-type mediating particle, a mediating particle called the Higgs boson. This doublet φ has the limit $\varphi_{vacuum} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \frac{\mu}{\lambda} \end{pmatrix}$.

Note: we find the fundamental idea of H. Yukawa which associates a mediating particle, a wave function, and a potential (or a gauge field).

On the model of the scalar meson of the Yukawa interaction, the Lagrangian of the Higgs scalar boson is written:

$$L_{Higgs}(\varphi) = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - V(\varphi)$$

with $V(\varphi) = \frac{1}{2} \mu^2 \varphi^2 + \lambda \varphi^4$ the self-interaction potential of the Higgs boson,

$M_H = \sqrt{2} \mu$ the mass of the Higgs boson.

Note: note that the Higgs boson is thus defined as a scalar meson particle mediating the strong Yukawa interaction. Since the strong interaction of Yukawa has a limited range, the Higgs boson has a mass M_H .

VIII.9.3 Assign mass to gauge bosons

The scalar field associated with the Higgs boson is coupled to the Gauge interaction. We obtain for the Lagrangian of the Higgs boson in the reference frame where we cancelled the gauge fields B^μ and $W^{a\mu}$:

$$[L_{Higgs}(\varphi)]_{R^{B+W}} = \frac{1}{2} D^\mu \varphi D_\mu \varphi - V(\varphi)$$

with $D^\mu \varphi = \partial^\mu \varphi + i g_Y \frac{Y_W}{2} \cdot B^\mu \varphi + i g_T T^a \cdot W^{a\mu} \varphi$ the covariant derivative of the Electroweak Model which applies to particles with a non-zero weak charge.

We assume a weak hypercharge $Y_W = -1$ for the Higgs boson.

We obtain for the covariant derivative of the doublet of wave functions of the Higgs boson:

$$D^\mu \varphi = \partial^\mu \varphi - i \frac{g_Y}{2} \cdot B^\mu \varphi + i g_T T^a \cdot W^{a\mu} \varphi$$

When we tend towards vacuum, we have:

$$D^\mu \varphi_{vacuum} = D^\mu \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mu \\ \lambda \end{pmatrix} \right)$$

We take for weak charges $T^a = \frac{\sigma^a}{2}$ ($a = 1,2,3$), with σ^a the 3 Pauli matrices.

We obtain for the covariant derivative:

$$D^\mu \varphi_{vacuum} = \frac{i}{2\sqrt{2}} \frac{\mu}{\lambda} (g_Y B^\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + g_T W^{1\mu} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + g_T W^{2\mu} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + g_T W^{3\mu} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}) \begin{pmatrix} 0 \\ \mu \\ \lambda \end{pmatrix}$$

$$D^\mu \varphi_{vacuum} = \frac{i}{2\sqrt{2}} \frac{\mu}{\lambda} \begin{pmatrix} g_T W^{1\mu} - i g_T W^{2\mu} \\ g_Y B^\mu - g_T W^{3\mu} \end{pmatrix}$$

We obtain for the Lagrangian of the Higgs boson:

$$[L_{Higgs}(\varphi_{vacuum})]_{R^{B+W}} = \frac{1}{8} \left(\frac{\mu}{\lambda} \right)^2 ((g_Y B^\mu - g_T W^{3\mu})^2 + (g_T W^{1\mu} - i g_T W^{2\mu})^2)$$

$$[L_{Higgs}(\varphi_{vacuum})]_{R^{B+W}} = \frac{1}{8} \left(\frac{\mu}{\lambda} \right)^2 ((g_Y B^\mu - g_T W^{3\mu})^2 + (g_T)^2 W^{1\mu} W_\mu^1 + (g_T)^2 W^{2\mu} W_\mu^2)$$

We take as before:

$$W^{+\mu} = \frac{1}{\sqrt{2}} (W^{1\mu} + i W^{2\mu})$$

$$W^{-\mu} = \frac{1}{\sqrt{2}} (W^{1\mu} - i W^{2\mu})$$

$$A^\mu = \frac{g_Y B^\mu + g_T W^{3\mu}}{\sqrt{g_T^2 + g_Y^2}}$$

$$Z^{0\mu} = \frac{g_Y B^\mu - g_T W^{3\mu}}{\sqrt{g_T^2 + g_Y^2}}$$

After some calculations, we deduce a Lagrangian of the masses of the type:

$$[L_{Higgs}(\varphi_{vacuum})]_{R^{B+W}} = \left[\left(\frac{g_T \mu}{2 \lambda} \right)^2 W^{+\mu} W_\mu^+ + \left(\frac{g_T \mu}{2 \lambda} \right)^2 W^{-\mu} W_\mu^- + \left(\frac{1}{2} \sqrt{g_T^2 + g_Y^2} \frac{\mu}{\lambda} \right)^2 Z^{0\mu} Z_\mu^0 \right]$$

We identify the terms in the Lagrangian of the masses, with the masses of the bosons W^\pm and Z^0 . The results are shown below.

Boson mass W^\pm :

$$M_{W^\pm} = \frac{g_T \mu}{2 \lambda}$$

Boson mass Z^0 :

$$M_{Z^0} = \frac{1}{2} \sqrt{g_T^2 + g_Y^2} \frac{\mu}{\lambda}$$

Mass ratio:

$$\frac{M_{W^\pm}}{M_{Z^0}} = \frac{g_T}{\sqrt{g_T^2 + g_Y^2}} = \frac{e}{g_Y} = \cos \theta_W$$

Note that photons with potential A^μ are eliminated from $[L_{Higgs}(\varphi_{vacuum})]_{R+B+W}$ and therefore have no mass.

Nothing is said about a possible mass of the boson B and we can assume, like the photon, that it does not have one.

VIII.9.4 Calculation of $\frac{\mu}{\lambda}$ with Fermi constant

At the limit of the electroweak model, we have seen that we can pose a link with the Fermi constant:

$$\frac{g_T^2}{8M_{W^\pm}^2} = \frac{G_F}{\sqrt{2}}$$

Note: Bosons W^\pm have a non-zero mass and a priori a zero momentum energy quadrivector. Indeed, the momentum energy quadrivector is tended towards 0, in order to find the Fermi constant G_F .

Digital application

We calculate:

$$\frac{\mu}{\lambda} = (\sqrt{2}G_F)^{-\frac{1}{2}} \approx (\sqrt{2} \times 1,166 \times 10^{-5})^{-\frac{1}{2}}$$

$$\frac{\mu}{\lambda} \approx 246 GeV$$

$$\frac{M_{W^\pm}}{g_T} \approx 123 GeV$$

$$g_T \approx 0,65$$

$$M_{W^\pm} \approx 79,6 GeV$$

$$M_{Z^0} = \frac{M_{W^\pm}}{\cos \theta_W} \approx \frac{79,6}{0,87} \approx 90,8 GeV$$

Note that the mechanism BEH makes no predictions here about the values of μ and λ taken separately and accordingly on the mass $M_H = \sqrt{2}\mu$ of the Higgs boson.

Nota: The Standard Model theoretically provides for (in 2022) a mass of 91,187 GeV for the boson Z^0 and a mass of 80,357 GeV for bosons W^\pm . For the latter, the mass measured and published in work in April 2022 is of the order of 80,433 GeV.

VIII.9.5 Assign mass to leptons

The BEH mechanism also makes it possible to assign mass to quarks and leptons. We explain only here the case of electrons, the simplest case.

We remind the Yukawa interaction between a scalar meson (called scalar field of wave function φ) and a fermion (called Dirac field of wave functions) ψ):

$$L_{Yukawa}(\varphi, \psi) = -g_l \bar{\psi} \varphi \psi$$

with g_l an arbitrary coupling constant.

The BEH mechanism uses a Yukawa interaction that connects the left and right components of the doublets of $SU(2)_T$:

$$L_{Yukawa}(\varphi, \psi) = -g_l (\bar{\psi}_L \varphi \psi_R + \bar{\psi}_R \varphi^\dagger \psi_L)$$

In the case of a coupling between a Higgs boson (called wave function scalar field ϕ_{vacuum}) and lepton doublets (called Dirac field), we have:

$$L_{H,Lepton} = -g_l \left(\overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} \phi_{vacuum} e_R + \bar{e}_R \phi_{vacuum} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right)$$

with $\phi_{vacuum}^\dagger = \phi_{vacuum} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mu \\ \lambda \end{pmatrix}$

$$L_{H,Lepton} = -\frac{g_l \mu}{\sqrt{2} \lambda} \left(\overline{\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L} e_R + \bar{e}_R \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right)$$

The electron acquires a mass of:

$$m_e = \frac{g_l \mu}{\sqrt{2} \lambda}$$

VIII.10 The Higgs boson H^0 , a composite particle?

VIII.10.1 Bosons, composite particles?

Since bosons have an integer spin and fermions a half-integer spin, some physicists hypothesize that bosons are actually particles composed of 2 fermions, whose half-integer spins add up to give an integer spin.

This would make it possible to understand that bosons do not follow the Pauli exclusion principle, since in this case, they form a system and within this system, both fermions follow the Pauli exclusion principle.

Some bosons are actually presented today as composite particles, such as mesons (formed by a quark and an antiquark) or Cooper pairs (formed by 2 electrons). Other bosons, such as photons, are generally not presented as composite particles. We are now looking at what could happen to the Higgs boson.

VIII.10.2 Reminder of the main characteristics of the Higgs boson

The Higgs boson H^0 has the following characteristics, it is:

- of integer spin (a boson), with zero spin: $(S, S^3) = (0, 0)$,
- of zero electric charge,

- meson-type, particle mediating the strong interaction according to the model of H. Yukawa. It has mass, thus explaining the limited range of the strong interaction in Space. As a zero electric charge meson, we can expect a strong charge or strong isospin equal to $(I, I^3) = (1, 0)$. It should be noted, however, that the Standard Model postulates for H^0 a strong charge or of zero color, and that it is not presented as a composite particle like other mesons,
- of type equal to its own antiparticle,
- of scalar meson type (a so-called scalar meson has $S = 0$ and $J^P = 0^+$, that is $P = +1$),
- non-zero weak charge $(T, T^3) = (\frac{1}{2}, -\frac{1}{2})$, this is why it is subject to weak interaction and couples with bosons W^\pm and Z^0 ,
- of non-zero weak hypercharge ($Y_W = 1$ and $Y_W = -1$ according to the initial assumptions), this is why it couples with the boson B.

On the first 4 characteristics, we can compare H^0 to the pion π^0 . It differs on:

- Parity (π^0 is a pseudoscalar meson, with $S = 0$ and $J^P = 0^-$, that is $P = -1$),
- non-zero weak charge,
- non-zero weak hypercharge.

They both have relatively similar masses: about $134,98 \text{ MeV} \cdot c^{-2}$ for π^0 and approximately $125,18 \text{ MeV} \cdot c^{-2}$ for H^0 .

VIII.10.3 Speculation about Higgs bosons

If we deviate from the Standard Model, who sees H^0 like an elementary particle, we can propose an analogy of the type between H^0 and π^0 for strong isospin, all 2 then composite particles:

$$\begin{aligned}
 |I = 1, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle + \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\
 &= \frac{1}{\sqrt{2}} (|u; \bar{u}\rangle + |d; -\bar{d}\rangle) = |\pi^0\rangle
 \end{aligned}$$

$$\begin{aligned}
 |I = 1, I^3 = 0\rangle &= \frac{1}{\sqrt{2}} \left(\left| i = \frac{1}{2}, i^3 = \frac{1}{2}; i' = \frac{1}{2}, i^{3'} = -\frac{1}{2} \right\rangle + \left| i = \frac{1}{2}, i^3 = -\frac{1}{2}; i' = \frac{1}{2}, i^{3'} = \frac{1}{2} \right\rangle \right) \\
 &= |H^0\rangle
 \end{aligned}$$

It is noted that the bound particles would however be different for H^0 and π^0 .

We can also suggest the existence of 3 Higgs bosons H^+ , H^- , H^0 in analogy with pions π^+ , π^- , π^0 .

We would have the following tables:

Pions	(S, S^3)	(I, I^3)	(T, T^3)	Q	Y_W
π^+	(0,0)	(1,1)	(0,0)	+1	2
π^-	(0,0)	(1,-1)	(0,0)	-1	-2
π^0	(0,0)	(1,0)	(0,0)	0	0

Higgs	(S, S^3)	(I, I^3)	(T, T^3)	Q	Y_W
H^+	(0,0)	(1,1)	$(\frac{1}{2}, -\frac{1}{2})$	+1	+3
H^-	(0,0)	(1,-1)	$(\frac{1}{2}, -\frac{1}{2})$	-1	-1

H^0	(0,0)	(1,0)	$(\frac{1}{2}, -\frac{1}{2})$	0	1
-------	-------	-------	-------------------------------	---	---

VIII.11 Analogies between the 3 interactions of the Electroweak Model

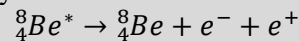
VIII.11.1 Questions about the boson B

The Electroweak Model provides little information about the boson B . A priori, this boson B must not have mass since it is not calculated via the BEH mechanism. Just like the photon γ , the boson B presumably has a non-zero momentum. In the next Memoir, it will also be proposed that it has an electric charge.

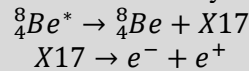
The boson B is the mediating particle of an interaction called hyper. Nothing is said about the scope (or range) of hyper interaction. But if the mass of the boson B is effectively zero, it can be assumed that the range of the hyper interaction is infinite like that of the electromagnetic interaction or the gravitational interaction.

Note, a relationship between the boson B and the boson $X17$?

In the 2010s, as part of research on dark matter (postulated because of a non-respect of Newtonian gravitation in galaxies), Hungarian physicist Attila Krasznahorkay and his colleagues at ATOMKI (*Atomki Nuclear Research Institute*) bombarded lithium $7\ ^3_3\text{Li}$ with protons. This creates unstable (or excited) beryllium $8\ ^4_4\text{Be}^*$ which decays into:



In an article signed in 2015, to explain during this decay, an angle of 140° between the electron and the positron, as well as an energy release of 17 MeV, the Hungarian team postulates the existence of a new particle $X17$ of mass 17 MeV, with intermediate decays:



In 2016, physicist Jonathan L. Feng suggested that this particle or boson $X17$ could be the mediating particle of a fifth fundamental interaction of very long range.

Finally, in 2019, the Hungarian team obtained similar results with no longer beryllium, but helium $4\ ^2_2\text{He}^*$ and a particle of mass almost identical to that of the $X17$.

This reminder of the events raises questions about a possible link between boson $X17$ and boson B of the Electroweak Model, which remains a "technical object" and undiscovered.

Note that the boson $X17$ has characteristics a priori different from those of the boson B , in particular a zero electric charge for the $X17$ and a priori non-zero electric charge for the B .

Nevertheless, the hypothesis that the boson B has zero mass and the observation that the boson $X17$ has a low mass, bring them both together in the idea that they would convey a new interaction of infinite range, or at least very long range. This interaction would have consequences on gravitation, especially at the scale of galaxies.

To be continued...

VIII.11.2 Analogies between electric charge Q , weak charge T^3 and weak hypercharge Y_W

The table below shows the analogies between electric charge Q , weak charge T^3 and weak hypercharge Y_W found in the Electroweak Model ($\mu = t, x, y, z$).

Quantum number conserved during a global gauge transform and generator of the	Charge Q	Weak charge T^3	Weak hypercharge Y_W
---	------------	-------------------	------------------------

transformation			
Related interaction	Electromagnetic interaction Infinite range	Weak interaction Low range	Hyper interaction Infinite range?
Charge function of quantum numbers	$Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$		$\frac{Y_W}{2} = Q - T^3$
Coupling constant, elementary charge	$\alpha_{em} = \alpha_Q$ $= \frac{e^2}{4\pi\epsilon_0\hbar c} e = g_Q$	$\alpha_T = \frac{g_T^2}{4\pi\epsilon_0\hbar c}$	$\alpha_Y = \frac{g_Y^2}{4\pi\epsilon_0\hbar c}$
Particle mediating the interaction	Photon γ transporting a spin S (quantum quantity with momentum)	Bosons W_1, W_2, W_3 transporting a quantum quantity with mass	Boson B transporting a quantum quantity with momentum?
Group of rotations, global and local gauge transformations	$U(1)_Q$	$SU(2)_T$	$U(1)_{Y_W}$
Local gauge transformation on the wave function	$\psi \rightarrow \exp(ieQ\alpha(x))\psi$	$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \exp(ig_T T^a \cdot \alpha_a(x)) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} a = 1,2,3$	$\psi \rightarrow \exp(ig_Y Y_W \alpha(x))\psi$
Covariant derivative of local gauge transformation	$D^\mu = \partial^\mu + ieQ \cdot A^\mu$	$D^\mu = \partial^\mu + ig_T T^a \cdot W^{a\mu} a = 1,2,3$	$D^\mu = \partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu$
Lagrangian fermions	$i\bar{\psi}\gamma^\mu(\partial^\mu + ieQ \cdot A^\mu)\psi - m\bar{\psi}\psi$	$i\bar{\psi}_L\gamma_\mu(\partial^\mu + ig_T T^a \cdot W^{a\mu})\psi_L - m\bar{\psi}_L\psi_L a = 1,2,3$	$i\bar{\psi}_L\gamma_\mu(\partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu)\psi_L - m\bar{\psi}_L\psi_L$ $i\bar{\psi}_R\gamma_\mu(\partial^\mu + ig_Y \frac{Y_W}{2} \cdot B^\mu)\psi_R - m\bar{\psi}_R\psi_R$
Potential quadrivectors related to mediating particles	A^μ	$W^{1\mu}, W^{2\mu}, W^{3\mu}$	B^μ
Currents quadrivectors	$j_Q^\mu = Q\bar{\psi}\gamma^\mu\psi$	$j_{T^a}^\mu = T^a\bar{\psi}_L\gamma^\mu\psi_L a = 1,2,3$	$j_{Y_W}^\mu = Y_W\bar{\psi}_L\gamma^\mu\psi_L + Y_W\bar{\psi}_R\gamma^\mu\psi_R$

VIII.12 Conclusion, on the mediating particles used by the Electroweak Model

The Electroweak Model involves 3 types of mediating particles (if we do not count the Higgs boson used to assign a mass and which would be included in the particles mediating the strong interaction).

First, the photon γ having non-zero momentum and zero mass. This photon γ is the mediating particle of the electromagnetic interaction of infinite range (hence the zero mass of the photon). It carries a spin S , which exchanged with a fermion, generates a change in the energy level of the fermion and a displacement of the fermion electric charge Q .

Next, bosons W^1 , W^2 , W^3 having zero momentum and non-zero mass. These bosons W^1 , W^2 , W^3 are the mediating particles of the weak interaction of limited range (hence the non-zero mass of these bosons according to the ideas of H. Yukawa).

Finally, the boson B possessing, a priori, a non-zero momentum and a zero mass. It is the mediating particle of an interaction called here hyper.

In the 3rd Memoir, several source field equations applied to gravitation were proposed. In particular, 4 equations were proposed with space-oriented sources: the first 2 with sources constructed from linear momentum density, the next 2 with sources constructed from linear mass density.

After this 4th Memoir, which remains essentially a course of particle physics in the 20th century, the 5th Memoir will again be very speculative. Its purpose will be to examine whether it is possible to reconcile 3 of these 4 source field equations with the 3 types of mediating particles involved in the Electroweak Model:

- the photon γ with a momentum,
- the bosons B with a momentum,
- the bosons W^1 , W^2 , W^3 with a mass.

For the last source field equation involving a mass, it will be examined whether it is possible to bring it closer to mass mediating particles of the strong interaction, also limited in scope.

Bibliography of Memoir 4

On the scattering of Ernest Rutherford

- [1] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007.
- [2] Wikipedia, Rutherford's experience

On Heisenberg isospin

- [3] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL. Téléchargeable sur Internet sur le site <http://www.sciences.ch/htmlfr/accueil.php>.
- [4] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007.

On Fermi's theory of weak interaction

- [5] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL.
- [6] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007.

On Yukawa's potential

- [7] Vincent Isoz, site internet sur la physique et les mathématiques, <http://www.sciences.ch/htmlfr/accueil.php>.
- [8] Wikipedia, Yukawa potential

On the scalar field

- [9] Mikhaïl Shaposhnikov et Sven Bachmann, Champs Quantiques Relativistes. Ecole Polytechnique Fédérale de Lausanne, 2005.

On groups $SU(n)$ and Gauge transformations

- [10] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL.
- [11] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007.
- [12] Wikipedia, Lie groups

On the transformations of the Yang-Mills Local Gauge

- [13] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [14] B. Gosselin-Cliche, Théorie de Jauge de Yang-Mills

On Lee Yang Wu's parity violation and the negative helicity of neutrino of Golhaber

- [15] Johann Collot, masses et oscillations de neutrinos, UJF Grenoble, années 2004-2005
- [16] Johann Collot, Symétries discrètes PCT règles de sélection, UJF Grenoble, années 2007-2008
- [17] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007

On scale operators

- [18] Wikipedia, Angular momentum (quantum mechanics)

On the Glashow-Weinberg-Salam electroweak model

- [19] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL.
- [20] Johann Collot, Edification de la théorie des interactions électrofaibles, UJF Grenoble, années 2007-2008

[21] Sheldon Glashow, *The charm of Physics*, The American Institute of Physics, 1991

[22] L. Marleau, *Particules élémentaires*. Université Laval, Québec, 2007

[23] P. Vilain, *Les Courants neutres et les Particules charmées en Physique du Neutrino*. Université Libre de Bruxelles, Avril 2005.

[24] Wikipedia, Yukawa interaction

Table of Contents

Memoir 4	Elements on nuclear interactions.....	247
Chapter I	Ernest Rutherford's scattering (1911) and particle collisions	248
I.1	The discovery of radioactivity by Henri Becquerel (1886).....	248
I.2	Rutherford's experiments (1909-1911).....	248
I.3	Rutherford scattering in classical mechanics	249
I.4	Rutherford scattering in the context of relativistic quantum electrodynamics	250
I.5	Conclusion of the chapter.....	252
Chapter II	The spin of Uhlenbeck and Goudsmit (1925) and the isospin of Heisenberg (1932) 253	
II.1	Le spin S by G. Uhlenbeck and S. Goudsmit (1925)	253
II.2	Heisenberg isospin (1932) defined as a strong nuclear charge of the neutron and proton 255	
II.3	Analogies between spin S reversal and isospin I reversal.....	257
II.4	Mediating particle and the notion of interaction	259
II.5	Conclusion.....	260
Chapter III	Fermi's theory about weak interaction (1933).....	261
III.1	Analogy between particle decays	261
III.2	Fermi theory	263
III.3	Foretaste of Glashow-Weinberg-Salam electroweak model and of quark model	264
III.4	Fermi process and Gamow Teller process.....	267
III.5	Conclusion of the chapter.....	268
Chapter IV	The Mesotron of Yukawa (1935).....	270
IV.1	Hideki Yukawa's theory to describe nuclear interactions, the mesotron or meson	270
IV.2	Analogy Schrödinger wave equation and Yukawa wave equation, case of an electron and a mesotron.....	271
IV.3	The potential of Yukawa	273
IV.4	On pions as mediating particles of the strong interaction, foretaste on Yang-Mills theory, the Quark Model, and the Yukawa interaction	276
IV.5	Conclusion of the chapter.....	281
Chapter V	Mathematical reminders about rotation groups $SU(n)$, physical applications to gauge transformations.....	282
V.1	General information on gauge theories and quantum numbers.....	282
V.2	Mathematical reminders about rotation groups $SU(n)$	284
V.3	Mathematical reminders about generators of group $SU(n)$	285
V.4	Physical application, charge defined as a generator of group $SU(n)$, multiplets of particles and wave functions.....	288
V.5	Physical application, Global Gauge transformations and conservation of the charges... 290	
V.6	Physical application, currents charge quadrivector conserved during a global gauge transform, Noether's theorem.....	293
V.7	Physical application, transformations of spin S and isospin I based on $SU(2)$	294
V.8	Conclusion of the chapter.....	296
Chapter VI	Yang-Mills local gauge theory on strong interaction (1954).....	297
VI.1	Preamble.....	297
VI.2	Local gauge transformation, analogy electromagnetic interaction and strong nuclear interaction.....	298
VI.3	Reminders on the Lagrangians used in electromagnetism and relativistic quantum electrodynamics	301

VI.4	Lagrangians involved in the strong interaction	303
VI.5	Chapter conclusion, summary on Global and Local Gauge Transformations.....	304
Chapter VII Lee Yang Wu's parity violation (1957) and the negative helicity of Golhaber's neutrino (1958)		
306		
VII.1	Reminders on helicity.....	306
VII.2	Historical reminders on parity violation and negative helicity of the neutrino	307
VII.3	Quantum numbers of particles.....	309
VII.4	Chapter conclusion.....	312
Chapter VIII Glashow-Weinberg-Salam electroweak model (1960s)		
313		
VIII.1	History of the Electroweak Model	313
VIII.2	Quantum numbers of the Electroweak Model, analogies between strong and weak interactions.....	313
VIII.3	Coupling constants	316
VIII.4	Local Gauge transformations and covariant derivatives	318
VIII.5	Study of Lagrangians involved in the Electroweak Model	319
VIII.6	Weak bosons W^+ , W^- and Z^0	321
VIII.7	Weak currents carrying a weak charge T	324
VIII.8	Study of interaction energies	325
VIII.9	Brout Englert Higgs mechanism (BEH), assign mass to bosons and fermions.....	330
VIII.10	The Higgs boson H^0 , a composite particle?	334
VIII.11	Analogies between the 3 interactions of the Electroweak Model.....	336
VIII.12	Conclusion, on the mediating particles used by the Electroweak Model	337
Bibliography of Memoir 4.....		339

Memoir 5 The effect of Einstein and the parable of Bohr

Objectives of the Memoir

Although it bears the names of Einstein and Bohr, this 5th Memoir will focus mainly on the work of the years 1950-1970 and will again be speculative. We will focus on the following 4 interactions present in the Standard Model: electromagnetic, hyper, weak and strong interactions (with two interactions of infinite range and two of finite range).

We will set aside the Higgs field, which in our opinion, would be closer to the strong interaction, since the Higgs boson intervenes as a meson mediator of the strong interaction, to assign a mass to the particles of the Electroweak Model. As for gravitational interaction, we will return to its case at the end of this Memoir.

Throughout this 5th Memoir, we will essentially try to show that there is a greater analogy between the first interaction: electromagnetic and the other 3: hyper, weak and strong, than the Standard Model usually presents.

Thus, its main objective will be to generalize the Einstein photoelectric effect and the Bohr parable to the 4 interactions mentioned above.

For this, we will first look for, for each of the 4 interactions, a quantum number analogous to that of the spin number S^a ($a = 1,2,3$) involved in electromagnetic interaction. We will qualify these 4 quantum numbers as a "source field", because for each, we suggest pairings with gravitational source field equations proposed in the 3rd Memoir. We will see that 2 quantum numbers are of impulse type and that the other 2 are of mass type.

We will then relate each quantum number "source field" to:

- reactions between fermion-type particles that reverse or modify these source field quantum numbers,
- Mediating particles carrying these 4 quantum numbers "source field": photon γ and boson B , both momentum boson type, bosons W^a and gluons G^a , both mass boson type.

Finally, from each quantum number source field, we will define a quantum number of charges X , complementary to the quantum number "source field" and function of the other 3 quantum numbers "source field".

Beyond the very speculative ideas put forward here, this Memoir will also be an opportunity to focus on the history of ideas in physics. We will start with a parallel between the law of areas of J. Kepler (1609), the notions of angular momentum, then quantum spin proposed by S. Goudsmit and G. Uhlenbeck (1925). We will discuss the proposals of new quantum numbers that are the strong hypercharge Y or strangeness St (K. Nishijima and M. Gell-Mann in the 1950s). We will also be interested in the Quark Model (M. Gell-Mann and G. Zweig from 1961 to 1964), a model based on the strong interaction between particles in the nucleus (hadrons).

We will then talk about the angle of N. Cabibbo (1963). This angle makes it possible to describe from the up and down quarks the electrically neutral and charged weak currents of fermions interacting with the weak bosons. We will also deal with the mechanism of S. L. Glashow-J. Illiopoulos-L. Maiani, mechanism that imagines the existence of a 4th quark, the quark c and which involves an electrically neutral weak current.

Through the work of Bruno Pontecorvo and Ziro Maki (from the 1960s), we will be interested in the oscillations of neutrinos: hypothesis of several families of neutrinos, existence of an angle (today called Pontecorvo angle similar for leptons to that of Cabibbo for quarks). The similarities between quarks and leptons in the weak and hyper interactions will be highlighted.

Finally, we will discuss Quantum Chromodynamics, a theory describing the strong interaction and based on the strong charge of color generating the group $SU(3)_{\text{colors}}$. We will also briefly discuss the notions of confinement and asymptotic freedom proposed in 1973 by H. David Politzer, Frank Wilczek and David Gross.

Chapter I Relating the de Broglie source field equation and the spin quantum number S

Purpose of the chapter

We try to show the link between the source equation Broglie field $\vec{p}^x = \hbar \vec{K}_{ty}^{1/x}$ and spin S .

More generally, we wish to propose that:

- the invariance of the source field equation of de Broglie corresponds to:
- conservation of orbital and spin angular momentum during reactions between particles.

I.1 Reminders: from Kepler's 2nd law to orbital angular momentum and orbital magnetic moment, then to spin

Here we make some reminders on the existing relations between Kepler's law of areas and the notion of angular momentum encountered in gravitation, magnetic moment encountered in electromagnetism and spin encountered in quantum physics.

I.1.1 Kepler's 2nd law

In 1609, Johannes Kepler proposed Kepler's 2nd law (also called Law of areas) which applies to the motion of the planets in the sky.

Let $A(t)$ be the area of the surface “swept” by a vector ray \vec{r} . According to Kepler's 2nd law, equal areas $A(t)$ are swept in equal times. For example, for a planet orbiting the Sun, this qualitatively means that its speed is greater the shorter its distance from the Sun.

We define an areal velocity, derived from the area $A(t)$ with respect to time:

$$\frac{dA(t)}{dt}$$

Using Kepler's 2nd law, we obtain a constant areal velocity:

$$\frac{dA(t)}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{const}$$

I.1.2 Relationship between areal velocity $\frac{dA(t)}{dt}$ and orbital angular momentum \vec{L}

We define an orbital angular momentum following z :

$$\vec{L}^z = \vec{y} \wedge \vec{p}^x$$

Note 1

x and y define a spatial plan here, y is the radial component and x the tangential component of a rotational motion.

Note 2

Usually, the term orbital angular momentum is rather reserved for quantum mechanics. In classical mechanics, we simply speak of angular momentum. It is used here to show the proximity between the classical notion and the quantum notion.

We obtain the relationship between the areal velocity and the orbital angular momentum by multiplying the areal velocity by twice the mass of the body studied:

$$\vec{L}^z = 2m \frac{dA(t)}{dt} \vec{z}$$

In the case of a constant mass m , Kepler's 2nd law can be rewritten with angular momentum:

$$\vec{L}^z = 2m \frac{dA(t)}{dt} \vec{z} = \text{const}$$

and thus translates the conservation over time of a constant quantity, the angular momentum.

Note, reminder on the conservation of angular momentum in the case of a central force motion

A central force movement is the movement of a material point subjected to a force passing through a fixed-point O. This is for example the case of the Earth, subject to the central gravitational force of the Sun, in the models of Kepler and Newton.

In the case of a central force motion, we have conservation of angular momentum. We have:

$$\vec{L}^z = \vec{OM} \wedge \vec{p}$$

Deriving with respect to time, we have:

$$\begin{aligned} \frac{d\vec{L}^z}{dt} &= \left(\frac{d\vec{OM}}{dt} \wedge \vec{p} \right) + \left(\vec{OM} \wedge \frac{d\vec{p}}{dt} \right) \\ \frac{d\vec{L}^z}{dt} &= (\vec{v} \wedge \vec{p}) + (\vec{OM} \wedge \vec{F}) \end{aligned}$$

$\vec{v} \wedge \vec{p} = \vec{0}$, because \vec{v} and \vec{p} are collinear

$\vec{OM} \wedge \vec{F} = \vec{0}$, because \vec{OM} and \vec{F} are collinear, because the motion is central force

We get:

$$\frac{d\vec{L}^z}{dt} = \vec{0}$$

Angular momentum \vec{L}^z is therefore constant during the movement.

I.1.3 Relationship between areal velocity $\frac{dA(t)}{dt}$, orbital angular momentum \vec{L} and orbital magnetic moment $\vec{\mu}_L$

We obtain the relationship between the areal velocity $\frac{dA(t)}{dt}$ and orbital magnetic moment $\vec{\mu}_L$ by multiplying the areal velocity by the charge of the body studied (A sign $-$ is introduced):

$$\vec{\mu}_L = -q \frac{dA(t)}{dt} \vec{z}$$

We have the relationship between orbital magnetic moment and orbital angular momentum:

$$\vec{\mu}_L = -\frac{q}{2m} \vec{L}$$

$\vec{\mu}_L = \gamma \vec{L}$ where $\gamma = -\frac{q}{2m}$ the gyromagnetic ratio of the magnetic dipole considered.

Note, reminder about the magnetic dipole

A magnetic dipole can be visualized as a small magnet. It is characterized by a magnetic moment $\vec{\mu}_L$. In the presence of a magnetic field \vec{B} , the magnetic dipole is subjected to a torque $\vec{\tau}$ and a force \vec{F} , to which potential energy is associated Ep .

We have the relationships:

$$\begin{aligned}\vec{\tau} &= \vec{\mu}_L \wedge \vec{B} \\ Ep &= -\vec{\mu}_L \cdot \vec{B} \\ \vec{F} &= -\vec{\nabla} Ep = -\vec{\nabla}(\vec{\mu}_L \cdot \vec{B})\end{aligned}$$

The small magnet tends to orient itself according to the magnetic field \vec{B} to minimize its torque $\vec{\tau}$ and maximize its Ep (in absolute value).

I.1.4 Relationship between orbital angular momentum \vec{L} and orbital magnetic moment $\vec{\mu}_L$, case of the electron

The electron of the Bohr model has an orbital angular momentum $\vec{L} = \hbar\vec{z}$ and an orbital magnetic moment $\vec{\mu}_{L-Bohr}$ called the Bohr-Procopiu magneton (in reference to Niels Bohr and Ștefan Procopiu).

We have the relationship between the orbital angular momentum and the orbital magnetic moment:

$$\begin{aligned}\vec{\mu}_{L-Bohr} &= \frac{e}{2m_e} \vec{L} = \frac{e\hbar}{2m_e} \vec{z} \\ \gamma_e &= \frac{e}{2m_e}\end{aligned}$$

I.1.5 Passage of orbital angular momentum \vec{L} to spin angular momentum \vec{S}

In Memoir 2, it was mentioned that the transition from an orbital rotation to a rotation on oneself (spin rotation) is done via a change of reference frames (for example from a terrestrial reference frame to a solar reference frame).

We can imagine that the passage of an orbital angular momentum \vec{L} to a spin angular momentum \vec{S} is also done via a change of reference frame.

We expect a same relationship between the spin magnetic moment $\vec{\mu}_S$ and the spin angular momentum \vec{S} for an elementary particle. This is the case for the electron with a good approximation. We'll look at that in the next paragraph.

Note

In the case of a point particle, the radius involved in the spin angular momentum does not really make sense. However, instead of thinking radius r , we can think wavelength λ and wave vector K as does wave mechanics.

I.1.6 Relationship between the spin magnetic moment $\vec{\mu}_S$ and spin angular momentum \vec{S} , case of the electron, Landé factor g

In quantum mechanics, spin angular momentum \vec{S} is defined with an additional factor $\frac{1}{2}$ relative to orbital angular momentum $\vec{L} = \hbar\vec{z}$. We have:

$$\vec{S} = \pm \frac{1}{2} \hbar\vec{z}$$

To explain:

- this 1st fact of an additional factor $\frac{1}{2}$ (case of the electron),
- as well as a 2nd fact concerning other particles (such as the proton or the neutron), which measures a ratio between the magnetic moment of spin and the angular momentum of spin, not depending only on the mass and charge of the particle,

Alfred Landé introduced in 1921 a factor g :

$$\vec{\mu}_S = g \frac{q}{2m} \vec{S}$$

For the electron, we have a Landé factor as expected: $g \approx -2$.

We find:

$$\vec{\mu}_S \approx \frac{e\hbar}{2m_e} \vec{z}$$

similar to Bohr's magneton $\vec{\mu}_{L-Bohr} = \frac{e\hbar}{2m_e} \vec{z}$.

Note 1

The Dirac wave equation of electron predicts $g = -2$. The experimental value is in fact $g \approx -2,002319$. To explain it, we speak in relativistic quantum electrodynamics of quantum vacuum with the appearance of virtual particles.

An anomaly is introduced a with:

$$g = 2(1 + a)$$

We have a power series of the electromagnetic coupling constant:

$$a = A_1\alpha_Q + A_2\alpha_Q^2 + A_3\alpha_Q^3 + o(\alpha_Q^3)$$

The power series is symbolized by virtual particles. This makes it possible to calculate A_1, A_2, A_3 , etc.

Note 2

The proton has a Landé factor $g \approx +5,586$ and the neutron $g \approx -3,826$, contrary therefore to the expectation which is true for the electron.

To explain this, it can be assumed that proton and neutron (both subject to strong interaction) are not elementary particles but composite particles. This is one of the strong arguments in favor of the Quark Model.

I.2 Combining the source field equation of de Broglie with orbital angular momentum and spin angular momentum

I.2.1 Quantization of orbital angular momentum L

We give the quantization proposed by N. Bohr of orbital angular momentum in his model of the electron:

$$L^z = yp^x = n\hbar$$

with n an integer

In the following lines, we remind the ideas proposed by L. de Broglie in his model of the electron wave, in order to explain the quantization of the orbital angular momentum of N. Bohr.

According to the source field equation of de Broglie, we have:

$$p^x = \frac{h}{\lambda} = \hbar K^{1/x}$$

We place ourselves in the reference frame R^Ω where the electron is motionless and is interpreted as a circular stationary (or standing) wave. To have a constructive interference of this wave, L. de Broglie proposes the condition:

$$\lambda = \frac{l}{n} = \frac{2\pi y}{n}$$

or $K^{1/x} = \frac{n}{y}$

Note on the wave vector

Since we reason on a circular wave (and not rectilinear), we do not have $K^{1/x}$ function of $\frac{1}{x}$ but of $\frac{1}{y}$, i.e., function of the radius.

We get:

$$p^x = \hbar K^{1/x} = n\hbar \frac{1}{y}$$

$$p^x = \frac{h}{\lambda} = n\hbar \frac{1}{y}$$

We find the Bohr quantification:

$$L^z = yp^x = n \frac{p^x}{K^{1/x}} = n\hbar$$

I.2.2 Define orbital angular momentum from the source field equation of de Broglie

From the above, we note that the source field equation of de Broglie:

$$\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt} = \frac{\hbar \vec{K}_{ty}^{1/x}}{\partial t}$$

$$\vec{p}^x = \hbar \vec{K}_{ty}^{1/x}$$

is associated with the quantization of orbital angular momentum.

It is proposed to define the orbital angular momentum L from the source field equation of de Broglie. We obtain the 3 components of the orbital angular momentum in the case where $n = 1$:

$$L^z = \hbar^z = \frac{\vec{p}^x}{\vec{K}_{ty}^{1/x}}$$

$$L^x = \hbar^x = \frac{\vec{p}^y}{\vec{K}_{tz}^{1/y}}$$

$$L^y = \hbar^y = \frac{\vec{p}^z}{\vec{K}_{tx}^{1/z}}$$

Note 1

Following the classical definition of angular momentum $\vec{L}^z = \vec{y} \wedge \vec{p}^x = \vec{y} \wedge m\vec{v}^x$, it is a linear speed of rotation \vec{v}^x and thus a rotation vector in a spatial plane $\vec{\Omega}$, which intervenes in the expression of angular momentum.

With this new definition of angular momentum, it is a generalized rotation vector in a spatiotemporal plane: \vec{K} , which intervenes in the expression of angular momentum

Note 2

This new definition clarifies the quantization in quantum mechanics of orbital angular momentum, whereas in classical mechanics orbital angular momentum has no reason to be quantized.

I.2.3 Same notion for orbital angular momentum L and spin angular momentum S , with change of reference frames?

Previously, it was suggested that orbital angular momentum L and spin angular momentum S correspond to the same notion with change of reference frames.

This idea of the same notion grouping together L and S , is also found at the operator level. In quantum mechanics, we define a total angular momentum, denoted vector operator \hat{J} , sum of an orbital angular momentum \hat{L} and of a spin angular momentum \hat{S} :

$$\hat{J} = \hat{L} + \hat{S}$$

The idea that L and S corresponds to the same notion (with change of reference frames), is also found in the (not relativist) electron model of Pauli (an adaptation of the electron model of de Broglie Schrödinger, with in addition the spin) when a photon (with spin angular momentum S) reacts with an electron and changes its orbital angular momentum L (and also its orbit) during an energy transition.

I.2.4 Quantization of spin angular momentum

In the Bohr electron model, only orbital angular momentum L is quantized (not the spin angular momentum S which is not included in the model).

Following the idea that L and S correspond to the same notion (with change of reference frames), we must also have a quantification of the spin S . This is precisely what we observe with:

$$S^z = n \frac{\hbar}{2}$$

with $n = \pm 1$ positive or negative integer

Note

In electron model of de Broglie Schrödinger, there are actually three quantum numbers:

- the principal quantum number n , which (roughly) corresponds to that of the Bohr model,
- the secondary quantum number l which corresponds to the quantization of the modulus of the orbital angular momentum,
- the magnetic quantum number m which corresponds to the quantization of the projection along the reference axis Oz of the orbital angular momentum vector,

To take into account the 4th quantum number, the spin s in a wave equation, we must use the electron model of Pauli.

I.2.5 Define spin angular momentum S à partir de l'équation source champ de Broglie

According to the finding that L and S are both quantified and correspond to the same notion (with change of reference frames), we also define the spin S from the source field equation of de Broglie:

$$S^z = \frac{1}{2} \hbar^z = \frac{1}{2} \frac{\vec{p}^x}{\bar{K}_{ty}^{1/x}}$$

$$S^x = \frac{1}{2} \hbar^x = \frac{1}{2} \frac{\vec{p}^y}{\bar{K}_{tz}^{1/y}}$$

$$S^y = \frac{1}{2} \hbar^y = \frac{1}{2} \frac{\vec{p}^z}{\bar{K}_{tx}^{1/z}}$$

I.3 Conclusion of the chapter

It has been proposed to link the source field equation of de Broglie to orbital and spin angular momentum, and thus to the electromagnetic interaction. This link helps explain the quantization in quantum mechanics of orbital angular momentum in integer multiples of \hbar , whereas this moment is not quantized in classical mechanics.

More broadly, we thus corresponded the invariance of a law of Nature: the source field equation of de Broglie, with conservation during reactions between particles of a quantum quantity: orbital and spin angular momentum.

Following this correspondence between the invariance of a law of Nature and conservation during reactions between particles of a quantum quantity, we wonder, would it be possible to find other examples and even to generalize more?

This is what will be examined in the next chapters., by searching in addition to bring the 4 interactions (electromagnetic, hyper, weak, and strong) closer in their form than does the Standard Model.

Chapter II The photoelectric effect of Einstein (1905) and the parable of Bohr (1913)

Purpose of the chapter

We present the main objective of this Memoir: that of generalizing the photoelectric effect of Einstein and the parable of Bohr applying to the electromagnetic interaction, to the 3 other interactions involved in the Standard Model: electromagnetic, hyper, weak and strong interactions.

II.1 The photoelectric effect of Einstein and the parable of Bohr

II.1.1 Reminders

In 1905, A. Einstein provided a quantum explanation for the photoelectric effect discovered 65 years earlier by Antoine Becquerel and Edmond Becquerel. The idea of A. Einstein is that particles of light, photons, carrying a quantized number today called the spin, exchange this spin with electrons. This spin S exchange generates a displacement of electrons and therefore an electric current of conserved electric charge Q .

Here we call parable of Bohr, the Bohr model of the electron proposed by N. Bohr in 1913. It is an extension of Einstein's photoelectric effect to the hydrogen atom. Following the parable, light waves of photons carrying spins S , can generate changes in the orbits of the electron and thus an electric current with charge Q .

The parable works in 2 senses. Indeed, changes in the orbit of the electron (change of orbit comparable to a displacement of conserved electric charge Q and therefore to an electric current), can radiate light waves composed of photons carrying spins S .

Note on quantum numbers S and Q involved in electromagnetic interaction

In relativistic quantum electrodynamics, the electromagnetic field is associated with the photon γ that is the spin carrier. The relationship between angular momentum and electric force, i.e., between quantum numbers S and Q , is found, for example, in N. Bohr's electron model, when he proposes:

$$rmv \times v = \frac{e^2}{4\pi\epsilon_0}$$

$$\hbar \times v = \frac{e^2}{4\pi\epsilon_0}$$

with \hbar the quantized orbital angular momentum L that is matched with S in the same notion of total angular momentum J , e the elementary electric charge of Q .

Historical note on Einstein's idea

All the electromagnetism of the 19th century developed by knowing only the quantum number of electric charge Q . The genius of A. Einstein is to have understood that to properly describe the electromagnetic interaction, it was necessary to add a second quantum number: the spin S .

Today, it may seem obvious that it is necessary to use both Q and S to describe the electromagnetic interaction, but at the time of A. Einstein little or nothing suggested it, since Q and S are so different.

In the figure above, the currents of electric charges correspond to the dotted arrows, when the electron passes from one orbit to another (i.e., from one level n to another level n).

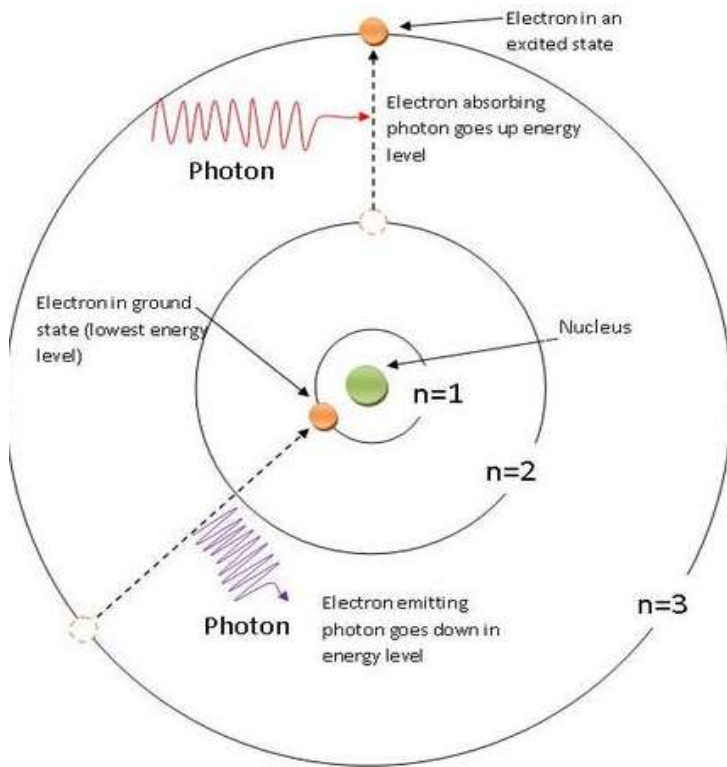


Figure 1: photon absorption and photon emission, current of electric charges

II.1.2 Photon absorption and photon emission, Feynman diagrams

Let's explain the 2 senses of Einstein's photoelectric effect and Bohr's parable, through Feynman's diagrams below.

A mediating particle generates a current of electric charges, photon absorption

We have the disappearance of a field and the mediating particle representing it, in the sense of Time.

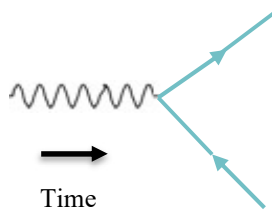


Figure 2: disappearance of the photon and creation of fermions

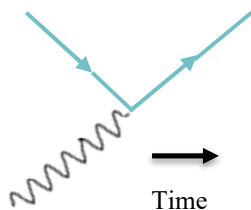


Figure 3: disappearance of the photon and scattering of fermions

A current of electric charges radiates a mediating particle, photon emission

We have the creation of the field or the mediating particle representing it, in the sense of Time.

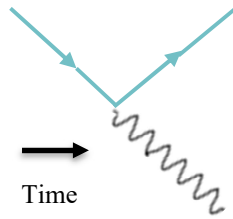


Figure 4: Photon creation and fermion scattering

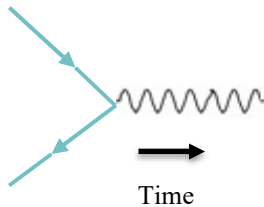


Figure 5bis: Photon creation and fermion annihilation

II.1.3 The main steps of Bohr's parable

Here we divide Bohr's parable into 6 steps:

1. A quantum number qualified here as a "source field" number: the orbital and spin angular momentum, which is partially summarized by spin S^a ($a = 1,2,3$),
2. light-mediating particles, photons γ that can appear and disappear, that carry an entire spin, and that are electrically neutral,
3. particles of matter (fermions) that persist (during a scattering reaction), which carry half-integer spins that can reverse, as well as a non-zero retained electric charge Q ,
4. a reaction between particles of light and matter, which consists of an exchange of spins between a photon γ and a fermion (for examples, an energy transition that changes the orbital angular momentum of the fermion, a hyperfine transition that reverses the spin angular momentum of the fermion, passage from e_R to e_L),
5. a change of orbit and therefore a displacement of the fermion which keeps its electric charge constant Q during this movement, i.e., an electromagnetic current j_Q^μ ,
6. in summary an electromagnetic interaction between photons and fermions, where we find the photoelectric effect of A. Einstein.

Main question

Is it the same for other Standard Model interactions? That is to say, can we find the 6 stages of Bohr parable and associate them with:

- quantum numbers qualified here as "source field" number, which reverse (or change) during reactions between particles,
- charges that move and are kept during that movement,
- interactions between boson-type mediating particles (which can appear and disappear) and fermion-type particles (which are generally persistent)?

Subsidiary question:

At the beginning of this Memoir, a new feature for electromagnetic interaction was proposed. Matching the invariance of a law of Nature: the source field equation of de Broglie:

$$\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt} = \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

with the conservation, during reactions between particles, of a quantum quantity: orbital and spin angular momentums. Is it possible to find such matches for the other 3 interactions?

II.2 Associate interactions, quantum numbers and bosons W^a and B intervening in beta decays

II.2.1 Program of work: search for analogies between the 3 electromagnetic, weak and hyper interactions

In the following chapters, we will examine whether it is possible to generalize the Einstein effect and the Bohr parable to the weak interaction and the hyper interaction.

Main question for weak interaction

Thus, we will study if it is possible to have for the weak interaction, the following Bohr parable:

1. a generation quantum number Ge qualified here as a "source field" number,
2. mediating particles: weak bosons W^a that carry Ge ,
3. left-handed fermions which carry a non-zero retained weak charge T^a and a generation quantum number Ge (which can be modified),
4. a reaction between bosons W^a and left-handed fermion, with a change in Ge of the fermion (for example, passage of omega baryon Ω_L^+ to proton p_L),
5. a change in generation of the fermion, which results in a "movement" of the left-handed fermion and its weak charge T^a , i.e., a weak current $j_{T^a}^\mu$,
6. a weak interaction between bosons and fermions.

Note on Ge

In this Memoir, the number of generations is defined as the sum of: Ge

$$Ge = St + C + \tilde{B} + T$$

We will come back to this at length later.

Subsidiary question for weak interaction

Is it possible, for the weak interaction, to match the invariance of a law of Nature: the source field equation of Compton:

$$\frac{c^2}{\hbar} \left(\frac{d\vec{m}^t}{dx} \right)_{tx}^y = - \frac{\partial \vec{K}_{ty}^{1/x}}{\partial t}$$

with the conservation, during reactions between particles, of a quantum quantity: the number of generations.

We would then have for the generation number Ge :

$$Ge = \hbar = - \left(\frac{d\vec{m}^t c^2}{dx} \right)_{tx}^y \frac{\partial t}{\partial \vec{K}_{ty}^{1/x}}$$

Main question for hyper interaction

We will study if it is possible to have for the hyper interaction, the following Bohr parable:

1. An isospin quantum number I^a qualified here as a "source field" number,
2. a boson B that carry I^a ,
3. fermions that carry a non-zero hypercharge Y_W and I^a (which can be reversed),
4. a reaction between boson B and fermions, with inversion of I^a of the fermion (for example, passing from n to p),
5. a change from neutron to proton, which results in a "movement" of the nucleon and its hypercharge Y_W , i.e., a hyper current $j_{Y_W}^\mu$,
6. a hyper interaction.

Subsidiary question for hyper interaction

Is it possible, for the hyper interaction, to match the invariance of a law of Nature: the source field equation of Newton:

$$\frac{1}{\hbar} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y = - \frac{\partial \vec{\Omega}_{xy}^{1/t}}{\partial x}$$

with the conservation, during reactions between particles, of a quantum quantity: the isospin number.

We would then have for isospin I :

$$I^x = \frac{1}{2} \hbar^x = - \frac{1}{2} \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \frac{\partial x}{\partial \vec{\Omega}_{xy}^{1/t}}$$

$$I^y = \frac{1}{2} \hbar^y = - \frac{1}{2} \left(\frac{d\vec{p}^y}{dt} \right)_{xt}^y \frac{\partial y}{\partial \vec{\Omega}_{yz}^{1/t}}$$

$$I^z = \frac{1}{2} \hbar^z = - \frac{1}{2} \left(\frac{d\vec{p}^z}{dt} \right)_{xt}^y \frac{\partial z}{\partial \vec{\Omega}_{zx}^{1/t}}$$

Note on $SU(2)$ (ou sur $SO(3)$)

The transformations of the orbital angular momentum (as well as the spin angular momentum) are based on the group of rotations $SU(2)$ (or $SO(3)$ if we work on a real space). Spin is a vector defined in the three dimensions of Space. Note that we find the three dimensions of Space at the level of momentum in the source field equation of de Broglie:

$$\vec{p}^a = \hbar \vec{K}^{1/a}$$

with $a = x, y, z$.

For source field equation of Newton:

$$\hbar^x = - \left(\frac{d\vec{p}^x}{dt} \right)_{xt}^y \frac{\partial x}{\partial \vec{\Omega}_{xy}^{1/t}}$$

which also involves a momentum in the 3 dimensions of Space, we must look for a quantum number whose transformations belong to $SU(2)$ (or to $SO(3)$). Isospin I therefore seems appropriate to be compared to the source field equation of Newton.

Main question for strong interaction

Later, at the end of the Memoir, we will study whether it is possible to have for the strong interaction, the following Bohr parable:

1. a baryonic number Ba qualified here as a "source field" number,
2. bosons that carry Ba ,
3. fermions that carry a strong charge noted Co^a and Ba (which can be modified),
4. a reaction between boson and fermion, with change in number Ba of the fermion,
5. a "movement" of the strong charge Co^a and a strong current $j_{Co^a}^\mu$,
6. a strong interaction.

Subsidiary question for strong interaction

Is it possible, for the strong interaction, to match the invariance of a law of Nature: source field equation of Einstein:

$$\hbar = \frac{\vec{m}^t c^2}{\vec{\Omega}_{xy}^{1/t}}$$

with the conservation, during reactions between particles, of a quantum quantity: the baryonic number.

We would then have for the baryonic number Ba :

$$Ba = \hbar = \left(\frac{d\vec{m}^t c^2}{dx} \right)_{tx}^y \frac{\partial x}{\partial \vec{\Omega}_{xy}^{1/t}}$$

II.2.2 Search for analogies between electric, hyper, weak, strong charges

For electromagnetic interaction, electric charge Q carried by a fermion is constructed according to the Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$$

from 3 quantum numbers I^3 , Ba and Ge which is proposed to be linked to the *other three* interactions of the Standard Model.

The electric charge Q appears as the complement of spin S , constructed from the *other three* source field quantum numbers: I^3 , Ba et Ge .

Inspired by Gell-Mann Nishijima relation, we will examine in this paper whether the charge Q can be expressed from a function f_Q such as:

$$Q = f_Q((I, I^3), Ba, Ge)?$$

Note

We expect a function f_Q more complicated than a simple sum as proposed by the Gell-Mann Nishijima relation. Indeed, we add terms of different natures, with on one side I^3 , and on the other side Ba and Ge .

For the hyper interaction, we have according to the Glashow relation of the electroweak model:

$$\frac{Y_W}{2} = Q - T^3$$

By analogy with electric charge Q , It will be examined whether the weak hypercharge Y_W can be complementary to isospin I , constructed from the other 3 source field quantum numbers. That is, do we have a function f_Y such as:

$$Y_W = f_Y((S, S^3), Ba, Ge)?$$

Similarly, we will examine whether the weak charge T^a can be complementary to the number of generations Ge , constructed from the other 3 source field quantum numbers. That is, do we have a function f_T such as:

$$(T, T^3) = f_T((S, S^3), (I, I^3), Ba)?$$

Similarly, we will examine whether the strong charge (noted here Co^a as color charge) can be complementary to Ba , constructed from the other 3 source field quantum numbers. That is, do we have a function f_{Co} such as:

$$(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)?$$

Note 1

It is assumed here that the weak charges T^a and strong charges Co^a are generators of groups of $SU(2)$. We'll come back to that.

Note 2

In the previous Memoir, we highlighted the existence of 2 types of quantum numbers:

- quantum numbers of charge X type that can radiate fields of interaction,

- quantum numbers involved in reactions between particles, which frequently reverse or change, and which cannot radiate fields of interaction. It is these quantum numbers that we propose to link to 4 source field equations of gravitation.

We will try here to show that the first (charge type) are functions of the second (source field type).

II.3 Conclusion of the chapter

The main objective of this thesis is to study the generalization of the Einstein photoelectric effect and the Bohr parable, from the electromagnetic interaction to the 3 other interactions involved in the Standard Model.

In the next chapters, we will look for more analogies between electromagnetic, strong, weak and hyper interactions than usually presented in the Standard Model. On the model of electromagnetic interaction, we will look for particles mediating weak and hyper interactions, non-charge X carrying, which by exchange of source field quantum quantities with charge carrying fermions, generate a displacement of charges and thus electromagnetic, weak, or hyper currents.

We will thus try to generalize the photoelectric effect of A. Einstein, to an effect bosons W - weak currents, then to an effect boson B - hyper currents.

For this, we will first study the possible connections between source field equations and quantum numbers carried by the mediating particles. We will return to the physics of the 1950s and 1960s, and the discovery of a new quantum number the strangeness St .

Chapter III New quantum numbers: the baryonic number Ba , the hypercharge Y and strangeness St (1950s)

Purpose of the chapter

We return here to the discovery in the 1950s of new quantum numbers such as the baryonic number Ba , the strong hypercharge Y and strangeness St , conserved globally during reactions between particles (except for St in some cases).

III.1 Historical Preamble, particle Accelerators

After the World War of 1939-1945 and following the use of atomic weapons, there was a rapid development of particle physics. States are ready to spend colossal sums on nuclear research, on what happens inside the nucleus of the atom. Particle accelerators are built, they make it possible to study the physics of particles of increasingly important energy, that is to say of smaller and smaller particle size (or wavelength smaller and smaller).

The experiments carried out in these particle accelerators will make it possible to discover new quantum numbers that are globally conserved during reactions.

III.2 The baryonic number Ba , the isospin I^a and the hypercharge Y

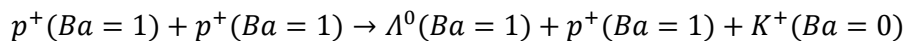
III.2.1 Baryonic number Ba

Fact 1: During a decay β^- involving the weak interaction (as well as the hyper interaction), a neutron decays into a proton. These two particles are called nucleons, they have close and relatively large masses for particles.

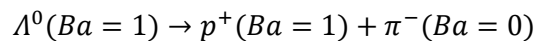
2nd fact: in reactions involving the strong interaction, we find that a proton never decays into a lighter particle. In the products of proton decay, there is always a particle of relatively large mass.

To clarify these 2 facts, as well as to mark the conservation of the number of nucleons (proton or neutron) and the close mass of these particles, the baryonic number Ba is introduced.

For example, we have the proton decay reaction, where the strong interaction intervenes, with the conservation of the baryonic number:



The lambda baryon Λ^0 decays in turn by involving the weak interaction, always with conservation of the baryonic number:



We assign a baryonic number $Ba = 1$ to baryons such as proton, neutron, or lambda baryon Λ^0 . We assign a baryonic number $Ba = -1$ to antibaryons (antiproton, antineutron, etc.).

Mesons (light particles) consisting of a quark and an antiquark (such as the K^+ or the π^-) have a baryonic number $Ba = 0$.

Note 1

In analogy to the baryonic number Ba for proton and neutron, there is a leptonic number $L_e = 1$ for the electron and neutrino, which is also conserved during reactions between particles.

For positron and antineutrino, we have a leptonic number $L_e = -1$.

When in this Memoir, we will refer to Ba as a source field quantum number, for leptons, it will be necessary to understand implicitly that it is a question of $-L_e$.

Note 2

The baryonic number is like the mass number A which counts the number of nucleons in the nucleus. The atomic number Z counts the number of protons.

III.2.2 Isospin I^a

In the previous Memoir, we have already mentioned isospin I^a ($a = 1,2,3$) proposed by W. Heisenberg in 1932 to explain a close mass and similarities in the behaviour of the proton and neutron vis-à-vis the strong interaction. Experiments in particle accelerators in the 1950s supported Heisenberg's ideas on isospin. It has been seen that in the theory of Yang-Mills (1954), isospin I^a is identified with the strong charge.

Like spin S^3 , isospin I^3 can be reversed during a reaction between particles, and yet remains globally conserved. For example, spin S^3 of an electron reverse during a hyperfine transition where electromagnetic interaction occurs. Isospin I^3 of a nucleon reverse during a decay β^- where hyper and weak interactions occur.

III.2.3 Hypercharge Y and Gell-Mann Nishijima relation

During a decay β^- , We have a transformation of the neutron into proton and inversion of the 3rd component of isospin I^3 which goes from $-\frac{1}{2}$ to $+\frac{1}{2}$. We also have the electric charge Q of the nucleon that goes from 0 to 1.

It is noted that isospin I^3 contains a share of electric charge Q . To link Q with I^3 , we postulate the existence of another quantum number, the hypercharge Y , which also contains a share of electric charge.

We have the so-called Gell-Mann Nishijima relation proposed independently by Kazuhiko Nishijima in 1953 and by Murray Gell-Mann in 1956:

$$Q = I^3 + \frac{Y}{2}$$

The proton and neutron have the same hypercharge $Y = 1$, conserved during decay β^- .

This makes it possible to find for the proton the electric charge:

$$Q_{pro} = \frac{1}{2} + \frac{1}{2} = 1$$

And for the neutron:

$$Q_{neu} = -\frac{1}{2} + \frac{1}{2} = 0$$

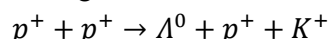
III.3 Strangeness St

III.3.1 Introduction of strangeness St

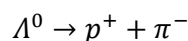
In the 1950s, experimenters discovered particles produced in collisions involving strong interactions. $\Delta t \approx 10^{-2}$ s (quick reactions), but with relatively long average lives $\tau \approx 10^{-9}$ s (slow reactions), that

is, a characteristic quantity of weak interactions. Thus, these particles (for example, the kaon meson K^+ ou le lambda baryon Λ^0) are produced by the strong way and disintegrate by the weak way.

We have the rapid reaction, where the strong interaction intervenes:

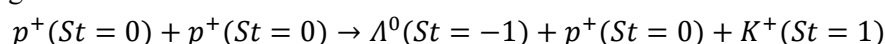


We have the slow reaction, where the weak interaction intervenes:

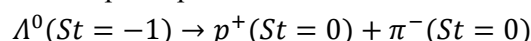


To explain these two facts, K. Nishijima and M. Gell-Mann introduced a new quantum number: strangeness, which will be noted here St to differentiate it from spin S .

Strangeness St is globally conserved in the strong interaction, where particles are produced in pairs of opposite strangeness:



The decay of strange particles into non-strange particles goes through the weak way, which explains why the strangeness does not seem a priori preserved in the weak interaction:



Note

The number is called strangeness precisely because it is conserved in the strong interaction and not in the weak interaction.

III.3.2 Relationship between the hypercharge Y , the baryonic number B and strangeness St

Physicists note that the hypercharge Y can sometimes be confused with strangeness St . For baryons, it is also necessary to take into account the baryonic number. We have the relationship:

$$Y = Ba + St$$

We obtain for the Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Y}{2}$$

$$Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$$

Note 1 on T and Y_W seen in the Electroweak Model

In the Electroweak Model, S. Glashow introduces 2 new quantum numbers, the weak charge T and weak hypercharge Y_W , by analogy respectively with isospin I (seen as the strong charge in Yang-Mills theory) and hypercharge Y (also called strong hypercharge).

To define the weak hypercharge, S. Glashow introduces the relationship between Q , T^3 and Y_W :

$$Q = T^3 + \frac{Y_W}{2}$$

We can speak of so-called strong quantum numbers for I and Y because they intervene in the strong interaction, and so-called weak quantum numbers for T and Y_W because they intervene in the weak interaction.

In this Memoir, however, we propose another distinction. We rank T and Y_W in charge type quantum numbers and isospin I in source field type quantum numbers. The same applies to hypercharge $Y = Ba + St$, classified here in the source field quantum number, because sum of 2 source field quantum numbers.

We will offer an explanation later.

Note 2 on I, Y, T, Y_W

We list the equalities (valid in some cases) between the so-called strong quantum numbers and the so-called weak quantum numbers.

For left-handed quarks of 1st generation (as well as for their antiparticles), we have:

$$I^3 = T^3$$

$$Y = Ba = Y_W$$

For left-handed quarks of 1st, 2nd, and 3rd generation (as well as for their antiparticles), we have:

$$Ba = Y_W$$

For left-handed leptons of 1st, 2nd, and 3rd generation (as well as for their antiparticles), we have:

$$-L_e = Y_W$$

For left-handed quarks and leptons of 1st, 2nd, and 3rd generation (as well as for their antiparticles), we have:

$$Ba - L_e = Y_W$$

Note 3

According to the relation $Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$, quantum numbers I^3 , Ba and St are electrically charged. That is, a particle carrying one of these three quantum numbers is electrically charged.

III.4 Conclusion of the chapter

Four quantum numbers have been mentioned: the baryonic number Ba , the isospin I^a , the strong hypercharge Y and strangeness St which are involved in reactions between particles, and which are generally conserved (except St in weak interaction).

Following the Gell-Mann-Nishijima relation, the electric charge Q is the sum of three of these quantum numbers: I^3 , Ba , St . In the next chapters, we will study the links between these three quantum numbers I^a , Ba , St and hyper, strong, and weak interactions. But first, we will look at the theories developed in the years 1950-1960 and describing the strong interaction.

Chapter IV The strong interaction, the Eightfold Way, and the Quark Model (1960s)

Purpose of the chapter

We first briefly describe the theories that followed one another from 1950 to 1970 to describe the strong interaction: the Yang-Mills theory with the strong charge generating $SU(2)_I$, the Eightfold Way and the Gell-Mann Quark Model with the strong charge generating $SU(3)_{flavor}$, finally quantum chromodynamics with the strong charge generating $SU(3)_{color}$.

We then return in more detail to the Eightfold Way and the Quark Model, two theories that use the strong quantum numbers mentioned in the previous chapter.

IV.1 Theories on strong interaction (1950-1970)

IV.1.1 Strong charge based on $SU(2)_I$ with pions as mediating particles (scale operator type for electrically charged pions)

In the 1950s, Chen Ning Yang and Robert Mills developed a theory of strong interaction, theory based on the group $SU(2)_I$, with isospin I as a strong charge.

The inspiration of the Yang-Mills theory is found in particular in the proton neutron couple, which attracts according to the strong interaction by their strong opposing charges, strong charges precisely equal to isospin $I^3 = -\frac{1}{2}$ for the neutron and isospin $I^3 = \frac{1}{2}$ for the proton.

In Yang-Mills theory, the mediating particles of the strong interaction are the 3 pions π^+ , π^- , π^0 . Unlike photons that do not carry electric charges, these 3 pions are strong charged particles in the sense that they carry an isospin. I . They can change the strong charge (respectively I^+ , I^- , I^0) of the fermion with which they interact.

Pions π^+ and π^- are referred to as scale operators (create and annihilate operators), that is, they can increase or decrease isospin I^3 of a fermion carrying a strong charge I^\pm with which they interact.

We have the relations between I^\pm and I^1 , I^2 :

$$I^+ = I^1 + iI^2$$

$$I^- = I^1 - iI^2$$

The pions π^+ and π^- are electrically charged, since isospin I^3 is electrically charged.

Let be $|m\rangle$ a common eigenvector for operators (I^2) and I^3 , and let be $\hbar m$ an eigenvalue of I^3 .

We have the following relationships, where the pions π^+ and π^- increase or decrease by I^3 the strong charge of the fermion with which they interact:

$$I^- |m\rangle = \hbar \sqrt{j(j+1) - m(m-1)} |m-1\rangle$$

$$I^+ |m\rangle = \hbar \sqrt{j(j+1) - m(m+1)} |m+1\rangle$$

Note

The 3 pions π^+ , π^- , π^0 of the strong interaction are the analogues of the 3 weak bosons W^+ , W^- , W^0 of the weak interaction. The first 2 W^+ and W^- are also scale operators and carry a weak charge T^3 . They interact with fermions of weak charges T^\pm by decreasing or increasing their weak charge by T^3 .

As a preview of the Quark Model, we remind the decomposition of the 3 pions into quarks and antiquarks of 1st generation:

$$|I = 1, I^3 = 1\rangle = -|u; \bar{d}\rangle = |\pi^+\rangle$$

$$|I = 1, I^3 = -1\rangle = |d; \bar{u}\rangle = |\pi^-\rangle$$

$$|I = 1, I^3 = 0\rangle = \frac{1}{\sqrt{2}}(|u; \bar{u}\rangle - |d; \bar{d}\rangle) = |\pi^0\rangle$$

IV.1.2 Strong charge based on $SU(3)_{flavor}$ with mesons as mediating particles (scale operator type for electrically charged mesons)

To include the quantum number of strangeness St , M. Gell-Mann proposed in the early 1960s in the Eightfold Way, then in the Quark Model (the second model being an extension of the first), that the strong charge is based on $SU(3)_{flavor}$, that is, on flavors u, d, s .

Note

The 2 flavors u, d are related to isospin I^3 . u corresponds to $I^3 = \frac{1}{2}$ and d to $I^3 = -\frac{1}{2}$. Flavour s is related to the number of strangeness $St = -1$.

The mediating particles are expanded from pions to mesons formed by a quark and an antiquark, to be chosen from the flavors u, d, s ($\bar{u}, \bar{d}, \bar{s}$ for antiquarks). Like the pions, the mesons of $SU(3)_{flavor}$ can change the strong charge of the fermion with which they interact.

We find electrically charged mesons whose electric charge is due to either isospin I^3 , or the strangeness number St , depending on the relationship:

$$Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$$

As for pions π^+, π^- , electrically charged mesons are of the scale operator type.

Note

The electrical charge of mesons cannot be due to Ba , since this quantum number is zero for mesons.

However, the symmetry between the different mesons only seems to be approached. For example, kaon mesons (strange particles) have a very different mass than pions. Thus, in the 1970s, quantum chromodynamics introduced a strong charge based on $SU(3)_{color}$.

IV.1.3 Strong charge based on $SU(3)_{color}$ with gluons as mediating particles (scale operator type)

Quantum Chromodynamics (QCD) was proposed in 1973 by H. David Politzer, Frank Wilczek and David Gross, in particular to obtain a theoretically perfect symmetry for strong charges vis-à-vis rotations of $SU(3)_{color}$. Strong charges are renamed color charges and based on the group $SU(3)_{color}$. We have 3 strong charges of colors: R, G, B , like red, green, and blue.

The particles mediating the interaction are no longer the mesons. These are the gluons G who carry a color charge and can also be considered as scale operators. They increase or decrease the strong color charge of the quark with which they interact.

Quantum chromodynamics attributes two important characteristics to quarks. A first called color confinement, a second called asymptotic freedom. These two characteristics are reminiscent of certain

characteristics of the electrostatic force and the Bohr electron model. We will return to these two characteristics at the end of this thesis in a final chapter devoted to strong interaction. In the immediate future, let's study the Eightfold Way and the Quark Model.

IV.2 The Eightfold Way

IV.2.1 The strong charge of flavor and the group $SU(3)_{flavor}$

In the Eightfold Way, the strong charge is the generator of the group $SU(3)_{flavor}$. The rotations of $SU(3)_{flavor}$ (with type Global Gauge transformation) act on triplets of particles with all three flavors.

We have the passage from one triplet to another triplet of particles associated with wave functions ψ :

$$\begin{pmatrix} \psi_u' \\ \psi_d' \\ \psi_s' \end{pmatrix} = U \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$$

with $U \in SU(3)_{flavor}$

U can be written $U(\alpha^1, \alpha^2, \dots, \alpha^8) = e^{i\frac{1}{2}(\alpha^1\lambda^1 + \alpha^2\lambda^2 + \dots + \alpha^8\lambda^8)}$ with $\lambda^1 \dots \lambda^8$ eight generators of $SU(3)_{flavor}$.

Starting from a hadron (e.g., wave function ψ_u), and applying to it a transformation $U \in SU(3)_{flavor}$, we end up with another hadron (e.g. wave function ψ_u') with a modified strong charge, but with in theory comparable properties (same mass, same orbital angular momentum, same spin, etc.).

Note 1

For example, when passing from neutron to proton, we have a strong modified charge ($I^3 = -\frac{1}{2}$ for the neutron, $I^3 = \frac{1}{2}$ for the proton), but comparable properties for the 2 nucleons (same mass, same orbital angular momentum, same spin, etc.).

Note 2

Hadrons are the particles of the atomic nucleus subject to the strong interaction. They are composed of baryons and mesons.

IV.2.2 Example of fundamental mesons

The Eightfold Way organizes hadrons according to their electric charge Q and their strong charge: isospin I^3 and strangeness St .

For example, we have nine fundamental mesons that rank among a singlet and an octet of $SU(3)_{flavor}$, hence the term Eightfold Way. We indicate them below following Q and St .

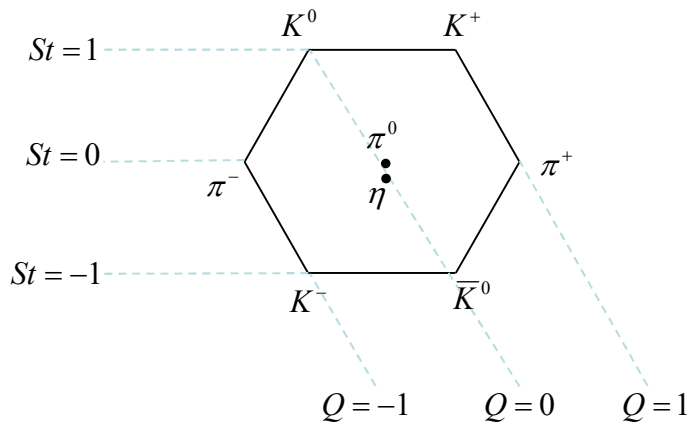


Figure 6: octet of the 8 fundamental mesons

IV.3 The Quark Model

IV.3.1 General

In 1964, M. Gell-Mann and George Zweig proposed that hadrons are not really elementary particles, but are made up of more fundamental components: quarks.

To account for the variety of hadrons known at the time, we need 3 types of quarks: u (up), d (down) and s (strange) which correspond to the three flavors used by the Eightfold Way in the group $SU(3)_{flavor}$.

In the Quark Model, baryons are formed by three quarks and have a positive baryonic number B_a . Anti-baryons are formed by three antiquarks and have a negative baryonic number B_a . Mesons consist of a quark and an antiquark and have a zero baryonic number B_a .

We have the following diagram that schematizes hadrons from the Quark Model.

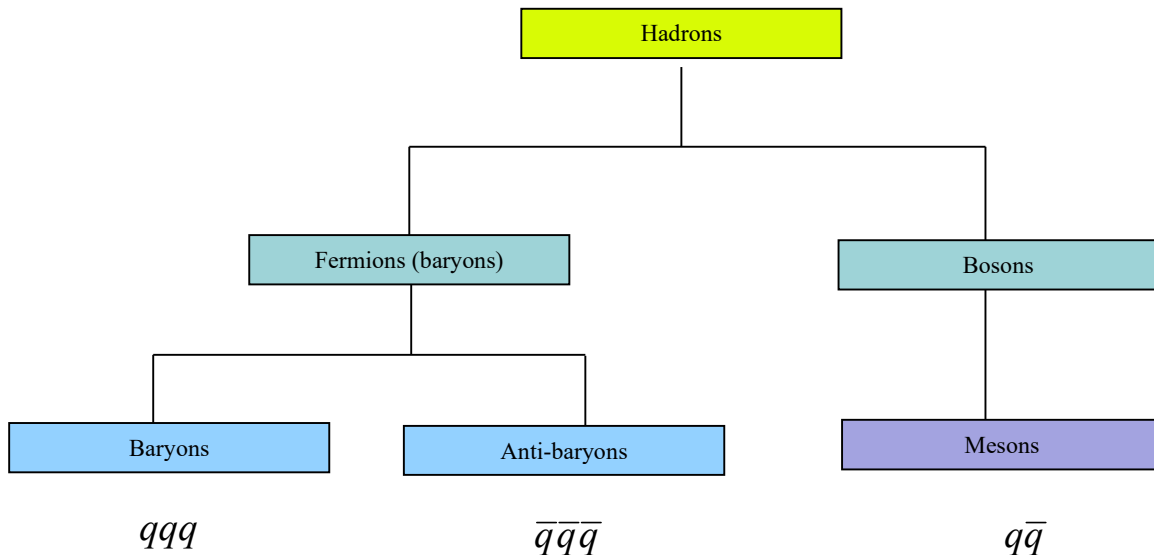


Figure 7: hadrons according to the Quark Model

Note 1

All hadrons are subject to strong interaction. This differentiates them from leptons that do not participate.

Note 2

The diagram is not exhaustive. Other fermions were later discovered, such as pentaquarks formed by 5 quarks.

IV.3.2 Examples of triplets of particles

We give the quantum numbers of a triplet of particles formed by quarks u , d , s .

Particle	Electric charge	Strong charge		Hypercharge $Y = Ba + St$	Baryonic number Ba
	$Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$	Isospin (I, I^3)	Strangeness St		
u	$+\frac{2}{3}$	$(\frac{1}{2}, +\frac{1}{2})$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
d	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	0	$+\frac{1}{3}$	$+\frac{1}{3}$
s	$-\frac{1}{3}$	(0,0)	-1	$-\frac{2}{3}$	$+\frac{1}{3}$

Note

Note that for the strong interaction, when modifying the strong charge, the baryonic number Ba is retained. For electromagnetic interaction, when we change the electric charge, it is the spin S that is retained.

We obtain the following triangle if we position the quarks u , d , s depending on their isospin I^3 and their hypercharge $Y = Ba + St$:

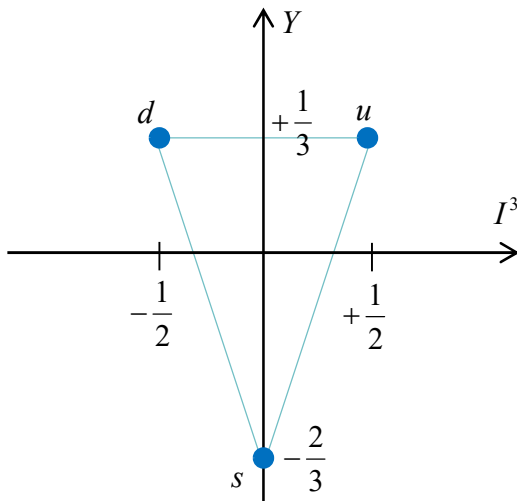


Figure 8: quarks u , d , s

Similarly, we give the quantum numbers of a triplet of particles formed by antiquarks \bar{u} , \bar{d} , \bar{s} .

Particle	Electric charge	Strong charge		Hypercharge $Y = Ba + St$	Baryonic number Ba
	$Q = I^3 + \frac{Ba}{2} + \frac{St}{2}$	Isospin (I, I^3)	Strangeness St		
\bar{u}	$-\frac{2}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	0	$-\frac{1}{3}$	$-\frac{1}{3}$

\bar{d}	$+\frac{1}{3}$	$(+\frac{1}{2}, +\frac{1}{2})$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
\bar{s}	$+\frac{1}{3}$	(0,0)	+1	$+\frac{2}{3}$	$-\frac{1}{3}$

We obtain the following triangle if we position the antiquarks \bar{u} , \bar{d} , \bar{s} depending on their isospin I^3 and their hypercharge Y :

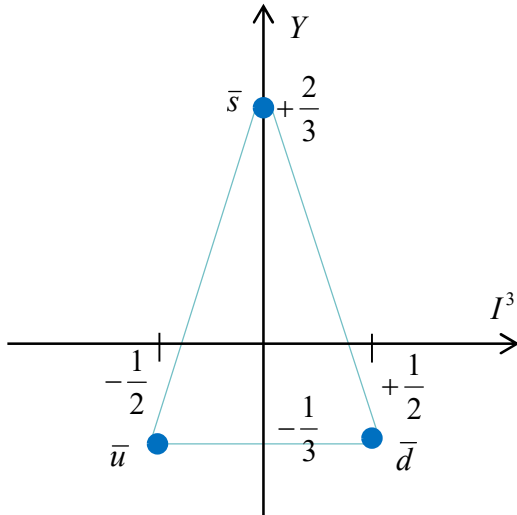


Figure 9: antiquarks \bar{u} , \bar{d} , \bar{s}

IV.3.3 6 flavors and 3 generations

In 1974, experimenters discovered the particle J/ψ , a linked state of $c\bar{c}$, that is, a 4th flavor of quarks called the charm flavor c . To preserve the Quark Model, the strong charge becomes the generator of the group $SU(4)_{flavor}$.

Nota

Meanwhile, in 1973, in quantum chromodynamics, it is proposed a strong charge of color that generates the group $SU(3)_{color}$.

In the decade that followed, two more quarks and 2 other flavors: b bottom and t top were discovered. Which makes 6 flavors in total. Flavour doublets are grouped by generation. So, we have 3 generations of 2 quarks each.

For leptons (particles orbiting the nucleus and not subject to strong interaction), we have a surprisingly similar pattern, also with 6 flavors and 3 generations.

We have 3 flavors of electrons: the electron e , the muon μ (kind of heavy electron, for the anecdote initially confused with the hypothetical mesotron of Yukawa), the tau τ (kind of electron even heavier), 3 flavors of neutrino: ν_e, ν_μ, ν_τ corresponding to the 3 electrons e, μ and τ .

These 6 quarks and 6 leptons are grouped into 3 generations of two quarks and two leptons. We have the following summary table:

Generations	I	II	III
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$
Leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

IV.3.4 Generalized Gell-Mann Nishijima relation, number of generations Ge

The addition of 3 new flavours c , b , t modifies the Gell-Mann Nishijima relation. We obtain the generalized Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Y}{2} = I^3 + \frac{Ba + St + C + \tilde{B} + T}{2}$$

with

Ba : baryonic number,

St : number of strangeness,

C : number of charms,

\tilde{B} : number of bottoms,

T : number of tops.

This Memoir defines the number of generations Ge as the sum:

$$Ge = St + C + \tilde{B} + T$$

Example for left-handed quarks of the 3 generations ($u_L, d_L, c_L, s_L, t_L, b_L$), we have the quantum numbers:

	Electric charge	(S, S^3)	Strong charge						Y	Ba	Weak charge (T, T^3)	Weak hypercharge Y_W
			(I, I^3)	Ge	St	C	\tilde{B}	T				
u_L	$\frac{2}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
d_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
c_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	0	+1	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
s_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	-1	-1	0	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
t_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	0	0	0	+1	$-\frac{2}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
b_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	-1	-1	0	-1	0	$-\frac{2}{3}$	$\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$

Note

The last two columns (weak charge (T, T^3) and weak hypercharge Y_W) are not about strong interaction. We find the weak charge and the weak hypercharge defined in the Electroweak Model. Right-handed quarks have the same quantum numbers as left-handed quarks, for the strong charge, the hypercharge Y and the baryonic number Ba .

IV.4 Conclusion of the chapter

Following experiments in particle accelerators, many particles were discovered in the 1950s such as baryons or mesons. The Quark Model, based on the strong interaction, makes it possible to order these particles and reduce the number of elementary components.

Via the Quark Model, we were interested in the strong interaction and the quantum numbers related to it. In the next chapter, we will focus on the quantum numbers found in the Electroweak Model and related to electromagnetic, weak, and hyper interactions. We will see that we often find the same quantum numbers as in the Quark Model.

Chapter V Quantum numbers of Electroweak model

Purpose of the chapter

We remind the quantum numbers used in the Electroweak Model. Some quantum numbers like (I, I^3) , Ba , Ge are used to describe the strong interaction in the Quark Model and are related to the Electroweak Model via charge Q .

We then examine the excited states of the particles vis-à-vis the spin. We propose an analogy between spin and baryonic number Ba , with quarks that would be excited states of nucleons, following the strong interaction, vis-à-vis the baryonic number.

We also examine how quantum numbers transform when we move from matter to antimatter.

V.1 Quantum numbers for 1st and 2nd generation particles

V.1.1 Quarks u , d , c , s of 1st and 2nd generation

In the table below, we give the quantum numbers used in the Electroweak Model, for left-handed and right-handed quarks of 1st and 2nd generation.

	Source field quantum numbers				Charges		
	$(S, S^3)H$	(I, I^3)	Ba	Ge	Electric charge Q $= I^3 + \frac{Ba}{2}$ $+ \frac{Ge}{2}$	Weak hypercharge $\frac{Y_W}{2} = Q - T^3$	Weak charge (T, T^3)
u_L	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
d_L	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
c_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
s_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
u_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{4}{3}$	(0,0)
d_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	(0,0)
c_R	$(\frac{1}{2}, \frac{1}{2})$	(0,0)	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	$+\frac{4}{3}$	(0,0)
s_R	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	$-\frac{2}{3}$	(0,0)

Note on momentum, spin, and helicity

We consider that the particles all have a momentum of the same direction and sense, that the antiparticles all have a momentum of the same direction and opposite sense (by parity operation P) compared to particles. This makes it possible to match in the table above spin S^3 and helicity H .

We fix the spin to $S^3 = -\frac{1}{2}$ for a left-handed electron u_L , we deduce the spin for the other particles.

V.1.2 Nucleons of 1st and 2nd generation

This table gives the quantum numbers used in the Electroweak Model for nucleons of 1st and 2nd generation.

	Source field quantum numbers				Charges		
	$(S, S^3)_H$	(I, I^3)	Ba	Ge	Electric charge Q $= I^3 + \frac{Ba}{2}$ $+ \frac{Ge}{2}$	Weak hypercharge $\frac{Y_W}{2}$ $= Q - T^3$	Weak charge (T, T^3)
$p_L uud$	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	+1	0	+1	+1	$(\frac{1}{2}, \frac{1}{2})$
$n_L udd$	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0	0	+1	$(\frac{1}{2}, -\frac{1}{2})$
Ω_L^+ CCS not observed	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	+1	+1	+1	$(\frac{1}{2}, \frac{1}{2})$
Ω_L^0 CSS	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	-1	0	+1	$(\frac{1}{2}, -\frac{1}{2})$
$p_R uud$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	+1	0	+1	+2	(0,0)
$n_R udd$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0	0	0	(0,0)
Ω_R^+ CCS not observed	$(\frac{1}{2}, \frac{1}{2})$	(0,0)	+1	+1	+1	+2	(0,0)
Ω_R^0 CSS	$(\frac{1}{2}, \frac{1}{2})$	(0,0)	+1	-1	0	0	(0,0)

Note 1 on the weak charge (T, T^3)

Weak charge (T, T^3) cancels out when switching from left-handed to right-handed particles. (T, T^3) is therefore a function of spin (S, S^3) .

For quarks and nucleons of 1st generation, we have $T^3 = I^3$.

T^3 reverses when reversed Ba (Transition from particle to antiparticle).

On the other hand, T^3 remains the same when passing from a proton with $Ba = 1$ to a quark with $Ba = \frac{1}{3}$.

T^3 is not modified when changing generations from $Ge = 0$ to $Ge = \pm 1$.

On the other hand, T^3 reverses when reversed Ge (for example, when changing the flavour for the same generation, or for a change from particle to antiparticle).

We therefore have a priori a function f_T with:

$$(T, T^3) = f_T((S, S^3), (I, I^3), \frac{Ba}{2})$$

Note 2 on weak hypercharge Y_W

The weak hypercharge Y_W changes when switching from left-handed to right-handed particles. Y_W is therefore a function of spin (S, S^3) .

Y_W reverses when reversed Ba (transition from particle to antiparticle).
For^{1st} generation left-handed quarks and nucleons, we have: $Y_W = Ba$.

Y_W is not modified when changing generations from $Ge = 0$ to $Ge = \pm 1$.
On the other hand, Y_W reverses when reversed Ge .

Y_W is not changed when reversed I^3 .
More generally, Y_W is not modified when changing the flavor for the same generation.

We therefore have a priori a function f_Y with:

$$Y_W = f_Y((S, S^3), \frac{Ba}{2})$$

Instead of $Y_W = f_Y((S, S^3), \frac{Ba}{2}, \frac{Ge}{2})$ expected

V.2 More and more excited states

V.2.1 Spin S

Following the model of the Bohr Goudsmit Uhlenbeck electron, when an electron is excited, it is brought in:

- either to occupy energy levels higher than that which it occupies in a stable way, that is to say to have higher orbital angular moments (Bohr model),
- or to know an inversion of its spin angular momentum (Goudsmit and Uhlenbeck model). For example, we go from a stable configuration of an electron and a proton of antiparallel spins $\uparrow\downarrow$ to an excited configuration of an electron and a proton of parallel spins $\uparrow\uparrow$.

Previously, it was proposed to group orbital angular momentum and spin angular momentum under one identic notion. It is proposed here to schematize the different excited states of a particle with respect to the angular momentum, according to the electromagnetic interaction, by a simplified series based simply on S^3 and of the type:

$$S^3 = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2} \dots$$

V.2.2 Isospin I

By analogy between spin and isospin I^3 , we propose for a simplified series of the same kind, with states increasingly excited vis-à-vis isospin:

$$I^3 = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2} \dots$$

For example, we have delta particles $\Delta^{++}(uuu)$ and $\Delta^{-}(ddd)$ which respectively have $I^3 = \frac{3}{2}$ and $I^3 = -\frac{3}{2}$.

	Source field quantum numbers			
	$(S, S^3)H$	(I, I^3)	Ba	Ge
$\Delta^{++}(uuu)$	$(\frac{3}{2}, \pm \frac{3}{2})$	$(\frac{3}{2}, +\frac{3}{2})$	+1	0
$\Delta^{-}(ddd)$	$(\frac{3}{2}, \pm \frac{3}{2})$	$(\frac{3}{2}, -\frac{3}{2})$	+1	0

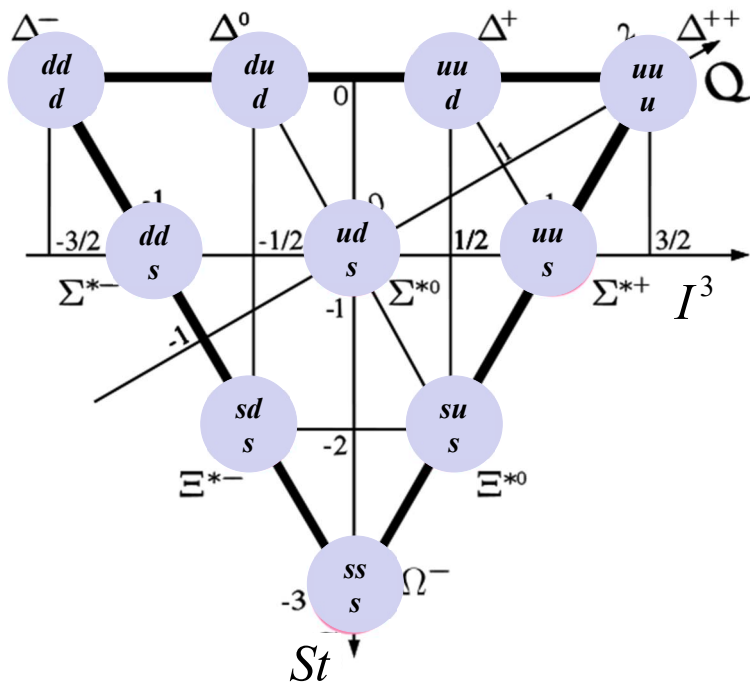


Figure 10: variation of I^3 by half-integer (source Wikipedia)

V.2.3 Baryonic number Ba

We give the table of the 4 source field quantum numbers for quarks and for their nucleon analogues formed of 3 quarks. We find the same values except for the baryonic number Ba .

	Source field quantum numbers					$(S, S^3)H$	(I, I^3)	Ba	Ge
u	$\frac{1}{2}, \pm \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	$+\frac{1}{3}$	0	$p\ uud$	$\frac{1}{2}, \pm \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$	+1	0
d	$\frac{1}{2}, \pm \frac{1}{2}$	$\frac{1}{2}, -\frac{1}{2}$	$+\frac{1}{3}$	0	$n\ udd$	$\frac{1}{2}, \pm \frac{1}{2}$	$\frac{1}{2}, -\frac{1}{2}$	+1	0
c	$\frac{1}{2}, \pm \frac{1}{2}$	(0,0)	$+\frac{1}{3}$	+1	Ω^+ ccs	$\frac{1}{2}, \pm \frac{1}{2}$	(0,0)	+1	+1
s	$\frac{1}{2}, \pm \frac{1}{2}$	(0,0)	$+\frac{1}{3}$	-1	Ω^0 css	$\frac{1}{2}, \pm \frac{1}{2}$	(0,0)	+1	-1

Note on the charges

We assume in this Memoir that the strong charge is not a function of Ba , but that the other 3 charges (electric, hyper, weak) are.

Quark and nucleon do not have the same electric charge Q and weak hypercharge Y_W , as expected since they have different baryonic numbers. They even have weak charge (T, T^3) (contrary to the expected) and even strong charge (as expected).

For the baryonic number Ba , we have for quarks $Ba = \frac{1}{3}$ and for nucleons (protons, neutrons) $Ba = 3 \times \frac{1}{3} = 1$.

In the Quark Model, the difference between quark and nucleon is explained by a non-elementary particle nucleon formed by 3 quarks. However, in an analogy with excited spin states $S^3 : \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$ and excited spin states $I^3 : \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$, We could think of the difference between quark and nucleon as a difference in excitation vis-à-vis Ba . Thus, quark and nucleon would in fact be the same particle, but would be distinguished by their excitation vis-à-vis Ba , following the strong interaction described by the Quark Model.

For spin S^3 (if we retain only the positive terms), We have a series of the type: $\frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$ For Ba , we expect by analogy to a series of the type: $\frac{1}{3}, \frac{3}{3}, \frac{5}{3} \dots$. The problem is that the nucleons of $Ba = \frac{3}{3} = 1$ are much more stable than quarks of $Ba = \frac{1}{3}$ (quarks that have never actually been observed).

To get out of this contraction, it is proposed for Ba a series of the type: $Ba = \frac{1}{1}, \frac{1}{3}, \frac{1}{5} \dots$

For the inverse of Ba , We have a series of the type: $\frac{1}{Ba} = 1, 3, 5 \dots$ with particles that are increasingly excited vis-à-vis Ba .

To check if the series $Ba = \frac{1}{1}, \frac{1}{3}, \frac{1}{5} \dots$ is more adequate than the series $Ba = \frac{1}{3}, \frac{3}{3}, \frac{5}{3} \dots$, let us examine whether there are no particles with $Ba = \frac{1}{5}$.

V.2.4 The pentaquarks

We give some examples of pentaquarks, very unstable particles formed of 5 quarks: $\Theta_s^+([ud][ud]\bar{s})$, $\Theta_c([ud][ud]\bar{c})$. In the table below, they have the same source field quantum number as their corresponding nucleons.

	Source field quantum numbers								
	$(S, S^3)H$	(I, I^3)	Ba	Ge		$(S, S^3)H$	(I, I^3)	Ba	Ge
$[cs][cs]\bar{d}$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	+1	0	$p\ uud$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	+1	0
$[cs][cs]\bar{u}$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0	$n\ udd$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0
$\Theta_s^+([ud][ud]\bar{s})$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	+1	$\Omega^+\ ccs$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	+1
$\Theta_c([ud][ud]\bar{c})$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	-1	$\Omega^0\ css$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	-1

Like a nucleon, a pentaquark has $Ba = 1$ (four quarks $Ba = +\frac{1}{3}$ and an antiquark $Ba = -\frac{1}{3}$). It can also be interpreted as consisting of 5 excited quarks of $Ba = \frac{1}{5}$.

In this case, we have a series $Ba = \frac{1}{1}, \frac{1}{3}, \frac{1}{5} \dots$. The steady state is the nucleon, then an excited state with quarks $Ba = \frac{1}{3}$, then an even more excited state with quarks $Ba = \frac{1}{5}$ (and forming between them a pentaquark).

We have the following table:

	Source field quantum numbers								
	$(S, S^3)H$	(I, I^3)	Ba	Ge		$(S, S^3)H$	(I, I^3)	Ba	Ge
Quark excited of $[cs][cs]\bar{d}$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{5}$	0	$p\ uud$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	+1	0

Quark excited of $[cs][cs]\bar{u}$	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{5}$	0	n udd	$(\frac{1}{2}, \pm\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0
Quark excited of $\theta_s^+([ud][ud]\bar{s})$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	$+\frac{1}{5}$	+1	Ω^+ ccs	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	+1
Quark excited of $\theta_c([ud][ud]\bar{c})$	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	$+\frac{1}{5}$	-1	Ω^0 css	$(\frac{1}{2}, \pm\frac{1}{2})$	(0,0)	+1	-1

Note on the Landé factor

As mentioned, interpreting the proton or neutron as a composite particle makes it possible to explain Landé factors not equal to 2 for nucleons.

To see if interpreting nucleon and quarks as states not composite, but states more and more excited vis-à-vis Ba , also helps to explain a Landé factor not equal to 2?

V.2.5 Table of excited states

We propose the following series with increasing or excited states for the 4 source field quantum numbers.

	Source field quantum numbers			
	(S, S^3)	(I, I^3)	$\frac{1}{Ba}$	$\frac{1}{Ge}$
u_L	$-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \dots$	$+\frac{1}{2}, +\frac{3}{2}, +\frac{5}{2} \dots$	+1, +3, +5...	0
d_L	$-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \dots$	$-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \dots$	+1, +3, +5...	0
c_L	$-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \dots$	0	+1, +3, +5...	+1, +3, +5...
s_L	$-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \dots$	0	+1, +3, +5...	-1, -3, -5...

Note 1

In the table above, quarks u_L , d_L , c_L , s_L correspond to the 1st state of excitation for spin S^3 , isospin I^3 , and to the 2nd state of excitation for Ba .

Note 2

Excited states vis-à-vis Ge indicated in the table above, are purely speculative and are given in analogy with Ba .

It is questionable whether the 3rd generation of quarks can correspond to an excited state vis-à-vis Ge ? This is not a priori the case since we find the same quantum numbers, especially for Ge .

	Source field quantum numbers				Charges		
	$(S, S^3)H$	(I, I^3)	Ba	Ge	Q $= I^3 + \frac{Ba}{2}$ $+\frac{Ge}{2}$	$\frac{Y_W}{2}$ $= Q - T^3$	(T, T^3)
t_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
b_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
c_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
s_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	-1	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$

V.3 Transition to antimatter

V.3.1 Operations C and P , transition from matter to antimatter

The parity operator P represents the inversion of the Space coordinates:

$$x \rightarrow x' = -x$$

The electric charge conjugation C (or operator C) is a transformation that reverses the electric charge of the particle.

Remind that the passage from matter to antimatter does not only correspond to the conjugation of charge C , but also to the parity operator P .

It can be seen in the diagram below, decay of a pion π^+ in $(\nu_\mu)_L$ and $(\mu^+)_L$. To obtain the passage from matter to antimatter, i.e., the decay of a pion π^- in $(\bar{\nu}_\mu)_R$ and $(\mu^-)_R$, we must carry out the operations C and P .

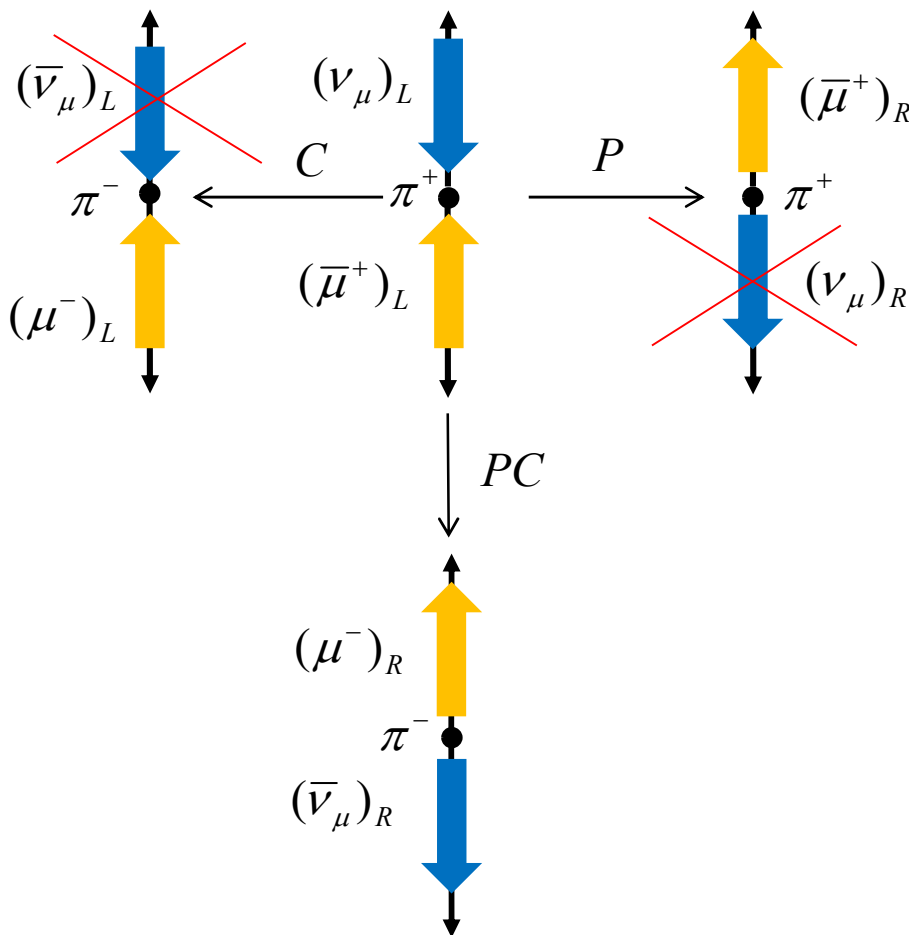


Figure 11: decay of a pion π^+ in $(\nu_\mu)_L$ and $(\mu^+)_L$, decay of a pion π^- in $(\bar{\nu}_\mu)_R$ and $(\mu^-)_R$

Note

The momentums of the particles are in fine and black arrows, the spins of the particles are in fat and colored arrows, we can deduce the helicities. Muon neutrino antiparticle $(\nu_\mu)_L$ is $(\bar{\nu}_\mu)_R$.

V.3.2 Inversion of quantum numbers Q, Ba, I^3, Ge

During the transition from matter to antimatter, there is reversal of the electric charge Q and the 3 quantum numbers from which it is built: Ba, I^3, Ge .

When passing from matter to antimatter, there is no spin reversal S^3 . This appears as the complement of Q and does not reverse during a transformation C or CP .

On the other hand, the operator P intervenes, with a reversal of the impulse. So, we have the helicity that reverses. Quarks u_L, d_L, c_L, s_L have for antiparticles $\bar{u}_L, \bar{d}_L, \bar{c}_L, \bar{s}_L$ with, for example, $\bar{u}_L = (\bar{u})_R$.

Reminder

Orbital angular momentum L and spin angular momentum S are not affected by the Parity operation. The energy and momentum are reversed. We have by parity operation P :

$$\begin{aligned} L &\rightarrow L' = L \\ S &\rightarrow S' = S \\ p &\rightarrow p' = -p \\ E &\rightarrow E' = -E \end{aligned}$$

V.3.3 Antiquarks $\bar{u}, \bar{d}, \bar{c}, \bar{s}$

In the table below, we give the quantum numbers used in the Electroweak Model, for left-handed and right-handed antiquarks of 1st and 2nd generation.

	Source field quantum numbers				Charges		
	$(S, S^3)_H$	(I, I^3)	Ba	Ge	Electric charge $Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	Weak hypercharge $\frac{Y_W}{2} = Q - T^3$	Weak charge (T, T^3)
\bar{u}_L	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{3}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
\bar{d}_L	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$-\frac{1}{3}$	0	$+\frac{1}{3}$	$-\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
\bar{c}_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$-\frac{1}{3}$	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
\bar{s}_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$-\frac{1}{3}$	+1	$+\frac{1}{3}$	$-\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
\bar{u}_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$-\frac{1}{3}$	0	$-\frac{2}{3}$	$-\frac{4}{3}$	(0,0)
\bar{d}_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$-\frac{1}{3}$	0	$+\frac{1}{3}$	$+\frac{2}{3}$	(0,0)
\bar{c}_R	$(\frac{1}{2}, \frac{1}{2})$	(0,0)	$-\frac{1}{3}$	-1	$-\frac{2}{3}$	$-\frac{4}{3}$	(0,0)
\bar{s}_R	$(\frac{1}{2}, \frac{1}{2})$	(0,0)	$-\frac{1}{3}$	+1	$+\frac{1}{3}$	$+\frac{2}{3}$	(0,0)

Note 1 on spin and helicity

Since helicity reverses and not spin when moving from matter to antimatter, helicity is not equal to spin in the table below.

Note 2 on Majorana particles

Majorana particles are their own antimatter. They have zero electric charge, zero isospin, zero baryonic number and zero number of generations. For example, we find the mesons $u\bar{u}, d\bar{d}$, etc. Majorana particles can have a non-zero and always integer spin.

Note 3 on antimatter, weak anti-charge, strong anti-charge, and hyper anti-charge

Antimatter concerns the electromagnetic interaction, it reverses 3 quantum numbers that are electrically charged and that intervene in the Gell-Mann Nishijima relation $Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$, the isospin I^3 , the baryonic number Ba et the number of generations Ge . On the other hand, it does not reverse the spin which is not electrically charged, and which is carried by the photon $S\gamma$.

By analogy with antimatter related to electromagnetic interaction (the antimatter matter association corresponds to an electromagnetic current j_Q^μ), It can be assumed that there is a weak anti-charge related to the weak interaction (the association weak charge weak anti-charge would correspond to a weak current $j_{T^a}^\mu$).

If we assume a relationship $(T, T^3) = f_T((S, S^3), (I, I^3), \frac{Ba}{2})$, 3 quantum numbers, the spin S^3 , the isospin I^3 and the baryonic number Ba are then weak charged. The transition from the weak charge to the weak anti-charge, must reverse these 3 quantum numbers.

Like spin S^3 transported by the photon (electrically neutral) γ , which is not changed during the transition from matter to antimatter, the source field quantum number carried by weak bosons W^a (no weak charged), shall not be changed when switching from weak charge to weak anti-charge. It will later be proposed that the source field quantum number transported by bosons W^a is the number of generations Ge .

We can see the operator switching to weak anti-charge as a transformation C_{T^a} (to see if there are also transformations P or T to be included?).

The same assumptions can be made for a transition from weak hypercharge to weak hyper anti-charge. If we assume a relationship: $Y_W = f_Y((S, S^3), Ba, Ge)$, we have a reversal or a change of (S, S^3) , Ba , Ge , and a conservation of (I, I^3) the source field quantum number that will be proposed transported by the boson B .

Ditto for a transition from the strong charge to the strong anti-charge. If we assume a relationship: $(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)$, we have a reversal or a change of (S, S^3) , (I, I^3) , Ge , and a conservation of Ba the source field quantum number that will be proposed transported by the particles mediating the strong interaction.

V.3.4 Cross channels and prescription of E. Stueckelberg and R. Feynman

According to the prescription of E. Stueckelberg and R. Feynman, we have an equivalence between:

- an antiparticle of positive energy,
- a particle of negative energy that goes back in time.

and conversely an equivalence between:

- a particle of positive energy,
- an antiparticle of negative energy that goes back in time.

This prescription applies to a 4-particle process and their 4 anti-particles:

$$A + B \rightarrow C + D$$

makes it possible to anticipate the existence of three other cross channels:

$$A + \bar{D} \rightarrow C + \bar{B} \text{ or } \bar{C} + B \rightarrow \bar{A} + D$$

$$\bar{D} + B \rightarrow C + \bar{A} \text{ or } A + \bar{C} \rightarrow \bar{B} + D$$

$$A \rightarrow \bar{B} + C + D \text{ or } B \rightarrow \bar{A} + C + D$$

Nota

These crossed channels are not true for spin, since spin does not reverse when moving from matter to antimatter. They apply only to quantum numbers and physical quantities that reverse during the transition from matter to antimatter.

V.4 Conclusion of the chapter

We reminded and commented for quarks and antiquarks the quantum numbers assigned to them by the Electroweak Model. We mentioned the 4 source field quantum numbers that we want to associate with the 4 interactions of the Standard Model. We also mentioned the 3 charges involved in electromagnetic, weak, and hyper interactions, charges that we propose to build from 3 source field quantum numbers. We also proposed to interpret quarks as excited states of nucleons, vis-à-vis the baryonic number, following the strong interaction. Finally, the transition from matter to antimatter was studied and the hypotheses of weak anti-charge, strong anti-charge and hyper anti-charge were suggested.

These data will allow us in the next chapter to examine the electromagnetic, weak, and hyper currents that we encounter in the Electroweak Model.

Chapter VI Electromagnetic currents, weak currents, and hyper currents, Cabibbo angle and GIM mechanism, case of quarks

Purpose of the chapter

In this chapter, we focus on the extensions of the Electroweak Model developed in the 1960s and 1970s. We will study the Cabibbo angle proposed by N. Cabibbo in 1963, then the GIM mechanism proposed by S. Glashow, J. Illiopoulos and L. Maiani in the early 1970s. From this, we will propose an analogy in the case of quarks between the electromagnetic current j_Q^μ that interacts with the photon γ , the weak currents j_{Ta}^μ that interact with weak bosons W^a , the hyper current $j_{Y_W}^\mu$ that interacts with the boson B .

VI.1 Electromagnetic current j_Q^μ

VI.1.1 Case of electrons

We remind the electromagnetic current of Dirac for an electron and a positron:

$$j_Q^\mu = Q\bar{e}\gamma^\mu e$$

with the wave function ψ associated with the particle which is here symbolized by the particle itself: e .

If we distinguish right and left helicity, we have:

$$j_Q^\mu = Q\bar{e}_L\gamma^\mu e_L + Q\bar{e}_R\gamma^\mu e_R$$

For left-handed particles, we have:

$$j_Q^\mu = Q\bar{e}_L\gamma^\mu e_L$$

We have the Feynman diagram of annihilation type (cancellation of the electric charge and addition of the spin):

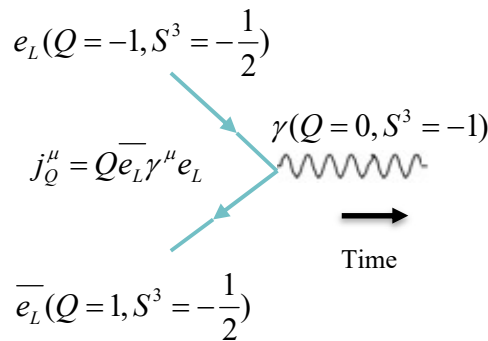


Figure 12: annihilation of electric charge

We have the Feynman diagram of scattering type (conservation of electric charge and spin reversal):

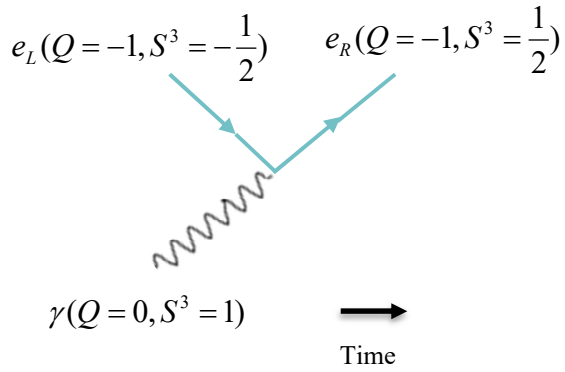


Figure 13: scattering of the electric charge, disappearance of the photon

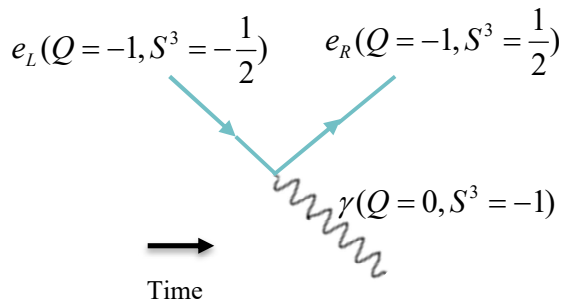


Figure 14: scattering of electric charge, appearance of photon

VI.1.2 Case of quarks

For a quark u_L and its antiquarks \bar{u}_L , we have the electromagnetic current:

$$j_Q^\mu = Q\bar{u}_L\gamma^\mu u_L$$

We have the Feynman diagram of annihilation type:

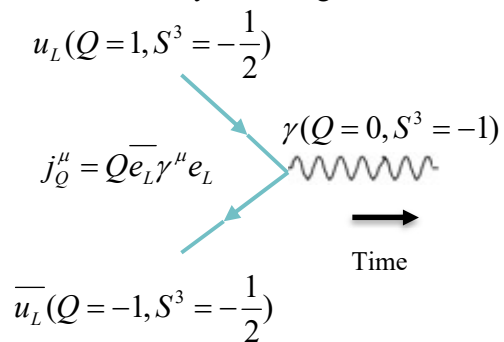


Figure 15: annihilation of electric charge

We have the Feynman diagrams of scattering type (hyperfine transition):

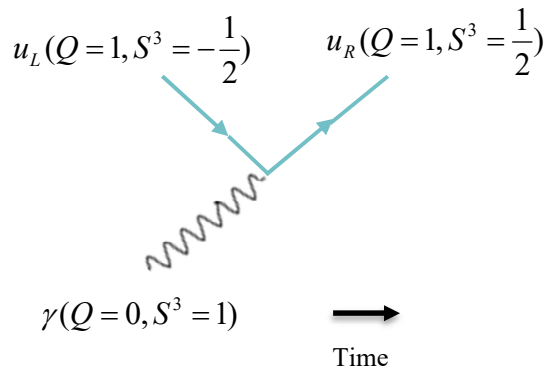


Figure 16: scattering of the electric charge, disappearance of the photon

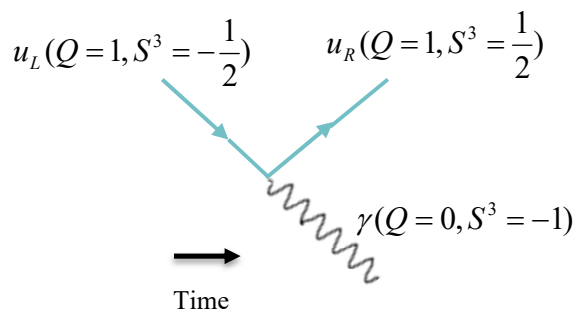


Figure 17: scattering of electric charge, appearance of photon

VI.1.3 Scattering: retained charge Q when reversing spin S^3

When the spin is reversed S^3 during a hyperfine transition, only the electric charge Q is conserved. The other 2 charges used in the Electroweak Model, Y_W et (T, T^3) are modified.

In the table below, we visualize the passage from a quark u_L to a quark u_R , with reversal of the spin S^3 , conservation of other source field quantum numbers, conservation of electrical charge Q , modification of charges Y_W and (T, T^3) .

	Source field numbers				Charges		
Conserved		X	X	X	X		
Modified	X					X	X
	(S, S^3)	(I, I^3)	Ba	Ge	$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	$\frac{Y_W}{2} = Q - T^3$	(T, T^3)
u_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
u_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{4}{3}$	(0,0)
d_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$

d_R	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$\left(\frac{1}{2}, -\frac{1}{2}\right)$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	(0,0)
-------	---	--	----------------	---	----------------	----------------	-------

VI.1.4 Annihilation: cancellation of charge Q et addition of spin S

We give the quantum numbers of the electromagnetic current $j_Q^\mu(\bar{u}_L, u_L)$:

	$j_Q^\mu(\bar{u}_L, u_L)$	\bar{u}_L	u_L	Total
Charge	Q	-1	1	0
Source field quantum number	S^3	$-\frac{1}{2}$	$\frac{1}{2}$	-1
	Ge	0	0	0
	Ba	$-\frac{1}{3}$	$\frac{1}{3}$	0
	I^3	$-\frac{1}{2}$	$\frac{1}{2}$	0

The electromagnetic current j_Q^μ corresponds to a couple $u_L \bar{u}_L$ where electrical charges Q cancel each other and where the spins S^a add up, to donate a photon γ of zero electric charge and carrying a spin S^a .

Following an analogy with the electromagnetic current, we will examine whether it is the same for weak currents and hyper current. That is to say, we will look at whether:

- we can match the weak current $j_{T^a}^\mu$ to a couple of quarks and antiquarks where the weak charges T^a cancel each other and where the generation numbers Ge add up to give a boson W^a of zero weak charge T^a and carrying a generation Ge ,
- we can match the hyper current $j_{Y_W}^\mu$ to a couple of quarks and antiquarks where weak hypercharges Y_W cancel each other and where the isospins I^a add up to give a boson B of zero weak hypercharge and carrying isospin I^a .

For this, we will first focus on the Cabibbo angle, the GIM mechanism and the different currents present in this mechanism.

VI.2 The model of N. Cabibbo

VI.2.1 Angle of Cabibbo

At the end of the 1950s, during experiments conducted in particle accelerators, it was observed that transitions with change of strangeness $|\Delta St| = 1$ occur at a much lower rate than transitions without a change in strangeness $|\Delta St| = 0$. This with a factor of about 20.

In the early 1960s, we simply have a model with 3 left-handed quarks that are subject to weak interactions: u_L , d_L and s_L .

In 1963, Nicola Cabibbo proposed the following explanation for the much lower rate of transitions with change of strangeness. Since quarks d_L and s_L have the same quantum numbers (except for flavor, as well as for mass), it's possible that it's, not the quark d_L that interacts with a weak boson W^a , but a mixture d_L' consisted of quarks d_L and s_L .

N. Cabibbo defines the mixture:

$$d_L' = -d_L \cos \theta_c + s_L \sin \theta_c$$

with θ_c the angle of Cabibbo.

Instead of a weak doublet: $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$, we have a weak doublet of the type:

$$\begin{pmatrix} u_L \\ d_L' \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \cos \theta_c + s_L \sin \theta_c \end{pmatrix}$$

that interacts with a boson W^a .

Coupling $u_L d_L$ characterized by $|\Delta St| = 0$ has a probability amplitude proportional to Zo^t . For its part, the coupling $u_L \gamma$ characterized by $|\Delta St| = 1$ has a probability amplitude proportional to $\sin \theta_c$.

We have approximately:

$$\frac{\sin^2 \theta_c}{\cos^2 \theta_c} \approx \frac{1}{20}$$

We get: $\theta_c \approx 0,23 \text{ radians}$ or $\theta_c \approx 13,04^\circ$

The fundamental idea of N. Cabibbo is that the eigenstates of masses (or energies), here quarks d_L and s_L , are not the eigenstates participating in reactions involving bosons W^a .

These are the quarks d_L' and s_L' , defined as the proper states of flavors, which participate in these reactions. They are related to quarks d_L and s_L by a constant angle, the angle of Cabibbo θ_c . The Cabibbo angle is symbolized by the following figure representing a rotation of angle θ_c .

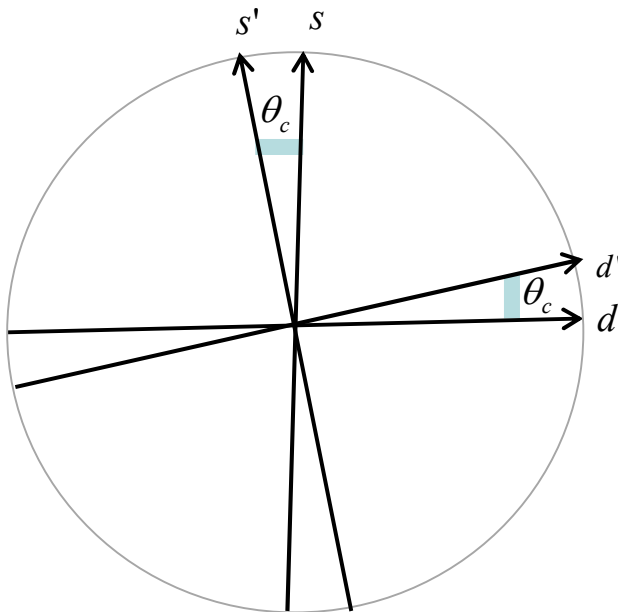


Figure 18: angle of Cabibbo

Note

Remind that flavor is a notion initially related to the strong nuclear charge.

We have the relationship between quarks d_L, s_L, d_L', s_L' via the Cabibbo matrix:

$$\begin{pmatrix} d_L' \\ s_L' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}$$

The vector on the left $\begin{pmatrix} d_L' \\ s_L' \end{pmatrix}$ is the eigenstate of flavors, i.e., the eigenstate of reactions involving bosons W^a .

The vector on the right $\begin{pmatrix} d_L \\ s_L \end{pmatrix}$ is the eigenstate of masses or energies.

VI.2.2 Cabibbo-Kobayashi-Maskawa matrix, hypothesis of a 3rd generation of quarks

In 1964, J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay highlighted the violation of symmetry CP (parity charge transformations, transition from matter to antimatter) by studying the properties of neutral kaons (strange particles of meson type) observed during decays β^\pm (we will come back to this in the next Memoir).

In 1973, Makoto Kobayashi and Toshihide Maskawa found that the violation of symmetry CP cannot be explained by a simple 4-quark model. Generalizing the Cabibbo matrix, they proposed a matrix M 3×3 of Cabibbo-Kobayashi-Maskawa (CKM matrix) defined from 6 quarks:

$$\begin{pmatrix} d_L' \\ s_L' \\ b_L' \end{pmatrix} = M_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

M. Kobayashi and T. Maskawa predicted a 3rd generation of quarks: the doublet of quarks b bottom and t top. The discovery of the bottom quark was confirmed a few years later by Fermilab in 1977.

There are several representations of the CKM matrix. A common representation is as follows: We define 3 Euler angles $\theta_{12} = \theta_c$, θ_{13} , θ_{23} and a phase δ .

We have:

$$\begin{aligned} c_{ij} &= \cos \theta_{ij} \\ s_{ij} &= \sin \theta_{ij} \end{aligned}$$

$$M_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

For example, the matrix breaks down as follows:

$$M_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is a combination of three rotation matrices, except for the presence of the phase δ which reflects the violation of symmetry CP .

Experimentally, we measure:

$\theta_{12} = \theta_c \approx 13,04^\circ$, $\theta_{13} \approx 0,20$, $\theta_{23} \approx 2,38^\circ$ and a phase $\delta \approx 1,20rad$.

$$M_{CKM} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ -0.22 & 0.97 & 0.04 \\ 0.008 & -0.04 & 0.99 \end{pmatrix}$$

The CKM matrix can be interpreted as the ease (probability) of a quark changing flavor, for example the passage from a quark d_L à un quark s_L' .

VI.3 Glashow-Illiopoulos-Maiani GIM mechanism and electrically neutral weak currents

VI.3.1 Finding of no change in strangeness in electrically neutral weak currents

Experimentally, it was noted the absence of change in strangeness $|\Delta St| = 0$ in reactions involving neutral weak currents $j_{T^0}^\mu$ and weak boson Z^0 . It was then said that there are no neutral weak currents changing the flavor (in the sense that strangeness St is one of the 3 flavors of quarks and that flavors u and d are also not modified).

For example, we had observed changes in strangeness. $|\Delta St| = 1$ in reactions involving charged weak currents $j_{T^\pm}^\mu$ and bosons W^\pm . This is the case in the disintegration of the kaon K^+ :

$$K^+(u\bar{s}) \rightarrow \pi^0(u\bar{u}) + e^- + \bar{\nu}_e$$

On the other hand, the kaon decay reaction K^+ with strangeness changes $|\Delta St| = 1$ and causing neutral weak currents to interact $j_{T^0}^\mu$ and bosons Z^0 is much rarer:

$$K^+(u\bar{s}) \rightarrow \pi^+(u\bar{d}) + \nu + \bar{\nu}$$

We have the report:

$$\frac{K^+(u\bar{s}) \rightarrow \pi^+(u\bar{d}) + \nu + \bar{\nu}}{K^+(u\bar{s}) \rightarrow \pi^0(u\bar{u}) + e^- + \bar{\nu}_e} < 10^{-5}$$

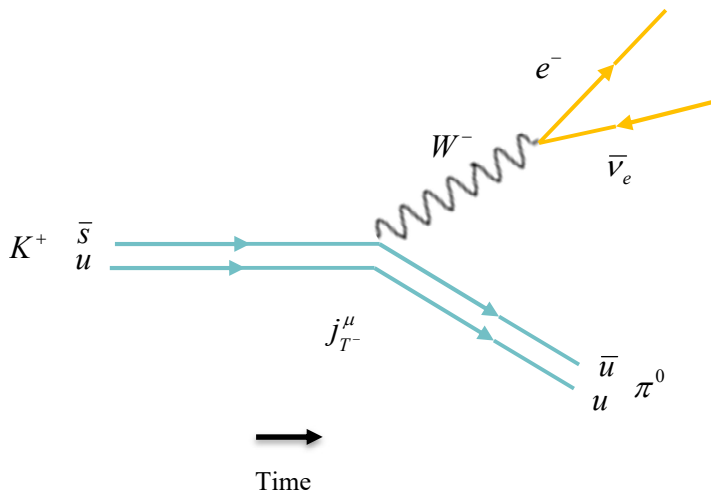


Figure 19: charged weak current $j_{T^-}^\mu$ interacting with W^-

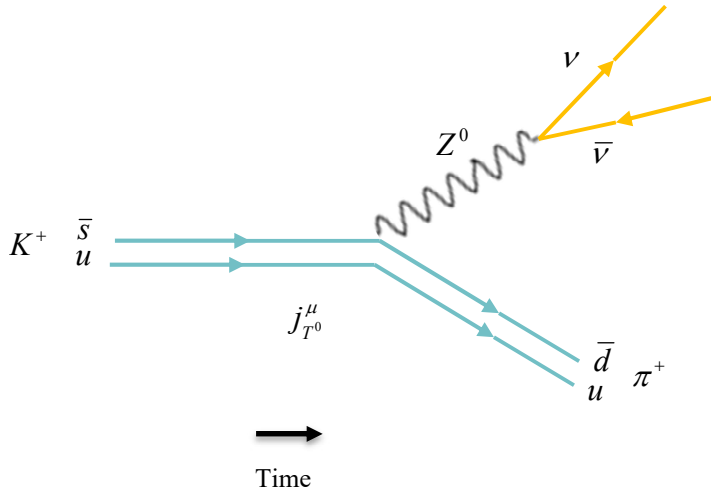


Figure 20: neutral weak current $j_{T^0}^\mu$ interacting with Z^0

Note

On the figure, ν and $\bar{\nu}$ represent several generations of neutrinos and antineutrinos.

A priori, these selection rules were poorly understood. To explain this, in the early 1970s, S. Glashow, J. Illiopoulos and L. Maiani postulated the existence of a 4th quark, the quark charm c , in what is called the GIM mechanism.

VI.3.2 Neutral weak current $j_{T^3}^\mu$ in the hypothesis of a 3-quark model

We remind the neutral weak current $j_{T^3}^\mu$ interacting with the boson W^3 , current used in the Electroweak Model, if we use the original hypothesis of a model with only 3 quarks u, d, s :

$$j_{T^3}^\mu = T^3 \overline{(u \quad -d')}_L \gamma^\mu \begin{pmatrix} u \\ d' \end{pmatrix}_L$$

$$j_{T^3}^\mu = T^3 \overline{(u \quad -d')}_L \gamma^\mu \begin{pmatrix} u \\ d' \end{pmatrix}_L$$

with:

$$T^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$$

T^a are matrices here 2×2 , defined from Pauli matrices $T^a = \frac{\tau^a}{2}$ and generators of the group $SU(2)_T$.

We have according to the Cabibbo matrix, the mixture:

$$d' = d \cos \theta_c + s \sin \theta_c$$

$$\bar{d}' = \bar{d} \cos \theta_c + \bar{s} \sin \theta_c$$

We get:

$$j_{T^3}^\mu = \frac{1}{2} \overline{(u \quad \cos \theta_c d + \sin \theta_c s)}_L \gamma^\mu \begin{pmatrix} u \\ \cos \theta_c d + \sin \theta_c s \end{pmatrix}_L$$

$$j_{T^3}^\mu = \frac{1}{2}(\overline{u_L}\gamma^\mu u_L + \cos^2 \theta_c \overline{d_L}\gamma^\mu d_L + \sin^2 \theta_c \overline{s_L}\gamma^\mu s_L + \cos \theta_c \sin \theta_c (\overline{d_L}\gamma^\mu s_L + \overline{s_L}\gamma^\mu d_L))$$

The 1st term $\overline{u_L}\gamma^\mu u_L + \cos^2 \theta_c \overline{d_L}\gamma^\mu d_L + \sin^2 \theta_c \overline{s_L}\gamma^\mu s_L$ corresponds to $|\Delta St| = 0$.

The 2nd term $\cos \theta_c \sin \theta_c (\overline{d_L}\gamma^\mu s_L + \overline{s_L}\gamma^\mu d_L)$ corresponds to $|\Delta St| = 1$. It is not or very little observed, which remains to be explained.

VI.3.3 Neutral weak current $j_{T^3}^\mu$ in the hypothesis of a 4-quark model

With a charm quark c , we have a new contribution and a neutral weak current of the form:

$$j_{T^3}^\mu = j_{T^3}^\mu(u, d')_L + j_{T^3}^\mu(c, s')_L$$

$$j_{T^3}^\mu = T^3 \overline{(u \quad -d')_L} \gamma^\mu \begin{pmatrix} u \\ d' \end{pmatrix}_L + T^3 \overline{(c \quad -s')_L} \gamma^\mu \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

$$d' = \cos \theta_c d + \sin \theta_c s$$

$$s' = \cos \theta_c s - \sin \theta_c d$$

Note

If we take into account three generations of quarks, we have two new contributions and a neutral weak current of the form:

$$j_{T^3}^\mu = T^3 \overline{(u \quad -d')_L} \gamma^\mu \begin{pmatrix} u \\ d' \end{pmatrix}_L + T^3 \overline{(c \quad -s')_L} \gamma^\mu \begin{pmatrix} c \\ s' \end{pmatrix}_L + T^3 \overline{(t \quad -b')_L} \gamma^\mu \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

We develop the new contribution:

$$j_{T^3}^\mu(c, s')_L = T^3 \overline{(c \quad -s')_L} \gamma^\mu \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

$$j_{T^3}^\mu(c, s')_L = \frac{1}{2} \overline{(c \quad \cos \theta_c s - \sin \theta_c d)_L} \gamma^\mu \begin{pmatrix} c \\ \cos \theta_c s - \sin \theta_c d \end{pmatrix}_L$$

$$j_{T^3}^\mu(c, s')_L = \frac{1}{2} (\overline{c_L}\gamma^\mu c_L + \cos^2 \theta_c \overline{s_L}\gamma^\mu s_L + \sin^2 \theta_c \overline{d_L}\gamma^\mu d_L - \cos \theta_c \sin \theta_c (\overline{d_L}\gamma^\mu s_L + \overline{s_L}\gamma^\mu d_L))$$

By adding $j_{T^3}^\mu(u, d')_L$ and $j_{T^3}^\mu(c, s')_L$, it follows that the terms in $|\Delta St| = 1$ cancel automatically, which explains their much greater rarity. We have:

$$j_{T^3}^\mu = j_{T^3}^\mu(u, d')_L + j_{T^3}^\mu(c, s')_L$$

$$j_{T^3}^\mu = \frac{1}{2} (\overline{u_L}\gamma^\mu u_L + (\cos^2 \theta_c + \sin^2 \theta_c) \overline{d_L}\gamma^\mu d_L + \overline{c_L}\gamma^\mu c_L + (\cos^2 \theta_c + \sin^2 \theta_c) \overline{s_L}\gamma^\mu s_L)$$

$$j_{T^3}^\mu = \frac{1}{2} (\overline{u_L}\gamma^\mu u_L + \overline{d_L}\gamma^\mu d_L + \overline{c_L}\gamma^\mu c_L + \overline{s_L}\gamma^\mu s_L)$$

Note

The Cabibbo angle disappears from the expression of the neutral weak current $j_{T^3}^\mu$

VI.3.4 "Real" weak currents $j_{T^3}^\mu(\overline{d}_L, s_L)$ and $j_{T^3}^\mu(\overline{s}_L, d_L)$

In the expression of $j_{T^3}^\mu$, it can be seen that the terms $\overline{d}_L \gamma^\mu s_L$ and $\overline{s}_L \gamma^\mu d_L$ in $|\Delta St| = 1$ correspond to what we are looking for. That is to say, currents that are called "true" weak currents noted $j_{T^3}^\mu(\overline{d}_L, s_L) = T^3 \overline{d}_L \gamma^\mu s_L$ and $j_{T^3}^\mu(\overline{s}_L, d_L) = T^3 \overline{s}_L \gamma^\mu d_L$, where during an annihilation, the weak charges T^3 cancel each other and generation numbers Ge add up.

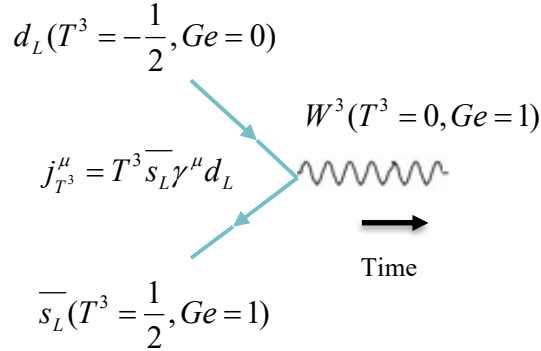


Figure 21: diagram of weak charge annihilation

During a scattering, we have conservation of the weak charge T^3 and change in the number of generations Ge .

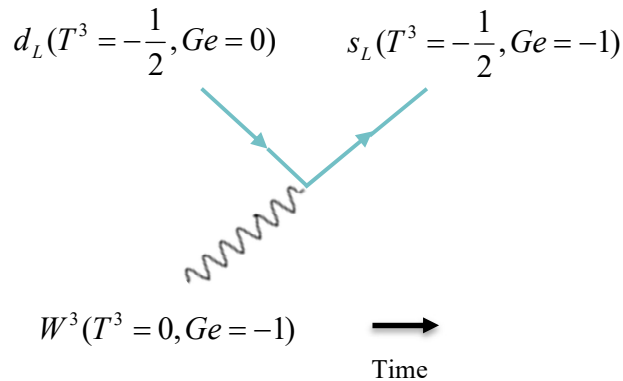


Figure 22: diagram of weak charge scattering

Let us now study the reactions of annihilation and scattering that involve these "real" weak currents. $j_{T^3}^\mu(\overline{d}_L, s_L)$ and $j_{T^3}^\mu(\overline{s}_L, d_L)$.

VI.3.5 Annihilation: weak charge cancellation T^3

Following a strict analogy with electromagnetic interaction, the objective is that during an annihilation reaction, the charge $(T, T^3) = f_T((S, S^3), (I, I^3), Ba)$ cancels and only the source field quantum number Ge , in order to generate a mediating particle.

For $j_{T^3}^\mu(\overline{d}_L, s_L)$, quantum numbers are computed during an annihilation reaction:

	$j_{T^3}^\mu(\overline{d}_L, s_L)$	\overline{d}_L	s_L	Total
Charge	T^3	$\frac{1}{2}$	$-\frac{1}{2}$	0
	Ge	0	-1	-1

Source field quantum numbers	Ba	$\frac{1}{3}$	$-\frac{1}{3}$	0
	I^3	$\frac{1}{2}$	0	$\frac{1}{2}$
	S^3	$-\frac{1}{2}$	$-\frac{1}{2}$	-1

For $j_{T^3}^\mu(\overline{s}_L, d_L)$, quantum numbers are computed during an annihilation reaction:

	$j_{T^3}^\mu(\overline{s}_L, d_L)$	\overline{s}_L	d_L	Total
Charge	T^3	$\frac{1}{2}$	$-\frac{1}{2}$	0
Source field quantum numbers	Ge	1	0	1
	Ba	$-\frac{1}{3}$	$\frac{1}{3}$	0
	I^3	0	$-\frac{1}{2}$	$-\frac{1}{2}$
	S^3	$-\frac{1}{2}$	$-\frac{1}{2}$	-1

Note 1

It can be seen that one cannot reproduce for the "real" weak current a strict analogy with the electromagnetic current. For this, it would be necessary to eliminate I^3 and S^3 in the "Total" column. To see why I^3 and S^3 are not cancelled?

Note 2

When changing the weak charge (T, T^3), we do not always have as expected conservation of Ge , for example when changing the generation of \overline{d}_L and s_L . If we want to follow a strict analogy with electromagnetism, we must when reversing the weak charge T^3 , stay in the same generation of quarks. To see why?

VI.3.6 Scattering: weak charge T^3 retained when changing generations

In the table below, we visualize the passage from a quark u_L to a quark c_L , scattering-type passage, with weak charge conservation (T, T^3) and modification of Ge .

	Source field numbers				Charges		
Conserved	X		X		X	X	X
Modified		X		X			
	(S, S^3)	(I, I^3)	Ba	Ge	$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	$\frac{Y_W}{2} = Q - T^3$	(T, T^3)
u_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
c_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	$+\frac{1}{3}$	+1	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$

It can be seen that we cannot repeat a strict analogy with the scattering of the electromagnetic interaction since:

- the source field quantum number (I, I^3) is modified when we wish it was not,
- the charges Q and Y_W are retained when they would like to be modified.

Note 1

To explain the 1st concern, to see if instead of taking Ge as a source field quantum number transported by the mediating particle, we should not rather take a quantum number function of Ge and (I, I^3) ?

To explain the 2nd concern, to see if instead of taking (T, T^3) as a weak charge conserved during a generation change, it is not necessary to associate the 3 charges instead: Q, Y_W and (T, T^3) in a “real” weak charge?

Note 2

It is observed that the weak charge (T, T^3) is not conserved when reversing the number of strangeness St (or the number of generations Ge). For example, when switching from s to \bar{s} , we reverse the weak charge from $T^3 = -\frac{1}{2}$ to $T^3 = +\frac{1}{2}$.

On the other hand, the weak charge T^3 remains well preserved, when moving from u_L to c_L , or from d_L to s_L , with T^3 which remains respectively equal to $T^3 = +\frac{1}{2}$ and $T^3 = -\frac{1}{2}$.

VI.4 Electrically charged weak currents

VI.5 Electrically charged weak currents $j_{T^+}^\mu$

The Electroweak Model defines a charged weak current $j_{T^+}^\mu$ (carrying a weak charge T^+), interacting with the boson W^+ (carrier of a positive weak charge T^3):

$$j_{T^+}^\mu = \overline{(u \quad c)}_L \gamma^\mu \begin{pmatrix} d' \\ s' \end{pmatrix}_L$$

$$j_{T^+}^\mu = \overline{(u \quad c)}_L \gamma^\mu \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L$$

By developing, we obtain:

$$j_{T^+}^\mu = \cos \theta_c \overline{u}_L \gamma^\mu d_L + \sin \theta_c \overline{u}_L \gamma^\mu s_L - \sin \theta_c \overline{c}_L \gamma^\mu d_L + \cos \theta_c \overline{c}_L \gamma^\mu s_L$$

$$j_{T^+}^\mu = \cos \theta_c (\overline{u}_L \gamma^\mu d_L + \overline{c}_L \gamma^\mu s_L) + \sin \theta_c (\overline{u}_L \gamma^\mu s_L - \overline{c}_L \gamma^\mu d_L)$$

Note 1

Charged weak current $j_{T^+}^\mu$ is given here from the Cabibbo matrix and for 2 generations of quarks.

Note 2

Like the photon γ that carries no electrical charge Q , the mediating particle W^3 does not carry any weak charge T^3 . It is neutral according to the weak interaction. However, unlike photons, bosons W^1, W^2, W^3 can react to each other via terms like $g_T \varepsilon^{123} W^{1\mu} W^{2\nu}$.

Bosons W^+ and W^- carry a respectively positive and negative weak charge T^3 . This weak charge T^3 is electrically charged, hence the electrical charge of W^+ and W^- .

Bosons W^+ and W^- are defined as scale operators that can increase or decrease the weak charge by one unit T^3 of fermions with weak charges T^+ and T^- , fermions with which these weak bosons interact.

For example, we have the interaction term between charged weak currents $j_{T^+}^\mu$ and $j_{T^-}^\mu$ (fermion representatives) and mediating particles W^+ and W^- :

$$E = -ig_T \left[\frac{1}{\sqrt{2}} (j_{T^+}^\mu W_+^\mu + j_{T^-}^\mu W_-^\mu) \right]$$

Note 3

We have for the weak current charged electrically and negatively:

$$j_{T^-}^\mu = \overline{(d' \ s')}_L \gamma^\mu \begin{pmatrix} u \\ c \end{pmatrix}_L$$

Note 4, how does the Electroweak Model define charged weak currents?

We use the Brout-Englert-Higgs mechanism to assign mass to particles. We also use what is called the Yukawa interaction (see the work of S. Weinberg in 1967 and A. Salam to assign mass to fermions).

We remind the Yukawa interaction used in the Electroweak Model between a scalar field φ and a fermion ψ :

$$L_{Yukawa} = -g_l (\bar{\psi}_L \varphi \psi_R + \bar{\psi}_R \varphi^\dagger \psi_L)$$

with g_l an arbitrary coupling constant.

By performing the appropriate matrix transformations, where the eigenstates are those where the quarks have mass, we find for the charged weak currents:

$$j_{T^+}^\mu = \overline{(u \ c)}_L \gamma^\mu \begin{pmatrix} d' \\ s' \end{pmatrix}_L$$

$$(\mu = t, x, y, z)$$

With three generations of quarks, we have:

$$j_{T^+}^\mu = \overline{(u \ c \ t)}_L \gamma^\mu \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L$$

Nota 5

Only left-handed particles and right-handed antiparticles are subject to the weak interaction. We have:

$$\bar{\psi}_L = \overline{(u \ c)}_L = ((\bar{u})_R \ (\bar{c})_R)$$

VI.5.1 "Real" hyper current $j_{Y_W}^\mu(\bar{\mathbf{u}}_L, \mathbf{d}_L)$

It is noted that the term $\bar{u}_L \gamma^\mu d_L$ corresponds to the "real" hyper current $j_{Y_W}^\mu$ that we are looking for, that is, a current where during annihilation, weak hypercharges Y_W cancel each other and isospins I^3 add up.

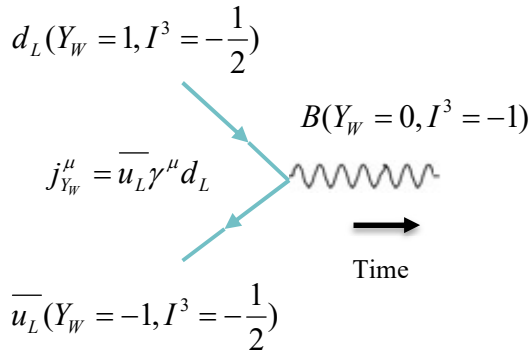


Figure 23: diagram of weak hypercharge annihilation

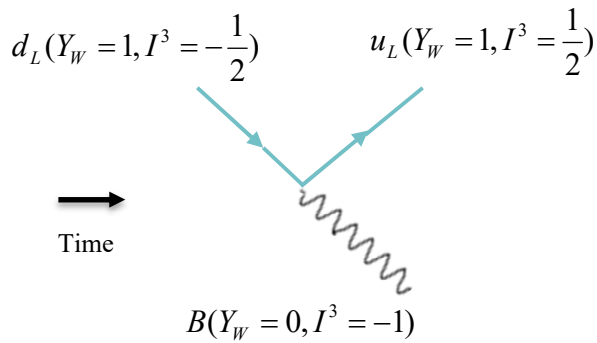


Figure 24: diagram of weak hypercharge scattering

VI.5.2 Annihilation: cancellation of weak hypercharge Y_W

Following a strict analogy with electromagnetism, the objective is that during an annihilation reaction, the charge $Y_W = f_{Co}((S, S^3), Ba, Ge)$ cancels and only the source field quantum number is retained (I, I^3), in order to generate a mediating particle.

For $j_{Y_W}^\mu$, we calculate the quantum numbers during an annihilation reaction:

	$j_{Y_W}^\mu(\bar{u}_L, d_L)$	\bar{u}_L	d_L	Total
Charge	Y_W	-1	1	0
	I^3	$-\frac{1}{2}$	$\frac{1}{2}$	-1
Source field quantum numbers	Ge	0	0	0
	Ba	$-\frac{1}{3}$	$\frac{1}{3}$	0
	S^3	$-\frac{1}{2}$	$\frac{1}{2}$	-1

During an annihilation reaction, it would therefore be necessary a priori to eliminate S^3 in the column "Total".

VI.5.3 Scattering: retained weak hypercharge Y_W when isospin is reversed I^3

In the table below, we visualize the passage from a quark u_L to a quark d_L , with isospin inversion I^3 , conservation of other source field quantum numbers, conservation of weak hypercharge Y_W , modification of charges Q and (T, T^3) .

	Source field numbers	Charges
--	----------------------	---------

Conserved	X	X	X	X	X	X	
Modified		X			X	X	
	(S, S^3)	(I, I^3)	Ba	Ge	$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	$\frac{Y_W}{2} = Q - T^3$	(T, T^3)
u_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, \frac{1}{2})$
d_L	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$+\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$
u_R ↓	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$	0	$+\frac{2}{3}$	$+\frac{4}{3}$	(0,0)
d_R	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	(0,0)

The conservation of weak hypercharge Y_W when passing from a quark d to a quark u (or from a neutron to a proton) only works for left-handed particles. That is, the particles that also participate in the weak interaction.

There is therefore a priori a strong link between hyper interaction, weak interaction, helicity, and spin.

VI.6 Conclusion of the chapter

In this chapter, it has been proposed to find in contributions of neutral weak currents, what is called here the "real" weak current. That is to say, in analogy with the electromagnetic current, a weak current formed by a pair of particles whose respective weak charges T^3 cancel each other out and whose respective generation numbers Ge add up.

Similarly, it has been proposed to find in contributions of charged weak currents, what is called the "real" hyper current. That is to say, in analogy with the electromagnetic current, a hyper current formed by a pair of particles whose respective hypercharges Y_W cancel each other out and whose respective isospin numbers I^3 add up.

In the next chapter, we will study whether what has just been proposed for quarks on electromagnetic currents, weak currents, and hyper currents, is also true for leptons, knowing that the latter are also subject to the electromagnetic, weak and hyper interactions.

Chapter VII Weak currents and hyper currents, angle of Pontecorvo, case of leptons

Objective of the chapter

Leptons are essentially distinguished from baryons by the fact that they do not participate in the strong interaction (zero strong charge for leptons).

We will first focus here on the oscillation of neutrinos, a phenomenon according to which a neutrino with a leptonic flavor (electron, muon, or tau neutrino) can be measured later with a different flavor. We will then study the Pontecorvo-Maki-Nakagawa-Sakata matrix, a matrix analogous for leptons to the Cabibbo-Kobayashi-Maskawa matrix for quarks, and which describes neutrino oscillations.

For the weak interaction and the hyper interaction, it should be emphasized that the similarities are qualitatively very large between baryons and leptons. This will make it possible to offer the same type of weak and hyper currents.

VII.1 Mass and oscillation of the neutrinos

Around 1957, B. Pontecorvo proposed the possibility of a low but not zero mass of neutrinos. He points out that there is no requirement that the eigenstates of neutrino flavors (i.e., the eigenstates of reactions involving bosons W^a), be equal to the eigenstates of masses (or energies). These eigenstates of flavors would rather be a linear combination of the eigenstates of masses.

He was inspired by the oscillation model of neutral kaons (de K^0 à \bar{K}^0) and proposed a model of neutrino oscillation, moving from one flavor to another.

VII.2 Matrix Pontecorvo-Maki-Nakagawa-Sakata (matrix PMNS)

VII.2.1 2-dimensional matrix and angle of Pontecorvo

In 1962, to describe the neutrino oscillations proposed by B. Pontecorvo, Z. Maki, M. Nakagawa et S. Sakata introduced the matrix of Pontecorvo-Maki-Nakagawa-Sakata (PMNS matrix), the analogue for leptons of the matrix of Cabibbo-Kobayashi-Maskawa (CKM matrix) for quarks.

Like the CKM matrix, the PMNS matrix can be interpreted as the ease (or as the probability) of a neutrino changing flavor.

We remind the Cabibbo matrix:

$$\begin{pmatrix} d_L' \\ s_L' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}$$

We have an analogue matrix (in two dimensions) of the Cabibbo matrix for neutrinos of 1st and 2nd generation:

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \end{pmatrix} = \begin{pmatrix} \cos \theta_p & \sin \theta_p \\ -\sin \theta_p & \cos \theta_p \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

with θ_p angle of Pontecorvo

The vector on the left $\begin{pmatrix} \nu_e' \\ \nu_\mu' \end{pmatrix}$ is the eigenstate of flavors, i.e., the eigenstate of reactions involving bosons W^a .

The vector on the right $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$ is the eigenstate of masses or energies.

VII.2.2 3-dimensional matrix

In 1962, the matrix introduced by Z. Maki, M. Nakagawa and S. Sakata involved only two generations of neutrinos. ν_e and ν_μ .

In 1973, M. Kobayashi and T. Maskawa predicted a 3rd generation of quarks and leptons, necessary to explain the CP violation. We also have a 3rd generation of electrons with τ and a 3rd generation of neutrinos with ν_τ .

We obtain the extension of the PMNS matrix of Pontecorvo-Maki-Nakagawa-Sakata with 3 generations of neutrinos:

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix} = M_{PMNS} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

As with the CKM matrix, there are several representations of the PMNS matrix. We use the same representation as for the CKM matrix.

We define 3 Euler angles $\theta_{12} = \theta_p$, θ_{13} , θ_{23} and a phase δ .

$$c_{ij} = \cos \theta_{ij}$$

$$s_{ij} = \sin \theta_{ij}$$

$$M_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

For example, the matrix breaks down as follows:

$$M_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is a combination of three rotation matrices, except the presence of the phase δ that reflects the violation of CP symmetry.

Experimentally, we measure (with values that vary significantly according to the experiments):

$$\theta_{12} = \theta_p \approx 33,62^\circ, \theta_{13} \approx 8,54^\circ, \theta_{23} \approx 47,2^\circ \text{ and a phase } \delta \approx 234^\circ$$

$$M_{PMNS} \approx \begin{pmatrix} 0.82 & 0.54 & -0.15 \\ -0.35 & 0.70 & 0.62 \\ 0.44 & -0.55 & 0.70 \end{pmatrix}$$

We had for the CKM matrix:

$$\theta_{12} = \theta_c \approx 13,04^\circ, \theta_{13} \approx 0,20^\circ, \theta_{23} \approx 2,38^\circ \text{ and a phase } \delta \approx 1,20rad$$

$$M_{CKM} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ -0.22 & 0.97 & 0.04 \\ 0.008 & -0.04 & 0.99 \end{pmatrix}$$

VII.3 Leptons e^- , ν_e , μ^- , ν_μ of 1st and 2nd generation

For leptons of 1st and 2nd generation, the following table is proposed in analogy with quarks:

	Source field quantum numbers				Charges		
	$(S, S^3)H$	(I, I^3)	Ba	Ge	Electric charge Q $= I^3 - \frac{Le}{2}$ $+ \frac{Ge}{2}$	Weak hypercharge $\frac{Y_W}{2} = Q - T^3$	Weak charge (T, T^3)
e_L^-	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0	-1	-1	$(\frac{1}{2}, -\frac{1}{2})$
ν_{eL}	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, +\frac{1}{2})$	+1	0	0	-1	$(\frac{1}{2}, +\frac{1}{2})$
μ_L^-	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	-1	-1	-1	$(\frac{1}{2}, -\frac{1}{2})$
$\nu_{\mu L}$	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	+1	+1	0	-1	$(\frac{1}{2}, +\frac{1}{2})$
e_R^-	$(\frac{1}{2}, +\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	+1	0	-1	-2	(0,0)
ν_{eR} not observed	$(\frac{1}{2}, +\frac{1}{2})$	$(\frac{1}{2}, +\frac{1}{2})$	+1	0	0	0	(0,0)
μ_R^-	$(\frac{1}{2}, +\frac{1}{2})$	(0,0)	+1	-1	-1	-2	(0,0)
$\nu_{\mu R}$ not observed	$(\frac{1}{2}, +\frac{1}{2})$	(0,0)	+1	+1	0	0	(0,0)

Note 1

Usually, we do not assign isospin (I, I^3) and Ge to leptons. Indeed, (I, I^3) and Ge are considered like the strong charge in the Quark Model. However, leptons are not subject to strong interaction and therefore have a zero strong charge.

In this Memoir, it is suggested to distinguish between strong charge Co , isospin (I, I^3) and Ge (by proposing nevertheless that the first quantum number of type charge is a function of the next two of type source field quantum number). It is then decided to assign an isospin (I, I^3) and a Ge to leptons in analogy with quarks. This verifies the relationship for leptons:

$$Q = I^3 - \frac{Le}{2} + \frac{Ge}{2}$$

Here, (I, I^3) and Ge are not treated in analogy with a charge X, but in analogy with the spin (S, S^3) , i.e., a source field quantum number which, being modified, generates a X charging current.

Note 2

Several reactions between particles indicate that quarks of 1st generation u and d , and leptons of 1st generation e^- et ν_e have a priori the same Ge .

For example, we have the decay reaction of a pion $\pi^0(u\bar{u}/d\bar{d})$ in an electron, a positron and a photon:

$$\pi^0(u\bar{u}/d\bar{d}) \rightarrow \gamma + e^- + \bar{e}^+$$

For the Ge to be identical on both sides, one of the same values is needed for quarks and leptons of 1st generation Ge .

Note 3 on hyper interaction, at once attractive, repulsive and infinite scope

According to S. Glashow's hypotheses, left-handed hadrons (particles of the nucleus) have a positive weak hypercharge Y_W and left-handed leptons have a negative weak hypercharge Y_W . Identical sign for weak hypercharge, left-handed hadrons repel each other. Similarly, of identical sign for weak hypercharge, left-handed leptons repel each other. Opposite sign for weak hypercharge, left-handed leptons and left-handed hadrons attract each other. For example, a left-handed neutron of weak hypercharge $Y_W = 1$ and a left-handed electron of hypercharge $Y_W = -1$ attract each other. Note that these attractions and repulsions have an infinite range, since the boson B carrying the hyper interaction is a priori presented without mass in the Electroweak Model.

Note that the case of right-handed particles is more complex, because according to the Electroweak Model some right-handed particles do not have Y_W and therefore do not participate in the hyper interaction.

VII.4 Electrically neutral weak currents

VII.4.1 Neutral weak current from PMNS matrix

With two generations of left-handed leptons, we have a neutral weak current of the form:

$$j_{T^3}^\mu = j_{T^3}^\mu(e^-, \nu_{e'})_L + j_{T^3}^\mu(\mu, \nu_{\mu'})_L$$

$$j_{T^3}^\mu = T^3 \overline{(e^- \quad -\nu_{e'})_L} \gamma^\mu \begin{pmatrix} e^- \\ \nu_{e'} \end{pmatrix}_L + T^3 \overline{(\mu \quad -\nu_{\mu'})_L} \gamma^\mu \begin{pmatrix} \mu \\ \nu_{\mu'} \end{pmatrix}_L$$

We have:

$$j_{T^3}^\mu(e^-, \nu_{e'})_L = \frac{1}{2} (\overline{e}_L \gamma^\mu e_L + \cos^2 \theta_p \overline{\nu_{eL}} \gamma^\mu \nu_{eL} + \sin^2 \theta_p \overline{\nu_{\mu L}} \gamma^\mu \nu_{\mu L} + \cos \theta_p \sin \theta_p (\overline{\nu_{eL}} \gamma^\mu \nu_{\mu L} + \overline{\nu_{\mu L}} \gamma^\mu \nu_{eL}))$$

The 1st term $\overline{e}_L \gamma^\mu e_L + \cos^2 \theta_p \overline{\nu_{eL}} \gamma^\mu \nu_{eL} + \sin^2 \theta_p \overline{\nu_{\mu L}} \gamma^\mu \nu_{\mu L}$ corresponds to $|\Delta St| = 0$.

The 2nd term $\cos \theta_p \sin \theta_p (\overline{\nu_{eL}} \gamma^\mu \nu_{\mu L} + \overline{\nu_{\mu L}} \gamma^\mu \nu_{eL})$ corresponds to $|\Delta St| = 1$.

VII.4.2 "Real" weak current $j_{T^3}^\mu(\overline{\nu_{eL}}, \nu_{\mu L})$

It is noted that the terms $\overline{\nu_{eL}} \gamma^\mu \nu_{\mu L}$ and $\overline{\nu_{\mu L}} \gamma^\mu \nu_{eL}$ in $|\Delta St| = 1$ correspond to what we are looking for, "real" weak currents noted $j_{T^3}^\mu(\overline{\nu_{eL}}, \nu_{\mu L})$ and $j_{T^3}^\mu(\overline{\nu_{\mu L}}, \nu_{eL})$, where during an annihilation, the weak charges T^3 cancel each other and generation numbers Ge add up.

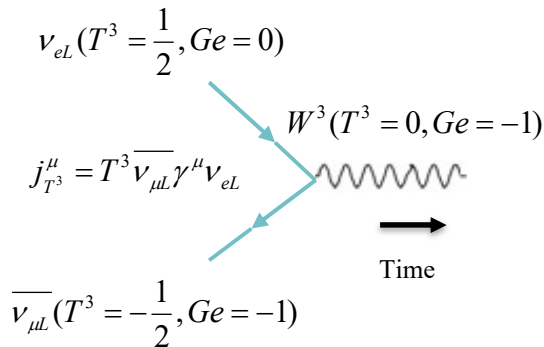


Figure 25: diagram of annihilation weak charge

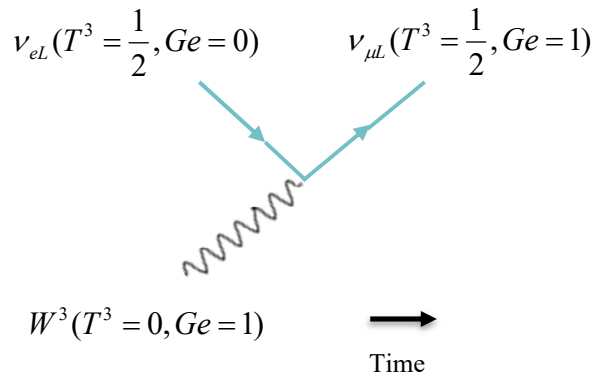


Figure 26: diagram of scattering weak charge

VII.4.3 Annihilation: cancellation of charge T^3

For $j_{T^3}^\mu(\overline{\nu_{eL}}, \nu_{\mu L})$, quantum numbers are computed during an annihilation reaction:

	$j_{T^3}^\mu(\overline{\nu_{eL}}, \nu_{\mu L})$	$\overline{\nu_{eL}}$	$\nu_{\mu L}$	Total
Charge	T^3	$-\frac{1}{2}$	$\frac{1}{2}$	0
Source field quantum numbers	Ge	0	1	1
	Le	-1	1	0
	I^3	$-\frac{1}{2}$	0	$-\frac{1}{2}$
	S^3	$-\frac{1}{2}$	$-\frac{1}{2}$	-1

Nota 1

As for quarks, during a scattering with change of Ge , the 3 charges $Q, Y_W, (T, T^3)$ are retained. To see why, since we wish that simply (T, T^3) is retained?

Nota 2

See also why I^3 and S^3 are not cancelled as expected?

VII.4.4 Scattering: conservation of weak charge T^3 when changing generations

In the table below, we visualize the passage from e_L to μ_L , scattering-type passage, with weak charge conservation (T, T^3) and modification of Ge .

	Source field quantum numbers				Charges		
Conserved	X		X		X	X	X
Modified		X		X			

	(S, S^3)	(I, I^3)	L_e	Ge	$Q = I^3 - \frac{Le}{2} + \frac{Ge}{2}$	$\frac{Y_W}{2} = Q - T^3$	(T, T^3)
e_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	1	0	-1	-1	$(\frac{1}{2}, -\frac{1}{2})$
μ_L	$(\frac{1}{2}, -\frac{1}{2})$	(0,0)	1	-1	-1	-1	$(\frac{1}{2}, \frac{1}{2})$

VII.5 Electrically charged weak currents

VII.5.1 Electrically charged weak currents $j_{T^-}^\mu$

The Electroweak Model determines a charged weak current $j_{T^-}^\mu$ carrying the weak charge T^- interacting with the boson W^- :

$$j_{T^-}^\mu = \overline{(e \ \mu)_L} \gamma^\mu \begin{pmatrix} \nu_{e'} \\ \nu_{\mu'} \end{pmatrix}_L$$

$$j_{T^-}^\mu = \overline{(e \ \mu)_L} \gamma^\mu \begin{pmatrix} \cos \theta_p & \sin \theta_p \\ -\sin \theta_p & \cos \theta_p \end{pmatrix} \begin{pmatrix} \nu_{e'} \\ \nu_{\mu'} \end{pmatrix}_L$$

By developing, we obtain:

$$j_{T^-}^\mu = (\cos \theta_p \overline{e}_L \gamma^\mu \nu_{e_L} + \sin \theta_p \overline{e}_L \gamma^\mu \nu_{\mu_L} - \sin \theta_p \overline{\mu}_L \gamma^\mu \nu_{e_L} + \cos \theta_p \overline{\mu}_L \gamma^\mu \nu_{\mu_L})$$

$$j_{T^-}^\mu = \cos \theta_p (\overline{e}_L \gamma^\mu \nu_{e_L} + \overline{\mu}_L \gamma^\mu \nu_{\mu_L}) + \sin \theta_p (\overline{e}_L \gamma^\mu \nu_{\mu_L} - \overline{\mu}_L \gamma^\mu \nu_{e_L})$$

Nota 1

The current is given from the PMNS matrix and for 2 generations of leptons.

Nota 2

We have the charged electrically and positively weak current:

$$j_{T^+}^\mu = \overline{(\nu_{e'} \ \nu_{\mu'})_L} \gamma^\mu \begin{pmatrix} e \\ \mu \end{pmatrix}_L$$

VII.5.2 Hyper currents $j_{Y_W}^\mu(\overline{e}_L, \nu_{e_L})$ and $j_{Y_W}^\mu(e_L, \overline{\nu}_{e_L})$

It is noted that the term $\overline{e}_L \gamma^\mu \nu_{e_L}$ corresponds to the hyper current $j_{Y_W}^\mu$ that we are looking for, that is to say a current where during an annihilation, the weak hypercharges Y_W cancel each other and isospins I^3 add up.

Feynman diagrams are given for the current $\overline{\nu}_{e_L} \gamma^\mu e_L$.

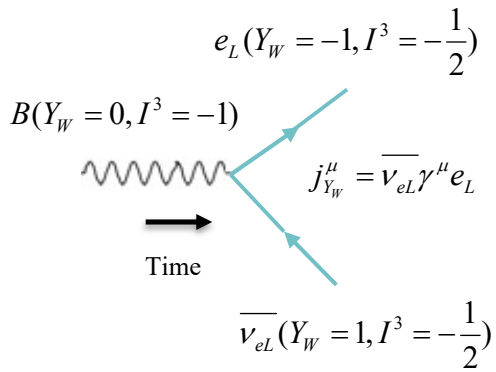


Figure 27: diagram of weak hypercharge creation

Note

Note the apparent instability of the boson B , which during a beta decay, interacts quickly with a couple electron e_L , antineutrino $\bar{\nu}_{eL}$.

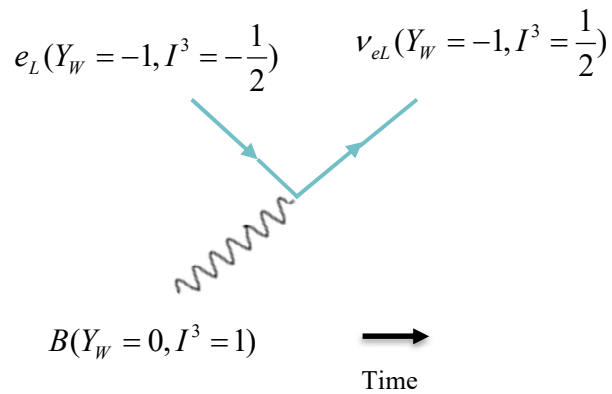


Figure 28: diagram of weak hypercharge scattering

As with quarks, this only works for left-handed leptons, with the weak hypercharge Y_W which remains constant when isospin I^3 is reversed. For right-handed leptons, the neutrino ν_{eR} is not observed.

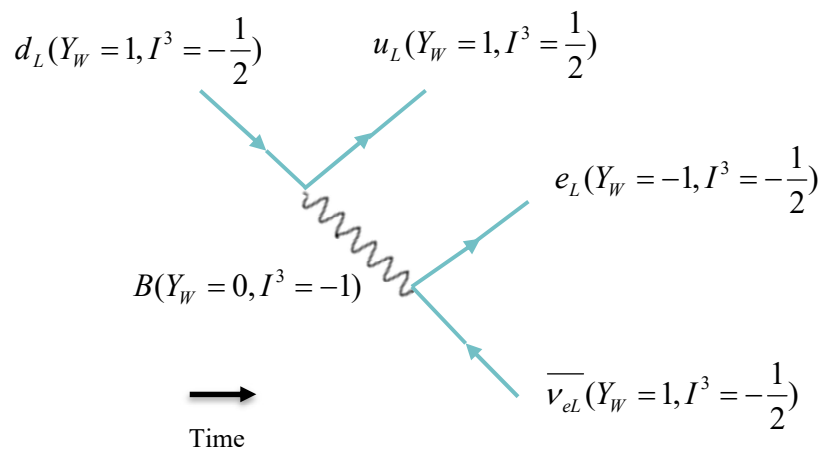


Figure 29: diagram of scattering for the quarks and creation for the leptons

VII.5.3 Scattering: weak hypercharge Y_W conserved when isospin is reversed I^3

In the table below, we visualize the passage from e_L to ν_{eL} , with isospin inversion I^3 , conservation of other source field quantum numbers, conservation of weak hypercharge Y_W , modification of charges Q and (T, T^3) .

	Source field numbers				Charges		
Conserved	X		X	X		X	
Modified		X			X		X
	(S, S^3)	(I, I^3)	L_e	Ge	$Q = I^3 - \frac{Le}{2} + \frac{Ge}{2}$	$\frac{Y_W}{2} = Q - T^3$	(T, T^3)
e_L ↓	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	1	0	-1	-1	$(\frac{1}{2}, -\frac{1}{2})$
ν_{eL}	$(\frac{1}{2}, -\frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	1	0	0	-1	$(\frac{1}{2}, \frac{1}{2})$

VII.5.4 Annihilation: cancellation of weak hypercharge Y_W

For $j_{Y_W}^\mu(\bar{e}_L, \nu_{eL})$, quantum numbers are computed during an annihilation reaction:

	$j_{Y_W}^\mu(\bar{e}_L, \nu_{eL})$	\bar{e}_L	ν_{eL}	Total
Charge	Y_W	1	-1	0
Source field quantum numbers	I^3	$\frac{1}{2}$	$\frac{1}{2}$	1
	Ge	0	0	0
	L_e	-1	1	0
	S^3	$-\frac{1}{2}$	$-\frac{1}{2}$	-1

VII.6 Conclusion

For weak and hyper interactions, the conclusion is qualitatively identical for quarks and leptons.

Neutral weak currents contain what is called here the "real" weak current, that is, in analogy with the electromagnetic current, a weak current formed by a pair of leptons whose respective weak charges T^3 cancel each other out and whose respective generation numbers Ge add up.

Charged weak currents contain what is called the "real" hyper current, that is, in analogy with the electromagnetic current, a hyper current formed by a pair of leptons whose respective weak hypercharges Y_W cancel each other out and whose respective isospin numbers I^3 add up.

In the next chapter, we will focus on the strong interaction and the particles of the nucleus formed of quarks, the hadrons, the only particles subject to this interaction.

Chapter VIII Proposal of a model for strong interaction in analogy with electromagnetic interaction

Purpose of the chapter

We first recall some generalities on two properties of the strong interaction: color charge confinement and asymptotic freedom. Theory on these 2 properties were developed in the early 1970s as part of Quantum Chromodynamics.

In strict analogy with the electromagnetic, weak and hyper interactions developed in the previous chapters, we then propose a model for the strong interaction based on a strong charge of color with mediating particles not charged with color, carrying a baryonic number, interacting with strong charged particles, modifying their baryonic number and generating a strong current J_{Co}^μ .

VIII.1 General on color charge confinement and asymptotic freedom

VIII.1.1 Limited range of a force explained by the notion of charge confinement

Charge confinement, it is the idea that opposing charges attract each other and that these charges attract each other so much that beyond a certain scale it is impossible to observe them separately. We then obtain, beyond this scale, zero, neutral or white charges to use the terminology of Quantum Chromodynamics. The resulting interaction (even if it has an infinite scope), no longer has any effect.

VIII.1.2 Limited range of a force explained by the rapid decrease of the potential

Both electrostatic force and strong interaction have charges that take opposite values and attract each other. The attractiveness of opposite charges of the electrostatic force and the strong interaction (following the ideas of H. Yukawa) is described respectively by:

- electric potential of Coulomb

$$qA^t(r) = \frac{-g_Q^2}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}$$

- strong potential of Yukawa

$$\psi(r) = -g_{Co}^2 \frac{e^{-\left(\frac{m_{mes}c}{\hbar}r\right)}}{r}$$

$g_Q^2 = e^2$ and g_{Co}^2 are the coupling constants of respectively the electrostatic force and the strong interaction, g_Q and g_{Co} are the elementary electric charges and elementary strong charges.

m_{mes} is the mass of the mesotron, the mass mediating particle proposed by H. Yukawa.

The Coulomb potential decreases in $\frac{1}{r}$ up to infinity, the range of the electrostatic force is infinite.

The Yukawa potential decreases much faster because of the term in $e^{-\left(\frac{m_{mes}c}{\hbar}r\right)}$, due to the mass of the mediating particle. This helps explain the small range of nuclear interactions that no longer act beyond the atomic nucleus.

VIII.1.3 Strong color charges confinement

There is an idea of confinement for both electrical charges and strong colored charges. For example, on the astronomical scale, when studying the motion of the planets, the electrostatic force no longer has any effect. The opposite electric charges compensate for each other. Only the gravitational force intervenes.

In the theory of quantum chromodynamics based on $SU(3)_{color}$, the scale of confinement of strong color charges is much smaller than that of electrostatic force. At the scale of nucleons or free mesons, the confinement of color charges already exists. It is impossible to observe free nucleons or free

mesons with charges of opposite colors (as on the astronomical scale, it is impossible to observe the effects of opposite electric charges). Free nucleons or mesons are said to have a white charge. The 3 colors R, G, B neutralize each other.

You have to go to a smaller scale, that of quarks, to find particles that carry a color charge and that manifest the effects of the strong interaction.

In quantum chromodynamics, the low scale of confinement of color charges is explained by variations in the coupling constant α_{Co} . This coupling constant α_{Co} is no longer a constant, it increases with distance (see later, in the paragraph Energy dependence of coupling constants).

Note 1

The scale of confinement is not a priori the same for $SU(2)_I$ and $SU(3)_{color}$. Indeed, in $SU(2)_I$ nucleons have a non-zero strong charge equal to (I, I^3) . In $SU(3)_{color}$, nucleons have a strong charge of zero or white color.

It is the same for pion mesons that have a strong charge equal to (I, I^3) in $SU(2)_I$ and a charge of white color in $SU(3)_{color}$.

Note 2 on the mass of gluons

Contrary to the ideas of H. Yukawa, in quantum chromodynamics, gluons G have zero mass. The small range of nuclear interaction between hadrons is explained solely by the confinement of color charges.

In the model that we will propose, we will return to the original idea of H. Yukawa. Gluons G will be attributed a quantum number, the baryonic number, containing a mass (or mass energy).

Note 2 on the color charge Co of the gluons

Unlike photons that carry no electric charge, the gluons G of quantum chromodynamics carry a color charge and can therefore change the color charge of the particle.

The gluons G are in this analogous to pions π^+ and π^- or to weak bosons W^+ and W^- , which are defined as scale operators, can increase or decrease by T^3 the weak charge of a fermion, itself carrying a weak charge T^\pm .

In the following, in analogy with γ and W^3 that do not carry respectively an electrical charge and a weak charge, we will define gluons G that do not carry a strong charge of color Co , and therefore cannot modify the strong charge of the fermion.

Like the photon γ who carry a spin S , and which can therefore modify the spin of a fermion, like the boson W^3 who carry a Ge and which can modify the generation of a quark, we will assign to the gluon a baryonic number Ba allowing it to modify the baryonic number of the fermion with which it interacts.

Note that we can have 2 types of gluons G (those carrying a strong charge and those not carrying one), as we can have the 2 types of bosons W (those carrying a weak charge and those not carrying a weak charge). What interests us here are photon γ like mediating particles, that is, those that do not carry strong charges but a source field quantum number.

VIII.1.4 Limit to attractiveness between 2 charges

In physics, when two charges (or masses) attract, it is often found that these two (usually opposite) charges attract each other only up to a certain stage. Beyond this stage, a compensatory phenomenon comes into action, which prevents the two charges from crashing against each other.

This idea is found in the Bohr electron model. The compensatory phenomenon to prevent the electron of electric charge $-$ and the proton of electric charge $+$ from attracting each other until they crash against each other, is the rotational movement of the electron around the proton. This movement is

interpreted as a centrifugal or repulsive inertia force, which compensates for the attractive electrostatic force, and "stabilizes" the electron on its orbit.

This idea is also found in Newton's theory of gravitation. The phenomenon compensating for gravitational attraction is the same as in the Bohr electron model. It is a rotational movement, for example that of the Moon around the Earth, which is interpreted as a repulsive force of inertia.

VIII.1.5 Asymptotic freedom of quarks

According to Quantum Chromodynamics, the compensatory phenomenon that prevents two quarks of opposite color charges from crashing against each other is asymptotic freedom.

To explain this asymptotic freedom, quantum chromodynamics proposes a coupling constant of the strong interaction α_{Co} , which decreases when distances become very small. Thus, quarks of opposite color charges no longer attract each other when the distances separating them become very small and that α_{Co} tends towards 0. At the limit, the strong interaction acting between them is so low, that quarks behave almost like free particles.

Why and how does this coupling constant α_{Co} vary? Let us now briefly study what Quantum Chromodynamics proposes.

VIII.2 Energy dependence of coupling constants

VIII.2.1 Case of Relativistic Quantum Electrodynamics

In relativistic quantum electrodynamics, to take into account the effects of the quantum vacuum, we assume a possible variation of the coupling constant as a function of the energy momentum quadrivector (here denoted q) of the virtual photon involved in the exchange mechanism. We call effective coupling constant, this constant $\alpha_Q(q^2)$ function of q^2 . We have:

$$\alpha_Q(q^2) \approx \frac{\alpha_Q}{1 - \frac{\alpha_Q}{3\pi} \ln\left(\frac{q^2}{4m_e^2}\right)}$$

We have at the limit:

$$\alpha_Q\left(\frac{q^2}{4m_e^2} \rightarrow 1\right) = \alpha_Q$$

with α_Q the electromagnetic coupling constant.

We note that the effective coupling constant $\alpha_Q(q^2)$ increases when q^2 increases.

VIII.2.2 Case of Quantum Chromodynamics

In quantum chromodynamics, we construct a coupling constant of the same kind:

$$\alpha_{Co}(q^2) \approx \frac{\alpha_{Co}}{1 + B\alpha_{Co} \ln\left(\frac{q^2}{q_0^2}\right)}$$

With $B = \frac{11N_C - 2N_S}{12}$,

N_C : 3 colors of quarks,

N_S : number of quark flavors involved,

q_0 reference value of q .

We have at the limit:

$$\alpha_{Co}\left(\frac{q^2}{q_0^2} \rightarrow 1\right) = \alpha_{Co}$$

We note that the effective coupling constant $\alpha_{Co}(q^2)$ decreases when q^2 increases.

Let's list the salient features of $\alpha_{Co}(q^2)$ according to Quantum chromodynamics.

To the low momentums energies $q \leq 200MeV$, i.e., at a large relative distance from quarks $r = \frac{\hbar c}{q} \geq 1 fm$, $\alpha_{Co} \geq 1$. The coupling constant α_{Co} increases rapidly with remoteness. The potential of the strong interaction becomes approximately linear like a spring: $V(r) \approx \lambda r$ with λ a constant of the order of $1GeV \times fm^{-1}$. This reflects the observed confinement effect of quarks.

To the great momentums energies $q \gg 1GeV$, i.e., at low relative distances $r \ll 1 fm$, we have $\alpha_{Co} \ll 1$. This makes quarks appear quasi-free. At the limit, we have $\alpha_{Co}(q^2 \rightarrow \infty) \rightarrow 0$, this is commonly referred to as asymptotic freedom.

VIII.3 Hypotheses for a model of strong interaction with uncharged mediating particles

After briefly mentioning some properties of quantum chromodynamics, we propose here a simple model for the strong interaction which is mainly inspired by $SU(2)_I$ (i.e., the Yang-Mills gauge theory on the strong interaction). Despite the great interest of coupling constants, they will therefore be left aside in the rest of this Memoir.

This model is in search of analogies with electromagnetic, hyper, and weak interactions. In particular, we will propose the existence of gluon-type mediating particles G which are non-carrier of strong charge and carry a quantum number source field, the baryonic number Ba .

VIII.3.1 Hypotheses on the strong charge and the group of rotations $SU(2)_{Co}$

Quantum chromodynamics is based on the group of rotations $SU(3)_{Color}$.

For simplicity and analogy with:

- the Yang-Mills theory is based on $SU(2)_I$ with strong charge (I, I^3) ,
- the Electroweak Model which is based on $SU(2)_T$ with weak charge (T, T^3) ,

we propose a model for strong interaction that is based on $SU(2)_{Co}$ with strong color charge (Co, Co^3) .

We propose a color charge (Co, Co^3) , with for strong charged fermions values of the type:

$$(Co, Co^3) = \left(\frac{1}{2}, \pm \frac{1}{2}\right)$$

Note 1 on the number of dimensions of the rotation group

If we accept the hypothesis of relating source field equations and interaction, there is no reason to differentiate strong and weak interaction by the number of dimensions of the associated rotation group. We choose here for simplicity to have strong and weak interactions based on $SU(2)$. But after all, these short-range nuclear interactions can be based on $SU(3)$, or on $SU(4)$, $SU(5)$...

Note 2 on the color charge of leptons

Leptons must have $(Co, Co^3) = (0,0)$, since they are not subject to strong interaction.

VIII.3.2 Hypotheses on the particles mediating the interaction and on the transported source field quantum number

Like the photon γ which is not electrically charged and carries a source field quantum number: the spin S , we define for the strong interaction of mediating particles, the gluons G , strong uncharged and carrying the source field quantum number Ba .

Note on the baryonic number

We will also take up the hypothesis proposed above that the quarks and quarks composing pentaquarks are in fact excited states of nucleons vis-à-vis the baryonic number, following the strong interaction. We have for nucleons $\frac{1}{Ba} = 1$, for quarks $\frac{1}{Ba} = 3$ and for quarks that are components of pentaquarks $\frac{1}{Ba} = 5$.

VIII.3.3 Hypotheses on the strong color charge Co

Inspired by the Gell-Mann Nishijima relation, it is suggested that the strong color charge is constructed from the 3 source field quantum numbers f , other than the one carried by the mediating particle.

Thus, it is assumed that the mediating particle, the gluon G carries a baryonic number Ba , and that the color charge Co is built from S , I and Ge .

We have a function f_{Co} with:

$$(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)$$

Note

In Yang-Mills theory, the strong charge is identified with isospin (I, I^3) and is generator of $SU(2)_I$. In the original Quark Model, the strong charge is identified with flavors u , d , s and is generator of $SU(3)_{flavor}$.

We find in u and d the isospin (I, I^3) and in s strangeness St , that is Ge .

To be verified experimentally, if for the strong interaction, like the weak interaction, there is not also a violation of parity, that is to say a different strong charge depending on the helicity of the particle? We would then have a strong charge also function of the spin (S, S^3) .

VIII.3.4 Hypotheses on strong currents

On the model of electromagnetic current j_Q^μ that carries an electrical charge Q , of weak current $j_{T^a}^\mu$ that carries a weak charge T^a or hyper current $j_{Y_W}^\mu$ that carries a weak hypercharge Y_W , we define a strong current $j_{Co^a}^\mu$ that carries a strong charge Co^a .

To explain this current, we take up Bohr's parable. The exchange of a source field quantum number between a fermion and a mediating particle carrying this quantum number, generates a move of the charge X (electric, hyper, weak or strong charge) and therefore a current.

In the other sense, a current or a charge X in motion radiates a mediating particle wave carrying the source field quantum number.

VIII.4 Application of the model to nucleons and quarks of 1st generation

VIII.4.1 Starting from the Yang-Mills theory for the strong interaction

It was proposed that:

$$(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)$$

If we take the Yangs-Mills theory for the strong interaction, in the case of nucleons and quarks of 1st generation, we can identify the strong charge Co^3 to isospin I^3 :

$$Co^3 = I^3$$

VIII.4.2 Scattering: strong charge (Co, Co^3) conserved when modifying Ba

During the passage from a proton p to a quark u , We have the modification of the baryonic number Ba and the conservation of the strong charge identified with isospin:

$$Co^3 = I^3 = \frac{1}{2}$$

	Source field numbers				Charges			
	Conserved	X	X	X	X	X	X	X
Modified			X		X	X		
	(S, S^3)	(I, I^3)	$\frac{1}{Ba}$	Ge	electric $Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	hyper $\frac{Y_W}{2} = Q - T^3$	weak (T, T^3)	strong (Co, Co^3)
$p \downarrow$	$(\frac{1}{2}, \pm \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	1	0	+1	+1 +2	$(\frac{1}{2}, \frac{1}{2})$ (0,0)	$(\frac{1}{2}, \frac{1}{2})$
u	$(\frac{1}{2}, \pm \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	3	0	$+\frac{2}{3}$	$+\frac{1}{3}$ $+\frac{4}{3}$	$(\frac{1}{2}, \frac{1}{2})$ (0,0)	$(\frac{1}{2}, \frac{1}{2})$

Note

La weak charge (T, T^3) is a priori preserved when we go from p to u whereas in the general case we would like that this weak charge is modified. To see why?

For left-handed nucleons and quarks of 1st generation, both the strong charge and the weak charge can be identified with isospin.

During the passage from a neutron n to a quark d , we have a change in the baryonic number Ba and conservation of the strong charge equal to isospin: $Co^3 = I^3 = -\frac{1}{2}$.

	Source field numbers				Charges			
	Conserved	X	X	X	X?	X?	X?	X

Modified			X		X	X		
	(S, S^3)	(I, I^3)	$\frac{1}{Ba}$	Ge	electric $Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$	hyper $\frac{Y_W}{2} = Q - T^3$	weak (T, T^3)	strong (Co, Co^3)
$n \downarrow$	$(\frac{1}{2}, \pm \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	1	0	0	+1 0	$(\frac{1}{2}, -\frac{1}{2})$ (0,0)	$(\frac{1}{2}, -\frac{1}{2})$
d	$(\frac{1}{2}, \pm \frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2})$	3	0	$-\frac{1}{3}$	$+\frac{1}{3}$ $-\frac{2}{3}$	$(\frac{1}{2}, -\frac{1}{2})$ (0,0)	$(\frac{1}{2}, -\frac{1}{2})$

We have Feynman diagrams of the scattering type:

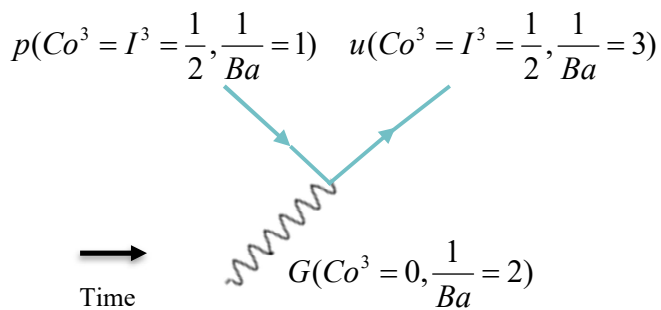


Figure 30: passage from a proton to an up quark, scattering of the strong charge

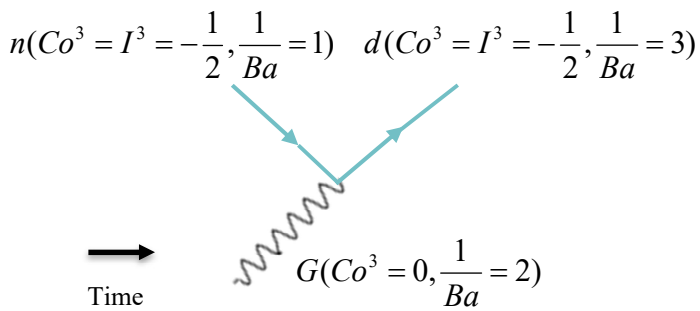


Figure 31: passage from a neutron to a down quark, scattering of the strong charge

Note

It is assumed here that it can be summed the terms in $\frac{1}{Ba}$ and not in Ba . This is purely speculative and needs to be verified experimentally.

VIII.4.3 Annihilation: cancellation of the strong charge Co^3

For a strong current of the type $j_{Co^3}^\mu(n, p) = Co^3 n \gamma^\mu p$, we calculate the quantum numbers during an annihilation:

	$j_{Co^3}^\mu(n, p)$	n	p	Total
--	----------------------	-----	-----	-------

Strong charge	$Co^3 = I^3$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
Source field quantum numbers	$\frac{1}{Ba}$	1	1	2
	Ge	0	0	0
	I^3	$-\frac{1}{2}$	$+\frac{1}{2}$	0
	S^3	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	0

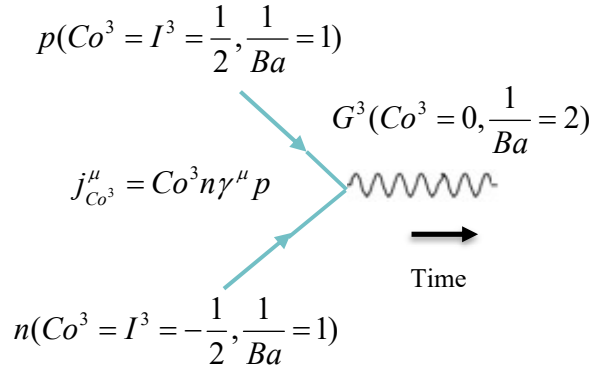


Figure 32: annihilation of the strong charge

Note 1, on uncharged mediating particles M^a

In this Memoir, we look for uncharged mediating particles M^a analogues of the photon γ , that do not change the charge X^a of the fermion, generate the displacement of the charge X^a when exchanging a source field quantum number, and create a current $j_{X^a}^\mu$.

We want to find Noether's theorem: a symmetry of the system, that is to say a quantity conserved by global gauge transformation, the charge X^a , implies the existence of a retained current $j_{X^a}^\mu$ carrying the charge X^a .

Like the photon γ , here proposed gluons G^1, G^2, G^3 do not carry a strong charge Co^a , $a = 1, 2, 3$. They interact with fermions with strong charge Co^a , by exchanging baryon numbers Ba .

For nuclear interactions, gluons G^1, G^2, G^3 are the analogues of the 3 weak bosons W^1, W^2, W^3 . The latter do not carry a weak charge T^a , $a = 1, 2, 3$. They interact with fermions with weak charge T^a , by exchanging generation numbers Ge .

Uncharged mediating particles associated with groups $SU(n)$ with $n \geq 2$, may nevertheless react to each other via terms such as $g\epsilon^{bca}A^{b\mu}A^{c\nu}$, and exchange source field quantum numbers. As these mediating particles do not possess a charge X^a , their interactions cannot generate charge movements, i.e., currents $j_{X^a}^\mu$.

Note 2, on charged mediating particles M^\pm

Mediating particles M^\pm , carriers of charge X^\pm , which may increase or decrease by X^\pm the charge X^a of a fermion, are referred to as scale operators. They are not presented here as the real particles mediating the interaction, in the sense that they are not like the photon, free of charge.

Following the Yang-Mills theory of strong interaction, pions π^+, π^- are the mediating particles of the scale operator type of the strong interaction, i.e., these pions carry a strong charge and can modify by I^3 the strong charge of a nucleon, while maintaining its baryonic number Ba .

According to the Quark Model of the strong interaction, electrically charged mesons formed of a quark and an antiquark (to choose from the flavors u , d and s) are the mediating particles of the scale operator type of the strong interaction, i.e., these mesons carry a strong charge and can modify the strong charge of a baryon, while maintaining its baryonic number Ba .

According to the Electroweak Model, weak bosons W^+ , W^- are the mediating particles of the scale operator type of the weak interaction, i.e., these weak bosons carry a weak charge and can modify by T^3 the weak charge of a left-handed fermion, while maintaining its number of generations (if we stay in the same generation of quarks).

For electromagnetic interaction, the analogue of these mediating particles of scale operator type which modify by X^\pm the charge X^a of a particle, is not the photon mediating particle γ , but rather protons H^+ and electrons e^- involved in chemical reactions (acid-base reactions, oxidation-reduction reactions, etc.) that modify the electric charge of a particle or a set of particles, while maintaining its spin number S .

VIII.5 Conclusion of the chapter

In previous chapters, we have tried to present more of the weak and hyper interactions encountered in the Electroweak Model in analogy with the electromagnetic interaction.

In this chapter, we have proposed a simple model of strong interaction, in strict analogy with electromagnetic, hyper, and weak interactions, with strong uncharged mediating particles.

Inspired by the Yang-Mills theory on the strong interaction, it has been proposed in the case of nucleons and quarks of 1st generation, a strong charge Co^3 equal to isospin I^3 . During a scattering with passage from a nucleon to a quark, it was pointed out that the strong charge $Co^3 = I^3 = \pm \frac{1}{2}$ is conserved and the baryonic number Ba is modified.

In the next chapter, we will summarize for each interaction, the source field equation and the quantum number that we propose to associate with it.

Chapter IX Source field equations, quantum numbers and interactions

Purpose of the chapter

In this chapter, we study for each of the 4 source field equations proposed for gravitation, its link with:

1. a source field quantum number S^a, I^a, Ge, Ba ,
2. a mediating particle carrying the quantum number,
3. a fermion carrying a charge X ,
4. an exchange of the source field quantum number between the mediating particle and the fermion,
5. a displacement of the charge X ,
6. an interaction.

IX.1 Source field equation of de Broglie, spin S , photon γ , electromagnetic interaction

At the beginning of this Memoir, it was proposed that the invariance of the source field equation of de Broglie corresponds to the conservation of orbital and spin angular momentum during reactions between particles. It is applied here to electromagnetic interaction. We will test in the next paragraphs a generalization on the other 3 interactions.

IX.1.1 Quantity exchanged between the mediating particle and the fermion: orbital angular momentum or spin angular momentum S

It is proposed to associate the spin S with the source field equation of de Broglie:

$$\frac{\hbar^z}{2} = \frac{1}{2} \frac{\vec{p}^x}{\vec{K}_{ty}^{1/x}}$$

$$\frac{\hbar^x}{2} = \frac{1}{2} \frac{\vec{p}^y}{\vec{K}_{tx}^{1/y}}$$

$$\frac{\hbar^y}{2} = \frac{1}{2} \frac{\vec{p}^z}{\vec{K}_{tx}^{1/z}}$$

Note

The 3 real dimensions or 2 complex dimensions of $SU(2)_S$ are here related to the 3 dimensions of the momentum \vec{p} in Space.

IX.1.2 Mediating particle, photon γ

The photon γ carries a spin S . Its other three quantum numbers (I, Ge, Ba) are zero. Following the Gell-Mann Nishijima relation $Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$, Its electric charge is zero: $Q = 0$.

The spin $\frac{\hbar^z}{2} = \frac{1}{2} \frac{\vec{p}^x}{\vec{K}_{ty}^{1/x}}$ of the photon γ breaks down into a momentum \vec{p}^x and a wavelength $\frac{1}{K_{ty}^{1/x}} = \frac{\lambda^x}{2\pi}$.

The photon γ has a momentum, a wavelength, but no mass.

Note 1 on the pulsation of the photon

For a photon γ propagating in a vacuum at speed c , we have $v_g = \frac{\partial \Omega}{\partial K} = v_\phi = \frac{\Omega}{K} = c$.

If we know its wave vector $K = \frac{p}{\hbar}$, we can easily deduce its pulsation $\Omega = Kc$.

Note 2 on bosons interpreted as composed of 2 particles

Like meson bosons composed of a quark and an antiquark, it could be interesting to imagine that each mediating particle is in fact composed of two fermion-type particles, opposite values for three source field quantum numbers and the same value for the quantum number characterizing them.

For example, a photon γ could be composed of:

Particle	Spin S^3	Isospin I^3	Generation Ge	Baryonic number Ba
u_L	$\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{3}$
\bar{u}_L	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$
γ	-1	0	0	0

We find very similar suggestions among many physicists. For example, L. de Broglie also proposes to interpret the photon as the assembly of 2 particles. Despite the speculative aspect of such a proposal, the interest is to reduce the number of elementary particles.

Moreover, as already mentioned, this would also explain why the Pauli exclusion principle does not apply to Bosons. Indeed, the latter would be composite particles formed of 2 particles, for which on the other hand the Pauli principle would apply.

IX.1.3 Particles subject to electromagnetic interaction

Particles subject to electromagnetic interaction are fermions with a spin S^3 with half-integer number (positive or negative) and an electric charge $Q \neq 0$.

By exchanging spins S with a photon γ , fermions change orbits. We have displacement of the electric charge Q and appearance of an electromagnetic current j_Q^μ .

IX.1.4 Examples of reactions

We give some reactions where the electromagnetic interaction intervenes:

- reversal of the spin of an electron (hyperfine transition), for example an electron of left helicity becomes an electron of right helicity (if same momentum for the 2 electrons),
- change in the energy level of an electron in an atom (change in its orbital angular momentum and change in its orbit),
- annihilation of an electron and a positron of the same spin with production of a photon γ .

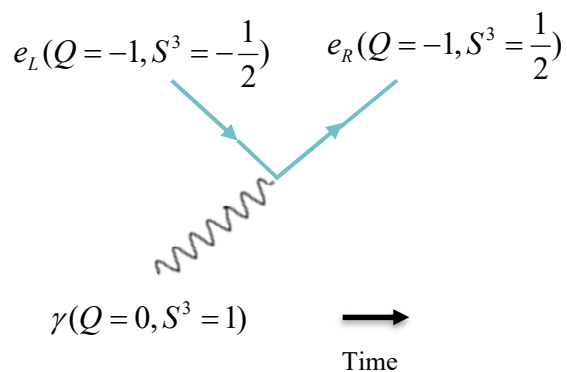


Figure 33: scattering of the electric charge, hyperfine transition

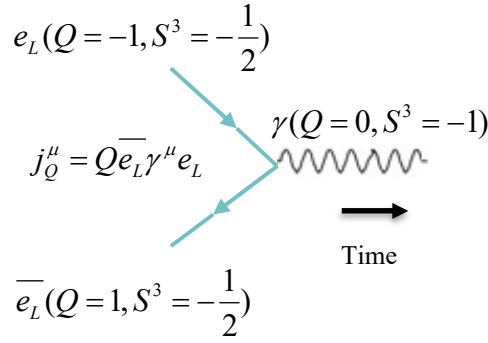


Figure 34: annihilation of the electric charge

IX.2 Newton's source field equation, isospin I , boson B , hyper interaction hyper

In analogy with what has been proposed for electromagnetism, we wish here to test whether the invariance of Newton's source field equation (or Newton's fundamental principle of dynamics), can correspond to the conservation of isospin (and its analogue in orbital) during reactions between particles.

IX.2.1 Quantity exchanged between the mediating particle and the fermion: isospin I

It is proposed to combine isospin I to Newton's source field equation:

$$\frac{\hbar^z}{2} = -\frac{1}{2} \left(\frac{d\vec{p}^x}{dt} \right)_{xt} \frac{\partial x}{\partial \vec{\Omega}_{xy}^{1/t}}$$

$$\frac{\hbar^x}{2} = -\frac{1}{2} \left(\frac{d\vec{p}^y}{dt} \right)_{yt} \frac{\partial y}{\partial \vec{\Omega}_{yz}^{1/t}}$$

$$\frac{\hbar^y}{2} = -\frac{1}{2} \left(\frac{d\vec{p}^z}{dt} \right)_{zt} \frac{\partial z}{\partial \vec{\Omega}_{yz}^{1/t}}$$

Note, propose a case where the isospin would be equal to the Newton's source field equation

Following the Gell-Mann Nishijima relation, we have:

$$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$$

In the event that $\frac{Ba}{2} + \frac{Ge}{2} = 0$, that is, where we eliminate the terms of mass, we get:

$$Q = I^3$$

Here we propose that isospin is the quantum number associated with Newton's source field equation:

$$I^3 = -\frac{dp}{dt} \times \frac{\partial r}{\partial \hbar \Omega}$$

We thus obtain:

$$Q = -\frac{dp}{dt} \times \frac{\partial r}{\partial \hbar \Omega}$$

We find the expression of the fundamental principle of dynamics for the electrostatic force:

$$\frac{dp}{dt} = -Q \frac{\partial \hbar \Omega}{\partial r}$$

$$\text{with } \hbar \Omega = eA^t$$

We propose a 2nd case a little further where we take into account the masses.

IX.2.2 Mediating particle, boson B

The boson B carry an isospin I . By analogy with the photon, its other three quantum numbers (S , Ge , Ba) are zero and its weak hypercharge Y_W is zero.

In addition to the S. Glashow relation for the Electroweak Model $Q = T^3 + \frac{Y_W}{2}$, we assume a relationship of the type:

$$Y_W = f_Y((S, S^3), Ge, Ba)$$

The boson B has a momentum, a pulsation, but no mass.

Note on the hypothetical characteristics of the boson B

Although the boson B is present in the Electroweak Model, it has not yet been discovered experimentally. It must be similar to the photon γ in the sense that both possess a momentum \vec{p} and no mass energy at rest $m^t c^2$.

Nevertheless, the boson B is electrically charged via its isospin I . It should be subject to the effects of an electric field. It is he who "collects" the negative electric charge during a decay β^- from a neutron to a proton.

Note that the boson B would be the only particle with an electric charge, but no rest mass.

IX.2.3 Particles subject to hyper interaction: fermions with a non-zero weak hypercharge Y_W

By analogy with spin, particles subject to hyper interaction are fermions with a half-integer isospin I (positive or negative) and a non-zero weak hypercharge Y_W .

By exchanging isospins I with the mediating particle B , fermions change their orbits, while keeping their weak hypercharge Y_W constant (invariance of Newton's fundamental principle of dynamics). We observe the displacement of the weak hypercharge Y_W and the appearance of a hyper current $j_{Y_W}^\mu$.

For example, during a decay β^- , by switching from a neutron to a proton, there is a change of orbit (taken in the broad sense) of the nucleon, displacement of the weak hypercharge Y_W carried by the nucleon and appearance of a hyper current $j_{Y_W}^\mu$.

IX.2.4 Example of reaction

Decay β^- , with transition from a left-handed neutron to a left-handed proton (or from a quark d_L to a quark u_L).

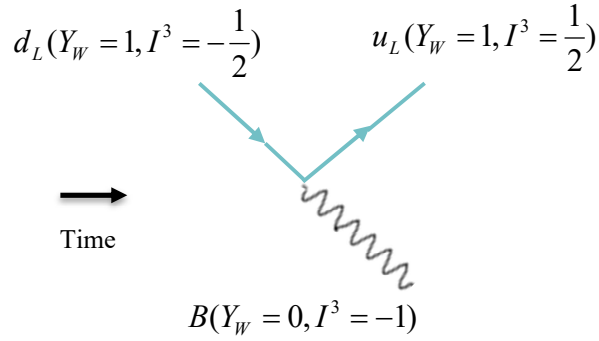


Figure 35: scattering of the weak hypercharge

Note

Conservation of weak hypercharge Y_W only works for left-handed fermions.

IX.3 Source field equation of Compton, number of generations Ge , Bosons W^a , weak interaction

In analogy with what has been proposed for electromagnetism, we wish here to test whether the invariance of the source equation Compton field can correspond to the conservation of the number of generations during reactions between particles.

IX.3.1 Quantity exchanged between the mediating particle and the fermion: number of generations Ge

It is proposed to associate the number of generations Ge to the Compton source field equation:

$$\hbar = -\left(\frac{d\vec{m}^t c^2}{dx}\right)_{tx} \frac{\partial t}{\partial K_{ty}^{1/x}}$$

IX.3.2 Mediating particles, weak bosons W^1, W^2, W^3

Weak bosons W^1, W^2, W^3 are carriers of the number of generations Ge .

By analogy with the photon, their other three quantum numbers $((S, S^3), (I, I^3), Ba)$ are zero and their weak charge (T, T^3) is zero.

According to the electroweak model, we have a relationship of the type between the 3 charges:

$$\frac{Y_W}{2} = Q - T^3$$

We also propose a relationship such as:

$$(T, T^3) = f_T((S, S^3), (I, I^3), Ba)$$

Weak bosons W^1, W^2, W^3 possess mass energy $p^t c = m^t c^2$ and wavelength $\lambda = \frac{2\pi}{K}$. Following the ideas of H. Yukawa, their mass energy explains the low range of the weak interaction.

Note, on internal reactions between weak bosons

Even if they do not have a weak charge T^a , weak bosons can react with each other via the term $g_T \varepsilon^{123} W^{1\mu} W^{2\nu}$.
 Since they do not carry a weak charge T^a , weak bosons W^1, W^2, W^3 do not generate weak currents $j_{T^a}^\mu$ during their displacement.

IX.3.3 Particles subject to weak interaction: fermions carrying a non-zero weak charge T^a

Particles subject to weak interaction are left-handed fermions and right-handed anti-fermions of non-zero weak charge T^a .

Right-handed fermions and left-handed anti-fermions have a zero weak charge T^a , they are not subject to weak interaction.

By exchanging Ge with the mediating particle, Left-handed fermions change "orbits" while keeping constant their weak charge T^a . We observe a displacement of the weak charge T^a and the appearance of a weak current $j_{T^a}^\mu$.

IX.3.4 Example of reaction

Decay of the kaon: $K^+(u\bar{s}) \rightarrow \pi^+(u\bar{d}) + \nu + \bar{\nu}$

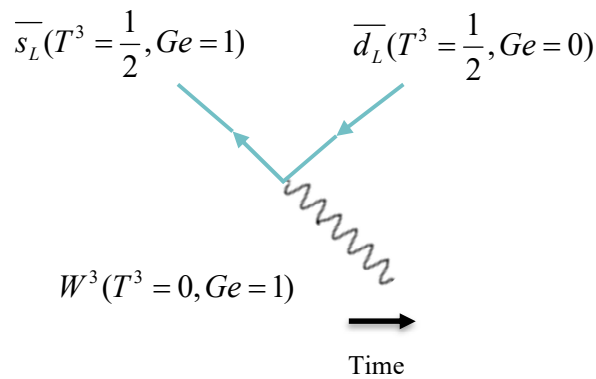


Figure 36: weak charge scattering

IX.4 Source field equation of Einstein, baryonic number Ba , gluons G^a , strong interaction

In analogy with what has been proposed for electromagnetism, we wish here to test whether the invariance of the source equation Einstein's field can correspond to the conservation of the baryonic number during reactions between particles.

IX.4.1 Quantity exchanged between the mediating particle and the fermion: baryonic number Ba

It is proposed to associate the baryonic number with Einstein's source field equation:

$$\hbar = \frac{\bar{m}^t c^2}{\bar{\Omega}_{xy}^{1/t}}$$

IX.4.2 Mediating particles, gluons G^1, G^2, G^3

Gluons G^1, G^2, G^3 are carriers of the baryonic number Ba .

By analogy with the photon, their other three quantum numbers $((S, S^3), (I, I^3), Ge)$ are zero and their strong charge Co is zero.

We also propose a relationship such as:

$$(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)$$

Gluons G^1, G^2, G^3 possess mass energy and pulsation.

IX.4.3 Particles subject to strong interaction: hadrons carrying a non-zero strong charge

The particles subject to strong interaction are hadrons, which have a non-zero strong charge.

IX.4.4 Example of reaction

A proton p transforms into a quark u .

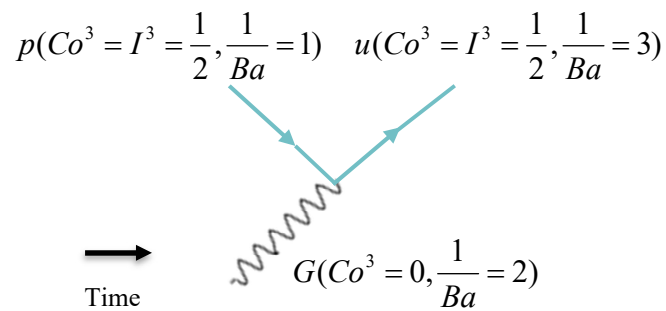


Figure 37: scattering of the strong charge

Note, propose a second case where the isospin would be equal to the source field equation of Newton

We are interested in nucleons. As a first approximation, the baryonic number corresponds to the mass number A :

$$Ba = A$$

We have the mass of the nucleus of an atom, for example a carbon atom of mass number $A = 12$, ^{12}C :

$$m_{^{12}C} = Am_{nu} = 12m_{nu}$$

with m_{nu} the mass of the nucleon particle.

It is proposed that Ba is associated with the Einstein source field equation. We have:

$$Ba = A = \frac{m_{nucleus}c^2}{\hbar\Omega_{nu}}$$

with Ω_{nu} the pulsation of the nucleon wave.

For the nucleus of ^{12}C , we have:

$$m_{^{12}C} = 12m_{nu} = \frac{m_{^{12}C}c^2}{\hbar\Omega_{nu}} m_{nu}$$

Following the Gell-Mann Nishijima relation, we have:

$$Q = I^3 + \frac{Ba}{2} + \frac{Ge}{2}$$

In the event that $\frac{Ge}{2} = 0$, that is, where we eliminate the terms of the weak interaction, we get:

$$Q = I^3 + \frac{Ba}{2}$$

$$I^3 = Q - \frac{Ba}{2}$$

$$I^3 = Q - \frac{A}{2}$$

It is proposed that isospin is the quantum number associated with Newton's source field equation. We have:

$$I^3 = -\frac{dp}{dt} \times \frac{\partial r}{\partial \hbar \Omega}$$

$$Q - \frac{A}{2} = -\frac{dp}{dt} \times \frac{\partial r}{\partial \hbar \Omega}$$

$$\frac{dp}{dt} = -Q \frac{\partial \hbar \Omega}{\partial r} + \frac{A}{2} \frac{\partial \hbar \Omega}{\partial r}$$

We suppose for energies:

$$\hbar \Omega = eA^t$$

$$\hbar \Omega = 2m_{nu}V^t$$

We then have:

$$\frac{dp}{dt} = -eQ \frac{\partial A^t}{\partial r} + m_{nu}A \frac{\partial V^t}{\partial r}$$

We find the expression of the fundamental principle of dynamics with an electrostatic force: $-eQ \frac{\partial A^t}{\partial r} = eQ \vec{E}s$ and a gravitational force: $m_{nu}A \frac{\partial V^t}{\partial r} = -m_{nu}A \vec{G}r$:

$$\frac{dp}{dt} = eQ \vec{E}s - m_{nu}A \vec{G}r$$

IX.5 Charge functions of 3 source field quantum numbers

Let us now recap the arguments that have been mentioned in this Memoir, to express each charge according to the 3 complementary source field quantum numbers, that is to say other than that carried by the particle mediating the interaction concerned.

IX.5.1 Electric charge $Q = f_Q((I, I^3), Ba, Ge)$?

We have the Gell-Mann Nishijima relation:

$$Q = I^3 + \frac{Ba}{2} +$$

$$\frac{Ge}{2} \text{ where we find } I^3, Ba \text{ and } Ge$$

A similar relationship has been proposed for leptons:

$$Q = I^3 - \frac{L_e}{2} + \frac{Ge}{2}$$

Nevertheless, we expect a function f_Q more complex than a simple sum since we add terms of different nature with on one side I^3 , and on the other Ba , L_e and Ge .

Note on spin

Electrical charge Q is not a function of spin (S, S^3), since the spin is not modified when moving from matter to antimatter.

IX.5.2 Strong charge $(C_0, C_0^3) = f_{C_0}((S, S^3), (I, I^3), Ge)$?

In Yang-Mills theory, the strong charge is identified with (I, I^3) .

In the Quark Model, the strong charge is identified with $(I, I^3) \times Ge$.

As mentioned, to be verified experimentally, if for the strong interaction, like the weak interaction, there is not also a different strong charge depending on the helicity of the particle? We would then have a strong charge also function of the spin (S, S^3).

Note 1 on the baryonic number

Strong charge (C_0, C_0^3) does not seem to be a function of the baryonic number Ba . In the Quark Model, nucleons and quarks have the same strong charge and a different baryonic number.

Note 2 on the pion π^0 carrying a strong charge as a function of spin?

In Yang-Mills theory, the pion π^0 carries a strong charge and can change the strong charge of a particle. The pion π^0 has a zero spin. Nevertheless, since π^0 is electrically neutral, the strong charge it carries, should be a function of spin, the only source field quantum number not electrically charged.

IX.5.3 Weak charge $(T, T^3) = f_T((S, S^3), (I, I^3), Ba)$?

In the weak interaction, there is a violation of parity which is explained by a different weak charge depending on the helicity of the particle. So, we have a weak charge as a function of spin (S, S^3).

For left-handed baryons and leptons of 1st generation, the weak charge is identified at (I, I^3) .

No arguments have been made in favour of a weak charge function of Ba or of L_e .

During the transition from matter to antimatter, T^3 and Ba both reverse.

Note 1 on the number of generations

Unlike the isospin (I, I^3) , the weak charge (T, T^3) remains the same regardless of the generation Ge of the particle. The weak charge (T, T^3) is therefore not a priori function of Ge .

On the other hand, during the transition from matter to antimatter, both T^3 and Ge reverse. For example, when switching from the quark c to the quark \bar{c} .

Note 2 on the boson Z^0

In the Electroweak Model, the boson Z^0 carries a weak charge and can change the weak charge of a particle. Unlike the pion π^0 , the boson Z^0 has a non-zero spin equal to 1. Since the boson Z^0 is electrically neutral, the weak charge it carries, should be a function of spin, the only source field quantum number not electrically charged.

IX.5.4 Weak hypercharge $Y_W = f_Y((S, S^3), Ba, Ge)$?

The weak hypercharge is different depending on the helicity of the particle. So, we have a weak hypercharge function of spin (S, S^3) .

For left-handed quarks and leptons of 1st, 2nd and 3rd generation (as well as for their antiparticles), we have:

$$Ba - L_e = Y_W$$

The weak hypercharge is therefore a function of Ba and of L_e .

Unlike (strong) hypercharge Y , the weak hypercharge Y_W remains the same regardless of the generation Ge of the particle. No arguments have been made in favour of a weak hypercharge function of Ge .

On the other hand, during the transition from matter to antimatter, both Y_W and Ge reverse.

Note on isospin

When I^3 varies, the weak hypercharge Y_W remains constant. Weak hypercharge Y_W is therefore not a priori function of (I, I^3) .

IX.5.5 Charges generators of groups $SU(n)$

The summary table is set out below:

	Source field equations with wavevector	Source field equations with pulsation
Source field equations with momentum	$\frac{\hbar^z}{2} = \frac{1}{2} \frac{\vec{p}^x}{\vec{K}_{ty}^{1/x}}$ Spin (S, S^3) generator of $SU(2)_S$ Electric charge Q generator of $U(1)_Q$	$\frac{\hbar^z}{2} = -\frac{1}{2} \left(\frac{d\vec{p}^x}{dt} \right)_{xt} \frac{\partial x}{\partial \vec{\Omega}_{xy}^{1/t}}$ Isospin (I, I^3) generator of $SU(2)_I$ Weak hypercharge Y_W generator of $U(1)_{Y_W}$
Source field equations with mass	$\hbar = -\left(\frac{d\vec{m}^t c^2}{dx} \right)_{tx} \frac{\partial t}{\partial \vec{K}_{ty}^{1/x}}$ Generation Ge generator of $U(1)_{Ge}$ Weak charge (T, T^3) generator de $SU(2)_T$	$\hbar = \frac{\vec{m}^t c^2}{\vec{\Omega}_{xy}^{1/t}}$ Baryonic number Ba generator of $U(1)_{Ba}$ Strong charge (Co, Co^3) generator de $SU(2)_{Co}$ (or of $SU(3)_{Co}$)

It is observed that:

- when the source field quantum number is a generator of $SU(2)$, its corresponding charge is generator of $U(1)$,
- When the source quantum number field is a generator of $U(1)$, its corresponding charge is a generator of $U(1)$.

This goes in the direction of relationships: charges X functions of the source field quantum numbers, which are proposed. We do indeed have a charge X generator of $SU(n)$ when 2 quantum numbers of which it is function, are generators of $SU(n)$.

$$Q = f_Q((I, I^3), Ba, Ge)$$

$$Y_W = f_Y((S, S^3), Ba, Ge)$$

$$(T, T^3) = f_T((S, S^3), (I, I^3), Ba)$$

$$(Co, Co^3) = f_{Co}((S, S^3), (I, I^3), Ge)$$

IX.5.6 Remarks and questions on charges X**Note 1, inversion or modification of a charge X, conservation of the associated source field quantum number**

During a transformation where a charge X is modified, the source field quantum number associated with the corresponding interaction must be retained.

For electromagnetic interaction, when passing for example from an electron e to an antielectron \bar{e} (reversal of electrical charge Q), the spin S^3 is retained. This corresponds to the electromagnetic current $\bar{e}\gamma^\mu e$.

For the strong interaction, when passing for example from a neutron n to a proton p (reversal of the strong charge $Co^3 = I^3$), the baryonic number Ba is retained. This corresponds to the strong current $n\gamma^\mu p$.

For hyper interaction, when passing for example from an antiquark \bar{u}_L to a quark d_L (reversal of the weak hypercharge Y_W), the isospin I^3 is retained. This corresponds to the hyper current $\bar{u}_L\gamma^\mu d_L$ (contribution to the weak charged current in the Electroweak Model).

Let's observe that it works less well for weak interaction. When passing from an antiquark \bar{s}_L to a quark d_L (reversal of the weak charge T^3), Ge is not retained since it changes from 1 to 0. This corresponds to the weak current $\bar{s}_L\gamma^\mu d_L$ (contribution to neutral weak current in the Electroweak Model).

On the other hand, it works for nucleons and quarks of 1st generation. When passing from a neutron n_L to a proton p_L or from a quark d_L to a quark u_L (reversal of the weak charge $T^3 = I^3$), the number of generations Ge is retained.

Note 2, moving charge = radiation of a progressive wave

For electromagnetic interaction, an electric charge in accelerated motion radiates an electromagnetic progressive wave (photon γ carrying a spin S following another representation of the phenomenon). This wave is interpreted as the variations in Space-Time of an electromagnetic field moving at speed c in the vacuum.

Following an analogy between the 4 interactions, a moving electric charge Q , a moving weak hypercharge Y_W , a moving weak charge T , a moving strong charge Co (that is, currents $j_Q^\mu, j_Y^\mu, j_T^\mu, j_{Co}^\mu$) respectively radiate an electromagnetic progressive wave (photon γ carrying a spin S), a hyper progressive wave (boson B carrying an isospin I), a weak progressive wave (bosons W carrying a Ge), a strong progressive wave (gluons G carrying a Ba).

These waves are interpreted as the variations in Space-Time respectively of an electromagnetic field, a hyper field, a weak field, a strong field, all these fields moving with a speed a priori equal to c in the vacuum.

Note 3, on source field equations

In the 3rd Memoir, source field equations for gravitation were proposed in analogy with Maxwell's source field equations based on electric charge. To see if there are also source field equations for the other 3 charges (weak, strong, and hyper)?

Note 4, How to make the link between the field (or potential) representing the mediating particle and the source field quantum quantity transported by the mediating particle?

In relativistic quantum electrodynamics, the link between the electromagnetic field, the photon and the spin S is one of the fundamentals of the theory.

However, spin does not appear in the gauge transform of the wave function: $\psi \rightarrow \psi' = \exp(ieQ\alpha(x))\psi$, nor in the covariant derivative: $\partial_\mu \rightarrow D_\mu = \partial_\mu + iqQA^\mu$, nor in the Lagrangian: $L = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A^\mu$.

The link is therefore not so obvious between the electromagnetic field and spin S .

It is the same for the other 3 interactions, where there is no obvious link between:

- the field or potential representing the mediating particle (as well as the Gauge transform and interaction term),
- the source field quantum number transported by the mediating particle.

This is why, in our opinion, that the Standard Model does not associate the boson B , the bosons W^1 , W^2 , W^3 , and the gluons G^1 , G^2 , G^3 respectively to I^a , Ge and Ba .

In the Bohr electron model, the link between angular momentum $\sigma = rmv = \hbar$ and electrostatic potential energy $E_p = eA^t = -\frac{e^2}{4\pi\epsilon_0 r}$ (in a way, the link between spin and the electromagnetic field) is done via the fundamental principle of Newton's dynamics. We have:

$$m \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\sigma = rmv = \hbar$$

$$rmv \times v = \frac{e^2}{4\pi\epsilon_0}$$

We obtain the expression of the velocity of the electron:

$$\hbar \times v = \frac{e^2}{4\pi\epsilon_0}$$

As well as the electromagnetic coupling constant:

$$\alpha_Q = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

To see therefore, for the 4 interactions, how to obtain a general link between:

- the field representing the mediating particle,
- the source field quantum quantity transported by the mediating particle?

IX.6 Stages of Bohr's parable

IX.6.1 Stages of Bohr's parable for the 4 interactions

In the table below, we summarize the steps of the Einstein effect and the Bohr parabola for the 4 interactions ($a = 1,2,3$).

Steps	Electromagnetic interaction	Hyper interaction	Weak interaction	Strong interaction
1- The mediating particle M and the source field quantity it carries	A photon γ carrier of spin S^a (electrically neutral quantity)	A boson B carrier of isospin I^a (electric charged quantity)	A boson W carrier of number of generations Ge (electric charged quantity)	A gluon G carrier of baryonic number Ba (electric charged quantity)

2- The fermion carrying a charge X	A fermion carrier of electric charge Q	A fermion carrier of weak hypercharge Y_W	A left-handed fermion carrier of weak charge T^a	A hadron carrier of strong charge $C o^a$
3- Exchange of the source field quantity between the mediating particle and the fermion	Exchange of spin S^a between photon and fermion	Exchange of isospin I^a between the boson B and the fermion	Exchange of number of generations Ge between the boson W and the left-handed fermion	Exchange of baryonic number Ba between the gluon G and the hadron
	Modification of a quantity specific to the fermion and function of its momentum. Modification that involves a change in its energy level (corresponding for example to a change of orbit)	Modification of a quantity specific to the fermion and function of its momentum. Does this imply a change in the orbit of the fermion?	Modification of a quantity specific to the left-handed fermion and function of its mass. Does this imply a change in the orbit of the left-handed fermion?	Modification of a quantity specific to the hadron and function of its mass. Does this imply a change in the orbit of the hadron?
4- Examples of observed reactions	Change in the energy level of the electron Hyperfine transition	Passage from a neutron n to a proton p , for example during a decay β^- Change of flavors for quark or lepton of the same generation	Change of fermion generations, for example during decay β with neutral currents (passage from a quark \bar{s} to a quark \bar{d})	Passage of the nucleon p to a quark u , or to a quark component of a pentaquark
5- Charge conserved during reaction	Q conserved during hyperfine transition, e_R and e_L have the same Q	Y_W conserved during decay β^- , n_L and p_L have the same Y_W	T^a conserved during generation change, \bar{s} and \bar{d} have the same T^a	$C o^a$ conserved during baryonic change, p and u have the same $C o^3 = I^3$
6- Effect, appearance of a current, observation of the interaction	Displacement in Space of the electric charge Q between 2 energy levels (or 2 orbits of the electron) Electromagnetic current j_Q^μ	Displacement in Space of the weak hypercharge Y_W Hyper current $j_{Y_W}^\mu$	“Displacement in Time” of the weak charge T^a Weak current $j_{T^a}^\mu$	“Displacement in Time” of the strong charge $C o^a$ Strong current $j_{C o^a}^\mu$

Note 1, on the 2 senses of the parable

As already mentioned, the stages of Bohr's parable work the other way around. That is, a current with charge X radiates a particle wave carrying a source field quantum number.

Note 2, defining an interaction

The notion of interaction is directly related to the displacement of the charge X associated with the interaction.

Following the exchange of a source field quantum number carried by a mediating particle, we observe an interaction when we have displacement of a charge X , which is a function of the source field quantum numbers other than that carried by the mediating particle.

For example, a photon γ can exchange spins. On the other hand, there is no displacement of the charge Q , since hers is null. A photon γ is therefore not subject to electromagnetic interaction.

Nota 3, change in mass and therefore in orbit?

To try to answer the question: "does the modification of a quantity specific to the fermion and function of its mass, imply a change of orbit of this fermion?", let's take up Kepler's 3rd law modified by I.

Newton:

$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M + m)}$$

with T the period of revolution,
 a the semi-major axis of the elliptical trajectory,
 M the mass of the orbited body,
 m the mass of the orbiting body.

Let us remember that according to this 3rd law, the variation of mass m of the orbiting body (for example when a hadron would pass from nucleon to quark, or when a left-handed fermion would change generations) modifies the ratio $\frac{T^2}{a^3}$ and therefore its orbit.

IX.6.2 Gauge transformation $SU(n)$

During a Gauge transformation of $SU(n)$, we have (with $a = 1,2,3$):

- potential quadrivectors $A^\mu, B^\mu, W^{a\mu}, G^{a\mu}$, representing respectively the mediating particles γ, B, W^a et G^a , transporting quantum numbers S^a, I^a, Ge and Ba , that are cancelled,
- the 4 quantum numbers of type charge Q, Y_W, T^a, Co^a , associated with elementary charge $g_Q = e, g_Y, g_T, g_{Co}$ and charge current quadrivector $j_Q^\mu, j_{Y_W}^\mu, j_{T^a}^\mu, j_{Co^a}^\mu$, that are conserved (in line with Noether's theorem),
- interaction energy terms between current quadrivectors and potential quadrivectors, such as $e j_Q^\mu \cdot A^\mu, g_Y j_{Y_W}^\mu \cdot B^\mu, g_T j_{T^a}^\mu \cdot W^{a\mu}, g_{Co} j_{Co^a}^\mu \cdot G^{a\mu}$, which are to be added in the Lagrangian in order to make invariant the great laws of Nature.

We summarize in the table below, for a Gauge transformation, the analogies between the 4 interactions, with the corresponding steps of the Bohr parable:

Stages of the parable	Quantum notions	Electromagnetic interaction	Hyper interaction	Weak interaction	Strong interaction
1-	Mediating particles	γ	B	W^a	G^a
1-	Cancelled potential quadrivectors	A^μ	B^μ	$W^{a\mu}$	$G^{a\mu}$
1-	Source field quantum numbers	S^a	I^a	Ge	Ba
2-	Quantum numbers of charge X	Q	Y_W	T^a	Co^a
2-	Elementary charge	$g_Q = e$	g_Y	g_T	g_{Co}
6-	Current charge quadrivectors	j_Q^μ	$j_{Y_W}^\mu$	$j_{T^a}^\mu$	$j_{Co^a}^\mu$
6-	Interaction energies to be added in the Lagrangian	$e j_Q^\mu \cdot A^\mu$	$g_Y j_{Y_W}^\mu \cdot B^\mu$	$g_T j_{T^a}^\mu \cdot W^{a\mu}$	$g_{Co} j_{Co^a}^\mu \cdot G^{a\mu}$

IX.7 Conclusion of the chapter, hypotheses to be confirmed

In this chapter we have summarized the analogies developed throughout this Memoir between the 4 interactions. We have listed the arguments in favour of a charge X , which is function of the 3 source field quantum numbers other than that carried by the mediating particle. Then, it was reminded how the Einstein effect and Bohr's parable could be applied to each interaction.

Nevertheless, if the similarities presented between the 4 interactions are often strong about this Bohr parable, the proposed link between source field equations and interactions is not, it is true, always convincing. For example, there is no concrete evidence that hyper interaction and isospin correspond to the fundamental principle of Newton's dynamics. As for the idea of constructing charges X from the complementary source field quantum numbers, it remains for the moment a hypothesis as long as the functions of the type f_Q, f_Y, f_T, f_{Co} will not be better known.

The following table is drawn to outline this conclusion:

	Electromagnetic interaction	Analogies with the other 3 interactions
Known characteristics of electromagnetism	Photoelectric effect and Bohr parable	Rather convincing
New features proposed in this Memoir for electromagnetism	Relationship between spin and source field equation of de Broglie	Moderately convincing for the other 3 interactions
	Charge Q function of (I, I^3) , Ba and Ge , i.e., complementary source field quantum numbers	Moderately convincing for the other 3 interactions

In the next chapter, we will look again at gravitational waves and gravitational interaction.

Chapter X Reflections on gravitational interaction

Purpose of the chapter

We examine the links between the 4 quantum interactions that we have just mentioned with a hypothetical quantum gravitational interaction.

X.1 Apply the Einstein effect and Bohr's parable to a possible quantum gravitational interaction

X.1.1 Preamble

In the previous chapters, 4 interactions were described: electromagnetic, hyper, weak and strong. In addition, there is another famous interaction: the gravitational interaction, an interaction that at the quantum level remains hypothetical. Indeed, it is not confirmed by experience and is not part of the Standard Model. For example, there is neither the Gauss gravitational source field equation nor the gravitational constant, G , nor the gravitational field Gr in the BEH mechanism assigning mass to fermions.

Note

If we find neither the Gauss gravitational source field equation, nor the gravitational constant G , nor the gravitational field Gr in the Electroweak Model, On the other hand, mass is omnipresent as a time component of the energy momentum quadrivector: $(p^t c = \gamma mc^2, p^x = \gamma mv^x, p^y, p^z)$.

In this chapter, we will examine whether the hyper interaction and the electromagnetic interaction can be compared to the gravitational interaction, since all three interactions have an infinite range.

It should first be noted that the notions of gravitational force developed in the 17th century by I. Newton, then of gravitational field developed in the 19th century under the influence of M. Faraday, are finally quite different notions from a hypothetical quantum gravitational interaction, which assumes the presence of mediating particles and the exchange of quantized quantities.

Note

The weak interaction and the strong interaction are not themselves forces, in the sense that they do not intervene in the fundamental principle of Newton's dynamics. Like electromagnetic interaction, they nevertheless have an interaction energy. For electromagnetic interaction, this interaction energy is precisely equal to a generalized potential energy, from which we can derive via the Euler Lagrange equation the electric and magnetic forces.

X.1.2 The gravospin and the graviton

If we generalize the Einstein effect and the Bohr parable to a quantum gravitational interaction, we have:

- a quantum number that could be called gravos or gravospin,
- mediating particles, gravitons that would carry this gravospin,
- Fermion-type particles carrying a charge: in this case a mass, and a gravospin that can be reversed,
- a reaction between particles that consists of an exchange of gravospins between a graviton and a fermion,
- an inversion of the fermion gravospin causing a displacement of the fermion, this fermion keeping its mass constant during this displacement,
- a mass current j_m^μ .

We would have an equivalent to the photoelectric effect of A. Einstein, it would be the graviton - mass current effect.

Note

Conversely, a mass current j_m^μ , that is, a moving mass, could generate a gravitational wave of gravitons carrying gravospins.

In the next paragraphs, we will suggest that these hypothetical gravospins and gravitons do not exist, in the sense that we find the mass current j_m^μ of Bohr's parable, both in hyper interaction and in electromagnetic interaction.

Indeed, in some cases, during a reversal of spin or isospin, we have a change of orbit, displacement of a particle, a mass that remains constant during the displacement, and therefore a mass current as desired.

Note on a "photo-mass" effect

We talk about photoelectric effect, as an effect of light and photons on the movement of electrons, of their electric charges, and appearance of an electric current.

Since electrons have mass, we could just as easily speak of a "photo-mass" effect, as an effect of light and photons on the movement of electrons, their masses, and appearance of a mass current.

This is also in line with what was proposed at the end of the 3rd Memoir, that is to say to interpret light as both an electromagnetic wave and a gravitational wave.

With the photoelectric effect and the "photo-mass" effect, electromagnetic and gravitational waves can generate accelerated movements of electric charges and masses, i.e., electric currents and mass currents, in "antennas".

X.2 Can we see electromagnetic interaction and hyper interaction as constituents of gravitational interaction?

X.2.1 Analogies between electromagnetic, hyper and gravitational interactions

We propose here a table of analogies, certainly to be completed, between the 3 electromagnetic, hyper and gravitational interactions.

	Electromagnetic interaction	Hyper interaction	Gravitational interaction
Mediating particles	Photon γ (electrically neutral)	Boson B (neutral for weak hypercharge)	
Source field quantum number	Spin S $\frac{\hbar}{2} = \frac{1}{2} \frac{p}{K}$	Isospin I $\frac{\hbar}{2} = -\frac{1}{2} \frac{p \partial x}{\partial Q}$	
Charge carried by the particle subject to the interaction	Electric charge Q	Weak hypercharge Y_W	Mass m or charge $X(m)$
Examples of reactions between particles	Hyperfine transition, spin reversal S , displacement with conservation of electrical charge Q	Beta decay, transition from neutron to proton, isospin reversal I , Displacement with weak hypercharge conservation Y_W	Hyperfine transition and beta decay, displacement with mass conservation

Constant	$\frac{1}{\epsilon_0}$		$-4\pi G$
Poisson relation	$\Delta A^t = \nabla^2 A^t = -\frac{\rho_{charge}}{\epsilon_0}$		$\Delta V^t = \nabla^2 V^t = 4\pi G \rho_{mass}$
Potential energy	$E_p = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$		$E_p = -G \frac{m_1 m_2}{r}$
Yukawa potential	$\psi(r) = -g^2 \frac{e^{-\frac{m_\gamma r}{\hbar}}}{r}$ with mass m_γ of the photon tending towards 0		$\psi(r) = -g^2 \frac{e^{-\frac{m r}{\hbar}}}{r}$ with masse m of the mediating particle tending towards 0
Coupling constant in Yukawa potential	$g^2 = \frac{-e^2}{4\pi\epsilon_0}$		$g^2 = G m_1 m_2$
Coupling constant in the Electroweak Model	$\alpha_Q = \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}$	$\alpha_Y = \frac{g_Y^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{100}$	It is also sometimes used: $\alpha_{X(m)} = \frac{G m_{proton}^2}{\hbar c}$
Non-relativistic energy balance	$E_m = E_c + E_p$		$E_m = E_c + E_p$
Non-relativistic model	Planetary model of the electron of Rutherford-Bohr-Broglie-Schrödinger-Pauli The electron is "stabilized" by the electrostatic attractive force generated by the proton and by the centrifugal inertia force.		Planetary model of Kepler-Newton La planète (ou le satellite) est « stabilisée » par la force attractive gravitationnelle générée par l'étoile et par la force d'inertie centrifuge.
Non-relativistic stationary wave equation	Schrödinger wave equation $\Delta\psi + \frac{2m_{elec}}{\hbar^2} (E_m - E_p)\psi = 0$ $\Delta\psi + \frac{2m_{elec}}{\hbar^2} (E_m - q_{elec} A^t)\psi = 0$ Poisson: $\Delta A^t = -\frac{q_{proton}}{\epsilon_0} \psi ^2$		Schrödinger – Newton wave equation $\Delta\psi + \frac{2m_{planet}}{\hbar^2} (E_m - E_p)\psi = 0$ $\Delta\psi + \frac{2m_{planet}}{\hbar^2} (E_m - m_{planet} V^t)\psi = 0$ Poisson: $\Delta V^t = 4\pi G m_{star} \psi ^2$

X.2.2 Study of charges

We recall the relations that we proposed, giving the 4 charges according to the 4 source field quantum numbers:

$$Q = f_Q((I, I^3), Ba, Ge)$$

$$\begin{aligned}
Y_W &= f_Y((S, S^3), Ba, Ge) \\
(T, T^3) &= f_T((S, S^3), (I, I^3), Ba) \\
(Co, Co^3) &= f_{Co}((S, S^3), (I, I^3), Ge)
\end{aligned}$$

We note that the charge Q and the weak hypercharge Y_W are the most mass charges, in the sense that they are both function of the 2 source field quantum numbers with mass energy $m^t c^2$: Ba and Ge .

Currents j_Q^μ and j_Y^μ can both be interpreted as currents carrying mass energy, i.e., mass displacements under the effect of an electromagnetic field and a hyper field.

These phenomena are reminiscent of the modification of the motion of a body of mass under the effect of a gravitational field, that is to say the gravitational force.

X.2.3 Study of transitions

In the case of electromagnetic interaction, during a spin reversal S d'un proton ou d'un électron, for example, we have displacement of the proton or the electron (with conservation of the mass of the particle during displacement).

We have displacement of an electric charge, but also of a mass because proton and electron are mass particles.

In the case of hyper interaction, during decay β^- from a neutron to proton with isospin inversion I^3 , we have displacement of the nucleon with a quasi-conservation of the mass (the neutron has a priori a mass very slightly higher than the proton).

We have displacement of the weak hypercharge Y_W , but not of the electric charge Q . For a global conservation of the electric charge, it is assumed that the boson B is electrically charged.

Note that for leptons, it works much less well. If we assume that the passage from a neutrino to an electron corresponds to an isospin inversion I^3 , we have a lepton displacement. However, the mass is not conserved, since the neutrino has a mass a priori much smaller than the electron.

To see if we can find a conservation of the mass, if we are interested globally in the 3 different generations of lepton: electronic, muonic and tauic?

To see also if we can find a conservation of the mass for baryons (or quarks), if we are globally interested in the 3 different generations of quarks? The concern of the slight difference in mass between proton and neutron could then be solved.

X.2.4 Decay β^- , case of left-handed particles of the nucleus of 1st generation (when $T^3 = I^3$)

We recall the relation of S. Glashow given in the Electroweak Model:

$$\frac{Y_W}{2} = Q - T^3$$

For left-handed particles of 1st generation, it is noted that the weak charge T^3 is equal to isospin I^3 . By substituting I^3 at T^3 , we get:

$$\frac{Y_W}{2} = Q - I^3$$

During the passage from a proton p_L to a neutron n_L , we can interpret the relationship $\frac{Y_W}{2} = Q - I^3$ valid for both the proton p_L and the neutron n_L , as follows:

Quantum number	$p_L \rightarrow n_L$	Description
I^3	$(+\frac{1}{2}) \rightarrow (-\frac{1}{2})$	The isospin I^3 reverses when switching from p_L to n_L , this generates the displacement of the nucleon and its weak hypercharge Y_W .
Y_W	$(+1) \rightarrow (+1)$ Conservation of Weak hypercharge. The hypercharge Y_W "sticks" to the nucleon.	The weak hypercharge Y_W moves and stays constant as you go from p_L to n_L with similar mass (the gravitational force has the effect of moving a mass that remains constant. Match in this case Y_W and mass).
Q	$(+1) \rightarrow (0)$	The electrical charge does not move when passing from p_L to n_L .

X.2.5 Hyperfine transition

We have in the table above the analogue for the electromagnetic interaction and the passage from an electron e_L to an electron e_R (we could also have taken: $p_L \rightarrow p_R$).

Quantum number	$e_L \rightarrow e_R$	Description
S^3	$(-\frac{1}{2}) \rightarrow (+\frac{1}{2})$	The spin reverses when we go from e_L to e_R , this generates a change in the orbit of the electron and the displacement of the electric charge Q .
Q	$(-1) \rightarrow (-1)$ Conservation of electrical charge. charge Q "sticks" to the electron.	The electric charge Q moves and stays constant as you go from e_L to e_R with identical mass (the electromagnetic force has the effect of displacing an electric charge and a mass that both remain constant).
Y_W	$(-1) \rightarrow (-2)$	Weak hypercharge Y_W does not move when moving from e_L to e_R .

For the **hyper interaction** and for left-handed particles of the 1st generation nucleus, we have:

$$\frac{Y_W}{2} = Q - I^3$$

The weak hypercharge is a function of the electric charge and the quantity transported by the boson B .

For **electromagnetic interaction** and for electrons e_L and e_R , the analogous relationship is:

$$\frac{Q}{2} = Y_W + 1 + S^3$$

The electric charge is a function of the weak hypercharge and the quantity carried by the photon γ .

X.2.6 What contribution to mass energy do quantum numbers Q and Y_W have?

The electrons e_L and e_R have the same mass (or mass energy $m^t c^2$). The contribution to mass energy of e_L and e_R appears essentially due to the electric charge which remains constant when one passes from e_L to e_R . This is true when examining the neutrino ν_{eL} , the analogue of the electron e_L for weak hypercharge ($Y_W = -1$), but for which the electric charge Q is zero. The mass of the neutrino ν_{eL} is very small.

On the other hand, for nucleons n_L and p_L , the situation is different. When examining the neutron n_L , the analogue of the proton p_L for weak hypercharge ($Y_W = 1$), but for which the electric charge Q is zero, its mass is slightly greater than that of the proton. The contribution to the mass energy of n_L and p_L appears mainly due to weak hypercharge Y_W , which remains constant when moving from n_L to p_L .

X.2.7 Case of strong and weak interactions

During a strong interaction between a fermion and a gluon G , with exchange of Ba and change of the Ba of the fermion, there is no conservation of the mass of the fermion during its "displacement" from nucleon to quark. Quarks of $Ba = \frac{1}{3}$ have, for example, a much smaller mass than the corresponding nucleons of $Ba = 1$.

There is therefore no possible rapprochement with a gravitational interaction and a mass current that is preserved during the "displacement" of the fermion from nucleon to quark.

The same is true for a weak interaction between a fermion and a boson W , with exchange of the number of generations Ge . There is no conservation of the mass of the quark when it passes from the 1st generation to the 2nd generation. A 1st generation quark is much less mass than a 2nd generation quark.

The contributions to the gravitational interaction therefore seem to come mainly from the electromagnetic interaction and the hyper interaction.

X.2.8 On the charge of the gravitational interaction and the mass current

If we accept that the gravitational interaction consists of electromagnetic and hyper interactions, the gravitational interaction must have as source field quantum numbers: the spin (S, S^3) and the isospin (I, I^3) .

Based on the above, the charge X of the gravitational interaction should be a function of the complementary source field numbers, i.e., the baryonic number Ba and the number of generations Ge . So, we have for the charge X of the gravitational interaction a function f_X of the type:

$$X(m) = f_X(Ba(m), Ge(m))$$

The baryonic number $Ba(m)$ and the number of generations $Ge(m)$ are both functions of mass m . We get as expected a charge $X(m)$ of gravitational interaction as a function of mass.

We remind the Gell-Mann Nishijima relation, for an electric charge Q carried by a hadron:

$$Q(m, p) = I^3(p) + \frac{Ba(m)}{2} + \frac{Ge(m)}{2}$$

To move from electromagnetic interaction to gravitational interaction, we eliminate in the electric charge $Q(m, p)$ the term with momentum p , here isospin $I^3(p)$. We can therefore expect, for a hadron, a relationship of the type:

$$X(m) = \frac{Ba(m)}{2} + \frac{Ge(m)}{2}$$

In the case of 1st generation hadrons, we have:

$$X(m) = \frac{Ba(m)}{2}$$

and therefore, a charge $X(m)$ identical (if we add a factor of 1/2) to the baryonic number.

For 1st generation leptons, we can expect:

$$X(m) = \frac{L_e(m)}{2}$$

In all cases, we would have a mass current carrying the conserved quantity $X(m)$.

Note on $X(m)$ negative or positive

The baryonic number $Ba(m)$, as well as the number of generations $Ge(m)$ can take positive or negative values. The same is therefore true for $X(m)$. To explain then why the mass m is always positive?

X.2.9 On the charge of nuclear interaction and a "momentum" current

By analogous reasoning, we obtain a charge $X(p)$ of nuclear interaction union of strong and weak nuclear interactions, function of spin (S, S^3) and isospin (I, I^3) . Both are functions of momentum p . We would therefore have for the charge $X(p)$ nuclear interaction, a function f_X such as:

$$X(p) = f_X((S, S^3)(p), (I, I^3)(p))$$

We would also have a "momentum" current carrying the conserved quantity $X(p)$.

X.2.10 Gravitational interaction and nuclear interaction, analogous to each other in Space and Time?

In Memoir 2, it has been proposed that the mass m^t is the analogue in Time, of the movement or speed $v^{x,y,z}$ in Space.

According to the same proposal, the gravitational interaction which conserves mass m^t and which varies the momentum $p^{x,y,z}$ would be the analogue in Space and Time of the nuclear interaction which varies the mass m^t and which would maintain the momentum $p^{x,y,z}$.

The gravitational interaction would have an infinite range in Space and the nuclear interaction an infinite range in Time.

	Retained quantity	Variable quantity
Gravitational interaction	Mass m^t	Momentum $p^{x,y,z}$ Movement or speed $v^{x,y,z}$
Nuclear interaction	Momentum $p^{x,y,z}$ Movement or speed $v^{x,y,z}$	Mass m^t

X.3 Finding the boson experimentally B?

Following what has been explained above, it is proposed here to draw up a list of characteristics for the boson B , in order to find it experimentally.

X.3.1 Characteristics according to the Electroweak Model

The Electroweak Model defines a boson B mediating particle of the hyper interaction. He does not attribute any mass to the boson B from the BEH mechanism. The boson B participates in decays β^\pm . During a decay β^- , It is very unstable, since it decays very quickly into an electron and an antineutrino. It has a priori a zero mass and the hyper interaction therefore has an infinite range.

X.3.2 New features proposed in this Memoir

In this Memoir, compared to the Electroweak Model, new characteristics for the boson B are proposed.

The boson B carries an isospin I^3 . Like the photon, it has a momentum. During a decay β^- , a boson B is emitted by a neutron when this neutron turns into a proton. We have the scattering reaction involving the hyper interaction, with inversion of I^3 and conservation of Y_W :

$$n_L(I^3 = -\frac{1}{2}, Y_W = 1, Q = 0) \rightarrow p_L(I^3 = \frac{1}{2}, Y_W = 1, Q = 1) + B(I^3 = -1, Y_W = 0, Q = -1)$$

The boson B also interacts with the couple electron e_L , antineutrino $\overline{\nu_{eL}}$, with inversion of I^3 and conservation of Y_W . We have the reactions of scattering and creation:

$$\nu_{eL}(I^3 = \frac{1}{2}, Y_W = -1, Q = 0) + B(I^3 = -1, Y_W = 0, Q = -1) \rightarrow e_L(I^3 = -\frac{1}{2}, Y_W = -1, Q = -1)$$

$$B(I^3 = -1, Y_W = 0, Q = -1) \rightarrow \overline{\nu_{eL}}(I^3 = -\frac{1}{2}, Y_W = 1, Q = 0) + e_L(I^3 = -\frac{1}{2}, Y_W = -1, Q = -1)$$

Following the Gell-Mann Nishijima relation, the isospin I^3 is electrically charged. The boson B that carries an isospin I^3 , is therefore electrically charged (this is what is indicated above with $Q = -1$).

As already pointed out in this chapter, when reversing isospin I^3 and passage from the neutron to the proton, we have conservation of the mass and appearance of a conserved mass current, as one would have for a gravitational force. So, we have the hyper interaction and the boson B that intervene in the gravitational interaction. Boson B et photon γ are the two particle waves constituent of gravitational waves.

Note 1 on gravitational waves

If we assume that a mass current radiates a gravitational wave, and that the electromagnetic and hyper interactions are both contributions to the gravitational interaction and thus to the existence of a mass current, both electromagnetic and hyper waves must be contributions to gravitational waves.

Nota 2 on mediating particles subject to other interactions?

It is proposed in this Memoir that a mediating particle is charged for the 3 interactions, other than the one for which it is the mediator. In this case, to check experimentally if the particle mediating of an interaction is subject or not to the other 3 interactions?

If this is the case, by interacting, a mediating particle must gain a source field quantum number and transform into a fermion.

In this regard, let us note that a photon is sensitive to gravitation, since a light ray is deflected by a gravitational field.

Note 3 on rest mass

Note that what is zero, both for the photon γ and for the boson B , it is their rest mass m_0^t with respect to the observer. When they have a speed c , we don't know. This explains in particular their sensitivity to gravitation or participation in the Compton scattering of the photon γ .

We will come back to this at the end of the next Memoir.

In summary, here are the qualitative characteristics of the boson B which would make it possible to find it experimentally. The boson B has zero rest mass, a momentum, an isospin I^3 , an electric charge, a strong charge, and a weak charge. It is found in decays β^\pm . It should contribute, like the photon γ , to gravitational interaction.

X.4 Charge of particles according to their participation in the interactions

In the table below, we list the particles according to their participation in the interactions.

	Gravitational interaction		Nuclear interaction	
	Electromagnetic interaction	Hyper interaction	Weak interaction	Strong interaction
The particle does not participate in the interaction if its charge is zero	Cohesion of the atom	Cohesion of the atom	Cohesion of nucleus + cohesion of lepton	Cohesion of nucleus
Left-handed proton	+X	+X	+X ³	+X ³

Left-handed neutron		+X	$-X^3$	$-X^3$
Right-handed proton	+X	+X		$+X^3$
Right-handed neutron		+X		$-X^3$
Left-handed electron	-X	-X	$-X^3$	
Left-handed neutrino		-X	$+X^3$	
Right-handed electron	-X	-X		
Right-handed or sterile neutrino (not observed)		-X?		

Note 1 on the sterile neutrino

In 1967, B. Pontecorvo proposed the existence of a sterile neutrino, of the right-handed neutrino or left-handed antineutrino type, not subject to electromagnetic interactions, strong and weak. Indeed, its electric charge, strong charge and weak charge are zero.

According to B. Pontecorvo, this sterile neutrino should take part in the phenomena of oscillation of neutrinos, it would have a mass and would therefore be subject to gravitational interaction. So, far, this sterile neutrino has never been detected.

If we go back to Glashow's relationship $Q = T^3 + \frac{Y_W}{2}$, a right-handed neutrino has a zero weak hypercharge Y_W , is therefore not subject to hyper interaction.

However, if we accept that neutrinos have mass and that the hyper interaction contributes to the gravitational interaction, a right-handed neutrino could still be subject to the hyper interaction and possess a non-zero weak hypercharge. Glashow's relationship would then have to be amended.

To confirm or deny this by experiment, in order to better understand the gravitational interaction and this sterile neutrino.

Note 2 on the characterization of particles from their participation or not in interactions

Inspired by this table, it can be suggested that any particle is *first* characterized by the interactions in which it participates or not.

Thus, according to this idea, a proton or an electron differ respectively from a neutron or a neutrino *essentially* because proton and electron are subject to electromagnetic interaction, whereas neutron and neutrinos are not.

Similarly, particle hadrons in the nucleus differ from particle leptons outside the nucleus *essentially* because hadrons are subject to strong interaction, whereas leptons are not.

Similarly, left-handed particles differ from right-handed particles *essentially* because the former are subject to the weak interaction, whereas the latter are not.

What remains to be explained is why when we cancel the electric charge of a particle, that is to say when we make it insensitive to the electromagnetic interaction, for example by passing from a proton to a neutron, other characteristics of the particle change? Passing from proton to neutron, isospin, as well as strong and weak charges are reversed, the mass also varies very slightly...

Similarly, when cancelling the strong charge of a particle, i.e., when making it insensitive to the strong interaction, for example by passing from a proton to an electron, why do other characteristics of the

particle change? By passing from the proton to the electron, the electric charge is reversed, the mass varies very strongly...

Similarly, when cancelling the weak charge of a particle, for example by switching from a left-handed electron to a right-handed electron, why do other characteristics of the particle change? By switching from a left-handed electron to a right-handed electron, the weak hypercharge varies (but a priori not the mass).

Note 3 on particles of zero hypercharge?

The electromagnetic, strong and weak interactions each make it possible to distinguish two categories of particles (particles subject to interaction and those not subject because of zero charge). For hyper interaction, are there also two categories of particles: those subject to hyper interaction and non-subjects because of zero weak hypercharge?

Note 4 on the attractions of the different interactions

It should be noted that electromagnetic and hyper interactions are attractive between hadrons (particles of the nucleus) and lepton (particles around the nucleus) and therefore ensure the cohesion of the atom. Protons and electrons have opposite electric charges and therefore attract each other. Hadrons and leptons have opposite weak hypercharges and therefore attract each other.

The strong nuclear interaction is attractive between proton and neutrons and therefore ensures the cohesion of the nucleus. Protons and neutrons have opposite strong nuclear charges and therefore attract each other.

The weak nuclear interaction ensures cohesion among the particles of the nucleus: hadrons and among the particles around the nucleus: leptons.

X.5 Proposal of a synthesis in "Coriolis" format with "generalized" rotation cancelled when changing reference frames

In the table below, following an approach in "Coriolis" format, we give the quantities of inertia to be added in the "laws of Nature", so that they remain invariant during a change of reference frames.

Inertial acceleration or inertial interaction	Range in Space	Range in Time	Quantity retained when changing reference frames	"Generalized" rotation cancelled when changing reference frames	Modified quantity	"Coriolis" format	Orientation
Coriolis acceleration	Infinite		$\vec{v}^{x,y,z}$ speed	$2\vec{\Omega}_{xy}^{1/t}$	\vec{m}^t	$\vec{a}_{Cor\ ty}^x = -\vec{v}^y \wedge 2\vec{\Omega}_{xy}^{1/t}$	Space
Electromotor field	Infinite		$\vec{v}^{x,y,z}$ speed	$\vec{B}_{xy}^{1/t}$	\vec{m}^t	$\vec{E}m_{ty}^x = \vec{v}^y \wedge \vec{B}_{xy}^{1/t}$	Space
"hypermotor" field	Infinite		$\vec{v}^{x,y,z}$ speed	$\vec{B}h_{xy}^{1/t}$	\vec{m}^t	$\vec{H}m_{ty}^x = \vec{v}^y \wedge \vec{B}h_{xy}^{1/t}$	Space
Gravitation	Infinite		\vec{m}^t mass	$\vec{K}_{tx}^{1/y}$	$\vec{v}^{x,y,z}$	$\vec{F}_{K\ ty}^x = \vec{m}^t \wedge 2c^2\vec{K}_{tx}^{1/y}$	Space
Electric	Infinite		$\vec{q}^t(\vec{m}^t)$ electric charge	$\vec{E}l_{tx}^{1/y}$	$\vec{S}^{x,y,z}(\vec{v}^{x,y,z})$ spin, angular momentum	$\vec{F}_{El\ ty}^x = \vec{q}^t \wedge \vec{E}l_{tx}^{1/y}$	Space

Hyper	Infinite		$\vec{Y}_W^t(\vec{m}^t)$ hypercharge	$\vec{B}h_{tx}^{1/y}$	$\vec{I}^{x,y,z}(\vec{v}^{x,y,z})$ isospin	$\vec{F}_{Bh_{ty}}^x$ $= \vec{Y}_W^t \wedge \vec{B}h_{tx}^{1/y}$	Space
Nuclear		Infinite	$\vec{v}^{x,y,z}$ speed	$\vec{K}_{tx}^{1/y}$	\vec{m}^t	$\vec{I}n_{K_{ty}}^t$ $= \vec{v}^x \wedge 2c^2 \vec{K}_{tx}^{1/y}$	Time
Strong nuclear		Infinite	$\vec{C}o^{x,y,z}(\vec{v}^{x,y,z})$ strong charge	$\vec{S}_{tx}^{1/y}$	$\vec{B}a^t(\vec{m}^t)$	$\vec{I}n_{S_{ty}}^t = \vec{C}o^x \wedge \vec{S}_{tx}^{1/y}$	Time
Weak nuclear		Infinite	$\vec{T}^{x,y,z}(\vec{v}^{x,y,z})$ weak faible	$\vec{W}_{tx}^{1/y}$	$\vec{G}e^t(\vec{m}^t)$	$\vec{I}n_{W_{ty}}^t = \vec{T}^x \wedge \vec{W}_{tx}^{1/y}$	Time

Note that there are 3 types of inertial quantities to add:

1. type of inertial accelerations,
2. type of inertial forces or gravitational, electromagnetic and hyper interactions,
3. type of strong and weak nuclear interactions.

The 3 types of inertia quantities to be added are distinguished by the orientation, in Space or in Time, of the quantity conserved and the "generalized" rotation canceled when changing reference frames.

We have 3 possible combinations: Space Time, Time Space, Space Space (with for the latter combination, 2 different dimensions of Space perpendicular to each other). The Time Time combination is impossible, because it would take 2 dimensions of Time perpendicular to each other.

Inertial acceleration or inertial interaction	Quantity retained when changing reference frames	"Generalized" rotation cancelled when changing reference frames	Orientation of inertial acceleration or inertial interaction
Inertial accelerations	Space-oriented type $\vec{v}^{x,y,z}$	Rotation in a spatial plane, Time-oriented type $2\vec{\Omega}_{xy}^{1/t}$	Space-oriented type $\vec{v}^{x,y,z} \wedge 2\vec{\Omega}_{xy}^{1/t}$
Gravitational, electromagnetic, and hyper interactions	Time-oriented type \vec{m}^t	Rotation in a spatiotemporal plane, Space-oriented type $2\vec{K}_{tx}^{1/y}$	Space-oriented type $\vec{m}^t \wedge 2c^2 \vec{K}_{tx}^{1/y}$
Strong and weak nuclear interactions	Space-oriented type $\vec{v}^{x,y,z}$	Rotation in a spatiotemporal plane, Space-oriented type $2\vec{K}_{tx}^{1/y}$	Time-oriented type $\vec{v}^{x,y,z} \wedge 2c^2 \vec{K}_{tx}^{1/y}$

Note 1

It is explicitly assumed here:

- that a local gauge transformation,
 - and that a change of reference frames where a "generalized" rotation is cancelled,
- are the same thing.

Note 2

To study forces and interactions, we must be able to restrict ourselves to "generalized" rotations in a *spatiotemporal* plane cancelled during a change of reference frames. Indeed, when we perform a change of reference frames where we cancel a rotation "generalized" in a *spatial* plane and the quantity conserved is a velocity, we obtain for example an acceleration or an electromotor field. If, in addition, we have as quantity conserved a mass or an electric charge when changing reference frames, we obtain a force. This can then always be obtained by cancelling a "generalized" rotation in a *spatiotemporal* plane.

In summary, forces and interactions are to be added in the Laws of Nature, when one cancels during a change of reference frames, a generalized rotation in a *spatiotemporal* plane and that the quantity conserved is of charge-type for the 4 fundamental interactions, of mass-type for the gravitational interaction and a priori of velocity-type for the nuclear interaction (to be confirmed for the latter).

X.6 Conclusion of the chapter, on the contribution to gravitational interaction

In conclusion of this chapter, Bohr's parable does not seem to apply to gravitational interaction. Indeed, there is no mediating particle, nor quantum number or gravospin-type for gravitation. On the other hand, understanding gravitational interaction as constituted by hyper interaction and electromagnetic interaction, seems a promising track. Indeed, under the effect of these 2 interactions, for example during spin reversals S^3 or isospin reversals I^3 , we observe the displacement of a fermion with conservation (or quasi-conservation) of its mass. This is the desired effect for gravitational interaction.

As for the strong and weak interactions, they do not seem to contribute to the gravitational interaction, since during a change of Ba or of Ge , the mass of the fermion is not conserved, with impossibility to obtain the desired mass current.

X.7 Conclusion of the Memoir, from the infinitely small to the infinitely large, on dark matter and the hypothesis of a new interaction

In physics, the 2 main fields of experimentation were on the one hand the observation of celestial bodies which tended little by little towards an infinitely large in Space and towards a distant past (study of phenomena of infinite scope in Space), on the other hand the observation of the "field", which tended little by little towards particle physics and an infinitely small in Space (study of phenomena of infinite scope in Time?).

Naively, an observer of the past could have assumed that the observations and laws of the stars had nothing to do with the observations and laws of the "field". The scales are indeed so different that commonalities cannot exist.

Yet if you look at the history of physics, it has been quite the opposite. These incessant back and forth between the observations of the stars and those of the field, these analogies, even these identities observed between the laws of the stars and the laws of the field, have often proved singularly fruitful in the understanding of the Universe.

One can think of the famous parable of the apple of I. Newton, field observation, which combined with the observation of the stars, made it possible to state the law of universal gravitation. But there are many other cases.

The enigma of dark matter, i.e., the non-respect of Newtonian gravitation at the scale of galaxies, is another good example to show the fruitful alliance of experiments conducted on the infinitely large and those conducted on the infinitely small, in the understanding of the Universe.

To explain dark matter, we can currently distinguish 3 tracks, according to the proposed hypotheses. The first track is the proposal of additional matter, in this case dark matter itself. The second track is the modification of Newton's laws on a large scale. This is for example the path followed by Mordehai Milgrom, in the empirical model called *Mond* (Modified Newtonian dynamics). The third track is the hypothesis of a new long-range interaction that would compensate for certain aspects of gravitation. This is the path followed by Attila Krasznahorkay, in his research on particles, with a new interaction carried by the X17 boson.

The tracks followed to understand dark matter, are therefore both towards the infinitely large and the infinitely small. No doubt this fruitful alliance will one day succeed in solving the enigma.

In Memoir 4, it was proposed to explain dark matter by an analogue in gravitation, induction phenomena in electromagnetism. In a way, a modification of Newton's "usual" gravitational laws, and a rapprochement with the track of Mr. Milgrom.

In this Memoir, it is suggested that the gravitational interaction consists of 2 interactions, including a hyper interaction, which can be described as "new", even if it is explicitly present in the Electroweak Model. We would then join the track of A. Krasznahorkay.

In the end, this Memoir had two objectives. First, to show that the 4 interactions present in the Standard Model were more similar than the latter presented, in particular by following Bohr's parable. Then, that of giving a reality to this hyper interaction, transforming it into an interaction comparable to the other 3, that it is not a simple "technical artifact" as it appears in Electroweak Model.

The proposals made in this Memoir may not be accepted. In any case, the understanding of gravitation, dark matter, the quest for a new interaction, will certainly be part of the major challenges of Physics in the 21st century.

According to the ideas proposed here, the analogies between the laws of the stars and the laws of the field, between the laws of the infinitely large and the laws of the particles have often proved singularly fruitful in the understanding of the Universe, precisely because of the analogies between Space and Time.

Another major challenge will be a better understanding of what Time is. Since the beginning of this Memoir, a temporal dimension identical to the three spatial dimensions has been consistently used. This approach to Time is in fact constantly found in Quantum Physics and in the theory of Relativity. In the next Memoir, we will focus on the following question: why does the Time of Physics seem so different from our felt Time?

Bibliography of Memoir 5

On new quantum numbers

- [1] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL. Téléchargeable sur Internet sur le site <http://www.sciences.ch/htmlfr/accueil.php>
- [2] D. Fontaine, Mélange des saveurs selon le modèle standard, Liège, 2007
- [3] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007

On the strong interaction, the Eightfold Way, and the Quark Model

- [4] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [5] Johann Collot, Classification des particules subatomiques Propriétés des interactions fondamentales, UJF Grenoble, années 2004-2005
- [6] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007

On the quantum numbers of the Electroweak Model

- [7] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [8] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007
- [9] Wikipedia, Mesons, pions

On antimatter

- [10] Johann Collot, Cinématique, taux de désintégration et sections efficaces de réactions, UJF Grenoble, années 2005-2006

On angle of Cabibbo, neutral and charged weak currents

- [11] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [12] D. Fontaine, Mélange des saveurs selon le modèle standard, Liège, 2007
- [13] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007
- [14] P. Vilain, Les Courants neutres et les Particules charmées en Physique du Neutrino. Université Libre de Bruxelles, Avril 2005
- [15] Wikipedia, CKM matrix

On leptons and angle of Pontecorvo

- [16] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [17] Johann Collot, Masse et oscillations de neutrinos, UJF Grenoble, années 2004-2005
- [18] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007
- [19] P. Vilain, Les Courants neutres et les Particules charmées en Physique du Neutrino. Université Libre de Bruxelles, Avril 2005
- [20] Wikipedia, PMNS matrix (neutrino oscillation)

On the strong interaction and quantum chromodynamics

- [21] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [22] R. Feynman, QED: The Strange Theory of Light and Matter, Princeton University Press, 1985

Table of Contents

Memoir 5	The effect of Einstein and the parable of Bohr	343
Chapter I	Relating the de Broglie source field equation and the spin quantum number S	345
I.1	Reminders: from Kepler's 2 nd law to orbital angular momentum and orbital magnetic moment, then to spin.....	345
I.2	Combining the source field equation of de Broglie with orbital angular momentum and spin angular momentum	348
I.3	Conclusion of the chapter.....	351
Chapter II	The photoelectric effect of Einstein (1905) and the parable of Bohr (1913).....	352
II.1	The photoelectric effect of Einstein and the parable of Bohr.....	352
II.2	Associate interactions, quantum numbers and bosons W_a and B intervening in beta decays	355
II.3	Conclusion of the chapter.....	358
Chapter III	New quantum numbers: the baryonic number B_a, the hypercharge Y and strangeness St (1950s)	359
III.1	Historical Preamble, particle Accelerators.....	359
III.2	The baryonic number B_a , the isospin I_a and the hypercharge Y	359
III.3	Strangeness St	360
III.4	Conclusion of the chapter.....	362
Chapter IV	The strong interaction, the Eightfold Way, and the Quark Model (1960s).....	363
IV.1	Theories on strong interaction (1950-1970).....	363
IV.2	The Eightfold Way.....	365
IV.3	The Quark Model	366
IV.4	Conclusion of the chapter.....	369
Chapter V	Quantum numbers of Electroweak model	370
V.1	Quantum numbers for 1 st and 2 nd generation particles	370
V.2	More and more excited states.....	372
V.3	Transition to antimatter	376
V.4	Conclusion of the chapter.....	379
Chapter VI	Electromagnetic currents, weak currents, and hyper currents, Cabibbo angle and GIM mechanism, case of quarks.....	380
VI.1	Electromagnetic current $jQ\mu$	380
VI.2	The model of N. Cabibbo	383
VI.3	Glashow-Illiopoulos-Maiani GIM mechanism and electrically neutral weak currents... 386	
VI.4	Electrically charged weak currents.....	391
VI.5	Electrically charged weak currents $jT + \mu$	391
VI.6	Conclusion of the chapter.....	394
Chapter VII	Weak currents and hyper currents, angle of Pontecorvo, case of leptons. 395	
VII.1	Mass and oscillation of the neutrinos.....	395
VII.2	Matrix Pontecorvo-Maki-Nakagawa-Sakata (matrix PMNS)	395
VII.3	Leptons $e - , \nu_e , \mu - , \nu_\mu$ of 1 st and 2 nd generation.....	397
VII.4	Electrically neutral weak currents	398
VII.5	Electrically charged weak currents.....	400
VII.6	Conclusion.....	402

Chapter VIII	Proposal of a model for strong interaction in analogy with electromagnetic interaction	403
VIII.1	General on color charge confinement and asymptotic freedom	403
VIII.2	Energy dependence of coupling constants	405
VIII.3	Hypotheses for a model of strong interaction with uncharged mediating particles.....	406
VIII.4	Application of the model to nucleons and quarks of 1st generation.....	408
VIII.5	Conclusion of the chapter.....	411
Chapter IX	Source field equations, quantum numbers and interactions	412
IX.1	Source field equation of de Broglie, spin S , photon γ , electromagnetic interaction	412
IX.2	Newton's source field equation, isospin I , boson B , hyper interaction hyper	414
IX.3	Source field equation of Compton, number of generations Ge , Bosons Wa , weak interaction.....	416
IX.4	Source field equation of Einstein, baryonic number Ba , gluons Ga , strong interaction	417
IX.5	Charge functions of 3 source field quantum numbers	419
IX.6	Stages of Bohr's parable	423
IX.7	Conclusion of the chapter, hypotheses to be confirmed.....	426
Chapter X	Reflections on gravitational interaction	427
X.1	Apply the Einstein effect and Bohr's parable to a possible quantum gravitational interaction.....	427
X.2	Can we see electromagnetic interaction and hyper interaction as constituents of gravitational interaction?.....	428
X.3	Finding the boson experimentally B ?.....	433
X.4	Charge of particles according to their participation in the interactions.....	434
X.5	Proposal of a synthesis in "Coriolis" format with "generalized" rotation cancelled when changing reference frames	436
X.6	Conclusion of the chapter, on the contribution to gravitational interaction	438
X.7	Conclusion of the Memoir, from the infinitely small to the infinitely large, on dark matter and the hypothesis of a new interaction	438
Bibliography of Memoir 5		440

Memoir 6 Time felt and Time in Physics

Summary of Memoir

We wonder here about the differences between Felt Time (Time oriented like an arrow), which can also be described as Psychological Time, and Time used in physics (Time often described mathematically as a spatial dimension).

To answer this question, we will study the most varied concepts. John Wheeler and Richard Feynman's idea of an electron that goes back in time (idea evoked in R. Feynman's Nobel Prize acceptance speech in 1965). Works by Rudolf Clausius on entropy (1865), Ludwig Boltzmann on statistical entropy (circa 1870), Claude Shannon on the entropy of information (1950s). Chien-Shiung Wu (1957) experiments on parity violation during cobalt-60 beta decay. General Relativity (A. Einstein around 1915). First models of Big-bang that result, models proposed by Willem de Sitter, Alexandre Friedmann and Georges Lemaître in the 1920s. Discoveries in 1920 by Edwin Hubble of an expanding Universe, then in 1965 by Arno Penzias and Robert Wilson of a cosmic microwave background, two discoveries that confirm the models of the Big Bang and make it a theory. Analogies in the 1970s between "classical" thermodynamics and that of black holes, by Jacob Bekenstein and Stephen Hawking.

Finally, we will broaden our reflections to other fields than physics such as the phenomenologist approach of Edmund Husserl (early 20th century) or the Gaia hypothesis of James Lovelock (1979).

Chapter I Why does Time used in Physics seem so different from Time felt?

Purpose of the chapter

We highlight the contradiction between Time frequently used in Physics: a Time with 2 senses, treated in equations as a spatial dimension, and Time felt by us human beings which is oriented towards the future. To highlight this contradiction, the physicist A. Eddington introduced in 1927 the concept of the arrow of Time.

I.1 The Time of Physics, the Time Felt and the Arrow of Time

In previous memoirs, the dimension of Time and the three dimensions of Space have been treated identically. For example, the notion of rotation vector in a spatiotemporal plane has been proposed on the model of a rotation vector in a spatial plane. To obtain the source field equations applied to gravitation, we used a Time with 2 senses, a Time similar to a spatial dimension. A Time finally very different from the Time felt, in which we live.

In fact, this observation applies to many other physical models and theories, where the characteristics of the Time used often appear quite different from those of the Time felt. In particular, most of the fundamental equations of physics are invariant by reversing Time.

By contrast, the Time felt, in which we live, is oriented and does not resemble a spatial dimension. It seems to flow roughly linearly (although we all perceive an acceleration of flow as we age). In 1927, to characterize this felt and oriented Time, the physicist A. Eddington introduced the notion of arrow of Time. A notion that insists on the observation that Time, our Time, always seems to flow in the same direction and in the same sense. The arrow of Time is frequently referred to as a psychological arrow because it refers to our mind.

Thus, one of the fundamental questions of physics is this: why from mathematical equations most often reversible in Time, does our felt Time take a particular direction and a particular sense?

Note on the concrete duration of Henri Bergson (1859 – 1942)

Many scientists and philosophers have proposed to distinguish two forms of Time. Even if the terms used are often different from those of Felt Time and Physics Time, we can find similar or at least similar meanings.

For example, in *Essai sur les données immédiates de la conscience* (1889), H. Bergson opposes two notions of Time. The first notion is the Time that our intelligence represents itself. This Time is similar to a geometric scheme, it is homogeneous and conceived as a one-dimensional continuous.

The second notion is for H. Bergson the true nature of Time. It is real and has a concrete duration which is real progress, creation of new forms and continuous invention.

In this memoir, unlike H. Bergson, it is not suggested that one of the two Times is real and the other a simple representation. By contrast, we emphasize this demarcation between a Time of Physics (according to H. Bergson, the one represented by our intelligence) and a felt Time (according to H. Bergson, the one that is real and in continuous invention mode).

I.2 Time in the theory of Relativity, often presented as similar to a spatial dimension

In the scientific writings of A. Einstein, an idea keeps coming back. It is the treatment of the Time dimension in the image of the three dimensions of Space.

Despite the difficulty of intuitively accepting such a resemblance between Time and Space, A. Einstein and his successors found that the analogy between Time and Space is often fruitful in attempts to understand the Universe.

For example, in the theory of Relativity, we find an identical treatment between Time and Space in the proposal of a four-dimensional Space-Time continuum, with deformations of distances like those of durations, or in the local variable ct treated (nearly) as the local variables, x, y, z .

It is also observed in the notion of quadrivector Space-Time. In geometry, in a three-dimensional space, we usually use the notion of vector. Mathematicians generalize this notion of vector in a space with n identical dimensions.

We owe to H. Minkowski and A. Einstein the introduction of the Space-Time quadrivector in the theory of Relativity. Instead of working in a three-dimensional space, using vectors, or rather space tri-vectors to use analogous terminology, we work in a four-dimensional space-time, using Space-Time quadrivectors. This implicitly assumes a temporal dimension of the same nature as the three spatial dimensions.

The theory of Relativity and its extension to electromagnetism and quantum physics, relativistic Quantum Electrodynamics (QED), use much space-time quadrivectors: the quadrivector energy momentum (generalization of the tri-vector momentum), the quadrivector pulsation wave vector, the quadrivector electromagnetic potential vector, the quadrivector electric charge current densities, etc. All these quadrivectors manifest an identity of approach for Time and Space.

I.3 Most physical theories are reversible over time

Through a few examples, let us now emphasize that most physical theories, from Newtonian gravitation, through the theory of Relativity, to relativistic Quantum Electrodynamics are reversible in Time.

I.3.1 Theory of Gravitation of I. Newton

Let's start with the following thought experiment. Imagine a flying saucer A filmed by an experimenter E. The flying saucer has a muzzle velocity, then moves away from the Earth with engines off.

During the (usual) passage of the film towards the future, the experimenter E visualizes a decelerated motion of the flying saucer A under the effect of the attractive gravitational force of the Earth, in accordance with the theory of gravitation of I. Newton.

During the film's backward passage to the past, experimenter E visualizes an accelerated movement of flying saucer A approaching Earth. This flying saucer A undergoes an attractive gravitational force and accelerated motion, also in accordance with the theory of I. Newton.

This thought experiment emphasizes the reversibility in Time of the Gravitational theory of I. Newton. Whether the film has moved to the future or to the past, the images visualized are in agreement with the theory.

I.3.2 Theory of Relativity of A. Einstein and antimatter interpretation by P. A. Dirac

Previously, it was pointed out that the theory of Relativity treats Time and Space as if they were similar in nature. However, initially, the theory of Relativity does not give Time 2 senses as it does for spatial dimensions. This appears in the choice of a positive energy for the solutions of the equation:

$$E^2 = c^2 p^2 + m^2 c^4$$

The question of a temporal reversibility of the theory of Relativity arises from 1928, with the introduction by P. A. Dirac of a wave equation requiring energy solutions both positive and negative, and therefore implicitly reversible in Time:

$$E = \pm \sqrt{c^2 p^2 + m^2 c^4}$$

P. A. Dirac proposes to interpret negative energy as antimatter, that is to say antiparticles of the same mass and electric charge opposite to the corresponding particle of matter. For example, the antielectron (also called the positron) is the antiparticle of the electron.

At the time, the proposal of P. A. Dirac left most physicists incredulous. However, a few years later, in 1932, Carl David Anderson discovered positively charged electrons in the cosmic rays of the sky. The hypothesis of an antimatter becomes widely accepted by the scientific community.

Note

This hypothesis of antimatter is all the more interesting because it suggests an energy conservation on the scale of the Universe. If matter and antimatter are in the same quantity, the total energy balance of the Universe is zero.

I.3.3 The positron, the electron that goes back in Time and relativistic Quantum Electrodynamics

In the 1950s, R. Feynman proposed the following idea: the positron (or antielectron) is similar to an electron going back in Time. R. Feynman associates the suffix anti with the property of going back in Time.

This combination is called the prescription of E. Stueckelberg and R. Feynman, also referring to E. Stueckelberg who had a similar idea a few years earlier.

R. Feynman introduces this association into the equations of relativistic Quantum Electrodynamics. It should be noted that it is already found in the equations of electromagnetism of the 19th century.

Let be a particle of electric charge $-q$ going back in Time in an electromagnetic field. Let us show that this particle undergoes the same effects as another particle of charge q , of identical mass m , advancing in Time in the same direction as us, that is to say towards the future.

In an electromagnetic field, an electric charge responds to the equation of motion:

$$m \frac{d^2 \vec{X}}{d^2 t} = q \vec{E} l + q \frac{d\vec{X}}{dt} \wedge \vec{B}$$

The fields $\vec{E}l$ and \vec{B} derive from a potential quadrivector \vec{A} . We obtain the equation of motion:

$$m \frac{d^2 \vec{X}}{d^2 t} = (q) \frac{\partial \vec{A}}{\partial t} + (q) \frac{d\vec{X}}{dt} \times (\vec{V} \cdot \vec{A})$$

equivalent to:

$$m \frac{d^2 \vec{X}}{d(-t)^2} = (-q) \frac{\partial \vec{A}}{\partial(-t)} + (-q) \frac{d\vec{X}}{d(-t)} \times (\vec{V} \cdot \vec{A})$$

It can therefore be seen that a charge q , advancing in Time following moments t , presents an equation of motion equivalent to a charge $-q$, with the same mass m , and going back in Time following instants $-t$.

During the acceptance speech of his Nobel Prize, R. Feynman explains how the idea of an electron going back in Time, germinated in his mind. He himself was inspired by an idea by his former professor J. A. Wheeler, seeking to understand why all electrons in the Universe have the same mass and electric charge.

"Feynman!" J. A. Wheeler once exclaimed over the phone, "I know why all electrons have the same mass and charge."

"Why?" asked R. Feynman.

"Because they are all the same electrons!"

J. A. Wheeler then explained that all electrons were the same particle that goes back and forth in Time. When going to the future, it is an electron and when returning a positron.

"But, sir," said R. Feynman, "there are not so many positrons and electrons!"

"Oh, maybe they're hidden in protons or something," he replied.

R. Feynman retains only part of the original idea of his former teacher and applies it to the antielectron.

The drawing below complements R. Feynman's Nobel speech. It represents an electron moving in Space and Time. At points A and C, electron and positron annihilate. This annihilation can be interpreted as a U-turn in the Time of the electron, then transforming into a positron.

At the instant t_0 , We observe 3 electrons and 2 positrons. These back and forth in Time make it possible to generate matter and antimatter.

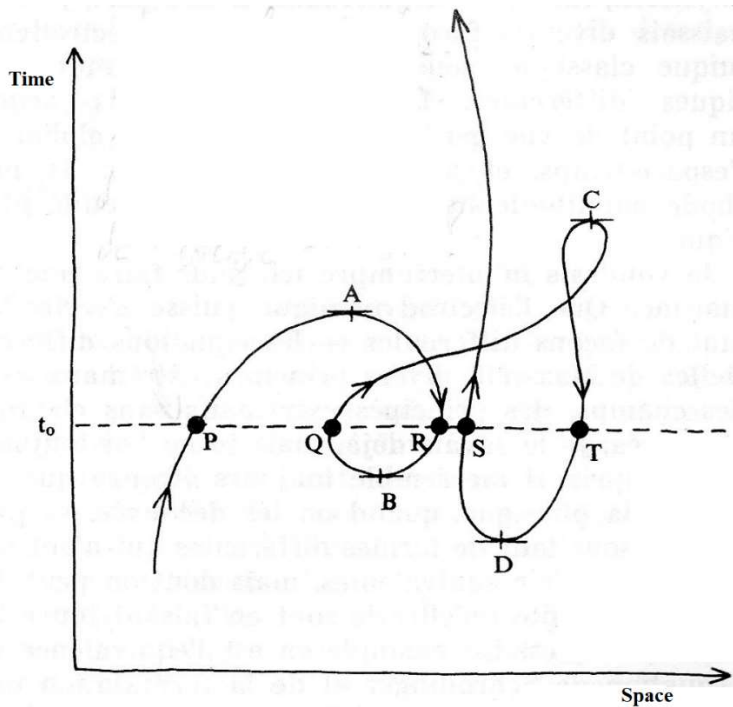


Figure 1: drawing by R. Feynman, the Universe line of an electron capable of going back in Time

Note, generate new chemical elements or isotopes by increasing the number of round trips in Time of a proton or nucleon?

We can have fun using the idea of J. A. Wheeler, to explain the different atomic numbers of atoms. Each nucleus of an atom would be generated by a single proton (of electric charge $Q = +1$) going back and forth in Time and therefore appearing as multiplied to an observer according to the number of round trips made. Uranium ${}_{92}\text{U}$ would be different from hydrogen ${}_{1}\text{H}$ per 92 round trips of the proton instead of 1. Nevertheless, we would have the same question as R. Feynman, where the electric charges $Q = -1$ have gone? Can they be hidden in electrons like what J. A. Wheeler retorts?

To try to explain neutrons, instead of seeing electric charges $Q = +1$ go into the future and electric charges $Q = -1$ return to the past, this would be the weak hypercharges $Y_w = +1$ that would go into the future and weak hypercharges $Y_w = -1$ who would return to the past. Each nucleus of an atom

would be generated by a single nucleon (of weak hypercharge $Y_w = +1$) going back and forth in Time and therefore appearing as multiplied to an observer according to the number of round trips made. Uranium isotope ${}^{235}_{92}\text{U}$ would be distinguished from uranium isotope ${}^{232}_{92}\text{U}$ by 235 back and forth of the nucleon instead of 232. However, we would have the same question again: where have the weak hypercharges gone? $Y_w = -1$? Can they be hidden in electrons and neutrinos both of $Y_w = -1$ and going into the past?

I.3.4 Repulsion of opposite charges and attraction of identical charges

From R. Feynman's idea of the electron that goes back in time, it is possible to understand in a very intuitive way why two opposite charges repel each other and why two identical charges attract.

In Quantum Electrodynamics, the electromagnetic interaction manifests itself for example during a photon exchange between two electrons of identical electric charges (with as a consequence of this exchange, a variation in the motion of the 2 electrons).

Intuitively, we can accept that two bodies that exchange a third, are pushed to move away according to the principle of conservation of momentum. Two electrons of the same electric charge, which exchange a photon, are thus pushed away (note that the 3 bodies have a momentum).

This is described in the figure below:

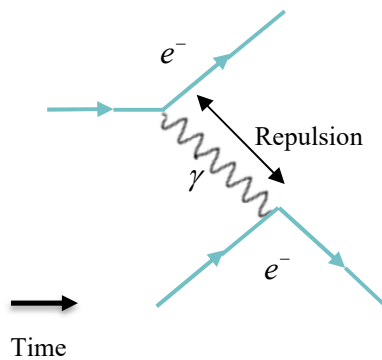


Figure 2: 2 bodies of the same electric charges repel each other2

The electromagnetic interaction also manifests itself during a photon exchange between an electron that advances in time and an electron that goes back in time, that is to say a positron.

As before, we can always accept that two bodies that exchange a third, are pushed to move away. Thus, in the sense of the past, electron and positron repel each other. And in the direction of the future, repulsion becomes attraction: electron and positron of opposite electric charges attract.

This is described in the figure below:

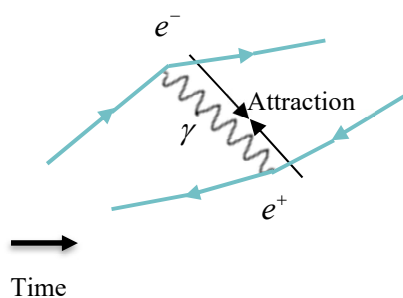


Figure 3: 2 bodies of opposite electric charges attract each other3

Note

Following the model of the Bohr atom, a first excited electron emits a photon, it sees its mechanical energy decrease by ΔE_m , it changes circular orbit and approaches, for example, the nucleus of the atom. The emitted photon has energy $E = \Delta E_m = h\nu$ and a momentum $p = \frac{E}{c} = \frac{\Delta E_m}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$. This photon is then absorbed by a second electron. This electron sees its mechanical energy increase by ΔE_m , it changes circular orbit and for example, moves away from the nucleus of the atom. We thus observe a repulsive electromagnetic interaction between the two electrons of the same electric charge, with the photon as an intermediate particle.

I.4 2-sens Time and principle of least action

I.4.1 Advantages and disadvantages of a 2-sens Time

The 2-sens Time hypothesis, identical to the three spatial dimensions has an undeniable advantage. Indeed, the model proposed for Space-Time is not complexified. On the contrary, it is simplified. The introduction of Time into mathematical equations is even singularly trivial, since it is identical to the 3 spatial dimensions.

The hypothesis of a two-way Time nevertheless presents a major handicap. It is the loss in passing of a principle a priori fundamental: the principle of causality difficult to conceive with a reversible Time.

To compensate for the loss of this principle of causality, the supporters of the reversibility of Time, in particular R. Feynman, insist on another principle frequently encountered in physics: the principle of least action.

I.4.2 The principle of least action

The first real physical approach to the principle of least action can be dated to the 17th century and to Louis Fermat, in the context of geometric optics and the study of light. L. Fermat insisted on minimizing the trajectory of light.

A century later, in 1744, Pierre Louis Moreau de Maupertuis extended this principle to Newtonian mechanics and the study of forces.

In 1756, Joseph-Louis Lagrange proposed a mathematical description of the principle and applied it to various phenomena in physics. For example, the trajectory of light in dispersive media, the classical laws of Newtonian dynamics.

In his 1942 thesis, R. Feynman was interested in this principle, he studied its adaptation to quantum mechanics.

To explain the principle of least action, let us take up the tasty parable of R. Feynman reported in his book *The Nature of Physics*.

On a sunny beach in the ocean, a lifeguard runs to rescue a girl bather who is drowning about twenty meters from the shore. The lifeguard symbolizes light or any other system that seeks to minimize its efforts. Athletic, the lifeguard nevertheless remains an earthling, he runs much faster than he swims. To minimize his efforts and travel time, he makes a much longer journey on the sand than in the water.

Let be v_{beach} the speed of the lifeguard on the beach, v_{ocean} the speed of the lifeguard in the ocean and c a reference speed. We have:

$$c > v_{beach} > v_{ocean}$$

The indices are defined:

$$n_1 = \frac{c}{v_{beach}}$$

$$n_2 = \frac{c}{v_{ocean}}$$

$$n_1 < n_2$$

The lifeguard's trajectory forms an angle at the level of the ocean beach passage (sand/water). We find the Snell-Descartes law for the refraction of light:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

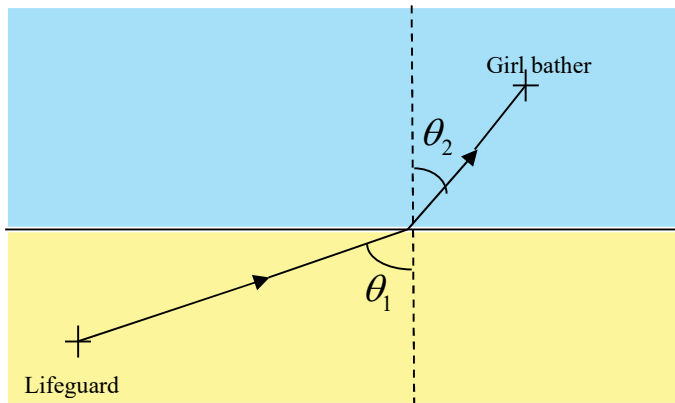


Figure 4: minimization of lifeguard action

In classical analytic mechanics, we define the action $S[q_2(t + \varepsilon), q_1(t)]$ from the integral of the Lagrangian $L(q, \dot{q}, t)$ over a period of time ε :

$$S[q_2(t + \varepsilon), q_1(t)] = \int_t^{t+\varepsilon} L(q, \dot{q}, t) dt$$

According to the mathematical expression of the principle of least action, with the action considered as an extremum, we have:

$$\delta S[q] = 0$$

We deduce the Euler-Lagrange equation:

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = 0$$

Note 1

In the hypothesis of a 2-sense Time, the principle of least action could favourably replace that of causality as the constitutive principle of the Universe. The debate obviously remains open.

Note 2

From the principle of least action, then from the Euler-Lagrange equation, it is possible to find the fundamental principle of Newton's dynamics. To see if it is also possible from this principle of least action to find other source field equations?

I.4.3 Spin magnetic moment and path integrals of R. Feynman

One of the great successes of relativistic quantum electrodynamics is to recover with great precision the experimental value of the spin magnetic moment $\vec{\mu}_s$ of the electron. This magnetic moment is described using the Landé factor g and the spin kinetic moment (or spin angular momentum) \vec{S} :

$$\vec{\mu}_s = g \frac{-e}{2m_e} \vec{S}$$

To recover this experimental value, we can use the path integrals of R. Feynman, built on the model of the action S . We introduce in relativistic quantum electrodynamics an analogous formulation of:

$$S[q_2(t + \varepsilon), q_1(t)] = \int_t^{t+\varepsilon} L(q, \dot{q}, t) dt$$

The use of R. Feynman's path integrals requires considering the 2 *senses* of Time, in order to find the experimentally measured value of $\vec{\mu}_S$. Indeed, it is necessary to introduce both particles that advance in Time, and antiparticles that go back in Time.

I.5 Conclusion of the chapter

Most physical theories are reversible in time. In Newtonian mechanics, in the theory of Relativity or in relativistic quantum Electrodynamics, the analogies Time Space are often promising. Above all, the hypothesis of a 2-sens Time makes it possible to find experimental results whose obtaining would be impossible otherwise.

This contradicts the felt Time in which we live, which presents an arrow frequently referred to as a psychological arrow, because it refers to our mind.

This contradiction between Time used in Physics and Time felt, appears today as one of the great enigmas of Physics.

In the next 3 chapters, we will nevertheless see that there are some cases in physics, theories or experimental facts, which do not seem reversible in Time. They are found, for example, in thermodynamics with the notion of entropy, in certain elementary particle experiments involving weak interactions, as well as in the cosmological theory of the Big Bang. For these three cases, we speak respectively of thermodynamic arrow, microscopic arrow, and cosmological arrow. It is these three arrows that we will now study.

Chapter II Existence of a thermodynamic arrow?

Objective of the chapter

We are interested here in the existence of a thermodynamic arrow that manifests itself in the notion of entropy.

II.1 Definitions of entropy

II.1.1 Clausius entropy

If most physical theories present a possible temporal reversibility, one of them, thermodynamics is an exception. Temporal irreversibility manifests itself in the notion of entropy, that evolves increasing in Time.

We remind the formulas of entropy of a system introduced in 1865 by Rudolf Clausius as part of the 2nd law of thermodynamics, according to the work of Sadi Carnot:

- for a reversible transformation in Time, we have:

$$dS_{sys} = \frac{\delta Q_{rev}}{T}$$

- for an irreversible transformation over time, we have:

$$dS_{sys} > \frac{\delta Q_{irrev}}{T}$$

Q_{rev} is the amount of heat received by a thermodynamic system during a reversible reaction,
 Q_{irrev} is the amount of heat received by a thermodynamic system during an irreversible reaction,
 T is the temperature of the thermodynamic system.

It is noted that during an irreversible reaction in Time, entropy S_{sys} can only grow.

II.1.2 Boltzmann statistical entropy, the measure of disorder

In the 1870s, Ludwig Boltzmann developed statistical thermodynamics. He shed new light on the notion of entropy by proposing statistical entropy of a system:

$$S_{sys} = k_B \log w$$

with $k_B \approx 1,38064 \times 10^{-23} m^2 kg \times s^{-2} K^{-1}$ the Boltzmann constant and w the number of possible states of the system studied.

Statistical entropy describes the probability of the existence of a system. It measures the degree of disorder of this system at the microscopic level. The higher the entropy of the system, the greater the number of possible states, the greater its probability of existence, the less its elements are ordered, related to each other. The share of energy that cannot be used to obtain a work is growing. According to statistical thermodynamics, if nothing is done, the system naturally tends towards disorder.

II.2 Time oriented in thermodynamics, bringing together entropy and information?

II.2.1 Maxwell's demon

In parallel with L. Boltzmann, J. C. Maxwell is also interested in statistical thermodynamics. He seems to have been the first to have made an implicit link between entropy and information through a thought experiment called Maxwell's demon.

Maxwell's demon is a hypothetical being that processes information and memorizes it. The demon measures the speed of gas molecules that reach the entrance of two compartments. It opens or closes a valve between the two compartments depending on the speed of the molecules. If the molecules go fast, he sends them to the 1st compartment. If they are slow, he sends them to the 2nd. This action builds two different compartments, one hot and the other cold.

Thus, the demon can reduce the entropy of a homogeneous gas at a given temperature. He reverses the course of thermodynamic Time, apparently acting against the second law of thermodynamics.

II.2.2 Shannon entropy, the measure of missing information for a receptor

Maxwell's demon has raised many studies and questions, in order to better understand how it can reverse the course of thermodynamic Time.

In 1929, Leo Szilard argued that creating information requires energy to account for how Maxwell's demon acts. In the 1950s, John von Neumann and Claude Shannon explicitly linked information and entropy.

Initially, C. Shannon did not know the work of L. Boltzmann on statistical entropy. He developed his own work on statistical information. We have for Shannon's formula on information:

$$I = k \log\left(\frac{1}{p}\right)$$

with I the amount of information contained in a source, missing information for a receiver and therefore to be acquired,

p the probability of an event,

k a constant.

If all events, in number w , are also likely, the probability of each is:

$$p = \frac{1}{w}$$

We then obtain the formula of the missing information to be acquired for a receiver:

$$I = k \log w$$

In examining the work of C. Shannon, J. von Neumann made the connection with that of L. Boltzmann. He then brought together entropy and missing information. Thus, I is now called Shannon entropy or information entropy.

II.3 Conclusion of the chapter

The temporal irreversibility of thermodynamics is manifested in the notion of entropy. Historically, this notion of entropy was introduced as a measure of disorder, which can only grow. At the end of the 19th century, J. C. Maxwell is the first to make the link between entropy and information via his demon. In the 1950s, disorder, entropy, missing information and to be acquired was associated. It was then possible to connect the arrow of thermodynamics to an arrow of missing information, arrows against which Maxwell's demon can apparently oppose by processing and memorizing the information.

Chapter III Existence of a microscopic arrow of the particles?

Objective of the chapter

We are interested here in the hypothesis of an arrow of Time at the level of elementary particles. We will focus on some breakthroughs in physics from the 1950s and 1960s. This physics has particularly studied symmetries P , C and T through particle decays.

III.1 Existence of a microscopic arrow of Time?

In the 1st chapter, it was pointed out that the laws of classical mechanics, as well as those of electromagnetism were possibly reversible in Time. Temporal reversibility even seems necessary for the relativistic quantum electrodynamics of the 1950s.

However, during the 1950s and 1960s, by studying collisions between elementary particles in an increasingly detailed way, some physicists detected signs of temporal irreversibility. This is what we will now study starting with some reminders about operators P , C and T .

III.1.1 Reminders on operators P and T

The operator P is a transformation that reverses the coordinates of Space (operation also called parity).

$$x \rightarrow x' = -x$$

Note

The orbital angular momentum L and the spin angular momentum S are not affected by the Parity operation. The energy and momentum are reversed. By inverting the coordinates of Space, we have:

$$\begin{aligned} L &\rightarrow L' = L \\ S &\rightarrow S' = S \\ p &\rightarrow p' = -p \\ E &\rightarrow E' = -E \end{aligned}$$

The operator T is a transformation that reverses the sense of Time.

$$t \rightarrow t' = -t$$

Note

By reversing the coordinates of Time, we have:

$$\begin{aligned} L &\rightarrow L' = -L \\ S &\rightarrow S' = -S \\ p &\rightarrow p' = -p \\ E &\rightarrow E' = -E \end{aligned}$$

III.1.2 Reminders on electric charge conjugation C

Conjugation of electric charge C is a transformation that reverses the electric charge of a particle. It also changes quantum quantities related to electric charge. We find orbital and spin magnetic moments, isospin I , hypercharge Y (and related numbers: baryonic number Ba , strangeness St , lepton number L_e , etc.).

Charge conjugation C does not affect a priori orbital and spin angular momentum, mass, momentum, average life of a particle.

Note 1

The only particles that remain invariant by charge conjugation C are the truly electrically neutral particles, i.e., those that have null their electric charge and bound quantum quantities (magnetic moment, isospin, hypercharge, etc.).

We find the photon γ , the boson Z^0 , and states formed of a particle and its antiparticle: $e^- \bar{e}^+$, the meson pion π^0 formed from a mixture of quark pairs $u\bar{u}$ and $d\bar{d}$.

Note that the neutron is not really electrically neutral since it has a positive hypercharge Y and a negative isospin I . There is therefore an electrically neutral antineutron of opposite hypercharge (negative) and opposite isospin (positive).

Note 2

The equations of electromagnetic interaction, as well as Maxwell's equations, are charge-conjugation invariant.

III.1.3 Symmetry and violation reminders

This is called symmetry of P , C or T , a conservation of the laws of motion of a particle by transformation P , C or T .

This is called a violation of P , C or T , a non-conservation of the laws of motion of a particle by transformation P , C or T .

III.1.4 Theorem CPT

The theorem CPT states that a transformation $R = CPT$, i.e., a combined operation of C , P and T leaves the laws of motion invariant for a particle.

The theorem CPT appeared for the first time in 1951, implicitly, in the work of J. Schwinger. J. Schwinger then sought to prove the correlation between spin and statistics. He tried to make the link between the statistics of Fermi-Dirac and Bose-Einstein. To understand why the electron has a half-integer spin while the photon has an integer spin, he turned his attention to the theorem CPT .

In 1954, G. Lüders and W. Pauli established a proof of the theorem CPT . Thus, this theorem is sometimes called the Lüders–Pauli theorem. At the same time and independently, the theorem is also proved by J. S. Bell. His proofs are based on the validity of the invariance of major physical laws vis-à-vis the Lorentz transform and the principle of locality in the interaction of quantum fields.

These demonstrations are based on the premise that the world in which we live is quantum, relativistic and causal at the same time. The experimental proof or refutation of the theorem CPT remains an open question today.

Note 1

If we accept the theorem CPT , a violation of CP involves a violation of T .

Note 2

Related to the theorem CPT , every particle has an antiparticle (which can sometimes be confused with itself, as in the case of the photon), of opposite or zero charge depending on the transformation C , of opposite momentum and helicity depending on the transformation P , which propagates by going back in Time depending on the transformation T .

III.2 Decay reactions β^\pm , violation of P and C , conservation of symmetry CP

III.2.1 Decay reaction β^- of cobalt-60, violation of symmetry P

In classical physics and relativistic quantum electrodynamics, the equations of motion are separately invariant by transformations P , C and T .

However, in the years 1956-57, it is surprising to note that the invariance by parity P at the spin level is not a priori satisfied in reactions involving the weak interaction.

Physicists Tsung-Dao Lee and Chen Ning Yang became interested in the conservation of parity in all fundamental interactions.. They propose to test their ideas to the experimental physicist C.-S. Wu Chien-Shiung Wu on decay reaction β^- of cobalt-60.

The decay reaction is shown schematically in the figure below. The momentums of the particles are in fine and black arrows, the spins of the particles are in fat and colored arrows. We deduce the helicities for electrons and antineutrinos products of decay β^- .

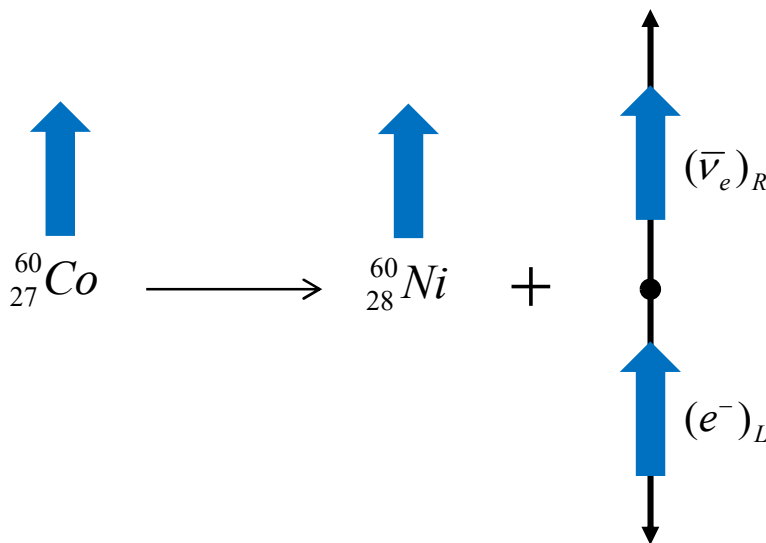


Figure 5: decay reaction β^- from cobalt to nickel

If parity P is retained, one must obtain as a product of the reaction as many electrons of right helicity $(e^-)_R$ as of left helicity $(e^-)_L$. However, C.-S. Wu only got left helicity electrons $(e^-)_L$ and right helicity antineutrinos $(\bar{\nu}_e)_R$ (the only ones that seem to exist).

For the decay β^- Cobalt-60, parity is therefore violated to the maximum, that is, no left helicity antineutrino is observed $(\bar{\nu}_e)_L$.

III.2.2 Reminders on the pions

Following the Yang-Mills theory, the pions π^+ , π^- and π^0 are the strong charged mediating particles of the strong interaction. They carry a strong charge and can change the strong charge of a nucleon.

The pions π^+ , π^- are of the scale operator type, they can increase or decrease by a I^3 the strong charge of a nucleon.

According to the Quark Model, the pions π^+ , π^- and π^0 are formed quarks u and d (and antiquarks \bar{u} and \bar{d}) from 1st generation. All have an isospin $I = 1$, a zero spin and a zero hypercharge. We have the following table where they are distinguished by the 3rd component of their isospin I^3 .

$\pi^+ u\bar{d}$	$\pi^0 (u\bar{u} - d\bar{d})/\sqrt{2}$	$\pi^- \bar{u}d$
------------------	--	------------------

$(I, I^3) = (1, +1)$	$(I, I^3) = (1, 0)$	$(I, I^3) = (1, -1)$
----------------------	---------------------	----------------------

III.2.3 Decay reaction β^\pm of pions π^+ and π^- , 1st generation particles, conservation of symmetry CP

The figure below shows the decays β^\pm observed from charged pions π^+ and π^- to anti-muon, muon neutrino, muon and muon antineutrino.

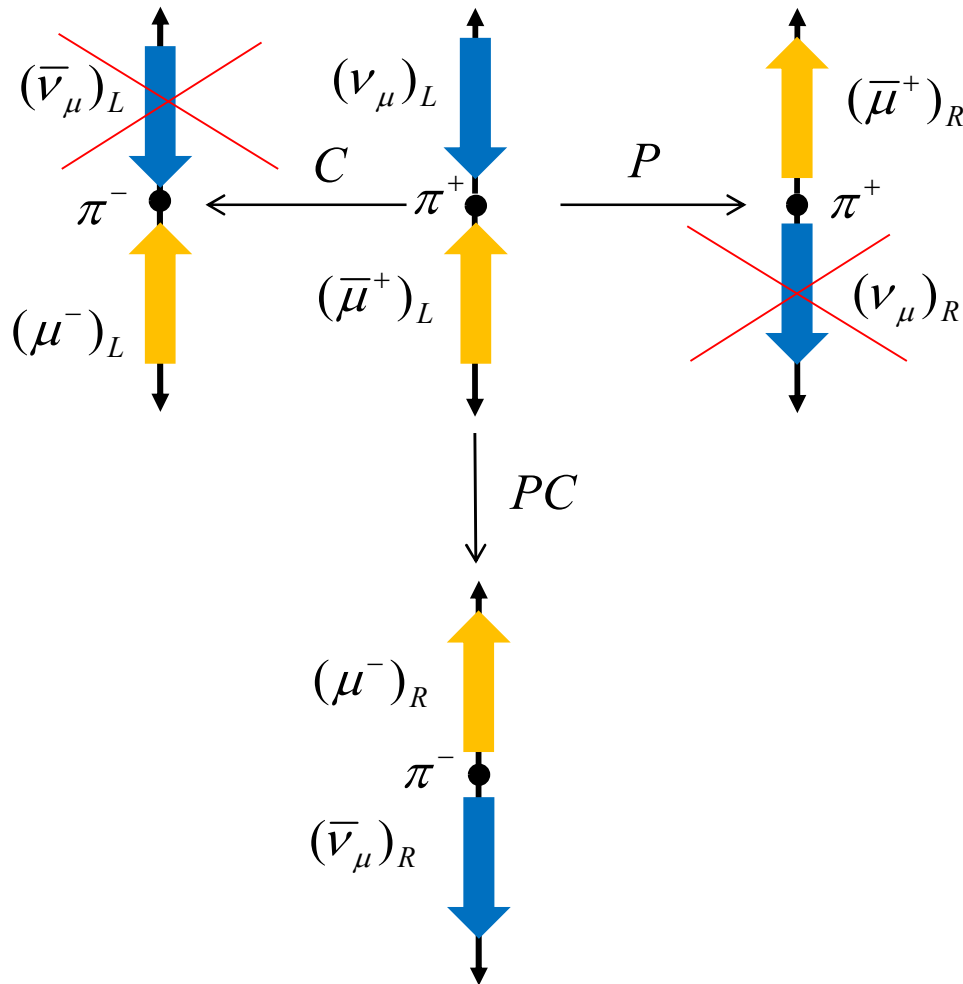


Figure 6: decay reactions β^+ and β^- of pions π^+ and π^-

We start from decay reaction from β^+ from a pion π^+ to a left-handed muon neutrino $(\nu_\mu)_L$ and a left-handed anti-muon $(\bar{\mu}^+)_L$. It is noted that symmetry P is violated to the maximum, that is, we do not observe any right-handed muon neutrino $(\nu_\mu)_R$ during decay reaction β^+ . Similarly, symmetry C is violated to the maximum, that is, left-handed muon anti-neutrino $(\bar{\nu}_\mu)_L$ is not observed during decay reactions β^- of a pion π^- .

By contrast, we are seeing a decay reaction β^- from a pion π^- to a right-handed muon anti-neutrino $(\bar{\nu}_\mu)_R$ and right-handed muon $(\mu^-)_R$. The passage from a decay reaction β^+ of a pion π^+ to a decay reaction β^- of a pion π^- corresponds to a transformation CP .

Thus, the transformation C only is impossible because left-handed muon anti-neutrinos do not exist. Similarly, the transformation P only is impossible because right-handed muon neutrinos do not exist. Since symmetries C and P are both violated to the maximum, the symmetry CP is retained. We find the conservation of the laws of motion of a particle by transformation CP .

In the case of pions, we stop at 1st generation quarks. We will now focus on 2nd generation quarks, that possess several non-zero strangeness St . We will study the physics of kaons and find in this case a violation of symmetry CP .

III.3 Physics of kaons, violation of symmetry CP

III.3.1 Reminders about kaons

According to the Quark Model, kaon mesons belong to the mediating particles of the strong interaction (i.e., kaons carry a strong charge and can modify the strong charge of a baryon).

Kaons are made up of quarks u , d and s (and their antiquarks). They are at the origin of the proposal of the number of strangeness St .

For kaons K^+ , K^- , K^0 , \bar{K}^0 , we have the following decomposition into quarks:

$K^+ u\bar{s}$	$K^0 d\bar{s}$
$\bar{K}^0 \bar{d}s$	$K^- \bar{u}s$

Let's take a closer look at neutral kaons and antikaons K^0 and \bar{K}^0 .

III.3.2 Decay reactions β^\pm of neutral kaons

The neutral kaons and antikaons K^0 and \bar{K}^0 can disintegrate into pions π^+ , π^- , π^0 . For example, we have decay reactions β^\pm where the weak interaction comes in:

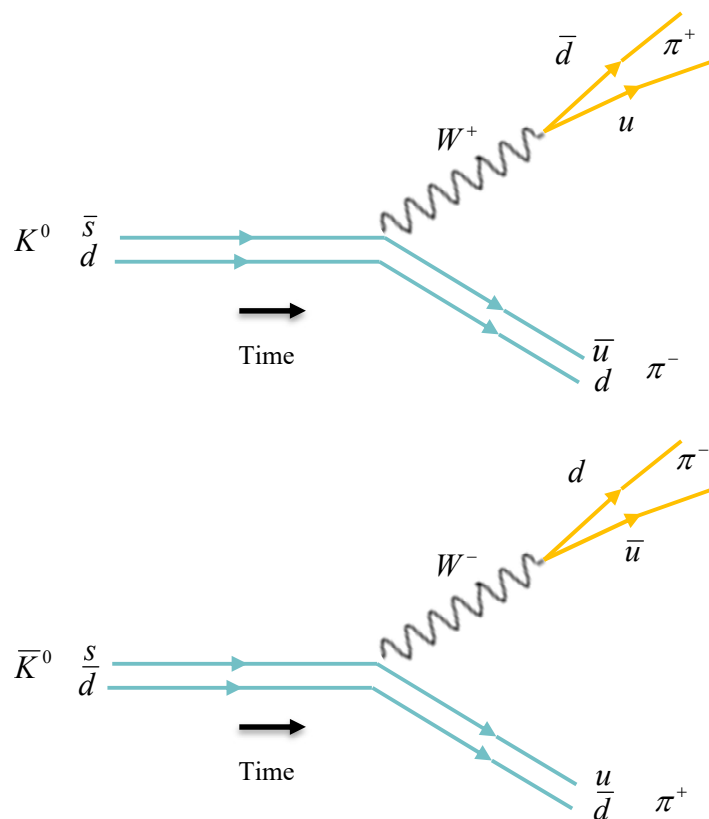


Figure 7: decay reaction β^+ of K^0 and decay reaction β^- of \bar{K}^0

Note
 Note that the number of strangeness St is not preserved during these 2 disintegrations.

In 1964, J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay demonstrated the violation of symmetry in the weak interaction by studying the properties of neutral CP kaons . Let us recall here their model.

III.3.3 Symmetry violation CP in weak interaction, model of Christenson-Cronin-Fitch-Turlay

Kaons are produced by strong interaction and decay by weak interaction. Just like pions, they have negative parity. We have the following transformations by the parity operator P :

$$P|K^0\rangle = -|K^0\rangle$$

$$P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

By charge conjugation C , we have:

$$C|K^0\rangle = |\bar{K}^0\rangle$$

$$C|\bar{K}^0\rangle = |K^0\rangle$$

By combining the two transformations CP , we get:

$$CP|K^0\rangle = -|\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = -|K^0\rangle$$

Let be $\eta_{CP} = 1$ and $\eta_{CP} = -1$, the eigenvalues of the charge conjugation operator C and parity operator P , associated with eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$. We have the relationships:

$$CP|K_1^0\rangle = |K_1^0\rangle$$

$$\text{with } \eta_{CP} = 1$$

$$CP|K_2^0\rangle = -|K_2^0\rangle$$

$$\text{with } \eta_{CP} = -1$$

We give the eigenstates of the transformation CP $|K_1^0\rangle$ and $|K_2^0\rangle$ depending on the states produced by strong interaction $|K^0\rangle$ and $|\bar{K}^0\rangle$:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

If symmetry CP is conserved in the weak interaction, the states $|K_1^0\rangle$ and $|K_2^0\rangle$ must represent observed particles, i.e., the eigenstates of the weak interaction. These particles decay by the way $\eta_{CP} = 1$ for the state $|K_1^0\rangle$ and by the way $\eta_{CP} = -1$ for the state $|K_2^0\rangle$, i.e., in two and three pions respectively.

Note, brief explanation to understand the decays of kaons K_1^0 and K_2^0 in 2 and 3 pions respectively

Let be a 2-pion system in a state of relative orbital angular momentum L , we have:

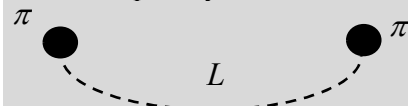


Figure 8: 2-pion system

We have the proper value of the transformation CP given by the formula:

$$\eta_{CP}(\pi\pi) = (-1)^2(-1)^L = (-1)^L$$

Analysis of angular distributions of pions reveals that $L = 0$.

So, we have:

$$\eta_{CP}(\pi\pi) = 1$$

For a three-pions system, with l and L the orbital angular momentum, we have:

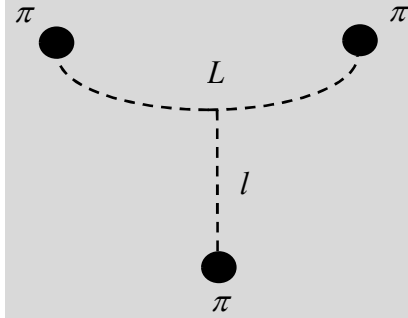


Figure 9: 3-pions system

We have the proper value of the transformation CP given by the formula:

$$\eta_{CP}(\pi\pi\pi) = (-1)(-1)^l(-1)^L$$

Analysis of angular distributions of pions reveals that $L = l$.

So, we have:

$$\eta_{CP}(\pi\pi\pi) = -1$$

Since the 2-pion system corresponds to $\eta_{CP}(\pi\pi) = 1$, K_1^0 must disintegrate into 2 pions. Similarly, since the 3-pion system corresponds to $\eta_{CP}(\pi\pi\pi) = -1$, K_2^0 must disintegrate into 3 pions.

Experimentally, we distinguish a long neutral kaon K_L with a long lifespan from a short neutral kaon K_S with a short lifespan. The kaons observed K_L and K_S are both eigenstates of the weak interaction.

If symmetry CP is conserved by weak interaction, we should observe a state $K_L = |K_2^0\rangle$ which decays only into 3 pions and a state $K_S = |K_1^0\rangle$ which decays only into two pions. We have the 2 decay reactions:

$$K_L \rightarrow \pi^+ + \pi^- + \pi^0$$

$$K_S \rightarrow \pi^+ + \pi^-$$

However, with a very low but not zero probability, there is a decay of K_L in two pions.

We have indeed the relations:

$$|K_S\rangle = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (|K_1^0\rangle - \varepsilon|K_2^0\rangle)$$

$$|K_L\rangle = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (\varepsilon|K_1^0\rangle + |K_2^0\rangle)$$

with $\varepsilon = 2,284 \pm 0,014 \times 10^{-3}$

Therefore, the 2 eigenstates of the weak interaction $|K_S\rangle$ and $|K_L\rangle$ are not exactly identical to the 2 eigenstates of the transformation CP $|K_1^0\rangle$ and $|K_2^0\rangle$. This indicates that symmetry CP is violated by weak interaction.

Note 1 on the CKM matrix

To consider the violation of symmetry CP in weak interaction, Makato Kobayashi and Toshihide Maskawa proposed in 1973 a 3rd generation of quarks. They introduced a complex phase δ in the CKM matrix, Cabibbo-Kobayashi-Maskawa matrix comprising 3 generations of quarks. The phase δ introduces a term that violates the symmetry of the transformation CP .

$$M_{CKM} = \begin{pmatrix} c_1 & c_3 s_1 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + c_3 s_2 e^{i\delta} \\ s_1 s_2 & -c_1 c_3 s_2 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

Note that with the Cabibbo matrix with two generations of quarks, the violation of symmetry CP is not considered.

Note 2 on the theorem CPT

If we accept the theorem CPT , a violation of symmetry CP involves an (indirect) violation of T-symmetry.

Note 3, direct violation of T-symmetry?

There are experiments that suggest a direct violation of T-symmetry. This violation was observed in 1998 during the CPLEAR experiment at CERN. It also covers neutral kaons.

The CPLEAR experiment shows that the probability of oscillation of a neutral kaon in its antikaon is not strictly equal to its image by time reversal: the probability of oscillation of an anti-kaon in kaons. Schematically, we have:

$$K^0 \rightarrow \bar{K}^0 \neq \bar{K}^0 \rightarrow K^0$$

Note 4, on the weak anti-charge

In Memoir 5, it was pointed out that the passage from matter to antimatter, i.e., the operation CP is related to electromagnetic interaction. The analogue for the weak interaction would be the operation that reverses the weak charge C_T (and perhaps also the operation P).

As mentioned, the spin S is retained when passing from matter to antimatter, whereas it should be changed when switching from weak charge to weak anti-charge.

Similarly, the number of generations Ge reverses when passing from matter to antimatter, whereas it should be retained when passing from weak charge to weak anti-charge.

In this case, we can reasonably accept for the weak interaction, a conservation of symmetry $C_T P$, a violation of symmetry CP , while refusing a violation of symmetry T .

The theorem CPT would then only apply to electromagnetic interaction.

III.4 Conclusion of the chapter, existence or not of a microscopic arrow?

In conclusion, the question of the existence or not of a microscopic arrow is still much debated. In this chapter, we have discussed the Christenson-Cronin-Fitch-Turlay model which highlights in neutral kaons a violation of symmetry CP in weak interaction.

Nevertheless, in this Memoir, we will lean towards the absence of violation of symmetry T at the level of elementary particles. That is, we will apply the theorem CPT only to electromagnetic interaction. Anyway, the question remains open, and no real new argument will be brought here.

In the next chapter, we will focus on the cosmological arrow and the origins of the Big Bang theory.

Chapter IV Cosmological arrow and elements on the theories of General Relativity, the Big Bang, and black holes

Purpose of the chapter

We give some elements on the theories of General Relativity, the Big Bang and black holes, highlighting the temporal irreversibility encountered.

The Big Bang theory is first evoked, from the beginnings in the 1910s with the theory of general relativity, until the discovery in the 1960s of fossil electromagnetic radiation. We conclude with some great ideas of the thermodynamics of black holes proposed in the 1970s.

IV.1 Historical Reminders about the Theory of General Relativity

IV.1.1 The General Relativity of A. Einstein (1910s)

During the 1910s, A. Einstein developed the theory of General Relativity that applies to gravitation. His main objective was to respect the following principle: the laws of Nature remain the same during any change of reference frames (for example an accelerated movement, a rotational movement, etc.).

To develop his theory of gravitation and find an equation comparable to Newton's universal law of gravitation $\vec{F}_{Gr} = -G \frac{mM}{r^3} \vec{r} = m\vec{G}r$, A. Einstein started from an analogy with the potential source equation of Poisson gravitation:

$$\nabla^2 V^t = 4\pi G\rho$$

V^t is the Newtonian potential,

G the gravitational constant,

ρ the mass density.

The idea of A. Einstein is to extend the Poisson gravitational equation to the 4 dimensions of Space-Time. For this, in the theory of General Relativity, $\nabla^2 V^t$ is equated with the Ricci tensor R_{ij} (the terms of $\nabla^2 V^t$ and of R_{ij} both have second derivatives). The mass density ρ is assimilated to the tensor T_{ij} , the energy-momentum tensor.

A. Einstein proposed a first equation:

$$G_{ij} = R_{ij} = \frac{4\pi G}{c^4} T_{ij}$$

G_{ij} is called the Einstein tensor.

The problem is that in analogy with $\nabla^2 V^t$ which has a zero derivative, Einstein's tensor $G_{ij} = R_{ij}$ thus defined does not have a null covariant derivative.

In order to obtain a zero covariant derivative, the Einstein tensor is then redefined with:

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$$

R is the scalar curvature.

g_{ij} is the metric tensor.

A. Einstein proposed a second equation:

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = \frac{8\pi G}{c^4} T_{ij}$$

Note 1, Einstein's gravitation equation seen as a generalization of Gauss gravitational, from Space to Space-Time

Einstein's gravitation equation is an equation between a source (Energy momentum tensor T_{ij}) potential (G_{ij}) in a 4-dimensional Space-Time. It can be interpreted as a generalization of the 3 dimensions of Space to the 4 dimensions of Space-Time of the source potential of Poisson gravitation: $\nabla^2 V^t = 4\pi G\rho$ or of the "usual" source field Gauss gravitational equation:

$$\oiint_{\Sigma} \vec{G}r(M) \cdot d^2 \vec{S} = \iiint_V -4\pi G\rho_{mass} dV = -4\pi GM_{int}$$

$$div \vec{G}r = -4\pi G\rho_{mass}$$

Note 2, differences with the source field equations proposed in this essay

A first difference is at the level of the energy momentum tensor T_{ij} , for all that is momentum density or energy flow. In the source field equations proposed here, for momentum densities, we have replaced the constant G by the constant $\frac{l_p^2 c}{\hbar}$ or $\frac{1}{\hbar}$ (with l_p the Planck length).

Another difference is at the level of the Ricci tensor R_{ij} , Einstein's equation has no terms of the type $-\frac{\partial V^x}{\partial t}$ present for example in the wave vector $K_{xt}^{1/y}$. There is therefore no consideration of gravitational phenomena analogous to those of electromagnetic induction $-\frac{\partial A^x}{\partial t}$.

IV.1.2 The Minkowski metric

We remind the metric g_{ij} given in its differential and general form:

$$ds^2 = g_{ij} dx^i dx^j$$

Following the ideas of A. Einstein, the space-time interval ds^2 must be retained regardless of the change of reference frames.

In the case of Special Relativity, we stop at the changes of Galilean (or inertial) reference frames. The space-time interval to be kept is:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

We have the so-called Minkowski metric:

$$g_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV.1.3 The K. Schwarzschild metric (1915)

In 1915, Karl Schwarzschild proposed a simple solution of Einstein's equation in the case of spherical symmetry.: gravitational field generated by a spherical mass, static (without rotation) not charged and surrounded by vacuum.

This mass can be a star, a planet or a Schwarzschild black hole. The resolution is given in spherical coordinates (Schwarzschild metric in polar coordinates):

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 dS$$

where:

- ds is the space-time interval of an infinitesimal displacement dt , dr , $d\theta$, $d\phi$ from the point P center of spherical symmetry,
- t , r , θ , ϕ are the so-called Schwarzschild coordinates of the point P in Space-Time,
- t is the Time coordinate at which we consider the point P (measured by a clock located at an infinite distance from the massive object),
- r is the radial coordinate of the point P ,
- θ is the colatitude of the point P in radians,
- ϕ is the longitude of the point P in radians,
- $dS = d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ the space interval on the surface sphere S can grow or decrease,
- $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius of the massive object, with G the gravitational constant, M the mass of the object, and c the speed of light.

Note 1 on the Schwarzschild radius or radius of the horizon of a black hole

We also often talk about $r_s = \frac{2GM}{c^2}$ of the horizon radius of a black hole. According to General Relativity, photons, within the radius of this black hole, cannot escape.

Note 2 on the Schwarzschild radius, Planck mass and quantum micro black holes

Planck mass is sometimes defined as the mass of a particle, whose reduced Compton wavelength would be equal to the Schwarzschild half-radius.

The reduced Compton length is equal to:

$$\frac{\lambda_c}{2\pi} = \frac{\hbar}{m_p c}$$

and the Schwarzschild half-radius is equal to:

$$\frac{r_s}{2} = \frac{Gm_p}{c^2}$$

We have:

$$\frac{\lambda_c}{2\pi} = \frac{r_s}{2}$$

$$\frac{\hbar}{m_p c} = \frac{Gm_p}{c^2}$$

We obtain the Planck mass:

$$m_p = \sqrt{\frac{\hbar c}{G}}$$

Note that the reduced Compton length and the Schwarzschild half-radius are then both equal to the Planck length:

$$\frac{\lambda_c}{2\pi} = \frac{r_s}{2} = \sqrt{\frac{G^2 \hbar c}{c^4 G}} = \sqrt{\frac{G \hbar}{c^3}} = l_p$$

For this type of particle, we speak of quantum micro black holes or Planck particles.

IV.2 Historical Reminders on the Big Bang Theory

IV.2.1 Model of the Static Universe and Cosmological Constant (1917)

Starting from Newtonian mechanics and the Poisson gravitational equation, there had already been attempts to describe the general state of the Universe. In 1917, in a famous article entitled *Cosmological Considerations on the Theory of General Relativity*, A. Einstein proposed a cosmological model of the Universe, based on the equation he had proposed a few years earlier.

A. Einstein used what is now called the cosmological principle. This principle states that man does not occupy a privileged position in the Universe. This results in a homogeneous and isotropic Universe, that is to say similar to itself regardless of the place and direction of observation.

To the cosmological principle, A. Einstein implicitly added another hypothesis, that the Universe is static, that is to say does not evolve with Time. He proposed to transform the Poisson equation $\nabla^2 V^t = 4\pi G\rho$, by adding what he called a Universal constant Λ (called today cosmological constant). He obtained:

$$\nabla^2 V^t - \Lambda V^t = 4\pi G\rho$$

Note on the spring universe

If we change the sign of Λ and that the source be cancelled, with $\nabla^2 V^t + \Lambda V^t = 0$, we can interpret the introduction of Λ , as if the Universe were likened to a big spring, making oscillations but not extending infinitely (i.e., contrary to the idea of an expanding Universe).

For a spring:

$$\frac{d^2 x}{dt^2} + \omega_0^2 x = 0$$

$$\text{with } \omega_0 = \sqrt{\frac{k}{m}}$$

We have a solution like:

$$x = A \cos(\omega_0 t) + B \sin(\omega_0 t)$$

A. Einstein made a similar approach for his equation $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi}{c^4}T_{ij}$. He transformed it by introducing a cosmological constant. Λ and obtained:

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R - g_{ij}\Lambda = \frac{8\pi G}{c^4}T_{ij}$$

Note

The following form is also often found, with Λ of contrary sign:

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

IV.2.2 Expanding Universe model, Friedmann-Lemaître-Robertson-Walker metric (1920s)

In the 1920s, Willem de Sitter, Alexandre Friedmann and Georges Lemaître used Einstein's equation (the one without cosmological constant) to develop a model of the universe that was no longer static, but expanding. It was the 1st model of Big Bang, even if at the time, this name was not yet attributed to it.

This model of an expanding universe is described by the so-called Friedmann-Lemaître-Robertson-Walker metric, which partly uses the Schwarzschild metric (especially for space intervals):

$$ds^2 = c^2 dt^2 - R(t)^2 \left(\frac{1}{1 - kr^2} dr^2 + r^2 dS \right)$$

where:

- $R(t)$ is the radius of the Universe. The sign of $\dot{R}(t)$ provides information on the evolution of the Universe. $\dot{R}(t) > 0$ for an expanding Universe, $\dot{R}(t) < 0$ for a shrinking Universe and $\dot{R}(t) = 0$ for a static Universe, all considered at the moments t .
- k is the curvature of Space, $k = \{-1, 0, +1\}$. If $k = 0$ the model is said to be flat (corresponding to the Euclidean space of Special Relativity), if $k = -1$ the model is said to be open (corresponding to hyperbolic geometry), and if $k = +1$ the model is said to be closed (corresponding to a spherical geometry).
- $dS = d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ the space interval on the surface sphere S that can grow or decrease.
- t is Cosmic Time.

Looking at this metric, we find that it contains a number of implicit assumptions:

- that the Space interval varies in proportion to the factor $R(t)^2$, which is a function of the moments,
- finally, that **Time is oriented** from an origin.

Unlike the majority of physical theories (including General Relativity) that are possibly reversible in Time, this 1st Big Bang model is postulated non-reversible in Time.

The idea of a growth (or decrease) of the Universe from an initial moment is implicitly contained in the Friedmann-Lemaître-Robertson-Walker metric. This will be found in the other models of Big bang.

IV.2.3 Redshift (1929)

The ideas of W. de Sitter, A. Friedmann and G. Lemaître of an expanding Universe met above all skepticism, even indifference. In 1927, G. Lemaître published an article whose audience remained confined. He proposed the existence of a constant relationship between distance and speed of distance from certain nebulae (now called galaxies).

In 1929, Edwin Hubble published a long and detailed article, this time managing to reach a wide scientific audience. He found a redshift in the discontinuous spectrum of light emitted by the chemical elements of galaxies. The redshift is proportional to the distance of galaxies from our Galaxy, the Milky Way. This shift is reminiscent of the Doppler-Fizeau effect of a light wave emitted by a source that moves away from the observer.

E. Hubble then proposed that galaxies move away from the Milky Way, with a speed proportional to their distance from our Galaxy, this is Hubble's law.

IV.2.4 The cosmic microwave background predicted in the 1940s and discovered in 1965

In the 1950s, for the first time, the model was referred to as the ironic Big Bang by physicist Fred Hoyle, who himself defended static state models.

In 1965, the Big Bang model received further experimental confirmation with the discovery of the cosmic microwave background. To fully understand, we must go back a few years earlier, to the Big Bang model proposed by George Gamow, Ralph Alpher and Robert Herman.

The expansion of the Universe naturally induces a greater density in the past. Just as a gas heats up when compressed, the universe must also have been hotter in the past.

In the 1940s, G. Gamow, R. Alpher and R. Herman established that the Universe must be filled with electromagnetic radiation that loses energy due to expansion. This electromagnetic radiation must

have been all the more intense as the Universe was dense. It must still exist today, although considerably weakened.

Since the work of Gustav Kirchhoff in 1862, Joseph Stefan in 1879, Ludwig Boltzmann in 1884, Wilhelm Wien in 1896, as well as Max Planck in 1900, we link electromagnetic radiation emitted by a source (called black body when considered an ideal object) and its temperature T .

We recall below the law of radiation of Planck, correction of that of Wien for low frequencies, which is also valid for its historical interest because it introduces the Planck's constant h :

$$L_\lambda = \frac{C_1}{\lambda^5} \frac{1}{e^{\frac{C_2}{\lambda T}} - 1}$$

L_λ monochromatic energy luminance,

λ wavelength,

T temperature in kelvin,

$C_1 = 2\pi hc^2$ with c the speed of light and h the Planck's constant,

$C_2 = \frac{hc}{k_B}$ with k_B the Boltzmann's constant.

In their work, G. Gamow, R. Alpher and R. Herman suggested that the current temperature of electromagnetic radiation in the Universe can be calculated from knowledge of the age of the Universe (about 13.8 billion years), the density of matter, and the abundance of helium. .

In 1964-1965, Arno Allan Penzias and Robert Wilson discovered electromagnetic radiation, now called the cosmic microwave background or fossil radiation. According to the predictions of G. Gamow, R. Alpher and R. Herman, the radiation corresponds to that of a black body at low temperature (2.7 kelvins) (the predicted temperature being however significantly different).

IV.3 Issues raised by the Big Bang, sources of inspiration

Following its experimental confirmations, the Big Bang model acquires the status of a theory. Nevertheless, like many fascinating theories, the Big Bang raises more questions than it solves. Here we cite the problem of the observer, we then address the sources of inspiration of the theory.

IV.3.1 The observer's problem

« The observer is as essential to the creation of the Universe as the Universe is to the creation of the observer. »

J. A. Wheeler

To continue with this sentence by J. A. Wheeler, one of the main questions raised by the Big Bang theory is this paradox of a relativistic theory, which is built without observers, and which is verified by observations (redshift, cosmic microwave background) that require an observer.

An answer to this contradiction will be suggested in the following chapters.

IV.3.2 Sources of inspiration

There are two types of influence in the Big Bang theory, some of mystical inspiration, others of scientific inspiration.

We first note the idea of an original design, with a moment of creation of the Universe (and therefore possibly a Creator), then a march of the Universe in perpetual growth towards a destiny traced, even mystical.

We then note the influence of evolutionary theories of species (theories developed especially in the 19th century by Jean-Baptiste de Lamarck, Charles Darwin, etc.). The universe is described as a huge

organism that would have been built little by little. In all these theories, we note the principle of a History and a Time oriented.

IV.4 Thermodynamics of black holes

We are mainly interested here in the simplest black holes: Schwarzschild black holes. They use the Schwarzschild metric seen earlier. They have a Schwarzschild radius $r_s = \frac{2G}{c^2}$, a mass M , but no charge Q or angular momentum J like Kerr-Newman black holes. The Kerr-Newman black holes respond to a modified metric (relative to that of Schwarzschild) to solve the equation of A. Einstein.

IV.4.1 Principles of black hole thermodynamics

In the 1970s, Jacob Bekenstein and Stephen Hawking clashed in a fruitful debate of ideas, involving analogies between "classical" thermodynamics and black holes, "cosmic objects" theorized in General Relativity. This debate and other work will give rise to the thermodynamics of black holes. This paragraph summarizes some of the main ideas of this debate. We will see that these analogies are based in particular on the identification between:

- 2 intensive quantities: temperature T of a system and surface gravity κ of a black hole,
- 2 extensive quantities: entropy S of a system and the surface A of a black hole.

For a black hole at rest, according to A. Einstein, we ask an internal energy:

$$U = Mc^2$$

with M the mass of the black hole.

Following the formula proposed in 1972 by L. Smarr, we calculate the surface variation A of a black hole, to which we inject a small amount of energy matter dMc^2 and a small amount of work δW_{BH} :

$$dMc^2 = \frac{c^2\kappa}{8\pi G} dA + \delta W_{BH}$$

with κ the surface gravity of the black hole.

In 1972, J. Bekenstein proposed the notion of black hole entropy S_{BH} (BH like black hole or Bekenstein Hawking), with entropy S_{BH} proportional to the surface A of the black hole.

S. Hawking and J. Bekenstein found that when two black holes merged, the surface A of the resulting black hole is always greater than the sum of the surfaces of the two initial black holes. We therefore have, like entropy $dS \geq 0$, $dA \geq 0$.

From 1973, J. Bekenstein, S. Hawking, James M. Bardeen and Brandon Carter developed analogies between the 4 principles of thermodynamics and principles applying to black holes. A table of these analogies is given below.

	Thermodynamics	Black holes
Principle zero	The temperature T of a system is the same everywhere when it is at thermal equilibrium	Surface gravity κ is constant over the entire surface of the black hole
First principle	$dU = TdS + \delta W$ $dU = \delta Q + \delta W$	$dMc^2 = \frac{c^2\kappa}{8\pi G} dA + \delta W_{BH}$ $dMc^2 = TdS_{BH} + \delta W_{BH}$ $dMc^2 = \delta Q_{BH} + \delta W_{BH}$
Second principle	The variation of entropy dS is positive (or zero) in any transformation involving a closed system	Surface variation dA is positive (or zero) in any transformation involving black holes

Third principle	Inability to obtain $T = 0$ by a physical process	Inability to obtain $\kappa = 0$ by a physical process
------------------------	---	--

Note on state functions

We speak of state function of a system, when it does not depend on the different possible paths travelled by the system in the past: it depends only on the current state of the system. In other words, the history of a system does not affect the state functions that characterize it.

In thermodynamics, internal energy U or entropy S are good examples of state functions.

Mathematically, we represent their infinitesimal variations by an exact total differential: dU and dS . Work W is not a state function, it depends on the path taken by the system. Mathematically, we represent its infinitesimal variations by a partial differential: δU .

If we strictly follow the analogy between thermodynamics and black holes, the surface A or entropy S_{BH} must be state functions. Their infinitesimal variations must be represented by exact total differentials: dA and dS_{BH} .

IV.4.2 Temperature and radiation of black holes

In 1974, S. Hawking proposed that black holes radiate and inspired by Planck's radiation law, that they have a temperature T_{BH} proportional to κ :

$$T_{BH} = \frac{\hbar\kappa}{k_B 2\pi c}$$

with k_B the Boltzmann's constant.

This temperature T_{BH} proportional to κ , validates the identification of T to κ .

Note, temperature of a Schwarzschild black hole

In the case of a Schwarzschild black hole, we have:

$$r_s = \frac{2GM}{c^2}$$

According to Newton's universal law of gravitation, we have for the surface gravity of a spherical body:

$$\kappa = \frac{GM}{r_s^2}$$

In the case of a Schwarzschild black hole, we have:

$$\kappa = \frac{c^4}{4GM}$$

We obtain the temperature of a Schwarzschild black hole:

$$T_{BH} = \frac{\hbar c^3}{8\pi k_B GM}$$

From the identification of $\frac{c^2\kappa}{8\pi G} dA$ to δQ_{BH} :

$$\delta Q_{BH} = \frac{c^2\kappa}{8\pi G} dA$$

and the temperature of a black hole:

$$T_{BH} = \frac{\hbar\kappa}{k_B 2\pi c}$$

S. Hawking calculates the entropy of a black hole:

$$dS_{BH} = \frac{\delta Q_{BH}}{T_{BH}}$$

By replacing with the appropriate terms:

$$dS_{BH} = \frac{\delta Q_{BH}}{T_{BH}} = \frac{c^2 k dA}{8\pi G} \frac{k_B 2\pi c}{\hbar k} = \frac{k_B c^3 dA}{4G\hbar}$$

he gets as a function of the surface A and Planck length l_p :

$$S_{BH} = \frac{k_B A c^3}{4G\hbar} = \frac{k_B A}{4l_p^2} = k_B \ln w$$

with $w = \exp(\frac{A}{4l_p^2})$ the number of possible states of the system studied (not to be confused here with work).

Note, entropy of a Schwarzschild black hole

In the case of a Schwarzschild black hole, we have:

$$r_s = \frac{2GM}{c^2}$$

$$A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4}$$

$$S_{BH} = \frac{k_B A}{4l_p^2} = \frac{4\pi k_B G M^2}{\hbar c}$$

The term $S \leq \frac{4\pi k_B G M^2}{\hbar c}$ corresponds to the Bekenstein limit. According to J. Bekenstein, if the entropy of a system exceeds this limit, the system becomes a black hole.

IV.5 Conclusion of the chapter

We have discussed in this chapter two "cosmic objects" directly derived from General Relativity and the ideas of A. Einstein.

The first, the Big Bang is one of the few physical theories postulated irreversible in Time. The theory presents both astonishing predictions that have received confirmation and a conceptual problem with the absence of an observer.

The second, the black hole is not postulated irreversible in Time. On the other hand, it has surfaces A and entropies S_{BH} which can only grow over time, and which also contain temporal irreversibilities.

Chapter V Demarcation between Life and non-Life, the phenomenological approach, the Gaia hypothesis

Purpose of the chapter

To answer the distinction between the Time of physics and the Time felt, it is proposed to characterize Life by an oriented Time. We will call this oriented Time, the biological Time.

V.1 Demarcation between Life and non-Life

V.1.1 A characterization of Life?

According to biologist Ladislav Robert, a researcher specializing in immunology and aging, "Time has existed in biology since the appearance of Life on Earth."

This relationship between Time and Life challenges us in our initial question posed in this Memoir: why does Time used in physical equations seem as different as Time felt or that used in Biology?

This leads us to propose the following hypothesis: can the demarcation between temporal reversibility and temporal irreversibility be the same as that between non-living and living?

This hypothesis has the advantage of proposing a simple answer to our initial question. What would characterize the non-living, would be a possible temporal reversibility. And as has been pointed out, most physical theories have such freedom.

What would characterize the living, would be a temporal irreversibility, created from original conditions, with loss of freedoms such as that of temporal reversibility, but offering in return the principle of causality, the possibility of a History and that of a destiny.

When we observe in the three dimensions of Space material bodies, they are rarely symmetrical in the three dimensions of Space like spheres. Spatial symmetry is simply possible, it is only very rarely required. By analogy between Time and Space, temporal symmetry is only possible.

In summary, what would distinguish the non-living from the living is for the first a possible temporal symmetry and for the second, a characteristic temporal asymmetry, which is called the arrow of Time.

Note on the film of a living being

When we play the film of a living being upside down, we quickly notice the anomaly. On the contrary, when you play the film of a planet or asteroid in motion, it is almost impossible to distinguish right sense from bad sense.

For some natural phenomena such as rain or snow, we also see if they have passed upside down or in the place. To see if these natural phenomena cannot actually be assimilated to Life?

V.1.2 Characterization of life by L. Pasteur

The demarcation between the non-living and the living is not easy, as the living has the same molecules, the same atoms as the non-living. In the second half of the 19th century, Louis Pasteur proposed the most famous, and probably the most enigmatic, characterization of the living. Taking up the work of E. Mitscherlich, he found that tartrate modifies the polarization of light, while a substance a priori identical: paratartrate does not modify this polarization.

To understand this, remember that tartrate comes from living matter, while paratartrate comes from mineral matter. Tartrate and paratartrate are both so-called chiral substances, i.e., composed of non-superposable molecules with their symmetric in a mirror.

Tartrate, capable of altering the polarization of light, is composed of a single chiral type of molecules. We speak of a substance of molecules, either dextrorotatory or levorotatory. The tartrate is for example dextrorotatory.

Paratartrate, unable to change the polarization of light, is composed of both chiral types of molecules. We are talking about a substance of molecules that are both levorotatory and dextrorotatory.

A precursor, L. Pasteur assumed that the ability to modify the polarization of light (i.e., being molecules of a single chiral type) is a characteristic of living (or formerly living) matter. He thus interpreted as living entities that were not previously considered in this way. These are yeasts, microorganisms, bacteria... Following this, L. Pasteur developed vaccination and experiments on the immune system. This will also pave the way for antibiotics.

This characterization of Life (of a single chiral type) implies the existence of a spatial orientation specific to Life. To see if there is also a spatiotemporal orientation specific to Life and a link between a single chiral type of molecules of life and the arrow of biological Time?

V.1.3 Dextrorotatory, levorotatory, series D and L

A dextrorotatory molecule ("which turns right", from the Latin dexter, right) thus has the property of deflecting the polarization plan of polarized light, to the right of an observer receiving light. This observer sees the polarization plan rotates clockwise.

A levorotatory molecule ("which turns left", from the Latin laevus, left) thus has the property of deflecting the polarization plan of polarized light, to the left of an observer receiving light. This observer sees the polarization plan rotates counterclockwise.

In biochemistry, we speak of D or L series. The series D or L are differentiated, according to the Fisher representation, by the order of the groups carried by the asymmetric central carbon C:

- Aldehyde group COH or carboxylic group $COOH$, upstairs,
- hydroxyl group OH or amino acid group NH_2 , right or left depending on whether the molecule is dextrorotatory or levorotatory.

Carbohydrates are series D. We have for example for Glyceraldehyde, with the form encountered in the living circled in green:

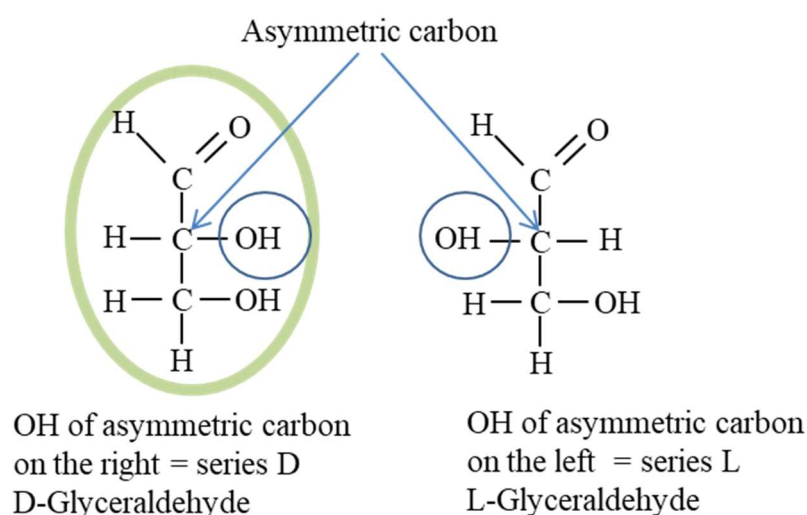


Figure 10: chiral forms of glyceraldehyde

Amino acids, constituting proteins, are L-series. We have for example for Alanine, with the form encountered in the living circled in green:

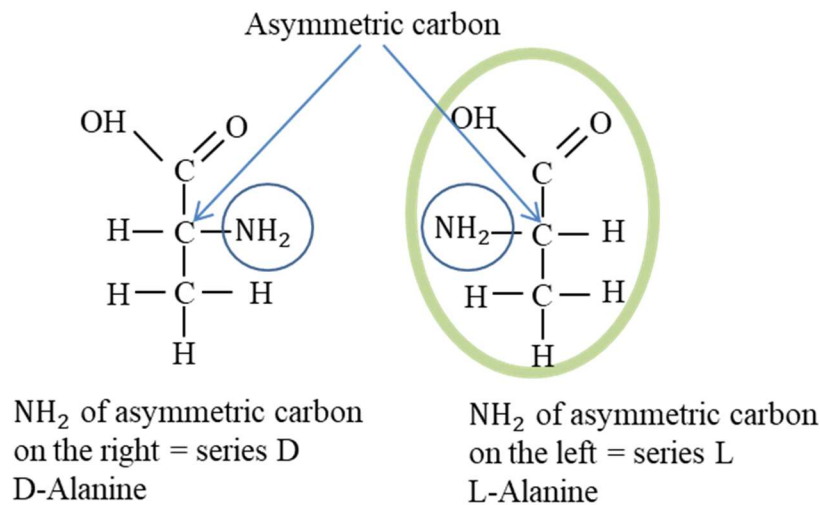


Figure 11: chiral forms of Alanine

Nevertheless, the D or L series do not always correspond to the dextrorotatory or levorotatory properties of the molecule. For example, the amino acid L-Serine is indeed levorotatory, but the amino acid L-Alanine is dextrorotatory!

Let us remember that the living selects certain series: D for carbohydrates and L for amino acids, and that this remains for the moment unexplained.

V.1.4 The illusion of the passage of Time? Is it Time passing or us passing?

In most physical theories or models (except thermodynamics with entropy or the Big Bang), there is no need to introduce a flow of time. For this reason, some physicists, and philosophers, such as recently Carlo Rovelli or Thibault Damour, have spoken of the illusion of the passage of Time, partly joining Plato's idea of a "timeless reality".

In current theories of physics, it has already been pointed out that the assumption of a Time similar to the three dimensions of Space works surprisingly well. This hypothesis makes it possible to obtain much better results than the hypothesis of a differentiated Time of Space. The hypothesis of a Time similar to the three dimensions of Space is also a strong argument in favor of the illusion of the flow of Time.

Yet, in our daily lives, we have a strong feeling of the passage of Time. It is even according to Jean-Paul Sartre, our existence (in a way our immersion in the flow of Time), which precedes our essence and thus defines us.

It should also be noted that in life sciences such as biology, there is little or no reference to a Time similar to the three dimensions of Space. The hypothesis of a Time that passes, different from Space, is usually necessary and implicitly postulated.

It is this contradiction, it must be admitted disturbing, between the hypothesis made by contemporary physics of a Time similar to the three dimensions of Space and the observation of the flow of Time in our daily life (joining the necessary hypothesis of a flow of Time in biology), that we try here to understand.

It is suggested that the flow of Time in one direction, that is, the arrow of Time, is the first characteristic of Life. To paraphrase Irish author Ken Bruen (and many others before him). In reality, time does not pass. We are the ones who pass.

V.2 The phenomenological approach

V.2.1 Ontological knowledge

The idea of bringing the arrow of Time closer to Life, and consequently to the subject, is part of a vast current of ideas, between science and philosophy, called phenomenology. The initiators of this current of ideas were at the beginning of the 20th century the philosophers Edmund Husserl, Martin Heidegger or in France Maurice Merleau-Ponty. The interest of phenomenology is the richness of the applications offered, to all possible fields.

The basic idea of phenomenology is that the phenomenon, a priori external to the subject, actually contains the main information about him.

The frequently cited example is that of musical notes. For a simple tape recorder, these notes are only a series of sound waves, but for a human subject, they express a melody, sensations, memories, beliefs, hopes.

Phenomenological philosophers are wont to say that knowledge of a phenomenon is ontological (from ancient Greek, onto, what is). That is to say, the phenomenon teaches us more about the subject, the being who observes and thinks, than about the object that is observed.

Phenomenological ideas developed from the 1910s. Originally, they were probably influenced by the physical ideas of the time, in particular by the principle of relativity, brought up to date by H. Poincaré and A. Einstein.

For example, according to the principle of relativity, there is no absolute velocity, but a difference in velocity between an object and a subject. Velocity measurement learns as much about the measured object as it does about the measuring subject.

In the years 1920-1930, phenomenological ideas in turn influenced the quantum physics of N. Bohr, W. Pauli or W. Heisenberg, for example on the dependence of the result of the experiment on the subject.

At the time, thanks to the Schrödinger equation, physics and chemistry were unified. This provoked contagious optimism among scientists. On this impulse, W. Pauli predicted a unification of the study of the psyche and physics (i.e., psychology and physics) into a new and forthcoming science.

V.2.2 The example of common sense

In a short essay, entitled *Invitations to Cognitive Science*, biologist Francisco Varela focuses on cognitive science, artificial intelligence, and how the human mind works. He proposes answers that are part of the phenomenological approach, answers ultimately close to a subjective explanation of the arrow of Time.

To introduce phenomenological ideas into cognitive science, F. Varela emphasizes the inadequacy of approaches containing a predefined Universe to subjects and their minds. He takes as an example and argument common sense, which he defines as an identical judgment, that we human beings bear in the face of an event (or more generally a phenomenon) experienced collectively.

A part of achievements is detectable in the common sense: human beings with the same culture, the same history, are predisposed to make identical judgments in the face of an event.

There is also a part of innate in common sense, a part of innate a priori enigmatic, which does not make it possible to understand why human beings make identical judgments in the face of an event.

To explain this part of innate in common sense, F. Varela suggests that reality and the perception that we have of it, is due as much, if not more, to our nature as human beings, than to external Nature.

Thus, since reality and the perception that we have of it, are mainly built through our nature as human beings, and since we human beings share an identical nature, this explains the existence of identical judgments carried by the common sense of human beings.

V.2.3 Can the activity of Life create reality?

In his essay, F. Varela extends his reasoning. His idea is not only that reality is partly defined mentally by the subject himself, but that it is also physically created by the subject.

To explain this, F. Varela forges the concept of enaction. A concept that suggests that the activity of Life, during its History, physically transforms and creates reality.

Indeed, on a daily basis, we see that our activities (artistic, sporting, scientific or other) modify the reality that surrounds us. Through our decisions and actions, we have the means to physically transform a part of our reality.

V.3 One application, the Gaia hypothesis

V.3.1 Life builds an environment conducive to its development

In the 1970s, chemist James Lovelock proposed that the Earth's surface and the atmospheric layer around it form a hyper organism that he named Gaia. He then referred to an Earth goddess from Greek mythology.

Following an analogy with each of our organisms, which are made up of multiple cells and microorganisms interacting with each other, Gaia would consist of all living things on Earth (as well as their production), interacting with each other.

In his book *The Earth is a Living Being*, the Gaia Hypothesis, J. Lovelock reports that the source of inspiration for the Gaia hypothesis is its questions about how to detect the presence or absence of Life on Earth's sister planets: Mars and Venus.

He answers his questions by suggesting to analyse the atmospheric composition of these two planets. Mars contains 96,0 % of CO_2 , and Venus 96,5 % of CO_2 , a composition a priori similar to the primitive Earth of about 4 billion years ago. In view of these compositions, J. Lovelock deduces that no action of Life intervenes in the atmospheric composition of the two planets. He retains that Life is certainly not present on Mars and Venus.

Note

Like L. Pasteur, J. Lovelock also seeks to characterize Life. The first does it by molecules of a single chiral type, the second does it by the action of Life on the atmospheric composition.

Subsequently, J. Lovelock develops the idea that the atmosphere of the Earth has gradually differentiated from that of its two sister planets by the action of Life on Earth. As a result, he refutes the idea that the conditions of the early Earth were especially more advantageous than those of Venus or Mars.

According to him, to explain the presence of Life on Earth, we cannot simply argue that our planet was well positioned, just at the right distance from the Sun, in order to benefit from temperatures conducive to Life.

He maintains that Life has gradually built on Earth an environment favourable to its own development. As in a globally virtuous retroactive loop, the earth's environment has been transformed by the action of Life. This has favoured the appearance of living forms that are increasingly rich, more and more diversified, more and more complex.

V.3.2 Major step in the transformation of the Earth's environment

According to J. Lovelock, we can distinguish several major stages of transformation of the terrestrial environment.

As a major step, J. Lovelock cites the appearance of photosynthesis with two important consequences for Gaia. On the one hand, photosynthesis used solar energy, CO_2 and water to produce organic matter. Through this new mechanism, organic matter has grown much faster than in the past.

On the other hand, photosynthesis has greatly increased the level of dioxygen O_2 present in the atmosphere.

Dioxygen O_2 has a great oxidation power, it caused the disappearance of many microorganisms that could not resist it. However, resistant microorganisms have been able to use this power to produce their own energy. It was the appearance of breathing. By "burning" glucose (and other organic substances) from O_2 , the cells produced energy much more easily and increased their ability to move.

J. Lovelock compares the appearance of O_2 in the atmosphere to high-voltage power lines. These high-voltage lines are much more dangerous than low-voltage, but they can transport larger amounts of energy, much further, without much energy loss by joule effect.

Note 1

Antioxidants are often presented as active agents against the aging of our cells. And it's probably true! The fact that our cells are still highly sensitive to oxidation and that this accelerates their aging, is the mark that the ancestors of our cells once developed in an atmosphere that contained little or no O_2 .

Note 2

In the Darwinian model of natural selection, individuals advantaged by environmental conditions have a reproductive advantage and are therefore more likely to have significant offspring.

J. Lovelock complicates the Darwinian model by adding a feedback loop that gives individuals the ability to modify the environment. Environmental modification can take a direction that either disadvantages or advantages individuals.

V.3.3 Many definitions and characterizations of Life

There are many ways to define and characterize Life and its productions (e.g., termite mounds, coral reefs, megacities like Tokyo or New York, and if we accept the Gaia hypothesis, Gaia herself).

For thermodynamic biologists, Life is characterized by its ability to generate negative entropy, that is, order.

For molecular biologists who study DNA, RNA of our cells, Life is characterized by its ability to perform replication, transcription, and translation autonomously. Thus, according to them, a virus is not alive, in the sense that it needs the machinery of a host cell to perform replication, transcription or translation.

For L. Pasteur, Life is characterized by molecules of a single chiral type. This enigmatic characterization is certainly the most fascinating.

For his part, J. Lovelock seeks to characterize Life by the atmospheric derivatives it produces (such as O_2) and which would not be found in the same proportions in an environment without Life.

In this chapter, we propose to characterize Life by an oriented Time. For example, when we film living beings or their productions, and we play this film backward or forward, the original sense of the film is easily identifiable, especially thanks to the chronological succession of causes and effects to be respected.

V.4 Conclusion of the chapter

In this chapter, it has been proposed to characterize Life by its ability to orient one of the four spatiotemporal dimensions, advancing continuously in one direction and creating what is called "biological" Time. This biological Time would contain the precious principle of causality and allowed the constitution of a History. That would have been to advantage of the Life.

A parallel was then drawn between this characterization of Life, phenomenological ideas and the Gaia hypothesis. According to this hypothesis, Life on Earth, during a virtuous circle, would have modified its environment to its advantage for billions of years.

In the next chapter, we return to the Big Bang, a rare physical theory to postulate itself irreversible in Time.

Chapter VI The origin of Life, the Big Bang a biological phenomenon?

Purpose of the chapter

We are interested in the origin of Life, we wonder if the Big Bang can be interpreted as a pure biological phenomenon.

VI.1 Origin of Life according to biologists

VI.1.1 Biological phenomenon and time-oriented phenomenon

In the previous chapter, it was proposed that any time-oriented phenomenon is a characteristic of Life, that is, a biological phenomenon.

Previously, we talked about 3 arrows in physics: the thermodynamic arrow, the microscopic arrow and the cosmological arrow.

Regarding the thermodynamic arrow, like Maxwell's demon, Life is able to go against the growth of disorder, against the growth of entropy and against the growth of missing information. By acting against the second law of thermodynamics, Life can reverse the course of thermodynamic Time, and follow its own Time: biological Time.

Regarding the microscopic arrow, we suppose in this Memoir for its absence at the level of elementary particles. The CPT theorem is a priori valid only for the electromagnetic interaction and not for the weak interaction.

It now remains to try to understand the cosmological arrow, the Big Bang theory, and the entropy of black holes. This is what we will study in this chapter, first through the origin of Life.

VI.1.2 Proteins and DNA, metabolism, and information

Life is mainly made up of the symbiosis of two radically different entities. On the one hand, there are chains of amino acids more commonly known as proteins. Their main function is to catalyze chemical reactions in the body, i.e., the metabolism of this organism, and thus to ensure a certain thermodynamic stability.

On the other hand, there are chains of nucleotides of which RNA and DNA are the most famous representatives. According to our current knowledge, these are the only carriers of genetic information.

Models on the origin of Life can be divided into two broad categories. On the one hand, there are models that postulate the primacy of nucleotide, RNA or DNA chains. These models are related to information theories, since RNA and DNA are carriers of genetic information.

On the other hand, there are models based on the self-regulating power of living entities, on chemical reactions and therefore more on proteins. This power of self-regulation can be interpreted as a restoration of order against disorder, that is to say a fight against the 2nd law of thermodynamics (the irreversible growth of entropy). These models are related to thermodynamic theories. They also have a link to information if we associate the growth of entropy with the growth of missing information.

Today, RNA models are favoured by biologists. Indeed, in the 1980s, Tom Cech and Sidney Altman independently discovered that certain RNAs, then called ribozymes, have a catalytic role like proteins. Following this, the hypothesis of the RNA world developed, with RNA considered as the primordial entity of Life.

VI.1.3 Prerequisites for the appearance of Life

In his book *The major transitions in Evolution*, biologist and former aeronautical engineer John Maynard Smith defines in chronological order the major stages of Life:

1. Replication (production of clones thanks to the RNA world hypothesis),

2. Catalysis (first RNA as both gene and enzymes, then DNA as a gene and proteins as enzymes),
3. Symbiosis (for example, symbiotic union of prokaryotic cells without a nucleus, and appearance of eukaryotic cells with nucleus),
4. Sexuality, gene exchange to train new individuals,
5. Appearance of multicellular living beings,
6. Emergence of societies of individuals,
7. Emergence of human societies endowed with culture and language.

We note that J. M. Smith places replication before catalysis. He also gives primacy to theories of information and RNA, over thermodynamic and protein theories. He also takes up the ideas of Lynn Margulis and gives importance to the symbiosis between living entities in the major transitions of Evolution.

Note on the RNA World

There are still many questions about the RNA world. For example, how did natural selection play out to create RNA? Were there several competing genetic codes?

VI.1.4 Physical approach to a biological problem

Biologists have stuck to the RNA world hypothesis, but RNA is already emerging as a formidable complexity for physicists studying the Quark Model. It seems that between quarks and RNA, there are many steps to take. We need other prerequisites for the appearance of Life, much further upstream. Without claiming to be exhaustive, we can mention:

- the problem of Time,
- the primacy of matter, in the face of antimatter,
- the constitution of the atom, chemical elements, and molecules.

VI.1.5 The evolving Universe, seen as an increasing complexity with Life gradually emerging

Another way of approaching the problem of the origin of Life is not to pose a fundamental difference between the living and the non-living. The Universe appears as an evolving entity, with an increase in complexity over biological time. There is no discontinuity between a Universe that was once inert and without subjects, and a Universe endowed with Life and therefore subjects.

Basically, Life differs from inert matter by an increasing complexity in its organization. In a way, we solve the problem of origin, since the origin of Life merges with the origin of the Universe as the initial instant point of the beginning of complexification. Life is not radically different from inert matter: it is already present during the Big Bang, or at least underlying.

We can call this hypothesis animistic in the sense that everything in the Universe is alive, or is not radically different from it. While this approach shares similarities with the ideas presented here, it nevertheless presents a contradiction with what has been proposed previously. Indeed, in this case, Life would not be radically distinguished from matter by its orientation in Time.

VI.2 The Big Bang, the origin of Life?

« Some believe that from matter is born Life, but in 10,000 years of that, will not realize that from Life is born matter? »

Louis Pasteur

In this paragraph, inspired by this sentence of L. Pasteur, we will suggest that it is not from matter that Life is born, but on the contrary, that it is from Life that matter is born.

VI.2.1 Find a subject for the Big Bang theory

We have already mentioned the discovery in 1965 by A. Penzias and R. Wilson of a cosmic microwave background, which spectacularly confirms the predictions of Gamow Alpher Herman's Big Bang model.

However, according to phenomenological ideas, the Big Bang theory and its experimental confirmation pose a problem. Indeed, in the first, the subject is absent, only the object exists, or rather begins to exist, while in the second, subject and objects exist, highlighting an inconsistency between theoretical prediction and experimental confirmation.

One can even wonder if the discovery of a cosmic microwave background, with subject, can be the confirmation of a theory with no subject. The disadvantage is that in some phenomenological approaches, the object without subject does not really make sense. It remains indefinable.

However, since this cosmic microwave background must mean something, and it seems to confirm in an astonishing way the theory of the Big Bang, it is tempting to postulate for the era of the Big Bang the existence of a subject. This then makes its coherence between theoretical prediction and experimental verification, both including an object subject relationship.

This subject is obviously not a human being, it could even be the very beginning of Life, this phenomenon then merging with the Big Bang.

VI.2.2 A deep connection with our ancestors that allows us to perceive the cosmic microwave background?

We are reminded of the ideas of the biologist Ernst Haeckel, a fervent admirer of C. Darwin and evolutionary ideas.

At the end of the 19th century, E. Haeckel proposed a fascinating hypothesis: that ontogenesis summarizes phylogeny. That is to say, every living being summarizes during the first moments of its life, the evolution of species, or the long road that its ancestors once accomplished. Think, for example, of webbed fingers that appear in the human foetus and then disappear, each living being would trace an evolution in shorthand to that of its ancestors.

Even though the hypothesis of E. Haeckel has sometimes been put in difficulty, it seems today rather confirmed by the results of molecular biology.

For example, most animals can be classified as triploblastic, 3 germ layers in Greek (also called triploblastic or bilaterian). For triploblastics, all organs derive from one of these 3 germ layers, each formed by a single layer of cells.

These 3 leaflets appear early during embryogenesis, during gastrulation. A hollow ball, called the blastula and formed by a single layer of cells, folds in on itself and gives the gastrula formed by 3 layers of cells.

These successive states, blastula, gastrula, neurula for vertebrates, are crossed by all living beings of triploblastic type, and correspond according to the hypothesis of E. Haeckel, to ancestral beings, that is to say to the evolutionary stages of the species to which the living being belongs.

According to this hypothesis, we human beings would still carry within us a part of the most primitive characteristics of Life. Characteristics that would connect us to the very first moments of Life, characteristics that would still allow us today to perceive the cosmic microwave background?

VI.2.3 Links between cosmological models (temporally irreversible) and biology?

Like the postulate of a temporal irreversibility of the Big Bang, which introduces a link between cosmological models and biology, do black holes and their entropies with temporal irreversibility, also introduce a link between cosmological models, classical entropy, information, and biology?

Big bang, cosmological black holes or quantum micro black holes, can also be grouped in what are called gravitational singularities, that is to say that they correspond to regions of Space-Time where according to Einstein's equation:

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = \frac{8\pi G}{c^4} T_{ij}$$

the gravitational field diverges infinitely.

Nevertheless, there are still many elements to be clarified, to understand the links between gravitational singularities, the origin of Life and that of each living being. The question remains open.

VI.2.4 Life builds a Universe favourable to its flourishing

Although it may only be beautiful fiction, let us now tell the following story that would have begun more than 13 billion years ago.

From a very free Universe, with 4 identical dimensions and without stories, Life would have been constituted by building the conditions necessary for its existence. It would have had the desire to advance continuously in one direction, in one of the 4 existing dimensions. It would have created a Universe of Life adapted to its development. Time oriented and organized matter would have appeared.

The so-called reasonable Universe of Life would also be very close to that imagined by G. Lemaître, G. Gamow, and many others, containing temporal irreversibility, the principle of causality, and singularities in Space-Time, such as these black holes or this Big Bang.

We would have gone from physics with a 2-sense Time, based on the principle of least action, to History with an oriented Time, based on the principle of causality.

The bang would be even more fascinating, it would have exploded not for the appearance of the Universe, but for that of Life.

The arrows of Time, that of the living (often described as psychological) and that of the Big Bang (often described as cosmological) would have a single origin: the living. To these two arrows, we could add the cognitive arrow, that is to say the arrow of intelligence and knowledge, which would oppose the arrow of entropy and missing information.

To generate this Universe of Life, living systems, to live together, would have advanced in the same way in Time, at every moment. They would have gradually shaped the Universe of Life as we know it today.

According to this hypothesis, Gaia would not only be the surface of the Earth and its atmosphere, but the Universe of Life as a whole.

Note 1 on cyclic systems in Time

There are many cyclic systems in space. Examples include aromatic molecules (benzene, etc.), food chains or rings networks. Following an analogy between Space and Time, we can imagine cyclical systems in Time, with a possible return to the past.

These cyclic systems in Time would not concern a priori living beings, except perhaps at the very beginning of Life.

Let us imagine that at the beginning of Life (actually just before the Big Bang and the Oriented Time), the precursors of living beings did not quite advance in one direction of Time, that this would have come later. These precursors could present cyclical systems in Time.

This kind of (a little crazy) idea could solve the chicken and egg problem, or the RNA and protein problem. One does not have to generate the other, since these two entities could once be associated in a cyclical system in Time.

In this case, the question is whether there are traces of these cyclical systems in Time, precursors of living beings. How could these cyclical systems in Time manifest themselves in what we see of the Universe today?

Note 2 on the existence of other life forms

Since the 1990s, many exoplanets have been discovered across our Galaxy. Some scientists assume the existence of life forms on these exoplanets. If so, do these life forms share a common origin with life on Earth or are they completely independent?

The question is somewhat reminiscent of the debates conducted in the 19th century on spontaneous generation. On the one hand, we have Félix Archimède Pouchet as a figurehead, who believes in the possible appearance of multiple forms of life. On the other, we have L. Pasteur, who believes in the uniqueness of the origin of Life.

In this Memoir, we propose to link the Big Bang to our lineage of life. This implies a singularity of our lineage and the virtual impossibility of discovering other independent lineages.

If we detect other living entities on other planets, then they will share with us a common origin, and they will have similarities.

Nevertheless, after all, perhaps there are other forms of life? But their time is not ours, their Big Bang is not ours. For us, they simply live in other Universes.

VI.3 My Big Bang

VI.3.1 The feeling of biological time

In the beautiful book *Me and the Others: Introduction to Genetics*, the geneticist and mathematician Albert Jacquard talks about my Big Bang, that is to say for everyone, of his original moment, to be interpreted differently according to the cultures.

For some the moment when the father's sperm meets the mother's egg, for others the birth, for still others the baptism a week after birth (in the latter case, perhaps a remnant of tradition in what once allowed to eliminate the most serious pathological cases?).

In a very accurate way, A. Jacquard compares the Big Bang of each, the beginning of an individual life, with the famous Big Bang of physicists, seen as the original moment of the Universe, and interpreted here as the beginning of Life.

In his book, A. Jacquard emphasizes that the feeling of biological Time is measured as a logarithmic scale. In the figure below, we have, for example, the linear scale of measured durations (in years) on the y-axis, and the corresponding logarithmic scale of the durations felt on the x-axis.

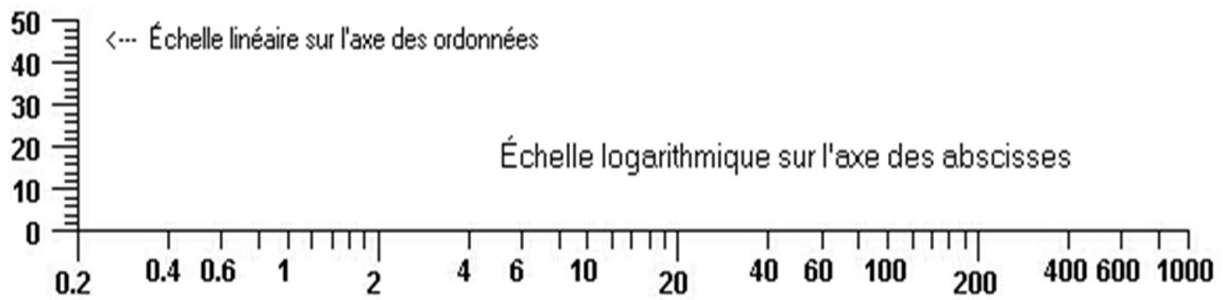


Figure 12: logarithmic scale (source Wikipedia)

As we get older, Time seems to pass faster and faster, the years go by. Similarly, the birth seems to date back an eternity, to come from an infinitely distant past. We don't even remember it anymore!

For now 0 of each, we have according to the logarithmic function: $\log 0 = -\infty$. The $-\infty$ of the birth of each individual seems to merge with the $-\infty$ of the Big Bang or the appearance of Life.

Again, this is reminiscent of the hypothesis of E. Haeckel, the recapitulation by ontogenesis of phylogeny. By reproducing the major stages of evolution, the living being traces the long path of Life, in a Time a priori accelerated according to a linear scale, but in fact with longer and longer durations when they tend towards the moment 0. This allows the living being to join the rest of the living, in a present where we all advance concomitantly at the same Time.

Note on the absence of a logarithmic scale to describe Time in Physics, digression on cosmic inflation

Let us observe that in no theory of current physics, a logarithmic scale is used to describe Time, even if the model of cosmic inflation developed in particular by Robert Brout, François Englert, Edgar Gunzig, Alexei Starobinskia, Alan Guth, a model that fits into the Big Bang theory, can still strongly think about it.

VI.4 Conclusion of the chapter

In the previous chapter, it was proposed to characterize Life by its oriented Time. Thus, any physical phenomenon in Oriented Time would in fact be a biological phenomenon.

In this chapter, it has been proposed to interpret the Big Bang as a biological phenomenon¹. It would correspond to the beginning of Life, that is, to the moment when Life had the desire to advance continuously in the same direction and in one direction. Each of us would also come from an individual Big Bang.

In the next chapter, we will focus on the processes implemented by living beings to keep into the same Time, and thus live together.

¹ Some may see it as an idea similar to those developed by Lee Smolin in his book *Life of the Cosmos*. L. Smolin proposes in particular to apply natural selection to the birth of universes.

Chapter VII How to live together at the same time? The Light Time Hypothesis

Objective of the chapter

In this chapter, we first return to the ideas developed by A. Einstein in the theory of Special Relativity on the deformations of distances and durations. Starting from the idea of a deformable Time, we then focus on the processes used by living beings to fit into the same moments, that is to say to live together.

Idea 9. The speed of light in Space, always measured constant regardless of the movement of the observer, could be a characteristic of the advance in Time, identical and shared, of human beings living together.

VII.1 The apparent incompatibility between the principle of relativity stated by G. Galileo and the propagation of light

VII.1.1 Preamble

In 1916, A. Einstein published a small popular work entitled *Relativity* where he recalled his vision of Special and General Relativity. For the writing of this book, he was inspired by a long review article also published in 1916 *The foundations of the theory of General Relativity*² taking up many ideas developed since his first articles in 1905.

In chapter 7 of *Relativity*, A. Einstein returns to the apparent incompatibility between the principle of relativity stated by G. Galileo and the experiments carried out at the end of the 19th century by A. Michelson and E. Morley, on the propagation of light and the search for a lumiferous ether. According to these experiments, the speed of light is always measured constant, regardless of the motion of the observer, thus contradicting the principle of Galilean relativity.

In chapters 8 to 13, one can only be won over by the solution proposed by A. Einstein to settle this apparent incompatibility. He first questions the notion of simultaneity, then proposes the notions of deformations of durations and distances (deformations of Space-Time), thus reconciling the principle of Relativity and the propagation of light.

Note

This translates mathematically into the replacement of the Galileo transform by the Lorentz transform seen in the 1st Memoir.

In the following paragraphs, we will propose an alternative and intuitive approach to the notions of deformations of durations and distances, in order to find the main results of Special Relativity. The approach will also make it possible to better understand the notions of deformations of durations and distances, at least to explain them differently.

Note, consider interstellar travel from one end of the Milky Way to the other?

Related to the question of "why a speed limit c of light?", we have the question of "what is the profound difference between:

- phenomena such as light or electromagnetic waves, which have a propagation speed limited by c in space,
- and phenomena such as the gravitational force, the electromagnetic force, and apparently quantum entanglement, which have an a priori instantaneous effect from one point to another in space?»

² "Die Grundlage der allgemeinen Relativitätstheorie" Annalen der Physik, vol XLIX, 1916, pp. 769-882

We will not venture here to answer this question, even if it seems to contain as some fundamental things. Its resolution would in any case make it possible to envisage interstellar travel, from one end of the Milky Way to the other.

VII.1.2 Experience, light signal speeds

Let be a mirror and an experimenter $E1$. Mirror and $E1$ are fixed relative to each other. The experimenter $E1$ sends a light signal to the mirror. $E1$ knows the distance separating him from the mirror and measures the time taken by the light signal to make the round trip. He deduces the speed of the light signal:

$$c = \frac{dx}{dt}$$

Let be a motionless mirror and an experimenter $E2$, who according to $E1$ advances with speed v along this mirror. $E2$ sends a light signal to the mirror. We are looking for the speed of light $\frac{dx}{dt}$ measured by $E2$ according to $E1$?

If we apply the principle of Galilean relativity, we have the velocity diagram:

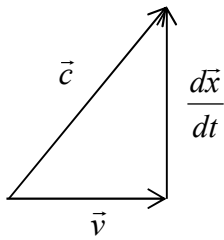


Figure 13: speed diagram

According to Pythagoras' theorem, we have:

$$c^2 = \left(\frac{dx}{dt}\right)^2 + v^2$$

According to $E1$, $E2$ measures for light a speed lower than that measured by him:

$$\frac{dx}{dt} = \sqrt{c^2 - v^2}$$

Experimentally, $E2$ also measures a speed of light equal to:

$$c = \frac{dx}{dt}$$

VII.1.3 Deformation of durations

To resolve the apparent incompatibility between the Galilean principle of relativity and this experiment reminiscent of those of A. Michelson and E. Morley, A. Einstein imagines that the proper durations of $E2$ (who considers himself immobile): dt_0 , are different from those of $E1$ (who considers $E2$ moving at a speed v): dt .

In the time of $E1$, $E2$ measures: $\frac{dx}{dt} = \sqrt{c^2 - v^2}$, and in his own time, $E2$ measures: $\frac{dx}{dt_0} = c$.

We get a speed c measured by $E2$ and a Time that becomes deformable with the introduction of dt_0 . Time is no longer an absolute. Its flow is related to the speed of the experimenter.

To find the relationship between the durations of $E1$ and those of $E2$, we start from dx which remains the same for $E1$ and $E2$:

$$dx = \sqrt{c^2 - v^2} dt = dx = c dt_0$$

We obtain the relationship between the proper durations dt_0 of $E2$ and durations dt of $E1$:

$$dt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt_0$$

We generally ask:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$$

The durations of a body (here $E2$) who considers himself immobile are therefore always smaller than those of a body (here $E1$) who considers it in motion v compared to him.

The following notations are introduced:

- the durations of $E2$ compared to $E1$ (durations of $E2$ in the reference frame of $E1$): $dt = [dt_{E2}]_{R^{E1}} = dt_{E2/E1}$,
- the durations of $E2$ compared to $E2$ (durations of $E2$ in the reference frame of $E2$): $dt_0 = [dt_{E2}]_{R^{E2}} = dt_{E2/E2}$.

We have the relationship:

$$dt_{E2/E1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt_{E2/E2}$$

Note:

It is assumed that the proper durations are the same for everyone. So, we have for experimenters $E1$, $E2$ or for any body A :

$$dt_0 = dt_{E2/E2} = dt_{E1/E1} = dt_{A/A}$$

So, we also have:

$$dt_{E2/E1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt_{E1/E1}$$

VII.1.4 Deformation of distances

The same reasoning can be applied to distances. To resolve the apparent incompatibility between the Galilean principle of relativity and experiment, we imagine that the proper distances of $E2$ dx_0 , are different from those of $E1$ dx .

In the space of $E1$, $E2$ measures: $\frac{dx}{dt} = \sqrt{c^2 - v^2}$, and in his own space, $E2$ measures: $\frac{dx_0}{dt} = c$.

We obtain the same speed c measured by $E2$ and a Space that becomes deformable with the introduction of dx_0 .

To find the relationship between the distances of $E1$ and those of $E2$, we start from dt which remains the same here for $E1$ and $E2$:

$$\frac{dx}{\sqrt{c^2 - v^2}} = dt = \frac{dx_0}{c} = dt$$

$$\frac{dx}{dx_0} = \frac{\sqrt{c^2 - v^2}}{c}$$

We get:

$$dx = \sqrt{1 - \frac{v^2}{c^2}} dx_0$$

$$\text{with } \sqrt{1 - \frac{v^2}{c^2}} < 1$$

The distances of a body (here $E2$) who considers himself motionless are therefore always larger than that of a body (here $E1$) who considers him in motion v in relation to him.

We introduce the following notations:

- the distances of $E2$ compared to $E1$ (distances of $E2$ in the reference frame of $E1$): $dx = [dx_{E2}]_{R^{E1}} = dx_{E2/E1}$,
- the distances of $E2$ compared to $E2$ (distances of $E2$ in the reference frame of $E2$): $dx_0 = [dx_{E2}]_{R^{E2}} = dx_{E2/E2}$.

We have the relationship:

$$dx_{E2/E1} = \sqrt{1 - \frac{v^2}{c^2}} dx_{E2/E2}$$

Nota

It is assumed that the proper distances are the same for everyone. So, we have for experimenters $E1$, $E2$ or for any body A :

$$dx_0 = dx_{E2/E2} = dx_{E1/E1} = dx_{A/A}$$

$$dx_{E2/E1} = \sqrt{1 - \frac{v^2}{c^2}} dx_{E1/E1}$$

VII.1.5 Deformation of distances and durations

We can imagine having both deformations of distances and durations.

The proper durations and proper distances must then verify the relationship:

$$\frac{dt_0}{dt} \times \frac{dx}{dx_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{dt_{E2/E2}}{dt_{E2/E1}} \times \frac{dx_{E2/E1}}{dx_{E2/E2}} = \sqrt{1 - \frac{v^2}{c^2}}$$

VII.2 Concept of distance deformation or advance in Space

VII.2.1 Definition

We have the following relationship between two experimenters $E1$ and $E2$:

$$dx_{E2/E1} = \sqrt{1 - \frac{v^2}{c^2}} dx_{E1/E1}$$

We have a similar relationship for an experimenter E who studies a body A:

$$dx_{A/E} = \sqrt{1 - \frac{v^2}{c^2}} dx_{E/E}$$

We define the deformation of distances, which can be interpreted as an advance in space:

$$\alpha_{A/E}^x = \frac{dx_{A/E}}{dx_{E/E}} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\text{with } \alpha_{A/E}^x < 1$$

VII.2.2 Study of the speed of a body A

Let $v_{A/E}$ the speed of body A when E considers himself advancing at the speed of light.

Let $v_{E/E} = c$ the speed of the body E when E is considered to be advancing at the speed of light.

Let $v_{A/E \div E} = v$ the speed of the body A when E considers himself motionless.

We place ourselves in the case where only distances are deformed. By definition of speed, we have:

$$v_{A/E} = \frac{dx_{A/E}}{dt_{E/E}}$$

We get:

$$v_{A/E} = \frac{dx_{A/E}}{dt_{E/E}} = \frac{dx_{A/E}}{dx_{E/E}} \times \frac{dx_{E/E}}{dt_{E/E}} = \alpha_{A/E}^x v_{E/E}$$

We have the relationship between the deformation of distances and the velocity:

$$v_{A/E} = \alpha_{A/E}^x v_{E/E} = \sqrt{1 - \frac{v^2}{c^2}} c = \sqrt{1 - \frac{v_{A/E \div E}^2}{v_{E/E}^2}} v_{E/E}$$

with $\alpha_{A/E}^x < 1$

E observes for all bodies A a speed lower than his own $v_{E/E}$.

VII.2.3 Multiplication of distance deformations, composition relation

Let a body A advancing at initial velocity $v_{A/E}^0 = \alpha_{A/E}^{x0} \times v_{E/E}$, measured by the experimenter E, with an initial distance deformation equal to $\alpha_{A/E}^{x0}$.

Body A enters a body B where the distances are contracted by $\alpha_{B/E}^x$.

The penetration of body A into body B is interpreted as follows: the deformation of the distances of body A, initially measured with respect to the experimenter E, is now measured with respect to body B, with a deformation of the distances of body A retaining the same value.

The translation into equation is:

$$\alpha_{A/B}^{x1} = \alpha_{A/E}^{x0}$$

We now want to calculate the new contraction of the distances of the body A with respect to the experimenter E. We have:

$$\alpha_{A/E}^{x1} = \alpha_{A/B}^{x1} \times \alpha_{B/E}^{x1} = \alpha_{A/E}^{x0} \times \alpha_{B/E}^{x0}$$

We ask:

$$\alpha_{A/E}^{x1} = \alpha_{A \times B/E}^x$$

$$\alpha_{A \times B/E}^x = \alpha_{A/E}^x \times \alpha_{B/E}^x$$

The velocity of body A in body B, measured according to the distances from the experimenter E is equal to:

$$v_{A \times B/E} = \alpha_{A/E}^{x1} v_{E/E}$$

We get:

$$v_{A \times B/E} = \alpha_{A/E}^x \times \alpha_{B/E}^x v_{E/E}$$

VII.3 Notion of deformation of durations or advance in Time

VII.3.1 Definition

We have the following relationship between two experimenters $E1$ and $E2$:

$$dt_{E2/E1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt_{E1/E1}$$

We have a similar relationship for an experimenter E who studies a body A:

$$dt_{A/E} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} dt_{E/E}$$

We define the deformation of durations, which can be interpreted as an advance in Time:

$$\alpha_{A/E}^t = \frac{dt_{A/E}}{dt_{E/E}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{with } \alpha_{A/E}^t > 1$$

VII.3.2 Study of the velocity of a body A

We place ourselves in the case where only the durations are deformed. By definition of speed, we have:

$$v_{A/E} = \frac{dx_{A/E}}{dt_{E/E}}$$

We get:

$$v_{A/E} = \frac{dx_{E/E}}{dt_{A/E}} = \frac{dt_{E/E}}{dt_{A/E}} \times \frac{dx_{E/E}}{dt_{E/E}} = \frac{1}{\alpha_{A/E}^t} v_{E/E}$$

We have the relationship between the deformation of durations and the speed:

$$v_{A/E} = \frac{1}{\alpha_{A/E}^t} v_{E/E} = \sqrt{1 - \frac{v^2}{c^2}} c = \sqrt{1 - \frac{v_{A/E+E}^2}{v_{E/E}^2}} v_{E/E}$$

with $\alpha_{A/E}^t > 1$

E therefore observes for all bodies A a speed lower than his own $v_{E/E}$.

VII.3.3 Composition relationship

We have the same composition relation for duration deformations as for distance deformations:

$$\alpha_{A \times B/E}^t = \alpha_{A/E}^t \times \alpha_{B/E}^t$$

We get:

$$v_{A \times B/E} = \frac{1}{\alpha_{A/E}^t \times \alpha_{B/E}^t} v_{E/E}$$

VII.4 Concomitant deformations of durations and distances

By definition of speed, we have:

$$v_{A/E} = \frac{dx_{A/E}}{dt_{A/E}}$$

Placing ourselves in the case where distances and durations are deformed, we have:

$$v_{A/E} = \frac{dx_{A/E}}{dx_{E/E}} \times \frac{dt_{E/E}}{dt_{A/E}} \times \frac{dx_{E/E}}{dt_{E/E}}$$

$$v_{A/E} = \frac{\alpha_{A/E}^x}{\alpha_{A/E}^t} v_{E/E}$$

$$\text{with } v_{E/E} = c \text{ et } \frac{\alpha_{A/E}^x}{\alpha_{A/E}^t} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{v_{A/E+E}^2}{v_{E/E}^2}} < 1$$

We get:

$$v_{A/E} = c \sqrt{1 - \frac{v^2}{c^2}}$$

In Special Relativity, we use the notation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So, we have:

$$\gamma_{A/E} = \frac{\alpha_{A/E}^t}{\alpha_{A/E}^x}$$

Or in more compact notation:

$$\gamma = \frac{\alpha^t}{\alpha^x}$$

Note 1, on the speed result of a geometric mean

We have:

$$v_{A/E} = c \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{c^2 - v^2} = \sqrt{(c - v)(c + v)}$$

$$v_{A/E}^2 = (c - v)(c + v)$$

Let us observe that $c - v$ and $c + v$ correspond to the two velocities of a body A, obtained according to the 2 Galileo transforms, that is to say to the 2 changes of reference frames: from a reference frame where the experimenter considers himself stationary to a reference frame where the experimenter considers himself advancing at speed c , taking into account the 2 possible directions of v with respect to E.

To see why, to get the speed $v_{A/E}$, It is necessary to calculate the geometric mean of the 2 speeds of A: $c - v$ and $c + v$ obtained by these 2 transformed Galileo?

Note 2, querying the meaning of the constant c in $E^t = m^t c^2$?

Why in mass energy $E^t = m^t c^2$, measured for bodies at rest in Space in relation to an experimenter, we introduce a velocity equal to c ?

In Special Relativity, would not the reference speed of an experimenter, according to him at rest, be implicitly c , and in this case would the reference speed of light be implicitly zero?

VII.5 Thinking on conventions for the description of motion and their study intervals

VII.5.1 Choice of conventions for the description of movement

To study the speed of light, an experimenter E can consider himself:

- or as motionless and the light advancing at a speed c ,
- or as advancing at speed c and stationary light.

The following table summarizes these two conventions, with the notations used in this Memoir:

	Speed of light	Speed of E	Speed of A	Study interval for speed of A
First Convention: E motionless and light L advancing at speed c	$v_{L/E \div E} = c$	$v_{E/E \div E} = 0$	$v_{A/E \div E} = v$	$[0, c]$
Second Convention: E advancing at speed c and light L motionless	$v_{L/E} = 0$	$v_{E/E} = c$	$v_{A/E} = \sqrt{c^2 - v^2}$ $v_{A/E} = c \sqrt{1 - \frac{v^2}{c^2}}$ If $v = c$ then $v_{L/E} = 0$ If $v = 0$ then $v_{\frac{E}{E}} = c$	$[0, c]$

VII.5.2 How to integrate into the second convention the speeds going in the opposite direction of Space?

In the second convention, we have an experimenter E considering himself advancing at speed c , observed bodies that have a speed lower than his, and a minimum for light L which has zero velocity.

To integrate into this second convention, the speeds going in the opposite direction of Space, a first option is to have for these speeds, negative velocities and an experimenter, in this case, considering himself advancing at the speed $-c$.

	Speed of light	Speed of E	Speed of A	Full study interval for speed of A
First Convention: E motionless and light L advancing at speed $\pm c$	$v_{L/E \div E} = \pm c$	$v_{E/E \div E} = 0$	$v_{A/E \div E} = v$	$[-c, 0] \cup [0, +c]$
Second Convention: E advancing at speed $\pm c$ and light L motionless	$v_{L/E} = 0$	$v_{E/E} = \pm c$	$v_{A/E} = \pm \sqrt{c^2 - v^2}$ $v_{A/E} = \pm c \sqrt{1 - \frac{v^2}{c^2}}$ If $v = c$ then $v_{L/E} = 0$ If $v = 0$ then $v_{E/E} = \pm c$	$[-c, 0] \cup [0, +c]$

This is a possible option, but not very satisfying for the mind. Indeed, the reference experimenter E at two possible speeds $\pm c$ according to the bodies studied.

Let us look for another convention where the experimenter E would maintain, according to him, always the same reference speed c .

In the second convention, in the case where a body A goes half as fast in space as an experimenter E, the latter measures a velocity $v_{A/E} = \frac{1}{2}c$. The experimenter E advances with velocity c in one direction of Space, and body A advances with velocity $v_{A/E} = \frac{1}{2}c$ in the same sense of Space. The body A advances a priori slower than the experimenter E and it appears between E and A, a distance gap that continues to grow.

According to a third convention (in fact, an extension of the second convention to bodies going in the opposite direction of Space), if the distance gap between E and A keeps getting bigger, it is because A goes twice as fast as E, with $v_{A/E} = 2c$, in the opposite direction of Space.

In this third convention, E advances at speed c in Space. The bodies studied all have the same sense of velocity in Space (the same as E). For E, bodies whose velocity belongs to the interval $[0, c]$, advance in Space slower than him. Bodies whose velocity belongs to the interval $[c, +\infty[$, advance in Space faster than him. The complete study interval is $[0, c] \cup [c, +\infty[$.

The table above summarizes the values obtained, with the first and third conventions, always with the notations used in this Memoir:

	Speed of light	Speed of E	Speed of A	Study interval for speed of A	Full study interval for speed of A
First Convention: E immobile et	$v_{L/E \div E} = \pm c$	$v_{E/E \div E} = 0$	$v_{A/E \div E} = v$	$[-c, 0] \cup [0, +c]$	$[-c, 0] \cup [0, +c]$

Light L advancing at speed $\pm c$					
Third Convention: E advancing at speed c and light L motionless or advancing at speed $+\infty$	$v_{L/E} = 0$	$v_{E/E} = c$	$v_{A/E} = c \sqrt{1 - \frac{v^2}{c^2}}$ If $v = c$ then $v_{L/E} = 0$ If $v = 0$ then $v_{E/E} = c$	$[0, +c]$	$[0, c] \cup$ $[c, +\infty[$
	$v_{L/E} = +\infty$	$v_{E/E} = c$	$v_{A/E} = \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}$ If $v = c$ then $v_{L/E} = +\infty$ If $v = 0$ then $v_{E/E} = c$	$[c, +\infty[$	

Let us retain from these different conventions or points of view, that there is not really an unsurpassable speed c of light, but rather a bound c present in all study intervals.

VII.6 Mass, a kind of "speed" in Time?

VII.6.1 The relativistic mass γm^t

In the preceding paragraphs, we have taken up the idea of A. Einstein of a deformable Space and Time. An alternative and intuitive approach to the notions of distance and duration deformations has been proposed.

In Memoir 2, we also suggested that mass m^t could be interpreted as a kind of "speed" in Time.

In Special Relativity, the notion of relativistic mass is introduced. γm^t , with:

- m^t the resting mass of a body,
- γm^t its mass when this body has a velocity v .

Based on the above, we have:

$$\gamma m^t = \frac{\alpha^t}{\alpha^x} m^t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m^t \text{ in the general case,}$$

$$\gamma m^t = \alpha^t m^t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m^t \text{ if absence of distance distortions.}$$

We can interpret:

- the mass m^t as "a speed" in Time for a body at rest in Space, relative to an observer,
- the relativistic mass $\gamma m^t = \alpha^t m^t$ as a "speed" in Time for a moving body at the speed v in Space, relative to an observer.

VII.6.2 The case of light and photons

Recall that according to Special Relativity, a particle of momentum p , of speed v and of mass m^t has a total energy:

$$E^t = \gamma m^t c^2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m^t c^2$$

In the case of a photon, we have a velocity:

So, we have:

$$v \rightarrow c$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow +\infty$$

We have for its mass at rest:

$$m^t \rightarrow 0$$

We obtain an indeterminate form for the total energy:

$$E^t = \gamma m^t c^2 = 0 \times \infty$$

if it is calculated from its mass m^t and from γ .

We also have for its relativistic mass an indeterminate form:

$$\gamma m^t = 0 \times \infty$$

According to Special Relativity, a particle of momentum p and of mass m^t has a total energy:

$$(E^t)^2 = p^2 c^2 + (m^t)^2 c^4$$

In the case of a photon, we have:

$$(E^t)^2 = p^2 c^2 + 0$$

We get its total energy from its momentum p and its frequency f :

$$E^t = pc = hf$$

We therefore have for a photon:

- if it is at rest in Space relative to us, a zero mass $m^t = 0$,
- a speed c in Space,
- a total energy that is indeterminate, if calculated from its mass m^t and from γ ,
- a total energy that is determinate, if calculated from its momentum or its frequency: $E^t = pc = hf$

In conclusion, it is often said that light and photons have zero mass. This is not entirely accurate. A photon has zero mass, if it were at rest relative to the observer. When it is at speed c , we do not know. But it can be assumed that its mass is not zero in relation to the observer. This explains in particular its sensitivity to gravitation or its participation in Compton scattering.

VII.6.3 Why is there no "shock" when we superimpose 2 beams of light?

Light is sometimes presented as wave, sometimes as corpuscular. In the second case, we can expect to observe "shocks" between two photons when we superimpose 2 beams of Light. This is not experimentally the case.

We can intuit it by saying that two photons have a zero velocity for each other, and therefore for each other a zero mass. So, there is no "shock".

It can be objected that two photons crossing in opposite directions, measure for each other a speed $2c$. Let us then resume the results of the experiments of A. Michelson and E. Morley: Whatever the frame of reference, an experimenter measures for light a velocity c . Let's transpose this result to the photon: whatever the reference frames, one photon measures for another photon a zero velocity.

On the other hand, a collision between a photon and an electron is observed experimentally, for example during Compton scattering. The photon therefore has, in a way, a mass for the electron.

Note

To be complete, a theory proposed by W. Heisenberg in the 1930s provides for a possible interaction between two photons *via* the exchange of an electron. Extensive experiments are being conducted on the subject and could corroborate W. Heisenberg's hypothesis.

VII.6.4 Why do we always measure a speed c for Light?

Why for any phenomenon L having:

- a "speed" in Time which is null: $m^t = 0$, if this phenomenon was at rest in Space relative to an observer,
- a speed in Space not equal to that of the observer, this observer considering himself motionless in Space,

the observer **necessarily** measures a velocity in Space equal to c ?

According to Special Relativity, we have the total energy of a particle:

$$E^t = \gamma m^t c^2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m^t c^2$$

It is noted that for any phenomenon L having a rest mass of zero limit $m^t \rightarrow 0$, it is possible to have a non-zero energy E^t , on the **necessary** condition that the speed of the phenomenon L has the limit $v \rightarrow c$ and therefore $\gamma \rightarrow \infty$.

We have indeed:

- $v \rightarrow c$ if the observer considers himself motionless in Space,
- $v_{L/obs} = \gamma c \rightarrow \infty$ if the observer considers himself to be advancing at speed c in Space.

VII.7 The Light Time Hypothesis, Light Time is it also our Time?

VII.7.1 Do we share the same Time with these phenomena or bodies of which we measure a zero mass (if they were at rest in Space) and a speed c ?

As mentioned in the previous paragraph, we measure for certain phenomena or bodies, such as light or photons:

- if they were at rest in Space relative to us, a zero mass $m^t = 0$,
- a speed c in Space.

Following a relativistic approach to movements, would these phenomena or bodies, if they were at rest in Space in relation to us, have a zero "speed" in Time in relation to us, that is to say a "speed" in Time identical to ours, and therefore would they share the same Time as us, human beings?

We poetically baptize this question, the *Light Time hypothesis*, a Light Time that would be that of light (if it were at rest in Space in relation to us), and *also our biological Time*. In other words, the "speed" of Light in Time and the "speed" of living beings in Time would be the same.

VII.7.2 How do living beings "settle" on the same moments?

In the hypothesis of a deformable Time, where all bodies do not have the same "speed" m^t in Time, the question is how do living beings "settle" on the same moments? In other words, how do living beings live together?

VII.7.3 What is the point of an always constant speed c of light?

Let us opt for a functionalist approach in this paragraph. That is to say, let's interpret the Universe as a big machine where all the elements would have a function (we can even talk about utility). Following

this approach, do we ask ourselves about the function of a constant speed c of light regardless of the reference frame? What is its interest?

In the previous paragraphs, it has been pointed out that if Light had zero speed in Space relative to an observer, its mass γm^t would be zero and its "speed" in Time as well. The light would therefore have the same Time as the observer.

Following a phenomenological approach, this finding implicitly introduces a link between all observers measuring the same speed c of light.

Thus, would this constant c indicate the existence of an identical link shared by all observers living together? Would this link, loss of freedom, allow observers to live together, to advance concomitantly in Time?

We began poetically this essay by imagining rotations in spatiotemporal planes. It ends just as poetically by imagining the constant c , as an identical characteristic shared by human beings living together, allowing them to position themselves on the same moments. This constant c is great, but it is true that life passes so quickly!

Our initial concern was uselessness a priori of a constant and unsurpassable speed c of light, with the annoyance of not being able to exceed a certain value (moreover hardly compatible with the ideas of infinity and continuity).

Our argument is the usefulness of a quantity shared and identical to all human beings, so that we can live together.

Note 1 on memories and memory function

Here we have suggested that human beings must move identically in Time to live together. Nevertheless, our being does not necessarily form a whole. We can imagine certain parts of our being that advance in Time identically to those facing us, and other parts that go back in Time or that remain frozen at a certain time (this is what suggests the measurement of a non-zero mass m^t for our body and especially our brain). These parts relive moments, giving them a kind of eternity. They can also bring back forgotten memories.

So, when we look back on our past life, are we entirely in the present moment, or are certain parts of ourselves in the moment that we remember? Does our memory store all the information or does it only point to moments, which we must return to, to find the information?

Note 2 on a biological agent which makes identically advance in Time the beings living together

A biological agent can be defined by the utility and functions it performs within an organism. For example, DNA is an agent for storing and transmitting genetic information.

Similarly, what could be the biological agent (we can speak of a biological clock) that would allow beings living together to advance identically in Time?

Perhaps this biological agent is not spatially localized like DNA and that it has to do with L. Pasteur's discovery for the characterization of Life: molecules of a single chiral type?

Note 3 on Life and Death

If Life is characterized by the measurement of a constant c and an identical "speed" in Time, death could be characterized by the end of this identical "speed" in Time.

Following this hypothesis, and in the quest for a biological agent that advances living beings identically in Time, one track would be to study the functions that disappear during the passage from life to death. Could the biological agent be linked to one of these functions?

Note 4, materialism or dualism?

In the traditional debate between materialist or monist philosophers (inseparable matter and spirit) and dualistic philosophers (dissociated matter and spirit), let us remember that the hypotheses proposed here do not allow a distinction.

We can reconcile both materialism and dualism, with a matter interpreted as a "speed" in Time, a Light Time hypothesis and an arrow of Time which would be the first characteristic of Life. A form of life beyond would not be inherently impossible, except that it would no longer have Light Time.

VII.8 Conclusion of the essay, the question of Time

We began this essay by questioning the similarities between electromagnetism and gravitation. In the 3rd Memoir, it was proposed to unify Newtonian gravitation with quantum physics (in a restricted sense, however, i.e., according to the equations $E = h\nu$ and $p = \frac{h}{\lambda}$), as J. C. Maxwell did in the 19th century with the unification of electrostatics and magnetism.

In the 5th Memoir, we were first interested in bringing electromagnetic, hyper, and nuclear interactions closer together, then in studying how gravitation could be integrated with these interactions. We simply proposed tracks.

That is, the initial question of the similarities between electromagnetism and gravitation, as well as that of the unification of all these interactions, remained unresolved. It therefore remains to understand the main one, in particular the link between electric charge and mass. All electric charges also have a priori mass, but why, for example, proton and electron have identical electric charges in absolute value, and masses so different?

For the last major unification project of current physics, that of General Relativity with Quantum Physics, only a few remarks have been outlined in the 6th Memoir. Rather, it was pointed out that General Relativity, and especially 2 of its cosmological developments: Big Bang and black holes, had similarities with biology. The idea is not that General Relativity and Quantum Physics will not one day be fused. The universe is only one and therefore all theories should be able to join together like the branches of a huge tree. The idea is that General Relativity and Quantum Physics will first have to merge with other models or theories before they come together.

To conclude, if we had to propose a central theme to this essay, it would undoubtedly be the question of Time. The question of Time was mainly addressed from a physical point of view: trying to understand how it intervenes in the equations of gravitation, electromagnetism, and nuclear interactions? It was nevertheless surprising that the Time of Physics is so similar to a dimension of Space and does not require, in its equations, the notion of flow.

Because what interests us all, more than the Time of physics, is the Time felt, our Time that takes us in a mad race. A race similar to a turbulent torrent made of currents, rapids and eddies. A race of which internally, we can go back the course, in order to relive once again some wonderful moments. A race that externally, is without possible return, with terminus stop at the ocean.

To better understand something, it is sometimes relevant not to apprehend it directly, but to study similar things. As the systemic ones point out, we do not understand something in our own right, we understand it through its relationship with others, its differences, its flaws, its inconsistencies. Through this long journey dedicated to the study of Space and Time in physics, our goal was also to better understand our felt Time. What could characterize it and make it its singularity vis-à-vis the Time of Physics?

Bibliography of the Memoir 6

On 2-sense time in physical theories

- [1] R. Feynman, The principle of least action in Quantum Mechanics, thèse, Princeton University, 1942
- [2] R. Feynman, Conference Nobel, 1965
- [3] R. Feynman, The Character of Physical Law, 1965

On the arrow of Time

- [4] Etienne Klein, Michel Spiro, Le Temps et sa Flèche. Colloques de la Société Française de Physique: Physique et Interrogations fondamentales. Première Rencontre sur « le Temps et sa Flèche », Editions frontières, Paris, 1994
- [5] Wikipedia, Arrow of Time

On the arrow of thermodynamic Time

- [6] Roger Balian, Le Temps macroscopique. Colloques de la Société Française de Physique: Physique et Interrogations fondamentales. Première Rencontre sur « le Temps et sa Flèche », Editions frontières, Paris, 1994
- [7] Wikipedia, Maxwell's demon

On the arrow of Microscopic Time

- [8] L. Bay, Introduction à la physique des particules. Laboratoire de Physique des Hautes Energies, EPFL
- [9] J. Collot, Symétries discrètes P C T Règles de sélection. UJF Grenoble, années 2007-2008
- [10] Maurice Jacob, Le temps des processus élémentaires II. Colloques de la Société Française de Physique: Physique et Interrogations fondamentales. Première Rencontre sur « le Temps et sa Flèche », Editions frontières, Paris, 1994
- [11] L. Marleau, Particules élémentaires. Université Laval, Québec, 2007

On the arrow of cosmological time

- [12] L. Smolin, Life of the Cosmos, New York, Oxford University Press, 1997
- [13] L. Smolin, Time Reborn, Houghton Mifflin Harcourt, 2013
- [14] Wikipédia, Big bang
- [15] Wikipédia, Schwarzschild metric
- [16] Wikipédia, Thermodynamics of black holes

On the arrow of biological time

- [17] L. Robert, Le Temps en biologie. Colloques de la Société Française de Physique: Physique et Interrogations fondamentales. Première Rencontre sur « le Temps et sa Flèche », Editions frontières, Paris, 1994

On Life, the study of the human mind and knowledge

- [18] R. Dawkins, The Ancestor's Tale, Mifflin Compagny, New-York, 2004
- [19] J. Lovelock, The Earth is a living being, Oxford University Press, Oxford, 1979
- [20] L. Margulis, D. Sagan. The Bacterial Universe, New-York, 1986
- [21] J. M. Smith, The Major Transitions in Evolution, Oxford University Press, Oxford, 1995
- [22] L. Pasteur, Ecrits scientifiques et médicaux, Flammarion, Paris, 2010
- [23] F. Varela, Invitation aux sciences cognitives. Editions du Seuil, Paris, 1996, pour la traduction et l'édition françaises.
- [24] Wikipédia, Origin of life

On the theory of Relativity

[25] A. Einstein, *Relativity: the Special and the General Theory*, 1916 for the 1st edition in German

Table des matières

Memoir 6	Time felt and Time in Physics	443
Chapter I	Why does Time used in Physics seem so different from Time felt?	444
I.1	The Time of Physics, the Time Felt and the Arrow of Time	444
I.2	Time in the theory of Relativity, often presented as similar to a spatial dimension	444
I.3	Most physical theories are reversible over time	445
I.4	2-sens Time and principle of least action	449
I.5	Conclusion of the chapter	451
Chapter II	Existence of a thermodynamic arrow?	452
II.1	Definitions of entropy	452
II.2	Time oriented in thermodynamics, bringing together entropy and information?	452
II.3	Conclusion of the chapter	453
Chapter III	Existence of a microscopic arrow of the particles?	454
III.1	Existence of a microscopic arrow of Time?	454
III.2	Decay reactions $\beta \pm$, violation of P and C , conservation of symmetry CP	456
III.3	Physics of kaons, violation of symmetry CP	458
III.4	Conclusion of the chapter, existence or not of a microscopic arrow?	461
Chapter IV	Cosmological arrow and elements on the theories of General Relativity, the Big Bang, and black holes	463
IV.1	Historical Reminders about the Theory of General Relativity	463
IV.2	Historical Reminders on the Big Bang Theory	466
IV.3	Issues raised by the Big Bang, sources of inspiration	468
IV.4	Thermodynamics of black holes	469
IV.5	Conclusion of the chapter	471
Chapter V	Demarcation between Life and non-Life, the phenomenological approach, the Gaia hypothesis	472
V.1	Demarcation between Life and non-Life	472
V.2	The phenomenological approach	475
V.3	One application, the Gaia hypothesis	476
V.4	Conclusion of the chapter	477
Chapter VI	The origin of Life, the Big Bang a biological phenomenon?	479
VI.1	Origin of Life according to biologists	479
VI.2	The Big Bang, the origin of Life?	480
VI.3	My Big Bang	483
VI.4	Conclusion of the chapter	484
Chapter VII	How to live together at the same time? The Light Time Hypothesis	485
VII.1	The apparent incompatibility between the principle of relativity stated by G. Galileo and the propagation of light	485
VII.2	Concept of distance deformation or advance in Space	488
VII.3	Notion of deformation of durations or advance in Time	490
VII.4	Concomitant deformations of durations and distances	491
VII.5	Thinking on conventions for the description of motion and their study intervals	492
VII.6	Mass, a kind of "speed" in Time?	494
VII.7	The Light Time Hypothesis, Light Time is it also our Time?	496
VII.8	Conclusion of the essay, the question of Time	498
Bibliography of the Memoir 6		499

