



HAL
open science

Sequential Harmonic Component Tracking For Underdetermined Blind Source Separation in a Multi-Target Tracking Framework

Romain Delabeye, Martin Ghienne, Jean-Luc Dion

► **To cite this version:**

Romain Delabeye, Martin Ghienne, Jean-Luc Dion. Sequential Harmonic Component Tracking For Underdetermined Blind Source Separation in a Multi-Target Tracking Framework. 2023. hal-03912257v1

HAL Id: hal-03912257

<https://hal.science/hal-03912257v1>

Preprint submitted on 9 Feb 2023 (v1), last revised 1 Mar 2023 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Sequential Harmonic Component Tracking For Underdetermined Blind Source Separation in a Multi-Target Tracking Framework

Romain Delabeye¹, Martin Ghienne¹, and Jean-Luc Dion¹

¹Department of Vibrations, Acoustics and Structures
Quartz Laboratory, EA7393, ISAE-Supméca, Saint-Ouen, France

ABSTRACT

Smart factories are composed of heterogeneous cyber-physical systems. In light of their complexity and the lack of transparency in their design, monitoring the health of these machines in real-time is made possible by the use of non-intrusive sensors. Such sensors produce mixed signals capturing component-specific signatures. Retrieving the activation statuses of the components (over the different operating modes of a machine) is essential for estimating their associated performance indicators. This is a special case of Underdetermined Blind Source Separation (UBSS), yet a sensor fusion perspective is adopted in this paper. A harmonic component detector produces observations in the Time-Frequency (TF) domain, inherently entailing noise-induced false alarms. The main contribution of this paper consists in a clutter-resilient multi-harmonic component tracking algorithm, based on the Sequential Monte-Carlo Probability Hypothesis Density (SMC-PHD) filter. Additionally, this paper presents a track association algorithm adapting the results obtained in the multi-target tracking framework for unsupervised multi-label classification. The combination of the two algorithms mitigates typical difficulties encountered in traditional UBSS problems, such as non-stationary and partially-coupled mode decomposition. The performance of the proposed technique is assessed upon synthetic data.

Keywords: Harmonic Component Tracking, Multi-Target Tracking, Sensor Fusion, Underdetermined Blind Source Separation, SMC-PHD Filter

INTRODUCTION

Energy sustainability is one of the greatest challenges faced by the manufacturing industry. Manufacturing industry is energy-intensive by nature, making it worthwhile to put the emphasis on energy efficiency when aiming for substantial discounts in energy usage and associated carbon emissions. From a physical point of view, energy efficiency boils down to minimizing dissipated energy for a given production. General indicators such as the Specific Energy Consumption (SEC), i.e., the total energy consumption per unit of output, only allow for a shallow analysis of a system's energy efficiency. A key success factor in enhancing a production system's energy sustainability lies in the ability to allocate Energy Performance Indicators (EnPI) to dedicated active components, actuators and operating modes, designated as *components*, *actuators* and *operations* respectively. In this context, an actuator consists of a group of physical components always active simultaneously (e.g., a rotor and bearings), and an operation relates to the accomplishment of a task using a fixed group of actuators (e.g., drilling would use two motors to rotate and advance the drill). A machine thus obeys the same dynamics throughout an operation. This dynamics is more specifically made up of the actuators' dynamics and possible couplings between them. In practice, though it is common to monitor a manufacturing machine's total energy consumption (a mandatory requirement to compute EnPI), the activation sequences of (i) the actuators composing this machine and (ii) the different operations performed by the machine are seldom available. Hence the times at which components, actuators and operations are active need to be inferred from sensor data, without any prior regarding the studied system (no physical nor process models). Furthermore, in order for such a process identification technique to scale, the use of non-intrusive sensors is preferred (e.g., accelerometers or current sensors). These

sensors have the particularity of sensing much information from multiple remote sources, resulting in coupled dynamics from a sensor's point of view. Signal processing is thus required to uncouple these sources. This constitutes an Underdetermined Blind Source Separation (UBSS) problem, yet only activation statuses are sought rather than mixed signal. By putting the emphasis on the activation statuses of the components, actuators and operations rather than source signal recovery, the latter aspect can be performed using an independent physics-informed regression algorithm instead of a statistical decomposition. Only the former aspect is considered here. Moreover, data are represented in the Time-Frequency (TF) domain.

UBSS problems are traditionally tackled using decomposition algorithms, either identifying a mixing matrix and source signals, or learning a sparse representation from a dictionary of representative vectors built iteratively. Such processes (subspace methods in particular) are very efficient when data are piecewise stationary [1], yet this assumption is too restrictive in this context since this would not cover controlled systems.

This motivates the use of Multi-Target Tracking (MTT) for estimating both the state and the number of active components in a signal. The main contribution of this paper hence consists of an MTT formulation allowing for tracking harmonic components over time. We propose a simple peak-based harmonic component detector (stemming from the signal's power spectral density). A particularity of the problem on hand is the time-varying number of false alarms per scan, which depends on the level of noise associated with each source signal. We hence propose a Feature-Aided Tracking (FAT) formulation, based on the spectral kurtosis, increasing the clutter-resilience of our tracking filter.

Related works focusing on MTT and frequency tracking are presented in the first section. Problem formulation is detailed in a second section, together with background knowledge regarding the techniques used in this paper. The enhancements made to the tracker in order to increase its resilience to clutter are presented in a third section. A fourth section details how this MTT formulation is adapted to solve an unsupervised multi-label classification problem. The verification of the developed technique is performed on synthetic data, the results are presented and discussed in a fifth section.

RELATED WORKS

In blind source separation, Independent Component Analysis (ICA) and its extension to the underdetermined case [2] has received great attention over the years, yet this method cannot separate sub-Gaussian distributions and is not well suited to discrete event data. Diverse techniques exist to estimate the number of source signals [3], often based on subspace methods. For the problem on hand, the number of actuators cannot be recovered using these methods [4], but the number of distinct operations can be retrieved. A traditional approach to tackle UBSS problems consists in clustering time series data into chunks over which the number of sources is assumed to be constant, the signal is then factorized into a mixing matrix and unmixed source signals [5–7]. Another formulation consists in factorizing data as a dictionary of atoms (representing relevant modes) and a representation (linear combination of atoms) [8]. For these linear combinations to truly represent the sought labels, the representation must remain binary as in the semi-binary Non-negative Matrix Factorization [9, 10]. Alternatively, Dynamic Time Warping (DTW) can be coupled with hidden Markov models when clustering signals with different shapes [11, 12].

In this paper, we adopt a multi-target tracking framework in which the number and states of target harmonic components are sought. Multi-frequency tracking has been extensively studied in the literature as a data assimilation problem, using Kalman filters [13, 14] and particle filters [15, 16] in particular. In these applications, tracks are initiated heuristically, tracking is then treated solely as a state estimation problem.

Two additional challenges arise in the class of applications considered in this paper. First, the number of harmonic components to track is not known a priori, evolves over time and is not necessarily detected at each time step. A second important aspect is the presence of a time-varying number of false alarms (clutter). Suitable real-time compatible data association filters (responsible for mitigating the effect of clutter on state estimation performance) include the Joint Probabilistic Data Association (JPDA) filter, probabilistically associating measurements to tracks, and Probability Hypothesis Density (PHD) filters [17, 18], implicitly fusing all states with all measurements at each time step. In-between these two types of filter, a set JPDA has been proposed in [19].

With a view to increase state estimation performance, feature-aided tracking has been investigated in the literature. The association probability of the JPDA was refined using Signal-to-Noise Ratio (SNR) [20], radar High Resolution Range (HRR) [21] and wavelet-based spectral features [22]. Target doppler and down-range extent were implemented in feature-aided PHD filters [23, 24]. In these applications, the ingenious integration of feature information – essentially in the data association part of the filter – resulted in better tracking performance and exhibited clutter-resilient behaviors.

PROBLEM FORMULATION

Among existing frameworks, the MTT formulation has the potential to track time-varying spectral components while detecting when a new component appears or disappears, thus alleviating major limitations in subspace decomposition methods.

As a machine operates through its manufacturing process, an actuator a with status $\delta_t^a \in \{0, 1\}$ at a time step t can switch on (1) or off (0) components with status $\delta_t^c \in \{0, 1\}$, producing sudden changes in sensor data, and bringing the machine into a new operating mode with status $\delta_t^o \in \{0, 1\}$. From a set of H_t harmonic components (targets) with states $\{\mathbf{x}_t^h\}_{h=1}^{H_t}$ at time step t in the TF domain, a set $\{\mathbf{z}_t^m\}_{m=1}^{M_t}$ of M_t measurements is produced from a single sensor. Targets follow a Markov transition model $p(\mathbf{x}_t^h | \mathbf{x}_{t-1}^h)$.

A simple harmonic component detector is proposed. The Discrete Fourier Transform (DFT) is evaluated on successive overlapping windows with time index t . Then a detection $\mathbf{z}_t^m = [a_t^m, b_t^m, A_t^m, \omega_t^m]^T + \mathbf{w}_t$ is triggered every time the power spectral density reaches a peak above a threshold τ , where $\mathbf{w}_t \sim \mathcal{N}(\cdot; 0; \text{diag}([\sigma_a, \sigma_b, \sigma_A, \sigma_\omega]))$; $A_t^m, \omega_t^m, a_t^m, b_t^m$ are a harmonic component's amplitude, pulse and complex coefficients respectively, and $\sigma_A, \sigma_\omega, \sigma_a, \sigma_b$ denote their corresponding standard deviations. This measurement m corresponds to a complex yet undamped modulated sine wave supplemented with Gaussian noise $w_t^s \sim \mathcal{N}(\cdot; 0; \sigma_s)$, i.e., $s_t^m = A_t^m \exp(i \omega_t^m t) + w_t^s = a_t^m + i b_t^m$. An immediate consequence of this detection technique is the time-varying nature of the number of false alarms per scan λ_{FA} . In this formulation, λ_{FA} is implicitly defined by the noise level σ_s . The underlying detection probability p_D is unknown, yet close to unity.

The trajectory $\mathbf{T}_{t_i:t_f}^h$ of a target h is made of associated states \mathbf{x}_t^h (defined by the same variables as the measurement vector) between times t_i and t_f . For simplicity, the same target index h is kept over time.

For interpretation, a trajectory represents the behavior of a physical component, unless its frequency can be expressed as a positive integer multiple of another trajectory's (i.e., as an harmonic of the fundamental frequency).

Formally, an occurrence of an operation o thus corresponds to a set of trajectories $\mathbf{\Omega}_{t_i^o:t_f^o}^o = \{\mathbf{T}_{t_i^o:t_f^o}^h\}_{h=1}^{H^o}$ made of H^o components between times t_i^o and t_f^o . Similarly, each time an actuator a is activated, it induces a set of trajectories $\mathbf{\Omega}_{t_i^a:t_f^a}^a = \{\mathbf{T}_{t_i^a:t_f^a}^h\}_{h=1}^{H^a}$ composed of H^a components between times t_i^a and t_f^a .

FEATURE-AIDED SMC-PHD FOR HARMONIC COMPONENT TRACKING

The SMC-PHD filter is considered in this paper, due to its inherent clutter-resilience and computational efficiency (from a data association point of view) [17]. Indeed, in this MTT framework, target dynamics is nonlinear and much clutter (false alarms) is expected, yet the detection probability is high. The SMC-PHD filter is particularly well suited to such problems. This filter consists of five steps: (i) particle sampling, (ii) prediction, (iii) update, (iv) resampling, (v) clustering and (vi) assignment. Particle sampling is responsible for exploring the state space and spotting new targets. Prediction, update, resampling and clustering constitute the multi-target state estimation activities. Assignment binds tracks (states and associated covariances) to one another across time steps, resulting in trajectories. The SMC-PHD filter relies on two assumptions [24]:

Assumption 1 *the targets are independent from one another and generate at most one measurement per scan each.*

Assumption 2 *clutter and target birth distributions are Poisson and target-independent.*

We apply uniform particle sampling over the Field Of View (FOV), i.e. the whole spectrum. This degrades the state estimation performance but allows to locate any target appearing within the FOV.

State estimation relies on a transition model, here the amplitude and the frequency follow a generic random walk, whereas the complex coefficients rotate at the harmonic component's angular frequency. The transition equation is given by [25]:

$$\begin{bmatrix} a_{t+1}^h \\ b_{t+1}^h \\ A_{t+1}^h \\ \omega_{t+1}^h \end{bmatrix} = \begin{bmatrix} \cos(\omega_t^h \Delta t) & -\sin(\omega_t^h \Delta t) & 0 & 0 \\ \sin(\omega_t^h \Delta t) & \cos(\omega_t^h \Delta t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_t^h \\ b_t^h \\ A_t^h \\ \omega_t^h \end{bmatrix} + \mathbf{v}_t \quad (1)$$

where \mathbf{v}_t is a zero-mean Gaussian noise and Δt is the duration of a time step.

Similarly to [24], we augment the likelihood (equation 3a) as well as the PHD of the posterior (equation 3b) using a feature likelihood, i.e., the probability for a measurement to be target-originated, based on the spectral kurtosis. The *spectral kurtosis* κ_t^m is obtained by evaluating the kurtosis on a window t filtered using a band-pass filter (BPF) with a $\Delta\omega_{bp}$ bandwidth centered around ω_t^m , a more detailed definition is presented in [26, 27]. In order to interpret the spectral kurtosis and elicit a feature likelihood, the following assumption is made [27]:

Assumption 3 *Noise and excitations highlighting normal modes are assumed to be mesokurtic or leptokurtic.*

That is, the deterministic part of the signals of interest (in their bandwidth) must have a kurtosis strictly below 3 (kurtosis of a Gaussian distribution, i.e., noise in this context). This assumption also leads to a restriction on transient responses. Fast and spiky transients will be leptokurtic and difficult to distinguish from noise and exceptional events. Such transients will thus be treated as clutter. For this reason, we use the spectral kurtosis to correct the estimation in a probabilistic framework rather than triggering measurements solely based on this information.

With a view to lower the weight of clutter-influenced particles, a feature likelihood (assessing the extent to which a measurement was target- or clutter-originated) is elicited from the spectral kurtosis. To this end, we fit a gamma distribution such that the cumulative probability function (cdf) reaches 95% at $\kappa = 3$ (shape parameter $\alpha = 2.615$ and scale parameter $\theta = 0.525$), resulting in the spectral kurtosis likelihood:

$$p_f(\kappa_t^m) = \frac{1}{\Gamma(\alpha)\theta^\alpha} (\kappa_t^m)^{\alpha-1} \exp\left(-\frac{\kappa_t^m}{\theta}\right) \quad (2)$$

This feature likelihood is assumed to be independent from the kinematic likelihood $g_t(\mathbf{z}_t^m | \mathbf{x}_t^h)$. It refines the weights $\{w_{t|t}^{(p)}\}_{p=1}^{L_t}$ of the L_t particles approximating the PHD of the posterior at time step t , expressed with respect to prior weights $\{w_{t|t-1}^{(p)}\}_{p=1}^{L_t}$:

$$g_t(\mathbf{z}_t^m | \mathbf{x}_t^h) \leftarrow g_t(\mathbf{z}_t^m | \mathbf{x}_t^h) p_f(\kappa_t^m) \quad (3a)$$

$$w_{t|t}^{(p)} = \left[1 - p_D + \sum_{m \in \llbracket 1, M_t \rrbracket} \frac{p_D g_k(\mathbf{z}_t^m | \mathbf{x}_t^{(p)})}{K + \sum_{p'=1}^{L_t} p_D g_t(\mathbf{z}_t^m | \mathbf{x}_t^{(p')}) w_{t|t-1}^{(p')}} \right] w_{t|t-1}^{(p)} \quad (3b)$$

where the detection probability p_D and the clutter spatial density K are assumed to be constant and uniformly distributed over the FOV. These weights are also further scaled up or down (and carefully re-normalized to their original mass) according to their probability $p_f(\kappa_t^{(p)})$ to represent a target, i.e., $w_{t|t}^{(p)} \leftarrow w_{t|t}^{(p)} p_f(\kappa_t^{(p)})$. However, evaluating the spectral kurtosis of each particle at their estimated frequency would be computationally intractable. Instead, the spectral kurtosis of a particle is computed as a linear interpolation of the one calculated during preprocessing (for each frequency bin of the DFT). Alternatively, this step can be skipped for real-time applications.

This feature-aided SMC-PHD filter finally yields tracks (states and associated covariances) $\{\mathbf{T}^h\}_{h=1}^H$, where H is the total number of harmonic components detected in a dataset.

MULTI-TARGET TRACKING TO UNSUPERVISED MULTI-LABEL CLASSIFICATION

The tracks obtained in the previous section at most represent the behavior of physical components. In this section, a technique is proposed to process and interpret these tracking results in order to recover the activation sequences of the actuators, and the operations they perform.

A first step consists in grouping harmonic components according to their trajectories' states. For simplicity, a descriptive vector is computed for each component, namely $\boldsymbol{\mu}^h = [A^h, \omega^h, \sigma_A^h, \sigma_\omega^h]^T$, corresponding to the average amplitude, angular frequency, and associated covariances over a trajectory \mathbf{T}^h . Pairwise distances are computed, allowing for harmonic components to be grouped with one another according to these descriptors. Euclidean distance upon standardized features was considered in this paper. In more complex cases than those considered, other metrics can be used to associate the tracks between them. For instance, Fréchet distance [28] takes the shape of the trajectory into account, and distances based on Gaussian Processes (GP) [29] can take advantage of uncertainty information (i.e., state covariance along each trajectory) provided by the MTT framework. Furthermore, actual harmonics are grouped together. That is, given two components h and h' , h is paired with h' as one of its harmonics if there exist $k \in \mathbb{N}$ such that $\omega_i^h \approx k \omega_i^{h'}$ throughout the tracks' lifespans. This step results in sets of trajectories $\boldsymbol{\Omega}^c$ for each component c . The activation statuses $\boldsymbol{\delta}^c \in \{0, 1\}^{1 \times T}$ over T time steps are immediately deduced from these groups.

In a second step, components are grouped together according to their activation sequences. Pairwise similarities are computed to identify components that are always simultaneously active. The *Jaccard index* [30] is considered here, yet other clustering metrics can be used to compare label sequences with each other. This step results in the activation statuses $\boldsymbol{\delta}^a$ of the actuators. At last, each operation can be characterized by a set of actuators simultaneously active. Operation activation statuses $\boldsymbol{\delta}^o$ hence immediately stem from the actuators' statuses.

Interestingly, the transition from multi-target tracking to unsupervised multi-label classification follows a bottom up approach (gradually building the operation activation statuses from atomic components), whereas traditional underdetermined blind source separation methods use top-down approaches (from operation clustering to their decomposition into atoms) [4, 5].

NUMERICAL SIMULATIONS AND DISCUSSION

A study has been conducted to assess the ability of the proposed approach to identify a machine's production process. A representative synthetic scenario has been designed. A univariate signal was composed as the superimposition of actuator-originated signals, according to the pattern "AC – AB – BC – ABC", where A, B, C denote both actuators and atomic operations (i.e., originated by a single actuator).

These actuators produce a 50Hz-triangle wave, a 700 – 800Hz second order (with rise time $\tau_1^a = 0.3s$ and damping $\zeta = 0.3$) and a 400 – 500Hz first order (with rise time $\tau_2^a = 4s$) modulated sine waves as source signals respectively, with amplitudes 1, 1 and 3 units; noise standard deviations $\sigma_{s,0}^a = 0.2$, $\sigma_{s,1}^a = 0.8$, $\sigma_{s,2}^a = 1.2$. The signal is sampled at $f_s = 6250Hz$, and windowed at $w = 0.3s$ with a 50% overlap for Short-Time Fourier Transform (STFT) and spectral kurtosis computation. Measurements are generated according to a detection threshold $\tau = -5.5$ on the log power spectral density.

To compute the spectral kurtosis, a second order Butterworth band-pass filter is used with a bandwidth $\Delta\omega_{bp} = 3\Delta f$, depending on the spectral resolution $\Delta f = 1/w$.

This scenario has several specificities. An actuator is never active alone to begin with. This case would typically be misinterpreted by traditional decomposition algorithms, in that the pairs AB, BC and AC would form atomic unseparated modes. Furthermore, the second order source signal is characterized by its fast rise time and important overshoot. This signal highlights the expected difficulties encountered by the filter when presented such transients, as mentioned in the third section.

The SMC-PHD is parametered by a clutter rate $\hat{\lambda}_{FA} = 20$ false alarms per scan, a probability of detection $p_D = 99\%$, noise standard deviations $\sigma_A = 0.3$, $\sigma_\omega = 2 \cdot 2\pi \text{ rad} \cdot s^{-1}$, and 1500 particles per expected target. As a birth model, particles are sampled uniformly over the FOV in order to spot targets as they appear. Due to the high frequency resolution and the fast convergence of target states, we apply the roughening strategy proposed in [31] in order to limit the risk of sample impoverishment.

Despite obvious difficulties with fast rising transients, we observe that smooth transients are correctly tracked. In comparison to other UBSS frameworks in which data is represented as successive vectors, the orthogonality between the dimensions would make the associated techniques fail (e.g., Singular Value Decomposition (SVD) or sparse regression).

The results of the proposed approach are presented in Figure 1.

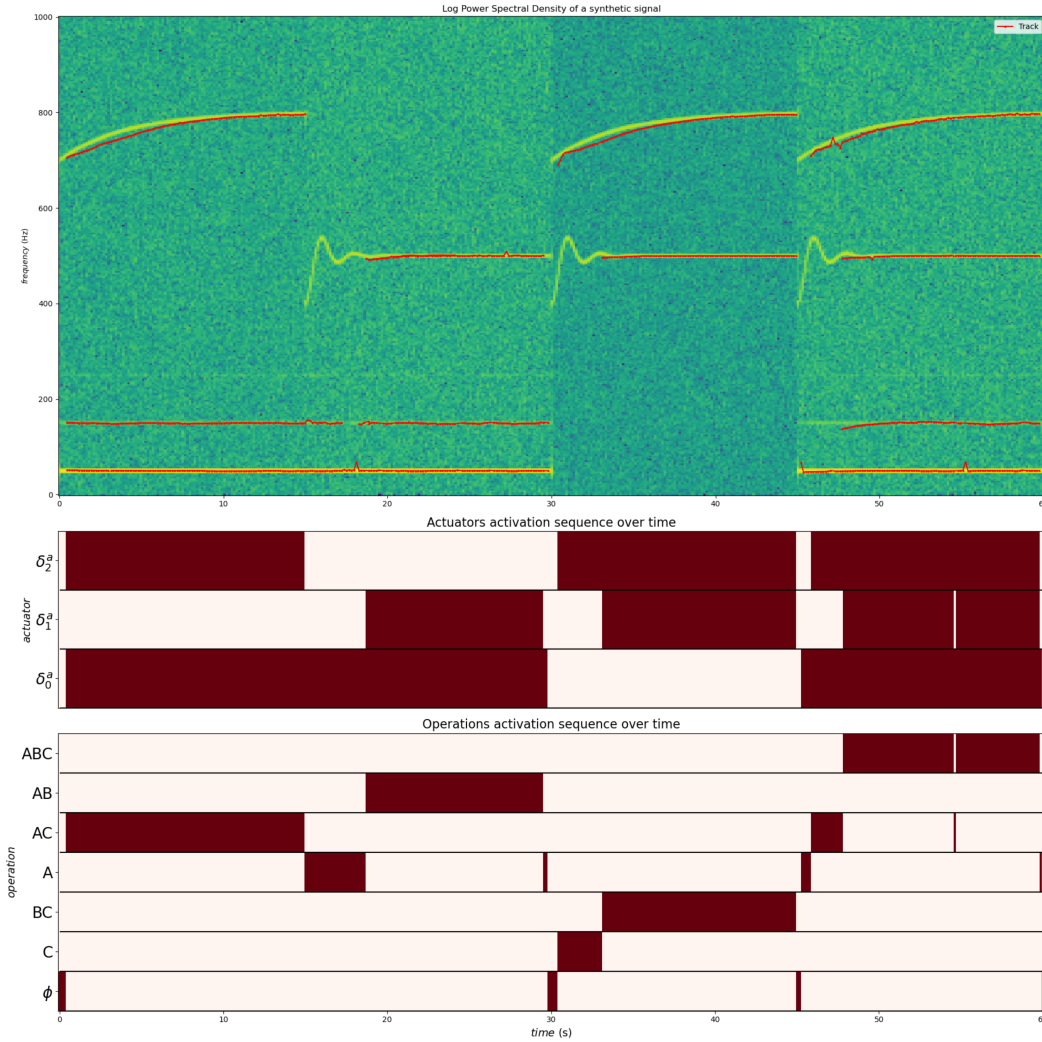


Figure 1: Tracking and decomposition results on a synthetic use case ("AB – BC – AC – ABC" sequence of operations with actuators $A = \delta_0^a$, $B = \delta_1^a$, $C = \delta_2^a$; active operations and actuators are shown in red.).

Numerical experiments highlighted the little sensitivity the SMC-PHD filter has with respect to its estimated clutter rate. Furthermore, slightly over-estimating $\hat{\lambda}_{FA}$ results in better estimation performance, yet at this stage, artifacts remained. This motivated the use of the spectral kurtosis feature to make the filter less dependent on the true (noise-induced) clutter rate across the different operations. Although this enhancement had a very positive effect on clutter resilience, the formulation proposed to elicit a feature likelihood out of the spectral kurtosis experimentally suffered the (theoretically expected) drawback of preventing the tracker to pick up on fast transient responses. Additionally, the birth model is corrected immediately after particle sampling using the proposed feature likelihood. This prevents erroneous tracks from being generated.

Another major advantage of the proposed method for UBSS problems is the ability to decompose a signal despite nonlinearly mixed signals. In practice, actuators emit component-specific signatures, i.e. harmonic components that uniquely define them in a machine. Hence by removing coupled harmonics (same frequencies, but different amplitudes), actuators can be well separated regardless of the way they were aggregated in the first place by the remote non-intrusive sensor.

CONCLUSIONS

In this paper, a feature-aided SMC-PHD was proposed to track harmonic components, using the spectral kurtosis to distinguish targets from noise-originated clutter. An algorithm was developed to convert MTT results (trajectories, states and covariances) into component, actuator and operation activation sequences. This end-to-end unsupervised process identification approach was verified on synthetic data, in a non-trivial scenario in which typical UBSS methods would underperform.

Future work will focus on increasing clutter resilience, handle stronger nonlinearities and transient responses, and validate the generalizability of the approach. Indeed, real-world signals often exhibit a variety of mixed behaviors, from stationary to non-stationary and nonlinear harmonic components. The use of heterogeneous models will be investigated. Moreover, uncertainty information provided by the MTT framework will be leveraged, using statistical distances between tracked harmonic components, for a better multi-label clustering performance.

REFERENCES

- [1] A Sadhu, S Narasimhan, and J Antoni. “A review of output-only structural mode identification literature employing blind source separation methods”. *Mechanical Systems and Signal Processing*, 94:415–431, 2017.
- [2] Sang Gyun Kim and Chang D Yoo. “Underdetermined independent component analysis by data generation”. In *International Conference on Independent Component Analysis and Signal Separation*, pages 445–452. Springer, 2004.
- [3] J Antoni and S Chauhan. “Second order blind source separation techniques (so-bss) and their relation to stochastic subspace identification (ssi) algorithm”. In *Structural Dynamics, Volume 3*, pages 177–187. Springer, 2011.
- [4] Romain Delabeye, Martin Ghienne, Arkadiusz Kosecki, and Jean-Luc Dion. “Unsupervised manufacturing process identification using non-intrusive sensors”. In *CFM 2022-25ème Congrès Français de Mécanique*, 2022.
- [5] Qingyi Wang, Yiqiong Zhang, Shuai Yin, Yuduo Wang, and Genping Wu. “A novel underdetermined blind source separation method based on optics and subspace projection”. *Symmetry*, 13(9):1677, 2021.
- [6] Benedikt Loesch and Bin Yang. “Source number estimation and clustering for underdetermined blind source separation”. In *Proc. IWAENC*, 2008.
- [7] Yuan Xie, Kan Xie, Zongze Wu, and Shengli Xie. “Underdetermined blind source separation of speech mixtures based on k-means clustering”. In *2019 Chinese Control Conference (CCC)*, pages 42–46. IEEE, 2019.
- [8] Julien Mairal, Francis Bach, Jean Ponce, and Guillermo Sapiro. “Online dictionary learning for sparse coding”. In *Proceedings of the 26th annual international conference on machine learning*, pages 689–696, 2009.
- [9] Jacek Wodecki, Rafał Zdunek, Agnieszka Wyłomańska, and Radosław Zimroz. “Local fault detection of rolling element bearing components by spectrogram clustering with semi-binary nmf”. *Diagnostyka*, 18, 2017.
- [10] Masako Matsumoto, Yu Fujimoto, and Yasuhiro Hayashi. “Energy disaggregation based on semi-binary nmf”. In *International Conference on Machine Learning and Data Mining in Pattern Recognition*, pages 401–414. Springer, 2016.
- [11] Ying Yao, Xiaohua Zhao, Yiping Wu, Yunlong Zhang, and Jian Rong. “Clustering driver behavior using dynamic time warping and hidden markov model”. *Journal of Intelligent Transportation Systems*, 25(3):249–262, 2021.
- [12] Kanghang He, Vladimir Stankovic, and Lina Stankovic. “Building a graph signal processing model using dynamic time warping for load disaggregation”. *Sensors*, 20(22):6628, 2020.
- [13] Xiaohua Nie. “Detection of grid voltage fundamental and harmonic components using kalman filter based on dynamic tracking model”. *IEEE Transactions on Industrial Electronics*, 67(2):1191–1200, 2019.
- [14] Chenchen Wu, Mario E Magaña, and Eduardo Cotilla-Sánchez. “Dynamic frequency and amplitude estimation for three-phase unbalanced power systems using the unscented kalman filter”. *IEEE Transactions on Instrumentation and Measurement*, 68(9):3387–3395, 2018.
- [15] Vicente Climente-Alarcon, Jose Alfonso Antonino-Daviu, Ari Haavisto, and Antero Arkkio. “Particle filter-based estimation of instantaneous frequency for the diagnosis of electrical asymmetries in induction machines”. *IEEE Transactions on Instrumentation and Measurement*, 63(10):2454–2463, 2014.

- [16] Sunghan Kim, Lars Holmstrom, and James McNames. "Multiharmonic tracking using marginalized particle filters". In *2008 30th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, pages 29–33. IEEE, 2008.
- [17] Ba-Ngu Vo, Sumeetpal Singh, Arnaud Doucet, et al. "Sequential monte carlo implementation of the phd filter for multi-target tracking". In *Proc. Int'l Conf. on Information Fusion*, pages 792–799, 2003.
- [18] Daniel E Clark, Kusha Panta, and Ba-Ngu Vo. "The gm-phd filter multiple target tracker". In *2006 9th International Conference on Information Fusion*, pages 1–8. IEEE, 2006.
- [19] Lennart Svensson, Daniel Svensson, Marco Guerriero, and Peter Willett. "Set jpd filter for multitarget tracking". *IEEE Transactions on Signal Processing*, 59(10):4677–4691, 2011.
- [20] Shaoming He, Hyo-Sang Shin, and Antonios Tsourdos. "Multi-sensor multi-target tracking using domain knowledge and clustering". *IEEE Sensors Journal*, 18(19):8074–8084, 2018.
- [21] Yanhua Ruan and Lang Hong. "Feature-aided tracking with gmt and hrr measurements via mixture density estimation". *IEE Proceedings-Control Theory and Applications*, 153(3):342–356, 2006.
- [22] Donald W Pace, M Mallick, and W Eldredge. "Spectral feature-aided multi-target multi-sensor passive sonar tracking". In *Oceans 2003. Celebrating the Past... Teaming Toward the Future (IEEE Cat. No. 03CH37492)*, volume 4, pages 2120–2126. IEEE, 2003.
- [23] Chen Ying, Cheng Zhen, and Wen Shuliang. "Feature aided gaussian mixture probability hypothesis density filter with modified 2d assignment". In *Proceedings of 2011 IEEE CIE International Conference on Radar*, volume 1, pages 800–803. IEEE, 2011.
- [24] Romain Delabeye, Hyo-Sang Shin, and Gokhan Inalhan. "Feature-aided smc-phd filter for nonlinear multi-target tracking in cluttered environments". In *International Conference on Robot Intelligence Technology and Applications*, pages 351–362. Springer, 2022.
- [25] Cyrille Stephan, Hugo Festjens, Franck Renaud, and Jean-Luc Dion. "Poles tracking of weakly nonlinear structures using a bayesian smoothing method". *Mechanical Systems and Signal Processing*, 84:136–151, 2017.
- [26] Jérôme Antoni. "The spectral kurtosis: a useful tool for characterising non-stationary signals". *Mechanical systems and signal processing*, 20(2):282–307, 2006.
- [27] J-L Dion, Imad Tawfiq, and Gaël Chevallier. "Harmonic component detection: Optimized spectral kurtosis for operational modal analysis". *Mechanical Systems and Signal Processing*, 26:24–33, 2012.
- [28] Thomas Devogele, Laurent Etienne, Maxence Esnault, and Florian Lardy. "Optimized discrete fréchet distance between trajectories". In *Proceedings of the 6th ACM SIGSPATIAL Workshop on Analytics for Big Geospatial Data*, pages 11–19, 2017.
- [29] Fred C Schweppe. "State space evaluation of the bhattacharyya distance between two gaussian processes". *Information and Control*, 11(3):352–372, 1967.
- [30] Luciano da F Costa. "Further generalizations of the jaccard index". *arXiv preprint arXiv:2110.09619*, 2021.
- [31] Tiancheng Li, Juan M Corchado, Shudong Sun, and Hongqi Fan. "Multi-eap: Extended eap for multi-estimate extraction for smc-phd filter". *Chinese Journal of Aeronautics*, 30(1):368–379, 2017.