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## Monte Carlo simulation of a scintillation crystal read by a SiPM with GATE

Brahim Mehadji<sup>a,b</sup>, Mathieu Dupont<sup>a</sup>, Denis Fougeron<sup>a</sup>, and Christian Morel<sup>a</sup>

<sup>a</sup>Aix-Marseille Univ, CNRS/IN2P3, CPPM, Marseille, France <sup>b</sup>present address: Aix Marseille Univ, APHM, Hôpital de la Timone et Nord, Médecine Nucléaire, Marseille, France

#### **Abstract**

Silicon photomultipliers (SiPMs) have recently emerged as a replacement for photomultiplier tubes (PMTs) for light detection in many applications, including high-energy physics and medical imaging. Recently, detailed Monte Carlo simulation of SiPMs has been implemented in GATE to quantify the impact of SiPM specifications on the linearity, energy and time resolution of a scintillation crystal read by a SiPM. In this paper, GATE simulations of a LYSO crystal coupled to a SiPM are compared to measurements. The energy spectra of the <sup>241</sup>Am and <sup>22</sup>Na radioactive sources are found to agree with less than 2% difference. The linearity of the SiPM response is duly affected by the SiPM saturation and, as seen above 511 keV with our configuration, it is slightly enhanced by the generation of crosstalk. Furthermore, with an over-voltage close to that recommended by the manufacturer, all sources of SiPM noise contribute about equally to the degradation of the energy resolution at low energies, which is downgraded by more than 15% at 60 keV, but have less impact at higher energies. In addition, the GATE simulations show that crosstalk plays an important role on the time resolution of the installation.

Keywords: Monte Carlo simulation, GATE, SiPM, CTR, gamma-ray spectrometry.

#### 1. Introduction

Silicon photomultipliers (SiPMs) replace photomultiplier tubes (PMTs) for light detection in many applications, incuding high-energy physics and medical imaging [1] [2] [3]. SiPMs are composed of thousands of micro avalanche photodiodes [4]. Their sizes vary from one to tens of millimeters depending on the SiPM model. The detection of an optical photon by one micro avalanche photodiode (also called micro-cell) induces an electric signal (pulse), which approximately follows a double exponential decay. The sum of all the pulses issued by the micro-cells gives the SiPM signal, which is very sensitive to temperature and over-voltage [5]. Various sources of correlated noise — crosstalks (micro-cells that fire due to infrared excitation resulting from the avalanche of another neighbor micro-cell), after-pulses (a micro-cell that fires again a few nanoseconds after the bulk interaction resulting from a first avalanche within the micro-cell), delayed-crosstalks (a micro-cell that fires a few nanoseconds after the bulk interaction resulting from a first avalanche in another micro-cell) — and uncorrelated noise — dark counts (a random firing of a micro-cell due to thermic excitation) and white noise — perturb this signal.

Recently, detailed simulation of analog SiPMs has been implemented in GATE [6] [7] to quantify the impact of SiPM specifications on the linearity, energy and temporal resolution of a scintillation crystal read by a SiPM.

In this paper, GATE simulations of a LYSO crystal coupled to a SiPM or to a PMT are compared to measurements. The comparison between the simulated and measured PMT energy spectra (i.e., in the absence of significant sources of correlated and uncorrelated noise) of <sup>241</sup>Am and <sup>22</sup>Na radioactive sources is used to normalize the SiPM data for the intrinsic linearity and resolution of the LYSO scintillation response. This allows the non-linearity of the SiPM response due to its limited number of cells to be assessed on the simulated data and the impact of SiPM noise on energy and time resolution to be quantified using an over-voltage close to that recommended by the manufacturer.

#### 4 2. Measured and simulated data of a LYSO crystal read by a PMT

The experimental setup consists of a polished  $3 \times 3 \times 5$  mm<sup>3</sup> scintillating crystal of lutetium-yttrium oxyorthosilicate (LYSO) from Crystal Photonics. The unwrapped crystal was positioned without optical coupling on the photocathode surface of a PMT 9125WB (11 dynodes, photon detection efficiency (PDE) @ 420 nm: 28%, single electron resolution (SER): 7.5 ns FWHM) from ET Enterprises, connected to an oscilloscope HDO9404-MS from LeCroy. The crystal was irradiated with 60, 511 and 1275 keV gamma rays emitted by  $^{241}$ Am and  $^{22}$ Na radioactive sources.

#### 2.1. Photo-peak measurements

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The PMT signals were integrated and results were plotted in histograms to get energy spectra. Figure 2 shows the measured photo-peaks at 60, 511 and 1275 keV. Photo-peak positions and resolutions (FWHM) result from fitting a Gaussian superimposed on a linear background. Photo-peak positions show a non linearity of 18% of the LYSO scintillation response at 60 keV compared to 511 keV, while it is negligible (< 1%) at 1275 keV [8]. These values will be used to set the scintillation light yield of LYSO at 60, 511 and 1275 keV in GATE.

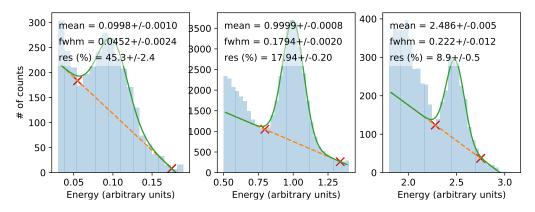


Fig. 1. Measured energy spectra for a  $3 \times 3 \times 5$  mm<sup>3</sup> LYSO crystal read by a PMT for incident gamma rays of (left) 60 keV, (center) 511 keV and (right) 1275 keV. Energy scale (in arbitrary units) is normalized to the 511 keV photo-peak position. The peak positions and resolutions result from fitting (in green) a Gaussian superimposed on a linear background.

#### 2.2. GATE simulations

To model our experimental setup with GATE, the PDE of the PMT and the crystal composition (Lu<sub>1.8</sub>Y<sub>0.2</sub>Si<sub>1.0</sub>O<sub>5.0</sub>), density (7.05 g/cm<sup>3</sup>), refractive index (1.81), and scintillation light yield ( $LY = 28\,000\,\text{ph/MeV}$ ) and fall-off decay time ( $\tau_d = 42\,\text{ns}$ ) were instantiated from specifications provided in the literature [9] and by the manufacturers. The scintillation rise time constant was set to 57 ps with a resolution of 20 ps FWHM according

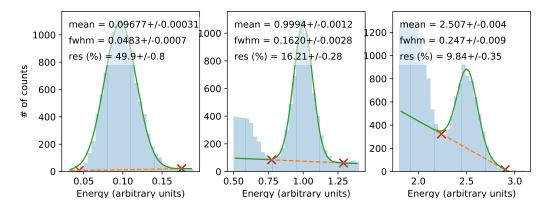


Fig. 2. Simulated energy spectra for a  $3 \times 3 \times 5$  mm<sup>3</sup> LYSO crystal read by a PMT for incident gamma rays of (left) 60 keV, (center) 511 keV and (right) 1275 keV. Energy scale (in arbitrary units) is normalized to the 511 keV photo-peak position. The peak positions and resolutions result from fitting (in green) a Gaussian superimposed on a linear background.

to [10]. The non linearity of scintillation response of the crystal was set according to the measurements described in the previous section. Finally, we used the *Rought\_LuT* surface type from the University of California Davis Look Up Table [11] [12].

In GATE, the number of scintillation photons generated for an energy E deposited in the crystal is distributed with a Gaussian of mean  $E \times LY$  and standard deviation  $RESOLUTIONS\,CALE \times \sqrt{E \times LY}$ . The parameter  $RESOLUTIONS\,CALE$  accounts for the impact of the non-linearity of the light yield at low energy, which degrades energy resolution because of the balance between photoelectric interaction and Compton scattering while collecting the total energy deposit in the crystal that results from the sum of contributions at different energies [8]. Once set to 4.41, the parameter  $RESOLUTIONS\,CALE$  is reproducing at best the measured energy resolutions.

#### 2.3. Comparison of measured and simulated data of a LYSO crystal read by a PMT

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Table 1 lists the measured and simulated photo-peak resolutions (FWHM) for incident gamma rays of 60, 511 and 1275 keV. Despite the adjustment of the scintillation light yields from the measured photo-peak positions, significant differences are still observed between the simulated and measured energy resolutions that are probably due to the large intrinsic resolution contribution of the scintillation crystal [13], which is not taken into account by GATE.

E (keV)	Measured res. (%)	Simulated res. (%)	Relative diff. (%)		
60	$45.3 \pm 2.4$	$49.9 \pm 0.8$	$10.2 \pm 5.6$		
511	$17.9 \pm 0.2$	$16.2 \pm 0.4$	$-9.5 \pm 2.2$		
1275	$8.9 \pm 0.5$	$9.8 \pm 0.4$	10.1 ± 7.19		

Table 1. Comparison of the measured and simulated photo-peak resolutions (FWHM) for 60, 511 and 1275 keV gamma rays falling on a LYSO crystal read by a PMT.

#### 3. Measured and simulated data of a LYSO crystal read by a SiPM

We used here the same experimental setup as in Section 2, except that the PMT was replaced by a  $3 \times 3 \,\mathrm{mm^2}$  SiPM MPPC S13360-3050CS (micro-cell pitch:  $50 \,\mu\mathrm{m}$ , #cells: 3558, PDE @ 420 nm: 40%, gain @  $V_{ov} = 3 \,V : 1.7 \times 10^6$ , recovery time:  $28.5 \,\mathrm{ns}$  [6], crosstalk probability: 5.4% [6]) from Hamamatsu Photonics powered with an over-voltage of 3 V with the C12332-01 card, which contains an OPA846 signal amplifier from Texas Instrument. The SiPM was coupled to the crystal with an optical grease from Saint-Gobain with a refractive index of 1.51 and operated at a room temperature of  $21\,^{\circ}\mathrm{C}$ .

#### 3.1. Photo-peak measurements

The amplifier was deactivated for measurements at 511 keV and 1275 keV, because otherwise the signal amplitude saturates. The SiPM signal were numerically integrated during 400 ns with a sampling frequency of 5 GHz and results were binned in histograms to get energy spectra. Figure 4 (top row) presents the photopeaks measured at 60, 511 and 1275 keV. Photo-peak positions and resolutions (FWHM) result from fitting a Gaussian superimposed on a linear background.

#### 3.2. GATE simulations

We used the same values for the specifications of the LYSO crystal as in Section 2.2. All the parameters used for the Monte Carlo simulation of the SiPM with GATE are instantiated in the files SiPM.XML and Surface.XML, which are described in [6]. The SiPM equivalent circuit has a cell capacitance  $Cp=84.5\,pf$ , a cell parasitic capacitance  $Cq=16.8\,pF$ , a SiPM grid capacitance  $Cg=16.8\,pF$  and a cell quenching resistance  $Rq=300\,k\Omega$ , but we do not simulate it to generate the SiPM signal in GATE. Instead, the pulse amplitude and shape of a micro-cell stored in the file SiPM.XML corresponds to the average pulse obtained by illuminating the entire surface of the SiPM with a 20 ps FWHM laser pulse divided by the number of micro-cells (3558). Figure 3 presents the measured (blue curve) and simulated (orange curve) mean shapes and dispersions of SiPM signals for 60, 511 and 1275 keV incident gamma rays.

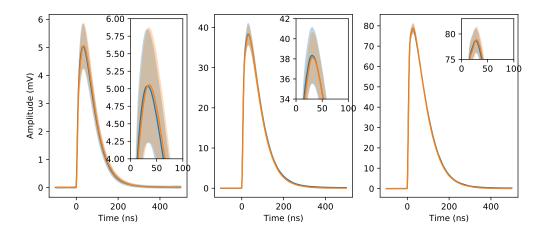


Fig. 3. Measured (blue curve) and simulated (orange curve) mean shapes and dispersions of SiPM signals for (left) 60 keV, (center) 511 keV, and (right) 1275 keV gamma rays falling on a LYSO crystal read by a SiPM.

#### 3.3. Comparison of measured and simulated data of a LYSO crystal read by a SiPM

Measured and simulated energy spectra are compared in Figure 4. Note that for 511 keV gamma rays, the peak-to-valley depth is smaller for measured data than for simulated data because measurements were performed using a  $^{22}$ Na radioactive source, which implies a Compton background from the 1275 keV gamma rays below the 511 keV photo-peak. Nevertheless, the non linearity of SiPM due to the limited number of micro-cells [14] is correctly simulated by GATE as shown in Figure 5 (left). The non-linearity of SiPM has a major contribution to the energy resolution since it improves faster with energy than it would with a linear response. In fact, following [14], the non linear response of SiPM can be modeled by  $V(E) = A \times (1 - \exp(-E/(N_{\text{cells}} \times B)))$  where E is the energy and E and E are free parameters. A linear response of the SiPM E corresponding to the first-order approximation of E can then be estimated by E by E and E are E and E are free parameters. A linear response of the SiPM E corresponding to the first-order approximation of E can then be estimated by E by E and E are free parameters.

The photo-peak resolutions of measured and simulated data differ by about 10% for gamma rays of 60 and 511 keV. Indeed, as we noted in Section 2.3, the simulated resolutions do not take into account

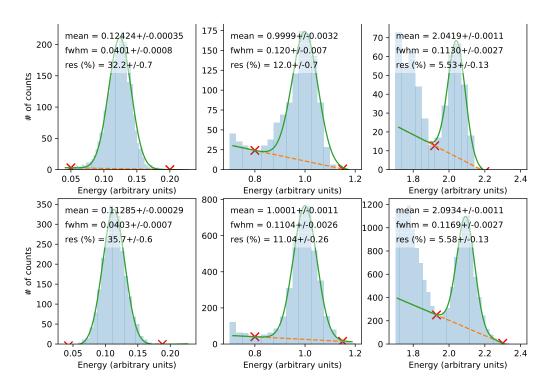


Fig. 4. Measured (top row) and simulated (bottom row) energy spectra for a  $3 \times 3 \times 5$  mm<sup>3</sup> LYSO crystal read by a SiPM for incident gamma rays of (left) 60 keV, (middle) 511 keV, (right) 1275 keV. Energy scale (in arbitrary units) is normalized to the 511 keV photopeak position. The peak positions and resolutions result from fitting (in green) a Gaussian superimposed on a linear background.

the intrinsic resolution of the scintillation crystal. For a fair comparison of the measured and simulated SiPM resolutions, we need to normalize the simulated SiPM resolutions by the ratio of the measured and simulated PMT resolutions. However, as the PMT energy spectra are not subject to saturation effects, this normalization requires first correcting the SiPM spectra for the SiPM non-lineartiy as proposed in [14] (see Figure 5 (right)). Table 2 shows the positions and resolutions (FWHM) of the measured and simulated photopeaks. A good agreement of less than 1.5% is obtained between the measured and simulated resolutions.

	Mea	surements	Sin	nulations	Relative diff. (%)		
E (keV)	pos.	res. (%)	pos.	res. (%)	pos.	res.	
60	0.11	$32.6 \pm 0.7$	0.11	$32.9 \pm 0.5$	0	0.9	
511	1.00	$13.5 \pm 0.8$	1.00	$13.3 \pm 0.3$	0	1.5	
1275	2.48	$8.6 \pm 0.6$	2.49	$8.5 \pm 0.7$	0.4	1.2	

Table 2. Comparison of the measured and simulated photo-peak positions (in arbitrary units normalized to the 511 keV photo-peak position) and resolutions (FWHM) for incident gamma rays of 60, 511 and 1275 keV after correction of the SiPM non-linearity and normalization of the resolutions by the ratio of the measured and simulated PMT resolutions.

#### 3.4. Impact of SiPM noise sources on energy resolution

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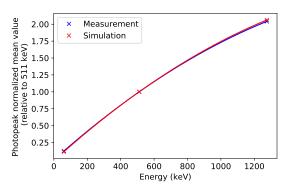
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In order to determine the effect of the various SiPM noise sources (dark counts, after-pulses and cross-talks) on the energy spectra, simulation studies were carried out with all noise sources disabled or with only one noise source disabled at a time. Table 3 lists the photo-peak positions and resolutions (FWHM) resulting from this study. A significant impact of SiPM noise is observed on the photo-peak resolution for incident gamma rays of 60 keV: disabling the SiPM noise improves the resolution by 15%. A moderate contribution



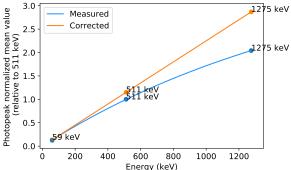


Fig. 5. (left) Comparison of SiPM measured (blue) and simulated (red) photo-peak positions (in arbitrary units normalized to the 511 keV photo-peak position) as a function of the incident gamma ray energy. (right) Correction of the non-linearity of the SiPM energy spectra as proposed in [14].

from the SiPM non-linearity can also be seen for the incident gamma rays of 1275 keV: disabling the SiPM noise increases the position of the photo peak by 2.5%.

	With noise		No after-pulses		No crosstalks		No dark counts		No noise	
E (keV)	pos.	res. (%)	pos.	res. (%)	pos.	res. (%)	pos.	res. (%)	pos.	res. (%)
60	0.11	$32.4 \pm 0.5$	0.11	$28.4 \pm 1.2$	0.11	$28.7 \pm 0.9$	0.11	$28.4 \pm 1.2$	0.11	$27.7 \pm 1.2$
511	1.00	$11.0 \pm 0.3$	1.00	$10.9 \pm 0.6$	1.00	$10.2 \pm 0.6$	1.00	$10.3 \pm 0.6$	1.00	$10.3 \pm 0.5$
1275	2.09	$5.6 \pm 0.1$	2.10	$5.3 \pm 0.4$	2.13	$5.3 \pm 0.3$	2.09	$5.5 \pm 0.3$	2.14	$5.2 \pm 0.6$

Table 3. Simulated photo-peak positions (in arbitrary units normalized to the 511 keV photo-peak position) and resolutions (FWHM) for incident gamma rays of 60, 511 and 1275 keV.

Figure 6 displays the average signal shapes obtained by disabling all or only one SiPM noise source at a time. The average signal amplitude decreases when no noise is simulated. At 3 V over-voltage, crosstalk makes the largest contribution to the signal, while dark counts make the least.

#### 3.5. Measured and simulated coincidence time resolutions (CTR)

Simulation of the SiPM time resolution is evaluated by comparing the measured and simulated coincidence time resolutions (CTR) for the detection of 511 keV annihilation pairs by two  $3 \times 3 \times 5$  mm<sup>3</sup> LYSO crystals coupled to SiPMs with optical grease and wrapped in Teflon. Each SiPM was connected to an amplification circuit optimized for timing applications as described in [15]. A <sup>68</sup>Ge source was placed between the two detectors and the oscilloscope was set to trigger coincidentally with a threshold level three times above the electric noise. The time difference between the two signals was plotted in a histogram and the CTR (FWHM) was estimated using the NEMA protocol [16]. Coincidence time measurements were performed for several SiPM over-voltages from 4 to 9 V.

The method proposed by Rosaldo and Hidalgo [17], which was used in [6] to evaluate the paramaters necessary to simulated the SiPM noise sources, i.e.,  $\epsilon_c$  (the probability to have at least one crosstalk),  $C_{\rm AP}$  (the probability that an after-pulse occurs at t=0),  $C_{\rm CT}$  (a constant expressed in unit of [time]<sup>-1/2</sup> involved in the probability of having a delayed-crosstalk) and DCR (dark count rate), cannot be used to simulate the SiPM operating at high over-voltages. Instead, these four parameters can be extrapolated from measurements obtained with lower over-voltages (below 5 V) using the following equations:

$$\epsilon_c = 1 - e^{-\left(\frac{V_{\text{ov}}}{k}\right)^{1+\alpha}}$$
 
$$AP = \left(\frac{V_{\text{ov}}}{k_{\text{AP}}}\right)^{1+\beta}$$

where AP stands for either  $C_{AP}$ ,  $C_{CT}$  or DCR, and k,  $\alpha$ ,  $k_{AP}$  and  $\beta$  are fitting parameters. Also note that the PDE is over-voltage dependent and must be instantiated in GATE using the manufacturer's specifications.

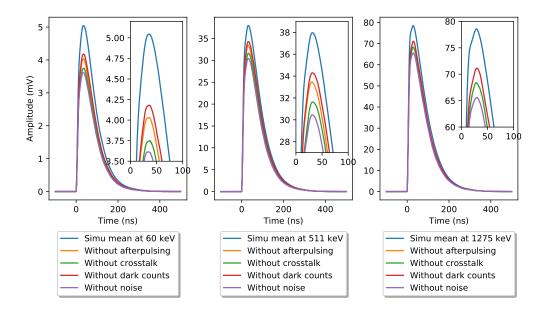


Fig. 6. Average signal shapes obtained by removing all or one SiPM noise source at a time for incident gamma rays of (left) 60 keV, (center) 511 keV and (right) 1275 keV.

Finally, we used the *Rough\_Teflon* surface type of the UNIFIED model [18] with a reflectivity of 52% to describe the crystal surfaces as in [11]. Figure 7 (left) shows the comparison of histograms of measured and simulated coincidence time differences at an over-voltage of 9 V. The measured and simulated CTR as a function of SiPM over-voltage presented in Figure 7 (right) are in good agreement.

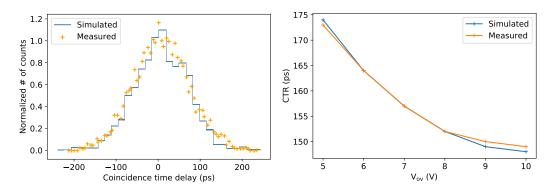


Fig. 7. (left) Comparison of histograms of measured (crosses) and simulated (continuous line) coincidence time differences at an over-voltage of 9 V. (right) Measured and simulated CTR as a function of SiPM over-voltage.

#### 3.6. Impact of SiPM noise on CTR

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To understand the contribution of crosstalk on CTR, a simulation was performed without crosstalk with a SiPM over-voltage of 9 V. A degradation of CTR of about 4% due to crosstalk is observed. This contribution increases with over-voltage since crosstalk increases with over-voltage [19]. For instance, Stefan Gundacker et al., showed in [10] that the impact of the crosstalk on CTR becomes significant above  $\epsilon_c = 0.65$ .

The dependence of CTR as a function of delayed-crosstalks and afterpulses is not studied here since these latter appear after the SiPM response to detected optical photons. As far as the DCR is concerned, its contribution is less intuitive. When a dark count appears shortly before the detection of a gamma ray, the estimated time of triggering will be that of the dark count. The time distribution of the DCR being uniform, it will add uniform noise to the CTR. However, if the DCR is very large, it is no longer possible to keep the threshold close to the electrical noise. In this case, the measured CTR will also be impacted.

#### 4. Conclusion

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GATE simulations of a scintillation crystal read by a SiPM were compared successfully to measurements. The energy spectra of the  $^{241}$ Am and  $^{22}$ Na radioactive sources for a  $3 \times 3 \times 5$  mm $^3$  LYSO crystal coupled to a  $3 \times 3$  mm $^2$  SiPM MPPC S13360-3050CS are in agreement with less than 2% difference, and CTRs estimated as a function of the SiMP over-voltage are in good agreement between measurements and simulations.

In a second step, the simulations were used to quantify the impact of SiPM noise sources on the energy and time resolutions. With an over-voltage close to that recommended by the manufacturer, all SiPM noise sources contribute about equally to the degradation of the energy resolution at low energies, which is degraded by more than 15% at 60 keV, but have less impact at higher energies. In addition, the GATE simulations show that crosstalk plays an important role on the time resolution of the setup.

#### 155 Acknowledgements

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