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Michael Eichhorn

Tiziano Bellini

Daniel Mayenberger

# **Reverse Stress Testing in Banking**

**A Comprehensive Guide**

De Gruyter

Claudio Albanese<sup>19</sup>, Stéphane Crépey<sup>20</sup> and Stefano Iabichino<sup>21</sup>

## 17 Capital and Collateral Simulation for Reverse Stress Testing

Chapter 16 introduced a holistic framework for Reverse Stress Testing (RST), based on accurate data mining of large-scale synthetic scenarios. The current section takes a step closer to the operational reality of a market maker, shedding light on the theoretical and computational framework for the analysis of extreme events in the form of forward-looking RST. The framework builds on a large-scale Monte Carlo simulation, thus, targeting the bespoke characteristics of the portfolio held. The Monte Carlo, relying on models calibrated to option prices, is used to propagate all the risk factors the portfolio is exposed to, while the RST aims to gather those future economic states under which bank solvency could be impaired. Specific and General Wrong Way Risks effects are injected through dynamic market-credit and credit-credit correlations.

Throughout this chapter, which provides a roadmap on the implementation of Albanese, C., S. Crépey, and S. Iabichino (2020), we introduce the main ingredients and mechanics required to achieve such a holistic framework, which coherency extends to the entire banking-book. While driving the reader throughout the framework's alchemy, we will present numerical results collected on a representative portfolio. We refer the interested reader to Albanese, C., S. Crépey, and S. Iabichino (2020) for details on the possible scope of applicability, of the presented framework, to the reality of a market-maker.

### 17.1 Setting the Stage

For the sake of simplicity, we use the risk-free (OIS rate) asset growing as a numeraire, and we assume equality between physical and pricing measures, denoted by  $P$  and  $Q$  respectively. The latter assumption is justifiable by the absence of reliable knowledge about the discrepancy between  $P$  and  $Q$  at the large horizons, which are typical of an OTC derivative portfolio. Furthermore, we indicate with  $E^Q[\cdot | F_t] = E_t^Q[\cdot]$  the conditional (to the  $\sigma$ -algebra  $F_t$ ) risk-neutral expectation.

Consider a portfolio composed by a set  $N_c$  of counterparties. A counterparty can be characterised either by an individual netting-set, or by a collection of netting sets<sup>22</sup>, for which a Credit Support Annex (CSA) characterises collateral posting obligations.

For each counterparty, the characteristics of an individual netting-set are analysed in terms of risk-factors drivers and future cash-flows. This information is then sourced to the Monte Carlo based environment to generate counterparty specific CCR, and portfolio-wide funding exposure profiles. Therefore, for each counterparty, the distribution of future CCR losses as well as its present expected value i.e., the CVA (Counterparty Valuation Adjustment), are computed.

The (unilateral) CVA for a counterparty  $c$  is defined as

$$\begin{aligned} UCVA_{t_0}(c) &= E_{t_0}^Q \left[ e^{-\int_{t_0}^{\tau_c} r_{OIS}(s) ds} \mathbf{1}_{\{t_0 < \tau_c < T\}} LGD(c) (V_{\tau_c}(c) - CSA_{\tau_c}(c))^+ \right] \\ &= E_{t_0}^Q \int_{t_0}^T e^{-\int_{t_0}^t r_{OIS}(s) ds} \mathbf{1}_{\{t_0 < t < T\}} LGD(c) (V_t(c) - CSA_t(c))^+ \delta_{\tau_c}(dt) \end{aligned}$$

where:

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<sup>20</sup> University of Evry in Paris-Saclay. The research of Stéphane Crépey benefited from the support of the Chair Stress Test, RISK Management and Financial Steering, led by the French Ecole polytechnique and its Foundation and sponsored by BNP Paribas.

<sup>21</sup> JP Morgan, London. This paper represents the opinions of the authors and it is not meant to represent the position or opinions of JP Morgan or their members.

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<sup>22</sup> However, for the sake of narration simplicity, in the following, we will adopt the simplifying view that a counterparty has only one netting set with a CSA.

- $t_0$  and  $T$  are the computation date and the final maturity of the portfolio;
- $\tau_c$  is the default time of the counterparty  $c$ ;
- $r_{OIS}$  is the risk-free rate;
- $1_A$  is an indicator function, subject to the A condition;
- $LGD(c)$  is the counterparty-specific Loss Given Default, i.e. one minus the counterparty-specific recovery rate;
- $V_t(c)$  is the counterparty specific aggregated Mark-to-Market;
- $CSA_t(c)$  is the collateral posted by the client  $c$  according to the counterparty specific Credit Support Annex;
- $v^+ = \max(v, 0)$ .

Additionally, the present value of the Funding Valuation Adjustment FVA, computed at the funding-set level, is determined. Following Albanese, Andersen, and Iabichino (2015), the FVA is defined as:

$$FVA_{t_0} = E_{t_0}^Q \int_{t_0}^T e^{-\int_{t_0}^t r_{OIS(s)} ds} S_t(B) \left( \sum_{c=1}^{N_c} (V_t(c) - CSA_t(c)) 1_{t < \tau_c} \right)^+ 1_{t < \tau_B} dt$$

where:

- $\tau_B$  is the default time of the bank;
- $S_t(B)$  is the funding spread of the bank.

### 17.1.1 Tensor algebra, CVA, and FVA computations

Define a set of epoch dates  $t_h$ , with  $h \in \{1, 2, \dots, N_h\}$ , and a set of primary scenarios  $s \in [1, N_s]$ . For each risk-factor characterising the portfolio, define a set of discrete state variables  $x$ , with  $x \in \{1, 2, \dots, N_x\}$ . The dynamic of each risk-factor are described in terms of a transition probability tensor, representing a collection of kernels  $(T_{\delta t}(x, x'; t_h))$  whose elements specify the transition probabilities to evolve from the state  $x$  to the state  $x'$  at any given epoch-date  $t_h$ . The validity conditions for a transition probability tensor are:

1. Positivity:  $T_{\delta t}(x, x'; t_h) \geq 0, \quad \forall x;$
2. Probability Conservation:  $\sum_x T_{\delta t}(x, x'; t_h) = 1, \quad \forall t_h,$

The first condition implies the non-negativity of the elements of the tensor, while, for each epoch-date  $t_h$ , the second imposes the additivity to one of the kernels.

For each epoch-date, counterparty, and scenario, we store in-memory all the relevant counterparty specific information e.g. conditional default probabilities  $(PD(h, c, s))$ , OIS discounted exposures  $(\hat{V}(h, c, s))$  and collateral amounts  $(\overline{CSA}(h, c, s))$ .

In order to compute the CVA, following Albanese and Pietronero (2010), define a loss lattice  $l \in \{1, 2, \dots, N_l\}$ . At each epochs, the scenario conditional portfolio default loss distribution  $(\Lambda(h, s, l))$  is calculated taking the convolution product over individual counterparty losses conditioned to a scenario (assuming that the individual counterparty losses are independent given the realisation of a scenario):

$$\Lambda(h, s, l) = \Lambda(h, s, c_1, l) * \Lambda(h, s, c_2, l) * \dots * \Lambda(h, s, c_{N_c}, l)$$

where

$$\Lambda(h, s, c_i, l) = (1 - PD(h, c_i, s)) \delta(l) + PD(h, c_i, s) \delta\left(l - \left(\hat{V}(h, c_i, s) - \overline{CSA}(h, c_i, s)\right)\right)$$

is the probability that the loss triggered by the possible default of counterparty  $c_i$  at time  $h$  equals  $l$ , conditional on the realization  $s$  of the economic drivers at time  $h$  (with  $\delta$  for the Dirac measure at 0).

The (unconditional) point-in-time Default-Loss-Distribution, is deduced as:

$$\Lambda(h, l) = \frac{1}{N_s} \sum_{s=1}^{N_s} \Lambda(h, s, l)$$

from which cumulative portfolio losses between time 0 and  $H$  are computed by summation between epochs  $h = 0$  and  $H$  (see Figure 16.1). Figure 16.1 represents the expected loss level, for each epochs, using white pins, which converges to the time 0 CVA in the large horizon limit.

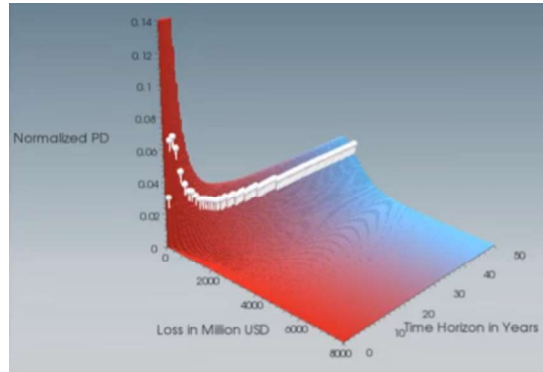


Figure 17.1 Cumulative Portfolio Default Loss Distribution. The white pins indicate the corresponding expected portfolio Default Loss at each epoch date

At this point, the definition of a suitable scenario ranking criterion, i.e. a merit function, takes a pillar role.

## 17.2 The KVA

A holistic RST framework, which scope of applicability extends to the entire derivative book, requires a merit function sensitive to all the risk sources characterising the book, including CVA and FVA volatility. As we will discuss, the KVA owns all the desired properties.

The Regulatory Capital Valuation Adjustment (KVA) is a risk margin metric representing the expected remuneration of bank shareholders for their posted loss-absorbing capital, at some hurdle rate ( $h$ ). However, instead of considering Pillar I regulatory capital models, following Albanese, Caenazzo, and Crépey (2016, 2017), Crépey, Sabbagh, and Song (2020), Albanese, Crépey, Hoskinson, and Saadeddine (2019), and consistently with IFRS17 (see IASB (2017)), we use a more comprehensive definition of KVA in terms of Pillar II economic capital.

The reason to favour a KVA defined in compliance with the Pillar II economic capital sits on the fact that, while Pillar I Regulatory Capital is consistent in intent with the Pillar II economic capital, Pillar I Regulatory Capital is characterized by several modelling assumptions. These assumptions, proposed to reduce computation times, strongly shrink the risk factors which the Pillar I Regulatory Capital is able to capture. For example, the Expected Effective Positive Exposure (EEPE)<sup>23</sup>, based on an individual counterparty viewpoint, neglects General Wrong-Way Risk (GWWR), default clustering, as well as complex tail dependencies, while the usage of Gaussian copulas (in the default RWA) highly

<sup>23</sup> The EEPE, for an unsecured portfolio, is defined as:  $\int_{t_0}^{1 \text{ Year}} \max_{s \in [t_0, t]} E[(V_s)^+] dt$

stylizes default occurrences. Remarkably, the possibility of creating doom loops (e.g., between Capital Requirements and CCR hedges) and the losses triggered by the volatility of CVA and FVA are completely neglected.

Instead, Pillar II economic capital strongly tights to the shareholder’s Core Equity Tier 1 capital (CET1), which is allocated to absorb any form of unexpected losses. CET1 depletions are given by

$$-dCET1_t = dD(t) + dF(t) + dUCVA_t + dFVA_t \tag{17.1}$$

where  $dD(t)$  and  $dF(t)$  are the shorthand notation for the integrands in the  $E_{t_0}^Q \int_{t_0}^T \dots$  formulations of the CVA and FVA above, i.e. the future losses realized by the bank due to defaults and funding expenses. The Pillar II economic capital (EC) profile at time  $t$  is specified as the 1 year 97.5% expected shortfall depletion of CET1, i.e.

$$EC(t) = E^Q[CET1_t - CET1_{t+1} \mid CET1_t - CET1_{t+1} \geq VaR(t)] \tag{17.2}$$

where  $VaR(t)$  is the 97.5% Value-at-Risk of the  $CET1_t - CET1_{t+1}$  distribution. The present value of the KVA is then given by

$$KVA_0 = h \int_0^\infty EC(t) dt \tag{17.3}$$

where  $h$  is the inter-temporal hurdle rate, representing shareholder’s expected reward for unit of capital invested.

Therefore, the  $KVA_0$ , which equals zero if and only if the CET1 is a constant function through time, represents the present value of the weighted average of future economic capital needs. It is, therefore, an organic metric to express the inter-temporal risk of the bank.

The computation of equations (17.1)-( 17.2) requires the knowledge of the conditional future  $UCVA_t$  and  $FVA_t$  terms. These future conditional metrics are computed through nested Monte Carlo, where, for each epoch-date and primary scenario, a set of secondary scenarios are branched-off (ref. Figure 17.2 Nested Monte Carlo).

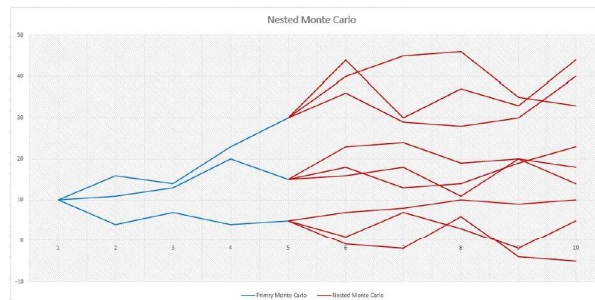


Figure 17.2 Nested Monte Carlo

See Figure 17.3 for the resulting CET1 distribution at one epoch date.

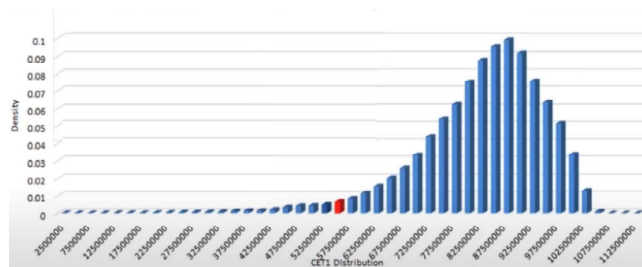


Figure 17.3 The CET1 distribution for one epoch date. The red bin marks the EC profile at the 97.5% confidence level.

### 17.3 The KVA as Merit Function and the Reversing Problem

The definition of KVA, anchoring to those CET1 losses exceeding their 97.5% VaR, naturally focuses on those stress scenarios where the whole portfolio suffers from severe default and/or market losses. However, before risk-factor data-mining, all the scenarios that composed the CET1 distribution need to be ranked in terms of their riskiness.

We define the scenario ( $s \in [1, N_s]$ ) incremental KVA as the marginal contribution of a given scenario to the overall  $KVA_0$ , i.e. as:

$$\delta_s KVA_0 = KVA_0 - KVA_0(-s) \quad \forall s \tag{17.4}$$

In other words, the scenario incremental KVA is defined as the difference between the KVA and the KVA recomputed excluding scenario  $s$ .  $\delta_s KVA_0$  thus, captures the dividend stream accrual requested by the capital market to lend capital to the bank to cover unexpected losses. Therefore,  $\delta_s KVA_0$  represents a suitable merit function, as each incremental scenario will collect the additional capital requirement needed to cover any type of future losses.

#### 17.3.1 Risk-Factors Inversion and Data Mining

Among the pool of scenarios, we select stress scenarios in the confidence interval [99%,99.9%]. We, thus, select the 200 most adverse scenarios out of the original pool of primary scenarios (ref. to Figure 17.4)

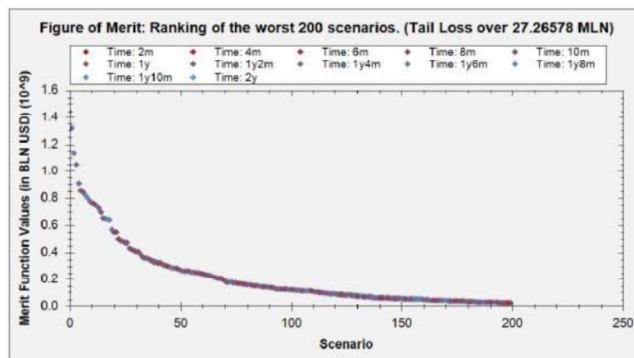


Figure 17.4 Figure of Merit Ranking criterion

An interval-by-interval drill-down analysis, among the pool of gathered CET1 absorbing scenarios, identifies the points in time where the most significant CET1 depletions occur (ref. to Figure 17.5).

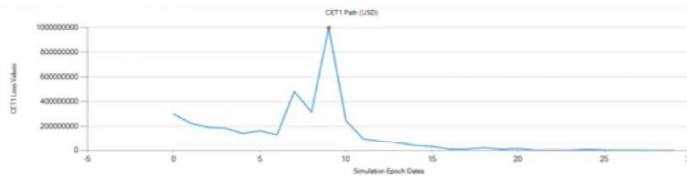


Figure 17.5 Time Point where the most significant CET1 loss materialises

The knowledge of the scenario-epoch date tuple, representing the most substantial CET1 depletion, allows to perform a scenario drill-down analysis. We, therefore, gather the set collecting the risk-factor state underneath the CET1 depletion spike (see Figure 17.6).

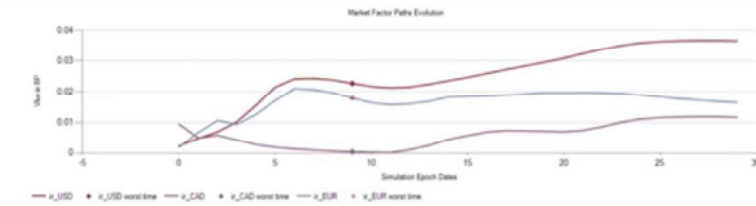


Figure 17.6: Risk-Factor state driving the CET1 loss

Figure 17.7 depicts a bivariate projection of such an analysis. Each rhomboid pinpoints the risk factor state where the most material CET1 depletions manifest.

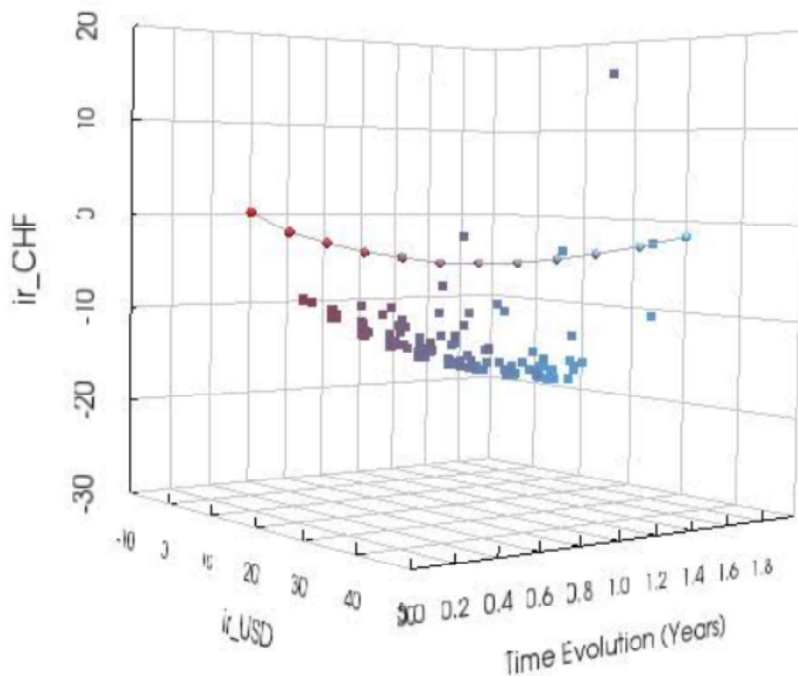


Figure 17.7: USD OIS vs. CHF overnight rates (in BPS) for stress scenarios on a representative fixed-income portfolio.

### 17.3.2 Counterparty Inversion and Data Mining

After identifying the most adverse economic states, we can proceed with a further data-mining analysis. We, therefore, expand the analysis to identify the counterparties which exacerbate the CET1 depletion in the identified states (see Figure 17.8).



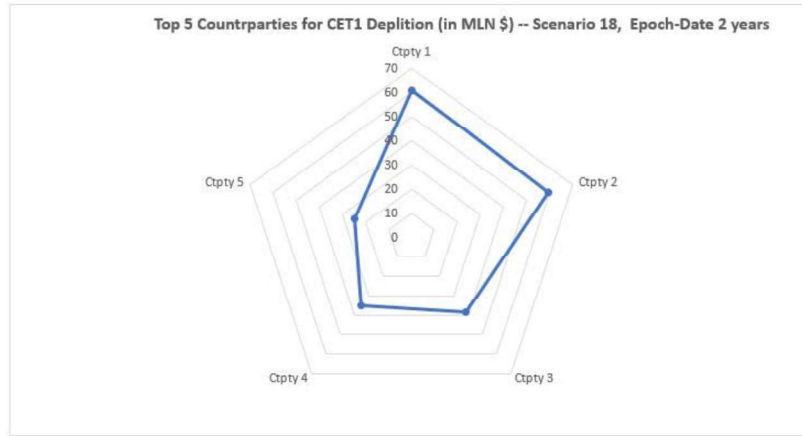


Figure 17.8: Extreme stress scenario – top name

The possibility to identify the counterparties that impair the capital demand (see Figure 17.8) sets the base to formulate a new credit limit metric based on counterparty incremental KVA. Taking the difference between the  $KVA_0$  of the entire portfolio and the KVA of the portfolio obtained by removing the counterparty  $c$ , we compute the incremental counterparty KVA ( $\Delta_c KVA_0$ ).

Similarly to the potential future exposure (PFE)<sup>24</sup>, the  $\Delta_c KVA_0$ , assumes going-concern, and a run-off basis portfolio holding. However, differently from the PFE<sup>25</sup>, which concerns only MtM losses, the  $\Delta_c KVA_0$  reflects all the potential sources of future losses, and it carries the intuitive meaning of the incremental demand for CET1 capital generated by a counterparty (see Figure 17.9: Incremental Counterparty KVA).  $\Delta_c KVA_0$  could, thus, take naturally a fundamental spot for setting limits on idiosyncratic risk appetites. Accounting for the full-term structure of economic capital, as opposed to just the current value, is in line with regulatory directives, such as CRR 292.7(b) (see European Banking Association (2013)). The CRR 292.7(b) explicitly prescribes to monitor and potentially capitalize on the concentration of exposure beyond the first year of credit exposure.

<sup>24</sup> An extreme quantile (typically ranging between 95% and 99%) of the counterparty specific time- $t$  MtM distribution defines the PFE. The risk metric used for credit limit monitoring is typically the maximum PFE ( $\max_{t \in [0, T]} (PFE(t))$ ), where  $T$  is either the residual life of the netting set or its first year of credit exposure.

<sup>25</sup> The PFE, currently used to impose Credit Limits, is based on primordial RST exercises, as it concerns potential future extreme MtM losses. To prevent risk concentrations, which might threaten bank solvency, credit risk officers set limits based on extreme exposure levels. These limits are used to bound the accepted total exposure over the lifetime of a netting set.

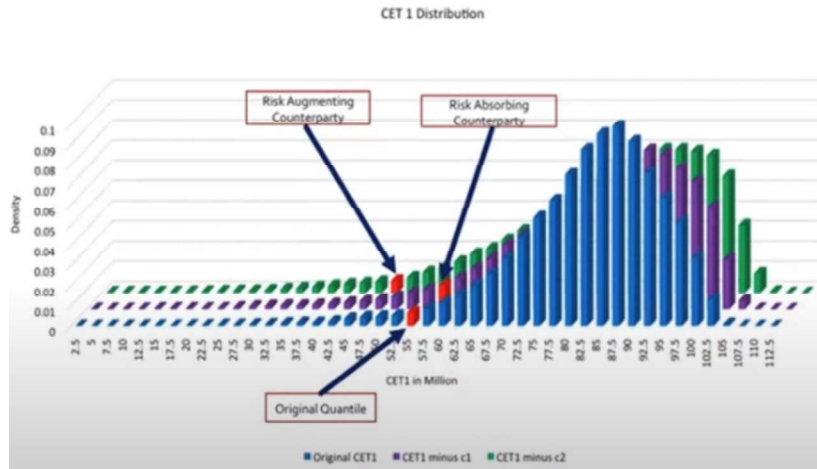


Figure 17.9: Incremental Counterparty KVA

Current Credit Limit practices rely on metrics which are limited, as often based on a counterparty-centric view. As for Regulatory Capital metrics, correlated losses triggered by GWWR, or convexity concentrations for exotic derivative, are neglected. The lack of a comprehensive risk view, typical of MtM centric metrics, can, thus, obfuscate solvency risk<sup>26</sup>. The higher risk sensitivity of  $\Delta_c KVA$  makes KVA based credit limits a substantial addition to a PFE based system.

<sup>26</sup> For example, fully collateralized derivative portfolios result in a zero PEE, although there is collateral funding risk, so that the risk embedded could be far different from zero.

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