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► **To cite this version:**

V. Filipe Martins-Da-Rocha, Toan Phan, Yiannis Vailakis. Pecuniary Externalities in Competitive Economies with Limited Pledgeability. 2022. hal-03909596

**HAL Id: hal-03909596**

**<https://hal.science/hal-03909596>**

Preprint submitted on 21 Dec 2022

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# Pecuniary Externalities in Competitive Economies with Limited Pledgeability\*

V. Filipe Martins-da-Rocha<sup>†</sup>    Toan Phan<sup>‡</sup>    Yiannis Vailakis<sup>§</sup>

September 10, 2022

## Abstract

We analyze the efficiency properties of competitive economies with strategic default and limited pledgeability. We show that laissez-faire equilibria can be constrained *sub-optimal*: under certain conditions, imposing *tighter* borrowing constraints (relative to the laissez-faire regime) can make everybody in the economy better off. The inefficiency is due to the interaction between debt pricing and the default option, which generates a pecuniary externality. We also show that a Pigouvian subsidy on net financial positions may induce borrowers to internalize this externality and increase welfare.

**Keywords:** Limited pledgeability; debt constraints; constrained inefficiency; macro-prudential interventions.

**JEL codes:** E00; E10; F00.

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\*We thank Kartik Athreya, Piero Gottardi, Bernardo Guimaraes, Anton Korinek, Dirk Krueger, Guido Lorenzoni, Pietro Peretto, Nico Trachter, Jan Werner, and various seminar/conference participants at the Federal Reserve Bank of Richmond, Virginia Tech, Duke University Triangle Dynamic Macro Workshop, University of Tokyo, University of California–Santa Cruz, Santa Clara University, University of Virginia, the Economics School of Louvain, the Sao Paulo School of Economics, the Canon Institute for Global Studies, and the 71st European Meeting of the Econometric Society for helpful comments and suggestions.

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# 1 Introduction

Understanding whether competitive economies with financial frictions are vulnerable to potential inefficiencies or market failures is an important question in macroeconomics with many relevant implications. In particular, it helps us understand whether and when policy interventions are warranted. However, asserting that equilibria might be inefficient from a second-best point of view turns out to be more nuanced than it may appear.

There are broadly two strands of the literature that provide different answers and implications. On the one hand, a large and growing body of research has emphasized the presence of pecuniary externalities as a fundamental source of inefficiency, especially in settings where contractual arrangements are subject to limited commitment and/or informational asymmetries.<sup>1</sup> There, the frictions take the form of borrowing constraints that depend on market prices of goods or assets. Private agents fail to take into account the general equilibrium effects of their individual decisions on market prices, and that failure could lead, for instance, to excessive borrowing in equilibrium. On the other hand, standard general equilibrium models with self-enforcing debt constraints have found it generally harder to show that competitive equilibria are constrained suboptimal. In the well-known class of single-commodity models, which is widely used in applications and where debt constraints are microfounded by the threat of financial autarky, the competitive equilibria are indeed constrained efficient. This is despite the fact that the debt constraints depend on market prices. For instance, in the seminal work of [Alvarez and Jermann \(2000, 2001\)](#)<sup>2</sup> borrowing is subject to debt limits that are set at the largest possible levels such that the value of repayment (that depends on asset prices) equals the autarkic value.

In addition to the nontrivial problem of establishing inefficiency, how the resulting externalities are related to the precise nature of the underlying financial frictions is less obvious than commonly understood. For example, little theoretical work has explored whether dif-

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<sup>1</sup>A nonexhaustive list includes, among others, [Kiyotaki and Moore \(1997\)](#), [Gromb and Vayanos \(2002\)](#), [Golosov and Tsyvinski \(2007\)](#), [Lorenzoni \(2008\)](#), [Farhi et al. \(2009\)](#), [Bianchi \(2011\)](#), [Bianchi and Mendoza \(2011\)](#), and [Dávila and Korinek \(2018\)](#).

<sup>2</sup>See also [Kehoe and Levine \(1993, 2001\)](#), [Kocherlakota \(1996\)](#), [Bloise and Reichlin \(2011\)](#).

ferent debt enforcement mechanisms lead to different types of inefficiency.

In this paper, we revisit these issues in the context of a standard dynamic general equilibrium model with microfounded borrowing constraints. More precisely, we study endowment economies in which agents cannot commit to honor their liabilities and debt repayment is sustained because a part of the private resources is pledgeable, and/or due to exclusion from credit markets upon default. Pledgeable resources represent output contraction in the case of sovereign default, or recourse and seized collateral in the case of consumer and corporate default. Exclusion from credit reflects the adverse effects on debtors' reputation in financial markets. Agents can smooth their consumption by trading one-period-ahead contingent claims (Arrow securities), but their borrowing is subject to endogenous borrowing constraints induced by the default punishment. Following [Alvarez and Jermann \(2000\)](#), we consider laissez-faire equilibria where debt limits are *not too tight*, i.e., they are set at the largest possible levels so that repayment is always individually rational. This setup serves our purposes well as it encompasses economies where debt repudiation leads to dead weight losses and exclusion from the credit market, as well as economies with collateral constraints.<sup>3</sup>

Our main result is to show that in economies with limited pledgeability, laissez-faire equilibria might be constrained inefficient, in the sense that restricting the amount of credit private agents can obtain may lead to Pareto improvement. More precisely, we consider policy interventions where a regulator imposes tighter debt constraints than the not-too-tight

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<sup>3</sup>There are many variations of models with collateralized or reputation debt, and the literature is too vast to be summarized here. The reputation mechanism strand of the literature was significantly spurred by the early contributions of [Eaton and Gersovitz \(1981\)](#) and [Bulow and Rogoff \(1989\)](#) and embedded to general equilibrium by [Kehoe and Levine \(1993\)](#), [Zhang \(1997\)](#), [Alvarez and Jermann \(2000\)](#), [Kehoe and Levine \(2001\)](#) and [Hellwig and Lorenzoni \(2009\)](#). It also relates to several contributions in the optimal contracting literature (for instance, [Krueger and Uhlig \(2006\)](#)) that endogenize agents' outside options by assuming that defaulting agents can start a new credit relationship with a competing principal. The seminal papers of [Geanakoplos \(1997\)](#), [Kiyotaki and Moore \(1997\)](#), and [Geanakoplos and Zame \(1997\)](#) brought together collateral with rigorous general equilibrium theory. Recent contributions include, among others, [Kubler and Schmedders \(2003\)](#), [Azariadis and Kaas \(2007\)](#), [Fostel and Geanakoplos \(2008\)](#), [Chien and Lustig \(2010\)](#), and [Gottardi and Kubler \(2015\)](#). The papers of [Hellwig and Lorenzoni \(2009\)](#) and [Chien and Lustig \(2010\)](#) provide the closest settings to our setup.

constraints. We interpret such interventions as a parsimonious representation of regulatory or prudential policies that aim to constrain leverage in the financial markets. We show that, under certain conditions, the policy intervention can increase the ex-ante welfare of all agents in the economy.

Intuitively, though all agents are fully rational and forward looking, they fail to internalize how changes in the severity of credit restrictions in the future feedback on equilibrium prices and, most crucially, the effect of changes in market prices on the default option. In particular, tightening the debt constraints from some period  $\tau$  onward might increase bond prices, or equivalently, lower the implied interest rates. In the setting where defaulters are subject to endowment losses and exclusion from credit (à la [Bulow and Rogoff 1989](#), and [Hellwig and Lorenzoni 2009](#)), this tightening might reduce the value of the default in periods  $t < \tau$ , since it is now more costly to smooth consumption over time by saving only. As a consequence, the not-too-tight debt limits increase at periods  $t < \tau$ , and this opens the possibility for Pareto improvement: the benefits from the relaxed debt constraints at periods  $t < \tau$  may compensate for the costs of facing tighter constraints in subsequent periods. In a setting with collateral constraints (à la [Chien and Lustig 2010](#)), the argument is similar. There, lower interest rates raise the value of pledgeable income and increase trade opportunities in the periods that precede the tightening of the collateral constraints. Again, this opens the possibility for Pareto improvement. This is the essence of the mechanism we explore in this paper.

Our analysis exploits an intuitive and powerful characterization of not-too-tight debt limits in economies with limited pledgeability: debt limits are always decomposed into a component that equals the present value of pledgeable resources, and a credit bubble component that is interpreted as the amount of credit agents can rollover indefinitely.<sup>4</sup> This characterization serves our purposes well as it substantially simplifies the computation of the laissez-faire equilibria, ruling out complications related to the fixed-point determination of the debt limits. Importantly, it allows us to map the set of laissez-faire equilibria in the

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<sup>4</sup>Though this decomposition can be seen as the analogue of [Hellwig and Lorenzoni \(2009\)](#)'s characterization result in an augmented setup with output losses, the result cannot be derived by a simple adaptation of their argument. It rather builds on novel insights that have no analogue in the absence of output losses.

environment with reputation debt to the set of equilibria in the environment with collateralized debt and vice versa. This equivalent mapping between different equilibrium concepts offers a useful benchmark upon which we can carry out our policy interventions.

To provide more clarity on the underlying mechanism, we concentrate to a simple example of an economy with two agents facing uncertainty only at the initial period. Once uncertainty is resolved, the economy is a deterministic one in which endowments switch from a high value to a low value between periods. We further assume that pledgeable resources are time-invariant and identical for both agents. Within this setting, we restrict attention to symmetric Markov laissez-faire equilibria where, by an appeal to our characterization result, debt limits are bubble-free and equal to the present value of pledgeable resources. The policy intervention takes the form of tightening debt limits by a fraction  $\varepsilon$  from some period  $\tau$  onward and analyzing the feedback effect of such a distortion on equilibrium prices and the default option.

It is worth remarking two important features of our policy experiment. First, the intervention in financial markets is not equivalent to modifying pledgeable resources, which remain fixed throughout the paper. The reallocation is induced by tightening the borrowing limits to levels that are lower than those in the laissez-faire equilibrium. Second, the equivalence between the set of equilibria in economies with reputation debt and in economies with collateralized debt breaks down in the post-intervention economy, where the debt limits are no longer not-too-tight. Therefore, we do have to conduct our analysis and compute the new equilibrium variables in each setup separately. Interestingly enough, though the source of inefficiency is common in both environments, Pareto-improving interventions have qualitatively different properties. For instance, we show that delaying the intervention in financial markets in the economy with collateral constraints can lead to equilibria that are close to the first-best outcome.

The fact that private agents fail to internalize the pecuniary externality at the competitive equilibrium with limited pledgeability implies that there is room for government intervention by means of macroprudential controls on financial markets in the lines of [Jeanne and Korinek \(2010, 2019\)](#) and [Farhi and Werning \(2016\)](#). We show that the externality discussed above can be tackled by means of corrective Pigouvian subsidies on net financial positions supported

by lump-sum taxes. In particular, we show that a planner who has flexibility in the choice of the subsidy rate can improve welfare without intervening in each individual decision made by each agent. The distortion created by the subsidy leads to a wedge in marginal rates of substitution between the high-income and the low-income agents. When compared to the laissez-faire equilibrium, the wedge generates higher prices and looser debt limits that can reduce the extent of market failure. An interesting observation is that the equivalence mapping between the reputation debt model and the collateral debt model is not distorted by this type of intervention, and this permits to study in a unified way whether macroprudential controls can be welfare improving.

**Related Literature.** The idea that economies with limited commitment are prone to market failures dates back to [Kehoe and Levine \(1993\)](#). When there is more than one commodity and default cannot exclude agents from trading in spot markets, constrained efficiency might fail because private contracts cannot internalize their effect on relative prices and the default option. The logic there is conceptually the same as in incomplete markets economies where a redistribution of asset holdings, through the induced price changes, affects the spanning properties of the limited assets ([Hart 1975](#), [Stiglitz 1982](#), [Geanakoplos and Polemarchakis 1986](#)). Similar results also obtain in settings where contracting is subject to private information ([Greenwald and Stiglitz 1986](#)). In the single good model studied here, however, there are no spot markets or private information, and as a result, this mechanism is absent. Moreover, [Alvarez and Jermann \(2000, 2001\)](#) show that competitive equilibria are constrained efficient when the default option is autarky. We instead show that constrained inefficiency obtains in economies with a single commodity when debt enforcement relies on the limited pledgeability of private resources and/or a weak form of exclusion (i.e., one-sided exclusion) from financial markets. Changes in the severity of credit restrictions induce price changes in bond markets. These price changes, in turn, affect the value of default and, therefore, the extent of risk sharing, potentially improving efficiency. This source of inefficiency is not present in [Alvarez and Jermann \(2000\)](#)'s framework, since the value of default does not respond to changes in bond prices.

Our work is related to a well-developed literature studying the emergence of pecuniary

externalities in production economies with collateral constraints. [Gromb and Vayanos \(2002\)](#) show that both distributive and collateral externalities can emerge due to market segmentation. [Lorenzoni \(2008\)](#) shows that financial distress might lead to fire sales whose effects on asset prices are not internalized by highly leveraged investors. [Dávila and Korinek \(2018\)](#) characterize pecuniary externalities in dynamic settings that are subject to reduced-form, price-dependent collateral constraints. They distinguish between distributive and collateral externalities and show that each of these two types can be quantified as a function of intuitive sufficient statistics. In all of these works, because of capital accumulation, the reallocation of resources is induced by a change in the level of investment. A planner can overcome the market failure by reducing aggregate investment ex ante and, therefore, the size of the asset sales in bad states. In contrast, in our pure exchange setup, this channel is absent as aggregate resources are fixed and only their distribution can vary. The reallocation of resources is solely induced by the tightening of the endogenously determined debt constraints. This relates to the work of [Guerrieri and Lorenzoni \(2017\)](#), which studies the effects of unexpected credit contractions in Bewley-type economies with incomplete markets and exogenous borrowing limits, and the work of [Aguiar et al. \(2022\)](#), which studies Pareto-improving fiscal policies in this kind of environment when the interest rate on the government bond is below the growth rate.

[Gottardi and Kubler \(2015\)](#) provide an antecedent to our paper by analyzing constrained suboptimality in a collateral economy à la [Chien and Lustig \(2010\)](#). Our analysis differs from theirs in two important aspects. First, they assume that the intervention is unexpectedly announced at the initial period after all trades have taken place. We instead assume that the intervention is fully anticipated by private agents. Second, their policy experiment exploits an equivalence between equilibria in the economy with collateral constraints and equilibria of an auxiliary economy with financial intermediaries where agents can only take long positions on contingent trees. They show that Pareto improvement obtains in the auxiliary economy, however, they do not show whether the established equivalence is preserved post intervention, so that the equilibrium with financial intermediaries is mapped back to the equilibrium with tighter collateral constraints.<sup>5</sup>

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<sup>5</sup>The online supplemental material of this paper offers a more detailed discussion on how our analysis is



Finally, our work is related to a complementary strand of literature that focuses on macroprudential controls that take the form of Pigouvian taxes or subsidies to reduce pecuniary externalities. [Park \(2014\)](#) studies optimal taxation in an [Alvarez and Jermann \(2000\)](#) production economy. There, individuals do not take into account that their labor and saving decisions affect aggregate labor and capital supply and wages, and thus the value of autarky. [Jeanne and Korinek \(2010, 2019\)](#) and [Dávila and Korinek \(2018\)](#) provide a welfare rational for the taxation of capital flows to mitigate the financial amplification effects of fire sales in economies with collateral constraints. In [Farhi and Werning \(2016\)](#), the focus is on demand externalities that are associated with the presence of nominal price rigidities. Though such externalities are qualitatively different from the pecuniary externalities that we study here, [Korinek and Simsek \(2016\)](#) argue that the two types of externalities interact and may mutually reinforce each other. We show that, in an exchange setup, Pigouvian corrective subsidies on net financial deliveries can be welfare improving because they induce a wedge in marginal rates of substitutions that results in inflating bond prices and relaxing credit conditions.

The plan of the paper is as follows. Section 2 describes the baseline model environment. Section 3 provides a characterization of not-too-tight debt limits in two environments with microfounded borrowing constraints. Section 4 shows that laissez-faire equilibria can be Pareto inferior to equilibria with tighter debt constraints. Section 5 shows that corrective Pigouvian subsidies can mitigate the extent of market failure. Section 6 concludes. The proofs of the results and details of the technical arguments are presented in the online appendix, where additional issues are also discussed.

## 2 General Model

Consider an infinite-horizon endowment economy with a single nonstorable consumption good at each date. Time and uncertainty are both discrete. We use an event tree  $\Sigma$  to describe the revelation of information over an infinite horizon. There is a unique initial date-0 event  $s^0 \in \Sigma$ , and for each date  $t \in \{0, 1, 2, \dots\}$ , there is a finite set  $S^t \subseteq \Sigma$  of date- $t$

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distinct from [Gottardi and Kubler \(2015\)](#).

events  $s^t$ . Each  $s^t$  has a unique predecessor  $\sigma(s^t)$  in  $S^{t-1}$  and a finite number of successors  $s^{t+1}$  in  $S^{t+1}$  for which  $\sigma(s^{t+1}) = s^t$ . The notation  $s^{t+1} \succ s^t$  specifies that  $s^{t+1}$  is a successor of  $s^t$ . The event  $s^{t+\tau}$  is said to follow event  $s^t$ , also denoted  $s^{t+\tau} \succ s^t$ , if  $\sigma^{(\tau)}(s^{t+\tau}) = s^t$ .<sup>6</sup> The set  $S^{t+\tau}(s^t) := \{s^{t+\tau} \in S^{t+\tau} : s^{t+\tau} \succ s^t\}$  denotes the collection of all date- $(t+\tau)$  events following  $s^t$ . Abusing notation, we let  $S^t(s^t) := \{s^t\}$ . The subtree starting at event  $s^t$  is then given by:

$$\Sigma(s^t) := \bigcup_{\tau \geq 0} S^{t+\tau}(s^t).$$

We use the notation  $s^\tau \succeq s^t$  when  $s^\tau \succ s^t$  or  $s^\tau = s^t$ . In particular, we have  $\Sigma(s^t) = \{s^\tau \in \Sigma : s^\tau \succeq s^t\}$ .

There is a finite set  $I$  of household types, each consisting of a unit measure of identical, infinitely lived agents who consume the single perishable good. Preferences over (nonnegative) consumption processes  $c = (c(s^t))_{s^t \succeq s^0}$  are represented by the lifetime expected and discounted utility:

$$U(c) := \sum_{t \geq 0} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c(s^t)),$$

where  $\beta \in (0, 1)$  is the discount factor,  $\pi(s^t)$  is the unconditional probability of  $s^t$ , and  $u : [0, \infty) \rightarrow \mathbb{R}$  is a utility function that is strictly increasing, strictly concave, continuous on  $[0, \infty)$ , differentiable on  $(0, \infty)$ , and satisfies Inada's condition  $\lim_{\varepsilon \rightarrow 0} [u(\varepsilon) - u(0)]/\varepsilon = \infty$ . To further simplify the exposition of the theoretical results in Section 3, we assume that  $u$  is also bounded. This restriction ensures that the lifetime utility  $U$  is continuous (for the product topology), and the demand set is nonempty.<sup>7</sup> Given an event  $s^t$ , we denote by

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<sup>6</sup>Formally,  $\sigma$  is a mapping from  $\Sigma \setminus \{s^0\}$  to  $\Sigma$  such that  $\sigma(S^{t+1}) = S^t$  for every  $t \geq 0$ . We pose  $\sigma^{(1)} := \sigma$  and  $\sigma^{(\tau+1)} := \sigma \circ \sigma^{(\tau)}$  for every  $\tau \geq 1$ .

<sup>7</sup>The characterization of debt limits can be extended, and our results in this section continue to hold even when  $u$  is unbounded. In particular, when  $u$  belongs to the class of constant relative risk aversion utility functions  $u(c) = c^{1-\alpha}/(1-\alpha)$  with  $\alpha > 0$ . In fact, in the simple economy we consider illustrating our main result, we assume a logarithmic period utility function, i.e.,  $\alpha = 1$ . A general treatment of unbounded utility functions requires some additional technical assumptions on endowment processes together with a suitable modification of the utility function  $u$  outside a specific interval such that the equilibrium outcomes remain unaffected. For a detailed discussion, see [Martins-da-Rocha and Santos \(2019\)](#).

$U(c|s^t)$  the lifetime continuation utility conditional on  $s^t$ , as defined by:

$$U(c|s^t) := u(c(s^t)) + \sum_{\tau \geq 1} \beta^\tau \sum_{s^{t+\tau} \succ s^t} \pi(s^{t+\tau}|s^t) u(c(s^{t+\tau})),$$

where  $\pi(s^{t+\tau}|s^t) := \pi(s^{t+\tau})/\pi(s^t)$  is the conditional probability of  $s^{t+\tau}$  given  $s^t$ . Agents' endowments are subject to random shocks. We denote by  $y^i = (y^i(s^t))_{s^t \succeq s^0}$  the process of positive endowments  $y^i(s^t) > 0$  of a representative agent of type  $i$ . For notational convenience, we have written the primitives as if agents' preferences and beliefs are homogeneous. However, all of our arguments remain valid when agents have heterogeneous preferences and beliefs, and the only necessary change is to replace  $(u, \beta, \pi)$  with  $(u^i, \beta^i, \pi^i)$ . In fact, we explicitly consider a setting with heterogeneous beliefs in the example analyzed in Sections 4 and 5.

## 2.1 Debt-Constrained Asset Markets

At any event  $s^t$ , agents can issue and trade state-contingent one-period bonds, each one promising to pay one unit of the consumption good contingent on the realization of a successor event  $s^{t+1} \succ s^t$ . Let  $q(s^{t+1}) > 0$  denote the price, at event  $s^t$ , of the  $s^{t+1}$ -contingent bond (the inverse of  $q$  is the interest rate between  $s^t$  and  $s^{t+1}$ ). Agent  $i$ 's bond holdings are  $a^i = (a^i(s^t))_{s^t \succeq s^0}$ , where  $a^i(s^t) \leq 0$  is a liability, and  $a^i(s^t) \geq 0$  is a claim. Each agent's debt is observable and subject to certain (state-contingent, nonnegative, and finite) debt limits  $D^i = (D^i(s^t))_{s^t \succeq s^0}$ . Given an initial bond holding  $a^i(s^0)$  and debt limits  $D^i$ , we denote by  $B^i(D^i, a^i(s^0)|s^0)$  the budget set of an agent who never defaults. It consists of all pairs  $(c^i, a^i)$  of consumption and bond holdings satisfying the following budget flows and debt constraints: for all  $s^t \succeq s^0$ ,

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1}) a^i(s^{t+1}) \leq y^i(s^t) + a^i(s^t), \quad (2.1)$$

and

$$a^i(s^{t+1}) \geq -D^i(s^{t+1}), \quad \text{for all } s^{t+1} \succ s^t. \quad (2.2)$$

We naturally restrict attention to allocations where the initial asset holdings clear the market, i.e.,  $\sum_{i \in I} a^i(s^0) = 0$ , and satisfy the debt constraints, i.e.,  $a^i(s^0) \geq -D^i(s^0)$  for each  $i$ . Similarly, contingent to an event  $s^\tau$ , we let  $B^i(D^i, x|s^\tau)$  be the set of all plans  $(c^i, a^i)$  satisfying

restrictions (2.1) and (2.2) at every successor node  $s^t \succeq s^\tau$  with initial claim  $a^i(s^\tau) = x$ . The contingent value function at event  $s^\tau$ , when agent  $i$  starts with financial wealth  $x$ , is denoted by  $V^i(D^i, x|s^\tau)$ . It is defined as the largest continuation utility  $U(c^i|s^\tau)$  among all budget feasible plans  $(c^i, a^i) \in B^i(D^i, x|s^\tau)$ . When  $x = a^i(s^\tau)$ , this will be the equilibrium value, i.e., the payoff to each agent  $i$  along the equilibrium path following any event  $s^\tau$ .

**Definition 2.1.** Given initial asset holdings  $(a^i(s^0))_{i \in I}$  satisfying  $\sum_{i \in I} a^i(s^0) = 0$ , an *equilibrium*  $(q, (c^i, a^i, D^i)_{i \in I})$  is a collection of state-contingent bond prices  $q$ , a consumption allocation  $(c^i)_{i \in I}$ , a bond holdings allocation  $(a^i)_{i \in I}$ , and a family of nonnegative and finite debt limits  $(D^i)_{i \in I}$  satisfying:

- (a) each agent  $i$ , taking prices and the debt limits as given, chooses a plan  $(c^i, a^i)$  that is optimal among budget feasible plans in  $B^i(D^i, a^i(s^0)|s^0)$ ;
- (b) markets clear:  $\sum_{i \in I} c^i = \sum_{i \in I} y^i$  and  $\sum_{i \in I} a^i = 0$ .

So far, debt limits are arbitrary. We now move to the endogenous determination of the debt limits, which are a critical determinant of equilibrium allocations and equilibrium payoffs.

## 2.2 Debt Limits

The limits represent the maximal amount of debt that borrowers can issue. In general equilibrium, they also represent the maximal amount of liquidity (or storage of value) that savers have access to. For reasons and microfoundations that will soon be provided, we specify that debt limits satisfy the following general decomposition property:

$$D^i(s^t) = \ell^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})D^i(s^{t+1}), \quad \text{for all } s^t \succ s^0, \quad (2.3)$$

where the first term  $\ell^i(s^t) \in [0, y^i(s^t)]$  represents the amount of endowment that can be pledged, and the second term is the maximum amount the agent can get from rolling over debt. We use the terms *pledgeable endowment* and *endowment loss* interchangeably when referring to the process  $\ell^i = (\ell^i(s^t))_{s^t \succeq s^0}$ .

It is straightforward that a process of debt limits  $D^i$  satisfies property (2.3) if, and only if, it can be decomposed into a *fundamental* and a *bubble* component:

$$D^i(s^t) = \underbrace{\text{PV}(\ell^i|s^t)}_{\text{fundamental}} + \underbrace{M^i(s^t)}_{\text{bubble}}, \quad \text{for all } s^t \succeq s^0. \quad (2.4)$$

Here, the fundamental component is simply the present value of pledgeable income:

$$\text{PV}(\ell^i|s^t) := \frac{1}{p(s^t)} \sum_{s^\tau \succeq s^t} p(s^\tau) \ell^i(s^\tau),^8$$

where  $p(s^t)$  is the date-0 price of consumption at event  $s^t$ .<sup>9</sup> The bubble component of  $D^i$  is a nonnegative process satisfying the following *exact rollover* property:

$$M^i(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1}) M^i(s^{t+1}), \quad \text{for all } s^t \succ s^0.$$

We now have the following equilibrium definition:

**Definition 2.2.** Given pledgeable endowment processes  $(\ell^i)_{i \in I}$ , we call an *equilibrium with limited pledgeability* any equilibrium  $(q, (c^i, a^i, D^i)_{i \in I})$  such that the debt limits  $D^i$  of each agent  $i$  satisfy Condition (2.3), or equivalently, Condition (2.4).

Our setting nests several important benchmarks. When the whole endowment is pledgeable, i.e.,  $\ell^i = y^i$  for each agent  $i$ , the debt limits coincide with the natural debt limits, i.e.,  $D^i = \text{PV}(y^i)$ .<sup>10</sup> When no endowment is pledgeable, i.e.,  $\ell^i = 0$  for each agent  $i$ , then our setting collapses to Hellwig and Lorenzoni (2009), where debt is necessarily rolled over as a credit bubble. In Martins-da-Rocha et al. (2021), we provide an example of an equilibrium where the fundamental and bubble components coexist, i.e.,  $\text{PV}(\ell^i) > 0$  and  $M^i > 0$ . We also provide conditions on primitives sufficient to guarantee the existence of an equilibrium with limited pledgeability.

When can we rule out bubbly equilibria where the credit bubble component  $M^i$  is positive? The following proposition shows that this is the case when the pledgeable resources constitute a *nonnegligible* fraction of aggregate resources:

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<sup>8</sup>Similarly, the *wealth* of an agent at event  $s^t$  is defined as the present value  $\text{PV}(y^i|s^t)$  of his endowments.

<sup>9</sup>Formally,  $p(s^t)$  is defined recursively by  $p(s^0) = 1$  and  $p(s^{t+1}) = q(s^{t+1})p(s^t)$  for all  $s^{t+1} \succ s^t$ .

<sup>10</sup>Proposition 2.1 below implies that when  $\ell^i = y^i$  for each agent  $i$ , the bubble component is necessarily zero in equilibrium, and hence  $D^i = \text{PV}(y^i)$ .

**Proposition 2.1.** *If pledgeable resources are a nonnegligible fraction of aggregate resources, in the sense that there exists  $\varepsilon > 0$  such that:*

$$\sum_{i \in I} \ell^i(s^t) \geq \varepsilon \sum_{i \in I} y^i(s^t), \quad \text{for all } s^t \succ s^0,$$

*then in any equilibrium with limited pledgeability, the bubble component is necessarily zero. As a consequence,  $D^i = \text{PV}(\ell^i)$  for every agent  $i$ .*

### 3 Microfoundations for Debt Limits

So far, we have exogenously imposed decomposition condition (2.3) on debt limits. It turns out that this decomposition property arises endogenously in environments with limited commitment. To see this, consider an environment where agents cannot commit to their financial contracts and may opt for default. We denote by  $V_{\text{def}}^i(s^t)$  agent  $i$ 's value of the default option at event  $s^t$ . Following Alvarez and Jermann (2000), we impose that the debt limits reflect the fact that repayment is always individually rational. Specifically, we say that debt limits  $D^i$  are *self-enforcing* if debtors prefer to repay even the maximum debt allowed, i.e.,

$$V^i(D^i, -D^i(s^t)|s^t) \geq V_{\text{def}}^i(s^t), \quad \text{for all } s^t \succeq s^0. \quad (3.1)$$

We say that  $D^i$  are *not too tight* if (3.1) always holds with equality, i.e., borrowers are indifferent between repaying and defaulting:

$$V^i(D^i, -D^i(s^t)|s^t) = V_{\text{def}}^i(s^t), \quad \text{for all } s^t \succeq s^0. \quad (3.2)$$

Given future debt limits  $(D^i(s^\tau))_{s^\tau \succ s^t}$ , the level  $D^i(s^t)$  satisfying (3.2) is interpreted as the largest self-enforcing debt limit contingent to event  $s^t$ . We say that  $D^i$  are *too tight* if they are self enforcing and (3.1) holds with strict inequality at some event  $s^t \succ s^0$ .

**Definition 3.1.** Given a family of default value functions  $(V_{\text{def}}^i)_{i \in I}$ , we call a *self-enforcing equilibrium* any equilibrium  $(q, (c^i, a^i, D^i)_{i \in I})$  such that the debt limits  $D^i$  of each agent  $i$  satisfy condition (3.1). When the debt limits satisfy the not-too-tight condition (3.2), we use the term *not-too-tight equilibrium*. Similarly, when the debt limits are too tight, we use the term *too-tight equilibrium*.

It is reasonable to expect that in a competitive market, competition among lenders should naturally lead them to offer as much credit as possible, without violating borrowers' incentive to repay.<sup>11</sup> Hence, we will also use the term *laissez-faire equilibrium* as a synonym for not-too-tight equilibrium.

The value of default  $V_{\text{def}}$  is the key object that determines the not-too-tight debt limits. We now analyze two well-established frameworks: a *reputation debt* environment in which default entails restricted market participation and a loss of endowments (Bulow and Rogoff 1989 and Hellwig and Lorenzoni 2009), and a *collateralized debt* environment in which the only consequence of default is the seizure of a collateral asset (Chien and Lustig 2010 and Gottardi and Kubler 2015). For each possible value of the default option  $V_{\text{def}}$ , we characterize the corresponding not-too-tight debt limits.

### 3.1 Reputation Debt

First, we consider a framework à la Bulow and Rogoff (1989) where all assets are seized upon default, and debtors lose access to credit while retaining the ability to save (by purchasing other people's debt). In addition, default causes a (dead weight) endowment loss: if agent  $i$  defaults at  $s^\tau$ , then his endowments will reduce to  $y^i(s^t) - \ell^i(s^t)$  for all successor events  $s^t \succeq s^\tau$ , with  $\ell^i(s^t) \in [0, y^i(s^t)]$  exogenously given.<sup>12</sup> As a consequence, the value of default for any agent  $i$  at any event  $s^t$  is given by:

$$V_{\text{def}}^i(s^t) = V_{\ell^i}^i(0, 0|s^t) := \sup\{U(c^i|s^t) : (c^i, a^i) \in B_{\ell^i}^i(0, 0|s^t)\}, \quad (3.3)$$

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<sup>11</sup>Krueger and Uhlig (2006) show that, absence of transition costs, the contracting problem of financial intermediaries that compete for customers by offering risk-sharing contracts is equivalent to a consumption-saving problem with not-too-tight borrowing limits of zero.

<sup>12</sup>In the context of consumer credit, the endowment loss is a parsimonious way to capture recourse and other legal consequences of default (e.g., Chatterjee et al. 2007, Livshits et al. 2007, Livshits 2015). In the context of sovereign debt, the endowment loss parsimoniously captures the negative effects of default on domestic production (e.g., Eaton and Gersovitz 1981, Bulow and Rogoff 1989, Cole and Kehoe 2000, Aguiar and Gopinath 2006, Arellano 2008).

where  $B_{\ell^i}^i(0, 0|s^t)$  is the budget set of any agent  $i$  who has zero liabilities, cannot borrow, and is endowed with  $y^i - \ell^i$  resources. The condition (3.2) then reads as follows:

$$V^i(D^i, -D^i(s^t)|s^t) = V_{\ell^i}^i(0, 0|s^t), \quad \text{for all } s^t \succeq s^0. \quad (3.4)$$

The following result shows that not-too-tight debt limits can be decomposed into a fundamental component and a credit bubble component that captures the possibility of rolling over a fraction of debt indefinitely. It provides our first microfoundation for the decomposition property (2.4).

**Theorem 3.1.** *In the reputation debt framework, where the value of default is given by (3.3), any process of not-too-tight debt limits  $D^i$  can be decomposed as the sum of the present value of the endowment loss process  $\ell^i$  and a bubble component  $M^i$ , i.e.,  $D^i = \text{PV}(\ell^i) + M^i$ , where  $M^i$  is a nonnegative exact rollover process.*

Intuitively, the bubble component reflects the fact that credit beyond the fundamental component is sustainable only if agents can rollover their debt. A crucial and nontrivial step to prove the result is to show that the process  $\text{PV}(\ell^i)$  is a lower bound to any sequence of not-too-tight debt limits. A second step, based on a translation invariance of the flow budget constraints, then shows that the process  $\text{PV}(\ell^i)$  is itself not too tight. Finally, the result follows from the well-known fact that the difference between two processes of not-too-tight debt limits necessarily satisfies the exact rollover property (see [Martins-da-Rocha and Santos 2019](#)).

Besides providing a microfoundation for our specification of debt limits in Section 2, Theorem 3.1 is also useful for the computation of equilibria. It eliminates the usual complications related to the fixed-point process of determining not-too-tight debt limits, where the value of default depends on prices (as defaulting agents can still save), which in turn depend on equilibrium allocations and hence the debt limits. The usefulness will become clear when we conduct our policy intervention experiments in Section 4.

## 3.2 Collateralized Debt

In this section, we shift our attention to an environment where all borrowing and lending is fully secured by collateral. As in [Chien and Lustig \(2010\)](#) and [Gottardi and Kubler \(2015\)](#),



we assume that agents back their promises by means of trading a long-lived asset (or Lucas tree). In contrast to the economy studied in the previous section, debt repudiation does not induce any form of exclusion from financial markets. Upon default, debtors lose their collateralizable assets which are handed over to creditors, but they still maintain access to financial markets. Within this framework, our aim is to provide another microfoundation for the specification of debt limits in Section 2. Furthermore, we will use Theorem 3.1 to establish an intriguing and nontrivial equivalence mapping between the two settings. This equivalence unravels an interesting link between credit limits and asset prices.

Consider an economy where each agent  $i$  receives an endowment of  $e^i(s^t) \geq 0$  units of the consumption good at event  $s^t$ . At the initial period, each agent  $i$  is also endowed with an exogenous share  $\alpha^i(s^{-1}) \geq 0$  of a Lucas tree. The tree is an infinitely lived physical asset that pays a dividend of  $\delta(s^t) \geq 0$  units of the consumption good at event  $s^t$ . Agent  $i$ 's total endowment is therefore  $y^i(s^t) := e^i(s^t) + \alpha^i(s^{-1})\delta(s^t)$  at event  $s^t$ . The tree exists in unit supply, and its shares can be traded at the ex-dividend price  $P(s^t)$ , determined in equilibrium. We denote by  $\alpha^i(s^t) \geq 0$  the post-trade tree holding of agent  $i$  at event  $s^t$ . Agents can also trade one-period-ahead contingent bonds at any event  $s^t$ . Let  $b^i(s^{t+1}) \in \mathbb{R}$  denote the position on the bond paying at event  $s^{t+1}$ , whose price, expressed in units of  $s^t$ -consumption, is  $q(s^{t+1})$ . For each agent  $i$ , given an initial contingent claim  $b^i(s^0)$ , the initial financial wealth is given by  $a^i(s^0) := b^i(s^0) + \alpha^i(s^{-1})[P(s^0) + \delta(s^0)]$ .

Since the tree holdings can be seized by creditors, it is intuitive to assume that debt limits are imposed on the net asset position.<sup>13</sup> Formally, we let  $\tilde{B}^i(\tilde{D}^i, a^i(s^0)|s^0)$  denote the budget set consisting of all triples  $(c^i, \alpha^i, b^i)$  of consumption processes  $c^i = (c^i(s^t))_{s^t \succeq s^0}$ , non-negative tree holdings  $\alpha^i = (\alpha^i(s^t))_{s^t \succ s^0}$ , and contingent claims  $b^i = (b^i(s^t))_{s^t \succ s^0}$  satisfying the following flow budget constraints and debt constraints: for all  $s^t \succeq s^0$ ,<sup>14</sup>

$$c^i(s^t) + P(s^t)\alpha^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})b^i(s^{t+1}) \leq e^i(s^t) + b^i(s^t) + \alpha^i(\sigma(s^t)) [\delta(s^t) + P(s^t)], \quad (3.5)$$

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<sup>13</sup>This follows [Kocherlakota \(2008\)](#).

<sup>14</sup>To keep notational consistency, we extend the domain of the predecessor function  $\sigma$  to the whole tree  $\Sigma$  by posing  $\sigma(s^0) := s^{-1}$ .

and

$$b^i(s^{t+1}) + \alpha^i(s^t) [\delta(s^{t+1}) + P(s^{t+1})] \geq -\tilde{D}^i(s^{t+1}), \quad \text{for all } s^{t+1} \succ s^t. \quad (3.6)$$

Since we have more markets than in the environment described in Section 2, we need to modify Definition 2.1 as follows.

**Definition 3.2.** Given initial contingent claims  $(b^i(s^0))_{i \in I}$  satisfying  $\sum_{i \in I} b^i(s^0) = 0$ , and initial shares  $(\alpha^i(s^{-1}))_{i \in I}$  satisfying  $\sum_{i \in I} \alpha^i(s^{-1}) = 1$ , an equilibrium  $(q, P, (c^i, \alpha^i, b^i, \tilde{D}^i)_{i \in I})$  is a collection of state-contingent bond prices  $q$ , tree prices  $P$ , a consumption allocation  $(c^i)_{i \in I}$ , an allocation of tree holdings  $(\alpha^i)_{i \in I}$ , an allocation of contingent claims  $(b^i)_{i \in I}$ , and finite debt limits  $(\tilde{D}^i)_{i \in I}$  such that:

- (a) each agent  $i$ , taking prices and the debt limits as given, chooses a plan  $(c^i, \alpha^i, b^i)$  that is optimal among budget feasible plans in  $\tilde{B}^i(\tilde{D}^i, a^i(s^0)|s^0)$ ;<sup>15</sup>
- (b) all markets clear:  $\sum_{i \in I} c^i = \sum_{i \in I} y^i$ ,  $\sum_{i \in I} b^i = 0$  and  $\sum_{i \in I} \alpha^i = 1$ .

For every event  $s^\tau \succ s^0$  and every beginning-of-period net financial wealth  $x \in \mathbb{R}$ , we let  $\tilde{B}^i(\tilde{D}^i, x|s^\tau)$  be the set of triples  $(c^i, \alpha^i, b^i)$  satisfying the flow budget constraint (3.5) and the debt constraints (3.6) for all successor events  $s^t \succeq s^\tau$ , together with the initial wealth condition:

$$x = b^i(s^\tau) + \alpha^i(\sigma(s^\tau)) [\delta(s^\tau) + P(s^\tau)].$$

The continuation value conditional on no default is then given by:

$$\tilde{V}^i(\tilde{D}^i, x|s^\tau) := \sup\{U(c^i|s^\tau) : (c^i, \alpha^i, b^i) \in \tilde{B}^i(\tilde{D}^i, x|s^\tau)\}.$$

At any contingency, debtors have the option to renege on their contracts and file for bankruptcy. In this case, all tree holdings and current period dividends are seized and transferred to lenders to redeem their debt. The part  $e^i(s^t)$  of total endowment  $y^i(s^t) = e^i(s^t) + \alpha^i(s^{-1})\delta(s^t)$  cannot be seized, and defaulters still maintain access to financial markets. We refer to the process  $(\alpha^i(s^{-1})\delta(s^t))_{s^t \succeq s^0}$  as the *collateralizable income*. The residual  $e^i(s^t)$  constitutes the nonpledgeable component of the total endowment  $y^i(s^t)$ , since it cannot be

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<sup>15</sup>Recall that the initial financial wealth is given by  $a^i(s^0) = b^i(s^0) + \alpha^i(s^{-1})[P(s^0) + \delta(s^0)]$ .

sold in advance to finance consumption or savings at any date before the endowment is received. This specification of the default punishment leads to the following value of default:

$$\tilde{V}_{\text{def}}^i(s^\tau) := \tilde{V}^i(\tilde{D}^i, 0|s^\tau), \quad (3.7)$$

and the condition (3.2) for debt limits  $\tilde{D}^i$  to be not too tight becomes:

$$\tilde{V}^i(\tilde{D}^i, -\tilde{D}^i(s^t)|s^t) = \tilde{V}^i(\tilde{D}^i, 0|s^t) \quad \text{for all } s^t \succeq s^0. \quad (3.8)$$

Comparing condition (3.8) and its counterpart (3.4) reveals that it is simpler to solve for the not-too-tight debt limits in the collateral model than in the reputation model. Indeed, we have the following immediate result: when the value of default is given by (3.7), any process of not-too-tight debt limits must be equal to zero.<sup>16</sup> This in turn implies that the not-too-tight debt constraints (3.6) are equivalent to the collateral constraints:

$$b^i(s^{t+1}) \geq -\alpha^i(s^t) [P(s^{t+1}) + \delta(s^{t+1})], \quad \text{for all } s^{t+1} \succ s^t. \quad (3.9)$$

To connect the above constraint to the decomposition of debt limits in (2.4), we recall the following standard asset-pricing result:

$$\delta + P = \text{PV}(\delta) + M, \quad (3.10)$$

where  $M$  is a nonnegative exact rollover process.<sup>17</sup> Asset pricing equation (3.10) implies that tree holdings are indeterminate, since what matters for consumption smoothing purposes is the net financial position

$$\theta^i(s^t) := b^i(s^t) + \alpha^i(\sigma(s^t)) [P(s^t) + \delta(s^t)].$$

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<sup>16</sup>In the reputation debt framework, the debt limit  $D^i(s^t)$  is not too tight when  $V^i(D^i, -D^i(s^t)|s^t) = V_{\ell^i}^i(0, 0|s^t)$ . Since future debt limits only appear in the LHS of the equation, the determination of  $D^i(s^t)$  depends on the value of the future debt limits  $(D^i(s^\tau))_{s^\tau \succ s^t}$ . Additionally, since we require future debt limits to be not too tight, this involves a fixed point in the space of debt limits processes. In the collateralized debt environment, however, future debt limits appear both in the RHS and LHS of (3.8). Since the mapping  $x \mapsto \tilde{V}^i(\tilde{D}^i, x|s^t)$  is strictly increasing, we deduce that  $\tilde{D}^i(s^t) = 0$  is the only possible solution of (3.8). Observe that this property is valid even if future debt limits were too tight. This will be crucial in the analysis conducted in Section 4.4.

<sup>17</sup>We refer to Appendix A.3 for a detailed derivation of this equation.

Indeed, given Equation (3.10), the flow budget constraint (3.5) can be written as:

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})\theta^i(s^{t+1}) \leq e^i(s^t) + \theta^i(s^t).$$

Therefore, adjusting contingent claims  $b^i$  if necessary, we can assume without any loss of generality that agents do not trade their equity shares, i.e.,  $\alpha^i(s^t) = \alpha^i(s^{-1})$  for every  $s^t$ . The flow budget constraint (3.5) then becomes:

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})b^i(s^{t+1}) \leq y^i(s^t) + b^i(s^t) \quad (3.11)$$

while the debt constraint is stated as:

$$b^i(s^{t+1}) \geq -\alpha^i(s^{-1})P(s^{t+1}) = -[\text{PV}(\alpha^i(s^{-1})\delta|s^{t+1}) + \alpha^i(s^{-1})M(s^{t+1})]. \quad (3.12)$$

Fix an arbitrary decomposition of the process  $M = \sum_{i \in I} M^i$  where each  $M^i$  is a nonnegative exact rollover process. Consider the allocation  $(a^i)_{i \in I}$  given by:

$$a^i(s^t) := b^i(s^t) + \alpha^i(s^{-1})M(s^t) - M^i(s^t).$$

We can check that the flow budget constraint (3.11) is equivalent to:

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a^i(s^{t+1}) \leq y^i(s^t) + a^i(s^t),$$

and the debt constraint (3.12) is equivalent to:

$$a^i(s^{t+1}) \geq -[\text{PV}(\alpha^i(s^{-1})\delta|s^{t+1}) + M^i(s^{t+1})].$$

The above implies that agents' borrowing capacity in the economy with collateralized debt is decomposed into a fundamental and a bubble component exactly the same way it is decomposed in the economy with reputation debt. We can now present the following equivalence theorem.

**Theorem 3.2.** *A consumption allocation is the outcome of a laissez-faire equilibrium in the collateralized debt framework, where the tree's dividend process is  $\delta$ , and the tree's initial holdings are  $(\alpha^i(s^{-1}))_{i \in I}$ , if, and only if, it is the outcome of a laissez-faire equilibrium in the reputation debt framework, where the endowment losses are  $(\alpha^i(s^{-1})\delta)_{i \in I}$ .*

Our equivalence result has implications for the effects of vanishing pledgeable income on borrowing capacity and intertemporal trade. Assume, as in [Chien and Lustig \(2010\)](#), that endowments are bounded and that collateralizable income represents a constant fraction of the endowment, i.e., there exists  $\delta \geq 0$  such that for all  $s^t$ ,  $\delta(s^t) = \delta \geq 0$ . When  $\delta > 0$ , [Proposition 2.1](#) implies that the present value of pledgeable resources is finite, and assets are priced at their fundamental value, so prices are bubble-free. One may think that when  $\delta = 0$  (i.e., assets pay no dividends), asset prices must equal zero, so autarky is the only equilibrium outcome. But such a claim presupposes that the aggregate wealth is still finite, or equivalently, that the implied interest rates remain positive (higher than the growth rate) when passing to the limit. However, as documented by [Hellwig and Lorenzoni \(2009\)](#), when  $\delta = 0$ , equilibrium interest rates can be sufficiently low (equal to zero in the absence of growth) so that the economy's aggregate wealth is infinite. The implication for the collateral equilibrium, is that, even if the trees pay no dividend, assets may be priced as a speculative bubble. Indeed, it is sufficient to appeal to [Theorem 3.2](#) and translate the bubbly equilibrium of [Hellwig and Lorenzoni \(2009\)](#) in the environment of [Chien and Lustig \(2010\)](#). The intuition for this discrepancy relies on the dual role of collateral as a source of liquidity. As dividends become negligible (i.e.,  $\delta$  approaches zero), the value of the asset increases to compensate for the decreased investment value. In the limit, the value of the collateral asset is still positive, reflecting purely a bubble, even though there is no collateral in the market anymore.

## 4 Tightening Debt Constraints

Are allocations with not-too-tight debt limits constrained efficient? The common belief in models where financial frictions are due to limited commitment is that borrowing should be subject to not-too-tight debt limits. As mentioned before, not-too-tight debt limits should arise naturally in a competitive credit market where competition among lenders will eventually permit borrowers to issue the largest amount of debt compatible with repayment incentives. This view is further reinforced by the misguided intuition that not-too-tight debt limits allow for maximum risk-sharing. Though this is trivially true in a partial equilibrium framework where prices are fixed, this intuition is questionable in general equilibrium set-

tings where both prices and debt limits are determined endogenously. We should therefore investigate whether competition necessarily lead to maximum constrained risk-sharing in the economy. In other words, is it possible that equilibria with *too-tight* debt limits (see Definition 3.1) can Pareto dominate laissez-faire equilibria? To study this question, we adopt the following concept of *debt-constrained efficiency*:

**Definition 4.1.** Fix a feasible allocation  $(a^i(s^0))_{i \in I}$  of initial financial claims and a self-enforcing equilibrium  $(q, (c^i, a^i, D^i)_{i \in I})$ .<sup>18</sup> This equilibrium is said to be *debt-constrained efficient* if there does not exist another self-enforcing equilibrium  $(\hat{q}, (\hat{c}^i, \hat{a}^i, \hat{D}^i)_{i \in I})$  with, possibly, a different feasible allocation  $(\hat{a}^i(s^0))$  of initial financial claims such that the consumption allocation  $(\hat{c}^i)_{i \in I}$  Pareto dominates  $(c^i)_{i \in I}$ .<sup>19</sup>

We hereafter explore whether laissez-faire equilibria in economies with limited commitment, like those analyzed in the previous sections, are debt-constrained efficient and whether policy interventions are warranted. In doing so, it is useful to first revisit a well-known benchmark where debt-constrained efficiency is unambiguous, despite the fact that equilibrium debt limits depend on market prices: the [Alvarez and Jermann \(2000\)](#) model, where default induces complete financial autarky and nonnegligible dead weight losses. There, we have the following result:

**Theorem 4.1.** *Assume that for each agent  $i$ , the value of the default option is financial autarky with endowment losses, i.e.,  $V_{\text{def}}^i(s^t) := U^i(y^i - \ell^i | s^t)$  for all  $s^t \succ s^0$ , where the endowment losses  $(\ell^i)_{i \in I}$  are a nonnegligible fraction of aggregate resources (see Proposition 2.1). Then any laissez-faire equilibrium is debt-constrained efficient.*

This section's main contribution is to overturn this efficiency result in economies where agents can still save upon default. We show that, in the environments of Sections 3.1 and 3.2, a policy intervention that tightens the debt constraints can Pareto improve upon the laissez-faire allocation. Specifically, we allow for a credit agency or the government to impose

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<sup>18</sup>In the sense that debt limits  $D^i$  satisfy the condition (3.1) with (possibly) strict inequality in some contingencies. Recall that laissez-faire equilibria are a particular case of self-enforcing equilibria.

<sup>19</sup>In the sense that  $U(\hat{c}^i) > U(c^i)$  for each agent  $i \in I$ .

too-tight debt limits and provide an example where such policy intervention can lead to a Pareto improvement. We interpret such interventions as a parsimonious representation of regulatory or prudential policies that aim to constrain leverage in the financial markets.

Intuitively, when do we expect laissez-faire allocations to be debt-constrained inefficient? When the value of default depends on market prices, there is a pecuniary externality that is not internalized by agents in a competitive environment. In particular, we will show that a reduction of the borrowing capacity from a period  $\tau$  onward reduces the credit volume and increases bond prices, or equivalently, lowers the implied interest rates. This impact on prices has a negative feedback effect on the value of the default option at periods  $t < \tau$ , since it is now more costly to smooth consumption over time by saving only. This implies that the not-too-tight debt limits at periods  $t < \tau$  must be looser compared to their level before the intervention. Pareto improvement can be obtained when the benefits from the relaxed credit conditions at periods  $t < \tau$  compensate the costs of the tighter credit conditions in subsequent periods.

To illustrate the intuition above in the simplest possible manner, we consider an economy with two agents facing uncertainty only at the initial period. The economy is thereafter a deterministic one in which every other period agents' endowments switch from a high value to a low value. Within this setting, we perform the following exercise. We first construct a Markov laissez-faire equilibrium  $(q, (c^i, a^i, D^i)_{i \in I})$ , in which after the realization of uncertainty, the economy settles in a cyclical and symmetric steady-state equilibrium where debt limits are not too tight. We then construct another equilibrium  $(\hat{q}, (\hat{c}^i, \hat{a}^i, \hat{D}^i)_{i \in I})$ , supported by the same allocation of initial financial claims, but with debt constraints that are too tight. We then show that the consumption allocation  $(\hat{c}^i)_{i \in I}$  Pareto dominates the consumption allocation  $(c^i)_{i \in I}$  of the laissez-faire equilibrium. Our analysis makes extensive use of our decomposition result (Theorem 3.1) and equivalence result (Theorem 3.2).

## 4.1 Primitives of the Example

There are two agents  $I = \{a, b\}$  who enter the market with an identical endowment  $y_0 > 0$  and no financial claims (i.e.,  $a^a(s^0) = a^b(s^0) = 0$ ). There is uncertainty only at the initial period  $t = 0$ , described by two possible states  $z^a \neq z^b$ . After the realization of the

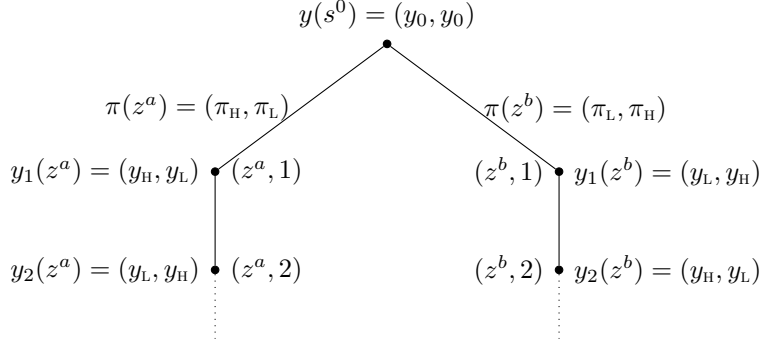


Figure 4.1: Event tree and endowments.

state  $z^i$ , the economy becomes deterministic where agents endowments' switch between a high value  $y_H$  and a low value  $y_L$  with  $y_H > y_L$ . The realization of state  $z^i$  means that it is the agent  $i$  who starts with the high endowment at  $t = 1$ . The beliefs are heterogeneous, with each agent assigning a probability  $\pi_H < 1/2$  ( $\pi_L := 1 - \pi_H$ , respectively) of getting the high (low, respectively) endowment at  $t = 1$ .

Since there is uncertainty only at the initial period, we simplify notation by writing a generic process  $(x(s^t))_{s^t \succeq s^0}$  as follows:  $x(s^0) = x_0$  and  $x(s^t) = x_t(z)$  if  $s^t \succeq (s^0, z)$  with  $z \in \{z^a, z^b\}$ .<sup>20</sup> The representation of the event tree is as in Figure 4.1.

For future reference, we point out that the *symmetric first-best allocation* of this economy obtains when both agents consume their endowment at  $t = 0$  and, conditional on the realization of state  $z^i$ , agent  $i$  consumes  $\underline{c}^{\text{fb}}$  while agent  $j \neq i$  consumes  $\bar{c}^{\text{fb}}$  at every period  $t \geq 1$ . The consumption levels  $\bar{c}^{\text{fb}}$  and  $\underline{c}^{\text{fb}}$  solve the following system of equations:

$$\pi_H u'(\underline{c}^{\text{fb}}) = \pi_L u'(\bar{c}^{\text{fb}}) \quad \text{and} \quad \bar{c}^{\text{fb}} + \underline{c}^{\text{fb}} = y_H + y_L. \quad (4.1)$$

Observe that  $\bar{c}^{\text{fb}} > \underline{c}^{\text{fb}}$ : since both agents believe that reaching the low endowment state at  $t = 1$  has a higher likelihood  $\pi_L > \pi_H$ , they will trade to implement the larger consumption level  $\bar{c}^{\text{fb}}$  contingent to this event.

We specify the following parametrization in the subsequent analysis:  $u = \ln$ ,  $y_0 = 1$ ,  $y_L = 2$ ,  $y_H = 2.5$ ,  $\beta = 0.9$  and  $\pi_H = 0.35$ . Setting specific values for the primitives serves the purpose to provide a graphical illustration of our policy interventions. It also helps to

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<sup>20</sup>The event tree  $\Sigma$  can be formally defined as follows:  $S^0 := \{s^0\}$  and for every  $t \geq 1$ ,  $S^t = \{(z^a, t), (z^b, t)\}$ . The binary relation  $\succ$  is defined as follows:  $(z, 1) \succ s^0$  and  $(z, \tau) \succ (\zeta, t)$  when  $z = \zeta$  and  $\tau > t$ .



verify straightaway the validity of the first-order optimality conditions which is an essential part of the construction of Pareto-improving equilibria. It is straightforward to show that, given our assumptions (continuity), the analysis is valid for an open set of parameter values around the values specified above.

## 4.2 Laissez-Faire Equilibrium

Suppose that pledgeable endowment is time-invariant and identical for both agents, i.e.,  $\ell^i(s^t) = \ell$  for all agent  $i$  and event  $s^t$ . Within this framework, we restrict attention to symmetric Markov equilibria with limited pledgeability and recall that equilibrium debt limits are equal to the present value of pledgeable endowment.<sup>21</sup>

We first notice that the first-best allocation can be implemented as an equilibrium when the level  $\ell$  of pledgeable endowment is larger than the threshold  $\ell^{\text{fb}} := [(y_{\text{H}} - \underline{c}^{\text{fb}}) - \beta(\underline{c}^{\text{fb}} - y_{\text{L}})]/(1 + \beta)$ . Consider next the following lower level of pledgeable endowment:

$$\ell^* := \frac{1 - \beta}{1 + \beta} \times \frac{y_{\text{H}} - y_{\text{L}}}{2}. \quad (4.2)$$

We assume the parameters such that  $\ell^* < y_{\text{L}}$ .<sup>22</sup> The pledgeable income level  $\ell^*$  supports an equilibrium with the following characteristics: at period  $t = 0$ , both agents borrow against their high-income state and save contingent to their low-income state. After the resolution of the uncertainty at period  $t = 1$ , the economy settles in a cyclical steady-state where the low-income agent borrows up to the not-too-tight debt limit, the high-income agent saves, and consumption is constant and equal to  $c^{\text{lf}} := (y_{\text{L}} + y_{\text{L}})/2$  for every  $t \geq 1$ .

**Claim 4.1.** *Let  $\ell^*$  be specified as in (4.2) and denote  $q(0) := \beta$  and  $d(0) := \ell^*/(1 - q(0))$ . There exists an equilibrium with limited pledgeability  $(q, (c^i, a^i, D^i)_{i \in I})$  where for each  $z \in \{z^a, z^b\}$  and every  $i \in I$ :*

- (i) *debt limits equal  $D_t^i(z) = d(0)$ , for  $t \geq 1$ ;*
- (ii) *consumption is risk-free:  $c_0^i = y_0$  and  $c_t^i(z^a) = c_t^i(z^b) = c^{\text{lf}}$ , for  $t \geq 1$ ;*

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<sup>21</sup>When  $\ell > 0$ , it follows from Proposition 2.1 that debt limits in an equilibrium with limited pledgeability cannot display bubbles.

<sup>22</sup>This is the case when  $\beta$  is sufficiently close to 1, or  $y_{\text{H}} - y_{\text{L}}$  is sufficiently small.

- (iii) net asset positions are  $a_t^i(z) = -d(0)$  (i.e., the debt limit binds) if  $y_t^i(z) = y_H$ , and  $a_t^i(z) = d(0)$  if  $y_t^i(z) = y_L$ , for  $t \geq 1$ ;
- (iv) prices are given by  $q_1(z) = \beta \pi_L u'(c^{\text{lf}})/u'(y_0)$  and  $q_{t+1}(z) = q(0)$ ;
- (v) the continuation utility at period  $t = 1$  is  $U_1^{\text{lf}} := u(c^{\text{lf}})/(1 - \beta)$  and the expected utility at period  $t = 0$  is  $U_0^{\text{lf}} := u(y_0) + \beta U_1^{\text{lf}}$ .

We omit the straightforward proof of the claim and conclude by noting that Theorem 3.1 and Theorem 3.2 imply that the equilibrium described above can be supported as a *laissez-faire* equilibrium where debt limits are not-too-tight, as is the case in the model with reputation debt and the model with collateralized debt. Indeed, the collateral equilibrium is obtained when the dividend process  $\delta$  of the Lucas tree is constant and equal to  $2\ell^*$ , and the initial tree holdings are symmetric, i.e.,  $\alpha^i(s^{-1}) = 1/2$  for each  $i$ . Nonpledgeable endowment is then given by  $e^i(s^t) := y^i(s^t) - \ell^* > 0$ .

### 4.3 Tightening Debt Limits in the Reputation Debt Environment

We now proceed to show that a policy intervention that tightens the debt constraints can potentially Pareto improve upon the *laissez-faire* allocation, starting with the reputation debt framework from Section 3.1 (Section 4.4 will analyze the collateral debt framework from Section 3.2). Formally, for each tightening parameter  $\varepsilon \in [0, 1]$ , we will show that there is an equilibrium  $(q^\varepsilon, (c^{i,\varepsilon}, a^{i,\varepsilon}, D^{i,\varepsilon})_{i \in I})$ , where, for every state  $z \in \{z^a, z^b\}$ , the debt limits  $D_1^{i,\varepsilon}(z)$  satisfy the not-too-tight condition (3.2), but the debt limits in subsequent periods are too-tight and equal to:

$$D_t^{i,\varepsilon}(z) = (1 - \varepsilon) \text{PV}_t^\varepsilon(\ell^*|z), \text{ for } t \geq 2. \quad (4.3)$$

Our aim is to show that for some values of  $\varepsilon$ , the new consumption allocation  $(c^{i,\varepsilon})_{i \in I}$  Pareto dominates the *laissez-faire* consumption allocation  $(c^i)_{i \in I}$  in Claim 4.1. To facilitate the exposition of this policy experiment, we split the argument in several steps.

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<sup>23</sup>The notation  $\text{PV}_t^\varepsilon(\ell^*|z)$  represents the present value  $\text{PV}(\ell^*|s^t)$  computed with the price process  $q^\varepsilon$  and conditional to time- $t$  event  $s^t = (z, t)$ .

### 4.3.1 Steady-State Phase

The first step amounts to show that the debt limits in (4.3) support a cyclical steady-state from period  $t = 2$  onward. For each tightening parameter  $\varepsilon$ , denote

$$q(\varepsilon) := \beta \frac{u'(c_L(\varepsilon))}{u'(c_H(\varepsilon))} \quad \text{and} \quad d(\varepsilon) := (1 - \varepsilon) \frac{\ell^*}{1 - q(\varepsilon)}, \quad (4.4)$$

where  $c_H(\varepsilon) := y_H - (1 + q(\varepsilon))d(\varepsilon)$  and  $c_L(\varepsilon) := y_L + (1 + q(\varepsilon))d(\varepsilon)$ . We have the following result:

**Claim 4.2.** *Contingent to any state  $z \in \{z^a, z^b\}$ , the economy reaches a steady-state phase at  $t = 2$  with the following characteristics: for all  $t \geq 2$ :*

- (i) *debt limits  $D_t^{i,\varepsilon}(z) = d(\varepsilon)$  are too tight;*
- (ii) *consumption levels are  $c_t^{i,\varepsilon}(z) = c_H(\varepsilon)$  if  $y_t^i(z) = y_H$ , and  $c_t^{i,\varepsilon}(z) = c_L(\varepsilon)$  if  $y_t^i(z) = y_L$ ;*
- (iii) *net asset positions are  $a_t^{i,\varepsilon}(z) = -d(\varepsilon)$  if  $y_t^i(z) = y_H$ , and  $a_t^{i,\varepsilon}(z) = d(\varepsilon)$  if  $y_t^i(z) = y_L$ ;*
- (iv) *prices are given by  $q_{t+1}^\varepsilon(z) = q(\varepsilon)$ .*

In words, agents borrow the amount  $d(\varepsilon)$  when their income is low and save the amount  $d(\varepsilon)$  when their income is high. We show below (see Figures 4.2(a) and 4.2(b)) that the higher the tightening coefficient  $\varepsilon$ , the tighter the debt limits (i.e., the function  $\varepsilon \mapsto d(\varepsilon)$  is decreasing), and the higher the steady-state price  $q(\varepsilon)$  (or, equivalently, the lower the steady-state interest rate). In the limit, when  $\varepsilon$  tends to 1, the interest rate is zero (i.e.,  $\lim_{\varepsilon \rightarrow 1} q(\varepsilon) = 1$ ), and debt limits form a bubble, i.e.,  $D_t^{i,1}(z) = d(1)$  where  $d(1)$  is determined by the equation:  $u'(y_H - 2d(1)) = \beta u'(y_L + 2d(1))$ .<sup>24</sup>

By construction, the steady-state variables satisfy market clearing. To be part of an equilibrium, they should also be optimal. This requires that the following inequality holds true:  $q(\varepsilon) \geq \beta u'(c_H(\varepsilon))/u'(c_L(\varepsilon))$ .<sup>25</sup>

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<sup>24</sup>We notice that under the chosen values for the primitives, we have that  $\beta u'(y_L)/u'(y_H) = 1.125 > 1$ , which ensures the existence of a pure bubbly equilibrium when there is no endowment loss.

<sup>25</sup>For the values of the primitives we consider, Figure 4.3(c) shows that  $c_H(\varepsilon) \geq c_L(\varepsilon)$ . Since Figure 4.2(b) shows that  $q(\varepsilon) \geq \beta$ , the inequality is satisfied.

The first claim that  $D_t^{i,\varepsilon}(z)$  is too tight relies on the following observation. From the decomposition result (Theorem 3.1), we infer that the equilibrium described in Claim 4.2 is in fact a laissez-faire equilibrium of another economy where the endowment loss upon default is  $(1 - \varepsilon)\ell^*$ . Indeed, we have  $d(\varepsilon) = \text{PV}_t^\varepsilon((1 - \varepsilon)\ell^*|z)$  for every  $z \in \{z^a, z^b\}$  and  $s^t = (z, t)$  with  $t > 0$ . So  $D^{i,\varepsilon}(s^t) = d(\varepsilon)$  satisfies the not-too-tight condition  $V^i(D^{i,\varepsilon}, -D^{i,\varepsilon}(s^t)|s^t) = V_{(1-\varepsilon)\ell^*}^i(0, 0|s^t)$ . Since in the actual economy, the endowment loss equals  $\ell^*$ , we deduce that:

$$V^i(D^{i,\varepsilon}, -D^{i,\varepsilon}(s^t)|s^t) = V_{(1-\varepsilon)\ell^*}^i(0, 0|s^t) > V_{\ell^*}^i(0, 0|s^t) = V_{\text{def}}^i(s^t),$$

which proves the claim.

### 4.3.2 Transition Phase

The second step is to determine the equilibrium variables for the transition periods  $t = 0$  and  $t = 1$ . This is nontrivial since we have to compute the not-too-tight debt limits  $D_1^{i,\varepsilon}(z)$  without being able to appeal to our decomposition result (Theorem 3.1).<sup>26</sup> For the moment, fix a parameter  $d_1 \in [0, y_H)$  representing the debt issued at period  $t = 0$ , and look for an equilibrium such that  $c_0^{i,\varepsilon} = y_0$ ,  $a_1^{i,\varepsilon}(z^i) = -d_1$ ,  $a_1^{i,\varepsilon}(z^j) = d_1$  for  $j \neq i$ , and  $D_1^{i,\varepsilon}(z^i) = d_1$ . That is, at the initial period, both agents borrow against next period's high-income state and save contingent to the low-income state. Since at period  $t = 2$ , the economy settles in the cyclical steady-state described in Claim 4.2, bond holdings at the end of period  $t = 1$  should be  $a_2^{i,\varepsilon}(z) = d(\varepsilon)$  if  $y_2^i(z) = y_L$ , and  $a_2^{i,\varepsilon}(z) = -d(\varepsilon)$  if  $y_2^i(z) = y_H$ . This in turn implies that the corresponding consumption levels at  $t = 1$  are given by  $c_1^{i,\varepsilon}(z^i) = c_{1,H}(\varepsilon, d_1)$  and  $c_1^{i,\varepsilon}(z^j) = c_{1,L}(\varepsilon, d_1)$  for  $j \neq i$  where

$$c_{1,H}(\varepsilon, d_1) := y_H - d_1 - q_2^\varepsilon(z)d(\varepsilon) \quad \text{and} \quad c_{1,L}(\varepsilon, d_1) := y_H - d_1 - q_2^\varepsilon(z)d(\varepsilon).$$

The bond prices  $q_2^\varepsilon(z)$  at period  $t = 1$  are determined by the first-order conditions:

$$q_2^\varepsilon(z) = \beta \frac{u'(c_L(\varepsilon))}{u'(c_{1,H}(\varepsilon, d_1))} =: q_2(\varepsilon, d_1), \quad \text{for } z \in \{z^a, z^b\}.$$

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<sup>26</sup>Note that we only need to specify the debt limit  $D_1^{i,\varepsilon}(z^i)$  as the debt limit  $D_1^{i,\varepsilon}(z^j)$  that is contingent to the low-income state will be nonbinding.

Similarly, the bond prices at period  $t = 0$  are determined by the following first-order conditions:

$$q_1^\varepsilon(z) = \beta \pi_L \frac{u'(c_{1,L}(\varepsilon, d_1))}{u'(y_0)} =: q_1(\varepsilon, d_1), \quad \text{for } z \in \{z^a, z^b\}.$$

Optimality requires validity of some inequalities derived from the first-order conditions associated to the borrowing decisions at  $t = 0$  and  $t = 1$ . We show in the online appendix (Section B.2) that these inequalities are satisfied for the chosen parameter values.

We next identify the level of  $d_1$  (that is not too tight) given that the debt limits at all successor periods  $t \geq 2$  are set to be too tight (see Claim 4.2). Let us denote by  $d_1(\varepsilon)$  this level and remark that we cannot appeal to Theorem 3.1 to claim that  $d_1(\varepsilon) = \text{PV}_1^\varepsilon(\ell^*|z)$ . This would be the case if future debt limits were also not-too-tight (i.e., they are also equal to the present value of pledgeable income), which we have ruled out by construction. Therefore, the determination of  $d_1(\varepsilon)$  requires that we do compute the value functions associated to equilibrium and out-of-equilibrium paths. For this purpose, we introduce the following notations.

Let  $U_{1,H}(\varepsilon, d_1) := V^i(D^{i,\varepsilon}, -d_1|(z^i, 1))$  denote the value function that corresponds to the largest continuation utility when the debt of the high-income agent at period  $t = 1$  equals to  $d_1$ . Let also  $W_{1,H}(\varepsilon, d_1) := V_{\ell^*}^i(0, 0|(z^i, 1))$  denote the default option of the high-income agent at  $t = 1$ .<sup>27</sup> Note that the default option depends indirectly on the debt level  $d_1$ , as  $d_1$  affects the bond prices  $q_2(\varepsilon, d_1)$ . Figure 4.2(a) plots the debt level  $d_1(\varepsilon)$  obtained as the solution to the following not-too-tight condition

$$U_{1,H}(\varepsilon, d_1) = W_{1,H}(\varepsilon, d_1). \tag{4.5}$$

For comparison, we also plot the equilibrium too-tight debt level  $d(\varepsilon)$  as defined in (4.4). We see that  $\varepsilon \mapsto d_1(\varepsilon)$  is an increasing function while  $\varepsilon \mapsto d(\varepsilon)$  is a decreasing function. With the determination of  $d_1(\varepsilon)$  well understood, we can simplify the notation for the equilibrium variables along the transition as follows: for  $t \in \{1, 2\}$ ,

$$c_{1,H}(\varepsilon) := c_{1,H}(\varepsilon, d_1(\varepsilon)), \quad c_{1,L}(\varepsilon) := c_{1,L}(\varepsilon, d_1(\varepsilon)), \quad \text{and} \quad q_t(\varepsilon) := q_t(\varepsilon, d_1(\varepsilon)).$$

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<sup>27</sup>The online appendix (Section B.2) provides the details for the solution of  $W_{1,H}$ .

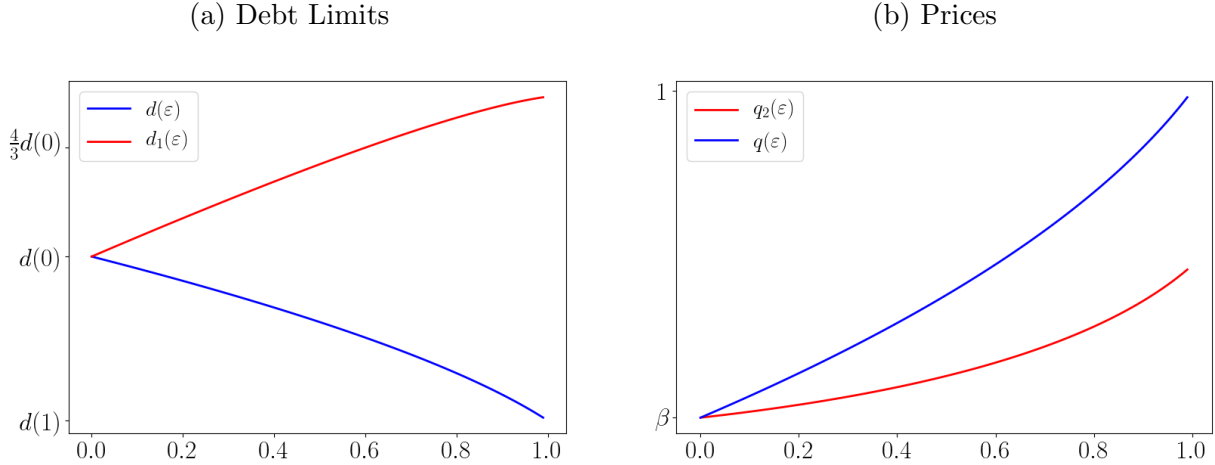


Figure 4.2: Equilibrium debt limits and prices as functions of the tightening coefficient  $\varepsilon$ .

To understand why the policy intervention might be Pareto-improving, it is useful to disentangle the effects it has on the not-too-tight debt limit  $D_1^i(z^i)$ . Before the intervention ( $\varepsilon = 0$ ), the economy is at the laissez-faire equilibrium where  $D_1^i(z^i)$  equals to  $d(0)$ . If we ignore the impact on prices, the deterioration of future credit conditions has a first-order effect: restricting borrowing in the future (i.e.,  $d(\varepsilon)$  falls below  $d(0)$  as  $\varepsilon$  increases) reduces the value of honoring the debt  $d(0)$  at period  $t = 1$  while it leaves the default option unaffected. This implies that  $D_1^i(z^i)$  has to decrease below  $d(0)$  for the not-too-tight condition (3.1) to be satisfied. Taking into account the feedback on prices produces a second-order effect: as  $\varepsilon$  increases, both the period-1 bond price  $q_2(\varepsilon)$  and the steady-state price  $q(\varepsilon)$  increase, as shown in Figure 4.2(b). Thus the intervention makes the option of default less appealing by reducing the interest rate on saving. The impact on the value for honoring the debt  $d(0)$  is, however, ambiguous since along the equilibrium path, the agents both save and borrow. Figure 4.3(a) shows that the value  $U_{1,H}(\varepsilon, d(0))$  of repaying the debt level  $d(0)$  is strictly above the value  $W_{1,H}(\varepsilon, d(0))$  of the default option, so  $D_1^i(z^i)$  has to increase above  $d(0)$  for the not-too-tight condition (3.1) to be satisfied. The overall effect of policy intervention on the level of  $D_1^i(z^i)$  is reflected in the value of  $d_1(\varepsilon)$ , the new not-too-tight debt level. As shown in Figure 4.2(a),  $d_1(\varepsilon) > d(0)$ , so the second-order effect outweighs the first-order effect.

The following claim summarizes the construction of the equilibrium with too-tight debt

constraints.

**Claim 4.3.** *The consumption allocations, bond holdings, and debt limits of the steady-state and transition phases support a competitive equilibrium with not-too-tight reputation debt at  $t = 1$  and too-tight reputation debt at every subsequent date  $t \geq 2$ .*

### 4.3.3 Pareto Improvement

We now numerically show that the equilibrium described in Claim 4.3 Pareto dominates the laissez-faire equilibrium. To identify the overall impact on expected utility, we introduce the following notations. Let  $U_{1,H}(\varepsilon)$  and  $U_{1,L}(\varepsilon)$  be the continuation utilities contingent to high and low income at  $t = 1$  when the debt limit is  $d_1(\varepsilon)$ . That is,  $U_{1,H}(\varepsilon) = u(c_{1,H}(\varepsilon)) + \beta U_L(\varepsilon)$  and  $U_{1,L}(\varepsilon) = u(c_{1,L}(\varepsilon)) + \beta U_H(\varepsilon)$ . where

$$U_H(\varepsilon) := \frac{u(c_H(\varepsilon)) + \beta u(c_L(\varepsilon))}{1 - \beta^2} \quad \text{and} \quad U_L(\varepsilon) := \frac{u(c_L(\varepsilon)) + \beta u(c_H(\varepsilon))}{1 - \beta^2}$$

are the steady-state continuation utilities. Time-0 utility  $U_0(\varepsilon)$  is then given by

$$U_0(\varepsilon) = u(y_0) + \beta [\pi_H U_{1,H}(\varepsilon) + \pi_L U_{1,L}(\varepsilon)].$$

Since the equilibrium is symmetric, we have  $U^i(c^{i,\varepsilon}|s^0) = U_0(\varepsilon)$  for each agent  $i \in I$ . It is straightforward to verify that if  $\varepsilon = 0$ , then we recover the laissez-faire equilibrium with not-too-tight debt limits, that is  $(q^0, (c^{i,0}, a^{i,0}, D^{i,0})_{i \in I}) = (q, (c^i, a^i, D^i)_{i \in I})$  and we deduce that  $U_0(0) = U^i(c^i|s^0)$ . Therefore, to show that the consumption allocation  $(c^{i,\varepsilon})_{i \in I}$  Pareto dominates the consumption allocation  $(c^i)_{i \in I}$ , it is sufficient to show that  $U_0(\varepsilon) > U_0(0)$  for some value of  $\varepsilon$ .

Figure 4.3(b) shows that the steady-state utility of the high-income (low-income, respectively) agent increases (decreases, respectively) with  $\varepsilon$ . We can also see from Figure 4.3(c) that the period  $t = 1$  consumption of the high-income (low-income, respectively) agent decreases (increases, respectively). The above impacts the ex-ante (i.e., at  $t = 0$ ) utility in two ways. There is a negative effect due to the decrease of period  $t = 1$  continuation utility of the high-income agent, and a positive effect due to the increase of period  $t = 1$  continuation utility of the low-income agent. This is illustrated in Figure 4.3(d). Since both agents assign a higher probability on low-income state than on high-income state, i.e.,  $\pi_L > \pi_H$ ,

it is possible that, for some values of  $\varepsilon$ , the positive effect might offset the negative effect, so that the ex-ante utility increases. Figure 4.3(e) confirms this conjecture: for the values of the primitives, we consider the benefit due to the increased borrowing capacity at  $t = 0$  can outweigh the cost of reduced borrowing opportunities at each  $t \geq 1$ . In summary, our numerical analysis has shown that for some values of  $\varepsilon$ , the intervention of tightening the debt limits can *increase* the ex-ante utility for both agents.

It is crucial that the intervention does not occur at the initial period  $t = 0$ . Indeed, without the transition phase, the symmetric ex-ante expected utility associated to this naive intervention is  $U_0^n(\varepsilon) := u(y_0) + \beta [\pi_H U_H(\varepsilon) + \pi_L U_L(\varepsilon)]$ . Mathematically, this corresponds to a laissez-faire equilibrium where the pledgeable endowment is reduced by  $\varepsilon$ . As depicted in Figure 4.3(f), such intervention does not improve welfare.

#### 4.4 Tightening Debt Limits in the Collateral Debt Environment

We now show that tightening debt limits can also increase welfare in the environment of Section 3.2 where debt is collateralized. We mentioned before that the laissez-faire equilibrium described in Claim 4.1 can be supported as an equilibrium with collateralized debt when the dividend of the Lucas tree is constant and equal to  $2\ell^*$ , and the initial tree holdings are symmetric, i.e.,  $\alpha^i(s^{-1}) = 1/2$  for each agent  $i$ . Nonpledgeable endowment is then given by  $e^i(s^t) := y^i(s^t) - \ell^*$ . We recall from Section 3.2 that debt limits are self-enforcing at event  $s^t = (z, t)$  when

$$\tilde{V}_t^i(\tilde{D}^i, -\tilde{D}_t^i(z)|z) \geq \tilde{V}_t^i(\tilde{D}^i, 0|z).^{28}$$

Independently of the level (not too tight or too tight) of future debt limits  $\tilde{D}_\tau^i(z)$  for  $\tau > t$ , the above condition is satisfied with equality at date  $t$  if, and only if,  $\tilde{D}_t^i(z) = 0$ , whereas a strict inequality obtains if, and only if,  $\tilde{D}_t^i(z) < 0$ . Equivalently, in the collateral environment, the debt limit is too tight at some contingency if, and only if, it forces mandatory saving in net terms.

Our objective is to construct a collateral equilibrium with too tight debt limits at some events that Pareto dominates the laissez-faire equilibrium. To do this, we fix a sequence

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<sup>28</sup>We replace the notation  $\tilde{V}^i(\tilde{D}^i, x|s^t)$  by the simpler  $\tilde{V}_t^i(\tilde{D}^i, x|z)$  when  $s^t = (z, t)$ .



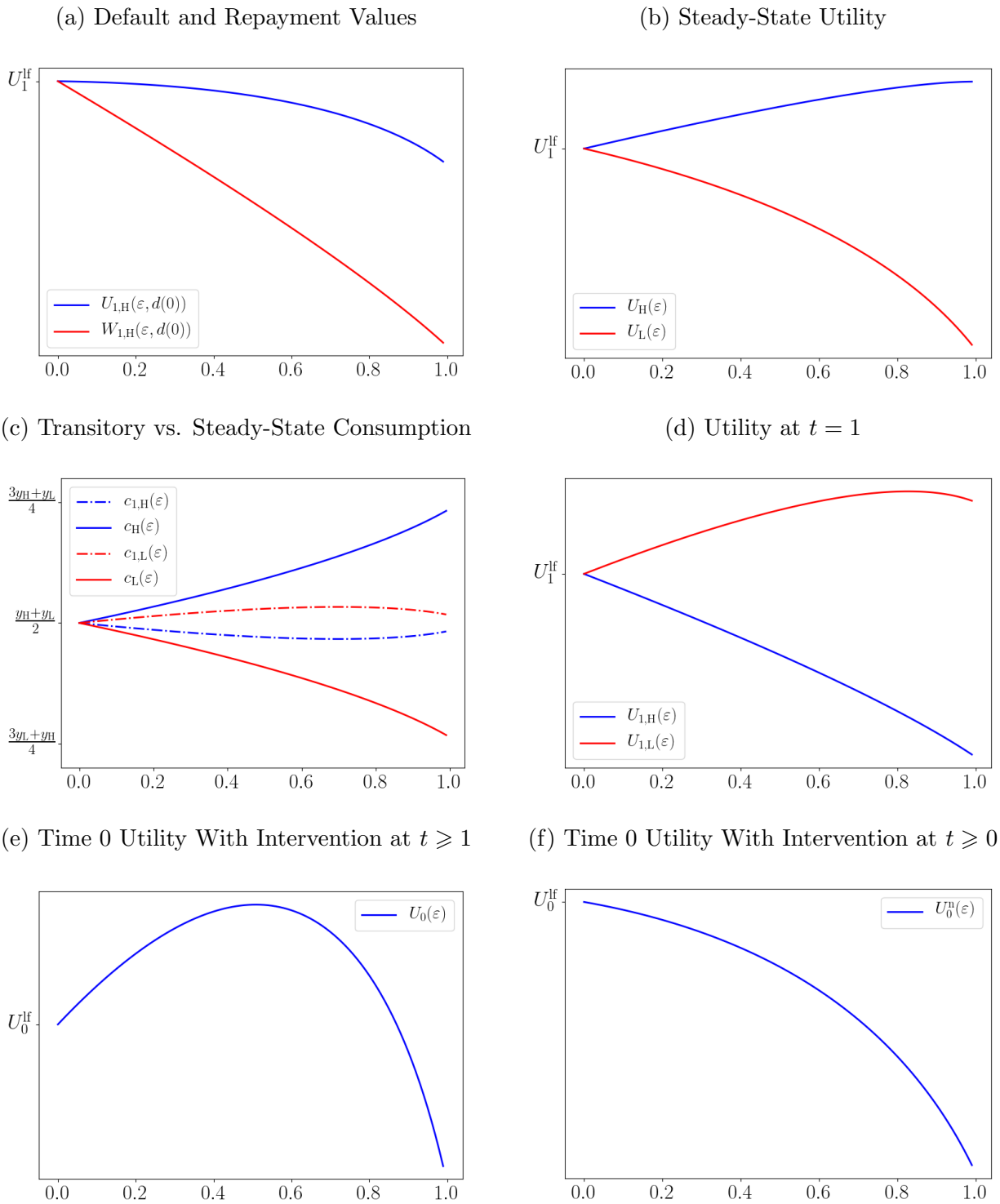


Figure 4.3: Consumption and utilities as functions of tightening coefficient  $\varepsilon$ .

$(\eta_t)_{t \geq 1}$  of tightening parameters  $\eta_t \in [0, 1]$  and set

$$\tilde{D}_t^i(z) = -\eta_t [P_t(z) + \ell^*], \quad (4.6)$$

where  $P_t(z)$  is the price of the tree at event  $s^t = (z, t)$ . When  $\eta_t = 0$ , the debt limit  $\tilde{D}_t^i(z) = 0$  is not too tight, whereas when  $\eta_t > 0$ , the debt limit  $\tilde{D}_t^i(z) < 0$  is too tight. The borrowing (collateral) constraints (3.6) now take the following form: for all  $z \in \{z^a, z^b\}$

$$b_t^i(z) + \alpha_{t-1}^i(z)[P_t(z) + \ell^*] \geq \eta_t [P_t(z) + \ell^*], \quad (4.7)$$

where  $0 \leq \eta_t \leq 1$  is interpreted as a margin requirement imposed by a regulatory agency or the government that requires agents to keep at least (the market value of) a fraction  $\eta_t$  of the physical asset in their balance sheet.

An important observation is that, even under the possibly too-tight debt limits (4.6), the asset pricing equation (3.10) remains valid, so we have that  $P_t(z) + \ell^* = \text{PV}_t(\ell^*|z)$ . This permits, without any loss of generality, to focus attention to the case where there is no trade in the equity market, i.e.,  $\alpha_t^i(z) = 1/2$  for each  $i$  and all  $t$ . In particular, as argued in Section 3.2, we can show that an equilibrium  $(q, P, (c^i, \alpha^i, b^i, \tilde{D}^i)_{i \in I})$  with self-enforcing (possibly too-tight) collateral constraints (4.7) is equivalent to an equilibrium with limited pledgeability  $(q, (c^i, a^i, D^i)_{i \in I})$  where the debt limits are given by  $D_t^i(z) := (1 - \eta_t) \text{PV}_t(\ell^*|z)$ . In the rest of the section, we compute such equilibria by considering a nonzero sequence of tightening coefficients, i.e.,  $(\eta_t)_{t \geq 1} \neq 0$ .

We perform two policy experiments that give rise to equilibria with different characteristics. We first look in the case where the margin requirements (or, equivalently, the too-tight collateral constraints) are imposed from period  $t = 1$  onward, similar to what we did in the economy with reputation debt. In the online appendix (Section B.3.2), we look in the case where the intervention takes place from period  $t = 2$  onward. Interestingly enough, the analysis reveals that delaying the tightening of the collateral constraints one period ahead generates higher welfare gains. We show that this is a general property in our example: the later in the future the intervention takes place, the higher the welfare gains. In the limit, if we delay the intervention for a sufficiently long time, we can get as close as we desire to the first-best regime (see Section B.3.3 in the online appendix).

Assume that  $\eta_1 = 0$  and  $\eta_t = \varepsilon > 0$  for every  $t \geq 2$ . We construct an equilibrium  $(q^\varepsilon, (c^{i,\varepsilon}, a^{i,\varepsilon}, D^{i,\varepsilon})_{i \in I})$  where the debt limits satisfy

$$D_1^{i,\varepsilon}(z) = \text{PV}_1^\varepsilon(\ell^*|z) \quad \text{and} \quad D_t^{i,\varepsilon}(z) = (1 - \varepsilon) \text{PV}_t^\varepsilon(\ell^*|z), \quad \text{for all } t \geq 2.$$

As argued above, such an equilibrium can be implemented as an equilibrium with not-too-tight collateral constraints at  $t = 0$  and too-tight collateral constraints at every  $t \geq 1$ .

The characteristics of the equilibrium are as follows: the economy reaches at period  $t = 3$  a cyclical steady-state  $(q(\varepsilon), c_H(\varepsilon), c_L(\varepsilon), d(\varepsilon))$  similar to the one obtained in the model with reputation debt (i.e., Claim 4.2 applies for  $t \geq 3$ ). In the transition periods  $t \in \{1, 2\}$ , consumption, asset holdings, and debt limits are symmetric, i.e., for any  $z \in \{z^a, z^b\}$ ,

$$c_t^{i,\varepsilon}(z) = \begin{cases} c_{t,H}(\varepsilon), & \text{if } y_t^i(z) = y_H, \\ c_{t,L}(\varepsilon), & \text{if } y_t^i(z) = y_L; \end{cases} \quad \text{and} \quad a_t^{i,\varepsilon}(z) = \begin{cases} -d_t(\varepsilon), & \text{if } y_t^i(z) = y_H, \\ d_t(\varepsilon), & \text{if } y_t^i(z) = y_L; \end{cases}$$

together with  $q_{t+1}^\varepsilon(z) =: q_{t+1}(\varepsilon)$  and  $D^{i,\varepsilon}(z) =: D_t(\varepsilon)$  where:

$$D_1(\varepsilon) = \ell^* \left[ 1 + q_2(\varepsilon) \left( 1 + q_3(\varepsilon) \frac{1}{1 - q(\varepsilon)} \right) \right] \quad \text{and} \quad D_2(\varepsilon) = (1 - \varepsilon) \ell^* \left[ 1 + q_3(\varepsilon) \frac{1}{1 - q(\varepsilon)} \right].$$

At  $t = 0$ , both agents consume their endowment  $c_0^{i,\varepsilon} = y_0$ , with asset prices given by:

$$q_1^\varepsilon(z) = \beta \pi_L \frac{u'(c_{1,L}(\varepsilon))}{u'(y_0)}, \quad \text{for each } z \in \{z^a, z^b\}.$$

An important feature of the cyclical steady state described in Claim 4.2 is that the interest rates tend to zero when  $\varepsilon$  converges to 1 (i.e.,  $\lim_{\varepsilon \rightarrow 1} q(\varepsilon) = 1$ ). This property has implications for the not-too-tight debt limit  $D_1(\varepsilon)$  and the too-tight debt limit  $D_2(\varepsilon)$ . In particular, Figure 4.4(a) shows that  $D_1(\varepsilon)$  explodes to infinite, while  $D_2(\varepsilon)$  decreases as  $\varepsilon$  increases. In turn, these features have implications for the determination of the equilibrium consumption, asset positions, and prices over the transition period. Specifically, they give rise to three threshold values  $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < 1$  over which equilibrium characteristics differ. We delegate the detailed equilibrium derivations to the online appendix of this paper (Section B.3.1) and present the main characteristics hereafter. A graphical illustration is given in Figure 4.4.

For  $\varepsilon \in [0, \varepsilon_1]$ , both agents borrow up to the debt limit against their high income at periods  $t \in \{1, 2\}$ , i.e.,  $d_1(\varepsilon) = D_1(\varepsilon)$  and  $d_2(\varepsilon) = D_2(\varepsilon)$ . For  $\varepsilon \in (0, \varepsilon_1)$ , we have  $c_{1,L}(\varepsilon) <$

$c_{2,H}(\varepsilon)$  and  $c_{1,H}(\varepsilon) > c_{2,L}(\varepsilon)$ . At the threshold value  $\varepsilon_1$ , the debt limit  $D_1(\varepsilon_1)$  is large enough so that agents' consumption levels at  $t = 1$  and  $t = 2$  are equalized, i.e.,  $c_{1,H}(\varepsilon_1) = c_{2,L}(\varepsilon_1)$  and  $c_{1,L}(\varepsilon_1) = c_{2,H}(\varepsilon_1)$ .

For  $\varepsilon \in (\varepsilon_1, \varepsilon_2]$ , both agents borrow up to the debt limit against their high-income state at period  $t = 1$ , i.e.,  $d_1(\varepsilon) = D_1(\varepsilon)$ . But now  $D_1(\varepsilon)$  is sufficiently large so that the low-income agent at period  $t = 1$  does not need to borrow up to the debt limit, i.e.,  $d_2(\varepsilon) < D_2(\varepsilon)$  to achieve perfect consumption smoothing between  $t = 1$  and  $t = 2$ . We then have  $c_{1,H}(\varepsilon) = c_{2,L}(\varepsilon)$  and  $c_{1,L}(\varepsilon) = c_{2,H}(\varepsilon)$ , which implies that  $q_2(\varepsilon) = \beta$ . As  $\varepsilon$  increases,  $D_1(\varepsilon)$  becomes so large that the high-income agent at period  $t = 1$  finds it optimal to borrow, i.e.,  $d_2(\varepsilon)$  becomes negative. The threshold value  $\varepsilon_2$  is determined by the binding constraint  $d_2(\varepsilon_2) = -D_2(\varepsilon_2)$ .

For  $\varepsilon \in (\varepsilon_2, \varepsilon_3]$ , both agents borrow up to the debt limit against their high-income state at  $t = 1$ , i.e.,  $d_1(\varepsilon) = D_1(\varepsilon)$ . However, perfect consumption smoothing between periods  $t = 1$  and  $t = 2$  is not feasible anymore, since the debt constraint of the high-income agent at period  $t = 1$  is binding, i.e., the agent continues to borrow up to  $d_2(\varepsilon) = -D_2(\varepsilon)$ . The consumption  $c_{1,L}(\varepsilon)$  of the low-income agent continues to increase with  $\varepsilon$  while the consumption  $c_{1,H}(\varepsilon)$  of the high-income agent continues to decrease. At the threshold level  $\varepsilon_3$ , the consumption levels  $c_{1,L}(\varepsilon_3)$  and  $c_{1,H}(\varepsilon_3)$  equal the first-best values  $\bar{c}^{\text{fb}}$  and  $\underline{c}^{\text{fb}}$ , so we have  $\pi_L u'(c_{1,L}(\varepsilon_3)) = \pi_H u'(c_{1,H}(\varepsilon_3))$ .

Finally, for  $\varepsilon \in (\varepsilon_3, 1]$ , the debt limit level  $D_1(\varepsilon)$  is so large that the debt constraint at  $t = 0$  is not binding, i.e.,  $d_1(\varepsilon) < D_1(\varepsilon)$ . The first-best allocation is implemented at period  $t = 1$ . The high-income agent continues to borrow up to the debt limit contingent to low income, i.e.,  $d_2(\varepsilon) = -D_2(\varepsilon)$ .

Figure 4.4(a) and Figure 4.4(b) plot the debt levels and equilibrium prices as functions of the tightening parameter  $\varepsilon$ . As in the model with reputation debt, the tightening of debt constraints at every  $t \geq 1$  leads to lower interest rates in the cyclical steady-state ( $\varepsilon \mapsto q(\varepsilon)$  is increasing). This has a positive feedback effect on the equity price at  $t = 1$ , which in turn relaxes the collateral constraints at  $t = 0$ : the price  $\text{PV}_1^\varepsilon(\ell^*|z)$  of the asset is increasing with  $\varepsilon$  and tends to infinite when  $\varepsilon$  converges to 1.

In terms of utility values, the tightening of debt constraints increases (decreases) the

steady-state continuation utility  $U_H(\varepsilon)$  ( $U_L(\varepsilon)$ ) of the high-income (low-income) agent. Figures 4.4(c) and 4.4(d) plot the consumption levels at dates  $t = 1$  and  $t = 2$ . The consumption  $(c_{1,L}(\varepsilon), c_{2,H}(\varepsilon))$  of the agent having low income at period  $t = 1$  increases with  $\varepsilon$ . As shown in Figure 4.4(e), this increase in consumption more than compensates for the lower steady-state utility value  $U_L(\varepsilon)$ , so the period-1 continuation utility  $U_{1,L}(\varepsilon)$  increases with  $\varepsilon$ . Symmetrically, the consumption  $(c_{1,H}(\varepsilon), c_{2,L}(\varepsilon))$  of the agent having high income at  $t = 1$  decreases with  $\varepsilon$ , and this outweighs the increase in the steady-state utility  $U_H(\varepsilon)$ , so the period-1 continuation utility  $U_{1,H}(\varepsilon)$  decreases with  $\varepsilon$ . The overall effect on ex-ante utility  $U_0(\varepsilon)$  is driven by the trade-off of period-1 continuation utility values. Since both agents assign a higher probability to low-income state than to high-income state, i.e.,  $\pi_L > \pi_H$ , we get Pareto improvement. This is illustrated in Figure 4.4(f).

When  $\varepsilon$  is close enough to 1 (formally,  $\varepsilon \geq \varepsilon_3$ ), interest rates are so low, and the value of collateral is so large that the debt constraints at  $t = 0$  are not binding anymore. Therefore, there is no gain (in terms of period  $t = 1$  and period  $t = 2$  consumption levels) from restricting trade in the future, and the ex-ante expected utility decreases with  $\varepsilon$ .

## 5 Pigouvian Subsidies

In the previous section, we have illustrated that private agents fail to internalize how their financial decisions affect the debt limits via prices. This gives room for a government intervention by means of macroprudential controls on financial markets in the lines of [Jeanne and Korinek \(2010, 2019\)](#) and [Farhi and Werning \(2016\)](#). Below, we explore the effects of such a policy experiment by considering corrective Pigouvian subsidies on net deliveries financed by lump-sum taxes.

Formally, each agent  $i$  maximizes  $U^i(c|s^0)$  among all plans  $(c, a)$  satisfying, for every event  $s^t$ , the flow budget constraint

$$T^i(s^t) + c(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a(s^{t+1}) \leq y^i(s^t) + a(s^t) + \kappa \left[ -a(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a(s^{t+1}) \right]^+ \quad (5.1)$$

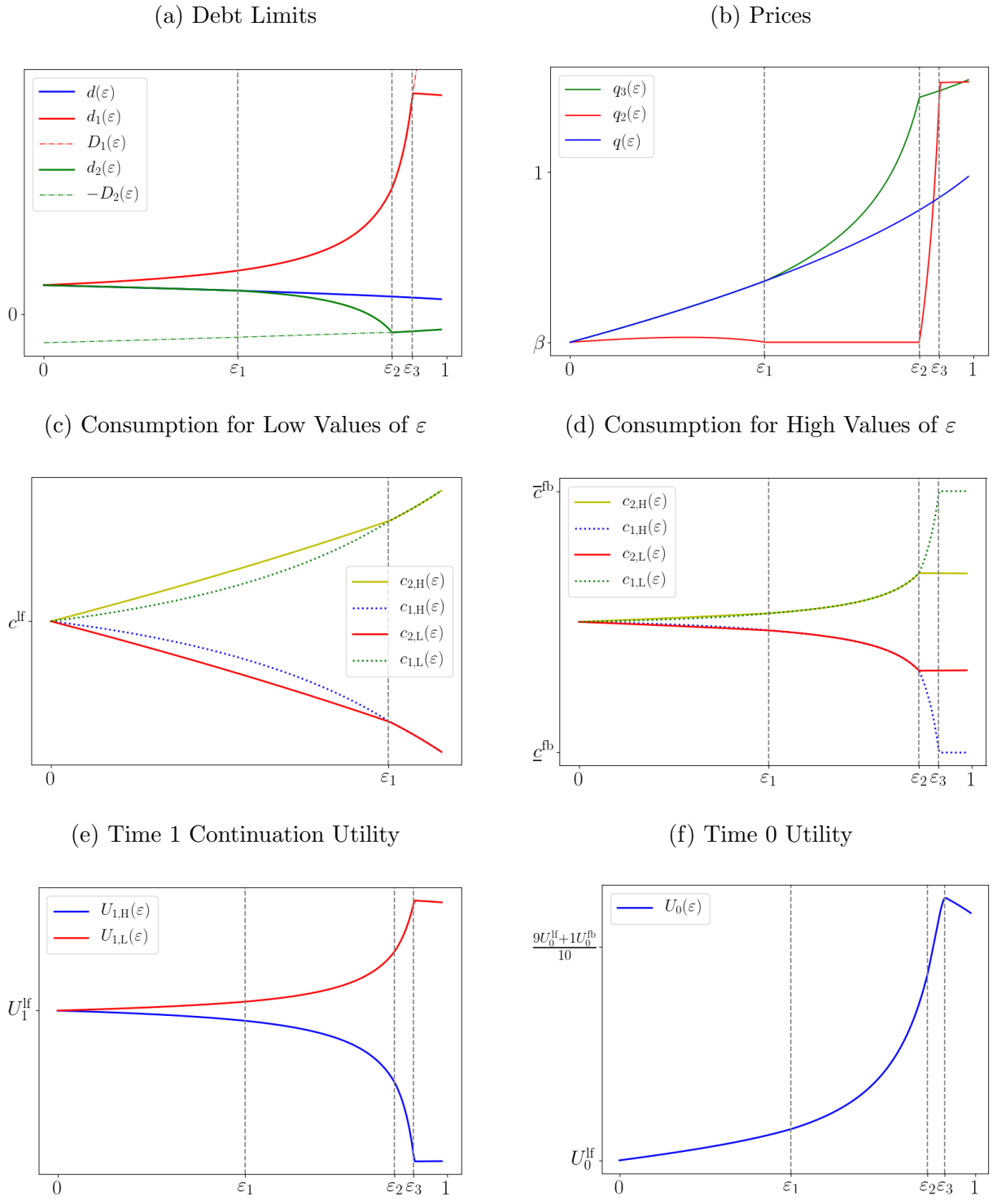


Figure 4.4: Equilibrium debt limits, prices, consumption, and utility as functions of the tightening coefficient  $\varepsilon$ .

while we keep unchanged the debt constraints

$$a(s^{t+1}) \geq -D^i(s^{t+1}), \quad \text{for all } s^{t+1} \succ s^t. \quad (5.2)$$

When maximizing his utility, agent  $i$  takes as given not only the price process  $(q(s^t))_{s^t \succ s^0}$  and the process  $(D^i(s^t))_{s^t \succ s^0}$  of debt limits, but also the subsidy coefficient  $\kappa \in [0, 1]$  and the process  $(T^i(s^t))_{s^t \geq s^0}$  of lump-sum taxes. The subsidy applies only when the net financial position  $-a(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a(s^{t+1})$  is positive. If the agent starts with some debt, i.e.,  $a(s^t) < 0$ , the net financial position is positive when the agent repays at least a part of his debt out of his endowment, or equivalently, when not all the current debt is rolled over. If, instead, the agent starts with some positive claim, i.e.,  $a(s^t) \geq 0$ , the net financial position is positive when the agent saves more than the value of his initial financial claim.

The following equilibrium concept is the analogue of Definition 2.2 in the current environment.

**Definition 5.1.** Given pledgeable endowment processes  $(\ell^i)_{i \in I}$ , a family  $(q, (c^i, a^i, D^i, T^i)_{i \in I})$  is a competitive equilibrium with limited pledgeability and Pigouvian subsidy rate  $\kappa \in [0, 1]$  when:

- (a) for each  $i$ , the plan  $(c^i, a^i)$  maximizes  $U^i(c|s^0)$  among all plans  $(c, a)$  satisfying the flow budget constraints (5.1) and the debt constraints (5.2);
- (b) for each  $i$ , there exists a nonnegative exact rollover process  $M^i$  such that the debt limits satisfy  $D^i = \text{PV}(\ell^i) + M^i$ ;
- (c) subsidies are financed by lump-sum taxes along the equilibrium path:

$$T^i(s^t) = \kappa \left[ -a^i(s^t) + \sum_{s^{t+1} \succ s^t} q^\kappa(s^{t+1})a^i(s^{t+1}) \right]^+; \quad (5.3)$$

- (d) markets clear.

Two observations are worth remarking. First, we only require that the tax revenue  $T^i(s^t)$  offsets the subsidy along the equilibrium path. Second, we notice that the microfoundations for limited pledgeability, discussed in Section 3, remain valid in the current environment.

This follows from our assumption that the subsidy only applies to the net financial position. The equivalence between the reputation debt model and the collateral debt model is preserved, and this permits us to study in a unified way whether macroprudential controls can be welfare improving.

We consider again the example of Section 4.1. For any possible value of the subsidy rate  $\kappa \in [0, 1]$ , we look for an equilibrium with limited pledgeability and subsidies on net deliveries having the following characteristics: at period  $t = 0$ , both agents borrow against their high-income state and save contingent to their low-income state. After the resolution of the uncertainty at period  $t = 1$ , the economy settles in a cyclical steady-state where the low-income agent borrows up to the not-too-tight debt limit, and the high-income agent saves. To describe the equilibrium variables, we denote by  $q(\kappa)$  the solution of the following equation:

$$q(\kappa) = \frac{\beta}{1 - \kappa} \times \frac{u'(c_L(\kappa))}{u'(c_H(\kappa))}, \quad (5.4)$$

where the consumption levels satisfy the following equations

$$c_H(\kappa) = y_H - (1 + q(\kappa))d(\kappa) \quad \text{and} \quad c_L(\kappa) = y_L + (1 + q(\kappa))d(\kappa), \quad (5.5)$$

and the level of debt satisfies

$$d(\kappa) = \frac{\ell^*}{1 - q(\kappa)}, \quad (5.6)$$

where  $\ell^*$  be specified as in (4.2). We claim that the above quantities support a competitive equilibrium provided that the subsidy rate is such that

$$\frac{u'(c_L(\kappa))}{u'(c_H(\kappa))} \geq 1 - \kappa. \quad (5.7)$$

**Proposition 5.1.** *Let  $q(\kappa)$ ,  $c_H(\kappa)$ ,  $c_L(\kappa)$  and  $d(\kappa)$  be specified as in (5.4), (5.5), (5.6), and assume that the rate  $\kappa$  satisfies condition (5.7). There exists a competitive equilibrium  $(q^\kappa, (c^{i,\kappa}, a^{i,\kappa}, D^{i,\kappa}, T^i)_{i \in I})$  with limited pledgeability and subsidy rate  $\kappa$  where for each  $z \in \{z^a, z^b\}$  and every  $t \geq 1$ :*

(i) *debt limits are  $D_t^{i,\kappa}(z) = d(\kappa)$ ;*

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<sup>29</sup>The inequality corresponds to the sufficient optimality condition for  $t \geq 1$ .



(ii) the consumption allocation is  $c_0^{i,\kappa} = y_0$ ,  $c_t^{i,\kappa}(z) = c_H(\kappa)$  if  $y_t^i(z) = y_H$ , and  $c_t^{i,\kappa}(z) = c_L(\kappa)$  if  $y_t^i(z) = y_L$ ;

(iii) Net asset positions are  $a_t^{i,\kappa}(z) = -d(\kappa)$  (i.e., the debt limit binds) if  $y_t^i(z) = y_H$ , and  $a_t^{i,\kappa}(z) = d(\kappa)$  if  $y_t^i(z) = y_L$ ;

(iv) prices are given by:

$$q_1^\kappa(z) = \pi_L \frac{\beta}{1 - \kappa} \times \frac{u'(c_L(\kappa))}{u'(y_0)} \quad \text{and} \quad q_{t+1}^\kappa(z) = q(\kappa);$$

(v) lump-sum taxes are  $T_t^{i,\kappa}(z) = \kappa(1+q(\kappa))d(\kappa)$  if  $y_t^i(z) = y_H$ , and  $T_t^{i,\kappa}(z) = 0$  if  $y_t^i(z) = y_L$ .

We delegate to the online appendix (Section C.2) the proof of the proposition. We here show numerically that there are values of  $\kappa$  such that the equilibrium described in Claim 5.1 Pareto dominates the laissez-faire equilibrium described in Claim 4.1. To this purpose, Figures 5.1(a), 5.1(b) and 5.1(c) plot the steady-state bond prices, debt levels and consumption allocations as a function of the subsidy rate  $\kappa$ . We also show in Figure 5.1(d) that the sufficient condition (5.7) is satisfied for the values of primitives we consider.

Given our specifications of the model, an equilibrium with subsidies can be seen as a standard equilibrium (without subsidies) where agents' time preference coefficient  $\beta$  is replaced by the higher  $\beta(\kappa) := \beta/(1 - \kappa)$  when agents' current income is high (and, consequently, future income is low). In other words, agents are more patient when their current income is high than when their current income is low. That is, the distortion created by the subsidies leads to a wedge in marginal rates of substitution between the high-income and the low-income agents. When compared to the laissez-faire equilibrium, this wedge allows for higher prices (Figure 5.1(a)), looser debt limits (Figure 5.1(b)), higher consumption when income is low and lower consumption when income is high (Figure 5.1(c)).

Let  $U_H(\kappa)$  and  $U_L(\kappa)$  be the continuation utilities when the agents' income is high and low, respectively. Observe that

$$U_H(\kappa) = \frac{u(c_H(\kappa)) + \beta u(c_L(\kappa))}{1 - \beta^2} \quad \text{and} \quad U_L(\kappa) = \frac{u(c_L(\kappa)) + \beta u(c_H(\kappa))}{1 - \beta^2}.$$

Let also  $U_0(\kappa) := u(y_0) + \beta [\pi_H U_H(\kappa) + \pi_L U_L(\kappa)]$ . Since the equilibrium is symmetric, for each agent  $i$ , the period-0 utility satisfies  $U^i(c^{i,\kappa}|s^0) = U_0(\kappa)$ . It is straightforward to verify that,

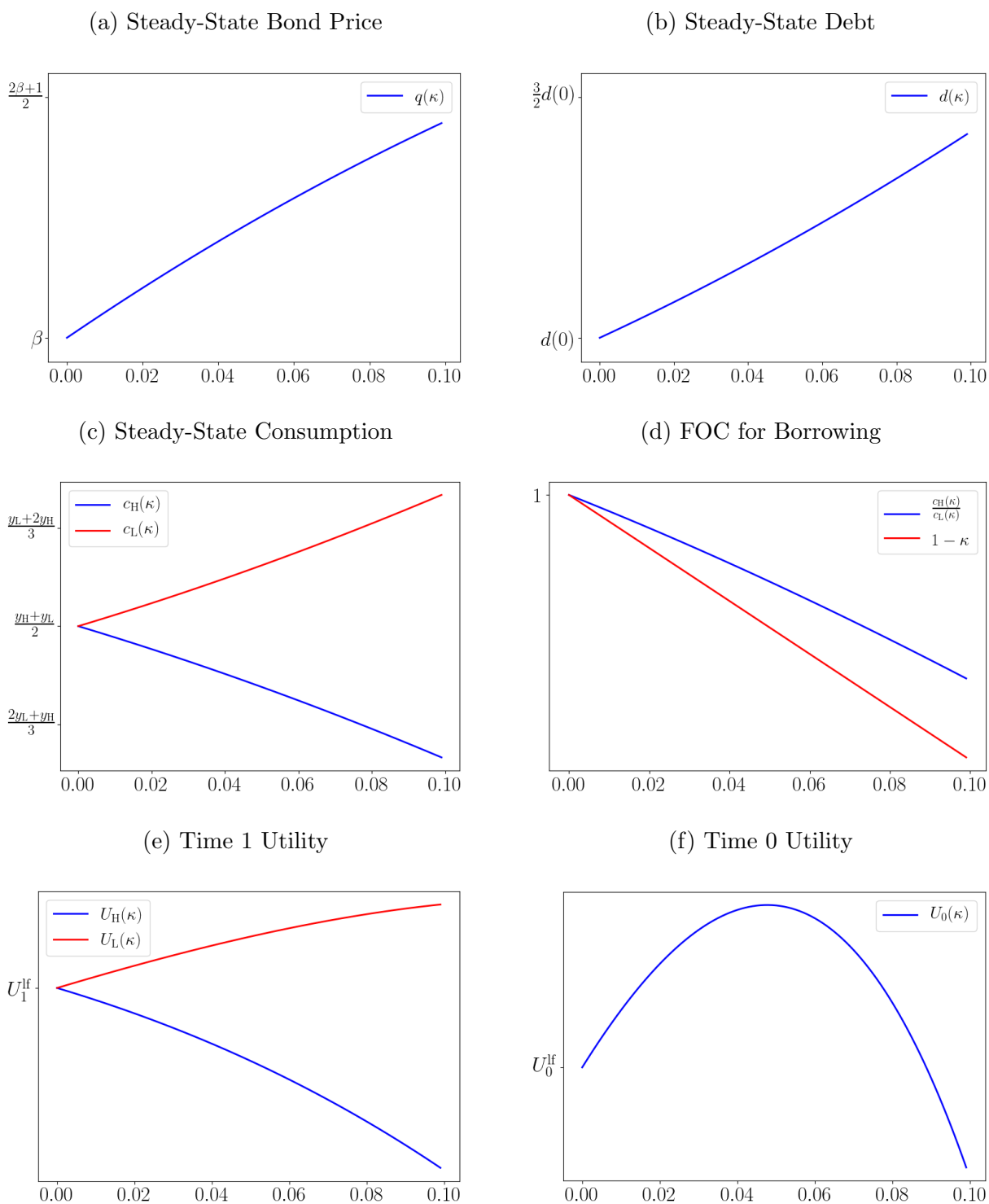


Figure 5.1: Equilibrium variables as functions of the tightening coefficient  $\varepsilon$ .

for  $\kappa = 0$ , we recover the laissez-faire equilibrium in Claim 4.1, that is  $(q^0, (c^{i,0}, a^{i,0}, D^{i,0})_{i \in I}) = (q, (c^i, a^i, D^i)_{i \in I})$ , and we deduce that  $U_0(0) = U^i(c^i | s^0) = U_0^{\text{lf}}$ .<sup>30</sup>

To show that the consumption allocation  $(c^{i,\kappa})_{i \in I}$  Pareto dominates the consumption allocation  $(c^i)_{i \in I}$ , it is sufficient to show that  $U_0(\kappa) > U_0(0)$  for some values of  $\kappa$ . Figure 5.1(c) shows that consumption contingent to low (high) income at  $t = 1$  increases (decreases) with  $\kappa$ . Figure 5.1(e) then shows that the continuation utility  $U_L(\kappa)$  ( $U_H(\kappa)$ ) contingent to low (high) income increases (decreases) with  $\kappa$ . Since agents believe that it is more likely that income is low at period  $t = 1$  ( $\pi_L > \pi_H$ ), in expectation, the increase of  $U_L(\kappa)$  more than compensates the loss of  $U_H(\kappa)$  as shown in Figure 5.1(f). This proves our claim.

## 6 Conclusion

There is a recent and growing literature in macroeconomics showing that competitive economies with price-dependent financial constraints are prone to pecuniary externalities. These claims are in sharp contrast with the renowned constrained efficiency results of Alvarez and Jermann (2000) in economies with limited commitment where financial constraints are microfounded as the largest self-enforcing debt limits. The contribution of this paper is to reconcile these two strands of the literature by showing that pecuniary externalities can emerge even in models à la Alvarez and Jermann (2000) when the autarkic default punishment is replaced by weaker punishments that allow agents to save upon default. A planner can improve upon the competitive outcomes by reducing credit in future periods, forcing the implied interest rates to decline. This can make saving after default less appealing and increase the borrowing capacity of constrained agents at earlier periods, leading to Pareto improvement. The analysis suggests that there is scope for macroprudential policies to reduce the extent of market failure. We show that imposing corrective Pigouvian subsidies on net financial positions supported by lump-sum taxes is an alternative Pareto-improving policy. The distortion created by the subsidy reduces interest rates and makes borrowing more permissive for the financially constrained agents.

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<sup>30</sup>Recall from Claim 4.1 that  $U_0^{\text{lf}} := u(y_0) + \beta u((y_H + y_L)/2)/(1 - \beta)$ .

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