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Social Networks

Weighting the transitivity of undirected weighted social networks with triadic edge dissimilarity scores --Manuscript Draft--

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First Author:	Guillaume PERON
Order of Authors:	Guillaume PERON
Abstract:	<p>When an individual is socially connected to two others, the resulting triplet can be closed (if the two social partners are themselves connected) or open (if they are not connected). The proportion of closed triplets, referred to as the binary network transitivity, is a classic measure of the level of interconnectedness of a social network. However, in any given triplet, if the closing link is weak, or indeed if any of the links in the triplet is weak, then the triplet should not contribute as much to network transitivity as if all three links were equally strong. I propose two ways to weight the contribution of each triplet according to the dissimilarity between the three links in the triplet. Empirically, the resulting new metrics conveyed information not picked up by any other network-level metric. I envision that this approach could prove useful in studies of triadic mechanisms, i.e., situations where pre-existing social ties influence the interactions with third parties. These metrics could also serve as repeatable synthetic variables that summarize information about the variability of the strength of social connections.</p>
Suggested Reviewers:	<p>Wey Tina tina.wey@gmail.com Specialist of animal contact networks</p> <p>Sal Pratha ps875@georgetown.edu Experience in comparative analysis of networks</p> <p>Eric Vander Waal eric.vanderwal@mun.ca Expert in the spread of infection through animal social networks</p>
Opposed Reviewers:	
Response to Reviewers:	

Response to reviews

I think this manuscript has been greatly improved. As you had mentioned, many of the reviewer comments were based around a "what's the point" narrative, and I feel you've done a great job addressing these concerns. Both the introduction and discussion provide the relevant level of details that were previously missing.

I appreciate that you have compared other SNA metrics to your new metrics to really showcase what your metrics are adding to the toolkit for SNA. You've done a great job highlighting when your new methods are best applied as well as assumptions about the new metrics. The results are much easier to follow, and there is good parallelism between the methods and results sections. The language is also much cleaner, which makes for a much better reading experience.

>> Thank you for taking the time for a second review and for your positive evaluation.

Minor Comments

Lines 113-134: I still disagree with the use of the word "excess". The sentence you've added for comparison, is more difficult to interpret than the original paper. I really think the sentence should instead read "This procedure generates mostly balanced triangles where either all three links were weak or all three links were strong."

>> I use that sentence now.

Lines 153-155: I feel that the sentence should read "I assessed whether the triadic edge dissimilarity scores or the PCA scores could discriminate between the 9 network types." I don't think you were assessing your own ability to discriminate between the networks, as the goal is for the new metrics to distinguish these various network types.

>> I use that sentence now

Lines 177-178: What does "deeper" into quadrant 2 look like? Towards the middle of the shape?

>> I reworded into "further away from the borders with the other quadrants"

Lines 237-239; You mention these permutation results are not shown. Is that something that could be included in one of the appendices?

>> To stay in line with the objective of streamlining the manuscript, I instead added a reference to another paper where these permutations are explained.

Line 297: What do you mean by classic moments?

>> I changed into “moment theory” to avoid any confusion with the vernacular meaning. There is also the Fisher 1930 reference to indicate that this is something standard.

Figure 1: I really appreciate the inclusion of the glossary. It makes interpretation in the text much easier.

>> Thank you for suggesting it in your first review.

Figure 2: The legend for Fig 2 was cut off, so I couldn't properly interpret 2b. I think though, based on what I can see, this is a better comparison than in the previous version of the manuscript. What I take away from this figure, is that yes, there are metrics that also show some dissimilarity between triadic edges, but having to incorporate 8 metrics into a PCA is more work, and also demonstrates less precise conclusions than your two new metrics. Which really lines up with the discussion, and drives home the point.

>> Apologies for the mishap. The reviewer correctly interpreted the main message of the figure.

- The transitivity of social networks is classically computed by summing over “triplet” contributions
- I weight the contribution of each triplet using the three links inside of it.
- The resulting new metrics effectively captured information otherwise conveyed by 8 metrics, and more.
- The new metrics are particularly geared towards studies into triadic mechanisms.

Weighting the transitivity of undirected weighted social networks with triadic edge dissimilarity scores

Guillaume Péron

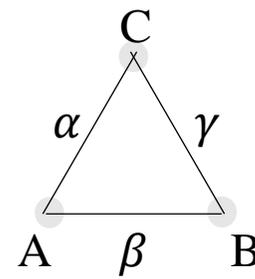
Univ Lyon, Université Lyon 1, CNRS, Laboratoire de Biométrie et Biologie Évolutive UMR5558, F-69622 Villeurbanne, France. guillaume.peron@univ-lyon1.fr

Running title: Triadic edge dissimilarity scores for undirected graphs

When an individual is socially connected to two others, the resulting triplet can be closed (if the two social partners are themselves connected) or open (if they are not connected). The proportion of closed triplets, referred to as the binary network transitivity, is a classic measure of the level of interconnectedness of a social network. However, in any given triplet, if the closing link is weak, or indeed if any of the links in the triplet is weak, then the triplet should not contribute as much to network transitivity as if all three links were equally strong. I propose two ways to weight the contribution of each triplet according to the dissimilarity between the three links in the triplet. Empirically, the resulting new metrics conveyed information not picked up by any other network-level metric. I envision that this approach could prove useful in studies of triadic mechanisms, i.e., situations where pre-existing social ties influence the interactions with third parties. These metrics could also serve as repeatable synthetic variables that summarize information about the variability of the strength of social connections.

Key-words: animal association network; indirect interactions; clustering coefficient; valence; graph theory; social style; intransitivity; socio-spatial structure

Glossary



With notation α, β, γ
 γ is the closing link

- Triad: $\alpha > 0$
- Triplet: $\alpha > 0$ and $\beta > 0$
- Closed triplet or triangle: $\alpha > 0, \beta > 0, \gamma > 0$

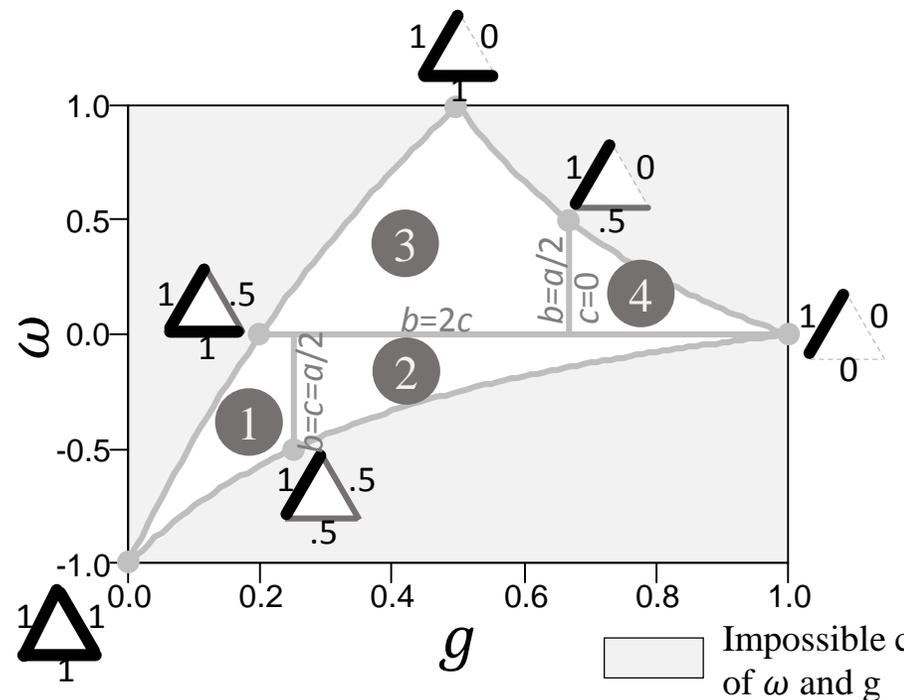
- Strong open triplet
- Weak open triplet
- False triplet

With notation a, b, c
 $1 \geq a \geq b \geq c \geq 0$



- Weak triplet closed by a strong link
- Strong triplet closed by a weak link

The domain of definition of ω and g



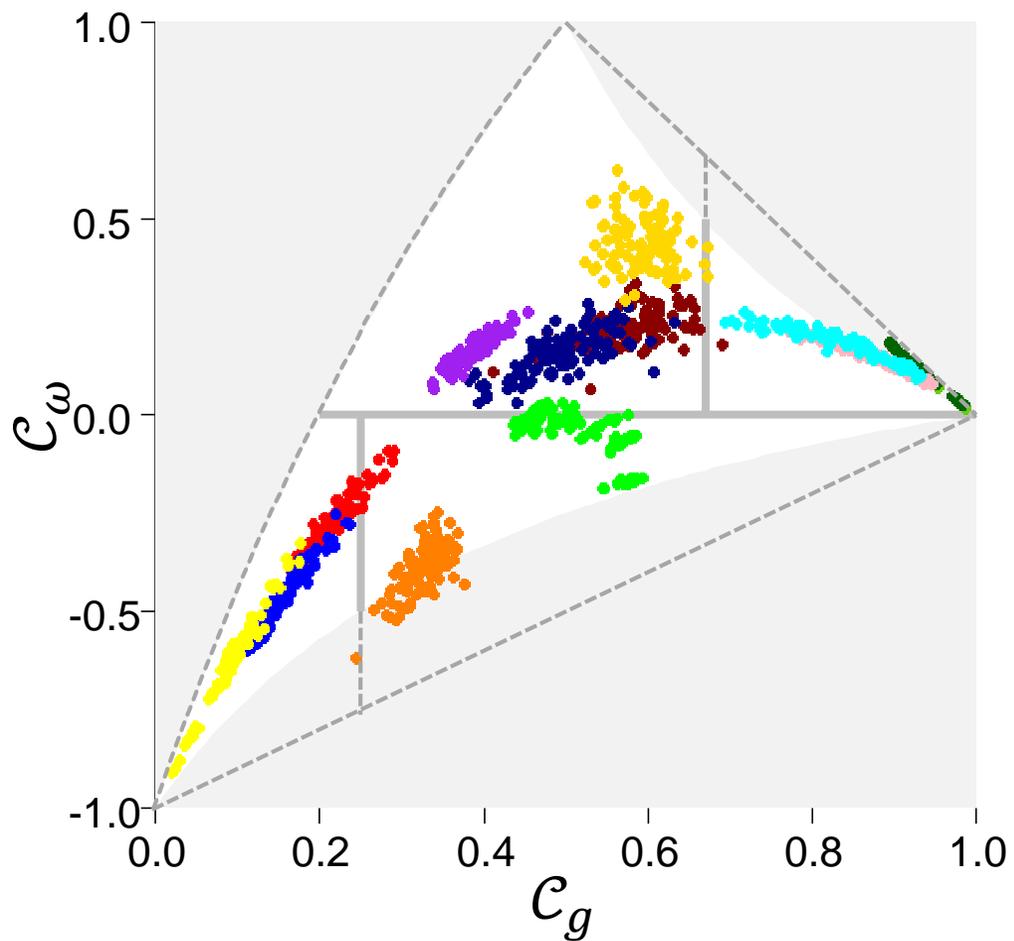
The four quadrants

- 1 Balanced triangles
- 2 Weak triplets closed by a strong link
- 3 Strong triplets closed by a weak link
- 4 Nearly false triplets

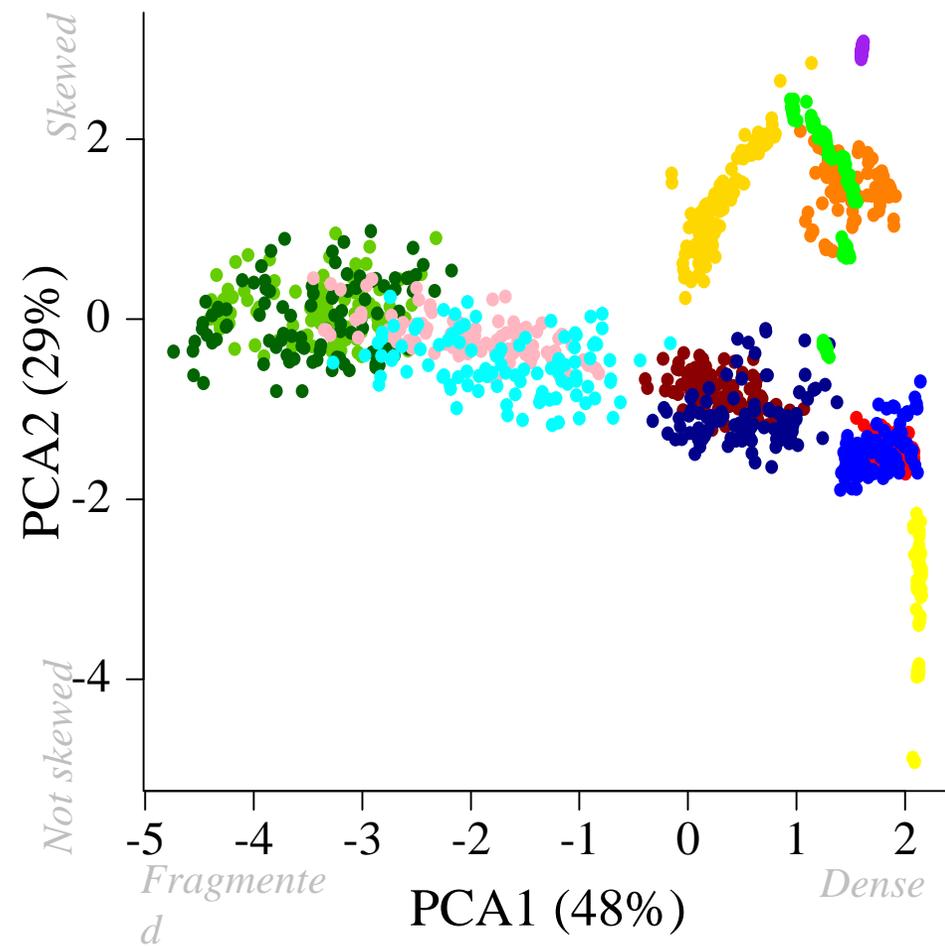
Impossible combinations of ω and g

- Scenario 1
- Scenario 2
- Scenario 3
- Scenario 4 - uniform
- Scenario 4 - Gaussian
- Full - uniform
- Full - Gaussian
- 50% - uniform
- 50% - Gaussian
- Lattice - uniform
- Lattice - Gaussian
- Modular + weak links
- Star + weak links

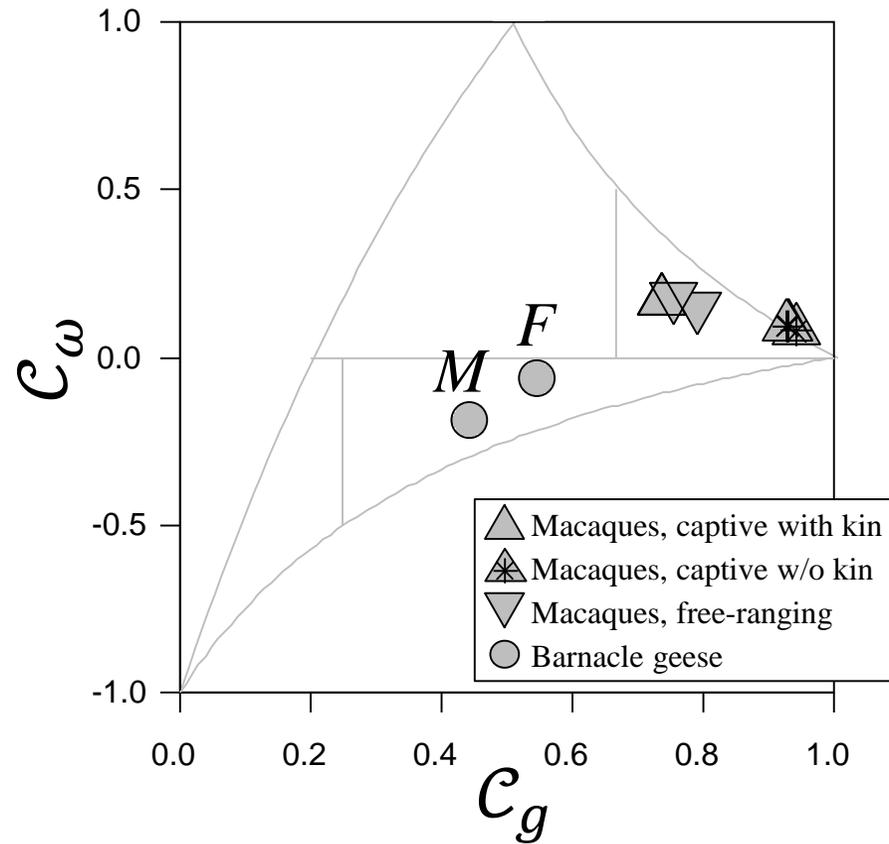
(a) Triadic edge dissimilarity scores



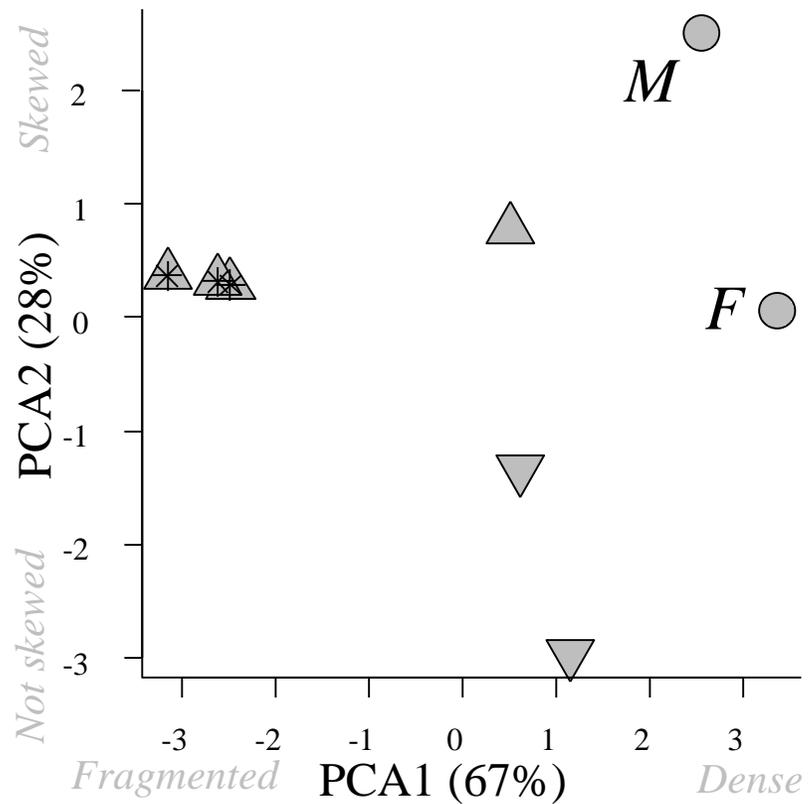
(b) PCA of 8 other metrics



(a) Triadic edge dissimilarity scores

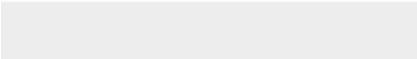
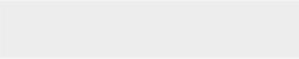


(b) PCA of 8 other metrics



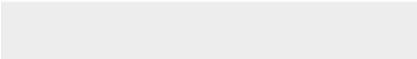
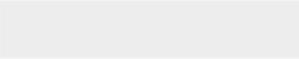


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Supplementary Material
Appendix S1.R





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Supplementary Material
Appendix S2.pdf



Weighting the transitivity of undirected weighted social networks with triadic edge dissimilarity scores

Running title: Triadic edge dissimilarity scores for undirected graphs

Abstract

When an individual is socially connected to two others, the resulting triplet can be closed (if the two social partners are themselves connected) or open (if they are not connected). The proportion of closed triplets, referred to as the binary network transitivity, is a classic measure of the level of interconnectedness of a social network. However, in any given triplet, if the closing link is weak, or indeed if any of the links in the triplet is weak, then the triplet should not contribute as much to network transitivity as if all three links were equally strong. I propose two ways to weight the contribution of each triplet according to the dissimilarity between the three links in the triplet. Empirically, the resulting new metrics conveyed information not picked up by any other network-level metric. I envision that this approach could prove useful in studies of triadic mechanisms, i.e., situations where pre-existing social ties influence the interactions with third parties. These metrics could also serve as repeatable synthetic variables that summarize information about the variability of the strength of social connections.

Key-words: animal association network; indirect interactions; clustering coefficient; valence; graph theory; social style; intransitivity; socio-spatial structure

1 Introduction

2 In undirected social networks, whenever an individual A is separately connected to two other
 3 individuals B and C, the resulting triplet can be closed (if B and C are also connected) or open (if B
 4 and C are not connected) (Fig. 1). The proportion of closed triplets measures the level of
 5 interconnectedness and is referred to as the binary network transitivity (Newman, 2008). Introducing
 6 the variable $\delta_t = 1$ if the triplet t is closed and $\delta_t = 0$ if the triplet t is open, the binary network
 7 transitivity \mathcal{C}_0 corresponds to the sum of δ_t across all the triplets (Newman, 2008).

Eq. 1

$$\mathcal{C}_0 = \frac{1}{\mathcal{T}} \sum_{t=1}^{\mathcal{T}} \delta_t$$

8 \mathcal{T} is the number of triplets in the network.

9 The binary network transitivity has become one of the most used metrics to characterize the
 10 structure of social networks. Many properties of social networks depend on transitivity, e.g., the
 11 occurrence of clusters of interconnected individuals, the expected length of the minimum path
 12 between a given pair of individuals, the expected number of different paths that connect a given pair
 13 of individuals. However, in the binary definition of transitivity, all the triplets that are closed
 14 contribute equally to the transitivity of the network. Yet, in many instances, the connections
 15 (synonyms: links, edges, bonds) vary in strength (synonyms: weight, valence). For example, not all
 16 connections are necessarily active at any given time. Intuitively, if the closing link is weak or rarely
 17 active, the triplet does not contribute as much to network transitivity compared to if the closing link
 18 is strong or frequently active. To address this issue, one might use the strength of the closing links to
 19 weight the contribution of the triplets to network transitivity (Eq. 2).

Eq. 2

$$\mathcal{C}_{0,closing} = \frac{1}{\mathcal{T}} \sum_{t=1}^{\mathcal{T}} \gamma_t$$

20 γ_t is the strength of the closing link in triplet t , rescaled to vary between 0 and 1. If the network was
 21 dichotomized by setting all the link strengths to 0 for a missing link or to 1 for a link with non null
 22 strength, then $\gamma_t = \delta_t$, and Eq. 2 would reduce to Eq. 1.

23 Alternatively, following Opsahl and Panzarasa (2009), one can weigh the contributions of the triplets
 24 according to the strength of their first two links, yielding the weighted network transitivity \mathcal{C}_1 . There
 25 are several variations (Eqs. 3a-3c).

Eq. 3a	$\mathcal{C}_{1,geom} = \frac{\sum_{t=1}^{\mathcal{T}} \sqrt{\alpha_t \beta_t} \cdot \delta_t}{\sum_{t=1}^{\mathcal{T}} \sqrt{\alpha_t \beta_t}}$
Eq. 3b	$\mathcal{C}_{1,min} = \frac{\sum_{t=1}^{\mathcal{T}} \min(\alpha_t, \beta_t) \cdot \delta_t}{\sum_{t=1}^{\mathcal{T}} \min(\alpha_t, \beta_t)}$
Eq. 3c	$\mathcal{C}_{1,max} = \frac{\sum_{t=1}^{\mathcal{T}} \max(\alpha_t, \beta_t) \cdot \delta_t}{\sum_{t=1}^{\mathcal{T}} \max(\alpha_t, \beta_t)}$

26 α_t and β_t are the strengths of the first two links in triplet t (Fig. 1). If the network was dichotomized
 27 by setting all the link strengths to 0 or to 1, then Eqs. 3a-3c would, like Eq. 2, reduce to Eq. 1.

28 Eqs. 3a-3c only use two of the three links in each triplet; Eq. 2 only used one. There does not exist yet
 29 a method that takes all three links into account. To address that issue, I quantified the dissimilarity
 30 between the three links in each triplet. First, I reordered and renamed the three link strengths
 31 $\alpha_t, \beta_t, \gamma_t$ into a_t, b_t, c_t with $a_t \geq b_t \geq c_t$ (Fig. 1). The notation change means that c_t does not
 32 compulsorily correspond to the closing link. I then computed the pairwise proportional difference in
 33 each triplet t (Eq. 4a) (Péron, 2022a).

Eq. 4a	$\omega_t = \frac{a_t - c_t}{a_t} + \frac{b_t - c_t}{a_t} - 1 = \frac{b_t - 2c_t}{a_t}$
--------	---

34 ω compares the weakest link c_t to the other two links. $\omega = 0$ for a false triplet ($b = c = 0$; Fig. 1),
 35 $\omega = 1$ for an open triplet made of two equally strong links ($a = b$ and $c = 0$), and $\omega = -1$ for a
 36 closed triangle made of three equally strong links ($a = b = c$) (Fig. 1). If the network was
 37 dichotomized, the largest link a_t would be set to 1 and b_t and c_t would take either value 0 or 1

38 (instead of varying between 0 and 1). Then the only possible values for ω_t would be -1, 0, or 1

39 (instead of varying between -1 and +1).

40 Next, I computed the Gini coefficient of the three link strengths (Péron, 2022a).

Eq. 4b	$g_t = \frac{1}{2} \frac{(a_t - b_t) + (b_t - c_t) + (a_t - c_t)}{a_t + b_t + c_t} = \frac{a_t - c_t}{a_t + b_t + c_t}$
--------	---

41 The Gini coefficient is a classic measure of inequality (Gini, 1936), often used to quantify how much
 42 larger is the largest quantile of a distribution. Applied to three values, $g = 1$ for a false triplet ($b =$
 43 $c = 0$), $g = 0.5$ for an open triplet made of two equally strong links ($a = b$ and $c = 0$), and $g = 0$
 44 for a closed triangle made of three equally strong links ($a = b = c$) (Fig. 1). If the network was
 45 dichotomized, the only possible values for g_t would be 0, 0.5, or 1 (instead of varying between 0 and
 46 1). Throughout, the strengths of the links a , b , and c were rescaled to vary between 0 (no link) and 1
 47 (strongest recorded link).

48 Compared to previous computations of network transitivity (Eqs. 1-3) (Newman, 2008; Opsahl and
 49 Panzarasa, 2009), my framework uses the terminology in a slightly different way (Fig. 1). (i) In my
 50 framework, missing links with a strength of 0 are special cases of weak links. By contrast, in Eqs.1 and
 51 3, as soon as even the most tenuous closing link was recorded, the triplet would change from open
 52 to closed. (ii) In my framework, Eqs. 4a-4b actually apply to “triads”, meaning that they only require
 53 one non-missing link (Fig. 1). By contrast, Eqs. 1-3 apply to “triplets” with at least two non-missing
 54 links. The difference is important. On the one hand “false triplets” (Fig. 1) count as triads but not as
 55 triplets (Fig. 1). On the other hand, a triangle made of three interconnected individuals and three
 56 links (a,b,c) contributes a single triad to the computation of Eq. 4, but three triplets to the
 57 computation of Eqs 1-3: triplets (a,b) closed by c , (a,c) closed by b , and (b,c) closed by a (Newman,
 58 2008; Opsahl and Panzarasa, 2009). (iii) As mentioned earlier, in my framework, I ordered the link
 59 strengths in each triad ($1 \geq a \geq b \geq c \geq 0$) before computing ω and g . Then I introduced phrases
 60 like “weak triplet closed by a strong link” to refer to specific triangular configurations (Fig. 1). A
 61 “weak triplet closed by a strong link” is a triangle (a,b,c) with c smaller than a and b . As mentioned

62 above, in Eqs. 2 and 3, that triangle would have contributed three triplets, each with a different
 63 weight. By contrast, in my framework, these three triplets all have the same weights ω and g . In
 64 summary, I associated specific meanings to the terms “triplet” and “triad”.

65 By computing ω and g for all the triads, I reduced the 3D space $\mathbb{A} = \{a, b, c \mid 1 \geq a \geq b \geq c \geq 0\}$
 66 into a 2D space $\mathbb{W} = \{\omega, g \mid 1 \geq a \geq b \geq c \geq 0\}$. Because of the condition that $a \geq b \geq c$ and
 67 because of the non independence between the formulae for ω and g , the domain \mathbb{W} exhibits a
 68 specific shape (Fig. 1). For interpretation purposes, I then divided the \mathbb{W} space into four quadrants
 69 (Fig. 1).

- 70 1. Quadrant 1 contained the “balanced triangles” where the three links are of similar strengths:
 71 $a \approx b \approx c$. In this situation, $\omega \rightarrow -1$ and $g \rightarrow 0$. I drew the limits of quadrant 1 by
 72 considering the two extreme cases $b = 2c$ yielding $\omega = 0$; and $b = c = a/2$ yielding $g =$
 73 $1/4$ (Fig. 1).
- 74 2. Quadrant 2 contained the “weak triplets closed by a strong link” where a is much larger than
 75 both b and c . I drew the limits of quadrant 2 by considering the two extreme cases $b = 2c$
 76 yielding $\omega = 0$; and $b = c = a/2$ yielding $g = 1/4$ (Fig. 1).
- 77 3. Quadrant 3 contained the “strong triplets closed by a weak link” where c is much smaller
 78 than both a and b . I drew the limits of quadrant 3 by considering the two extreme cases $b =$
 79 $2c$ yielding $\omega = 0$; and $c = 0$ plus $b = a/2$ yielding $g = 2/3$ (Fig. 1).
- 80 4. Quadrant 4 contained the “nearly false triplets” where a is extremely much larger than b and
 81 c . In this situation, $\omega \rightarrow 0$ and $g \rightarrow 1$. I drew the limits of quadrant 4 by considering the two
 82 extreme cases $b = 2c$ yielding $\omega = 0$; and $c = 0$ plus $b = a/2$ yielding $g = 2/3$ (Fig. 1).

83 Next, I averaged ω and g over all the triads in the network (Eq. 5a and 5b).

Eq. 5a	$C_\omega = \frac{1}{\bar{J}} \sum_{t=1}^{\bar{J}} \omega_t$
--------	--

Eq. 5b

$$C_g = \frac{1}{\tilde{\mathcal{T}}} \sum_{t=1}^{\tilde{\mathcal{T}}} g_t$$

84 $\tilde{\mathcal{T}}$ is the number of triads. The difference between $\tilde{\mathcal{T}}$ and the number of triplets \mathcal{T} depends on the
 85 number of missing links in the network. The R script to compute C_ω and C_g is provided in Appendix
 86 S1.

87 C_ω and C_g are new network-level metrics. In this paper, I performed the initial evaluation of their
 88 relevance. In particular, I assessed whether C_ω and C_g captured information not captured by other
 89 metrics. To do so, I simulated theoretical networks and I reanalyzed published animal association
 90 networks.

91 **Material and methods**

92 **Comparing with other network metrics**

93 First I considered three metrics that quantify the shape of the distribution of link strengths:

- 94 1. The variance in link strength denoted $Var(w)$
- 95 2. The skewness of the link strength distribution denoted $Skew(w)$
- 96 3. The Gini coefficient of the link strength distribution denoted $Gini(w)$

97 The Gini coefficient was here computed for all the links in the networks, whereas g_t from Eq. 4b is
 98 for three links only. Next I computed five classic network-level summary metrics:

- 99 4. The binary edge density \mathcal{D}_0 , corresponding to the proportion of non-missing links among all
 100 the possible links in the network.
- 101 5. The binary network transitivity, \mathcal{C}_0 (Eq. 1)
- 102 6. The weighted network transitivity with the geometric mean of link strengths as weight,
 103 $\mathcal{C}_{1,geom}$ (Eq. 3a)
- 104 7. The network modularity \mathcal{M} . I delineated modules of individuals that interact more among
 105 themselves than across modules using the short random walk community-finding algorithm:
 106 routine `cluster_walktrap` (Pons and Latapy, 2005) from `igraph` (Csardi and Nepusz,

107 2006). I took into account the weight of the links when delineating the modules (argument
108 `weights = E(graph)$weight`). I then computed the modularity score following the
109 usual formula (Newman et al., 2002).

110 8. The network fragmentation \mathcal{F} . I computed the network fragmentation as the overall network
111 size divided by the average size of the modules that contained more than one individual. A
112 large fragmentation score meant that there were many small modules.

113 **Simulating theoretical networks**

114 To demonstrate the type of information that the new metrics convey in a unique way, I designed
115 scenarios corresponding to each of the four quadrants. I predicted that, using the new metrics, the
116 resulting networks would be categorized as belonging to the correct quadrant.

117 1. Scenario 1 with many balanced triangles. I generated full networks with no missing links.

118 Then, at random, half of the links were strong (assigned a weight of 1) and half of the links
119 were weak (assigned a weight drawn at random between 0 and 0.5). This procedure
120 generated balanced triangles where either all three links were weak or all three links were
121 strong.

122 2. Scenario 2 with many weak triplets closed by a strong link. I generated full networks with no
123 missing links. Then, at random, 85% of the links were weak (assigned a weight of 0.1) and
124 15% of the links were strong (assigned a weight drawn at random between 0.75 and 1). This
125 procedure generated more triangles with two weak links and one strong link than if the link
126 strength distribution was unimodal for example.

127 3. Scenario 3 with many strong triplets closed by a weak link. First I generated networks with a
128 \mathcal{C}_0 -score of 0.8, using the `rguman` routine in R-package `sna` (Butts, 2020). Next, processing
129 each triangle one by one in a random order, I made one of the three links weaker than the
130 other two (by assigning a weight drawn at random between 0 and 0.1). This routine
131 generated more triangles with two strong links and one weak link than if I omitted the

132 reassignment step. Note that the routine processed each link multiple times because some
133 links belonged to several triangles.

134 4. Scenario 4 with many false triplets. I generated scale-free networks (Barabási and Albert,
135 1999) and then attributed random weights to the existing links. The weights were drawn
136 from either a Gaussian or uniform distribution. Scale-free networks feature more false
137 triplets (as defined in Fig. 1) than full networks with no missing link.

138 In addition to these 4 scenarios, I considered a few classic theoretical networks.

139 5. Full networks with no missing links (i.e., $\mathcal{D}_0 = 1$) in which I attributed random weights to the
140 existing links. The link strengths were drawn from either a Gaussian or uniform distribution.

141 6. Half-full networks in which I removed half of the links at random (i.e., $\mathcal{D}_0 = 0.5$), and then
142 assigned random weights to the remaining links. The link strengths were drawn from either a
143 Gaussian or uniform distribution.

144 7. Lattice graphs. In lattice graphs, each individual is connected to a fixed number of immediate
145 neighbors. I considered 2D, 3D, and 4D lattices. The link strengths were drawn from either a
146 Gaussian or uniform distribution.

147 8. Modular graphs made of a set number of full networks (from 1 to 8 full networks) of which
148 the link strengths varied between 0.75 and 1. These modules were then connected by weak
149 links varying between 0 and 0.25, thereby ultimately obtaining a full network with no missing
150 link, but a high modularity score as defined above.

151 9. Star graphs made of a central node connected to all other individuals by strong links varying
152 between 0.9 and 1, while the remaining links were all weak and varied between 0 and 0.1.

153 I generated 100 examples of these 9 types of theoretical networks using `igraph` for R (Csardi and
154 Nepusz, 2006). For each simulated network, I computed the two triadic edge dissimilarity scores and
155 the 8 comparison metrics. I then performed a principal component analysis (PCA) of the 8
156 comparison metrics to find axes of covariation between them. I assessed whether the triadic edge

157 dissimilarity scores or the PCA scores could discriminate between the 9 network types, i.e., if one
158 could determine the network type using only the triadic edge dissimilarity scores or only the PCA
159 scores. Lastly, using linear models, I computed the proportion of the variance in the new metrics that
160 was explained by the variation in the 8 comparison metrics.

161 **Using the new metrics to assess triadic mechanisms**

162 A “triadic mechanism” hereafter refers to any situation where an existing link influences the
163 occurrence or the strength of links with third parties (e.g., Perry et al., 2004; Wittig et al., 2014). If
164 that influence was negative, strong links would be surrounded by weak links, which in the parlance of
165 this paper corresponds to weak triplets closed by a strong link. The corresponding triads would score
166 into quadrant 2.

167 As an illustration, I reanalyzed the association networks of experimental flocks of captive barnacle
168 geese *Branta leucopsis* (Kurvers et al., 2013). I expected triadic mechanisms in geese flocks because
169 bonded individuals are dominant over non-bonded individuals (Black and Owen, 1989; Kurvers et al.,
170 2013). In free-ranging flocks, bonds correspond to family units or pairs. In the experiments of which I
171 reanalyzed the data, bonds were artificially created by raising goslings in separate subgroups, before
172 pooling the groups together, separately for males and females (Kurvers et al., 2013). In both sexes,
173 after the subgroups were pooled together, the goslings still bonded preferentially with members of
174 their initial subgroups (Kurvers et al., 2013). The recorded link strengths corresponded to the
175 frequency at which individuals fed on the same patch during standardized observation sessions in an
176 artificially patchy environment. There was no missing link in these networks, i.e., every dyad was
177 recorded to associate at least once. Before analysis, I applied a logit transformation on the link
178 strengths so they varied between 0 and 1 and had a median value of 0.5.

179 Because males are more aggressive and dominant than females in geese (Black and Owen, 1989), I
180 predicted that the effect of the triadic mechanism should be stronger in males than females. To
181 support that hypothesis, I quantified whether the male triads were more often in quadrant 2 or

182 deeper into quadrant 2 (i.e., further away from the borders with the other quadrants) than the
183 female triads using Hotelling's T-squared test (Hotelling, 1931).

184 **Using the new metrics as synthetic variables**

185 Even without triadic mechanism, the new metrics can prove useful because they summarize
186 information that would otherwise require many metrics.

187 As an illustration, I reanalyzed and compared 6 datasets of grooming interactions within groups of
188 rhesus macaques (*Macaca mulatta*), that I obtained through the ASNR repository (Sah et al., 2019).
189 Rhesus macaques exhibit an "intolerant social style" characterized in particular by strong preference
190 for kin in social interactions such as grooming (Thierry, 2007). Therefore, the kinship structure of the
191 group is expected to influence who grooms whom in a macaque group. Among the 6 datasets, there
192 were three groups of nonkin that met each other after weaning age (all from the same study:
193 Balasubramaniam et al., 2018). Then there were two free-ranging groups with naturally occurring
194 kinship structure (Griffin and Nunn, 2012; Puga-Gonzalez et al., 2018) (these are from the same
195 locality but different time periods). Another group was captive but featured locally born adult and
196 immature offspring, therefore similar to the natural kinship structure (Massen and Sterck, 2013). In
197 summary, I obtained data from free-ranging groups of kin, captive groups of kin, and captive groups
198 of nonkin, making it possible to decipher the influence of kinship and captivity. I used Hotelling's T2
199 statistic for this purpose.

200 Link strength corresponded to the frequency of recorded allogrooming interactions for each dyad
201 during standardized observation sessions. I did not distinguish who was the groomer and who was
202 the recipient. These networks featured many missing links, i.e., most of the dyads never groomed
203 each other. Based on the results of the simulation exercise (see below), this is a situation where the
204 overwhelming influence of missing links can cause binary metrics such as \mathcal{D}_0 and \mathcal{C}_0 to outperform
205 the new metrics in terms of discriminatory power. In other words, this exercise represented a

206 challenge for the new approach. Before analysis, I applied a logit transformation on the link strengths
207 so they varied between 0 and 1 and had a median value of 0.5.

208 **Results**

209 **Simulation study**

210 Visually, the two new metrics clearly discriminated among the 9 theoretical networks (Fig. 2a). In
211 particular, Scenarios 1-4 neatly fell in the expected quadrants (Fig. 2a).

212 The PCA of the 8 comparison metrics also discriminated among the 9 theoretical networks (Fig. 2b).

213 However, there was some overlap that did not occur with the new metrics, in particular between
214 Scenarios 2 and 3 and between modular networks and Scenario 2. All in all, the main advantage of
215 the new metrics in terms of discriminatory power was the dimension reduction (2 metrics instead of
216 8), the removal of the PCA step, and the possibility to interpret the scores using the quadrants.

217 29% of the variance in C_ω was left unexplained by any of the 8 comparison metrics. This means that
218 C_ω unambiguously picked up information not captured by any of the 8 comparison metrics. 4% of the
219 variance in C_g was left unexplained by any of the 8 comparison metrics. This latter result mostly
220 stemmed from the correlation between C_g and the edge density \mathcal{D}_0 (Appendix S2). However, for full
221 networks with no missing link, \mathcal{D}_0 was always equal to 1 but C_g was still very variable. In other
222 words, the new metrics were most informative when there were no or few missing links in the study
223 networks.

224 **Using the new metrics to assess triadic mechanisms**

225 Barnacle goose flocks of both sexes scored in quadrant 2, but as predicted, there were more weak
226 triplets closed by a strong link among males than females (Hotelling's T-squared on 2 and 2908
227 degrees of freedom: 65.3; $P < 0.001$; Fig. 3a: "M" vs. "F"). This suggests that bonded males were
228 more dominant over non-bonded males than bonded females were over non-bonded females.

229 The 8 comparison metrics were also different between the male and female networks (Fig. 3b). In
230 particular, the distribution of link strength was more skewed in males than females (5.4 vs. 2.8). This
231 suggests that the excess of weak triplets closed by a strong link in males relative to females might
232 simply stem from the rarity of strong links among males. Repeat samples would be necessary to
233 decipher the effect of the rarity of strong links from the triadic mechanism hypothesis.

234 **Using the new metrics as synthetic variables**

235 Among the macaque grooming networks, kinship explained more variation in the triadic edge
236 dissimilarity scores than captivity (Fig. 3a; Hotelling's T-squared for kin vs. nonkin: 3793.8 on 2 and
237 122780 degrees of freedom; for captive vs. free-ranging: 656.1; both $P < 0.001$).

238 The edge density alone would have been sufficient to discriminate between groups of kin and groups
239 of nonkin: $\mathcal{D}_0 = 0.62 \pm \text{SD } 0.02$ vs 0.19 ± 0.02 . This suggests that in this case, triadic dissimilarities
240 depended mostly on the frequency of missing links, and thereby reaffirm the above conclusion that
241 the new metrics are most informative when there are no or few of these missing links. This
242 conclusion was further verified by permutation exercises (Péron, 2022a): when I permuted the link
243 strengths while keeping the edge density constant, the triadic edge dissimilarity scores and the above
244 conclusion remained largely unchanged (not shown). Lastly, the captive groups exhibited more
245 skewed distributions of link strength than the free-ranging groups ($4.2 \pm \text{SD } 0.2$ vs. 2.8 ± 0.6 ; Fig. 3b).
246 The triadic edge dissimilarity scores did not pick up this information (Fig. 3a: the symbols for captive
247 groups are closer to each other than in Fig. 3b). This means that in the macaque case contrary to the
248 goose case, the rarity of strong links did not contribute much to the triadic edge dissimilarities. The
249 dissimilarities mostly came from missing links rather than from variation in the strength of non-
250 missing links.

251 **Discussion**

252 In this paper, I weighted the computation of network transitivity in a way that takes into account the
253 strength of all three links in each triplet. Previously, network transitivity was either not weighted for

254 link strength, yielding what I denoted the binary transitivity \mathcal{C}_0 (Newman et al., 2002). Alternatively,
 255 the transitivity was weighted but using only the strength of the first two links, yielding what I
 256 denoted the weighted transitivity \mathcal{C}_1 (Opsahl and Panzarasa, 2009). Both \mathcal{C}_0 and \mathcal{C}_1 ignore the
 257 strength of the closing link and the variation between links in the triplets. The new metrics in this
 258 paper are based on the dissimilarity between the three links. I propose the phrase “triadic edge
 259 dissimilarity scores”, the notation ω and g for triad-level scores, and \mathcal{C}_ω and \mathcal{C}_g for network-level
 260 scores. The two set of metrics are complementary and should be interpreted together.

261 **What’s the use?**

262 Based on the simulations and case studies, the new metrics are most relevant for full networks with
 263 no missing links. The main advantages of the new metrics appeared to be:

- 264 1) Ease of interpretation. The new metrics are geared to respond to the occurrence of triadic
 265 mechanisms. Of course, as illustrated by the geese case, strong inference about the actual
 266 occurrence of triadic mechanisms requires more than a point comparison. In particular,
 267 triadic patterns may emerge as a by-product of the rarity of strong links rather than as the
 268 consequence of the position of strong links relative to each other (Péron, 2022a).
 269 Nevertheless, in large comparative analyses, the new metrics should help deciphering the
 270 triadic mechanism hypothesis from alternative explanations. Studies into triadic mechanisms
 271 should in any case consider the new triadic edge dissimilarity scores to summarize the
 272 emerging triadic-level properties of the networks.
- 273 2) Repeatable dimension reduction. In both the geese and the macaque cases, the new metrics
 274 removed to need to explore a wide range of metrics or to train a multivariate model. Most
 275 importantly, in the simulation study, up to 30% of the information that the new metrics
 276 captured was not picked up by any of the 8 alternatives. This means that the new metrics
 277 indicated variation in the structure of weighed networks that the other metrics did not
 278 indicate. This was particularly the case in full networks with no missing link.

279 A major caveat is that, like all weighed network analysis techniques, the new triadic edge dissimilarity
280 scores are sensitive to the way the link strengths are computed and rescaled. In this paper, I used a
281 centered logit transformation, so that the link strengths varied between 0 and 1. Selecting another
282 rescaling method would have changed the point value of the triadic dissimilarity scores, and for
283 example changed the categorization in terms of quadrants. It is therefore important to standardize
284 the computation of link strength as much as possible.

285 Another caveat is the influence of missing links on the triadic edge dissimilarity scores. I recommend
286 sensitivity analyses where missing links are artificially replaced with increasingly strong links, or
287 where an increasingly high threshold is used to consider a link as missing. In the simulation study,
288 lattice graphs, which feature a lot of missing links, all grouped together tight into the rightmost
289 corner of the domain of definition (Fig. 2), providing another illustration of the overwhelming
290 influence of missing links on the dissimilarity scores. Lattice graphs have applications in behavioral
291 ecology, e.g., they can describe the interactions between neighbors in territorial species (Péron,
292 2022b). In this case, the appropriate level of investigation might be the individual rather than the
293 network, e.g., use the weighted degree centrality score, the individual transitivity score, etc. (Barrat
294 et al., 2004).

295 **Conclusion**

296 In this paper, I followed up on an intuition that the contributions of triplets to network transitivity
297 ought to be weighted according to the strength of all three links in the triplets. In doing so, I created
298 new network-level metrics. Whether these new metrics will prove useful remains to be established.
299 Indeed, pre-existing network metrics, if used together, appeared to already capture a lot of the
300 information conveyed by the new metrics. But not all of it. I envision that the new approach could
301 prove useful for studies into triadic mechanisms. Another, potentially widespread, use for these
302 metrics would be to convey the variability of the strength of social connections in a way that

303 complements the moment theory approach (Fisher, 1930) and the inequality approach (Gini, 1936)
 304 (Appendix S2, Fig. 3).

305 **Supplementary material**

306 Appendix S1: R script to compute the new metrics from the adjacency matrix of a social network

307 Appendix S2: Colinarity between the new metrics and the 8 comparison metrics in the simulation

308 study

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Figures

Fig. 1: Illustrated glossary and representation of the domain of definition of ω and g as well as the four quadrants to distinguish between four different types of weighted triadic interactions.

Fig. 2: Simulation study results. (a) Network-level triadic edge dissimilarity scores (new method). The network-level scores \mathcal{C}_ω and \mathcal{C}_g can fall outside of the domain of definition of the triad-level scores ω and g (\mathbb{W} , white shape). However, the network-level scores are always within the dotted lines. (b) Principal component analysis (PCA) of 8 other network-level metrics (see list in the main text).

Fig. 3: Application cases: barnacle geese and rhesus macaques. (a) Network-level triadic edge dissimilarity scores (new method). (b) Principal component analysis (PCA) of 8 other metrics (see list in the main text). The PCA coefficients and axes are different from Fig. 2b.

Weighting the transitivity of undirected weighted social networks with triadic edge dissimilarity scores

Running title: Triadic edge dissimilarity scores for undirected graphs

Abstract

When an individual is socially connected to two others, the resulting triplet can be closed (if the social partners are ~~also~~themselves connected) or open (if ~~the two~~they are not connected). The proportion of closed triplets, referred to as the binary network transitivity, is a classic measure of the level of interconnectedness of a social network. However, in any given triplet, if the closing link is weak, or indeed if any of the links in the triplet is weak, then the triplet should not contribute as much to network transitivity as ~~a strong triplet that would be closed by a consistently strong link. if all three links were equally strong.~~ I propose two ways to weight the contribution of each triplet according to the dissimilarity between the three links in the triplet. Empirically, ~~some of the resulting new metrics conveyed~~ information ~~conveyed by these network-level triadic edge dissimilarity scores~~ ~~was~~ not picked up by any other network-level metric. I envision that this approach could prove useful in studies of triadic mechanisms, i.e., situations where pre-existing social ties influence the interactions with third parties. These metrics could also serve as repeatable synthetic variables that summarize information about the variability of the strength of social connections.

Key-words: animal association network; indirect interactions; clustering coefficient; valence; graph theory; social style; intransitivity; socio-spatial structure

1 Introduction

2 In undirected social networks, whenever an individual A is separately connected to two other
 3 individuals B and C, the resulting triplet can be closed (if B and C are also connected) or open (if B
 4 and C are not connected) (Fig. 1). The proportion of closed triplets measures the level of
 5 interconnectedness and is referred to as the binary network transitivity (Newman, 2008). Introducing
 6 the variable $\delta_t = 1$ if the triplet t is closed and $\delta_t = 0$ if the triplet t is open, the binary network
 7 transitivity C_0 corresponds to the sum of δ_t across all the triplets ~~in the focal network~~ (Newman,
 8 2008).

Eq. 1

$$C_0 = \frac{1}{\mathcal{T}} \sum_{t=1}^{\mathcal{T}} \delta_t$$

9 ~~where~~ \mathcal{T} is the ~~total~~ number of triplets in the network.

10 The binary network transitivity has become one of the most used metrics to characterize the
 11 structure of social networks. Many properties of social networks depend on transitivity, e.g., the
 12 occurrence of clusters of interconnected individuals, the expected length of the minimum path
 13 between a given pair of individuals, the expected number of different paths that connect a given pair
 14 of individuals. However, in the binary definition of transitivity, all the triplets that are closed
 15 contribute equally to the transitivity of the network. Yet, in many instances, the connections
 16 (synonyms: links, edges, bonds) vary in strength (synonyms: weight, valence). For example, not all
 17 connections are necessarily active at any given time. Intuitively, if the closing link is weak or rarely
 18 active, the triplet does not contribute as much to network transitivity compared to if the closing link
 19 is strong- or frequently active. To address this issue, one might use the strength of the closing links to
 20 weight the contribution of the triplets to network transitivity (Eq. 2).

Eq. 2

$$C_{0,closing} = \frac{1}{\mathcal{T}} \sum_{t=1}^{\mathcal{T}} \gamma_t$$

21 γ_t is the strength of the closing link in triplet t , rescaled to vary between 0 and 1. If the network was
 22 dichotomized by setting all the link strengths to 0 for a missing link or to 1 for a link with non null
 23 strength, then $\gamma_t = \delta_t$, and Eq. 2 would reduce to Eq. 1.

24 Alternatively, following Opsahl and Panzarasa (2009), one can weigh the contributions of the triplets
 25 according to the strength of their first two links, yielding the weighted network transitivity \mathcal{C}_1 . There
 26 are several variations (Eqs. 3a-3c).

Eq. 3a	$\mathcal{C}_{1,geom} = \frac{\sum_{t=1}^T \sqrt{\alpha_t \beta_t} \cdot \delta_t}{\sum_{t=1}^T \sqrt{\alpha_t \beta_t}}$
Eq. 3b	$\mathcal{C}_{1,min} = \frac{\sum_{t=1}^T \min(\alpha_t, \beta_t) \cdot \delta_t}{\sum_{t=1}^T \min(\alpha_t, \beta_t)}$
Eq. 3c	$\mathcal{C}_{1,max} = \frac{\sum_{t=1}^T \max(\alpha_t, \beta_t) \cdot \delta_t}{\sum_{t=1}^T \max(\alpha_t, \beta_t)}$

27 α_t and β_t are the strengths of the first two links in triplet t , (Fig. 1). If the network was dichotomized
 28 by setting all the link strengths to 0 or to 1, then Eqs. 3a-3c would, like Eq. 2, reduce to Eq. 1.

29 Eqs. 3a-3c only use two of the three links in each triplet; Eq. 2 only used the third link. There does
 30 not exist yet a method that takes all three links into account. To address that issue, I quantified the
 31 dissimilarity between the three links in each triplet. First, I reordered and renamed the three link
 32 strengths $\alpha_t, \beta_t, \gamma_t$ into a_t, b_t, c_t with $a_t \geq b_t \geq c_t$ (Fig. 1). The notation change means that c_t does
 33 not compulsorily correspond to the closing link. I then computed the pairwise proportional
 34 difference in each triplet t (Eq. 4a) (Péron, 2022a).

Eq. 4a	$\omega_t = \frac{a_t - c_t}{a_t} + \frac{b_t - c_t}{a_t} - 1 = \frac{b_t - 2c_t}{a_t}$
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35 ω compares the weakest link c_t to the other two links. $\omega = 0$ for a false triplet ($b = c = 0$; Fig. 1),
 36 $\omega = 1$ for an open triplet made of two equally strong links ($a = b$ and $c = 0$), and $\omega = -1$ for a
 37 closed triangle made of three equally strong links ($a = b = c$) (Fig. 1). If the network was
 38 dichotomized, the largest link a_t would be set to 1 and b_t and c_t would take either value 0 or 1

39 (instead of varying between 0 and 1). Then the only possible values for ω_t would be -1, 0, or 1
 40 (instead of varying between -1 and +1).

41 Next, I computed the Gini coefficient of the three link strengths (Péron, 2022a).

Eq. 4b	$g_t = \frac{1}{2} \frac{(a_t - b_t) + (b_t - c_t) + (a_t - c_t)}{a_t + b_t + c_t} = \frac{a_t - c_t}{a_t + b_t + c_t}$
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42 The Gini coefficient is a classic measure of inequality (Gini, 1936), often used to quantify how much
 43 larger is the largest quantile of a distribution. Applied to three values, $g = 1$ for a false triplet ($b =$
 44 $c = 0$), $g = 0.5$ for an open triplet made of two equally strong links ($a = b$ and $c = 0$), and $g = 0$
 45 for a closed triangle made of three equally strong links ($a = b = c$) (Fig. 1). If the network was
 46 dichotomized, the only possible values for g_t would be 0, 0.5, or 1 (instead of varying between 0 and
 47 1). Throughout, the strengths of the links a , b , and c were rescaled to vary between 0 (no link) and 1
 48 (strongest recorded link).

49 Compared to previous computations of network transitivity (Eqs. 1-3) (Newman, 2008; Opsahl and
 50 Panzarasa, 2009), my framework uses the terminology in a slightly different way (Fig. 1). (i) In my
 51 framework, missing links with a strength of 0 are special cases of weak links. By contrast, in Eqs.1 and
 52 3, as soon as even the most tenuous closing link was recorded, the triplet would change from open
 53 to closed. (ii) In my framework, ~~Eqs. 4a-4b apply to "triads", meaning that they still require one non-~~
 54 ~~missing link (Fig. 1). By contrast, Eqs. 1-3 only apply to "triplets" with at least two non-missing links.~~
 55 ~~All triplets are triads, but not all triads are triplets (Fig. 1). 4a-4b actually apply to "triads", meaning~~
 56 ~~that they only require one non-missing link (Fig. 1). By contrast, Eqs. 1-3 apply to "triplets" with at~~
 57 ~~least two non-missing links. The difference is important. On the one hand "false triplets" (Fig. 1)~~
 58 ~~count as triads but not as triplets (Fig. 1). On the other hand, a triangle made of three interconnected~~
 59 ~~individuals and three links (a, b, c) contributes a single triad to the computation of Eq. 4, but three~~
 60 ~~triplets to the computation of Eqs 1-3: triplets (a, b) closed by c , (a, c) closed by b , and (b, c) closed by a~~
 61 ~~(Newman, 2008; Opsahl and Panzarasa, 2009).~~ (iii) As mentioned earlier, in my framework, I ordered
 62 the link strengths in each triad ($1 \geq a \geq b \geq c \geq 0$) before computing ω and g . Then I introduced

63 phrases like “weak triplet closed by a strong link” to refer to specific triangular configurations (Fig. 1).

64 A “weak triplet closed by a strong link” is a triangle (a,b,c) with c smaller than a and b . ~~In As~~
 65 ~~mentioned above, in Eqs. 4-2 and 3, that triangle would have contributed three triplets: (a,b) closed by c , (a,c) closed by b , and (b,c) closed by a . In Eqs. 3a-3c, these three triplets had~~
 66 ~~$a,b)$ closed by c , (a,c) closed by b , and (b,c) closed by a . In Eqs. 3a-3c, these three triplets had~~
 67 different ~~weights~~ weight. By contrast, in my framework, ~~they had~~ these three triplets all have the
 68 same weights ω and g . In summary, I associated specific meanings to the terms “triplet” and “triad”.

69 By computing ω and g for all the triads, I reduced the 3D space $\mathbb{A} = \{a, b, c \mid 1 \geq a \geq b \geq c \geq 0\}$
 70 into a 2D space $\mathbb{W} = \{\omega, g \mid 1 \geq a \geq b \geq c \geq 0\}$. Because of the condition that $a \geq b \geq c$ and
 71 because of the non independence between the formulae for ω and g , the domain \mathbb{W} exhibits a
 72 specific shape (Fig. 1). For interpretation purposes, I then divided the \mathbb{W} space into four quadrants
 73 (Fig. 1).

- 74 1. Quadrant 1 contained the “balanced triangles” where the three links are of similar strengths:
 75 $a \approx b \approx c$. In this situation, $\omega \rightarrow -1$ and $g \rightarrow 0$. I drew the limits of quadrant 1 by
 76 considering the two extreme cases $b = 2c$ yielding $\omega = 0$; and $b = c = a/2$ yielding $g =$
 77 $1/4$ (Fig. 1).
- 78 2. Quadrant 2 contained the “weak triplets closed by a strong link” where a is much larger than
 79 both b and c . I drew the limits of quadrant 2 by considering the two extreme cases $b = 2c$
 80 yielding $\omega = 0$; and $b = c = a/2$ yielding $g = 1/4$ (Fig. 1).
- 81 3. Quadrant 3 contained the “strong triplets closed by a weak link” where c is much smaller
 82 than both a and b . I drew the limits of quadrant 3 by considering the two extreme cases $b =$
 83 $2c$ yielding $\omega = 0$; and $c = 0$ plus $b = a/2$ yielding $g = 2/3$ (Fig. 1).
- 84 4. Quadrant 4 contained the “nearly false triplets” where a is extremely much larger than b and
 85 c . ~~Then~~ In this situation, $\omega \rightarrow 0$ and $g \rightarrow 1$. I drew the limits of quadrant 4 by considering the
 86 two extreme cases $b = 2c$ yielding $\omega = 0$; and $c = 0$ plus $b = a/2$ yielding $g = 2/3$ (Fig. 1).

87 Next, I averaged ω and g over all the triads in the network (Eq. 5a and 5b).

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Eq. 5a	$C_{\omega} = \frac{1}{\bar{T}} \sum_{t=1}^{\bar{T}} \omega_t$
Eq. 5b	$C_g = \frac{1}{\bar{T}} \sum_{t=1}^{\bar{T}} g_t$

88 where \bar{T} is the number of triads ($\bar{T} \geq T$). The difference between \bar{T} and the number of triplets T , C_{ω}
89 depends on the number of missing links in the network and C_g are new network-level metrics. The R script to
90 compute C_{ω} and C_g is provided in Appendix S1.

91 C_{ω} and C_g are new network-level metrics. In this paper, I performed the initial evaluation of ~~the~~ their
92 relevance ~~of these two network metrics~~. In particular, I assessed whether C_{ω} and C_g captured
93 information not captured by other metrics. To do so, I simulated theoretical networks and I
94 reanalyzed published animal association networks.

95 Material and methods

96 Comparing with other network metrics

97 First I considered three metrics that quantify the shape of the distribution of link strengths:

- 98 1. The variance in link strength denoted $Var(w)$
- 99 2. The skewness of the link strength distribution denoted $Skew(w)$
- 100 3. The Gini coefficient of the link strength distribution denoted $Gini(w)$

101 The Gini coefficient was here computed for all the links in the networks, whereas g_t from Eq. ~~5b4b~~ is
102 for three links only. Next I computed five classic network-level summary metrics:

- 103 4. The binary edge density \mathcal{D}_0 , corresponding to the proportion of non-missing links among all
104 the possible links in the network.
- 105 5. The binary network transitivity, C_0 (Eq. 1)
- 106 6. The weighted network transitivity with the geometric mean of link strengths as weight,
107 $C_{1,geom}$ (Eq. 3a)

- 108 7. The network modularity \mathcal{M} . I delineated modules of individuals that interact more among
 109 themselves than across modules using the short random walk community-finding algorithm:
 110 routine `cluster_walktrap` (~~Pons and Latapy, 2005~~)(Pons and Latapy, 2005) from
 111 `igraph` (~~Csardi and Nepusz, 2006~~)(Csardi and Nepusz, 2006). I took into account the weight
 112 of the links when delineating the modules (argument `weights = E(graph)$weight`). I
 113 then computed the modularity score following the usual formula (Newman et al., 2002).
 114 8. The network fragmentation \mathcal{F} . I computed the network fragmentation as the overall network
 115 size divided by the average size of the modules that contained more than one individual. A
 116 large fragmentation score meant that there were many small modules.

117 Simulating theoretical networks

118 To demonstrate the type of information that the new metrics convey in a unique way, I designed
 119 scenarios corresponding to each of the four quadrants. I predicted that, using the new metrics, the
 120 resulting networks would be categorized as belonging to the correct quadrant. ~~This is a piece of~~
 121 ~~information uniquely conveyed by the new approach.~~

- 122 1. Scenario 1 with many balanced triangles. I generated full networks with no missing links.
 123 Then, at random, half of the links were strong (assigned a weight of 1) and half of the links
 124 were weak (assigned a weight drawn at random between 0 and 0.5). ~~Compared to the same~~
 125 ~~networks but without this heterogeneity in link strength, this~~This procedure generated ~~an~~
 126 ~~excess of~~ balanced triangles where either all three links were weak or ~~either~~ all three links
 127 were strong.
- 128 2. Scenario 2 with many weak triplets closed by a strong link. I generated full networks with no
 129 missing links. Then, at random, 85% of the links were weak (assigned a weight of 0.1) and
 130 15% of the links were strong (assigned a weight drawn at random between 0.75 and 1).
 131 ~~Compared to the same networks but without this heterogeneity in link strength, this~~This
 132 procedure generated ~~an excess of~~more triangles with two weak links and one strong link
 133 than if the link strength distribution was unimodal for example.

134 3. Scenario 3 with many strong triplets closed by a weak link. First I generated networks with a
 135 C_0 -score of 0.8, using the `rguman` routine in R-package `sna` (Butts, 2020)(Butts, 2020).

136 Next, processing each triangle one by one in a random order, I made one of the three links
 137 weaker than the other two (by assigning a weight drawn at random between 0 and 0.1).

138 ~~Compared to the same networks but without the reassignment~~ This routine, this procedure
 139 generated ~~an excess of more~~ triangles with two strong links and one weak link ~~than if I~~
 140 ~~omitted the reassignment step~~. Note that the routine processed each link multiple times
 141 because some links belonged to several triangles.

142 4. Scenario 4 with many false triplets. I generated scale-free networks (Barabási and Albert,
 143 1999) and then attributed random weights to the existing links. The weights were drawn
 144 from either a Gaussian or uniform distribution. ~~Compared to a full network with no missing~~
 145 ~~link, scale~~ Scale-free networks feature ~~an excess of more~~ false triplets (as defined in Fig. 1)
 146 ~~than full networks with no missing link~~.

147 In addition to these 4 scenarios, I considered a few classic theoretical networks.

148 5. Full networks with no missing links (i.e., $\mathcal{D}_0 = 1$) in which I attributed random weights to the
 149 existing links. The link strengths were drawn from either a Gaussian or uniform distribution.

150 6. Half-full networks in which I removed half of the links at random (i.e., $\mathcal{D}_0 = 0.5$), and then
 151 assigned random weights to the remaining links. The link strengths were drawn from either a
 152 Gaussian or uniform distribution.

153 7. Lattice graphs. In lattice graphs, each individual is connected to a fixed number of immediate
 154 neighbors. I considered 2D, 3D, and 4D lattices. The link strengths were drawn from either a
 155 Gaussian or uniform distribution.

156 8. Modular graphs made of a set number of full networks (from 1 to 8 full networks) of which
 157 the link strengths varied between 0.75 and 1. These modules were then connected by weak

158 links varying between 0 and 0.25, thereby ultimately obtaining a full network with no missing
 159 link, but a high modularity score as defined above.

160 9. Star graphs made of a central node connected to all other individuals by strong links varying
 161 between 0.9 and 1, while the remaining links were all weak and varied between 0 and 0.1.

162 I generated 100 examples of these 9 types of theoretical networks using `igraph` for R (Csardi and
 163 Nepusz, 2006)(Csardi and Nepusz, 2006). For each simulated network, I computed the two triadic
 164 edge dissimilarity scores and the 8 comparison metrics. I then performed a principal component
 165 analysis (PCA) of the 8 comparison metrics to find axes of covariation ~~among~~between them. I
 166 assessed whether ~~the triadic edge dissimilarity scores or the PCA scores~~ could discriminate between
 167 the 9 network types, i.e., ~~if one could~~ determine the network type using only ~~the~~ triadic edge
 168 dissimilarity scores or only ~~the~~ PCA scores. Lastly, using linear models, I computed the proportion
 169 of the variance in the new metrics that was explained by the variation in the 8 comparison metrics.

170 Using the new metrics to assess triadic mechanisms

171 ~~A “triadic mechanism” hereafter refers to any situation where an existing link influences the~~
 172 ~~occurrence or the strength of links with third parties (e.g., Perry et al., 2004; Wittig et al., 2014). If~~
 173 ~~that influence was negative, strong links would be surrounded by weak links, which in the parlance of~~
 174 ~~this paper corresponds to weak triplets closed by a strong link. The corresponding triads would score~~
 175 ~~into quadrant 2.~~

176 ~~As an illustration, I reanalyzed the association networks of experimental flocks of captive barnacle~~
 177 ~~geese *Branta leucopsis* (Kurvers et al., 2013). I expected triadic mechanisms in geese flocks because~~
 178 ~~bonded individuals are dominant over non-bonded individuals (Black and Owen, 1989; Kurvers et al.,~~
 179 ~~2013). In free ranging flocks, bonds correspond to family units or pairs. In the experiments of which I~~
 180 ~~reanalyzed the data, bonds were artificially created by raising goslings in separate subgroups, before~~
 181 ~~pooling the groups together, separately for males and females (Kurvers et al., 2013). In both sexes,~~
 182 ~~after the subgroups were pooled together, the goslings still bonded preferentially with members of~~

183 ~~their initial subgroups (Kurvers et al., 2013). The link strengths corresponded to the frequency at~~
184 ~~which individuals were recorded feeding on the same feeding patch. There was no missing link in~~
185 ~~these networks, i.e., every dyad was recorded to associate at least once. Before analysis, I applied a~~
186 ~~logit transformation on the link strengths so they varied between 0 and 1 and had a median value of~~
187 ~~0.5.~~

188 A “triadic mechanism” hereafter refers to any situation where an existing link influences the
189 occurrence or the strength of links with third parties (e.g., Perry et al., 2004; Wittig et al., 2014). If
190 that influence was negative, strong links would be surrounded by weak links, which in the parlance of
191 this paper corresponds to weak triplets closed by a strong link. The corresponding triads would score
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193 As an illustration, I reanalyzed the association networks of experimental flocks of captive barnacle
194 geese *Branta leucopsis* (Kurvers et al., 2013). I expected triadic mechanisms in geese flocks because
195 bonded individuals are dominant over non-bonded individuals (Black and Owen, 1989; Kurvers et al.,
196 2013). In free-ranging flocks, bonds correspond to family units or pairs. In the experiments of which I
197 reanalyzed the data, bonds were artificially created by raising goslings in separate subgroups, before
198 pooling the groups together, separately for males and females (Kurvers et al., 2013). In both sexes,
199 after the subgroups were pooled together, the goslings still bonded preferentially with members of
200 their initial subgroups (Kurvers et al., 2013). The recorded link strengths corresponded to the
201 frequency at which individuals fed on the same patch during standardized observation sessions in an
202 artificially patchy environment. There was no missing link in these networks, i.e., every dyad was
203 recorded to associate at least once. Before analysis, I applied a logit transformation on the link
204 strengths so they varied between 0 and 1 and had a median value of 0.5.

205 Because males are more aggressive and dominant than females in geese (~~Black and Owen,~~
206 ~~1989~~Black and Owen, 1989), I predicted that the effect of the triadic mechanism should be stronger
207 in males than females. To support that hypothesis, I quantified whether the male triads were more

208 often in quadrant 2 or deeper into quadrant 2 ([i.e., further away from the borders with the other](#)
209 [quadrants](#)) than the female triads using Hotelling's T-squared test (Hotelling, 1931).

210 **Using the new metrics as synthetic variables**

211 Even without triadic mechanism, the new metrics can prove useful because they summarize
212 information that would otherwise require many metrics.

213 As an illustration, I reanalyzed and compared 6 datasets of grooming interactions within groups of
214 rhesus macaques, ([Macaca mulatta](#)), that I obtained through the ASNR repository (Sah et al., 2019).
215 Rhesus macaques ([Macaca mulatta](#)) exhibit an "intolerant social style" characterized in particular by
216 strong preference for kin in social interactions such as grooming ([Thierry, 2007](#))([Thierry, 2007](#)).

217 Therefore, the kinship structure of the group is expected to influence who grooms whom in a
218 macaque group. Among the 6 datasets, there were three groups of nonkin that met each other after
219 weaning age (all from the same study: [Balasubramaniam et al., 2018](#)),([Balasubramaniam et al., 2018](#)).

220 Then there were two free-ranging groups with naturally occurring kinship structure ([Griffin and](#)
221 [Nunn, 2012](#); [Puga-Gonzalez et al., 2018](#))([Griffin and Nunn, 2012](#); [Puga-Gonzalez et al., 2018](#)) (these
222 are from the same locality but different time periods). Another group was captive but featured

223 locally born adult and immature offspring, therefore similar to the natural kinship structure ([Massen](#)
224 [and Sterck, 2013](#))([Massen and Sterck, 2013](#)). In summary, I obtained data from free-ranging groups
225 of kin, captive groups of kin, and captive groups of nonkin, making it possible to decipher the
226 influence of kinship and captivity. I used Hotelling's T2 statistic for this purpose.

227 Link strength corresponded to the frequency of recorded allogrooming interactions for each dyad
228 during standardized observation sessions. I did not distinguish who was the groomer and who was
229 the recipient. These networks featured many missing links, i.e., most of the [dyad dyads](#) never
230 groomed each other. Based on the results of the simulation exercise (see below), this is a situation
231 where the overwhelming influence of missing links can cause binary metrics such as \mathcal{D}_0 and \mathcal{C}_0 to
232 outperform the new metrics in terms of discriminatory power. In other words, this exercise

233 represented a challenge for the new approach. Before analysis, I applied a logit transformation on
234 the link strengths so they varied between 0 and 1 and had a median value of 0.5.

235 **Results**

236 **Simulation study**

237 Visually, the two new metrics clearly discriminated among the 9 theoretical networks (Fig. 2a). In
238 particular, Scenarios 1-4 neatly fell in the expected quadrants (Fig. 2a).

239 The PCA of the 8 comparison metrics also discriminated among the 9 theoretical networks (Fig. 2b).

240 However, there was some overlap that did not occur with the new metrics, in particular between
241 Scenarios 2 and 3 and between modular networks and Scenario 2. All in all, the main advantage of
242 the new metrics in terms of discriminatory power was the dimension reduction (2 metrics instead of
243 8), the removal of the PCA step, and the possibility to interpret the scores using the quadrants.

244 29% of the variance in C_ω was left unexplained by any of the 8 comparison metrics. This means that
245 C_ω unambiguously picked up information not captured by any of the 8 comparison metrics. 4% of the
246 variance in C_g was left unexplained by any of the 8 comparison metrics. This latter result mostly
247 stemmed from the correlation between C_g and the edge density \mathcal{D}_0 (Appendix S2). However, for full
248 networks with no missing link, \mathcal{D}_0 was always equal to 1 but C_g was still very variable. In other
249 words, the new metrics were most informative when there were no or few missing links in the study
250 networks.

251 **Using the new metrics to assess triadic mechanisms**

252 Barnacle goose flocks of both sexes scored in quadrant 2, but as predicted, there were more weak
253 triplets closed by a strong link among males than females (Hotelling's T-squared on 2 and 2908
254 degrees of freedom: 65.3; $P < 0.001$; Fig. 3a: "M" vs. "F"). This suggests that bonded males were
255 more dominant over non-bonded males than bonded females were over non-bonded females.

256 The 8 comparison metrics were also different between the male and female networks (Fig. 3b). In
257 particular, the distribution of link strength was more skewed in males than females (5.4 vs. 2.8). This
258 suggests that the excess of weak triplets closed by a strong link in males relative to females might
259 simply stem from the rarity of strong links among males. Repeat samples would be necessary to
260 decipher the effect of the rarity of strong links from the triadic mechanism hypothesis.

261 **Using the new metrics as synthetic variables**

262 Among the macaque grooming networks, kinship explained more variation in the triadic edge
263 dissimilarity scores than captivity (Fig. 3a; Hotelling's T-squared for kin vs. nonkin: 3793.8 on 2 and
264 122780 degrees of freedom; for captive vs. free-ranging: 656.1; both $P < 0.001$).

265 The edge density alone would have been sufficient to discriminate between groups of kin and groups
266 of nonkin: $\mathcal{D}_0 = 0.62 \pm \text{SD } 0.02$ vs 0.19 ± 0.02 . This suggests that in this case, triadic dissimilarities
267 depended mostly on the frequency of missing links, and thereby reaffirm the above conclusion that
268 the new metrics are most informative when there are no or few of these missing links. This
269 conclusion was further verified by permutation exercises: [\(Péron, 2022a\)](#): when I permuted the link
270 strengths while keeping the edge density constant, the triadic edge dissimilarity scores and the above
271 conclusion remained largely unchanged (not shown). Lastly, the captive groups exhibited more
272 skewed distributions of link strength than the free-ranging groups ($4.2 \pm \text{SD } 0.2$ vs. 2.8 ± 0.6 ; Fig. 3b).
273 The triadic edge dissimilarity scores did not pick up this information (Fig. 3a: the symbols for captive
274 groups are closer to each other than in Fig. 3b). This means that in the macaque case contrary to the
275 goose case, the rarity of strong links did not contribute much to the triadic edge dissimilarities. The
276 dissimilarities mostly came from missing links rather than from variation in the strength of non-
277 missing links.

278 **Discussion**

279 In this paper, I weighted the computation of network transitivity in a way that takes into account the
280 strength of all three links in each triplet. Previously, network transitivity was either not weighted for

281 link strength, yielding what I denoted the binary transitivity C_0 (Newman et al., 2002). Alternatively,
282 the transitivity was weighted but using only the strength of the first two links, yielding what I
283 denoted the weighted transitivity C_1 (Opsahl and Panzarasa, 2009). Both C_0 and C_1 ignore the
284 strength of the closing link and the variation between links in the triplets. The new metrics in this
285 paper are based on the dissimilarity between the three links. I propose the phrase “triadic edge
286 dissimilarity scores”, the notation ω and g for triad-level scores, and C_ω and C_g for network-level
287 scores. The two set of metrics are complementary and should be interpreted together.

288 **What’s the use?**

289 Based on the simulations and case studies, the new metrics are most relevant for full networks with
290 no missing links. The main advantages of the new metrics appeared to be:

- 291 1) Ease of interpretation. The new metrics are geared to respond to the occurrence of triadic
292 mechanisms. Of course, as illustrated by the geese case, strong inference about the actual
293 occurrence of triadic mechanisms requires more than a point comparison. In particular,
294 triadic patterns may emerge as a by-product of the rarity of strong links rather than as the
295 consequence of the position of strong links relative to each other (Péron, 2022a).
296 Nevertheless, in large comparative analyses, the new metrics should help deciphering the
297 triadic mechanism hypothesis from alternative explanations. Studies into triadic mechanisms
298 should in any case consider the new triadic edge dissimilarity scores to summarize the
299 emerging triadic-level properties of the networks.
- 300 2) Repeatable dimension reduction. In both the geese and the macaque cases, the new metrics
301 removed to need to explore a wide range of metrics or to train a multivariate model. Most
302 importantly, in the simulation study, up to 30% of the information that the new metrics
303 captured was not picked up by any of the 8 alternatives. This means that the new metrics
304 indicated variation in the structure of weighed networks that the other metrics did not
305 indicate. This was particularly the case in full networks with no missing link.

306 A major caveat is that, like all weighed network analysis techniques, the new triadic edge dissimilarity
307 scores are sensitive to the way the link strengths are computed and rescaled. In this paper, I used a
308 centered logit transformation, so that the link strengths varied between 0 and 1. Selecting another
309 rescaling method would have changed the point value of the triadic dissimilarity scores, and for
310 example changed the categorization in terms of quadrants. It is therefore important to standardize
311 the computation of link strength as much as possible.

312 Another caveat is the ~~preponderant~~ influence of missing links on the triadic edge dissimilarity scores.
313 I recommend sensitivity analyses where missing links are artificially replaced with increasingly strong
314 links, or where an increasingly high threshold is used to consider a link as missing. In the simulation
315 study, lattice graphs, which feature a lot of missing links, all grouped together tight into the
316 rightmost corner of the domain of definition (Fig. 2), providing another illustration of the
317 overwhelming influence of missing links on the dissimilarity scores. Lattice graphs have applications
318 in behavioral ecology, e.g., they can describe the interactions between neighbors in territorial species
319 (Péron, 2022b). In this case, ~~a potentially more efficient approach would~~ the appropriate level of
320 investigation might be the individual-based rather than the network, e.g., use the weighted degree
321 centrality, score, the individual transitivity score, etc. (Barrat et al., 2004).

322 Conclusion

323 In this paper, I followed up on ~~the~~ an intuition that the contributions of triplets to network transitivity
324 ought to be weighted according to the strength of all three links in the triplets, ~~including the closing~~
325 ~~link~~. In doing so, I created new network-level metrics. Whether these new metrics will prove useful
326 remains to be established. Indeed, pre-existing network metrics, if used together, appeared to
327 already capture a lot of the information conveyed by the new metrics, ~~but~~. But not all of it. I envision
328 that the new approach could prove useful for studies into triadic mechanisms. Another, potentially
329 widespread, use for these metrics would be to convey the variability of the strength of social

330 connections in a way that complements the ~~classic moments~~moment theory approach (Fisher, 1930)
 331 and the inequality approach (Gini, 1936) ~~approaches~~ (Appendix S2, Fig. 3).

332 **Supplementary material**

333 Appendix S1: R script to compute the new metrics from the adjacency matrix of a social network

334 Appendix S2: Colinarity between the new metrics and the 8 comparison metrics in the simulation

335 study

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Figures

Fig. 1: Illustrated glossary and representation of the domain of definition of ω and g as well as the four quadrants to distinguish between four different types of weighted triadic interactions.

Fig. 2: Simulation study results. (a) Network-level triadic edge dissimilarity scores (new method). ~~The two new metrics were sufficient by themselves to discriminate among Scenarios 1-4 which I designed specifically so that they would fall into quadrants 1-4. NB:~~ The network-level scores \mathcal{C}_ω and \mathcal{C}_g can fall outside of the domain of definition of the triad-level scores ω and g (\mathbb{W} , white shape). However, the network-level scores are always within the dotted lines. (b) Principal component analysis (PCA) of 8 other network-level metrics (see list in the main text). ~~This approach also discriminated the scenarios, but less well and with more computation required.~~

Fig. 3: Application cases: barnacle geese and rhesus macaques. (a) Network-level triadic edge dissimilarity scores (new method). (b) Principal component analysis (PCA) of 8 other metrics (see list in the main text). ~~Note that the~~The PCA coefficients and axes are different from Fig. 2b.