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# Branch-Cut-and-Price Algorithm for an Integrated Storage Location Assignment and Picker Routing Problem

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## 1 Introduction

Warehousing is one of the main components of any supply chain, and an essential lever of performance optimization in a logistic system. Order picking (OP), i.e. the action of retrieving items from storage locations according to customers orders, is the most labor-intensive operation in a warehouse, and is considered as the main area for performance improvement [1]. Manual OP is still widely used for its involved flexibility, and due to the large investment cost of automation. The Operations Research literature on this topic focuses mainly on two main operations: storage and picker routing.

In this paper, we study the integration of these two problems: we aim at finding an efficient assignment of Stock Keeping Units (SKU, i.e. a collection of identical products) to storage locations that minimize the traveled distance for the OP. In the literature, the Storage Location Assignment Problem (SLAP) has mainly been tackled by using class-based storage and statistical information on SKU. The SLAP is thus mainly solved via policy-based heuristics, like the widely-used Cube-per-order-index (COI) [1]. These approaches have several advantages: **(i)** adaptable to many situations, **(ii)** easy to implement, and **(iii)** able to solve industrial-scale instances. However, the “pick-by-order” feature, where all items of one order are picked altogether in one route, is often not considered. Furthermore, the storage and routing problems are optimized separately, despite their strong connection. Few attempts have been made at tackling the problem with full information on demand and order structure. (Boysen and Stephan 2013 [2]) is one of the only example, proving the NP-hardness of the problem. Recently (Silva et al. 2020 [3]) tackled a

similar problem with a metaheuristic approach. The SLAP presents an academic interest, to get a more thorough comprehension of the interactions between storage, layout and routing. It also has a practical interest to model the forward area, where the storage assignment changes dynamically in a very versatile context, increasing in this way the impact of operational considerations. The problem under study is called the Integrated Storage Location Assignment and Picker Routing Problem (ISLAPRP). The present work aims at addressing these research gaps with an exact solution method, based on a Dantzig-Wolfe reformulation of the problem.

In the problem studied throughout this paper, the aim is to assign a set of SKU  $\mathcal{S}$  to a set of storage locations  $\mathcal{L}$ . This assignment is represented by the binary decision variables  $\xi_{ls}$ , equal to one if SKU  $s \in \mathcal{S}$  is stored in location  $l \in \mathcal{L}$ , 0 otherwise. The demand over the planning horizon is modeled by a set of orders  $\mathcal{O}$ , each order being defined by a set of SKU to collect. Order batching is not considered in the model, but can be optimized beforehand, and batches used as input instead of single orders. The picking policy is *pick-by-order*, where one picker route corresponds to the preparation of one order. The objective is then to minimize the total walking distance to retrieve all orders. The function  $f(o, \xi)$  represents the walking distance to retrieve all SKU from order  $o \in \mathcal{O}$ , according to storage assignment plan  $\xi$ . An explicit formulation of  $f$  require the introduction of new variables and constraints, depending on the routing policy and warehouse layout. The aforementioned model is given below.

$$\min \sum_{o \in \mathcal{O}} f(o, \xi) \tag{1}$$

$$s.t. \sum_{l \in \mathcal{L}} \xi_{ls} = 1 \quad \forall s \in \mathcal{S} \tag{2}$$

$$\sum_{s \in \mathcal{S}} \xi_{ls} \leq 1 \quad \forall l \in \mathcal{L} \tag{3}$$

$$\xi_{ls} \in \{0, 1\} \quad \forall l \in \mathcal{L}, s \in \mathcal{S} \tag{4}$$

The objective (1) is to minimize the total walking distance. Constraints (2) and (3) are assignment constraints. Constraints (4) define the definition domain of variables.

## 2 Methodology

### 2.1 Extended formulation

Formulated as a mixed-integer program, the ISLAPRP presented in model (1)-(4) has a very poor linear relaxation. Let us propose a novel extended formulation for this problem, based on a Dantzig-Wolfe reformulation. This extended formulation includes an exponential number of variables, added dynamically by a column generation technique. In this

formulation, the warehouse layout and picker routing policy are still considered implicitly, with the relevant details convexified into the subproblem.

For order  $o \in \mathcal{O}$ , we note  $\mathcal{S}(o) \subset \mathcal{S}$  the set of SKU included in order  $o$ , and  $\mathcal{R}_o$  the set of all possible routes to serve this order. A route  $r$  corresponds to a set of visited storage locations, and is associated with a cost  $c_{or}$ . This cost depends on the warehouse layout and the routing policy for pickers. A set of parameters  $a_{or}^l$  is associated with each route  $r \in \mathcal{R}_o$ , which equals to 1 if location  $l \in \mathcal{L}$  is visited by the route, and 0 otherwise. The binary decision variable  $\rho_{or}$  is associated with a route  $r \in \mathcal{R}_o$ , and equals to 1 if the route is selected in the plan, and 0 otherwise. The described model is presented below:

$$\min \sum_{o \in \mathcal{O}} \sum_{r \in \mathcal{R}_o} c_{or} \rho_{or} \quad (5)$$

$$s.t. \sum_{r \in \mathcal{R}_o} \rho_{or} \geq 1 \quad \forall o \in \mathcal{O} \quad (6)$$

$$\sum_{s \in \mathcal{S}} \xi_{ls} \leq 1 \quad \forall l \in \mathcal{L} \quad (7)$$

$$\sum_{l \in \mathcal{L}} \xi_{ls} = 1 \quad \forall s \in \mathcal{S} \quad (8)$$

$$\sum_{r \in \mathcal{R}_o} a_{or}^l \rho_{or} \geq \sum_{s \in \mathcal{S}(o)} \xi_{ls} \quad \forall l \in \mathcal{L}, o \in \mathcal{O} \quad (9)$$

$$\rho_{or} \in \{0, 1\} \quad \forall o \in \mathcal{O}, r \in \mathcal{R}_o \quad (10)$$

$$\xi_{ls} \in \{0, 1\} \quad \forall l \in \mathcal{L}, s \in \mathcal{S} \quad (11)$$

The objective (5) is to minimize the total walking distance of all the routes included in the plan. Constraints (6) state that, for every order  $o \in \mathcal{O}$ , at least one route is selected. Constraints (7) and (8) are the assignment constraints. Constraints (9) link the routes to the assignment variables. For each order  $o \in \mathcal{O}$  and location  $l \in \mathcal{L}$ , if one SKU from this order is assigned to location  $l$  (right-hand side), then there must be at least one route servicing  $o$  that visits location  $l$  (left-hand side). Constraints (10) and (11) are domain definition constraints for the binary decision variables.

## 2.2 Branch-and-price framework

The extended formulation (5)-(11) is well suited to column generation, since the exponential number of variables does not permit a tractable enumeration. The relaxation of the integrality constraints (10) and (11) leads to the master problem, then restricted to a subset of columns. Additional columns are integrated dynamically at each iteration, by looking for routes of negative reduced cost. The column generation is then integrated in a Branch-and-Bound framework to restore the variables integrality.

The highly structured constraint matrix of the problem leads to a natural decomposition for route generation. Indeed, one pricing problem needs to be solved for each order,

and all problems are independent. The warehouse layout and routing policy for order pickers appear explicitly in these problems, so the formulation of the pricing changes depending on the characteristics of the problem at hand. The subproblems are modeled with an adapted resource-constrained shortest path problem, solved by dynamic programming.

### 2.3 Valid inequalities

A family of valid inequalities has been identified to strengthen the dual bound provided by the column generation. These inequalities are quite similar to the linking constraints (9), since they link the storage and routing decisions. However their number is exponential, and as such they are generated dynamically by an efficient separation procedure. Since they are non-robust with the pricing problem, a particular attention is paid to the design of an efficient management of the cut pool.

## 3 Results and Discussions

To study the performance of the proposed approach, a set of heterogeneous instances of different sizes and parameters has been generated. The solution approaches discussed in this paper are implemented on Julia, with the use of CPLEX 12.10. Preliminary experiments are run with a time limit of 20 minutes. Table 1 presents a comparison of CPLEX with an MIP formulation, the branch-and-price with dual bounds computed using Column Generation (CG), with and without the addition of valid inequalities. The first column shows the number of instances solved to optimality (out of 30), and the second the average gap between primal and dual bounds when the algorithm stops.

Table 1: First results on a set of 30 benchmark instances

	# instances solved	Average gap
CPLEX	7	0.135
Branch-and-Price	1	0.107
Branch-Cut-and-Price	26	0.002

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