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Including Customer Lifetime Value in tree-based lapse management strategy

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ABSTRACT

A retention strategy based on an enlightened lapse modelization can be a powerful profitability lever for a life insurer. Some machine learning models are excellent at predicting lapse, but from the insurer's perspective, predicting which policyholder is likely to lapse is not enough to design a retention strategy. Changing the classical classification problem to a regression one with an appropriate validation metric based on Customer Lifetime Value (CLV) has recently been proposed. In our paper, we suggest several improvements and apply them to a sizeable real-world life insurance dataset.

We include the risk of death in the study through competing risk considerations in parametric and tree-based models and show that further individualization of the existing approach leads to increased performance. We show that survival tree-based models can outperform parametric approaches and that the actuarial literature can significantly benefit from them. Then, we compare how this framework leads to increased predicted gains for the insurer regardless of the retention strategy. Finally, we discuss the benefits of our modelization in terms of commercial and strategic decision-making for a life insurer.

1. Introduction

In life insurance, « lapse risk » or « persistency risk » is the risk that the policyholder will cancel the contract at a time other than when the issuer expected when pricing the contract (KPMG (2020)). This risk is not considered an insurance risk because the payment to the policyholder "is not contingent on an uncertain future event that adversely affects the policyholder". However, lapse management is still undoubtedly a primary concern for life insurers. Lapses may substantially affect a company's solvency, its future profits and cash flows (Buchardt (2014); Buchardt et al. (2015)) or its Asset and Liabilities Management (ALM) (Kim (2005); Gatzert and Schmeiser (2008); Eling and Kochanski (2013); Eling and Kiesenbauer (2014)). The importance of measuring lapse and churn behaviours is global; it goes from yielding individual estimations of the Customer Lifetime Values (CLV) to being an estimator of a firm's profitability (Gupta and Lehmann (2006); Gupta (2009)) or strength (Ascarza et al. (2018)). Therefore for a life insurer, an enlightened lapse management strategy (LMS) is critical for successful monitoring and steering.

Part of the literature on lapse management adopts an economiccentred point of view (Dar and Dodds (1989); Kuo et al. (2003); Kagraoka (2005); Cox and Lin; Kiesenbauer (2012); Russell et al. (2013); Sirak (2015); Vasudev et al.; Nolte and Schneider (2017); Poufinas and Michaelide (2018); Yu et al. (2019)); we refer the reader to the complete bibliometric

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analysis on this topic by Shamsuddin et al. (2022) for a summarized view of all these references. This economic-centred research aims to determine lapse factors like interest rates, gross domestic product, or unemployment rates. They are driven by economic hypotheses such as the emergency fund hypothesis (lapsing is a way of constituting an emergency fund), the policy replacement hypothesis (lapsing will occur when one changes its policy) or the interest rate hypothesis (lapsing depends strongly on rate change and arbitration).

On the other hand, a large part of the literature investigates the individual determinants of lapse with policyholdercentred approaches. Micro-oriented features such as policyholder's personal information or the policy characteristics have shown to give valuable insights into lapse behaviour (Renshaw and Haberman (1986); Milhaud et al. (2011); Eling and Kiesenbauer (2014); Hwang et al. (2022)). Ćurak et al. (2015) as well as Gemmo and Götz (2016)'s works indicate that policyholders' features such as age and the number of beneficiaries are significant lapse factors, whereas Sirak (2015) dismissed those results. More recently, Loisel et al. (2021) proposed a comparison of lapse management strategies based on an innovative evaluation metric derived from the Customer Lifetime Value (CLV). Hu et al. (2021) investigated the benefits of incorporating spatial analysis in lapse modelling, and Azzone et al. (2022) showed with an approach based on random forests that microeconomic features such as the company's commercial approach for instance - is determining in the lapse decision. In contrast, macro-economical features only have a limited effect. This variety of results – sometimes contradicting each other – demonstrates the active interest in this research problem.

This paper focuses on lapse management strategy and retention targeting. As in Ascarza et al. (2018); Loisel et al. (2021), our goal is not only to model the lapse behaviour

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but rather to select a retention strategy that would create more profitable policyholders whether the strategy targets them or not. This work shows that a well-chosen strategy, based on individualized CLV and directed toward a wellchosen target, increases the insurer's expected profitability. A critical concept that motivates many CLV-driven decisions is that customers should be judged as assets based on their future profitability for the insurer. Thus, since retention often serves as the basis for CLV modelizations (Gupta et al. (2006); Donkers et al. (2007); Lemmens and Gupta (2020) - sometimes specifically designed for targeting tasks (von Mutius and Huchzermeier (2021)) - and since CLV considerations should drive retention management, it seems natural to extend the existing life insurance application linking those topics together. We make decision-making a central concern of our work and suggest proactive lapse management tools allowing the insurer to undertake actions to prevent the causes of lapse; that is opposed to a reactive management approach where decisions are taken after lapses and aim at recapturing lost policyholders.

The goal of this paper is to create an individualized CLV model that will be used to enhance classical binary churn models. We will then have a model for lapse management strategy and retention targeting that we further improve with tree-based survival analysis and competing risks considerations. The global framework is directly inspired by Loisel et al. (2021). We try in this paper to build from that existing work and bring our contributions and extensions to that topic without dismantling the building blocks it represents. We model an individual future CLV with a survival model for which the risks of death and lapse are treated as mutually exclusive competing risks. For this purpose, we introduce parametric approaches - Cox cause-specific and subdistribution models - as well as tree-based survival models - Random Survival Forest (RSF) and Gradient boosting survival analysis. We focus here on tree-based models as they can often be considered state-of-the-art models (Grinsztajn et al. (2022)). Thus we introduce tree-based machine learning algorithms for binary prediction, including Classification and Regression Tree (CART), Random forests (RF), and Extreme Gradient Boosting (XGBoost) to lapse behaviour modelling. CART and XGBoost (Milhaud et al. (2011); Loisel et al. (2021)) were used in the literature for lapse modelling but have yet to be applied to predicting life insurance lapses in a competing risk setting. To our knowledge, while Random Survival Forest has been used for churn prediction recently (Routh et al. (2021)), both RSF and Gradient boosting survival analysis have never been used for that purpose before in an actuarial context. Our contribution to the actuarial literature is twofold. First, we detail a twostep lapse management modelization: we fit parametric and tree-based competing risk individual survival models to estimate individualized future CLVs that are part of an evaluation metric for tree-based lapse management models. Second, this work includes a business-oriented discussion of the results achieved by this framework, which is missing

from existing similar approaches.

The results and discussions show that a CLV-based lapse management strategy very often outperforms a more classical binary classification approach, even with competing risks and individualized considerations. When the latter yields profitable retention gain, the former can produce higher profits, up to more than 60%. If a loss-inducing retention strategy is considered, our methodology limits the loss considerably, often setting 0 as a floor limit or even turning it into a profit-inducing retention strategy. Sensitivity analysis explores the influence of conjectural and structural parameters.

The rest of this paper is structured as follows. We briefly outline the data used in our study in Section 2. In Section 3, we then introduce the binary classification models we selected and detail our study's methodology, describing the classical and CLV-based performance measures and discussing substantial parametrization improvements over existing approaches. Then, Section 4 details our two-step methodology, with the parametric and non-parametric modelizations of individual survival predictions, in a competing risks framework and then their implementation in the treebased classification approaches considered. Section 5 is dedicated to presenting the real-life application we considered and the different results it produces. Those results are analyzed and discussed in Section 6 with commercial and strategical decision-making orientations. Eventually, Section 7 concludes this paper.

2. Data

We apply our framework to a real-world insurance portfolio. For privacy reasons, all the data, statistics, product names and perimeters presented in this paper have been either anonymized or modified. All analyses, discussions and conclusions remain unchanged.

We illustrate our methodology with a life insurance portfolio from a french insurer contracted between 1997 and 2018. Each record in the data set represents a unique policy for a unique policyholder. In the following sections, we will often refer to a unique pair of policy and policyholder by the term "subject". The dataset contains 251,325 rows with 248,737 unique policies and 235,076 unique policyholders. It means that some policies are shared between several policyholders and that one individual can detain several insurance policies. The dataset contains 43 covariates described in Table 1.

Table 1Data set description

Covariates (Numerical, Categorical, Date)		Description			
ID	CDI_ID_PERSONNE CDI_ID_CONTRAT	Policyholder (PH) unique ID Policy unique ID			
PH-level informations	CDI_DT_NAISSANCE Age_souscription Nb_Contrats CDI_CD_SEXE CDI_DESTINATAIRE_COURRIER CDI_NUM_ET_NOM_VOIE CDI_CD_POSTAL CDI_COMMUNE CDI_COMMUNE CDI_TOP_ASSURE	PH's birth date (main PH when several poliholders owns one policy) PH's age at subscription Number of different policies owned by the policyholder PH's gender (1=Female; 2=Male; other=Non precised) Anonymised PH's name Anonymised PH's adress Anonymised PH's city of residence Binary: 1 if PH is the main PH on the policy, 0 otherwise			
Policy-level informations	CDI_TYPE_PRODUIT CDI_NOM_PRODUIT CDI_PARTENAIRE CDI_DATE_DEB_CONTRAT CDI_DATE_FIN_CONTRAT START_YEAR END_YEAR SENIORITY STATE YEAR	Type of product ("Top-end product" or "Classical product") Name of life insurance product ("Product 1", "Product 2" or "Product 3") Name of the insurance distributor Policy's start date Policy's start year Policy's seniority (final seniority if the policy is ended, current seniority otherwise) Policy's state ("Active", "Lapsed", or "Death" if the policy ended following PH's death) Last year of observation			
External data	DISCOUNT RATE	Discount rate corresponding to the last year of observation			
Policy's cumulated financial flows	TOTAL_PREMIUM_AMOUNT TOTAL_EURO_PREMIUM_AMOUNT TOTAL_UC_PREMIUM_AMOUNT ARBITRATION_EURO ARBITRATION_UC FEES_EURO FEES_UC OTHER_EURO OTHER_UC PREMIUM_EURO PREMIUM_UC PROFIT SHARING_EURO PROFIT SHARING_UC CLAIM_EURO CLAIM_UC	Total face amount of the policy Face amount of the policy in euros Face amount of the policy in units of account Cumulated arbitration amount of the policy in euros Cumulated arbitration amount of the policy in units of account Cumulated fees amount of the policy in units of account Cumulated fees amount of the policy in units of account Cumulated other part of the face amount of the policy in euros Cumulated other part of the face amount of the policy in units of account Cumulated payments amount of the policy in euros Cumulated payments amount of the policy in euros Cumulated profit sharing amount of the policy in euros Cumulated profit sharing amount of the policy in units of account Cumulated profit sharing amount of the policy in units of account Cumulated profit sharing amount of the policy in units of account Cumulated profit sharing amount of the policy in euros Cumulated partial or total lapsed amount of the policy in units of account			
Covariates derived from financial flows	%TOTAL_UC_PREMIUM_AMOUNT %TOTAL_EURO_PREMIUM_AMOUNT %CLAIM_UC %CLAIM_EURO %CLAIM	Percentage of the face amount in units of account Percentage of the face amount in euros Percentage of the face amount in units of account that was lapsed Percentage of the face amount in euros that was lapsed Percentage of the total face amount that was lapsed			
Target covariate	EVENT	Policy's state (0=Active, 1=Lapsed, 2 ended following PH's death)			

The data set represents policies that are majority owned by men (57.4%) for a mean censored seniority time of 13.4 years. Three products are present in the dataset. Product one was chosen by 72% of policyholders, product 2 by 25% and product 3 by 3%.

Regarding their state, 61% of the policies are still active, 22% lapsed, and 17% ended after the PH's death. We chose here to present the distribution of the variable SENIORITY as it is the response variable in our survival models. Its modelization has a critical influence on CLV, thus, on our lapse management strategy framework. We also chose to show the distribution of the variable TOTAL PREMIUM AMOUNT representing the most recent observed face amount for every subject, as it is a known determinant of lapse behaviour. We are aware that this covariate is a rather dynamic one as its value is updated at every payment, total or partial lapse, profit sharing, arbitration or even fees movements on a policy, and only considering its most recent value ignores a large part of the insights it can provide. Without any better option, we can only use TOTAL PREMIUM AMOUNT as it is and defer any dynamic considerations for future work.

The seniorities and most recent face amount recorded before the potential end of the policy are distributed as in Figure 1:



Figure 1: Seniorities and face amounts distributions

Without further analyzing the data, we can note several things. First, we can see that the mean censored seniority of 13.4 years is not equally distributed among our subjects. Active contracts tend to be older than lapsed ones, themselves older than policies that ended with the policyholder's death. That emphasizes the importance of several contributions, and the apparent difference in seniority regarding the cause of the policy's termination encourages a competing risks approach to analyze survival. Moreover, if we suspect that lapse and death are highly dependent on individual characteristics - such as the policyholder's age, for instance - this also supports an individualized survival analysis. Eventually, we can see that the last face amount observed is significantly lower for lapsed policies. It confirms our first intuitions, and the face amount will be included in our modelization.

Among the covariates introduced in Table 1, several ones play a central part in our two-step modelization:

For the competing risks survival analysis step, *SENIOR-ITY* will be the response variable, and all other covariates including individual data and financial flows are potential explanatory variables.

The binary classification second step aims at predicting the *EVENT* outcome with minor transformations explained in Section 3 below. This prediction is based on the covariates underlined in Table 1.

3. Framework

The next section describes a modelization that follows Loisel et al. (2021)'s work. Our contributions place our work in a framework that differs from it in ways that will be made explicit, yet we chose to use a majority of their existing notations here.

Usual lapse management models based on classification aim to predict whether a policyholder will lapse. They may perform very well at that specific task, but it only reflects some aspects of this economic problem. Indeed, we can easily imagine that many policyholders may be predicted as "lapsers" but may not be profitable to the insurance company. In that case, keeping such policyholders would be absurd, and an efficient model should not predict them as targets. Targeting policyholders is an economic problem that requires an economic measure to assess. We propose to consider a measure based on the discounted expected profit of all the policies, in other words, the sum of all (CLVs). Optimizing a lapse, churn or any other prediction task with business-related measures is not new. However, to our knowledge, none of the existing approaches uses individualized future CLVs and models the profit of retention strategy by accounting for competing risks or using survival tree-based models.

CLV is a well-studied subject in marketing and business economics in general and has also been modelled in an insurance context. For a given subject i, her future (CLV) at horizon *T* can be modelled as follows:

$${}^{F}CLV_{i}\left(\boldsymbol{p}_{i},\boldsymbol{F}_{i},\boldsymbol{r}_{i},\boldsymbol{d},T\right) = \sum_{t=0}^{T} \frac{p_{i,t}F_{i,t}r_{i,t}}{\left(1+d_{t}\right)^{t}}$$

With t in years, t = 0 represents the last observation point for subject i. The quantity $p_{i,t}$ is her profitability ratio as a proportion of $F_{i,t}$, representing her face amount observed at time t. The quantity $r_{i,t}$ is the i-th subject's probability of still being active at time t and naturally, d_t is the discount rate at time t, common to every subject. It is worth pointing out that FCLV_i does not represent the global profit generated by subject i from her policy's first year until time T; it represents the future T years of profit.

We suggest a modelization of the insurer's estimated profit - or loss - resulting from a lapse management strategy. In order to do that, we will compare the expected value of the portfolio before and after applying a given strategy. We are aware that there could be infinite ways to design a retention campaign: offering a punctual incentive, recurrent services or more profit sharing, for instance. Here, we define what we will consider a lapse management strategy (LMS):

Definition 1 (Lapse management strategy (LMS)). A T-years lapse management strategy is modelled by the offer of an incentive δ_i to subject i if she is targeted. The incentive is expressed as a percentage of her face amount and is accepted with probability γ_i . Contacting the targeted policyholder has a fixed cost c. A targeted subject who accepts the incentive will be considered as an "acceptant" who will never lapse, and her probability of being active at year $t \in [0,T]$ is denoted $r_{i,t}^{acceptant}$. Conversely, a subject who refuses the incentive and prefers to lapse will be considered as a "lapser", and her probability of being active at year t is denoted $r_{i,t}^{lapser}$. A lapse management strategy is uniquely defined by the parameters $(\mathbf{p}, \delta, \gamma, c, T)$

Even if this definition is already a simplification of any reallife insurance retention strategy, various constraints and the data and tools at the insurer's disposal do not always allow to conduct such a study. In the following section, we consider a simplified version of this framework by assuming that $p_{i,t}$, $F_{i,t}$, and d_t remain constant across time, and denoted p_i , F_i and d hereafter, with F_i being the most recent face amount observed for subject *i*. Moreover, we set γ_i and δ_i to be the same for all subjects and denoted as γ and δ hereafter. Finally, the last observed state of subject *i* is denoted y_i , with $y_i = 1$ if the policy is lapsed, $y_i = 0$ otherwise. With those considerations, we can then define the control portfolio's future value as follows:

$$FCPV(\boldsymbol{p}, \delta, \gamma, c, T) = \sum_{i=1}^{n} FCLV_i\left(p_i, F_i, \boldsymbol{r}_i^{\text{acceptant}}, d, T\right) \cdot \mathbf{1}\left(y_i = 0\right) + \sum_{i=1}^{n} FCLV_i\left(p_i, F_i, \boldsymbol{r}_i^{\text{lapser}}, d, T\right) \cdot \mathbf{1}\left(y_i = 1\right)$$

It represents the hypothetical value of the portfolio, considering that:

- every subject that did not lapse up to her last observation point - $y_i = 0$ at t = 0 - has a vector of retention probabilities of $\mathbf{r}_i^{\text{acceptant}}$;
- every subject that has been observed to lapse $y_i = 1$ at t = 0 - has a vector of retention probabilities of r_i^{lapser}

Remark 1. It is important to note that this does not reflect the actual future value of the portfolio - as the future CLV of lapsers should be 0 - but rather its hypothetical expected future value given the nature (lapser or not) of every subject but not their actual states (actually lapsed or not). It represents this hypothetical future CLV of all subjects if no customer relationship management about lapses is carried out.

A classification algorithm would take the lapse indicator y_i as a target variable and yield predictions \hat{y}_i . Given a lapse management strategy and such a classification algorithm, we define the lapse managed portfolio future value by:

$${}^{F}LMPV(p,\delta,\gamma,c,T) = \\ \sum_{i=1}^{n} {}^{F}CLV_{i}\left(p,F_{i},r_{i}^{\text{acceptant}},d,T\right) \cdot \mathbf{1}\left(y_{i}=0,\hat{y}_{i}=0\right) \\ + \sum_{i=1}^{n} {}^{F}CLV_{i}\left(p,F_{i},r_{i}^{\text{lapser}},d,T\right) \cdot \mathbf{1}\left(y_{i}=1,\hat{y}_{i}=0\right) \\ + \sum_{i=1}^{n} {}^{F}CLV_{i}\left(p-\delta,F_{i},r_{i}^{\text{acceptant}},d,T\right) \cdot \mathbf{1}\left(y_{i}=0,\hat{y}_{i}=1\right) \\ + \gamma \cdot \sum_{i=1}^{n} {}^{F}CLV_{i}\left(p-\delta,F_{i},r_{i}^{\text{acceptant}},d,T\right) \cdot \mathbf{1}\left(y_{i}=1,\hat{y}_{i}=1\right) \\ + (1-\gamma) \cdot \sum_{i=1}^{n} {}^{F}CLV_{i}\left(p,F_{i},r_{i}^{\text{lapser}},d,T\right) \cdot \mathbf{1}\left(y_{i}=1,\hat{y}_{i}=1\right) \\ - \sum_{i=1}^{n} {}^{c} \cdot \mathbf{1}\left(\hat{y}_{i}=1\right)$$

Clearly, the sums appearing in the formulas above could be grouped to make them more concise. We chose not to do so for the sake of visualization: we can distinctly see each possible case in each summand.

Then, we define the economic metric of the algorithm as the retention gain, the future profit generated by the retention management strategy over T years:

$$RG(\boldsymbol{p}, \delta, \gamma, c, T) = {}^{F} LMPV(\boldsymbol{p}, \delta, \gamma, c, T) - {}^{F} CPV(\boldsymbol{p}, \delta, \gamma, c, T)$$

that can be simplified as follows:

$$RG(\boldsymbol{p}, \boldsymbol{\delta}, \boldsymbol{\gamma}, \boldsymbol{c}, T) = \sum_{i=1}^{n} \left[\boldsymbol{\gamma} \left[{}^{F}CLV_{i} \left(\boldsymbol{p} - \boldsymbol{\delta}, F_{i}, \boldsymbol{r}_{i}^{\text{acceptant}}, \boldsymbol{d}, T \right) \right. \\ \left. - {}^{F}CLV_{i} \left(\boldsymbol{p}, F_{i}, \boldsymbol{r}_{i}^{\text{lapser}}, \boldsymbol{d}, T \right) \right] \cdot \mathbf{1} \left(\boldsymbol{y}_{i} = 1, \hat{\boldsymbol{y}}_{i} = 1 \right) \\ \left. - {}^{F}CLV_{i} \left(\boldsymbol{\delta}, F_{i}, \boldsymbol{r}_{\text{acceptant}}, \boldsymbol{d}, T \right) \cdot \mathbf{1} \left(\boldsymbol{y}_{i} = 0, \hat{\boldsymbol{y}}_{i} = 1 \right) \right] \\ \left. - \sum_{i=1}^{n} \boldsymbol{c} \cdot \mathbf{1} \left(\hat{\boldsymbol{y}}_{i} = 1 \right) \right]$$

This evaluation metric can now be derived into an individual retention gain measure. More specifically, we define z_i as:

$$z_{i} = \begin{cases} -{}^{F}CLV_{i}\left(\delta, F_{i}, \boldsymbol{r}_{i}^{\text{acceptant}}, d, T\right) - c & \text{if } y_{i} = 0\\ \gamma \cdot \left[{}^{F}CLV_{i}\left(\boldsymbol{p} - \delta, F_{i}, \boldsymbol{r}_{i}^{\text{acceptant}}, d, T\right) & \text{if } y_{i} = 1\\ -{}^{F}CLV_{i}\left(\boldsymbol{p}, F_{i}, \boldsymbol{r}_{i}^{\text{lapser}}, d, T\right)\right] - c & \end{cases}$$

That last equation can seem obscure. It only assigns to each individual the expected profit or loss that would result from targeting her with a given lapse management strategy. A positive amount for subject *i* means that targeting her would generate profit, whereas a negative one would lead to a loss for the insurer. We can take the example of a hypothetical scenario where $p_i = 3\%$, $\delta = 0.05\%$, $\gamma = 10\%$ and c = 10 euros. It would generate z_i s taking values from -234, $614 \notin$ to 53, $066 \notin$ with a mean of $-218 \notin$ and a median of $-55 \notin$. Different scenarios would result in very different distributions for the z_i 's.

Eventually, we define \tilde{y}_i as a binary target variable indicating for policyholder *i* if the individual expected retention gain resulting from a given retention strategy is a profit or a loss. More specifically, we define \tilde{y}_i as:

$$\tilde{y}_i = \left\{ \begin{array}{ll} 1 \text{ if } z_i > 0 \\ 0 \text{ if } z_i \leq 0 \end{array} \right.,$$

Remark 2. A subject in the dataset for which $y_i = 0$ would produce $\tilde{y}_i = 0$, whereas one for which $y_i = 1$ could produce $\tilde{y}_i = 0$ or $\tilde{y}_i = 1$. In other words, it is never profitable for the insurer to offer an incentive to a subject that would not have lapsed. On the other hand, offering that same incentive to a lapser can be profitable. However, depending on the subject's features and the lapse management strategy parameters, it can also lead to a loss.

We can now include \tilde{y}_i as a new binary target variable in our models and directly consider RG as the global evaluation metric in the tree-based models we consider.

We can now compare two models: the classical one with y_i as a target variable and accuracy as the evaluation metric; and the CLV-augmented one with \tilde{y}_i as a target variable and *RG* as the evaluation metric.

Intuitively, the former tries to predict whether a policyholder will lapse and tune its parameters by minimizing the misclassification rate. On the other hand, the latter aims at predicting whether applying a given retention strategy to the *i*-th individual will be profitable for the insurer and tune its parameters by maximizing the global expected retention gain.

4. Methodology

In Section 3, we described a business-oriented framework, augmenting lapse management strategy with an evaluation metric based on the future CLV of every subject. Evaluating this metric requires computing $r^{\text{acceptant}}$ and r^{lapser} , the matrices of size (n, T + 1) containing for every subject, survival probabilities that we detail below.

Given this framework, we propose a two-step methodology: firstly, we detail how this survival analysis is carried out to model those retention parameters, and secondly, we explain how we use them for training tree-based classification models.

4.1. Step 1: Modelling racceptant and rlapser

We recall that a given subject's policy can end with lapse or death, and the policy is considered active if competing events are yet to occur. Furthermore, while a lapser's policy can end with lapse or death, whatever comes first, an acceptant one can only end with death.

 r^{lapser} represents the probability that the policy of subject *i* is still active at time *t*, given that the subject is labeled as a lapser - EVENT = 1 - at *t* = 0. Predicting these overall conditional survival probabilities with competing risks can be achieved by creating a combined outcome: the policy ends with death or lapse, whichever comes first. In order to compute r^{lapser} in practice, we recode the competing events as a combined event. This approach is compatible with any survival analysis method regarding statistical guarantees.

Conversely, $r^{\text{acceptant}}$ represents the probability that the policy of subject *i* is still active at time *t*, given that the subject is not labelled as a lapser - EVENT = 0 or 2 - at *t* = 0. This modelization is more complex as we must dissociate the risks of lapse and death. These causes being mutually exclusive, a competing risks methodology is well-suited to estimate $r^{\text{acceptant}}$ (Laurent et al. (2016)).

4.1.1. Competing risks frameworks

We are aware that the context of our modelization requires competing risk setting. As detailed in Appendix A.5, several regression models exist to estimate the global hazard and the hazard of one risk in such settings: cause-specific and subdistribution modelizations. They account for competing risks differently, obtaining different hazard functions and thus have distinct advantages, drawbacks and interpretations. TheseThese differences are discussed in Milhaud and Dutang (2018), where the authors also considered a competing risk framework for lapse prediction.

After discussions detailed in Appendix A.5, the simplicity of

a cause-specific approach and the fact that it can be adapted to any survival method, including tree-based ones, oriented our choice towards it. We then computed $r^{\text{acceptant}}$ and r^{lapser} with three different methods - Cox model, Random Survival Forest and Gradient Boosting Survival Model - and retained the best one. These methods are shortly described in the following sections.

4.1.2. Cox proportional hazard model

One of the most common survival models is the Cox proportional hazard (CPH) model (Cox). It postulates that the hazard function can be modelized as the product of a time-dependent and a covariate-dependent functions. The hazard function at time t for subject i with covariate vector X_i , under Cox proportionnal hazard model can be expressed as:

$$\underbrace{\lambda(t|X_i^1, X_i^2, \dots)}_{\text{hazard function}} = \lambda(t|\mathbf{X}_i) = \underbrace{\lambda_0(t)}_{\text{baseline hazard}} \underbrace{e}_{\text{partial hazard}} \underbrace{e}_{\text{partial hazard}} \underbrace{(\mathbf{X}_i \cdot \boldsymbol{\beta}_i)}_{\text{partial hazard}}$$

It is crucial to note that in this model, the hazard function is the product of the baseline hazard, which only varies with time, and the partial hazard, which only varies depending on the covariates. The parameters of this model are the β , and they can easily be estimated with a maximum likelihood approach. Their estimation can be carried out without having to model $\lambda_0(t)$ - which is why CPH is considered semiparametric.

We use Python and lifelines (Davidson-Pilon (2019)) to implement it. We specify a spline estimation for the baseline hazard function. We select the covariates and model parameters using AIC (Akaike (1973)) and use the concordance index (Harrell et al. (1982)) to compare CPH to other models. The concordance index - or Harrel's c-index or simply c-index - is a metric to evaluate the predictions made by a survival model. It can be interpreted as a generalization of the area under a receiver operating characteristic (ROC) curve (Hanley and McNeil (1982)) - or AUC - in a survival setting with censored data.

4.1.3. Random Survival Forest

Survival trees have been extensively studied for a long time, and a complete review of such existing methods up to 2011 can be found in Bou-Hamad et al. (2011). The most important thing to understand is that a survival tree can be created by modifying the splitting criterion of a regular tree. Most survival tree algorithms are designed with a split function that aims to maximize the separation of the resulting child nodes in terms of survival profiles. This separation between nodes is estimated by maximizing the log-rank statistic (Mantel (1966); LeBlanc and Crowley (1993)). Each terminal node of a survival tree contains a survival profile from which we can derive the survival and cumulative hazard function. An RSF is the counterpart of a random forest (see Appendix A.6.2) for such survival trees. It has been developed in Ishwaran et al. (2008) and extended for competing risks a few years after (Ishwaran et al.). A prediction with RSF for a given subject is made by getting his/her survival profile in each tree in the forest. His/her corresponding survival and cumulative hazard function are estimated in each tree with Kaplan-Meier and Nelson-Aalen estimators, respectively. Eventually, the aggregation of those single-tree estimates constitutes the RSF's prediction.

We use Python and sksurv (Pölsterl (2020)) to implement RSF, and we tune and evaluate our model using the concordance index.

Remark 3. Sksurv allows us to use RSF with a causespecific consideration of the competing risks. To this day, sksurv does not have a subdistribution competing risks model, whereas its R implementation *randomForestSRC* does (Ishwaran and Kogalur (2007)).

Moreover, a severe limitation of that approach is that predictions can only be made at time points observed in the training set. Concretely, this prevents us from using RSF to extrapolate survival and hazard functions to unobserved time points.

4.1.4. Gradient Boosting Survival Model

In the same way Random Forest has a survival counterpart, this is also true for Gradient Boosting approaches. An essential distinction between classical boosting algorithms(see Appendix A.6.3) and Gradient Boosting Survival Model (GBSM) lies in its loss function. The loss function that we use with GBSM is the partial likelihood loss of a CPH model, and the optimization in such a model is achieved by maximizing a slightly modified log-partial likelihood function:

$$\arg\min_{f} \quad \sum_{i=1}^{n} \delta_{i} \left[f(\boldsymbol{X}_{i}) - \log \left(\sum_{j \in \geq_{i}} e^{(f(\boldsymbol{X}_{j}))} \right) \right]$$

Where δ_i is the event indicator and $f(X_i)$ is GBSM's prediction for the *i*-th subject, with a covariate vector X_i . g_i is the tree leaf including subject *i*.

Similarly to RSF, we use Python and sksurv (Pölsterl (2020)) to implement GBSM. We tune and evaluate our model using the concordance index. Remark 3 also applies here.

4.1.5. Final modelization choice

Our analysis shows that, based on concordance index, RSF and GSBM both outperformed a semiparametric Cox model in our study case. Regarding interpretability, we note that the feature importance analysis is very similar between the three models. All the details about the final concordance index scores, covariates importance and various plot for further analysis are available in Appendix A.6. In the following sections, we decide to retain **GBSM** for the modelization of $r^{\text{acceptant}}$ and r^{lapser} as it has the best concordance index.

Remark 4. As this study aims to be business-oriented and favour real-life decision-making, it is crucial to note that the computation times for fitting these different models are very different and could potentially be a huge constraint for real operational deployment. Specific computation times differ greatly depending on various factors, such as the number of subjects or features considered, the computation power or parallelization ability at disposal, for instance. However, we can still give here an order of magnitude for those differences. If the tuning and fitting process for CPH can last a few tens of seconds, it lasts hours for RSF and tens of hours for GBSM.

4.2. Step 2: Classification tasks

Our work focuses on lapse management with tree-based models. It aims to answer the question: which policyholders would be worth targeting with a lapse management strategy to maximize the expected T-year profit for the insurer? We will consider a single tree built with Breiman's CART algorithm, Random Forest, XGBoost, and RE-EM trees. The following sections detail how those different approaches work. Those models will be compared on two different classification tasks; and tuned with two different evaluation metrics, a statistical metric and a business-related one.

On y_i First, we will use a classical lapse prediction framework to model the policyholder's behaviour. Each policyholder will be labelled as a lapser or a non-lapser with a binary outcome y_i . Our first batch of models will be trained with y_i as a response variable and produce predictions \hat{y}_i . Accuracy (y, \hat{y}) , which is undoubtedly the most intuitive performance measure for binary classification, is defined as the proportion of correctly predicted observations over all observations. It is widely used for churn analysis and appears to be a satisfying performance measure for relatively balanced outcomes in binary classification problems. We will use it as an evaluation metric in a 10-fold crossvalidation step for tuning our models.

We know that more complex statistical evaluation metrics can be used for binary classification, such as the F_{β} Score family, the AUC under the ROC, the Brier Score, or the lift curve. In this paper, and similarly to a vast part of the applied binary classification literature, we will only select, evaluate and discuss the models in the light of the metrics we introduced.

On \tilde{y}_i Secondly, we will use the CLV-Augmented lapse prediction framework, detailed in Section 3. Each policy-holder will be labelled as a targeted lapser or a non-targeted policyholder with the binary outcome \tilde{y}_i and prediction for that outcome are denoted \hat{y}_i .

Remark 5. Note that whenever $y_i = 0$, we also have $\tilde{y}_i = 0$. In other words, if subject *i* does not intend to lapse, it is never worth proposing her an incentive: the subject will accept it with probability 1 and would not have lapsed.

On the other hand, when $y_i = 1$, it corresponds to either $\tilde{y}_i = 1$ or $\tilde{y}_i = 0$. In other words, if subject *i* is labelled as a lapser, it does not necessarily mean it is worth targeting her. From the insurer's point of view, some policies are better off lapsed. \tilde{y} can be seen as a more detailed version of y_i as it carries not only behavioural information regarding lapse but also a profitability one.

We thus train a second batch of models with \tilde{y}_i as a response variable. We use *RG* as an evaluation metric in a 10-fold cross-validation step for tuning these models.

Summary of our methodology: First, we train a CART, RF and XGBoost models with y_i as a binary target variable and accuracy as a tuning evaluation metric.

Then we train them with \tilde{y}_i as a binary target variable and *RG* as a tuning evaluation metric.

Finally, we train and test all six models on different random samples of our dataset and keep track of the model's classification performance on all of them and for various retention strategies for comparison's sake.

The sections below briefly introduce the tree-based model we selected before displaying how they performed in various lapse management scenarios.

4.2.1. CART

CART (*Classification And Regression Trees*) is an algorithm developed by Breiman et al. (1984) that consists of recursively partitioning the covariate space. It is a widespread, intuitive and flexible algorithm that handles regression and classification problems.

4.2.2. Random forest

A natural idea to correct CART's instability and enhance its prediction accuracy is the aggregation of a significant number of single trees, each grown on different subsamples of the dataset. A random forest (RF by Breiman (2001)) is a tree-based bagging procedure where each tree is grown on randomly drawn observations and contains splits considering only randomly drawn covariates.

4.2.3. XGBoost

Other tree-based approaches have been designed to reduce the instability of a single tree model. Model boosting is an adaptative technique, first developed by Freund et Shapire (Freund and Schapire (1996)), that does not rely on the aggregation of independent weaker models but rather on the aggregation of weak models built sequentially, one after the other. XGBoost (Chen and Guestrin (2016)) is a widespread and performant tree-boosting model that relies on a gradient boosting step and provides a very optimized parallelized procedure. It is considered a state-of-the-art library for various prediction problems.

The interested reader can find more detailed explanations about CART, RF and XGBoost mechanisms in Appendix A.1. For these modelizations, we used Python and sklearn (Pedregosa et al. (2011)).

5. Real-life application

Based on the real life-insurance dataset at our disposal (described in Section 2), we use the survival model we selected and estimate $r^{\text{acceptant}}$ and r^{lapser} for every individual. This allows us to compute the individual CLVs, RGs, z_i 's and \tilde{y}_i . We have already defined what a strategy is (see Definition 1), and we can thus apply our classification methodology to various retention strategies.

5.1. Considered lapse management strategies

The strategies considered are based on several criteria. First, we selected realistic strategy parameters and time horizons based on actual retention campaigns led by life insurers. Moreover, we chose to present strategies that illustrate the exhaustive list of conclusions and discussions that are carried out in the next section. Finally, we also incorporated strategies that are "obviously bad" in the sense that such strategies would necessarily lead to a loss for the insurer. Such extreme scenarios will supplement our discussions.

Results related to the 32 considered LMS are given in Section 5.2. Related comments on how to read these tables are given in Section 5.3.

Scenarios	р	δ	γ	c	d	Т
1	2.50%	0.04%	25%	10	1.50%	5
2	2.50%	0.04%	25%	100	1.50%	5
3	2.50%	0.04%	25%	100	1.50%	20
4	2.50%	0.04%	5%	10	1.50%	5
5	2.50%	0.04%	5%	100	1.50%	20
6	2.50%	0.10%	25%	10	1.50%	20
7	2.50%	0.10%	25%	100	1.50%	20
8	2.50%	0.10%	5%	100	1.50%	20
9	1.50%	0.04%	25%	10	1.50%	20
10	1.50%	0.04%	25%	100	1.50%	5
11	1.50%	0.04%	5%	10	1.50%	20
12	1.50%	0.04%	5%	100	1.50%	5
13	1.50%	0.20%	20%	10	1.50%	5
14	1.50%	0.10%	25%	100	1.50%	5
15	1.50%	0.10%	5%	10	1.50%	20
16	1.50%	0.10%	5%	100	1.50%	5
17	1.50%	0.10%	5%	100	1.50%	20
18	2.50%	0.08%	20%	10	1.50%	20
19	2.50%	0.08%	20%	100	1.50%	5
20	2.50%	0.08%	20%	100	1.50%	20
21	2.50%	0.08%	10%	10	1.50%	20
22	2.50%	0.08%	10%	100	1.50%	20
23	2.50%	0.20%	20%	100	1.50%	5
24	2.50%	0.20%	20%	100	1.50%	20
25	2.50%	0.20%	10%	10	1.50%	20
26	2.50%	0.20%	10%	100	1.50%	20
27	1.50%	0.08%	20%	10	1.50%	5
28	1.50%	0.08%	20%	100	1.50%	5
29	1.50%	0.08%	10%	100	1.50%	5
30	1.50%	0.20%	20%	100	1.50%	5
31	1.50%	0.20%	10%	10	1.50%	20
32	1.50%	0.20%	10%	100	1.50%	20

Table 2: All considered LMS

5.2. Numerical results

NTO	No dime (a) Madal		1.1 07 4	Accuracy		Retention gain		RG/target		T	
IN °	N° time (s)	Model	% target diff	y_i	\tilde{y}_i	V_i	\tilde{v}_i	V_i	\tilde{v}_i	Improvement	
		GLDT		00.00	0.5.000		210 (55	• 1	20.20	01.69	
		CART		92.3%	85.3%	114 661	219 655	4.48	38.20	91.6%	
1	4949	RF	62.6%	92.9%	85.4%	232 314	287 884	9.82	56.65	23.90%	
		XGB		93.4%	85.8%	243 365	324 952	9.61	54.64	33.50%	
		CART		92.3%	83.3%	- 2 187 622	- 8 224	- 85.52	- 31.09	99.60%	
2	4603	RF	93.5%	92.9%	83.4%	- 1 900 265	45 483	- 80.18	194.35	102.40%	
		XGB		93.4%	83.5%	- 2 032 650	77 481	- 80.39	174.44	103.80%	
		CART		92.3%	86.5%	4 789 814	5 117 844	187.00	577.74	6.80%	
2	5555	RF	55 101	92.9%	86.4%	4 463 796	4 255 175	188.47	566.05	-4.70%	
5	5555	XGB	55.470	93.4%	86.8%	5 032 706	5 433 366	198.92	610.26	8.00%	
		CART		92.3%	83.6%	- 514 477	- 112 372	- 20.08	- 86.48	78.20%	
4	4753	RF	86 7%	92.9%	83.4%	- 323 544	- 3 937	- 13.65	- 28.28	98.80%	
7	7755	XGB	00.770	93.4%	83.3%	- 383 004	-	- 15.14	-	100.00%	
		CART		92.3%	84.0%	- 1 966 473	- 46 323	- 76.83	- 22.31	97.60%	
5	5164	RF	87 80%	92.9%	84.0%	- 1 477 229	253 885	- 62.32	149.67	117.20%	
5	5104	XGB	02.070	93.4%	84.1%	- 1 621 796	273 243	- 64.14	117.83	116.80%	
		CART		92.3%	89.7%	4 614 513	4 483 831	180.36	266.33	-2.80%	
6	6075	RF	20.10	92.9%	89.9%	4 973 929	4 328 724	210.01	280.90	-13.00%	
0	0075	XGB	29.1%	93.4%	90.7%	5 354 770	5 368 917	211.69	301.57	0.30%	
		CART		92.3%	86.2%	2 312 231	3 310 314	90.36	412.71	43.20%	
7	5524	RF	57.001	92.9%	86.1%	2 841 351	3 129 652	120.01	465 74	10.10%	
/	5554	XGB	57.9%	93.4%	86.6%	3 078 755	3 825 920	121.69	475.53	24.30%	
		CADT		02.20	02.00/	2 778 022	724 772	147.40	110 50	20.60%	
				92.5%	03.0%	- 5 7 78 955	- /34 //3	- 14/.49	- 418.38	80.00%	
8	5096	KF	84.5%	92.9%	83.5%	- 2 513 261	8 914	- 106.03	20.13	100.40%	
		XGB		93.4%	83.6%	- 2 920 405	34 492	- 115.47	45.75	101.20%	
		CART		92.3%	91.3%	52 090 240	47 706 070	2 034.15	2 170.64	-8.40%	
9	6452	RF	12.1%	92.9%	91.9%	46 171 160	42 049 900	1 949.05	2 082.36	-8.90%	
		XGB		93.4%	92.5%	51 629 950	52 606 740	2 040.95	2 339.70	1.90%	
		CART		92.3%	85.2%	2 798 173	3 182 143	109.11	481.60	13.70%	
10	/013	RF	64.0%	92.9%	85.2%	2 502 903	2 743 070	105.69	554.76	9.60%	
10	4715	XGB	04.970	93.4%	85.6%	2 920 720	3 438 303	115.40	576.64	17.70%	
		CART		92.3%	90.2%	9 335 438	8 527 444	364.60	454.78	-8.70%	
11	6100	RF	23.0%	92.9%	90.6%	8 570 307	7 931 029	361.80	460.42	-7.50%	
11	0177	XGB	23.770	93.4%	91.2%	9 518 466	9 581 934	376.27	501.56	0.70%	
		CART		92.3%	83.6%	- 1 819 600	135 305	- 71.15	121.80	107.40%	
12	4601	RF	80.5%	92.9%	83.5%	- 1 575 489	159 620	- 66.48	215.65	110.10%	
12	4001	XGB	07.570	93.4%	83.7%	- 1 668 346	228 226	- 65.99	208.69	113.70%	
		CART		92.3%	89.2%	4 160 423	3 882 623	162.44	241.06	-6.70%	
13	5370	RF	31.0%	92.9%	89.5%	4 018 432	3 666 219	169.65	249.54	-8.80%	
15	5517	XGB	51.070	93.4%	90.0%	4 455 108	4 410 629	176.09	267.87	-1.00%	
		CART		92.3%	85.1%	1 858 140	2 575 538	72.44	442.86	38.60%	
14	1007	RF	66 701	92.9%	85.0%	1 885 853	2 387 018	79.65	531.25	26.60%	
14	400/	XGB	00.7%	93.4%	85.4%	2 179 093	2 879 880	86.09	544.35	32.20%	
		CART		92.3%	90.3%	7 522 978	7 058 487	293 94	382.06	-6 20%	
17	(170	RF	04 404	92.9%	90.6%	7 534 275	7 068 293	318.08	411.80	-6 20%	
15	01/9	XGR	24.4%	93.4%	91.0%	8 219 857	8 265 167	324 94	442.88	0.20%	
				00.20	02 601	0 217 037	120.002	07.05	101.44	0.0070	
			_	92.3% 02.0%	03.0% 02.50	- 2 JUD /49	- 139 983	- 97.93	- 121.44	94.40% 102 700	
16	4627	КГ	90.2%	92.9%	83.3%	- 1 909 304	75 101	- 85.10	111.49	103.70%	
	XGB		93.4%	83.6%	- 2 160 719	/6 641	- 85.45	93.28	103.50%		

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N TO					Accuracy		Retention gain		/target	Tanana
N°	time (s)	Model	% target diff	y_i	\tilde{y}_i	y_i	\tilde{y}_i	y_i	\tilde{y}_i	Improvement
		CART		02.3%	86.8%	5 220 695	5 811 833	203.94	583 55	11.3%
		DE	51.00	02.0%	86.0%	5 401 696	5 269 505	205.74	605.60	2 40%
17	5679	YCB	51.3%	03.1%	87 1%	5 9/3 8/1	6 682 230	220.00	670.03	12 40%
		AUD		95.470	07.470	5 945 041	0 082 230	234.94	070.05	12.4070
		CART		92.3%	89.7%	3 862 156	3 397 247	150.95	203.11	-12.00%
18	6074	RF	29.7%	92.9%	89.9%	4 127 224	3 550 730	174.26	230.67	-14.00%
		XGB		93.4%	90.6%	4 451 686	4 408 819	175.99	250.17	-1.00%
		CART		92.3%	83.2%	- 2 929 448	- 85 465	- 114.46	- 1 482.06	97.10%
10	1528	RF	96.6%	92.9%	83.3%	- 2 413 433	3 724	- 101.84	- 108.33	100.20%
19	4520	XGB	90.070	93.4%	83.3%	- 2 642 119	9 092	- 104.47	93.79	100.30%
		CART		02.3%	85.0%	1 559 874	2 471 262	60.95	329.63	58 40%
20	E 47C	RF	60.00	92.570	85.8%	1 994 645	2 517 111	84.26	422.45	26 20%
20	5476	XGB	60.9%	93.4%	86.3%	2 175 670	3 089 897	85.99	422.43	42 00%
)).+/(00.570	2 175 070	5 005 057	05.77	422.77	42.00%
		CART		92.3%	88.8%	705 721	922 490	27.69	60.21	30.70%
21	5906	RF	36.6%	92.9%	88.9%	1 352 182	1 269 349	57.11	97.63	-6.10%
		XGB		93.4%	89.6%	1 342 882	1 428 722	53.09	96.76	6.40%
		CART		92.3%	84.6%	- 1 596 562	169 852	- 62.31	41.78	110.60%
22	5278	RF	73.2%	92.9%	84.6%	- 780 396	637 625	- 32.89	194.52	181.70%
		XGB		93.4%	85.0%	- 933 133	780 845	- 36.91	188.79	183.70%
		CART		92.3%	83.3%	- 4 683 072	- 48 985	- 182.86	- 1 186.22	99.00%
23	1220	RF	00.2%	92.9%	83.3%	- 3 536 046	_	- 149.21	-	100.00%
23	4229	XGB	99.270	93.4%	83.3%	- 4 000 747	-	- 158.17	-	100.00%
		CADT		02.20/	85 00/	2 062 722	200 200	110.25	80.44	97 200/
				92.5%	0J.0%	- 5 002 732	- 300 209	- 119.55	- 00.44	07.30% 174.20%
24	5391	КГ VCP	66.8%	92.9%	04.170 95 201	- 937 039	710 000 924 109	- 40.29	163.99	174.30%
		AUD		95.4%	83.3%	- 1 404 301	034 190	- 55.52	105.88	139.40%
		CART		92.3%	87.8%	- 3 251 762	- 1 761 821	- 126.63	- 143.20	45.80%
25	5851	RF	43.0%	92.9%	86.4%	- 1 013 089	79 273	- 42.69	11.90	107.80%
		XGB		93.4%	83.3%	- 1 582 006	4 396	- 62.52	287.68	100.30%
		CART		92.3%	84.2%	- 5 554 044	- 1 491 522	- 216.63	- 549.23	73.10%
26	5182	RF	78.0%	92.9%	83.6%	- 3 145 668	52 475	- 132.69	84.54	101.70%
20	5102	XGB	70.070	93.4%	83.3%	- 3 858 022	-	- 152.52	-	100.00%
		CART		92.3%	88.9%	3 361 471	3 037 200	131 25	101 31	-9.60%
27	5204	RF	22.7%	92.570	89.3%	3 241 680	2 911 023	136.86	204.43	-10 20%
27	5324	XGB	32.1%	93.4%	89.6%	3 596 593	3 546 671	142.15	222.04	-1 40%
		GADT		00.00	04.7%	1 070 100	1 010 (01	41.05	222.01	71.00%
		CART		92.3%	84.7%	1 059 189	1 813 631	41.25	392.14	71.20%
28	4853	RF	70.3%	92.9%	84.8%	1 109 101	1 808 616	46.86	474.33	63.10%
		XGB		93.4%	85.0%	1 320 578	2 141 271	52.15	482.34	62.10%
		CART		92.3%	84.1%	- 1 165 404	458 223	- 45.60	172.83	139.30%
29	4746	RF	81.4%	92.9%	84.0%	- 855 770	525 335	- 36.09	288.55	161.40%
		XGB		93.4%	84.1%	- 890 871	645 445	- 35.26	310.86	172.50%
		CART		92.3%	84.3%	- 694 436	751 404	- 27.16	226.99	208.20%
30	/811	RF	73 0%	92.9%	84.4%	- 13 512	1 018 369	- 0.51	356.48	7637.00%
50	4011	XGB	13.970	93.4%	84.7%	- 38 050	1 253 252	- 1.55	345.94	3393.70%
		СЛРТ		07 20%	00 7%	14 747 500	13 838 380	576.22	600 22	6 200
		DE	• • • • • • •	72.3% 07.0%	90.7% 01.1%	14 /4/ 300	12 020 200	570.25 625 54	742.24	-0.20%
31	6243	КГ ХСР	20.5%	92.9% 03 101	91.1% 01 50/	14 010 030	15 578 400	629 20	143.34 Q11 Q0	-9.70%
		AUD		73.4%	71.3%	10 140 490	10 109 440	030.30	014.00	0.10%
		CART		92.3%	88.0%	12 445 210	11 693 070	486.23	884.49	-6.00%
32	5865	RF	41.4%	92.9%	88.3%	12 684 250	11 381 260	535.54	971.28	-10.30%
		XGB		93.4%	88.8%	13 870 470	14 101 470	548.30	1 048.38	1.70%

5.3. Comments

Several terms in the two previous tables need to be explained. "% target diff" represents how different y and \tilde{y} are. It is the percentage of subjects for which $y_i = 1$ and $\tilde{y}_i = 0$: in other words, the proportion of lapsers not worth targeting with a given strategy. Then the table shows the 10-fold cross-validated mean accuracies, retention gains and RG/target with two methodologies: the columns denoted y_i represent the metrics obtained by a model with y_i as a response variable and accuracy as an evaluation metric, and the columns denoted \tilde{y}_i represent the metric obtained by a model with \tilde{y}_i as a response variable and RG as an evaluation metric. RG/target represents the achieved retention gain for every targeted individual, for y_i , it is $RG/\sum_i \hat{y}_i$, for \tilde{y}_i it is $RG/\sum_i \hat{y}_i$. Eventually, "Improvement" represents the percentage of improvement between the RG obtained with a classification on y_i and the gain obtained with a classification on \tilde{y}_i .¹

Some LMS are worth focusing on. For every strategy, we can display its results when trained on random samples of the dataset: the boxplots below summarize some typical key results illustrated by several strategies. Those results will be discussed in the Section 6.



Figure 2: Strategy n°1: (Positive result on y_i and an improved result on \tilde{y}_i .)



Figure 3: Strategy n°4: (Very negative result on y_i and a losslimiting result on \tilde{y}_i .)



Figure 4: Strategy n°30: (Negative result on y_i and positive one on \tilde{y}_i)

¹In order to account for negative retention gains, the improvement is computed with an absolute value for the denominator. This leads to a rather unintuitive improvement measure whenever one of the models yields negative RG and the other positive RG.



Figure 5: Strategy n°21: (High positive result on y_i slightly improved with \tilde{y}_i .)



Figure 6: Strategy n°13: (Results on y_i better than results on \tilde{y}_i .)

Remark 6. With considerable computation power and great parallelization, the results for all strategies - see other strategies in Appendix A.7 - were obtained with a wall time of less than 4 days and a CPU time of more than 100 days.

6. Discussion

6.1. General statements

RF and XGBoost perform globally better than CART in terms of mean accuracy and RG. It is true for all 32 LMS considered in Table 1. Globally, XGBoost is more consistent and is the best model in most scenarios, both with and without the CLV-based measure. It is only outperformed by RF in strategies n°25 and 26.

The vast majority of strategies, including all the realistic ones, show that a classification on \tilde{y}_i produces a targeting that yields better RG than a classification on y_i . Conversely, a classification on y_i produces a targeting that yields better accuracies regarding whether a policyholder will churn than a classification on \tilde{y}_i . These results were expected because of the models' respective objectives. Even if it is not surprising, it once again shows that for an insurer, lapse prediction and lapse management strategy are two very different prediction problems, often treated as similar ones.

Our CLV-augmented modelization shows different behaviour depending on the strategy considered. As highlighted by Figure 2, a model on y_i is greatly improved by our framework regarding RG and RG/target. Conversely, its accuracy in lapse prediction is not optimal.

An attractive property of our framework can be observed in Figure 3: it yields loss-limiting targeting. When the LMS considered is too aggressive, it will usually prefer to predict that an LMS should not be applied at all $(\forall i, \hat{y}_i = 0)$, thus generating a *RG* around $0 \in$. This is made evident in some extreme strategies (like LMS n°23 for instance).

On less extreme strategies, it shows to yield substantial improvement when classification on y_i gives negative RG. That observation confirms what was already pointed out by Loisel et al. (2021): it can even turn a negative *RG* into a positive one (see LMS n°5, 12, 22, 24, 29 and 30 - Figure 4)

Our framework also improves a strategy where a classification on y_i gives high RG. However, the improvement decreases as the difference between the number of lapsers and the number of lapsers that would be profitable if retained is sizeable. An example of that is shown in Figure 5.

Finally, we can generate LMS for which our framework does not improve the expected RG. It is the case in LMS $n^{\circ}13$, 18 or 27 (See e.g Figure 6). In LMS $n^{\circ}13$, we can see that the mean of the RG is not improved, but the median is. In all those cases, the RG per target produced by the CLV-augmented model is greatly improved, indicating that a CLV-augmented strategy prefers to target fewer policyholders but only those who would generate high future profits.

This last observation explains why a CLV-augmented LMS generates higher RGs when the cost of contact c is considerable. Indeed, the more costly a contact is, the more precise and specific a targeting strategy should be.

Generally, if we consider our various LMS, excluding LMS $n^{\circ}30$ that has a very high improvement ratio:

The average observed RG improvement of a CLVaugmented framework over the classical lapse one is $57,9\%^a$. If we weight these results by the expected RGs, the average improvement is still 31,7%.

^aUsing XGBoost

6.2. Marketing decision making

We already pointed out that the improvement of a lapse management strategy including CLV grows with the proportion of lapsers with a negative CLV (see Figure 26 in Appendix A.7). Models resulting from our framework do not consider them as good targets. In fact, there is a Pearson correlation coefficient of 77% between RG improvement and the proportion of target differences among the LMS detailed in Table 1². Of course, as the improvement ratio has no clear interpretation in some cases, this analysis should be carried out in more depth, separating the cases where both RG - with and without the inclusion of CLV - are positive from the cases where one of them is negative. By doing so, we observe that the Pearson correlation coefficient for LMS yielding positive RG regardless of the inclusion of CLV is even higher: 83%.

In terms of targeting, it seems crucial to understand what differentiates a subject for which $y_i = 1$ and $\tilde{y}_i = 0$ from the others. An investigation of such policyholder's profile can be carried out for every lapse management strategy. We take the example of LMS n°1, where 62,6% of policyholders were in that case (see Section 5.2). With that strategy, the profile of non-targeted lapsers indicates that :

- 57.2% of them are men, similar to the entire dataset,
- 76.4% of them contracted product n°1 whereas 72% of all policyholders chose it,
- the mean seniority of their policy is 10.4 years compared to the 13.4 years for the complete dataset,
- the mean face amount of such policies is 12,156, whereas the average face amount for all considered policies is 40,263.

In that strategy, our framework indicates that marketing efforts on low seniority policyholders with low face amount policies are inefficient. Of course, this conclusion is only valid for the considered LMS; however, our framework allows us to conduct such analysis for any LMS and interpret the results at an individualized level.

6.3. Management rules decision making

Sensitivity analysis of those results can highly benefit management rules decision-making. This framework serves as a tool that compares future hypothetical lapse management strategies in order to choose the best one - among realistic scenarios -. It can also be used to tune a given strategy by answering questions like:

- For which incentive δ the retention strategy becomes profitable ?
- For which acceptance probability *γ* the retention strategy becomes profitable ?
- With a given budget, what is the optimal list of policies that should be targeted?
- At which horizon *T*, the retention strategy become profitable ? In other words, when can the insurer expect a return on investment?

Answering these questions constitutes a 1-parameter sensitivity analysis. In our framework, six parameters influence the expected retention gain $(p, \delta, \gamma, c, d, T)$.

We can argue that among them are three structural parameters that are insurer's dependent and not linked to the external state of the world: δ , γ and c. Among them, the contact cost c is more or less fixed and can not be easily changed by the insurer. Conversely, δ and γ are to be chosen by the insurer. Moreover, they also are correlated with management and commercial efficiency - an efficient campaign impacts the final γ - and correlated together: the higher the incentive δ , the higher the probability of acceptance γ .

By fixing all other parameters and trying various combinations of δ and γ we obtain the following 3D surfaces:



Figure 7: 3d plot (δ , γ , RG)

This surface is not surprising and indicates that the higher the acceptance rate and the lower the incentive, the higher the retention gain. The surface gradient can give powerful insights on the most efficient commercial efforts to make: is it better for the insurer to propose lower incentives and manage to conserve the same acceptance probability or to put commercial effort into improving the acceptance probability for the same proposed incentive? This surface directly addresses this question.

Remark 7. Of course, the interdependency of those parameters should make some part of this surface unrealistic from a management decision-making point of view. The insurer

 $^{^2} Taking the results of XGBoost and excluding LMS n°30 that has a very high improvement ratio.$

should consider such dependencies when designing a lapse management strategy.

Among the six parameters are also three conjectural parameters that depend on the external state of the world: the insurer's profitability p (that depends on competition, macroeconomic considerations or regulation), the discount rate d and the time horizon T (that can be driven by the insurer's vision but also by regulation³). Among them, we chose to fix p and let d and T vary. Moreover, p and T are obviously interdependent and considered through the management's prospective view of the conjecture's evolution. A given interest rate scenario should represent a curve on the following surface:





This surface is less smooth than the one displayed in Figure 7 and seems to indicate a more unstable relationship between RG and the conjectural parameters. An explanation of that behaviour can be that those surface points are generated by running our framework on a random subsample of our dataset, for computation time considerations. Generating the same surface with more policyholders is likely to give a smoother behaviour.

Remark 8. Of course, the interdependency of T and d should make some part of this surface unrealistic from an actuarial point of view. Actuarial rate projections would give precise plausible scenarios on this surface. Such considerations should be taken into account by the insurer when designing a lapse management strategy.

Remark 9. The insurer can also use our framework to measure the retention gain to be expected at different time horizons obtained by existing retention campaigns. In that case, the insurer would have to neutralize the effect of the existing LMS in order to estimate the control portfolio's future value. We leave this remark as future work for applied risk management research.

7. Conclusion, limitations and future work

The work carried out in this paper shows that including CLV in lapse management strategy can largely benefit an insurer's decision-making ability regarding lapse management strategy. We showed that survival tree-based models can outperform parametric approaches in such actuarial contexts. Then, our comparison of tree-based models on different lapse management strategies indicated that our CLV-based framework leads to increased predicted gains for any realistic scenario and acts as a loss-limiting targeting approach, regardless of the retention strategy. Finally, the discussion section highlighted the fact that our modelization can give insights to the life insurer regarding commercial and strategic decision-making.

The framework and methodologies described in this paper suffer some limitations. For instance, we can argue that following one single fixed strategy for every policyholder is unrealistic. We could easily imagine an extension of our models to individualized lapse management strategies that would vary between subjects, but that could also be adjusted with time. There is also room for improvement regarding the correlations of LMS parameters: gamma and delta are evidently interdependent parameters for an insurance company, and this interdependency could be considered.

This paper defines a practical management tool for life insurers as those models can be used to measure the *RG* and improve real strategies used in existing retention campaigns. Finally, our vision of CLV, and by extension, our whole methodology design could be improved by using longitudinal data that would yield time-dynamic results. We leave those two last observations for future work.

A real-life comparison between an actual retention strategy targeting and both the naïve and CLV-improved methodologies could be insightful for the insurer⁴.

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³The ORSA time horizon with the strategic and the long-term business planning time horizon should be both considered.

⁴On that point, it is to be noted that this use of our framework is not trivial. As RG is computed as the difference between a control portfolio value and a lapse-managed one, this setting would first require estimating a CPV that was never observed.

A. Appendix

A.1. CART

For visualization's sake, we can show below an example of a CART model partitioning (x^1, x^2) , the space defined by two covariates along different thresholds $(d_1, d_2, ...)$:



Generally speaking, there are far more than two covariates which makes the space partitioning representation difficult. Summarizing this cutting with a binary tree is obviously a very interpretable way of displaying the modelization: a node g is identified with a subregion of \mathcal{X} it represents. The first step of fitting a CART tree on a dataset $D = (\mathbf{x}_i, y_i)_{i=1}^n$, where \mathbf{x}_i is the vector of covariates for the *i*-th subject and y_i its response variable is applying a splitting procedure until a stopping criteria is met. Various splitting criteria exist depending on the nature of the prediction problem to solve. As we will work with both classification and regression settings, it seems natural to explicit both splitting mechanisms here.

In a classification context, let's define p_i , $i \in \{1, ..., P\}$ as the proportion of observation of class i in D. We extend this idea by defining $p_i(g)$ as the proportion of observation of class i in D(g), the collection of observations in node g.

An impurity function ϕ , is a function defined for p_i , $i \in \{1, ..., P\}$, with $p_i \ge 0$ and $\sum_i p_i = 1$ such that $\phi(p_1, ..., p_P)$ is positive, the minimum of ϕ is reached whenever any of the $p_i = 1$, then $\phi(p_1, ..., p_P) = 0$, its maximum is reached for $\phi(\frac{1}{P}, ..., \frac{1}{P})$ and it is symmetric with regard to its arguments.

For CART, usual classification impurities are:

- Entropy : $\phi(p_1, \dots, p_P) = -\sum_i p_i log(p_i)$
- Gini index: $\phi(p_1, \dots, p_P) = 1 \sum_{i < j} p_i p_j$

For a given node g with $\mathcal{N}(g)$ observations, the impurity of the node is given by $I(g) = \phi(p_1(g), \dots, p_P(g))$.

At each node of a CART, the optimal split is chosen as the split that reduces the impurity the most. That is to say, the split that maximizes the following gain function by splitting the parent node g_p into the two child nodes g_l and g_r :

$$G(g_p; g_l, g_r) = I(g_p) - \left(\frac{\mathcal{N}(g_l)}{\mathcal{N}(g_p)}I(g_l) + \frac{\mathcal{N}(g_r)}{\mathcal{N}(g_p)}I(g_r)\right),$$

This splitting procedure allows growing the tree until a splitting criterion - such as a maximal depth or a minimal number of observations in the terminal nodes - is met. A tree grown following this step is called a maximal tree.

Usually, the maximal tree is not an efficient model as it leads to a very thin partitioning of \mathcal{X} that does not produce good predictions for test observations. The bias of the maximal tree is low, but its variance is very high. Breiman suggested a method to select an optimal subtree of the maximum tree that would be deep enough to produce low-bias prediction but small enough to generalize well to new data. The idea is to penalize the performance of the subtrees by the number of terminal nodes they have, allowing to construct a sequence of optimal subtrees depending on the strength of the penalization. That pruning phase intends to find the sequence of best trees of 1, 2, ..., l leaves. The optimal tree is then selected among this sequence, by cross-validation.

This method leads to a clear and interpretable modelization that is not sensitive to outliers and can handle missing values. This makes CART a valid choice of modelization for real business applications. In contrast, the predictions obtained with CART are very sensitive to changes in the training set, which makes it an unstable model. This lack of stability led to the design of more elaborate procedures to design tree-based models, such as the *Bagging* and *Boosting* techniques detailed in the following sections.

A.2. Random forest

In the random forest algorithm, q covariates are randomly selected among the p available features at each node. At each node, only those q covariates are considered to find the optimal split.

Let's define Θ_{obs} , the random variable representing the bootstrap over observations of \mathcal{L} and $\Theta_{cov}(q)$, the random variable representing the random drawings of q covariates over the p available features at each node. We assume that Θ_{obs} and $\Theta_{cov}(q)$ are independent of \mathcal{L} . The *i*-th realisations



Figure 9: Random forest mechanism

of those bootstrap steps are denoted Θ_{obs}^{i} et $\Theta_{cov}^{i}(q)$.

 $\mathcal{L}(\Theta_{obs}^{i})$ would then be a subsample of \mathcal{L} and $\mathcal{T}(\mathcal{L}(\Theta_{obs}^{i}), \Theta_{cov}^{i}(q))$ the maximal tree grown with CART algorithm from this subsample with the *p* features considered at each node, selected with $\Theta_{cov}^{i}(q)$. The mechanism of a random forest built over N_{trees} trees is summarized below:

The parameter q is fixed in advance and is constant among trees of the same forest. The fact that the random feature selection happens at each node of a tree ensures that every tree deep enough is grown by considering almost surely every covariate. This algorithm, with two different bootstrap procedures, has been developed by Breiman (1996) and Amit and Geman (1997) and synthesized in Breiman (2001).

When predicting the forest outcome for an observation, the aggregation step consists of propagating the observation through every tree in the forest. The prediction of the forest is the majority vote among all trees of the mean outcomes of each individual tree respectively, in a classification or a regression context. In other words, let $h_i(x) = h_i(x, \mathcal{D}(\Theta_{obs}^i), \Theta_{cov}^i(q))$ denote the prediction of the observation x given by the *i*-th tree, grown with the bootstrap realizations Θ_{obs}^i and $\Theta_{cov}^i(q)$, the prediction of the random forest is RF(x):

$$RF(x) = mode(h_1(x), \dots, h_{N_{trees}}(x)), \text{ for classification}$$
$$RF(x) = \frac{1}{N_{trees}} \sum_{i=1}^{N_{trees}} h_i(x), \text{ for regression}$$

Theoretical guarantees on random forests exist and often rely on the hypothesis that all the trees within the forest are independent. The bootstrap procedure aims at diminishing the correlations between different trees to get closer to that assumption.

A.3. XGBoost

If tree-based boosting approaches share with bagging the global concept of aggregating weak predictions, the construction of each tree is very different. For boosting techniques, it relies on a recursive relation: a first tree is grown, then a second one that will put more weight on poorly predicted observations, and so on: single trees are grown iteratively such that each subsequent tree aims at reducing the errors of the previous one. Intuitively, the goal of boosting thought iterations is to focus on regions of the original dataset that yield poor predictions.

All those weak single trees build iteratively and then aggregated with a weighted mean, with weights related to their accuracy.

Generally, let $\boldsymbol{w}^{i} = (w_{1}, ..., w_{n}), \forall i \in [1, ..., N_{trees}]$ be the vector of observations weights during the *i*-th iteration of the boosting algorithm. We initialize: $\boldsymbol{w}^{1} = (\frac{1}{n}, ..., \frac{1}{n})$, let $\mathcal{D}^{\boldsymbol{w}^{i}}$ be the original dataset, with observations weighted by \boldsymbol{w}^{i} . Let $\mathcal{T}(\boldsymbol{w}^{i})^{5}$ be the tree grown from $\mathcal{D}^{\boldsymbol{w}^{i}}$. The final model is denoted $\mathcal{B}(\mathcal{D})$.

"The predictions are combined in an additive manner, where the addition of each base model improves (or "boosts") the overall model. Therefore, the overall model f is an additive model of the form:"

$$f(\mathbf{x}) = \sum_{m=1}^{M} \beta_m g(\mathbf{x}; \theta_m),$$

"where M>0 denotes the number of base learners, and $\beta_m \in \mathcal{R}$ is a weighting term. The function g refers to a base learner and is parameterized by the vector θ . Individual

⁵This notation is clearly simplified and does not take other parameters into account



Figure 10: Tree-boosting mechanism

base learners differ in the configuration of their parameters θ , which is indicated by a subscript m."

We can illustrate the mechanism of a boosting algorithm like so:

There are many boosting techniques adapted to trees, and the main difference between them can be their method of weighting training data points or the hypotheses they rely on. In this paper, we will only consider the library *XGBoost* as it is regarded as one of the most competitive boosting algorithms in terms of prediction accuracy, computation time, and flexibility *XGBoost* has been developed by Chen and GuestrinChen and Guestrin (2016) and relies on a gradient boosting procedure. One of the main advantages of *XGBoost* resides in its optimization, as it can be finely tuned through numerous parameters and benefits significantly from parallel computation.

A.4. Confusion matrix and evaluation metrics

Obviously, with a binary outcome, an observation can be well classified or misclassified in only four ways: a churned observation can be classified as such or misclassified, and an active observation can be classified as such or misclassified. A confusion matrix is essentially a table containing the number of observations N(j, k) in the test set that fall in each of those possibilities:

$$N(j,k) = \sum_{i=1}^{N} \mathbf{1} (y_i = j, \hat{y}_i = k).$$

	Real outcome y				
		1	0	Total	
Predicted outcome û	1	N(1, 1)	<i>N</i> (0, 1)	N(-, 1)	
riculticu outcome y	0	N(1, 0)	N(0, 0)	$N(-,0)^{6}$	
	Total	N(1, -)	N(-,0)	п	

N(1, 1), N(1, 0), N(0, 1), N(0, 0) are respectively called the number of true positive, false negative, false positive and true negative.

From this number, we can derive several well-used metrics:

$$\begin{aligned} \operatorname{Accuracy}(y, \hat{y}) &= \frac{N(1, 1) + N(0, 0)}{n} \\ \operatorname{Precision}(y, \hat{y}) &= \frac{N(1, 1)}{N(1, 1) + N(0, 1)} = \frac{N(1, 1)}{N(-, 1)} \\ \operatorname{Recall}(y, \hat{y}) &= \frac{N(1, 1)}{N(1, 1) + N(1, 0)} = \frac{N(1, 1)}{N(1, -)} \\ \operatorname{F}_1\operatorname{Score}(y, \hat{y}) &= \frac{N(1, 1)}{N(1, 1) + \frac{N(1, 0) + N(0, 1)}{2}} = \frac{2}{\frac{1}{\operatorname{recall}} + \frac{1}{\operatorname{precision}}} \\ &= 2 \cdot \frac{\operatorname{recall} \cdot \operatorname{precision}}{\operatorname{recall} + \operatorname{precision}} \end{aligned}$$

Accuracy (y, \hat{y}) is undoubtedly the most intuitive performance measure, and it is defined as the proportion of correctly predicted observations over all observations. It is widely used for churn analysis but appears to be a satisfying performance measure for balanced datasets, which is debatable in our case.

Precision(y, \hat{y}) measures the proportion of observations that actually churned among the observations predicted as churners: the higher this metric, the lower the false positive rate.

Recall(y, \hat{y}) measures the proportion of observations that were correctly identified as churners among the actually churned observations. The higher this metric, the lower the false-negative rate, which is of great interest for churn predictions that lead to the application of a retention strategy. The real-world retention strategies are often limited by budget or deployment constraints; a low recall would mean that a significant part of the retention budget was spent on policyholders that were not churners. It is often preferable to target a smaller number of policyholders with a more substantial proportion of churners at the risk of missing a lot of them than targeting a more significant number of policyholders with a lower proportion of churners, even if that means that more churners were identified as such. Lastly, F_1 Score can be seen as a weighted average between precision and recall. It measures a model's ability to predict churners in terms of both precision and recall. It is usually more efficient than accuracy in the presence of uneven class distribution. Accuracy seems to perform better if false positives and false negatives have similar costs, which is arguably not the case in our study.

A.5. Competing risk framework

There are several regression models to estimate the global hazard and the hazard of one risk in settings where competing risks are present: modeling the cause-specific hazard and the subdistribution hazard function. They account for competing risks differently, obtaining different hazard functions and thus distinct advantages, drawbacks, and interpretations. We will here quickly introduce those approaches' theoretical and practical implications and justify which one we will use in our modelizations.

In Cause-specific regression, each cause-specific hazard is estimated separately, in our case, the cause-specific hazards of lapse and death, by considering all subjects that experienced the competing event as censored: We remind that $J_T = 0$ corresponds to an active subject that did not experience lapse $J_T = 1$ or death $J_T = 2$. The cause-specific hazard rates regarding the *j*-th risk ($j \in [1, ..., J]$) are defined as:

$$\lambda_{T,j}(t) = \lim_{dt \to 0} \frac{P\left(t \le T < t + dt, J_T = j \mid T \ge t\right)}{dt}$$

We can recover the global hazard rate as $\lambda_{T,1}(t) + \cdots + \lambda_{T,J}(t) = \lambda_T(t)$, and derive the global survival distribution of *T* as

$$P(T > t) = 1 - F_T(t) = S_T(t)$$

= exp $\left(-\int_0^t \left(\lambda_{T,1}(s) + \dots + \lambda_{T,J}(s) \right) ds \right)$

This approach aims at analysing the cause-specific "distribution" function: $F_{T,j}(t) = P(T \le t, J_T = j)$. In practice, it is called the Cumulative Incidence Function (*CIF*) for cause *j* and not a distribution function since $F_{T,j}(t) \rightarrow P(J_T = j) \ne 1$ as $t \rightarrow +\infty$. By analogy with the classical survival framework, the *CIF* can be characterized as $F_{T,j}(t) = \int_0^t f_{T,j}(s) ds^7$, where $f_{T,j}$ is the improper⁸ density function for cause *j*. It follows that:

$$f_{T,j}(s) = \lim_{dt \to 0} \frac{P\left(t \le T < t + dt, J_T = j\right)}{dt} = \lambda_{T,j}(t)S_T(t)$$

The equation above is self-explanatory: the probability of experiencing cause j at time t is simply the product of

surviving the previous time periods by the cause-specific hazard at time *t*. We finally obtain the *CIF* for cause *j* as:

$$F_{T,j}(t) = \int_0^t \lambda_{T,j}(s) \exp\left(-\int_0^s \lambda_T(u) du\right) ds.$$

There are several advantages to that approach. First of all, cause-specific hazard models can be easily fit with any classical implementation of CPH by simply considering as censored any subject that experienced the competing event. Then the CIF is clearly interpretable and summable $P(T \le t) = F_{T,1}(s) + \cdots + F_{T,J}(s)^9$. On the other hand, the CIF estimation of one given cause depends on all other causes: it implies that the study of a specific cause requires estimating the global hazard rate, and interpreting the effects of covariates on this cause is difficult. Indeed, part of the effects on a specific cause comes from the competing causes, but in our setting, we are only interested in the prediction of the survival probabilities, not their interpretation as such.

We have introduced it at the beginning of this section; another approach is often considered to analyze competing risks and derive a cause-specific *CIF*. This other approach, called the subdistribution hazard function of Fine and Gray regression, works by considering a new competing risk process τ . Without loss of generality, let's consider death as our cause of interest:

$$\tau = T \times \mathbb{1}_{J_T = 2} + \infty \times \mathbb{1}_{J_T \neq 2}.$$

It has the same as *T* regarding the risk of death, $P(\tau \le t) = F_{T,2}(t)$ and a mass point at infinity $1 - F_{T,2}(\infty)$, probability to observe other causes $(J_T \ne 2)$ or not to observe any failure. In other words, if the previous approach considered every subject that experienced competing events as censored, this approach considers a new and artificial at-risk population. This last consideration is made clear when deriving the hazard rate of τ :

$$\lambda_{\tau}(t) = \lim_{dt \to 0} \frac{P\left(t \le T < t + dt, J_T = 2 \mid \{T \ge t\} \cup \{T \le t, J_T \neq 2\}\right)}{dt}.$$

Finally, we obtain the *CIF* for the risk of death as:

$$F_{T,2}(t) = 1 - \exp\left(-\int_0^t \lambda_\tau(s) ds\right)$$

This subdistribution approach resolves the most important drawback to cause-specific regression, as the coefficients resulting from it do have a direct relationship with the cumulative incidence: estimating the CIF for a specific cause does not depend on the other causes, which makes the interpretation of CIF easier. The subdistribution hazard models can be fit in R by using the crr function in the cmprsk package or using the timereg package. Still, to our knowledge, there is no implementation of a Fine and Gray model in Lifelines or, more generally, Python. We can also

⁷We suppose that T has a continuous distribution

⁸Because derived from the CIF, an improper cumulative distribution function

⁹unlike to the function $1 - \exp\left(-\int_0^t \lambda_{T,j}(u) du\right)$.

note that these two approaches are linked, Putter et al. and the link between $\lambda_{\tau}(t)$ and $\lambda_{T_i}(t)$ is given by:

$$\lambda_{\tau}(t) = r_j(t)\lambda_{T,j}(t), \text{ with } r_j(t) = \frac{P(J_T = 0)}{\sum_{p \neq j}^{J} P(J_T = p)}$$

In other words, if the probability of any competing risk is low, the two approaches give very close results.

A.6. Survival modelizations results

 $r_{i,t}^{lapser}$ represents the probability that the policy of subject *i* is still active at time *t*, given that it was active at its last observed time. Predicting the overall conditional survival with the competing risks, in that case, can be achieved by creating a combined outcome. The policy ends with death or lapse, whichever comes first, and to compute r^{lapser} , we recode the competing events as a combined event. In terms of statistical guarantees, this approach is compatible with any survival analysis method.

In the following sections of this appendix, $r_{i,t}^{acceptant}$ indicates the probability of survival for subject *i* at time *t* given that it will not lapse. In other words, it is the survival probability regarding only the risk of death. As detailed in Section 4.1.1, this corresponds to the cause-specific survival probability for death. It is to be noted that the density from which we derive our survival probabilities is improper as it derives itself from the *CIF*, which is not a proper distribution function.¹⁰. Therefore, any conclusion about those probabilities should be drawn with care. Our modelization is derived from a Cox cause-specific hazard regression with a spline baseline hazard from the Python library Lifelines. Similarly to r^{lapser} , covariates selection and tuning are performed by minimizing AIC.

A.6.1. Cox-model

We first decide to estimate survival with a Cox Proportional hazard model with a spline baseline hazard from the Python library Lifelines. Covariate selection and tuning are performed by minimizing AIC. Here is what $r^{acceptant}$, the vector of cause-specific probabilities, looks like, and we can compare it to r^{lapser} on some subjects:



Figure 11: 10 policyholders' survival curve for *r*^{acceptant} with Cox model



Figure 12: 10 policyholders' survival curve for r^{lapser}

¹⁰as it does not tend to 1 as t goes to $+\infty$

The effect of various covariates on the survival outcome can be found below:



Figure 13: Coefficient plot for r^{lapser}



Figure 14: rlapser trajectories for different products







Figure 16: r^{lapser} trajectories for different ages



Figure 17: rlapser trajectories for different face amounts



Figure 18: Coefficient plot for racceptant



Figure 20: racceptant trajectories for different ages



Figure 19: *r^{acceptant}* trajectories by gender



Figure 21: racceptant trajectories for different face amounts

A.6.2. RSF

We obtain better results than Cox in terms of concordance index at the cost of very high computation time for one training with one set of parameters - 5days without parallelisation, 4 hours with - compared to a few seconds for cox model.

Some of the results we obtain are displayed below:



Figure 22: 10 policyholders' survival curve for *r^{acceptant}* with RSF



Figure 23: 10 policyholders' survival curve for r^{lapser} with RSF

Weight			Feature		
0.3148	±	0.0064	Age_souscription		
0.0100	±	0.0008	CDI_CD_SEXE_1		
0.0091	±	0.0014	PRODUIT_2		
0.0077	±	0.0006	TOTAL_PREMIUM_AMOUNT		
0.0013	±	0.0004	Nb_Contrats		
0.0010	±	0.0003	PRODUIT_3		

Table 3: Covariates importance for $r^{acceptant}$ with RSFWeightFeature

0.1838	±	0.0045	Age_souscription
0.0415	±	0.0018	TOTAL_PREMIUM_AMOUNT
0.0083	±	0.0011	CDI_CD_SEXE_1
0.0026	±	0.0013	PRODUIT_2
0.0022	±	0.0006	PRODUIT_3
0.0020	±	0.0006	Nb_Contrats

Table 4: Covariates importance for r^{lapser} with RSF

A.6.3. XGSB

We obtain better results than Cox and slightly better results than RSF in terms of concordance index at the cost of even higher computation time for one training with one set of parameters - 10h with great parallelisation - compared to a few seconds for Cox model.

Some of the results we obtain are displayed below:



Figure 24: 10 policyholders' survival curve for *r*^{acceptant} with GBSM



Figure 25: 10 policyholders' survival curve for r^{lapser} with GBSM

Weight			Feature			
0.3274	±	0.0071	Age_souscription			
0.0104	±	0.0006	TOTAL_PREMIUM_AMOUNT			
0.0100	±	0.0008	CDI_CD_SEXE_1			
0.0025	±	0.0005	PRODUIT_2			
0.0005	±	0.0001	Nb_Contrats			
0.0000	±	0.0001	PRODUIT_3			

Table 5: Covariates importance for *r*^{acceptant} with GBSM

Weight			Feature			
0.1872	±	0.0039	Age_souscription			
0.0438	±	0.0020	TOTAL_PREMIUM_AMOUNT			
0.0134	±	0.0014	PRODUIT_2			
0.0076	±	0.0009	CDI_CD_SEXE_1			
0.0051	±	0.0006	PRODUIT_3			
0.0011	±	0.0004	Nb_Contrats			

Table 6: Covariates importance for r^{lapser} with GBSM

A.6.4. Final survival model

The final concordance index scores are displayed below:

	Concordance Index				
	r ^{lapser}	<i>r^{acceptant}</i>			
Cox model	69,5%	80,7%			
RSF	71,6%	83,7%			
GBSM	73,0%	84,1%			

Table 7: Survival models comparison)

A.7. Other LMS results

LMS	р	δ	γ	с	d	Т
A-1	2,50%	0,04%	25%	10	1,50%	5
A-2	2,50%	0,04%	25%	10	1,50%	20
A-3	2,50%	0,04%	25%	100	1,50%	5
A-4	2,50%	0,04%	25%	100	1,50%	20
A-5	2,50%	0,04%	5%	10	1,50%	5
A-6	2,50%	0,04%	5%	10	1,50%	20
A-7	2,50%	0,04%	5%	100	1,50%	5
A-8	2,50%	0,04%	5%	100	1,50%	20
A-9	2,50%	0,10%	25%	10	1,50%	5
A-10	2,50%	0,10%	25%	10	1,50%	20
A-11	2,50%	0,10%	25%	100	1,50%	5
A-12	2,50%	0,10%	25%	100	1,50%	20
A-13	2,50%	0,10%	5%	10	1,50%	5
A-14	2,50%	0,10%	5%	10	1,50%	20
A-15	2,50%	0,10%	5%	100	1,50%	5
A-16	2,50%	0,10%	5%	100	1,50%	20
A-17	1,50%	0,04%	25%	10	1,50%	5
A-18	1,50%	0,04%	25%	10	1,50%	20
A-19	1,50%	0,04%	25%	100	1,50%	5
A-20	1,50%	0,04%	25%	100	1,50%	20
A-21	1,50%	0,04%	5%	10	1,50%	5
A-22	1,50%	0,04%	5%	10	1,50%	20
A-23	1,50%	0,04%	5%	100	1,50%	5
A-24	1,50%	0,04%	5%	100	1,50%	20
A-25	1,50%	0,10%	25%	10	1,50%	5
A-26	1,50%	0,10%	25%	10	1,50%	20
A-27	1,50%	0,10%	25%	100	1,50%	5
A-28	1,50%	0,10%	25%	100	1,50%	20
A-29	1,50%	0,10%	5%	10	1,50%	5
A-30	1,50%	0,10%	5%	10	1,50%	20
A-31	1,50%	0,10%	5%	100	1,50%	5
A-32	1,50%	0,10%	5%	100	1,50%	20
B-1	2,50%	0,08%	20%	10	1,50%	5
B-2	2,50%	0,08%	20%	10	1,50%	20
B-3	2,50%	0,08%	20%	100	1,50%	5
B-4	2,50%	0,08%	20%	100	1,50%	20
B-5	2,50%	0,08%	10%	10	1,50%	5
B-6	2,50%	0,08%	10%	10	1,50%	20
B-7	2,50%	0,08%	10%	100	1,50%	5
B-8	2,50%	0,08%	10%	100	1,50%	20
B-9	2,50%	0,20%	20%	10	1,50%	5
B-10	2,50%	0,20%	20%	10	1,50%	20

р	δ	γ	c	d	Т
2,50%	0,20%	20%	100	1,50%	5
2,50%	0,20%	20%	100	1,50%	20
2,50%	0,20%	10%	10	1,50%	5
2,50%	0,20%	10%	10	1,50%	20
2,50%	0,20%	10%	100	1,50%	5
2,50%	0,20%	10%	100	1,50%	20
1,50%	0,08%	20%	10	1,50%	5
1,50%	0,08%	20%	10	1,50%	20
1,50%	0,08%	20%	100	1,50%	5
1,50%	0,08%	20%	100	1,50%	20
1,50%	0,08%	10%	10	1,50%	5
1,50%	0,08%	10%	10	1,50%	20
1,50%	0,08%	10%	100	1,50%	5
1,50%	0,08%	10%	100	1,50%	20
1,50%	0,20%	20%	10	1,50%	5
1,50%	0,20%	20%	10	1,50%	20
1,50%	0,20%	20%	100	1,50%	5
1,50%	0,20%	20%	100	1,50%	20
1,50%	0,20%	10%	10	1,50%	5
1,50%	0,20%	10%	10	1,50%	20
1,50%	0,20%	10%	100	1,50%	5
1,50%	0,20%	10%	100	1,50%	20
	p 2,50% 2,50% 2,50% 2,50% 2,50% 2,50% 1,50%	pδ2,50%0,20%2,50%0,20%2,50%0,20%2,50%0,20%2,50%0,20%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,08%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%1,50%0,20%	pδγ2,50%0,20%20%2,50%0,20%20%2,50%0,20%10%2,50%0,20%10%2,50%0,20%10%2,50%0,20%10%1,50%0,08%20%1,50%0,08%20%1,50%0,08%20%1,50%0,08%20%1,50%0,08%10%1,50%0,08%10%1,50%0,08%10%1,50%0,20%20%1,50%0,20%20%1,50%0,20%20%1,50%0,20%20%1,50%0,20%20%1,50%0,20%20%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%1,50%0,20%10%	pδγc2,50%0,20%20%1002,50%0,20%20%1002,50%0,20%10%102,50%0,20%10%102,50%0,20%10%1002,50%0,20%10%1001,50%0,20%10%1001,50%0,08%20%101,50%0,08%20%1001,50%0,08%20%1001,50%0,08%10%1001,50%0,08%10%1001,50%0,20%20%101,50%0,20%20%101,50%0,20%20%101,50%0,20%20%101,50%0,20%20%101,50%0,20%20%101,50%0,20%10%101,50%0,20%10%101,50%0,20%10%101,50%0,20%10%101,50%0,20%10%101,50%0,20%10%1001,50%0,20%10%1001,50%0,20%10%1001,50%0,20%10%1001,50%0,20%10%1001,50%0,20%10%1001,50%0,20%10%100	pδγcd2,50%0,20%20%1001,50%2,50%0,20%20%1001,50%2,50%0,20%10%101,50%2,50%0,20%10%101,50%2,50%0,20%10%1001,50%2,50%0,20%10%1001,50%2,50%0,20%10%1001,50%1,50%0,08%20%101,50%1,50%0,08%20%1001,50%1,50%0,08%20%1001,50%1,50%0,08%20%1001,50%1,50%0,08%10%101,50%1,50%0,08%10%101,50%1,50%0,20%20%101,50%1,50%0,20%20%101,50%1,50%0,20%20%101,50%1,50%0,20%20%1001,50%1,50%0,20%20%1001,50%1,50%0,20%10%101,50%1,50%0,20%10%101,50%1,50%0,20%10%101,50%1,50%0,20%10%101,50%1,50%0,20%10%101,50%1,50%0,20%10%1001,50%1,50%0,20%10%1001,50%1,50%0,20%10%1001,50%1,50%0,20%10%<

Table 8: More LMS



Figure 26: Correlation between the proportion of nontargeted lapsers and the improvement of a CLV-augmented LMS

NTO	·· ()	Model	% target diff	Accuracy		Retention gain		RG/target		T .
N°	time (s)			y_i	\tilde{y}_i	y_i	\widetilde{y}_i	y_i	\tilde{y}_i	improvement
		CART		92.3%	85 3%	114 661	219 655	4 48	38.20	91 57%
	10.10	PE		02,570	85 1%	232 314	217 055	0.82	56,65	23 02%
A-1	4949	YCB	62,58%	03 1%	85.8%	232 314	324 052	9,62	54.64	23,52%
		AUD		95,470	05,070	243 303	524 952	9,01	54,04	55,5270
		CART		92,3%	89,8%	7 092 097	6 142 119	277,00	353,83	-13,39%
A-2	6111	RF	26.66%	92,9%	90,2%	6 596 374	5 696 455	278,47	351,02	-13,64%
		XGB	,	93,4%	90,9%	7 308 721	7 432 688	288,92	404,84	1,70%
		CART		92.3%	83 3%	- 2 187 622	- 8 224	- 85 52	- 31 09	99.62%
• 2	1600	RF	02 5001	92,9%	83.4%	- 1 900 265	45 483	- 80 18	194 35	102 39%
A-3	4603	XGB	93,50%	93.4%	83 5%	- 2 032 650	77 481	- 80 39	174 44	103.81%
				22,170	00,070	1 500 014		105.00		105,0170
		CART		92,3%	86,5%	4 789 814	5 117 844	187,00	5/7,74	6,85%
A-4	5555	RF	55,37%	92,9%	86,4%	4 463 796	4 255 175	188,47	566,05	-4,67%
		XGB		93,4%	86,8%	5 032 706	5 433 366	198,92	610,26	7,96%
		CART		92,3%	83,6%	- 514 477	- 112 372	- 20,08	- 86,48	78,16%
A-5	4753	RF	86 72%	92,9%	83,4%	- 323 544	- 3 937	- 13,65	- 28,28	98,78%
110	1155	XGB	00,7270	93,4%	83,3%	- 383 004	-	- 15,14	-	100,00%
		CAPT		02.30%	87.0%	335.810	517 224	12 17	30.01	54 02%
		DE		92,370	07,970 87.0%	655 350	661 021	27.68	61 13	0.87%
A-6	5803		44,27%	92,970	01,970	654 210	720 403	27,00	58 22	11 510
		AUD		95,4%	00,0%	034 219	129 493	23,00	38,22	11,31%
		CART		92,3%	83,3%	- 2 816 759	- 10 205	- 110,08	- 384,04	99,64%
A-7	4241	RF	99,09%	92,9%	83,3%	- 2 456 122	1 013	- 103,65	66,30	100,04%
		XGB	,0,7,0	93,4%	83,3%	- 2 659 020	243	- 105,14	15,92	100,01%
		CART		92.3%	84.0%	- 1 966 473	- 46 323	- 76.83	- 22.31	97.64%
٨٥	5164	RF	07 700	92.9%	84.0%	- 1 477 229	253 885	- 62.32	149.67	117.19%
A-8	5164	XGB	02,70%	93.4%	84.1%	- 1 621 796	273 243	- 64.14	117.83	116.85%
		CADT		00.201	02.70	005 070	1(1 100	20.10	107.97	90,400
				92,3%	83,1%	- 825 372	- 161 100	- 32,19	- 127,87	80,48%
A-9	4781	KF	77,60%	92,9%	83,4%	- 384 / 36	8 596	- 10,22	32,12	102,23%
		XGB		93,4%	83,6%	- 498 263	22 337	- 19,70	35,47	104,48%
		CART		92,3%	89,7%	4 614 513	4 483 831	180,36	266,33	-2,83%
A-10	6075	RF	29.10%	92,9%	89,9%	4 973 929	4 328 724	210,01	280,90	-12,97%
	0070	XGB	_>,1070	93,4%	90,7%	5 354 770	5 368 917	211,69	301,57	0,26%
		CART		92.3%	83.2%	- 3 127 655	- 118 886	- 122 19	- 2 230 39	96.20%
	1506	RE	06 569	02,0%	83.3%	- 2 517 315	1 340	- 106 22	2 230,39 87 71	100.05%
A-11	4506	XGB	96,56%	03.4%	83.3%	- 2 774 278	736	- 100,22	52.00	100,03%
		AOD		JJ, + 70	05,570	-2774270	750	- 10),70	52,00	100,0570
		CART		92,3%	86,2%	2 312 231	3 310 314	90,36	412,71	43,17%
A-12	5534	RF	57,93%	92,9%	86,1%	2 841 351	3 129 652	120,01	465,74	10,15%
		XGB		93,4%	86,6%	3 078 755	3 825 920	121,69	475,53	24,27%
		CART		92,3%	83,3%	- 1 201 626	- 163 056	- 46,87	- 1 838,44	86,43%
A-13	4640	RF	92 91%	92,9%	83,3%	- 717 620	- 5 339	- 30,28	- 354,24	99,26%
11 15	1010	XGB	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	93,4%	83,3%	- 875 378	508	- 34,60	16,26	100,06%
		CAPT		02.3%	87.3%	1 476 651	831.010	57.40	77 00	13 77%
	FRA 0	RE	15 10 %	92,3 <i>1</i> 0 07.0%	86 D%	- 380 682	126 522	- 57,49 - 16.02	- 77,39	+3,1270
A-14	5739	YCP	47,12%	47,12% 92,9%	85,0%	- 611 220	20 202	- 10,05	21,14	101 560
		AUD		95,470	05,570	- 044 509	29 302	- 23,47	7,10	104,5070
		CART		92,3%	83,3%	- 3 503 908	- 97 263	- 136,87	- 2 354,34	97,22%
A-15	4216	RF	99,61%	92,9%	83,3%	- 2 850 198	-	- 120,28	-	100,00%
_	.210	XGB		93,4%	83,3%	- 3 151 393	-	- 124,60	-	100,00%
		CART		92.3%	83.8%	- 3 778 933	- 734 773	- 147.49	- 418.58	80.56%
A 16	5006	RF	81 160-	92.9%	83.5%	- 2 513 261	8 914	- 106.03	20.13	100.35%
A-10	2090	XGB	04,40%	93.4%	83.6%	- 2 920 405	34 492	- 115.47	45.75	101.18%
				,	, 5 / 6	. =		,	,	,.0,0

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	N 10		Model	% target diff	Accuracy		Retention gain		RG/target		Ŧ ,
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	IN ²	time (s)			y_i	\tilde{y}_i	y_i	\tilde{y}_i	y_i	\tilde{y}_i	improvement
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			CART		02.3%	89.5%	5 100 456	4 800 470	100 11	279.88	_3.04%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		5000	DE		02,5%	80.8%	1 635 482	4 226 648	105.60	275,00	-3,74% 8,87%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-17	5390	XGB	28,74%	93.4%	90.2%	5 196 736	5 138 253	205.40	270,00	-0,02%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			AUD		95,470	90,270	5 190 750	5 156 255	205,40	299,21	-1,1570
			CART		92,3%	91,3%	52 090 240	47 706 070	2 034,15	2 170,64	-8,42%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-18	6452	RF	12,12%	92,9%	91,9%	46 171 160	42 049 900	1 949,05	2 082,36	-8,93%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			XGB	,	93,4%	92,5%	51 629 950	52 606 740	2 040,95	2 339,70	1,89%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			CART		92.3%	85.2%	2 798 173	3 182 143	109 11	481.60	13 72%
	4 10	4012	RF	64.000	92,5%	85.2%	2 502 903	2 743 070	105,11	554 76	9.60%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-19	4913	XGB	64,89%	93.4%	85.6%	2 902 703	3 438 303	115.40	576.64	17 72%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			MOD		,,,,,,	05,070	2 920 720	5 150 505	115,10	570,01	17,7270
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			CART		92,3%	89,6%	49 787 960	45 366 730	1 944,15	2 616,32	-8,88%
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	A-20	6160	RF	29,03%	92,9%	90,0%	44 038 580	39 947 830	1 859,05	2 547,89	-9,29%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			XGB		93,4%	90,6%	49 353 940	49 789 670	1 950,95	2 796,17	0,88%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92.3%	86.8%	482 682	544 887	18.85	53,99	12.89%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A 21	5070	RF	51 60%	92.9%	86.8%	557 090	554 195	23.52	65.17	-0.52%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-21	3079	XGB	51,0970	93.4%	87.1%	607 670	624 556	24.01	64.79	2.78%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			CADT		00.00	00.001	0.225.420	0.507.444	264.60	454.70	0.000
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	90,2%	9 335 438	8 527 444	364,60	454,78	-8,66%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-22	6199	RF	23,94%	92,9%	90,6%	8 570 307	7 931 029	361,80	460,42	-7,46%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			XGB		93,4%	91,2%	9 518 466	9 581 934	376,27	501,56	0,67%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	83,6%	- 1 819 600	135 305	- 71,15	121,80	107,44%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	A-23	4601	RF	89 51%	92,9%	83,5%	- 1 575 489	159 620	- 66,48	215,65	110,13%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	11 25	4001	XGB	07,5170	93,4%	83,7%	- 1 668 346	228 226	- 65,99	208,69	113,68%
			CART		02.3%	87.0%	7 033 156	7 124 100	274.60	680.08	1 20%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				/	92,370	87,070 87,07	6 427 720	6 264 477	274,00	711.80	1,2970
A-255379CART NGB92,3%89,2%4 160 4233 882 623162,44241,06-6,68%A-255379RF XGB30,97%92,3%89,2%4 016 4233 666 219169,65249,54-8,76%A-266410RF XGB12,52%92,9%91,3%49 612 66045 948 6901 937,512 083,30-7,39%A-266410RF XGB12,52%92,9%91,9%44 548 72040 814 9601 880,592 029,68-8,38%A-274887RF XGB66,67%92,9%85,1%1 858 1402 575 53872,44442,8638,61%A-274887RF XGB66,67%92,9%85,0%1 885 8532 387 01879,65531,2526,57%A-286047RF 	A-24	5650	NI ⁻ VCP	50,83%	92,970	07,070 07 101	7 242 450	7 840 770	271,80	711,09	-1,1470
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			AUD		95,4%	87,4%	7 242 430	/ 840 / /0	280,27	//1,/1	8,20%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	89,2%	4 160 423	3 882 623	162,44	241,06	-6,68%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A-25	5379	RF	30,97%	92,9%	89,5%	4 018 432	3 666 219	169,65	249,54	-8,76%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			XGB	,	93,4%	90,0%	4 455 108	4 410 629	176,09	267,87	-1,00%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $			CART		92.3%	91 3%	49 612 660	45 948 690	1 937 51	2,083,30	-7 39%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1.00	(110	RF	10 500	92,9%	91,9%	44 548 720	40 814 960	1 880 59	2 029 68	-8 38%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A-20	6410	XGB	12,52%	93.4%	92 5%	49 676 000	50 549 740	1 963 72	2 260 20	1 76%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						2,570				2 200,20	1,70%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	85,1%	1 858 140	2 575 538	72,44	442,86	38,61%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A-27	4887	RF	66,67%	92,9%	85,0%	1 885 853	2 387 018	79,65	531,25	26,57%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			XGB		93,4%	85,4%	2 179 093	2 879 880	86,09	544,35	32,16%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	89,4%	47 310 370	43 168 880	1 847.51	2 519.41	-8,75%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A 28	6047	RF	20 12%	92,9%	89.9%	42 416 140	38 573 620	1 790.59	2 504,61	-9,06%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A-20	0047	XGB	29,4270	93,4%	90,5%	47 399 990	47 812 830	1 873,72	2 721,63	0,87%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			CADT		02.20	06 501	204 467	5 009	7.05	1.66	07.510
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					92,3%	80,5%	- 204 467	- 5 098	- 7,95	- 1,00	97,51%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	A-29	5070	KF	53,79%	92,9%	80,1%	163 014	273 435	6,90	40,30	67,74%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			XGB		93,4%	86,8%	115 297	248 982	4,55	28,64	115,95%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			CART		92,3%	90,3%	7 522 978	7 058 487	293,94	382,06	-6,17%
XGB 93,4% 91,2% 8 219 857 8 265 167 324,94 442,88 0,55% A-31 4627 RF XGB 90,18% 92,3% 83,6% - 2 506 749 - 139 983 - 97,95 - 121,44 94,42% A-31 4627 RF XGB 90,18% 92,9% 83,5% - 1 969 564 73 101 - 83,10 111,49 103,71% A-31 4627 RF XGB 90,18% 92,9% 83,5% - 2 160 719 76 641 - 85,45 93,28 103,55% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 220 695 5 811 833 203,94 583,55 11,32% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45%	A-30	6179	RF	24.36%	92,9%	90,6%	7 534 275	7 068 293	318,08	411,80	-6,18%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	11.00		XGB	27,5070	93,4%	91,2%	8 219 857	8 265 167	324,94	442,88	0,55%
A-314627RF XGB90,18%92,9%83,5% 93,4%-1 969 56473 101 76 641- 83,10111,49103,71%A-325679RF XGB51,25%92,9%86,8%5 220 6955 811 833203,94583,5511,32%A-325679RF XGB51,25%92,9%86,9%5 401 6965 269 505228,08605,69-2,45%A-325679RF XGB51,25%93,4%87,4%5 943 8416 682 230234,94670,0312,42%			СТЪРТ		92.3%	83.6%	- 2 506 749	- 130 082	_ 07 05	- 121 44	QA 17%
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.52=	RE	00 10~	07 00 ²	82 50/0	- 1 060 564	- 1 <i>39 9</i> 03 72 101	- 91,95	111 40	27,4270 103 710/
A-32 5679 RF XGB 51,25% 92,3% 86,8% 5 220 695 5 811 833 203,94 583,55 11,32% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45%	A-31	4627	XCP	90,18%	92,9% 02 101	82 601	- 1 202 J04 2 160 710	75 101	- 05,10	02 70	103,71%
A-32 5679 RF XGB 51,25% 92,3% 86,8% 5 220 695 5 811 833 203,94 583,55 11,32% A-32 5679 RF XGB 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45% 11,32% 93,4% 87,4% 5 943 841 6 682 230 234,94 670,03 12,42%			AUD		73,4%	05,0%	- 2 100 / 19	/0.041	- 05,45	93,28	105,55%
A-32 5679 RF 51,25% 92,9% 86,9% 5 401 696 5 269 505 228,08 605,69 -2,45% XGB 93,4% 87,4% 5 943 841 6 682 230 234,94 670,03 12,42%			CART		92,3%	86,8%	5 220 695	5 811 833	203,94	583,55	11,32%
XGB 93,4% 87,4% 5 943 841 6 682 230 234,94 670,03 12,42%	A-32	5679	RF	51,25%	92,9%	86,9%	5 401 696	5 269 505	228,08	605,69	-2,45%
			XGB	-	93,4%	87,4%	5 943 841	6 682 230	234,94	670,03	12,42%

			Model % target diff	Accuracy		Retention gain		RG/target		T
IN ²	time (s)	Model		y_i	\tilde{y}_i	y_i	\tilde{y}_i	y_i	\tilde{y}_i	improvement
		CART		92.3%	84 0%	- 627 165	- 148 913	- 24 46	- 65 19	76 26%
D 1	4770	RF	75.000	92,5%	83.7%	- 280 855	11 973	- 11 84	9 57	104 26%
B-1	4//8	XGB	/5,89%	92,970 93.4%	84.1%	- 366 103	25 099	- 14 47	12 30	104,20%
		AOD		JJ, 1 /0	04,170	- 500 105	25 077	- 14,47	12,50	100,0070
		CART		92,3%	89,7%	3 862 156	3 397 247	150,95	203,11	-12,04%
B-2	6074	RF	29,70%	92,9%	89,9%	4 127 224	3 550 730	174,26	230,67	-13,97%
		XGB		93,4%	90,6%	4 451 686	4 408 819	175,99	250,17	-0,96%
-		CART		92.3%	83.2%	- 2 929 448	- 85 465	- 114.46	- 1 482.06	97.08%
D 2	4528	RF	06 600	92.9%	83.3%	- 2 413 433	3 724	- 101.84	- 108.33	100.15%
D-3		XGB	90,00%	93,4%	83,3%	- 2 642 119	9 092	- 104,47	93,79	100,34%
		CAPT		07.3%	85.0%	1 550 874	2 471 262	60.05	320.63	58 13%
		DE	60.00%	02,5%	85.8%	1 00/ 6/5	2 471 202	84.26	422,45	26 10%
B-4	5476	XGB	60,93%	92,970 03 1%	86.3%	2 175 670	3 080 807	85.00	422,43	20,1970 42.02%
		AGD)J, + //	00,5%	2 175 070	5 005 057	05,77	422,77	42,0270
		CART		92,3%	83,4%	- 857 439	- 159 856	- 33,45	- 218,16	81,36%
B-5	4708	RF	84,33%	92,9%	83,3%	- 484 459	40	- 20,44	7,23	100,01%
		XGB		93,4%	83,3%	- 596 203	897	- 23,57	46,96	100,15%
		CART		92,3%	88,8%	705 721	922 490	27,69	60,21	30,72%
B-6	5906	RF	36 63%	92,9%	88,9%	1 352 182	1 269 349	57,11	97,63	-6,13%
D -0	5700	XGB	50,0570	93,4%	89,6%	1 342 882	1 428 722	53,09	96,76	6,39%
		CART		92.3%	83.2%	- 3 159 722	- 39 633	- 123 45	- 1 230 61	98 75%
D 7	4400	RF	00 400	92,9%	83.3%	- 2 617 037	1 024	- 110 44	0.56	100.04%
B-7	4400	XGB	98,49%	92,9 <i>%</i>	83.3%	- 2 872 219	295	- 113 57	10,30	100,04%
		AOD		JJ, + //	05,570	- 2 072 21)	275	- 115,57	17,51	100,0170
		CART		92,3%	84,6%	- 1 596 562	169 852	- 62,31	41,78	110,64%
B-8	5278	RF	73,18%	92,9%	84,6%	- 780 396	637 625	- 32,89	194,52	181,71%
		XGB		93,4%	85,0%	- 933 133	780 845	- 36,91	188,79	183,68%
	4601	CART		92,3%	83,3%	- 2 380 789	- 113 444	- 92,86	- 840,25	95,24%
R_0		RF	94 12%	92,9%	83,3%	- 1 403 468	317	- 59,21	7,96	100,02%
D-)	4001	XGB	77,1270	93,4%	83,3%	- 1 724 731	3 980	- 68,17	149,44	100,23%
		САРТ		02.20/	80.00/	760 440	420 106	20.25	20.80	156 1107
				92,5%	09,0 <i>%</i>	- 700 449	429 190	- 29,55	29,60	15 100
B-10	5947	КГ VCP	35,98%	92,9%	00, <i>37</i> 0 00, <i>00</i> /	1 175 540 971 455	1 334 131	49,71	06.25	13,19%
		AUD		95,4%	09,0%	0/1 433	1 430 080	54,40	90,23	07,09%
		CART		92,3%	83,3%	- 4 683 072	- 48 985	- 182,86	- 1 186,22	98,95%
B-11	4229	RF	99,16%	92,9%	83,3%	- 3 536 046	-	- 149,21	-	100,00%
		XGB		93,4%	83,3%	- 4 000 747	-	- 158,17	-	100,00%
		CART		92,3%	85,0%	- 3 062 732	- 388 289	- 119,35	- 80,44	87,32%
B-12	5301	RF	66 76%	92,9%	84,7%	- 957 039	710 688	- 40,29	220,55	174,26%
D 12	5571	XGB	00,7070	93,4%	85,3%	- 1 404 561	834 198	- 55,52	163,88	159,39%
		CART		92.3%	83.3%	- 2 358 179	- 159 922	- 91 98	- 2 793 13	93 22%
D 12	4402	RF	06 200	92,9%	83 3%	- 1 384 098		- 58 40		100.00%
B-13	4493	XGB	96,30%	93.4%	83 3%	- 1 705 577	_	- 67 42	_	100,00%
				02.00	03,570	2 251 5(2	1 7 (1 001	10((2	1 42 20	100,00%
B-14				92,3%	87,8%	- 3 251 /62	- 1 /61 821	- 126,63	- 143,20	45,82%
	5851	KF	42,98%	92,9%	86,4%	- 1 013 089	19 213	- 42,69	11,90	107,82%
		XGB		93,4%	83,3%	- 1 582 006	4 396	- 62,52	287,68	100,28%
B-15	4040	CART		92,3%	83,3%	- 4 660 462	- 38 969	- 181,98	- 2 075,03	99,16%
		RF	99.67%	92,9%	83,3%	- 3 516 676	-	- 148,40	-	100,00%
		XGB	,	93,4%	83,3%	- 3 981 592	161	- 157,42	10,53	100,00%
		CART		92.3%	84.2%	- 5 554 044	- 1 491 522	- 216 63	- 549 23	73 15%
D 16	5100	RF	77 070	92.9%	83.6%	- 3 145 668	52 475	- 132.69	84 54	101.67%
B-10	5182	XGB	11,91%	93.4%	83 3%	- 3 858 022		- 152,09		100.00%
					55,570	2 330 022				100,0070

NIO	·· · · · ·	Model	% target diff	Accuracy		Retention gain		RG/target		τ
IN ²	time (s)			y_i	\tilde{y}_i	y_i	\tilde{y}_i	y_i	\tilde{y}_i	mprovement
		CART		92.3%	88.9%	3 361 471	3 037 200	131.25	191 31	-9 65%
D 17	5204	RF	22 ((1)	92,5%	89.3%	3 241 680	2 911 023	136.86	204.43	-10 20%
B-1/	5324	XGB	32,66%	93.4%	89.6%	3 596 593	3 546 671	142 15	204,43	-1 39%
		AOD		JJ, + 70	07,070	5 570 575	5 540 071	142,15	222,04	-1,5770
		CART		92,3%	91,1%	39 860 670	37 695 680	1 556,66	1 778,71	-5,43%
B-18	6411	RF	13,83%	92,9%	91,7%	35 787 050	32 345 100	1 510,72	1 654,32	-9,62%
		XGB		93,4%	92,0%	39 908 670	40 886 810	1 577,61	1 848,71	2,45%
		CART		92,3%	84,7%	1 059 189	1 813 631	41,25	392,14	71,23%
B-10	4853	RF	70 34%	92,9%	84,8%	1 109 101	1 808 616	46,86	474,33	63,07%
D 1)	1000	XGB	70,5470	93,4%	85,0%	1 320 578	2 141 271	52,15	482,34	62,15%
		CART		92.3%	89.2%	37 558 390	34 068 550	1 466 66	2 125 97	-9 29%
D 20	5072	RF	21760	92,9%	89.4%	33 654 470	30.032.580	1 420 72	2 072 47	-10.76%
B-20	5973	XGB	31,76%	93.4%	90.1%	37 632 650	38 008 480	1 420,72	2 072,47	1 00%
				22,170	20,1%	1 12 6 050	1 150 005	1 107,01	2 277,17	1,00%
		CART		92,3%	87,7%	1 136 879	1 1/9 837	44,40	92,50	3,78%
B-21	5228	KF	41,79%	92,9%	88,1%	1 276 808	1 188 256	53,91	104,81	-6,94%
		XGB		93,4%	88,7%	1 385 145	1 356 864	54,74	104,76	-2,04%
		CART		92,3%	90,7%	18 704 980	17 177 190	730,55	852,81	-8,17%
B-22	6296	RF	19.52%	92,9%	91,1%	17 182 100	15 732 340	725,34	859,29	-8,44%
		XGB		93,4%	91,5%	19 071 370	19 050 020	753,90	939,00	-0,11%
		CART		92.3%	84.1%	- 1 165 404	458 223	- 45.60	172.83	139.32%
P 23	1716	RF	81 36%	92.9%	84.0%	- 855 770	525 335	- 36.09	288.55	161.39%
D-23	4740	XGB	81,50%	93,4%	84,1%	- 890 871	645 445	- 35,26	310,86	172,45%
		САРТ		02.20/	00 701	16 402 700	15 012 210	640.55	1 002 42	Q 1701
				92,5%	00,270 00 101	10 402 700	13 013 310	625 24	1 1095,45	-0,4770
B-24	5845	КГ VCP	40,47%	92,9%	00,4 <i>%</i> 00,00/	15 049 520	13 423 040	662.00	1 122,01	-10,61%
		AUD		93,470	00,970	10 795 500	17 144 200	003,90	1 247,30	2,0870
		CART		92,3%	88,6%	1 607 847	1 839 864	62,84	126,33	14,43%
B-25	5274	RF	37,42%	92,9%	88,7%	2 119 067	1 923 982	89,49	152,71	-9,21%
		XGB		93,4%	89,2%	2 237 965	2 194 469	88,45	155,54	-1,94%
		CART		92,3%	91,1%	35 238 060	32 690 970	1 376,37	1 558,26	-7,23%
B-26	6425	RF	14 83%	92,9%	91,6%	32 835 370	29 986 540	1 386,17	1 543,12	-8,68%
B 20	0125	XGB	11,0570	93,4%	92,0%	36 328 440	36 803 630	1 436,10	1 688,53	1,31%
		CART		92.3%	84 3%	- 694 436	751 404	- 27 16	226 99	208 20%
D 27	4011	RF	72 0.201	92,9%	84.4%	- 13 512	1 018 369	- 0 51	356.48	7636 98%
B-27	4811	XGB	13,92%	93.4%	84.7%	- 38 050	1 253 252	- 1.55	345.94	3393.68%
				00.20	00.10/	22.025.780	20.242.020	1 00(27	1 0 47 71	10.010
				92,3%	89,1%	32 935 780	29 342 930	1 286,37	1 847,71	-10,91%
B-28	5995	KF	32,61%	92,9%	89,4%	30 /02 /90	27 725 620	1 296,17	1 933,38	-9,70%
		XGB		93,4%	90,0%	34 052 420	34 390 060	1 346,10	2 094,90	0,99%
		CART		92,3%	87,3%	- 363 861	55 985	- 14,12	3,38	115,39%
B-29	5143	RF	47,03%	92,9%	87,4%	377 170	488 284	15,95	49,62	29,46%
		XGB	,	93,4%	88,0%	275 772	491 567	10,89	44,89	78,25%
B-30		CART		92,3%	90,7%	14 747 500	13 838 380	576,23	690,22	-6,16%
	6243	RF 20,4 XGB	20.47%	92,9%	91,1%	14 816 830	13 378 460	625,54	743,34	-9,71%
	0215		20,4770	93,4%	91,5%	16 146 490	16 169 440	638,30	814,80	0,14%
		CART		92.3%	83.7%	- 2 666 144	- 487 716	- 104 12	- 267 75	81 71%
D 21	4720	RF	02.02%	92,5%	83.7%	- 1 755 409	139 545	- 74 05	102.66	107.95%
B-31	4730	XGR	83,83%	93.4%	83.7%	- 2,000,244	134 100	- 79 11	130.13	106 71%
				00.00	00.0%	10 445 210	11 (02 050	106.22	004.40	
		CART		92,3%	88,0%	12 445 210	11 693 070	486,23	884,49	-6,04%
B-32	5865	5865 RF	41,41%	92,9%	88,3%	12 684 250	11 381 260	535,54	9/1,28	-10,27%
		XGB		93,4%	88,8%	13 8/0 4/0	14 101 470	548,30	1 048,38	1,67%

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