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► **To cite this version:**

Pedro Liguori, Ali Ridha Mahjoub, Guillaume Marques, Ruslan Sadykov, Eduardo Uchoa. Non-Robust Strong Knapsack Cuts for Capacitated Location-Routing and Related Problems. Première journée commune ROADEF-AIRO, Nov 2022, Virtual, France. hal-03899418

**HAL Id: hal-03899418**

**<https://hal.science/hal-03899418>**

Submitted on 14 Dec 2022

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# Non-Robust Strong Knapsack Cuts for Capacitated Location-Routing and Related Problems

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Online, 30/11/2022

# Capacitated Location-Routing Problem (CLRP)

## Data

- ▶  $I$  — set of potential depots with opening costs  $f_i$  and capacities  $W_i$ ,  $i \in I$
- ▶  $J$  — set of customers with demands  $d_j$ ,  $j \in J$
- ▶ Set of edges  $E = E_J \cup E_{IJ}$ :  $E_J = J \times J$ ,  $E_{IJ} = I \times J$
- ▶  $c_e$  — transportation cost of edge  $e \in E \cup F$
- ▶ An unlimited set of vehicles with capacity  $Q$ .

## The problem

- ▶ Decide which depots to open
- ▶ Assign every client to an open depot subject to depot capacity
- ▶ For every depot, divide assigned clients into routes subject to vehicle capacity
- ▶ Minimize the total depot opening and transportation cost

# CLRP: an illustration

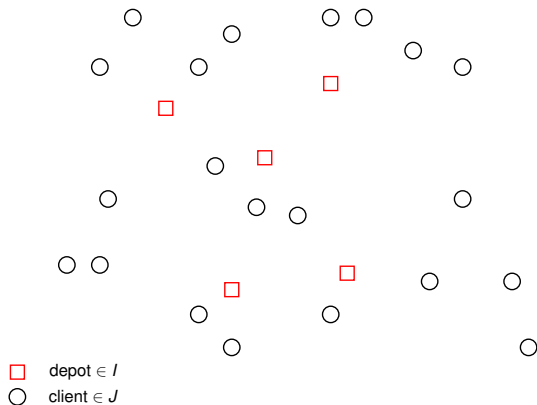
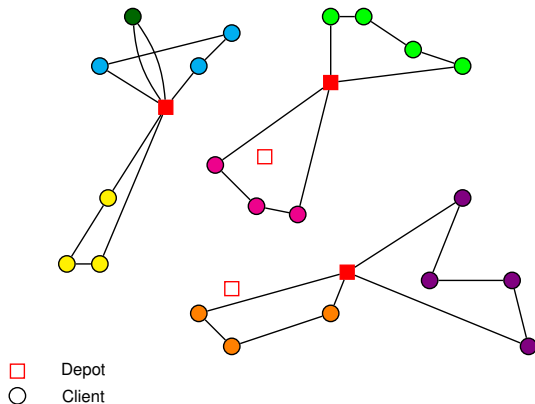


Figure: LRP instance:  $G = (I \cup J, E_J \cup E_{IJ})$

## CLRP: a solution



**Figure:** Location of depots must be jointly decided with vehicle routing.

# Literature on the CLRP

- ▶ A combination of **two central OR problems**
- ▶ **≈3700 papers** in Google Scholar with both “location” and “routing” in the title

## Some recent works on the standard CLRP

- ▶ [Belenguer et al. 2011] — important valid inequalities & Branch-and-Cut
- ▶ [Baldacci et al. 2011] — exact enumeration & column generation approach
- ▶ [Contardo et al. 2014]<sup>1</sup> — state-of-the-art exact algorithm
- ▶ [Arnold and Sörensen 2021; Schneider and Löffler 2019] — state-of-the-art heuristics
- ▶ [Schneider and Drexl 2017] — the latest survey

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<sup>1</sup>Claudio Contardo, Jean-François Cordeau, and Bernard Gendron (2014). “An Exact Algorithm Based on Cut-and-Column Generation for the Capacitated Location-Routing Problem”. In: *INFORMS Journal on Computing* 26.1, pp. 88–102.

# Our study

- ▶ Recently, **large improvement in exact solution** of classic VRP variants [Pecin et al. 2017] [Pecin et al. 2017] [Pessoa et al. 2018] [Sadykov et al. 2021]
- ▶ A **generic Branch-Cut-and-Price VRP solver** [Pessoa et al. 2020]<sup>2</sup> incorporates all recent advances

**[vrpsolver.math.u-bordeaux.fr](http://vrpsolver.math.u-bordeaux.fr)**

- ▶ This solver can be applied to the LRP, but **problem-specific cuts are necessary** for obtaining the state-of-the-art performance
- ▶ We propose **a new family of non-robust cuts**, which is shown to be useful for the LRP and some related problems

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<sup>2</sup>Artur Pessoa, Ruslan Sadykov, Eduardo Uchoa, and François Vanderbeck (2020). “A Generic Exact Solver for Vehicle Routing and Related Problems”. In: *Mathematical Programming* 183, pp. 483–523.

## Formulation

- ▶  $\lambda_r^i$ ,  $i \in I$ ,  $r \in R_i$ , equals 1 iff route  $r$  is used for depot  $i$
- ▶  $a_e^r$ ,  $e \in E_J \cup E_{IJ}$ ,  $r \in \cup_{i \in I} R_i$ , equals 1 iff edge  $e$  is used by  $r$
- ▶  $b_j^i$ ,  $j \in J$ ,  $r \in \cup_{i \in I} R_i$ , equals 1 iff client  $j$  is served by route  $r$
- ▶  $y_i$ ,  $i \in I$ , equals 1 iff route depot  $i$  is open
- ▶  $z_{ij}$ ,  $i \in I$ ,  $j \in J$ , equals 1 iff client  $j$  is assigned to depot  $i$

$$\min \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{r \in R_i} \sum_{e \in E_{IJ}} c_e a_e^r \lambda_r^i$$

$$\sum_{i \in I} z_{ij} = 1, \quad \forall j \in J,$$

$$\sum_{r \in R_i} b_j^i \lambda_r^i = z_{ij}, \quad \forall i \in I, j \in J$$

$$\sum_{j \in J} d_j z_{ij} \leq W_i y_i, \quad \forall i \in I,$$

$$z_{ij} \leq y_i, \quad \forall i \in I, j \in J,$$

$$(z, y, \lambda) \in \{0, 1\}^K$$



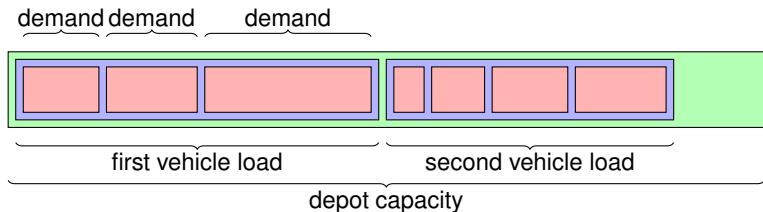
## Nested knapsack structure

- ▶  $\lambda_r^i$ ,  $i \in I$ ,  $r \in R_i$ , equals 1 iff route  $r$  is used for depot  $i$
- ▶  $b_j^r$ ,  $j \in J$ ,  $r \in \cup_{i \in I} R_i$ , equals 1 iff client  $j$  is served by route  $r$
- ▶  $z_{ij}$ ,  $i \in I$ ,  $j \in J$ , equals 1 iff client  $j$  is assigned to depot  $i$

$$\sum_{j \in J} d_j z_{ij} \leq W_i, \quad \forall i \in I,$$

↓

$$\sum_{r \in R_i} \left( \sum_{j \in J} \sum_{e \in \delta(j)} b_e^r d_j \right) \lambda_r^i \leq W_i, \quad \forall i \in I$$



# Master knapsack polytope

$$\mathcal{P}_{\text{MKP}}(W) = \text{conv} \left\{ t \in \mathbb{Z}_+^W : \sum_{q=1}^W q t_q \leq W \right\}.$$

Theorem ([Aráoz 1974]<sup>3</sup>)

*Each non-trivial facet of  $\mathcal{P}_{\text{MKP}}(W)$  can be described by an inequality of format  $\xi t \leq 1$  such that  $\xi \in \mathbb{R}_+^W$  is an extreme point of the following system of linear constraints:*

$$\begin{aligned} \xi_1 &= 0, & \xi_W &= 1, \\ \xi_q + \xi_{W-q} &= 1, & \forall 1 \leq q \leq W/2, \\ \xi_q + \xi_{q'} &\leq \xi_{q+q'}, & \forall q + q' \leq W. \end{aligned}$$

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<sup>3</sup>J. Aráoz (1974). “Polyhedral neopolarities”. PhD thesis. University of Waterloo, Department of Computer Science.

## Route Load Knapsack Cuts (RLKC)

$\theta_q^i \in \mathbb{Z}_+$  — # of routes in  $R_i$  with a total load of exactly  $q$  units.

### Theorem

$\xi t \leq 1$  defines a non-trivial facet of  $\mathcal{P}_{MKP}(W_i)$  if and only if

$\xi \theta \leq y_i$  defines a non-trivial facet of

$$\text{conv} \left\{ (\theta^i, y_i) \in \mathbb{Z}_+^{W_i} \times \{0, 1\} : \sum_{q=1}^{W_i} q \theta_q \leq W_i y_i \right\}.$$

### Definition

Given a depot  $i \in I$  and a vector  $\xi \in \mathbb{R}_+^{W_i}$  satisfying  $\xi_1 = 0$ ,  $\xi_{W_i} = 1$ , and  $\xi_q + \xi_{q'} \leq \xi_{q+q'}, \forall q + q' \leq W_i$ , the inequality

$$\sum_{q=1}^{W_i} \xi_q \theta_q^i \leq y_i$$

is known as a *Route Load Knapsack Cut* (RLKC).

### Theorem

A *Route Load Knapsack Cut* is valid for the location-routing formulation.

# Separation of RLKCs by Chvátal-Gomory rounding

For any depot  $i \in I$  and any multiplier  $\beta \in \mathbb{R}$  such that  $\beta \geq 1/W_i$ , the constraint

$$\sum_{q=1}^{W_i} \frac{\lfloor \beta q \rfloor}{\lfloor \beta W_i \rfloor} \theta_q^i \leq y_i$$

is a Route Load Knapsack Cut: superadditivity follows from  $\lfloor r \rfloor + \lfloor r' \rfloor \leq \lfloor r + r' \rfloor$  for all  $r, r' \in \mathbb{R}_+$ .

## Separation by enumeration

We consider all multipliers  $\beta = p/q$ , such that  $\bar{\theta}_q > 0$  and  $p = 1, \dots, q - 1$ .

# Separation of $1/k$ -facets

## Definition

A master knapsack facet  $\xi x \leq 1$  is called a  $1/k$ -facet if  $k$  is the smallest possible integer such that

$$\xi_q \in \{0/k, 1/k, 2/k, \dots, k/k\} \cup \{1/2\}.$$

## Separation by enumeration

We enumerate all possible  $1/6$ -,  $1/8$ -, and  $1/10$ -inequalities using the algorithm from [Chopra et al. 2015]<sup>4</sup>.

## Remark

An  $1/k'$ -inequality is also an  $1/k$ -inequality if  $k'$  is a divisor of  $k$ . So,  $1/2$ -,  $1/3$ -,  $1/4$ -, and  $1/5$ -inequalities are also separated.

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<sup>4</sup>Sunil Chopra, Sangho Shim, and Daniel E. Steffy (2015). “A few strong knapsack facets”. In: *Modeling and Optimization: Theory and Applications*. Ed. by Boris Defourny and Tamás Terlaky. Cham: Springer International Publishing, pp. 77–94.

## Route Load Knapsack Cuts: an example

Data (depot index  $i$  is omitted)

$Q = 70$ ,  $W = 140$ ,  $\bar{\theta}_{38} = 1/14$ ,  $\bar{\theta}_{53} = 1/2$ ,  $\bar{\theta}_{65} = 16/14$ ,  
 $\bar{\theta}_{70} = 1/2$ , and  $\bar{y} = 277/280$ .

Best RLKC obtained by rounding

Multiplier  $\beta = 1/53$  gives a RLKC with violation of  $\approx 0.082$ :

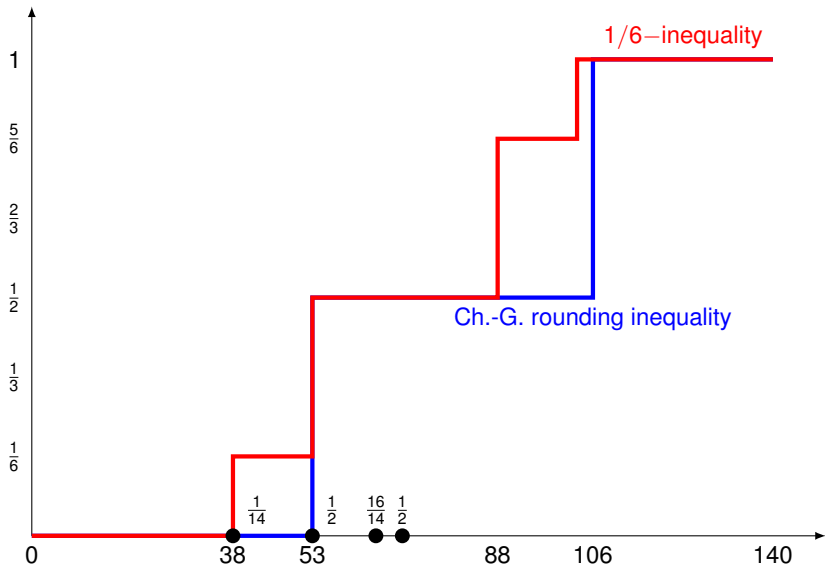
$$\sum_{q=53}^{105} \frac{1}{2} \theta_q + \sum_{q=106}^{140} \theta_q \leq y$$

A better facet-defining  $1/6$ -inequality

$$\sum_{q=38}^{52} \frac{1}{6} \theta_q + \sum_{q=53}^{87} \frac{3}{6} \theta_q + \sum_{q=88}^{102} \frac{5}{6} \theta_q + \sum_{q=103}^{140} \frac{6}{6} \theta_q \leq y$$

with violation  $\approx 0.094$ .

# Route Load Knapsack Cuts: an example



## RLKCs and the pricing: label domination

- ▶ Pricing problem: **Resource Constrained Shortest Path**
- ▶ It is solved by a labelling algorithm, each label  $L$  is  $(j^L, \bar{c}^L, q^L)$
- ▶ Dominance relation

$$L \succ L' \quad \text{if } j^L = j^{L'}, \bar{c}^L \leq \bar{c}^{L'}, q^L \leq q^{L'}$$

- ▶ Let  $\bar{\mu}(q)$  be the contribution of RLKCs to the reduced cost of a route variable with load  $q$
- ▶ The same dominance relation is still valid, as  $\bar{\mu}(q)$  is **non-decreasing**:

$$\bar{c}^L \leq \bar{c}^{L'}, q^L \leq q^{L'} \quad \Rightarrow \quad \bar{c}^L + \bar{\mu}(q^L) \leq \bar{c}^{L'} + \bar{\mu}(q^{L'}).$$



## RLKCs and the pricing: completion bounds

- ▶ A completion bound  $B(j, \tilde{\mathcal{L}})$  is valid if  $\bar{c}^{\vec{L}} + \bar{\mu}(q^{\vec{L}}) + B(j, \tilde{\mathcal{L}})$  gives a lower bound on the total reduced cost of paths obtained by concatenation of  $\vec{L}$  and any  $\vec{L} \in \tilde{\mathcal{L}}$  at node  $j$ .
- ▶ A weak completion bound

$$B_1(j, \tilde{\mathcal{L}}) = \min_{\vec{L} \in \tilde{\mathcal{L}}} \{ \bar{c}^{\vec{L}} \}$$

- ▶ A tighter completion bound

$$B_2(j, \tilde{\mathcal{L}}) = \min_{\vec{L} \in \tilde{\mathcal{L}}} \{ \bar{c}^{\vec{L}} + \bar{\mu}(q^{\vec{L}}) \}$$

is valid due to **super-additivity of  $\bar{\mu}(q)$** .

- ▶ Completion bounds are used to speed-up labels concatenation and prune labels by bound

# Rounded Capacity Cuts (RCC)<sup>5</sup>

Given a subset of clients  $C \subset J$ ,

$$\sum_{i \in I} \sum_{r \in R_i} \sum_{e \in \delta(C)} a_e^r \lambda_r^i \geq 2 \cdot \left\lceil \frac{\sum_{i \in C} d_i}{Q} \right\rceil.$$

## Separation

Heuristic algorithms by [Lysgaard et al. 2004]: connected components, max-flow based, greedy construction, local search on previously separated cuts (CVRPSEP reimplemented by us)

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<sup>5</sup>G. Laporte and Y. Nobert (1983). “A branch and bound algorithm for the capacitated vehicle routing problem”. In: *Operations-Research-Spektrum* 5.2, pp. 77–85.

## Depot Capacity Cuts (DCC)<sup>6</sup>

If a subset of clients  $C \subset J$  cannot be served by a subset of depots  $S \subset I$ ,

$$\sum_{j \in C} d_j > \sum_{i \in S} W_i,$$

then at least one vehicle from a depot  $i \in I \setminus S$  should visit  $C$ :

$$\sum_{i \in I \setminus S} \sum_{r \in R_i} \sum_{e \in \delta(C)} a_e^r \lambda_r^i \geq 2.$$

## Separation

A greedy construction heuristic starting from different seed vertices

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<sup>6</sup>José-Manuel Belenguer, Enrique Benavent, Christian Prins, Caroline Prodhon, and Roberto Wolfler Calvo (2011). "A Branch-and-Cut method for the Capacitated Location-Routing Problem". In: *Computers & Operations Research* 38.6, pp. 931–941.

## COVER inequalities for depots (COV)

Given a subset  $J' \subset J$  of customers, such that  $\sum_{j \in J'} d_j > W_i$ , the following inequality is valid

$$\sum_{j \in J'} z_{ij} \leq (|J'| - 1)y_i.$$

### Separation

We solve the MIP for each  $i \in I$  such that  $\bar{y}_i > 0$

$$\begin{aligned} \min \sum_{j \in J} (\bar{y}_i - \bar{z}_{ij}) w_j \\ \sum_{j \in J} d_j w_j \geq W_i + 1, \\ w_j \in \{0, 1\}, \quad \forall j \in J, \end{aligned}$$

to check if its solution is less than  $\bar{y}_i$ .

## Fenchel Cuts over $y$ variables (FC)<sup>7</sup>

$\hat{Y}$  is the set set of feasible depot configurations

$$\hat{Y} = \left\{ \hat{y} \in \{0, 1\}^{I|I} : \sum_{i \in I} w_i \hat{y}_i \geq \sum_{j \in J} d_j \right\}.$$

### Separation of $\bar{y} \in \text{conv}(\hat{Y})$

We try to find  $\alpha_i \in \mathbb{R}_+^{I|I}$  such that  $\sum_{i \in I} \alpha_i \bar{y} < 1$ , and  $\sum_{i \in I} \alpha_i \hat{y} \geq 1$  for all  $\hat{y} \in \hat{Y}$ , by solving the LP.

$$\begin{aligned} \min \quad & \sum_{i \in I} \bar{y}_i \alpha_i \\ & \sum_{i \in I} \hat{y}_i \alpha_i \geq 1, \quad \forall \hat{y} \in \hat{Y}, \\ & \alpha_i \geq 0, \quad \forall i \in I. \end{aligned}$$

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<sup>7</sup>Maurizio Boccia, Antonio Sforza, Claudio Sterle, and Igor Vasilyev (2008). "A Cut and Branch Approach for the Capacitated p-Median Problem Based on Fenchel Cutting Planes". In: *Journal of Mathematical Modelling and Algorithms* 7.1, pp. 43–58.

## Chvátal-Gomory Rank-1 Cuts [Jepsen et al. 2008]

Each cut is obtained by a **Chvátal-Gomory rounding of a set  $C \subseteq J$  of set packing constraints** using a vector of multipliers  $\rho$  ( $0 < \rho_j < 1, j \in C$ ):

$$\sum_{i \in I} \sum_{r \in R_i} \left[ \sum_{j \in C} \rho_j \sum_{e \in \delta(j)} \frac{1}{2} a_e^r \right] \lambda_r^i \leq \left[ \sum_{j \in C} \rho_j \right]$$

All non-dominated vectors  $\rho$  of multipliers for  $|C| \leq 5$  are given in [Pecin et al. 2017].

**Non-robust** in the terminology of [Pessoa et al. 2008]

### Separation

Enumeration for  $|C| \leq 3$  and a local search heuristic for each non-dominated vector of multipliers for  $|C| = \{4, 5\}$ .

## Other components of the BCP (through VRPSolver)

- ▶ Bucket graph-based labelling algorithm for the RCSP pricing [Righini and Salani 2006] [Sadykov et al. 2021]
- ▶ Partially elementary path (*ng*-path) relaxation [Baldacci et al. 2011]
- ▶ Automatic dual price smoothing stabilization [Wentges 1997] [Pessoa et al. 2018]
- ▶ Reduced cost fixing of (bucket) arcs in the pricing problem [Ibaraki and Nakamura 1994] [Irnich et al. 2010] [Sadykov et al. 2021]
- ▶ Enumeration of elementary routes [Baldacci et al. 2008]
- ▶ Multi-phase strong branching [Pecin et al. 2017]
  - ▶ On number of open depots in a subset of size at most 4 (largest priority)
  - ▶ On number of vehicles starting in a depot
  - ▶ On the total number of vehicles
  - ▶ On number of clients served from a depot
  - ▶ On assignment of clients to depots
  - ▶ On edges of the graph

## Computational results: impact of cuts

“Classic” CLRP test instances by [Prins et al. 2006]<sup>8</sup> with 5-10 depot locations and 50-200 clients. Time limit is 12 hours.

BCP<sub>0</sub> — “pure” VRPSolver (without problem-specific cuts)

| Variant                 | Root  |          | Nodes | Geomean  |        |
|-------------------------|-------|----------|-------|----------|--------|
|                         | Gap   | Time (s) |       | Time (s) | Solved |
| BCP <sub>0</sub>        | 4.46% | 57.9     | 19.2  | 758.7    | 24/26  |
| BCP <sub>all-GUB</sub>  | 3.08% | 99.0     | 9.0   | 481.0    | 25/26  |
| BCP <sub>all-DCC</sub>  | 0.85% | 101.0    | 9.1   | 504.6    | 24/26  |
| BCP <sub>all-FC</sub>   | 0.67% | 111.4    | 4.4   | 283.9    | 25/26  |
| BCP <sub>all-RLKC</sub> | 0.52% | 114.7    | 4.1   | 264.0    | 25/26  |
| BCP <sub>all-COV</sub>  | 0.49% | 114.4    | 4.6   | 273.4    | 25/26  |
| BCP <sub>all</sub>      | 0.48% | 115.0    | 4.1   | 265.5    | 25/26  |

<sup>8</sup>Christian Prins, Caroline Prodhon, and Roberto Wolfler Calvo (2006).

“Solving the capacitated location-routing problem by a GRASP complemented by a learning process and a path relinking”. In: *4OR* 4.3, pp. 221–238.



## Computational results: modified instances

$$\rho = \frac{Q}{\sum_{i \in I} w_i / |I|} \text{ — “vehicle capacity / depot capacity” ratio}$$

| Variant                 | $\rho$ | Root  |          | Nodes | Geomean  |        |
|-------------------------|--------|-------|----------|-------|----------|--------|
|                         |        | Gap   | Time (s) |       | Time (s) | Solved |
| BCP <sub>0</sub>        | 0.3    | 2.14% | 32.1     | 18.6  | 344.7    | 19/20  |
| BCP <sub>all-DCC</sub>  | 0.3    | 0.74% | 41.0     | 12.7  | 260.9    | 19/20  |
| BCP <sub>all-RLKC</sub> | 0.3    | 0.51% | 48.2     | 6.1   | 174.1    | 19/20  |
| BCP <sub>all</sub>      | 0.3    | 0.46% | 49.3     | 5.8   | 162.9    | 19/20  |
| BCP <sub>0</sub>        | 0.5    | 3.33% | 42.9     | 76.7  | 2513.6   | 13/17  |
| BCP <sub>all-DCC</sub>  | 0.5    | 2.09% | 108.4    | 35.0  | 1979.7   | 13/17  |
| BCP <sub>all-RLKC</sub> | 0.5    | 1.73% | 69.8     | 24.3  | 1059.3   | 14/17  |
| BCP <sub>all</sub>      | 0.5    | 1.26% | 120.3    | 13.2  | 813.6    | 15/17  |
| BCP <sub>0</sub>        | 0.7    | 5.94% | 51.6     | 255.3 | 10511.0  | 6/17   |
| BCP <sub>all-DCC</sub>  | 0.7    | 2.49% | 247.7    | 58.1  | 4531.6   | 12/17  |
| BCP <sub>all-RLKC</sub> | 0.7    | 3.91% | 83.0     | 89.4  | 5438.7   | 10/17  |
| BCP <sub>all</sub>      | 0.7    | 1.53% | 284.6    | 18.9  | 1734.7   | 14/17  |

## Cut generation statistics

# of generated cuts (# of active cuts at the end of the root)

| Cut family   | Original instances | Modified instances |                 |
|--------------|--------------------|--------------------|-----------------|
|              |                    | $\rho = 0.3$       | $\rho = 0.7$    |
| RCC          | 492.5 ( 11.6)      | 349.5 ( 7.2)       | 254.9 ( 1.9)    |
| Im-R1C       | 7044.0 (215.4)     | 22405.5 (153.0)    | 35610.9 (236.2) |
| COV          | 30.8 ( 0.2)        | 28.1 ( 0.1)        | 30.7 ( 0.1)     |
| FC           | 4.0 ( 0.7)         | 3.9 ( 0.6)         | 4.4 ( 0.5)      |
| GUB          | 338.5 ( 78.4)      | 282.4 ( 30.0)      | 203.2 ( 12.9)   |
| DCC          | 488.4 ( 9.9)       | 1135.1 ( 10.1)     | 1636.1 ( 11.4)  |
| RLKC (total) | 53.4 ( 1.2)        | 325.1 ( 3.9)       | 12785.2 ( 29.6) |
| RLKCround    | 26.8 ( 0.7)        | 296.1 ( 3.7)       | 109.9 ( 0.8)    |
| RLKC1/2      | 0.7 ( 0.0)         | 0.5 ( 0.0)         | 1401.8 ( 7.7)   |
| RLKC1/3      | 0.7 ( 0.0)         | 0.8 ( 0.0)         | 675.7 ( 1.6)    |
| RLKC1/4      | 1.2 ( 0.0)         | 1.4 ( 0.1)         | 418.2 ( 0.9)    |
| RLKC1/5      | 1.8 ( 0.0)         | 1.5 ( 0.0)         | 1738.4 ( 3.8)   |
| RLKC1/6      | 2.6 ( 0.0)         | 3.2 ( 0.1)         | 843.4 ( 1.7)    |
| RLKC1/8      | 5.7 ( 0.1)         | 5.0 ( 0.1)         | 2611.1 ( 4.9)   |
| RLKC1/10     | 13.8 ( 0.3)        | 16.6 ( 0.1)        | 4986.7 ( 8.2)   |

# Computational results: comparison with the literature

Time limit is 30 hours

| Instances | BCP <sub>all</sub> |      | [Contardo et al. 2014] |      |
|-----------|--------------------|------|------------------------|------|
|           | Solved             | Time | Solved                 | Time |
| PPW06     | 24/26              | 518  | 16/26                  | 836  |
| TB99      | 9/9                | 945  | 6/9                    | 5589 |

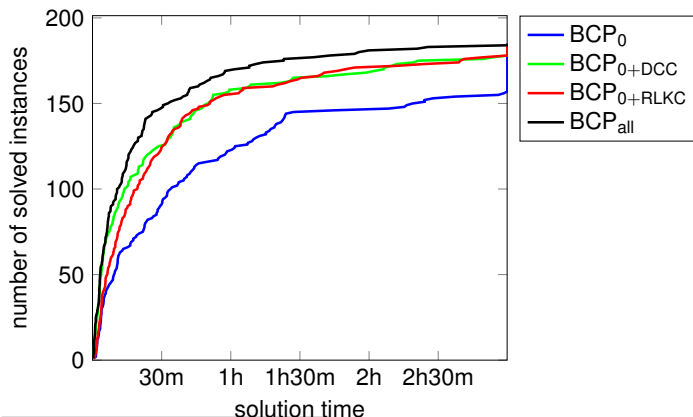
| Instances by [Schneider and Löffler 2019] <sup>9</sup> |       |        |              |             |  |
|--|-------|--------|--------------|-------------|--|
| $ I $  | $ J $ | Solved | Improved BKS | Improvement |  |
| 5  | 100   | 14/14  | 7/14         | 0.05%       |  |
| 10   | 100   | 14/14  | 5/14         | 0.11%       |  |
| 10   | 200   | 11/14  | 13/14        | 0.08%       |  |
| 15   | 200   | 15/20  | 18/20        | 0.12%       |  |
| 15   | 300   | 6/20   | 11/20        | 0.29%       |  |
| 20   | 300   | 4/20   | 8/20         | 0.91%       |  |

<sup>9</sup>Michael Schneider and Maximilian Löffler (2019). “Large Composite Neighborhoods for the Capacitated Location-Routing Problem”. In: *Transportation Science* 53.1, pp. 301–318.

## VRP-CMD instances

Instances from [Ben Mohamed et al. 2022]<sup>10</sup>, occur when solving the 2-echelon stochastic multi-period CLRP.

50 customers, 3-5 already opened depots

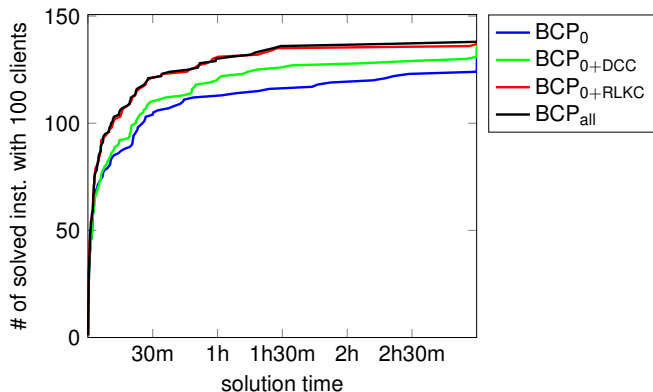


<sup>10</sup>Imen Ben Mohamed, Walid Klibi, Ruslan Sadykov, Halil Şen, and François Vanderbeck (2022). “The two-echelon stochastic multi-period capacitated location-routing problem”. In: *European Journal of Operational Research*.

# VRP with Time Windows and Shifts<sup>11</sup>

Instances with 25-100 customers and 3 shifts

Solved 421 from 504 instances in 30 min. ([Dabia et al. 2019]  
solved 280)






<sup>11</sup>Said Dabia, Stefan Ropke, and Tom van Woensel (2019). “Cover Inequalities for a Vehicle Routing Problem with Time Windows and Shifts”. In: *Transportation Science* 53.5, pp. 1354–1371.

# Conclusions

- ▶ A new **family of non-robust strong knapsack** cuts for the problems
- ▶ Exploited **monotonicity and superadditivity properties of cuts** to limit their impact on the pricing time
- ▶ These cuts make the **BCP algorithm more robust** (more harder instances can be solved)
- ▶ First exact algorithm for the CLRP which **can scale to instances with many depot locations**
- ▶ Good results for different problems with the **nested knapsack structure**.
- ▶ The paper has been accepted (subject to a minor revision) to the OR journal.

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