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Measuring information loss when modeling sick leave with weight mixture models

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Abstract

We consider a problem in actuarial sciences, in which we have M insurance companies (samples) and K groups of insured persons. We want to model sick leave for each group. However, the group label of each observation is not defined with certainty for confidential reasons. We assume that each insurance company can provide the proportion of each group in its database. In this case, we are going to compare two different Kaplan-Meier estimators for mixture models. We measure the information loss when modeling sick leaves with variable weight mixture models in censored duration models, comparing Ryzhov's and Maiboroda's methods, both using simulations and empirical data. The choice of the weights seems to have an impact on the quality of the estimation. We found that Ryzhov's estimator gives usually best results, especially in the case where the weights are quite close. But when the weights are further apart, it is the Maiboroda estimator that becomes the best, especially in the case of strong censorship. Using empirical data, the Maiboroda estimator also seems to give better results.

Keywords : mixture models, duration models, censored data, varying weights, sick leave

1 Introduction

Understanding the risks associated with sick leaves is very important for companies as well as for insurance organizations. Indeed, according to an IRDES paper [9], sick leave coverage plays an important health and economic role insofar as it allows individuals to have time to receive care and recover good health, while benefiting from the resources necessary to meet their basic needs. However, the drift away of sick leave creates additional costs that must be controlled. Modeling the duration of these leaves is therefore a central issue for good risk provisioning.

To analyse this actuarial data, it is recommended to have line-by-line data, with, for each leave that occurs, the start and end date of the sick leaves, the age and the average salary used (for the calculation of mathematical provisions), but also the socio-professional category and gender, to understand which populations are the most affected. However, these data are confidential and it can sometimes be difficult to access them, especially when they are not mandatory for the computation of the mathematical provisions.

Confidentiality protection and free information sharing are two major themes guiding data collection and dissemination, as explained by Duncan and Lambert [6]. To protect

privacy, one idea is to use ϵ-differential privacy, where the true data is perturbed by the addition of a random noise generated according to a chosen distribution, as studied by Dwork et al. [7]. The authors said that the goal of a privacy-preserving database is to analyse the properties of a population while protecting the privacy of the individual contributors. Duchi et al. [5] also worked in the local differential privacy framework and studied the tradeoff between privacy guarantees and the utility of the resulting statistical estimators.

For our part, we have chosen to imagine that we have access to the data but not, for each individual, to his or her socio-professional category or gender. Indeed, we can imagine that for confidential reasons, an insurance company cannot disclose the exact label of each insured. However, we can suppose that we know the proportion of each group in the samples. Let us assume that we have data from two companies, A and B, with, for each, the proportion of men and women in the database transmitted. We can then create a mixture model similar to the one used by Maiboroda [10].

We are going to consider a problem in actuarial sciences in which we have M=2 insurance companies (samples) and K=2 groups of insured persons (differentiated for example by gender), and we want to determine the behaviour of each group without knowing the exact label of each insured person. For that, we are going to determine the survival functions, taking into account the fact that the data related to sick leaves are censored data. We will compare two duration models using Kaplan-Meier estimator for mixture models, and see their specificities depending on the censure.

This paper is organized as follows. Section 2 will be about our methodology, recalling the Kaplan-Meier duration model and the mixture models, and comparing Ryzhov [20] and Maiboroda [15] models. Section 3 will present different simulations results for continuous and discrete data before applying the models to an empirical data set in Section 4. Finally, we will discuss the results and possible improvements.

2 Model description

2.1 Duration model: Kaplan-Meier estimator

We are going to use the Kaplan-Meier [8] estimator for our duration model. It is a non-parametric estimator of the survival function that is used in the presence of censoring and truncation. It is characterized graphically by a decreasing step-wise curve, the points of discontinuity corresponding to the points where an event occurs, in this case the end of the sick leave of an individual. The Kaplan-Meier estimator is based on the estimation of instantaneous survival rates. When a sufficiently large sample is used, it approximates the true survival function in that population.

Let Y be a random variable taking values in a set (which is countable but not necessarily finite) $\mathcal{T} = t_1, ..., t_n, ...$ In the case of censored data, $Y = min(X, C)$, with C the censoring and X the true value, which would have been observed without censoring. Let $\delta = \mathbb{1}_{X \leq C}$ be the censoring indicator.

Recall that the distribution of a random variable X is uniquely determined by its cumulative distribution function: $F_X(t) = \mathbb{P}(X \leq t)$. The survival function of X is defined by: $S(t) = 1 - F_X(t) = \mathbb{P}(X > t)$. $S(t)$ uniquely defines the distribution of X.

The hazard rate function of Y is defined as $\mu(t) = \frac{P(X=t)}{P(X \geq t)}$ in the discrete case. The hazard function is interpreted as the conditional probability of the failure of a device at age t, given that it did not fail before age t. In the discrete case, we note that $\mu(t) = \frac{S(t)-S(t+1)}{S(t)}$ and, therefore, $\frac{S(t+1)}{S(t)} = 1 - \mu(t)$. So we have $S(t) = \prod_{X_i<t}(1 - \mu(X_i))$. An estimator of the survival function is thus obtained from an estimator of the instantaneous rates by posing $\hat{S}(t) = \prod_{X_i<t}(1 - \hat{\mu}(X_i))$. The X_i are the values taken by the random variable X while the t are all the values that can be taken.

Let us now note that $M_0(t) = P(Y \leq t, \delta = 0), M_1(t) = P(Y \leq t, \delta = 1)$ and $M(t) = P(Y \leq t)$ correspond to the cumulative distribution function respectively of the censored data, the uncensored data and the data in the aggregate. These three quantities have easily computable estimators $\hat{M}_0(t) = \frac{1}{n}\sum_{i=1}^{n}(1-\delta_i)\mathbb{1}_{Y_i \leq t}$, $\hat{M}_1(t) = \frac{1}{n}\sum_{i=1}^{n}\delta_i\mathbb{1}_{Y_i \leq t}$ and $\hat{M}(t) = \hat{M}_0(t) + \hat{M}_1(t)$.

The instantaneous rates can be rewritten as $\mu(t) = \frac{dM_1(t)}{1-M(t)}$ and thus, $\hat{\mu}(t) = \frac{d\hat{M}_1(t)}{1-\hat{M}(t)}$, where $d\hat{M}_1(t)$ is the number of i where $Y_i = t$ and $\delta_i = 1$. With this notation, we have $\hat{S}(t) = \prod\limits_{Y_i \leq t}(1 - \hat{\mu}(Y_i))$. The Kaplan-Meier estimator can therefore be written, in the case with censorship and without ex-aequo for the Y_i, as :

$$\hat{S}(t) = \prod_{i:Y_i \leq t}(1 - \frac{\delta_i}{\sum_{j=1}^{n}\mathbb{1}_{Y_i \geq Y_j}})$$

2.2 Mixture models with varying mixing weights

A mixture model enables to model different sub-populations within the overall population without these sub-populations being identified in the data by an observed variable. McLachlan and Peel [16] are a reference in the literature. Using survival functions, let $X_1, ..., X_n$ be independent random variables such that, for any $1 \leq i \leq n$, the survival function of X_i on $\mathbb{R}$, denoted by $S(X_i)$, is a mixing density with $K(K \geq 2)$ components $S_1, ..., S_K$ and K non-negative mixing weights $p_1, ..., p_K$, i.e.. :

$$S(X_i) = \sum_{l=1}^{K} p_l S_l(X_i) \qquad \text{with} \qquad 0 \leq p_l \leq 1 \qquad \forall l \in (1, .., K) \qquad \text{et} \qquad \sum_{l=1}^{K} p_l = 1$$

The (p_l,S_l) pairs are unknown for all $l \in (1, .., K)$. The authors use a parametric approach for the S_l. That enable them to estimate the pairs (p_l,S_l).

Milhaud and al [17] also use two-component mixture distributions, but this time one of them is known. This type of model is particularly interesting when a known random phenomenon is contaminated by an unknown random effect. This article gives interesting ideas but the results concern semi-parametric models. Maiboroda and al. [12] use a finite mixture model in which the mixing probabilities vary from observation to observation. They assume a parametric model for one of the mixture component distribution, and non-parametric nuisance parameters for the others. Thus they propose Generalized Estimating Equations (GEE) for the semi-parametric estimation, and they applied their model to the sociological analysis of voting results.

If one wants to deal with the classical mixture model with unknown weights, it is necessary to make assumptions about the mixture components such as parametric families for the mixture components or some mixture components exactly known. From now on, we will focus on non-parametric models. The advantage of this type of statistics is that it does

not have to make any assumption about the mixture components as before but it requires the exact knowledge of the mixture weights.

Maiboroda and al. [13] analyses a mixture with varying concentrations for which they assume that the concentrations are known and the distributions of components are completely unknown. They propose a non-parametric technique for testing hypotheses on functional moments of components. Maiboroda and Sugakova [11] also use a finite mixture model and discuss the estimation of mixture components distributions, functional moments and densities. They propose tests for testing hypotheses on the moments, and applied their model to the analysis of DNA microarrays data.

Autin and Pouet [3] also use a variable weight mixture model (i.e. with different weights) to study the means of transportation to go to work either by bus/trolleybus or walking. For that, they use the means in the State of New York and the State of California, for people over or under 21 years old. Indeed, the authors consider two groups divided into several subgroups and are interested in comparing the means of two subgroups, one from each group. Samples are drawn from both groups and the subgroup label of each observation is not defined with certainty. The underlying mixture model is associated with known variable weights.

2.3 Mixture models with varying weights and censored data

Having recalled the basics of mixture models and duration models, we can now describe how these notions have been jointly studied in the literature. First, McLachlan and Peel [16] also studied the case of mixture models for failure time data, and thus the mixture of survival models. In this case, the author considers different types of failure ($K = 2$ as an example). An element fails as soon as the first failure occurs, whatever the cause, and we observe the time C until the failure and the type of failure i ($i = 1, 2$). In the case where the study ends before the failure, C is the censoring time and δ the censoring indicator is equal to zero to indicate that the failure time is right censored. However, it must be taken into account that the models considered in Mc Lachlan and Peel's book (Chapter 10) are parametric.

For non-parametric estimations, Ryzhov [20] addresses the problem of estimating the distribution functions of components in a mixture in the case of randomly censored observations. Indeed, he considers the duration model of each group j :

$$S_j(t) = p_1(j)S_1(t) + p_2(j)S_2(t) + ... + p_K(j)S_K(t)$$
$$\text{with, for all } j, \sum_{l=1}^{K} p_k(j) = 1 \text{ and } 0 \leq p_k(j) \leq 1,$$

Ryzhov studied estimators defined as linear combination of Kaplan-Meier estimators. Under mild conditions, he proved the asymptotic normality of these estimators with asymptotic variance depending on the coefficients of the linear combination and characteristic values of the cumulative distribution functions of the unknown mixture components. In the case of K mixture components and M subgroups (i.e. M different weight vectors) with $M > K$, several choices are possible for the coefficients in the linear combination. The optimal one depends as the asymptotic variance above on the mixing weights and on the characteristic features of the cumulative distribution functions of the mixture compo-

nents. Ryzhov proved that a completely data-driven procedure could be applied to mimic the optimal oracle estimator.

We are going to consider the case $K = M = 2$ as it allows us to already describe interesting new behaviours of the estimators. Under invertibility condition of the weight matrix, there is only one possible solution for the $a_j(l)$. As a matter of fact, the $a_j(l)$ are the solutions of

$$\begin{bmatrix} p_1(1) & p_1(2) \\ p_2(1) & p_2(2) \end{bmatrix} \begin{bmatrix} a_1(1) \\ a_2(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} p_1(1) & p_1(2) \\ p_2(1) & p_2(2) \end{bmatrix} \begin{bmatrix} a_1(2) \\ a_2(2) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, Maiboroda and Khyzanov [14] considered a modified Kaplan-Meier estimator for the distribution of components of a mixture with varying concentrations in the case of censored data.

We are now going to introduce the estimators of Ryzhov and Maiboroda. With a dataset divided into two samples, denoted A and B, and the mixture weights known for each sample, the estimator introduced by Ryzhov is based on a solution of the following equation system :

$$a_k(A)p_l(A) + a_k(B)p_l(B) = \delta_{kl}.$$

The estimator introduced by Ryzhov is

$$\hat{S}_l(t) = a_l(A)\hat{S}_A(t) + a_l(B)\hat{S}_B(t)$$

where $\hat{S}_A(t)$ and $\hat{S}_B(t)$ are the estimators directly computed from each sample.

The estimator introduced by Maiboroda is based on a solution of the following equation system

$$\frac{1}{n}\sum_{j=1}^{n} \tilde{a}_k(j)p_l(j) = \delta_{kl} \tag{1}$$

or equivalently

$$\frac{1}{n}\big[\sum_{j\in A} \tilde{a}_k(A)p_l(A) + \sum_{j\in B} \tilde{a}_k(B)p_l(B)\big] = \delta_{kl}.$$

In the uncensored case, i.e $Y_j = X_j$, the estimator introduced by Maiboroda is given by

$$\tilde{S}_l(t) = \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j>t}.$$

In the censored case, the estimator introduced by Maiboroda is :

$$\tilde{S}_l(t) = \prod_{j=1:Y_j\leq t}^{n} (1 - \frac{\tilde{a}_l(j)\delta_j}{n - \sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_j}\tilde{a}_l(i)}) \tag{2}$$

Remark *If the labels are exactly known, the coefficients $\tilde{a}_i(k)$ are equal to 0 or 1, and Maiboroda's estimator is equal to the usual Kaplan-Meier estimator computed for each sample.*

For the sake of completeness we state the following lemma dealing with the uncesored case.

Lemma 2.1. *Maiboroda estimator for censored duration model is the same as the estimator presented by Maiboroda and Dorin [13] in the uncensored case:*

$$1 - \tilde{S}_l(t) = \tilde{F}_l(t) = \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j<t}$$

Proof : The estimator proposed by Maiboroda and Doronin [13] is :

$$\tilde{F}_l(t) = \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j<t}$$

The estimator of the survival function corresponding to this estimator is

$$\begin{aligned}
\tilde{S}_l(t) &= 1 - \tilde{F}_l(t) \\
&= 1 - \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j<t} \\
&= \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j) - \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j<t} \\
&= \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)(1 - \mathbb{1}_{X_j<t}) = \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j>t}
\end{aligned}$$

Proposition 2.2. *In the uncensored case, the set of Ryzhov estimators is a subset of the set of Maiboroda estimators.*

Proof : Let us assume that the $\tilde{a}_k(j)$ are constant over each sample. Then the equation (1) becomes

$$\frac{n_A}{n}\tilde{a}_k(A)p_l(A) + \frac{n_B}{n}\tilde{a}_k(B)p_l(B) = \delta_{kl}$$

Let us state : $a_k(A) = \frac{n_A}{n}\tilde{a}_k(A)$ and $a_k(B) = \frac{n_B}{n}\tilde{a}_k(B)$.

Computing Maiboroda estimator with this relationship leads to

$$\begin{aligned}
\tilde{S}_l(t) &= \frac{1}{n}\sum_{j=1}^{n} \tilde{a}_l(j)\mathbb{1}_{X_j>t} \\
&= \frac{1}{n}\Big[\frac{n}{n_A}a_l(A)\sum_{j\in A}\mathbb{1}_{X_j>t} + \frac{n}{n_B}a_l(B)\sum_{j\in B}\mathbb{1}_{X_j>t}\Big] \\
&= a_l(A)\frac{1}{n_A}\sum_{j\in A}\mathbb{1}_{X_j>t} + a_l(B)\frac{1}{n_B}\sum_{j\in B}\mathbb{1}_{X_j>t} \\
&= a_l(A)\hat{S}_A(t) + a_l(B)\hat{S}_B(t).
\end{aligned}$$

The last line above is exactly Ryzhov estimator.

However this equality is no more valid in the case of censored data.

Proposition 2.3. *In the censored case for $M = K = 2$, Ryzhov and Maiboroda estimators can take different values.*

Proof : One needs only to describe a case with different values for the estimators. Let us assume the following pattern for the censored variable, δ_i :

- The first (according to ascending order) i observations are uncensored. The last observation from this subset is Y_{i_1}
- The first censored observation is Y_{i_2} and belongs to sample B
- The next observation just after Y_{i_2} is Y_{i_3}. We assume it is uncensored and belongs to the sample A.

Let us choose $t > Y_{i_3}$ such that there is no observation between t and Y_{i_3}. According to previous proposition, we have that $\hat{S}_l(s) = \tilde{S}_l(s)$ for any $s \leq Y_{i_1}$.

Ryzhov estimator at time t can be written as :

$$\begin{aligned}
\hat{S}_l(t) &= a_l(A)\hat{S}_A(t) + a_l(B)\hat{S}_B(t) \\
&= a_l(A)\hat{S}_A(s)\big(1 - \frac{1}{n_A - \sum_{i\in A} \mathbb{1}_{Y_{i_1}<Y_{i_3}}}\big) + a_l(B)\hat{S}_B(s) \\
&= \hat{S}_l(s) + a_l(A)\hat{S}_A(s)\frac{1}{n_A - \sum_{i\in A} \mathbb{1}_{Y_{i_1}<Y_{i_3}}}.
\end{aligned}$$

In fact, this estimator at point t is not different from the one we would obtain if observation Y_{i_2} did not exist. This observation has no influence on $\hat{S}_A(t)$ and $\hat{S}_B(t)$.

On the other hand, Maiboroda estimator at time t can be written as :

$$\begin{aligned}
\tilde{S}_l(t) &= \prod_{j=1:Y_j\leq t}^{n} (1 - \frac{\tilde{a}_l(j)\delta_j}{n - \sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_j}\tilde{a}_l(i)}) \\
&= \prod_{j=1:Y_j\leq s}^{n} (1 - \frac{\tilde{a}_l(j)\delta_j}{n - \sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_j}\tilde{a}_l(i)})(1 - \frac{\tilde{a}_l(i_3)\delta_{i_3}}{n - \sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_{i_3}}\tilde{a}_l(i)}) \\
&= \tilde{S}_l(s) - \tilde{S}_l(s)\frac{\tilde{a}_l(i_3)\delta_{i_3}}{n - \sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_{i_3}}\tilde{a}_l(i)} \\
&= \tilde{S}_l(s) - \tilde{S}_l(s)\frac{\tilde{a}_l(A)\delta_{i_3}}{n_A - \frac{n_A}{n}\sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_{i_3}}\tilde{a}_l(i)} \\
&= \tilde{S}_l(s) - \tilde{S}_l(s)\frac{\tilde{a}_l(A)}{n_A - \sum_{i\in A} \mathbb{1}_{Y_i<Y_{i_3}} + a_l(B)(\sum_{i\in A} \mathbb{1}_{Y_i<Y_{i_3}} - \frac{n_A}{n_B}\sum_{i=1}^{n} \mathbb{1}_{Y_i<Y_{i_3}})}
\end{aligned}$$

We can see that Ryzhov and Maiboroda estimators differs in the second term. For Ryzhov it only involves observations from the sample A whereas for Maiboroda it involves observations from the samples A and B.

3 Simulations

3.1 Continuous data

We start our analysis with continuous simulated data and we compare the Ryzhov [20] and Maiboroda [15] estimators. To measure the efficiency of our estimators, we measure the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$.

We restrict to the case of two insurers labelled A and B, that we are going to call samples, and two groups labelled 1 and 2. We also have in this case $n = n_A + n_B$ with balanced groups, that is to say $n_A = n_B = \frac{n}{2}$. The mixture weights is given by Table 1.

	Group 1	Group 2
Sample A	$p_1(A)$	$p_2(A)$
Sample B	$p_1(B)$	$p_2(B)$

Table 1: Mixing weights

Let us denote S_1 (resp. S_2) the survival function for the group 1 (resp. 2). Then the survival functions for the samples are given by

$$\hat{S}_A(t) = p_1(A)\hat{S}_1(t) + p_2(A)\hat{S}_2(t) \qquad \text{and} \qquad \hat{S}_B(t) = p_1(B)\hat{S}_1(t) + p_2(B)\hat{S}_2(t) \tag{3}$$

For the first simulation, we choose $n = 1000$ and $n_A = n_B = n/2 = 500$. Simulations are done using Python. We start by using exponential distributions of parameters $\lambda_1 = 50$ and $\lambda_2 = 30$ respectively. Exponential distributions are often used to model survival times because they are the simplest distributions that can be used to characterize survival data. This is because they are memoryless, and thus the hazard function is constant with time, as explained by Rodriguez [19] in the Chapter 7 of his lecture. The mixing weights are chosen equal to

$$p_1(A) = 0.7, p_2(A) = 0.3, p_1(B) = 0.1 \text{ and } p_2(B) = 0.9. \tag{4}$$

The sample A is simulated by the following model:

$$X_A = R_A X_{A1} + (1 - R_A)X_{A2},$$

with $Y_{A1} \sim \mathcal{E}(\lambda_1)$ and $Y_{A2} \sim \mathcal{E}(\lambda_2)$, of size n_A, and $R_A \sim \mathcal{B}ernoulli(p_1(A))$.

The sample B is simulated by a similar model where the index A is replaced by B:

$$Y_B = R_B Y_{B1} + (1 - R_B)Y_{B2},$$

with $X_{B1} \sim \mathcal{E}(\lambda_1)$ and $X_{B2} \sim \mathcal{E}(\lambda_2)$, of size n_B, and $R_B \sim \mathcal{B}ernoulli(p_1(B))$.

The censoring mechanism is modeled by the same exponential distribution for the two samples A and B: $\mathcal{E}(\lambda_c = 40)$. We denote C_A (resp. C_B the random variable modeling the censoring for insurer A (resp. B). We simulate the observed variables by $Y_A = min(C_A, X_A)$ and $Y_B = min(C_B, X_B)$.

The estimators are computed using scikit-survival module in Python [18], that allows doing survival analysis while utilizing the power of scikit-learn. For the Ryzhov estimator,

we use the distinction between the different samples. We estimate $\hat{S_A}(t)$ and $\hat{S_B}(t)$ using the Kaplan-Meier estimator. As the estimators $\hat{S}_A$ and $\hat{S}_B$ are not necessarily computed at the same points by the scikit-survival module in Python, we apply an interpolation of the obtained estimators at all the observed points whether they belong to sample A or sample B.

Then we compute the estimators of $\hat{S}_1$ and $\hat{S}_2$ according to the following linear system:

$$\hat{S}_A(t) = p_1(A)\hat{S}_1(t) + p_2(A)\hat{S}_2(t),$$
$$\hat{S}_B(t) = p_1(B)\hat{S}_1(t) + p_2(B)\hat{S}_2(t).$$

We can rewrite this system in vector form: $M.\hat{S}_{12} = \hat{S}_{AB}$ with M the weight matrix, $\hat{S}_{12} = [\hat{S}_1, \hat{S}_2]^t$ and $\hat{S}_{AB} = [\hat{S}_A, \hat{S}_B]^t$. As we have exactly two samples and two groups, we just have to invert the matrix M to obtain $\hat{S}_{12} = M^{-1}.\hat{S}_{AB}$. With the chosen weights, we obtain :

$$M^{-1} = \begin{pmatrix} -0.5 & 1.5 \\ 4.5 & -3.5 \end{pmatrix}$$

We can now compare $\hat{S}_1(t)$ with $S_1(t)$ and $\hat{S}_2(t)$ with $S_2(t)$. By computing the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$, we find, after 1000 repetitions, 0.109 for the estimate of S_1 and 0.380 for S_2. We then analyse the case with low censorship, using an exponential distribution of parameter $\lambda_c = 200$ and $\lambda_c = 1000$, that is almost a case without censorship. Results get better with less censorship as expected.

For the Maiboroda estimator, we concatenate the observations of X_A and X_B (X) and then C_A and C_B (C) and we study $Y = min(X, C)$. As we no longer have 2 distinct samples with a different proportion of each group for each sample, but a probability of belonging to each group for each individual. We thus seek the a_1 and a_2 such that:

$$\frac{1}{n}\sum_{k=1}^{n} \tilde{a}_1(k)p_1(k) = 1, \frac{1}{n}\sum_{k=1}^{n} \tilde{a}_1(k)p_2(k) = 0.$$
$$\frac{1}{n}\sum_{k=1}^{n} \tilde{a}_2(k)p_1(k) = 0, \frac{1}{n}\sum_{k=1}^{n} \tilde{a}_2(k)p_2(k) = 1.$$

As $n_A = n_B = n/2$ and as there is no reason to privilege one observation over another in each sample, we can choose for observation k in sample A

$$\tilde{a}_1(A) = \tilde{a}_1(k) = 2a_1(k) \quad \text{and} \quad \tilde{a}_2(A) = \tilde{a}_2(k) = 2a_2(k)$$

It is the same if observation k is in sample B. By computing again the mean of the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$, we find, after 1000 repetitions, 0.130 for the estimate of S_1 and 2e+57 for S_2. This last result is very surprising, since the sup-norm loss for the survival functions should be between 0 and 1.

We noticed that the $\tilde{a}_l$ could compensate each other in the denominator, and then we could have a division by 0 or by a very close number, which whould lead to a large result. Moreover, the chosen weights are $p_1(A) = 0.7, p_2(A) = 0.3, p_1(B) = 0.1$ and $p_2(B) = 0.9$, and in this case we have $\tilde{a}_1(A) = -1$, $\tilde{a}_1(B) = 3$, $\tilde{a}_2(A) = 9$ and $\tilde{a}_2(B) = -7$. In this case, is then possible that $\sum_{i:Y_i<Y_j} \tilde{a}_l(i)$ compensate if we have for example, for the group $l = 1$, 3 individuals from the sample A and 1 from the sample B.

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.281	0.109	0.364	0.130
$\lambda_c = 200$	0.191	0.063	0.207	0.065
$\lambda_c = 1000$	0.177	0.061	0.188	0.062

Table 2: Comparison Ryzhov and Maiboroda estimators for group 1 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.981	0.380	5e+31	2e+57
$\lambda_c = 200$	0.684	0.227	5e+28	9e+41
$\lambda_c = 1000$	0.623	0.218	1e+14	7e+38

Table 3: Comparison Ryzhov and Maiboroda estimators for group 2 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$

To my knowledge, this phenomenon had not been highlighted before. The choice of the weights can have a big impact on the results. That is why we have huge values for the mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ for the Maiboroda's estimator. We do not have the same kind of problem for the Ryzhov's estimators since the use of the weights is not done individually but directly on the survival function of the group.

Table 2 summarizes the results obtained for the selected weights. Other results have been obtained for different weights (available on request). The compensation problem in the denominator was also spotted for different couples of weights.

To have correct survival functions, we propose to perform a smoothing for Ryzhov and Maiboroda estimators. For this, we start by increasing the function by 1 and decreasing it by 0. We then smooth the survival function from step to step, starting with $t = 0$, and if $S(t+1) > S(t)$, giving $S(t+1)$ the value of $S(t)$.

In the particular case of $p_1(A) = 0.7, p_2(A) = 0.3, p_1(B) = 0.1$ and $p_2(B) = 0.9$, the mean of the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$ after 1000 repetitions is equal to 0.099 for the Ryzhov's estimator of S_1 and 0.228 for S_2, and equal to 0.103 for the Maiboroda's estimator of S_1 and 0.267 for S_2. Again, the results get better with less censorship. Table 4 shows the results obtained for this particular choice of mixing weights. In the Appendix, other results are given for different weights and sizes.

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.235	0.099	0.246	0.103
$\lambda_c = 200$	0.182	0.064	0.181	0.064
$\lambda_c = 1000$	0.170	0.059	0.174	0.059

Table 4: Comparison Ryzhov and Maiboroda smoothed estimators for group 1 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.456	0.228	0.496	0.267
$\lambda_c = 200$	0.432	0.199	0.440	0.204
$\lambda_c = 1000$	0.415	0.187	0.428	0.190

Table 5: Comparison Ryzhov and Maiboroda smoothed estimators for group 2 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$

The results obtained are quite close when λ_c is large, which is quite normal since we have seen that the estimators are the same in the uncensored case. The differences come from the fact that there is still a small amount of censoring and that there can be approximations with the Python calculation method.

3.2 Discrete data

Since the objective of our study is to be able to analyse real-life data such as sick leave data, which are discrete rather than continuous, we now turn to the case of discrete data. We propose a modified version of Maiboroda's estimator to take into account the fact that the data is now discrete. The modified Kaplan-Meier estimator is now given, with $\tilde{Y}_i$ denoting the observations without duplicates, by

$$\tilde{S}_l(t) = \prod_{\tilde{Y}_j \leq t} \Big(1 - \frac{\sum\limits_{Y_h = \tilde{Y}_j} \tilde{a}_l(h)\delta_k}{n - \sum\limits_{Y_i < \tilde{Y}_j} \tilde{a}_l(i)}\Big).$$

This estimator is similar to the Kaplan-Meier estimator in the discrete case. We obtain the same one in the case where the individuals are totally identified (with weights equal to 0 or 1): in this case, the $\tilde{a}_k(i)$ are either equal to $\frac{n}{n_k}$, or 0, which gives us the estimator seen in Section 2. Moreover, we never have $Y_k = \tilde{Y}_j$ in the continuous case except if $k = j$, so this estimator gives the same results that the one before in the continuous case. Therefore it extends Maiboroda estimator of the previous section.

We consider the same set-up as the one considered in the continuous case:

- same mixing weights

$$p_1(A) = 0.7, p_2(A) = 0.3, p_1(B) = 0.1 \text{ and } p_2(B) = 0.9,$$

- same sample sizes for samples A and B.

The mean of the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$ after 1000 repetitions is equal to 0.099 for the Ryzhov's estimator of S_1 and 0.222 for S_2, and equal to 0.1 for the Maiboroda's estimator of S_1 and 0.278 for S_2. The tables below summarize the results obtained for this particular choice of mixing weights for different n and λ_c :

In the Appendix, we compare $\hat{S}_1(t)$ with $S_1(t)$ and$\hat{S}_2(t)$ with $S_2(t)$ for different weights, different data sizes, and different values of the censoring parameter λ_c. The results are overall better than in the continuous case. Results have been obtained for different

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.238	0.099	0.244	0.100
$\lambda_c = 200$	0.177	0.063	0.183	0.067
$\lambda_c = 1000$	0.168	0.059	0.173	0.067

Table 6: Comparison Ryzhov and Maiboroda estimators for group 1 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ in the case of discrete data

	Ryzhov		Maiboroda	
	n = 100	n = 1000	n = 100	n = 1000
$\lambda_c = 40$	0.449	0.222	0.481	0.278
$\lambda_c = 200$	0.417	0.191	0.428	0.208
$\lambda_c = 1000$	0.408	0.189	0.412	0.195

Table 7: Comparison Ryzhov and Maiboroda estimators for group 2 for the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ in the case of discrete data

weights, different data sizes, and different censoring proportions, in the case of discrete data.

4 Application to an empirical database

4.1 Data description and application of our models

We are now going to apply our models to an anonymized dataset. We will use insurer data for which we have the duration of the sick leaves, the age of the persons stopped and their gender. We have chosen data with what is called a short deductible. The insurer intervenes directly in case of sick leaves, so we can consider that there is no truncation on the left. Our goal is to evaluated the loss of information caused by ignoring this data. We will use it only in a second step to compare our estimates of the duration models to those that we would have obtained by directly using the Kaplan-Meier estimator ($S^{(KM)}(t)$) on the data belonging to each group.

As we only have the data from one insurer, we consider samples A and B to be respectively the samples for people having an age less than or equal to 40 years, and an age above 40 years. The group 1 is the one of men and the group 2 is the one of women (as the first digit in the French social security identification number).

Age Group (Sample)	Gender (Group)	Number of people	Proportion
<= 40ans	M	6 339	0.385819
	F	10 091	0.614181
> 40ans	M	6 759	0.403643
	F	9 986	0.596357

Table 8: Proportion of each sample and group for our real database

We computed Ryzhov's and Maiboroda's estimators as described in Section 3 for discrete data. For the Maiboroda's estimator, we cannot use the same trick as before for the

computation of the $a_l(i)$ because the data size are not the same anymore. To obtain the weights $\tilde{a}_1$ and $\tilde{a}_2$, we use CVXPY [4], an open source Python-embedded modelling language for convex optimization problems. Figure 1 shows the results obtained for the estimated survival functions of group 1 using both estimators.

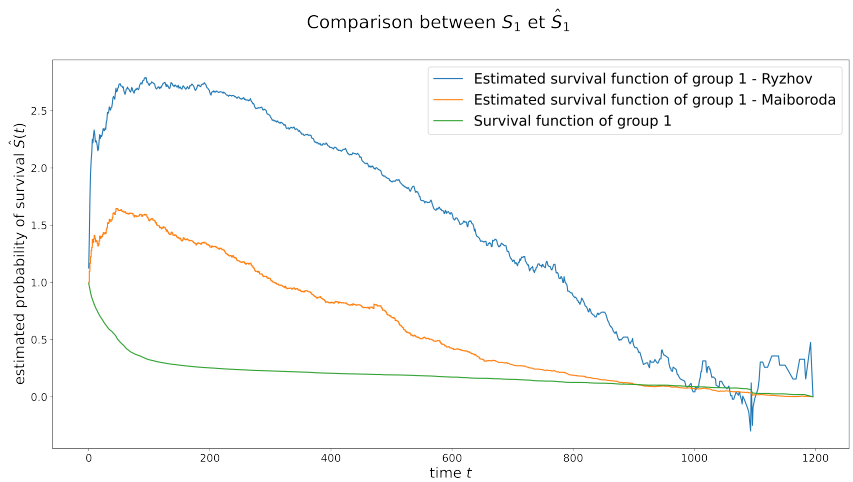


Figure 1: Estimated survival functions using Ryzhov's and Maiboroda's estimators

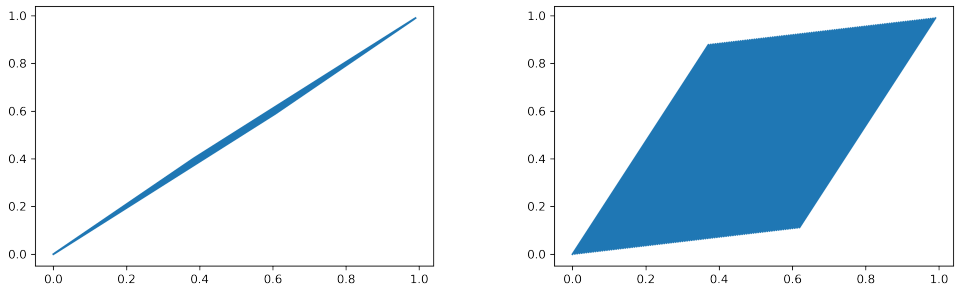
Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

We can see that the results are not as good as for simulated data. This was expected as the mixing weights are quite close and it is known that in this case the coefficients will be large entailing a large variance. Nevertheless they are better for the Maiboroda estimator. Using the real weights $p_1(A) = 0.385819, p_2(A) = 0.614181, p_1(B) = 0.403643$ and $p_2(B) = 0.596357$, we obtain, for the Ryzhov's coefficients:

$$M^{-1} = \begin{pmatrix} -33.45762024 & 34.45762024 \\ 22.64570951 & -21.64570951 \end{pmatrix}$$

For Maiboroda, the coefficients estimated using CVXPY are $\tilde{a}_1(A) = -67.55200551, \tilde{a}_2(A) = 45.72300539, \tilde{a}_1(B) = 68.26242083$ and $\tilde{a}_2(B) = -42.88179371$. We obtained the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S^{(KM)}(t)|$ equal to 2.489 for the Ryzhov's estimate of S_1 and to 1.623 for S_2, and equal to 1.260 for the Maiboroda's estimate of S_1 and equal to 1.212 for S_2. The figure for S_2 is given in the Appendix.

The results obtained are not surprising. Indeed, for the estimators of S_1 and S_2 to be between 0 and 1 at any time, the estimators of S_A and S_B would have to be part of the subspace $\{ M \begin{pmatrix} x \\ y \end{pmatrix}, (x, y) \in [0, 1]^2 \}$. Figure 2 shows the space of possible results. The distance between S_A ans S_B must be less than 0.15 in these case, according to the chosen weights. We see that the available space is small, so if the gap between the two estimators is a little too large, the results for the estimators of S_1 and S_2 will not always be in the interval [0,1]. This space becomes more important for the next choice of points, which explains the better results.



(a) Space of possible results with the original data weights

(b) Space of possible results with the chosen weights

Figure 2: Space of possible results

We analyse the results obtained using a limit of 360 days for the sick leaves. In fact, in France, sick leaves due to disability can last up to 3 years in case of LDD (long duration disease), but in general, the French Social Security stops compensating sick leaves outside LDD after one year. We obtained a sup-norm of 1.458 for the Ryzhov's estimate of S_1 and 0.953 for S_2, and 0.765 for the Maiboroda's estimate of S_1 and 0.787 for S_2.

As we have seen before, the choice of the weights can have a strong impact on the results, close weights give large values for the coefficients and therefore a large variance, which would give inadequate results. To get around this problem, one idea is to separate the data into two samples A and B, in order to force the proportion of each group (e.g. men/women) in each of these samples. In this way we can choose mixing weights that should give better results. We have chosen that in sample A there are 6000 women and 10000 men. Table 9 gives the new proportions.

Sample	Gender (Group)	Number of people	Proportion
A	M	10 000	0.625
	F	6 000	0.375
B	M	3 098	0.180376
	F	14 077	0.819622

Table 9: Proportion of each sample and group for our arranged database

We notice in Figure 3 that there are more very short breaks for sample B, while for breaks of more than 200 days, it is sample A that seems to predominate. These samples were created arbitrarily, but we can still notice their differences.

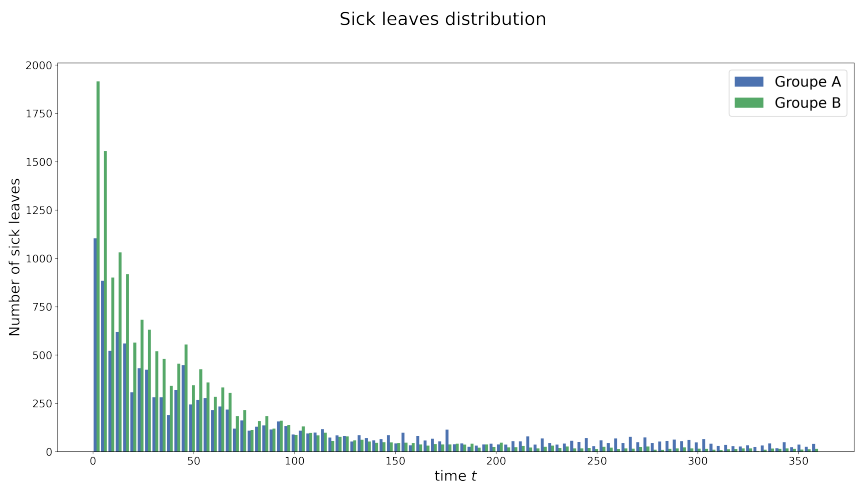


Figure 3: Sick leaves distribution

Note : Here we represent the distribution of the sick leave data. We stopped the representation at 360 days because there are only few leaves after this date.

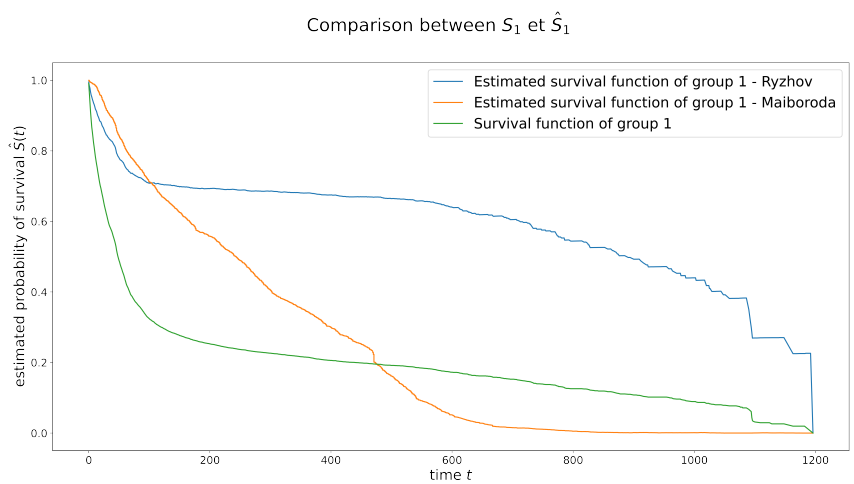


Figure 4: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

Figure 4 shows the results obtained. With the chosen weights, we obtain, for the Ryzhov's coefficients :

$$M^{-1} = \begin{pmatrix} 1.84341392 & -0.84341392 \\ -0.40568987 & 1.40568987 \end{pmatrix}$$

For Maiboroda, the coefficients estimated using CVXPY are $\tilde{a}_1(A) = 3.82218179, \tilde{a}_2(A) = -0.84121788, \tilde{a}_1(B) = -1.62911257$ and $\tilde{a}_2(B) = 2.7152399$

In this case, we obtained a sup-norm equal to 0.474 for the Ryzhov's estimate of S_1 and

equal to 0.285 for S_2, and equal to 0.396 for the Maiboroda's estimate of S_1 and equal to 0.234 for S_2. As we had better results with a limit of 360 days, we have done the same still choosing that in sample A there are 6000 women and 10000 men. The proportions are in this case :

Sample	Gender (Group)	Number of people	Proportion
A	M	10 000	0.625
	F	6 000	0.375
B	M	1 466	0.112795
	F	11 531	0.887205

Table 10: Proportion of each sample and group for our arranged database with a limit of 360 days

Figure 5 shows that the results are better than before. With the new weights, we obtain, for the Ryzhov's coefficients :

$$M^{-1} = \begin{pmatrix} 1.73212911 & -0.73212911 \\ -0.22021518 & 1.22021518 \end{pmatrix}$$

For Maiboroda, the coefficients estimated are $\tilde{a}_1(A) = 3.13919642, \tilde{a}_2(A) = -0.39913399, \tilde{a}_1(B) = -1.6334538$ and $\tilde{a}_2(B) = 2.72241359$. We have a sup-norm $\sup_{t \in R} |\hat{S}(t) - S^{(KM)}(t)|$ of 0.215 for the Ryzhov's estimate of S_1 and 0.131 for S_2, and 0.201 for Maibroda estimate of S_1 and 0.117 for S_2. We notice that with selected weights and with a maximum of 360 days, the results are particularly good.

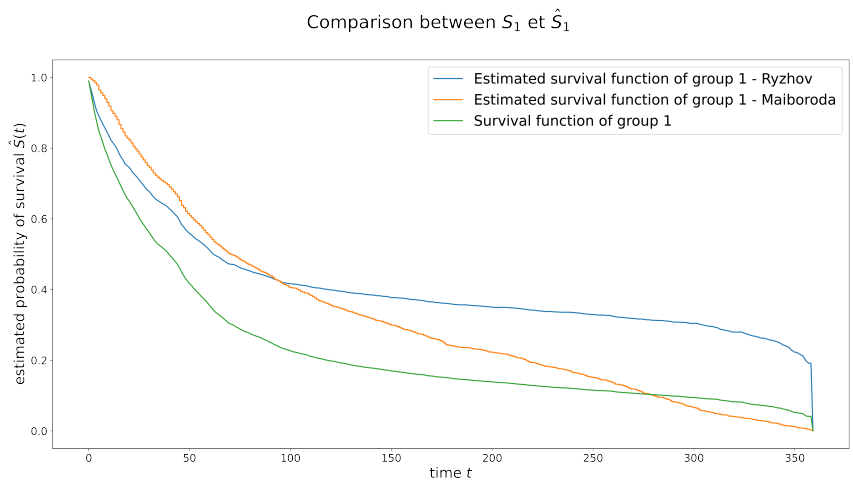


Figure 5: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

For the results for group 2, the figures can be found in the appendix. We notice that in the last case (with selected weights and with a maximum of 360 days), the results are particularly good.

Overall, we notice that in our empirical data case, Maiboroda's estimator is globally better than Ryzhov's. This result is quite different from what was obtained in the case of simulations using exponential functions. It can be explained by, first of all, the fact that the data do not follow the same law. This can therefore have an important impact on the results. Moreover, in the case with weak censoring, we had already seen that different weights gave good results for Maiboroda estimator.

4.2 Comparison with the results of simulated data with same characteristics

We now compare the results obtained with empirical data, with simulations having the same parameters, to know if the results remain similar. For this we will focus on the case with different weights, which, as we have seen earlier, gives better results.

For the simulation, exponential functions are again used, using the sick leaves means for each group as parameters. We have $\lambda_1 = 174.069$ and $\lambda_2 = 85.907$. We use a censorship parameter $\lambda_C = 500$. Using this parameter, the censorship of the whole group was of 18.74%, being of 20.84% in group A and 16.79% in group B.

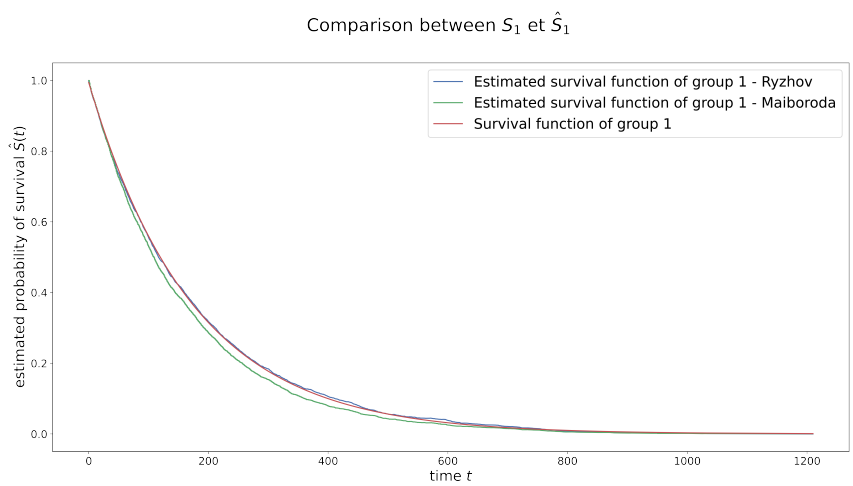


Figure 6: Estimated survival functions of simulated data using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions for the simulated data using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

Figure 6 shows that the results are very good for both estimators. We have a sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$ of 0.012 for the Ryzhov's estimate of S_1 and 0.008 for S_2. For Maiboroda, we have 0.038 for the estimate of S_1 and 0.040 for S_2. The difference between the estimators is 0.0353 for group 1 and 0.0375 for group 2.

We notice that the errors are larger when the λ_C parameter is smaller (and thus the censoring is stronger), which is in agreement with the results obtained in the previous section. On the other hand, the results improve by decreasing the censoring, but the Ryzhov estimator remains the closest to reality with the choice of weights made.

5 Conclusion and open questions

We have considered a problem in actuarial sciences, in which we wanted to model sick leave for K=2 groups, having some data from M=2 insurance companies (samples), but not defined with certainty for confidential reasons. Having the proportion of each group for each sample, we compare two different Kaplan-Meier estimators for mixture models. We measure the information loss when modelling sick leaves in censored duration models, comparing Ryzhov's and Maiboroda's methods. We proved that the Ryzhov and Maiboroda estimators were the same in the uncensored case, but could be quite different in cas of censorship.

For simulated data, we found that Ryzhov's model gives usually best results, especially in the case where the weights are quite close together. This result remains the same when simulating data with statistical main characterictics equal to the ones of the real dataset. However, when the weights are further apart, it is the Maiboroda estimator that becomes better, especially in the case of strong censorship. Using empirical data, we found that Maiboroda estimator gives best results. The results are related to the distribution of the data. It would be interesting in a future analysis to compare these two models on different distributions, in order to understand the impact they may have on these two estimators.

To go further, now that we have compared Maiboroda's and Ryzhov's methods in the case of 2 samples A and B and 2 groups, one might ask whether or not the results remain the same when there are more data sources (samples) and/or more groups. This study would still be interesting in terms of the anonymity of the data, since we would only need to know the membership of two groups of variables and the proportion of these groups in the overall population.

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Appendix

Simulation results with correction for Ryzhov's and Maiboroda's estimators

We compare $\hat{S}_1(t)$ with $S_1(t)$ and$\hat{S}_2(t)$ with $S_2(t)$ for different weights, different data sizes, and different values of the censoring parameter λ_c . We have computed the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$. All the simulation results are given for 1000 repetitions, with the correction mentioned on section 3 for Maiboroda's method.

Simulation results for $n = 100$ and $\lambda_c = 40$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.695	0.528	0.430	0.350	0.293	0.258	0.232	0.212	0.199	0.189
0.1	0.703	NaN	0.682	0.511	0.388	0.317	0.273	0.240	0.219	0.200	0.195
0.2	0.524	0.683	NaN	0.656	0.472	0.360	0.293	0.249	0.225	0.200	0.188
0.3	0.422	0.510	0.650	NaN	0.609	0.435	0.328	0.267	0.225	0.198	0.190
0.4	0.344	0.389	0.467	0.617	NaN	0.566	0.390	0.298	0.242	0.207	0.185
0.5	0.297	0.323	0.365	0.436	0.576	NaN	0.522	0.349	0.262	0.208	0.186
0.6	0.256	0.276	0.291	0.339	0.384	0.519	NaN	0.468	0.301	0.222	0.186
0.7	0.228	0.235	0.251	0.272	0.295	0.347	0.462	NaN	0.387	0.244	0.187
0.8	0.214	0.217	0.225	0.229	0.241	0.261	0.299	0.393	NaN	0.296	0.188
0.9	0.200	0.201	0.201	0.204	0.205	0.211	0.220	0.235	0.293	NaN	0.186
1.0	0.189	0.189	0.187	0.190	0.183	0.191	0.188	0.188	0.188	0.185	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.750	0.574	0.447	0.368	0.308	0.263	0.228	0.204	0.183	0.173
0.1	0.746	NaN	0.704	0.545	0.420	0.334	0.286	0.236	0.211	0.187	0.173
0.2	0.574	0.719	NaN	0.682	0.510	0.383	0.309	0.254	0.215	0.192	0.173
0.3	0.455	0.551	0.694	NaN	0.664	0.469	0.351	0.278	0.225	0.192	0.173
0.4	0.370	0.431	0.512	0.650	NaN	0.614	0.423	0.315	0.236	0.196	0.175
0.5	0.309	0.342	0.387	0.466	0.613	NaN	0.567	0.375	0.271	0.208	0.178
0.6	0.261	0.276	0.316	0.351	0.425	0.578	NaN	0.508	0.313	0.216	0.175
0.7	0.227	0.241	0.252	0.274	0.314	0.373	0.497	NaN	0.430	0.249	0.179
0.8	0.204	0.214	0.217	0.229	0.243	0.264	0.310	0.427	NaN	0.314	0.184
0.9	0.189	0.194	0.194	0.198	0.200	0.208	0.224	0.246	0.320	NaN	0.187
1.0	0.171	0.179	0.179	0.178	0.180	0.182	0.181	0.181	0.184	0.183	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.146	0.151	0.151	0.150	0.150	0.152	0.150	0.150	0.151	0.153
0.1	0.150	NaN	0.270	0.213	0.187	0.182	0.172	0.169	0.166	0.165	0.163
0.2	0.148	0.272	NaN	0.361	0.272	0.233	0.215	0.204	0.192	0.186	0.183
0.3	0.152	0.211	0.368	NaN	0.433	0.325	0.270	0.245	0.225	0.215	0.204
0.4	0.151	0.189	0.268	0.442	NaN	0.487	0.364	0.301	0.272	0.250	0.234
0.5	0.149	0.178	0.232	0.329	0.484	NaN	0.529	0.401	0.332	0.295	0.268
0.6	0.149	0.175	0.212	0.266	0.361	0.531	NaN	0.573	0.430	0.358	0.325
0.7	0.153	0.168	0.198	0.245	0.312	0.403	0.563	NaN	0.594	0.462	0.384
0.8	0.154	0.167	0.188	0.224	0.267	0.334	0.428	0.585	NaN	0.617	0.474
0.9	0.148	0.167	0.186	0.215	0.252	0.292	0.357	0.456	0.600	NaN	0.640
1.0	0.151	0.162	0.181	0.205	0.231	0.271	0.319	0.385	0.486	0.617	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.161	0.161	0.160	0.164	0.165	0.169	0.169	0.180	0.178	0.179
0.1	0.158	NaN	0.293	0.232	0.204	0.196	0.194	0.189	0.191	0.192	0.190
0.2	0.157	0.282	NaN	0.397	0.300	0.254	0.227	0.224	0.217	0.213	0.211
0.3	0.163	0.226	0.388	NaN	0.468	0.347	0.300	0.271	0.251	0.248	0.241
0.4	0.167	0.210	0.292	0.464	NaN	0.535	0.400	0.331	0.300	0.281	0.272
0.5	0.174	0.201	0.255	0.350	0.523	NaN	0.576	0.442	0.367	0.334	0.307
.6	0.178	0.195	0.229	0.299	0.392	0.568	NaN	0.608	0.468	0.411	0.358
0.7	0.180	0.194	0.223	0.273	0.336	0.433	0.606	NaN	0.632	0.494	0.421
0.8	0.183	0.196	0.218	0.254	0.305	0.380	0.466	0.627	NaN	0.636	0.519
0.9	0.185	0.196	0.218	0.244	0.287	0.333	0.404	0.496	0.662	NaN	0.655
1.0	0.195	0.204	0.216	0.241	0.267	0.310	0.360	0.420	0.523	0.660	NaN

Simulation results for $n = 100$ and $\lambda_c = 200$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.680	0.493	0.374	0.291	0.238	0.203	0.171	0.150	0.134	0.122
0.1	0.685	NaN	0.653	0.459	0.344	0.263	0.220	0.179	0.154	0.134	0.123
0.2	0.497	0.647	NaN	0.624	0.423	0.313	0.240	0.193	0.159	0.138	0.122
0.3	0.379	0.458	0.619	NaN	0.583	0.398	0.279	0.210	0.166	0.138	0.122
0.4	0.288	0.347	0.428	0.579	NaN	0.541	0.344	0.240	0.182	0.145	0.121
0.5	0.238	0.266	0.311	0.390	0.536	NaN	0.490	0.292	0.201	0.150	0.121
0.6	0.203	0.215	0.243	0.275	0.348	0.486	NaN	0.416	0.240	0.162	0.122
0.7	0.169	0.180	0.196	0.211	0.242	0.297	0.416	NaN	0.340	0.183	0.124
0.8	0.149	0.153	0.160	0.168	0.179	0.200	0.239	0.350	NaN	0.239	0.121
0.9	0.133	0.133	0.138	0.140	0.143	0.154	0.161	0.182	0.240	NaN	0.122
1.0	0.122	0.122	0.120	0.123	0.123	0.122	0.122	0.120	0.121	0.123	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.701	0.520	0.375	0.303	0.243	0.200	0.168	0.151	0.136	0.125
0.1	0.713	NaN	0.684	0.477	0.346	0.271	0.213	0.181	0.154	0.135	0.125
0.2	0.516	0.676	NaN	0.644	0.438	0.317	0.240	0.193	0.164	0.137	0.124
0.3	0.386	0.483	0.649	NaN	0.618	0.399	0.283	0.217	0.167	0.140	0.125
0.4	0.302	0.347	0.448	0.609	NaN	0.560	0.351	0.242	0.179	0.146	0.125
0.5	0.245	0.271	0.315	0.399	0.562	NaN	0.499	0.306	0.203	0.150	0.125
0.6	0.204	0.219	0.243	0.281	0.353	0.510	NaN	0.430	0.246	0.164	0.126
0.7	0.174	0.182	0.195	0.214	0.248	0.302	0.438	NaN	0.350	0.184	0.126
0.8	0.154	0.154	0.160	0.169	0.181	0.200	0.245	0.346	NaN	0.247	0.125
0.9	0.134	0.136	0.138	0.140	0.144	0.150	0.160	0.181	0.246	NaN	0.126
1.0	0.124	0.122	0.124	0.126	0.125	0.123	0.126	0.126	0.124	0.125	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.116	0.116	0.118	0.117	0.117	0.116	0.117	0.116	0.118	0.115
0.1	0.117	NaN	0.237	0.178	0.156	0.144	0.141	0.136	0.133	0.131	0.131
0.2	0.119	0.233	NaN	0.333	0.237	0.196	0.172	0.165	0.156	0.151	0.148
0.3	0.115	0.177	0.326	NaN	0.416	0.287	0.234	0.205	0.189	0.177	0.167
0.4	0.115	0.157	0.232	0.413	NaN	0.468	0.334	0.271	0.234	0.212	0.198
0.5	0.114	0.148	0.195	0.295	0.467	NaN	0.508	0.364	0.300	0.264	0.237
0.6	0.116	0.139	0.180	0.233	0.332	0.510	NaN	0.544	0.404	0.330	0.284
0.7	0.117	0.137	0.166	0.208	0.269	0.371	0.541	NaN	0.569	0.429	0.352
0.8	0.117	0.132	0.156	0.187	0.232	0.301	0.399	0.573	NaN	0.592	0.455
0.9	0.117	0.133	0.152	0.177	0.214	0.262	0.326	0.432	0.592	NaN	0.618
1.0	0.116	0.130	0.146	0.169	0.198	0.234	0.278	0.349	0.457	0.616	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.123	0.124	0.124	0.125	0.126	0.125	0.127	0.128	0.129	0.132
0.1	0.125	NaN	0.236	0.179	0.163	0.152	0.147	0.144	0.143	0.141	0.144
0.2	0.122	0.240	NaN	0.346	0.244	0.208	0.184	0.174	0.167	0.157	0.156
0.3	0.122	0.182	0.335	NaN	0.417	0.294	0.249	0.217	0.195	0.186	0.177
0.4	0.123	0.160	0.243	0.425	NaN	0.478	0.337	0.273	0.244	0.227	0.205
0.5	0.125	0.151	0.206	0.294	0.471	NaN	0.525	0.388	0.309	0.268	0.249
0.6	0.124	0.147	0.183	0.242	0.347	0.533	NaN	0.565	0.407	0.341	0.296
0.7	0.123	0.142	0.170	0.217	0.278	0.385	0.563	NaN	0.605	0.435	0.360
0.8	0.125	0.142	0.161	0.195	0.240	0.310	0.409	0.590	NaN	0.612	0.468
0.9	0.124	0.136	0.157	0.186	0.224	0.265	0.339	0.440	0.621	NaN	0.621
1.0	0.126	0.138	0.156	0.178	0.204	0.245	0.297	0.363	0.466	0.631	NaN

Simulation results for $n = 100$ and $\lambda_c = 1000$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.668	0.496	0.371	0.277	0.227	0.191	0.163	0.140	0.126	0.113
0.1	0.668	NaN	0.643	0.450	0.333	0.253	0.207	0.170	0.144	0.126	0.113
0.2	0.485	0.657	NaN	0.622	0.418	0.294	0.227	0.185	0.151	0.125	0.114
0.3	0.360	0.461	0.621	NaN	0.589	0.378	0.266	0.204	0.159	0.131	0.113
0.4	0.281	0.335	0.418	0.569	NaN	0.529	0.336	0.230	0.174	0.137	0.113
0.5	0.229	0.262	0.302	0.372	0.537	NaN	0.480	0.283	0.194	0.143	0.113
0.6	0.188	0.206	0.230	0.268	0.325	0.474	NaN	0.403	0.227	0.154	0.112
0.7	0.163	0.171	0.183	0.200	0.234	0.286	0.416	NaN	0.329	0.171	0.113
0.8	0.142	0.143	0.152	0.159	0.168	0.191	0.226	0.327	NaN	0.230	0.114
0.9	0.127	0.128	0.127	0.134	0.138	0.141	0.152	0.170	0.223	NaN	0.113
1.0	0.112	0.111	0.113	0.113	0.113	0.112	0.112	0.112	0.111	0.111	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.692	0.493	0.367	0.287	0.229	0.197	0.162	0.144	0.128	0.118
0.1	0.687	NaN	0.668	0.465	0.338	0.257	0.206	0.173	0.147	0.129	0.118
0.2	0.497	0.671	NaN	0.607	0.424	0.300	0.226	0.185	0.154	0.133	0.117
0.3	0.373	0.460	0.636	NaN	0.595	0.377	0.268	0.203	0.161	0.134	0.115
0.4	0.285	0.340	0.422	0.587	NaN	0.548	0.334	0.235	0.174	0.137	0.118
0.5	0.231	0.259	0.305	0.378	0.550	NaN	0.481	0.289	0.191	0.144	0.117
0.6	0.190	0.210	0.234	0.277	0.341	0.493	NaN	0.415	0.231	0.156	0.118
0.7	0.162	0.172	0.182	0.203	0.227	0.291	0.422	NaN	0.338	0.174	0.120
0.8	0.144	0.144	0.153	0.160	0.171	0.192	0.237	0.329	NaN	0.231	0.117
0.9	0.126	0.129	0.131	0.134	0.138	0.142	0.153	0.174	0.233	NaN	0.119
1.0	0.116	0.116	0.118	0.116	0.116	0.117	0.118	0.116	0.118	0.119	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.113	0.113	0.109	0.113	0.111	0.111	0.112	0.111	0.112	0.112
0.1	0.110	NaN	0.227	0.170	0.151	0.140	0.137	0.128	0.126	0.125	0.122
0.2	0.111	0.222	NaN	0.324	0.227	0.189	0.172	0.158	0.155	0.143	0.144
0.3	0.114	0.173	0.324	NaN	0.399	0.277	0.225	0.199	0.184	0.173	0.163
0.4	0.112	0.152	0.227	0.400	NaN	0.462	0.324	0.264	0.227	0.204	0.189
0.5	0.111	0.140	0.189	0.281	0.457	NaN	0.506	0.354	0.298	0.255	0.223
0.6	0.113	0.135	0.173	0.228	0.328	0.498	NaN	0.532	0.398	0.321	0.284
0.7	0.112	0.131	0.158	0.196	0.261	0.366	0.546	NaN	0.576	0.420	0.342
0.8	0.113	0.129	0.152	0.186	0.230	0.293	0.390	0.561	NaN	0.593	0.447
0.9	0.114	0.127	0.145	0.170	0.206	0.251	0.323	0.415	0.599	NaN	0.613
1.0	0.112	0.124	0.140	0.159	0.189	0.229	0.266	0.344	0.457	0.607	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.119	0.117	0.117	0.117	0.119	0.119	0.118	0.118	0.120	0.119
0.1	0.118	NaN	0.228	0.175	0.156	0.143	0.136	0.135	0.133	0.131	0.131
0.2	0.118	0.235	NaN	0.334	0.233	0.195	0.174	0.159	0.156	0.151	0.146
0.3	0.120	0.176	0.328	NaN	0.402	0.278	0.235	0.206	0.184	0.175	0.166
0.4	0.117	0.157	0.231	0.415	NaN	0.462	0.322	0.266	0.235	0.207	0.192
0.5	0.117	0.146	0.193	0.283	0.468	NaN	0.515	0.370	0.300	0.254	0.234
0.6	0.118	0.140	0.175	0.230	0.333	0.503	NaN	0.551	0.404	0.326	0.274
0.7	0.119	0.135	0.160	0.201	0.268	0.373	0.543	NaN	0.573	0.422	0.352
0.8	0.118	0.132	0.156	0.184	0.227	0.293	0.409	0.562	NaN	0.585	0.440
0.9	0.117	0.130	0.148	0.174	0.210	0.257	0.321	0.428	0.599	NaN	0.617
1.0	0.118	0.132	0.147	0.167	0.194	0.228	0.279	0.347	0.454	0.619	NaN

Simulation results for $n = 1000$ and $\lambda_c = 40$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.443	0.267	0.189	0.147	0.122	0.106	0.093	0.085	0.078	0.075
0.1	0.441	NaN	0.416	0.247	0.174	0.135	0.111	0.096	0.087	0.080	0.074
0.2	0.264	0.409	NaN	0.379	0.224	0.161	0.123	0.102	0.089	0.080	0.073
0.3	0.190	0.247	0.380	NaN	0.351	0.202	0.144	0.110	0.091	0.081	0.074
0.4	0.148	0.176	0.228	0.349	NaN	0.311	0.180	0.125	0.096	0.083	0.073
0.5	0.123	0.136	0.161	0.204	0.314	NaN	0.285	0.157	0.108	0.085	0.074
0.6	0.105	0.113	0.123	0.142	0.180	0.271	NaN	0.233	0.126	0.089	0.073
0.7	0.092	0.097	0.102	0.111	0.126	0.154	0.225	NaN	0.184	0.101	0.073
0.8	0.085	0.086	0.088	0.092	0.097	0.106	0.127	0.181	NaN	0.129	0.073
0.9	0.079	0.080	0.080	0.082	0.082	0.085	0.087	0.099	0.128	NaN	0.073
1.0	0.076	0.074	0.074	0.074	0.073	0.073	0.073	0.073	0.073	0.073	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.489	0.302	0.214	0.172	0.145	0.127	0.116	0.108	0.101	0.097
0.1	0.481	NaN	0.452	0.272	0.194	0.156	0.133	0.118	0.107	0.099	0.093
0.2	0.301	0.451	NaN	0.422	0.248	0.177	0.142	0.118	0.103	0.095	0.088
0.3	0.210	0.272	0.421	NaN	0.381	0.221	0.158	0.123	0.104	0.091	0.084
0.4	0.167	0.193	0.248	0.383	NaN	0.348	0.195	0.136	0.108	0.091	0.079
0.5	0.140	0.149	0.176	0.219	0.342	NaN	0.306	0.164	0.115	0.089	0.076
0.6	0.120	0.125	0.137	0.153	0.194	0.306	NaN	0.250	0.133	0.091	0.074
0.7	0.103	0.106	0.111	0.119	0.133	0.166	0.256	NaN	0.197	0.101	0.073
0.8	0.095	0.094	0.096	0.098	0.104	0.113	0.134	0.198	NaN	0.137	0.072
0.9	0.086	0.086	0.083	0.083	0.085	0.086	0.091	0.103	0.135	NaN	0.071
1.0	0.081	0.078	0.078	0.075	0.073	0.073	0.073	0.073	0.073	0.073	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.052	0.052	0.052	0.052	0.051	0.053	0.052	0.052	0.053	0.052
0.1	0.052	NaN	0.109	0.080	0.069	0.065	0.064	0.062	0.061	0.060	0.059
0.2	0.052	0.108	NaN	0.161	0.109	0.090	0.080	0.075	0.071	0.069	0.067
0.3	0.052	0.079	0.159	NaN	0.211	0.135	0.109	0.095	0.088	0.081	0.078
0.4	0.053	0.071	0.108	0.207	NaN	0.248	0.163	0.129	0.109	0.098	0.093
0.5	0.053	0.066	0.090	0.135	0.251	NaN	0.292	0.185	0.145	0.122	0.111
0.6	0.053	0.063	0.081	0.110	0.164	0.278	NaN	0.321	0.209	0.163	0.141
0.7	0.053	0.062	0.075	0.097	0.129	0.185	0.317	NaN	0.347	0.232	0.180
0.8	0.052	0.060	0.071	0.087	0.108	0.146	0.209	0.348	NaN	0.363	0.248
0.9	0.052	0.060	0.069	0.082	0.098	0.128	0.162	0.228	0.364	NaN	0.392
1.0	0.052	0.058	0.067	0.077	0.092	0.112	0.139	0.181	0.247	0.396	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.054	0.055	0.057	0.059	0.062	0.065	0.069	0.074	0.079	0.083
0.1	0.054	NaN	0.115	0.087	0.079	0.077	0.078	0.079	0.083	0.086	0.091
0.2	0.057	0.115	NaN	0.176	0.122	0.103	0.097	0.095	0.095	0.095	0.099
0.3	0.061	0.088	0.176	NaN	0.226	0.154	0.130	0.120	0.112	0.111	0.113
0.4	0.064	0.082	0.122	0.232	NaN	0.278	0.185	0.154	0.140	0.132	0.130
0.5	0.068	0.081	0.105	0.152	0.278	NaN	0.322	0.216	0.174	0.157	0.149
0.6	0.074	0.083	0.101	0.128	0.186	0.328	NaN	0.356	0.242	0.198	0.179
0.7	0.079	0.085	0.096	0.121	0.152	0.213	0.354	NaN	0.395	0.267	0.223
0.8	0.084	0.088	0.100	0.117	0.142	0.179	0.244	0.387	NaN	0.414	0.293
0.9	0.090	0.094	0.104	0.113	0.136	0.161	0.198	0.267	0.408	NaN	0.442
1.0	0.095	0.099	0.108	0.117	0.132	0.152	0.175	0.219	0.288	0.444	NaN

Simulation results for $n = 1000$ and $\lambda_c = 200$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.394	0.217	0.150	0.111	0.087	0.070	0.059	0.052	0.046	0.041
0.1	0.400	NaN	0.361	0.197	0.132	0.098	0.077	0.064	0.053	0.046	0.041
0.2	0.218	0.364	NaN	0.331	0.179	0.116	0.086	0.069	0.056	0.047	0.041
0.3	0.150	0.198	0.339	NaN	0.296	0.153	0.101	0.074	0.059	0.049	0.041
0.4	0.109	0.134	0.177	0.290	NaN	0.263	0.136	0.085	0.064	0.050	0.040
0.5	0.088	0.098	0.117	0.159	0.262	NaN	0.222	0.113	0.072	0.052	0.041
0.6	0.070	0.077	0.087	0.102	0.132	0.223	NaN	0.180	0.087	0.055	0.040
0.7	0.059	0.063	0.067	0.075	0.086	0.114	0.181	NaN	0.137	0.064	0.041
0.8	0.052	0.053	0.054	0.058	0.062	0.069	0.089	0.137	NaN	0.088	0.040
0.9	0.045	0.046	0.047	0.048	0.049	0.051	0.056	0.064	0.088	NaN	0.040
1.0	0.041	0.041	0.040	0.041	0.041	0.041	0.042	0.041	0.041	0.041	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.411	0.224	0.151	0.111	0.090	0.073	0.063	0.055	0.050	0.046
0.1	0.400	NaN	0.380	0.199	0.134	0.099	0.081	0.066	0.057	0.050	0.044
0.2	0.222	0.378	NaN	0.341	0.183	0.120	0.090	0.070	0.057	0.050	0.043
0.3	0.151	0.202	0.343	NaN	0.305	0.156	0.108	0.076	0.060	0.050	0.043
0.4	0.113	0.139	0.180	0.309	NaN	0.266	0.136	0.088	0.064	0.050	0.043
0.5	0.089	0.100	0.120	0.162	0.266	NaN	0.224	0.113	0.072	0.053	0.042
0.6	0.075	0.080	0.089	0.104	0.136	0.227	NaN	0.181	0.089	0.056	0.042
0.7	0.063	0.066	0.071	0.075	0.090	0.110	0.183	NaN	0.138	0.064	0.041
0.8	0.057	0.057	0.058	0.059	0.065	0.073	0.088	0.140	NaN	0.087	0.042
0.9	0.050	0.051	0.051	0.050	0.050	0.053	0.055	0.064	0.088	NaN	0.041
1.0	0.047	0.046	0.044	0.044	0.043	0.042	0.042	0.042	0.041	0.042	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.039	0.039	0.039	0.039	0.040	0.040	0.040	0.039	0.040	0.040
0.1	0.039	NaN	0.085	0.062	0.054	0.051	0.048	0.047	0.046	0.045	0.045
0.2	0.039	0.086	NaN	0.134	0.086	0.068	0.062	0.058	0.055	0.052	0.050
0.3	0.040	0.061	0.133	NaN	0.176	0.109	0.085	0.073	0.066	0.062	0.058
0.4	0.040	0.054	0.085	0.177	NaN	0.218	0.135	0.101	0.086	0.077	0.071
0.5	0.040	0.050	0.070	0.111	0.218	NaN	0.256	0.156	0.119	0.098	0.087
0.6	0.039	0.049	0.062	0.086	0.133	0.254	NaN	0.287	0.177	0.134	0.110
0.7	0.039	0.046	0.058	0.075	0.102	0.158	0.288	NaN	0.314	0.196	0.148
0.8	0.040	0.045	0.054	0.067	0.085	0.115	0.180	0.319	NaN	0.332	0.214
0.9	0.040	0.045	0.052	0.063	0.077	0.099	0.135	0.199	0.340	NaN	0.360
1.0	0.039	0.045	0.050	0.059	0.069	0.086	0.111	0.147	0.215	0.365	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.040	0.041	0.041	0.042	0.043	0.044	0.046	0.048	0.050	0.051
0.1	0.040	NaN	0.087	0.063	0.056	0.053	0.053	0.051	0.052	0.054	0.053
0.2	0.041	0.088	NaN	0.136	0.089	0.074	0.066	0.063	0.061	0.060	0.062
0.3	0.040	0.062	0.136	NaN	0.181	0.113	0.092	0.079	0.074	0.070	0.069
0.4	0.041	0.056	0.087	0.184	NaN	0.224	0.138	0.107	0.092	0.085	0.080
0.5	0.041	0.051	0.072	0.114	0.223	NaN	0.260	0.163	0.124	0.107	0.096
0.6	0.042	0.050	0.065	0.088	0.137	0.259	NaN	0.293	0.184	0.139	0.121
0.7	0.042	0.049	0.060	0.076	0.107	0.157	0.296	NaN	0.325	0.206	0.158
0.8	0.042	0.048	0.058	0.071	0.091	0.125	0.181	0.328	NaN	0.352	0.226
0.9	0.042	0.049	0.055	0.068	0.082	0.106	0.140	0.204	0.354	NaN	0.376
1.0	0.043	0.048	0.055	0.064	0.076	0.092	0.118	0.159	0.226	0.377	NaN

Simulation results for $n = 1000$ and $\lambda_c = 1000$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.391	0.209	0.140	0.108	0.084	0.067	0.056	0.048	0.043	0.039
0.1	0.385	NaN	0.359	0.193	0.129	0.094	0.074	0.059	0.050	0.043	0.038
0.2	0.208	0.355	NaN	0.325	0.171	0.115	0.082	0.064	0.053	0.044	0.038
0.3	0.142	0.191	0.320	NaN	0.288	0.151	0.098	0.071	0.055	0.045	0.039
0.4	0.106	0.129	0.171	0.290	NaN	0.255	0.127	0.084	0.060	0.046	0.038
0.5	0.081	0.093	0.112	0.150	0.252	NaN	0.215	0.106	0.067	0.049	0.038
0.6	0.068	0.072	0.082	0.098	0.132	0.219	NaN	0.174	0.082	0.053	0.037
0.7	0.056	0.060	0.063	0.071	0.082	0.106	0.171	NaN	0.130	0.059	0.038
0.8	0.047	0.050	0.052	0.056	0.060	0.067	0.084	0.129	NaN	0.082	0.038
0.9	0.043	0.043	0.045	0.045	0.046	0.049	0.052	0.059	0.083	NaN	0.038
1.0	0.039	0.038	0.038	0.038	0.038	0.039	0.038	0.038	0.039	0.038	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.393	0.216	0.141	0.105	0.083	0.067	0.057	0.049	0.043	0.039
0.1	0.398	NaN	0.357	0.189	0.127	0.095	0.074	0.059	0.050	0.044	0.039
0.2	0.214	0.357	NaN	0.322	0.175	0.111	0.081	0.063	0.052	0.044	0.039
0.3	0.141	0.194	0.328	NaN	0.287	0.149	0.099	0.072	0.055	0.045	0.038
0.4	0.106	0.130	0.170	0.304	NaN	0.250	0.127	0.084	0.060	0.046	0.038
0.5	0.083	0.094	0.114	0.150	0.257	NaN	0.212	0.107	0.067	0.049	0.038
0.6	0.067	0.073	0.083	0.098	0.127	0.216	NaN	0.175	0.083	0.053	0.039
0.7	0.057	0.059	0.065	0.071	0.083	0.106	0.173	NaN	0.131	0.060	0.038
0.8	0.049	0.051	0.052	0.055	0.060	0.067	0.082	0.129	NaN	0.081	0.039
0.9	0.043	0.044	0.044	0.045	0.046	0.049	0.052	0.059	0.085	NaN	0.039
1.0	0.039	0.039	0.039	0.039	0.039	0.038	0.039	0.039	0.039	0.039	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.038	0.038	0.037	0.038	0.038	0.038	0.038	0.038	0.038	0.037
0.1	0.038	NaN	0.083	0.061	0.052	0.049	0.046	0.044	0.043	0.043	0.042
0.2	0.037	0.082	NaN	0.129	0.082	0.068	0.060	0.055	0.051	0.050	0.048
0.3	0.038	0.059	0.128	NaN	0.174	0.107	0.084	0.072	0.064	0.060	0.056
0.4	0.038	0.053	0.082	0.172	NaN	0.214	0.129	0.098	0.082	0.072	0.066
0.5	0.038	0.048	0.067	0.105	0.213	NaN	0.248	0.150	0.112	0.093	0.082
0.6	0.038	0.046	0.060	0.081	0.132	0.252	NaN	0.280	0.169	0.129	0.105
0.7	0.038	0.045	0.054	0.071	0.098	0.151	0.283	NaN	0.312	0.188	0.142
0.8	0.037	0.043	0.052	0.064	0.083	0.113	0.173	0.309	NaN	0.330	0.209
0.9	0.038	0.043	0.049	0.060	0.074	0.095	0.126	0.187	0.330	NaN	0.351
1.0	0.038	0.042	0.048	0.057	0.067	0.083	0.106	0.141	0.213	0.355	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.040	0.039	0.038
0.1	0.039	NaN	0.084	0.059	0.052	0.048	0.047	0.045	0.044	0.044	0.044
0.2	0.038	0.084	NaN	0.129	0.084	0.067	0.059	0.056	0.053	0.050	0.050
0.3	0.038	0.061	0.130	NaN	0.171	0.106	0.085	0.072	0.064	0.060	0.057
0.4	0.039	0.053	0.083	0.177	NaN	0.213	0.128	0.100	0.083	0.075	0.068
0.5	0.038	0.048	0.067	0.107	0.213	NaN	0.247	0.153	0.115	0.095	0.085
0.6	0.038	0.046	0.060	0.084	0.129	0.252	NaN	0.280	0.171	0.131	0.106
0.7	0.038	0.046	0.056	0.072	0.100	0.152	0.281	NaN	0.314	0.193	0.145
0.8	0.039	0.044	0.053	0.066	0.083	0.115	0.172	0.311	NaN	0.333	0.211
0.9	0.039	0.043	0.050	0.060	0.075	0.096	0.131	0.190	0.336	NaN	0.360
1.0	0.038	0.043	0.049	0.058	0.067	0.084	0.109	0.144	0.210	0.366	NaN

Simulation results with correction for Ryzhov's and Maiboroda's estimators for discret data

We compare $\hat{S}_1(t)$ with $S_1(t)$ and$\hat{S}_2(t)$ with $S_2(t)$ for different weights, different data sizes, and different values of the censoring parameter λ_c . We have computed the sup-norm $\sup_{t\in\mathbb{R}} |\hat{S}(t) - S(t)|$. All the simulation results are given for 1000 repetitions, with the correction mentioned on section 3 for Maiboroda's method, and for discret data.

Simulation results for $n = 100$ and $\lambda_c = 40$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.676	0.514	0.415	0.339	0.287	0.254	0.230	0.210	0.199	0.189
0.1	0.668	NaN	0.650	0.495	0.382	0.317	0.269	0.234	0.214	0.197	0.184
0.2	0.518	0.648	NaN	0.624	0.453	0.352	0.285	0.244	0.216	0.200	0.186
0.3	0.417	0.497	0.626	NaN	0.588	0.430	0.320	0.260	0.227	0.199	0.190
0.4	0.345	0.383	0.458	0.586	NaN	0.557	0.383	0.289	0.237	0.202	0.192
0.5	0.286	0.319	0.351	0.416	0.553	NaN	0.508	0.343	0.254	0.209	0.185
0.6	0.258	0.266	0.293	0.325	0.384	0.509	NaN	0.450	0.292	0.217	0.185
0.7	0.227	0.237	0.245	0.264	0.288	0.337	0.453	NaN	0.381	0.240	0.183
0.8	0.210	0.215	0.219	0.226	0.239	0.253	0.289	0.391	NaN	0.294	0.183
0.9	0.196	0.199	0.198	0.203	0.204	0.208	0.218	0.238	0.292	NaN	0.184
1.0	0.188	0.186	0.186	0.187	0.184	0.183	0.187	0.185	0.185	0.188	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.676	0.544	0.429	0.347	0.287	0.253	0.221	0.191	0.179	0.166
0.1	0.674	NaN	0.664	0.520	0.407	0.324	0.273	0.225	0.200	0.180	0.164
0.2	0.535	0.663	NaN	0.637	0.476	0.368	0.300	0.240	0.203	0.185	0.171
0.3	0.430	0.524	0.650	NaN	0.612	0.453	0.330	0.268	0.218	0.189	0.171
0.4	0.350	0.412	0.480	0.619	NaN	0.575	0.407	0.302	0.232	0.193	0.174
0.5	0.294	0.327	0.361	0.439	0.591	NaN	0.540	0.367	0.260	0.199	0.173
0.6	0.252	0.265	0.291	0.336	0.402	0.535	NaN	0.479	0.305	0.217	0.178
0.7	0.217	0.229	0.237	0.265	0.297	0.358	0.491	NaN	0.406	0.239	0.178
0.8	0.193	0.200	0.211	0.219	0.236	0.262	0.304	0.422	NaN	0.314	0.184
0.9	0.180	0.182	0.189	0.191	0.198	0.207	0.218	0.244	0.307	NaN	0.189
1.0	0.175	0.174	0.176	0.179	0.180	0.182	0.185	0.186	0.185	0.190	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.149	0.147	0.150	0.148	0.147	0.148	0.151	0.152	0.149	0.152
0.1	0.148	NaN	0.264	0.206	0.188	0.178	0.171	0.164	0.162	0.161	0.160
0.2	0.147	0.267	NaN	0.361	0.263	0.230	0.211	0.200	0.186	0.181	0.177
0.3	0.148	0.207	0.351	NaN	0.417	0.313	0.265	0.235	0.221	0.212	0.198
0.4	0.149	0.190	0.260	0.424	NaN	0.475	0.348	0.301	0.262	0.245	0.225
0.5	0.149	0.178	0.226	0.311	0.466	NaN	0.510	0.386	0.323	0.286	0.263
0.6	0.147	0.174	0.210	0.262	0.356	0.515	NaN	0.539	0.411	0.349	0.315
0.7	0.152	0.167	0.193	0.238	0.291	0.383	0.549	NaN	0.566	0.435	0.366
0.8	0.146	0.165	0.188	0.223	0.266	0.320	0.416	0.571	NaN	0.588	0.464
0.9	0.147	0.162	0.180	0.210	0.242	0.289	0.344	0.449	0.590	NaN	0.610
1.0	0.149	0.159	0.177	0.201	0.233	0.270	0.311	0.369	0.465	0.602	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.166	0.170	0.175	0.175	0.179	0.184	0.190	0.196	0.195	0.204
0.1	0.167	NaN	0.280	0.226	0.212	0.205	0.203	0.200	0.202	0.206	0.211
0.2	0.171	0.280	NaN	0.380	0.288	0.260	0.240	0.231	0.229	0.228	0.228
0.3	0.176	0.230	0.376	NaN	0.444	0.333	0.299	0.271	0.261	0.250	0.251
0.4	0.178	0.218	0.289	0.439	NaN	0.491	0.379	0.335	0.297	0.289	0.275
0.5	0.183	0.206	0.255	0.348	0.495	NaN	0.522	0.417	0.354	0.323	0.312
0.6	0.186	0.207	0.240	0.299	0.388	0.526	NaN	0.544	0.437	0.383	0.358
0.7	0.195	0.203	0.232	0.272	0.331	0.414	0.565	NaN	0.576	0.469	0.411
0.8	0.197	0.209	0.227	0.258	0.303	0.360	0.450	0.588	NaN	0.597	0.499
0.9	0.201	0.212	0.224	0.255	0.285	0.330	0.378	0.481	0.603	NaN	0.601
1.0	0.205	0.209	0.233	0.244	0.279	0.308	0.358	0.409	0.490	0.599	NaN

Simulation results for $n = 100$ and $\lambda_c = 200$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.655	0.476	0.359	0.287	0.232	0.194	0.168	0.144	0.127	0.120
0.1	0.662	NaN	0.625	0.448	0.329	0.266	0.212	0.175	0.151	0.131	0.120
0.2	0.481	0.637	NaN	0.598	0.417	0.304	0.236	0.189	0.155	0.131	0.120
0.3	0.364	0.448	0.606	NaN	0.564	0.379	0.269	0.207	0.165	0.136	0.119
0.4	0.290	0.333	0.415	0.565	NaN	0.523	0.328	0.238	0.176	0.140	0.118
0.5	0.231	0.257	0.301	0.378	0.530	NaN	0.474	0.285	0.196	0.148	0.118
0.6	0.196	0.208	0.230	0.271	0.332	0.466	NaN	0.421	0.236	0.156	0.119
0.7	0.167	0.176	0.187	0.207	0.233	0.285	0.404	NaN	0.335	0.173	0.120
0.8	0.146	0.149	0.158	0.163	0.173	0.198	0.234	0.335	NaN	0.236	0.118
0.9	0.129	0.133	0.134	0.138	0.143	0.146	0.156	0.177	0.238	NaN	0.118
1.0	0.118	0.119	0.118	0.118	0.116	0.119	0.120	0.117	0.119	0.117	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.658	0.486	0.363	0.286	0.233	0.195	0.171	0.148	0.133	0.123
0.1	0.665	NaN	0.632	0.452	0.333	0.267	0.208	0.175	0.151	0.135	0.129
0.2	0.486	0.639	NaN	0.599	0.417	0.307	0.235	0.192	0.159	0.137	0.126
0.3	0.372	0.455	0.610	NaN	0.569	0.379	0.271	0.209	0.168	0.139	0.128
0.4	0.291	0.338	0.426	0.576	NaN	0.535	0.333	0.244	0.179	0.144	0.126
0.5	0.230	0.259	0.302	0.386	0.536	NaN	0.485	0.292	0.199	0.153	0.128
0.6	0.196	0.206	0.235	0.271	0.332	0.479	NaN	0.431	0.243	0.163	0.128
0.7	0.169	0.178	0.187	0.208	0.236	0.289	0.410	NaN	0.340	0.179	0.129
0.8	0.146	0.152	0.157	0.165	0.173	0.200	0.241	0.338	NaN	0.245	0.127
0.9	0.131	0.135	0.136	0.140	0.146	0.146	0.162	0.183	0.243	NaN	0.129
1.0	0.123	0.123	0.122	0.123	0.122	0.124	0.127	0.125	0.128	0.128	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.113	0.112	0.115	0.115	0.109	0.112	0.111	0.113	0.113	0.114
0.1	0.115	NaN	0.227	0.172	0.152	0.143	0.136	0.128	0.131	0.129	0.123
0.2	0.113	0.225	NaN	0.328	0.233	0.195	0.174	0.160	0.153	0.146	0.143
0.3	0.111	0.171	0.322	NaN	0.394	0.270	0.227	0.201	0.185	0.171	0.163
0.4	0.111	0.151	0.222	0.400	NaN	0.448	0.321	0.259	0.230	0.208	0.190
.5	0.111	0.143	0.193	0.281	0.453	NaN	0.486	0.360	0.291	0.254	0.225
0.6	0.112	0.136	0.169	0.228	0.323	0.494	NaN	0.529	0.389	0.318	0.271
0.7	0.111	0.132	0.159	0.201	0.261	0.360	0.526	NaN	0.561	0.414	0.336
0.8	0.112	0.127	0.154	0.183	0.231	0.293	0.388	0.555	NaN	0.580	0.446
0.9	0.114	0.125	0.148	0.171	0.209	0.249	0.321	0.417	0.577	NaN	0.601
1.0	0.113	0.126	0.140	0.167	0.191	0.232	0.274	0.348	0.439	0.588	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.132	0.134	0.138	0.139	0.134	0.140	0.143	0.143	0.145	0.149
0.1	0.132	NaN	0.237	0.186	0.169	0.162	0.158	0.154	0.157	0.159	0.155
0.2	0.134	0.236	NaN	0.332	0.244	0.208	0.188	0.181	0.172	0.171	0.171
0.3	0.131	0.184	0.333	NaN	0.402	0.279	0.242	0.221	0.208	0.190	0.188
0.4	0.134	0.168	0.236	0.404	NaN	0.455	0.329	0.271	0.247	0.228	0.215
0.5	0.132	0.162	0.206	0.287	0.463	NaN	0.492	0.365	0.302	0.271	0.244
0.6	0.137	0.154	0.185	0.239	0.333	0.486	NaN	0.531	0.400	0.328	0.295
0.7	0.133	0.149	0.175	0.215	0.274	0.369	0.526	NaN	0.557	0.424	0.347
0.8	0.136	0.147	0.172	0.200	0.247	0.299	0.400	0.555	NaN	0.576	0.453
0.9	0.135	0.146	0.168	0.186	0.224	0.263	0.332	0.428	0.572	NaN	0.597
1.0	0.137	0.147	0.162	0.185	0.208	0.250	0.291	0.354	0.440	0.585	NaN

Simulation results for $n = 100$ and $\lambda_c = 1000$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.649	0.475	0.355	0.275	0.226	0.186	0.162	0.137	0.122	0.107
0.1	0.654	NaN	0.618	0.446	0.324	0.249	0.199	0.170	0.141	0.126	0.112
0.2	0.470	0.628	NaN	0.594	0.400	0.289	0.221	0.178	0.146	0.127	0.109
0.3	0.353	0.444	0.609	NaN	0.561	0.368	0.255	0.192	0.157	0.127	0.109
0.4	0.271	0.323	0.402	0.560	NaN	0.526	0.324	0.226	0.169	0.134	0.109
0.5	0.221	0.250	0.295	0.366	0.516	NaN	0.463	0.277	0.185	0.138	0.110
0.6	0.187	0.196	0.221	0.260	0.321	0.464	NaN	0.401	0.221	0.147	0.111
0.7	0.158	0.165	0.180	0.196	0.223	0.283	0.398	NaN	0.319	0.169	0.106
0.8	0.137	0.141	0.147	0.154	0.166	0.189	0.224	0.329	NaN	0.225	0.110
0.9	0.123	0.123	0.128	0.129	0.130	0.138	0.146	0.168	0.227	NaN	0.111
1.0	0.109	0.110	0.109	0.112	0.108	0.109	0.107	0.110	0.108	0.109	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.651	0.475	0.356	0.275	0.227	0.190	0.165	0.144	0.131	0.118
0.1	0.655	NaN	0.626	0.450	0.325	0.252	0.204	0.173	0.148	0.134	0.123
0.2	0.468	0.628	NaN	0.595	0.401	0.288	0.221	0.182	0.155	0.137	0.119
0.3	0.351	0.448	0.604	NaN	0.558	0.367	0.255	0.196	0.161	0.135	0.121
0.4	0.274	0.321	0.409	0.563	NaN	0.526	0.324	0.231	0.171	0.140	0.121
0.5	0.222	0.251	0.297	0.364	0.515	NaN	0.459	0.276	0.188	0.146	0.121
0.6	0.191	0.195	0.224	0.259	0.319	0.467	NaN	0.398	0.227	0.153	0.123
0.7	0.161	0.170	0.181	0.200	0.226	0.285	0.400	NaN	0.321	0.175	0.122
0.8	0.142	0.146	0.153	0.159	0.170	0.190	0.227	0.327	NaN	0.231	0.122
0.9	0.131	0.130	0.135	0.134	0.138	0.144	0.151	0.173	0.232	NaN	0.124
1.0	0.118	0.118	0.120	0.122	0.120	0.122	0.119	0.120	0.121	0.120	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.108	0.108	0.107	0.109	0.107	0.108	0.107	0.106	0.107	0.107
0.1	0.107	NaN	0.218	0.170	0.144	0.136	0.130	0.126	0.125	0.121	0.119
0.2	0.109	0.224	NaN	0.312	0.222	0.189	0.164	0.155	0.146	0.142	0.136
0.3	0.106	0.166	0.321	NaN	0.390	0.273	0.220	0.192	0.176	0.166	0.158
0.4	0.107	0.146	0.224	0.392	NaN	0.449	0.311	0.255	0.220	0.201	0.182
0.5	0.106	0.133	0.186	0.271	0.451	NaN	0.486	0.353	0.289	0.247	0.219
0.6	0.107	0.127	0.168	0.222	0.313	0.481	NaN	0.520	0.383	0.311	0.270
0.7	0.108	0.127	0.155	0.193	0.255	0.356	0.526	NaN	0.542	0.412	0.338
0.8	0.106	0.124	0.146	0.180	0.219	0.289	0.375	0.555	NaN	0.564	0.434
0.9	0.109	0.119	0.141	0.163	0.200	0.245	0.314	0.408	0.563	NaN	0.594
1.0	0.108	0.119	0.138	0.160	0.183	0.216	0.270	0.330	0.434	0.583	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.128	0.131	0.131	0.132	0.130	0.131	0.131	0.132	0.132	0.133
0.1	0.130	NaN	0.226	0.182	0.159	0.153	0.149	0.146	0.147	0.142	0.140
0.2	0.128	0.232	NaN	0.317	0.231	0.199	0.178	0.170	0.160	0.158	0.156
0.3	0.127	0.176	0.322	NaN	0.391	0.275	0.229	0.203	0.189	0.180	0.174
0.4	0.129	0.160	0.233	0.392	NaN	0.441	0.316	0.262	0.230	0.211	0.195
0.5	0.129	0.149	0.196	0.276	0.445	NaN	0.482	0.352	0.292	0.252	0.228
0.6	0.128	0.145	0.179	0.228	0.315	0.482	NaN	0.510	0.384	0.316	0.278
0.7	0.130	0.145	0.168	0.207	0.263	0.357	0.521	NaN	0.538	0.410	0.344
0.8	0.129	0.141	0.162	0.192	0.228	0.293	0.377	0.552	NaN	0.559	0.438
0.9	0.132	0.140	0.156	0.179	0.210	0.251	0.322	0.412	0.559	NaN	0.586
1.0	0.128	0.140	0.156	0.173	0.195	0.228	0.279	0.334	0.431	0.580	NaN

Simulation results for $n = 1000$ and $\lambda_c = 40$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.420	0.266	0.186	0.146	0.122	0.106	0.095	0.085	0.079	0.075
0.1	0.419	NaN	0.402	0.240	0.172	0.134	0.112	0.098	0.086	0.080	0.076
0.2	0.257	0.396	NaN	0.368	0.220	0.158	0.123	0.103	0.089	0.080	0.074
0.3	0.186	0.241	0.370	NaN	0.337	0.200	0.141	0.109	0.093	0.082	0.075
0.4	0.144	0.174	0.224	0.339	NaN	0.303	0.176	0.125	0.097	0.083	0.074
0.5	0.120	0.135	0.154	0.198	0.305	NaN	0.270	0.154	0.109	0.085	0.074
0.6	0.103	0.112	0.123	0.141	0.174	0.266	NaN	0.225	0.125	0.089	0.074
0.7	0.094	0.097	0.102	0.111	0.125	0.152	0.223	NaN	0.179	0.099	0.074
0.8	0.086	0.086	0.090	0.094	0.098	0.108	0.126	0.180	NaN	0.128	0.075
0.9	0.081	0.080	0.081	0.083	0.083	0.085	0.089	0.099	0.127	NaN	0.075
1.0	0.075	0.076	0.075	0.075	0.075	0.074	0.074	0.075	0.075	0.074	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.444	0.275	0.197	0.152	0.125	0.112	0.101	0.092	0.087	0.082
0.1	0.440	NaN	0.424	0.252	0.178	0.138	0.116	0.102	0.090	0.084	0.079
0.2	0.270	0.421	NaN	0.395	0.229	0.163	0.127	0.104	0.089	0.082	0.074
0.3	0.191	0.253	0.390	NaN	0.360	0.208	0.145	0.111	0.092	0.080	0.074
0.4	0.149	0.174	0.233	0.358	NaN	0.323	0.184	0.127	0.099	0.080	0.071
0.5	0.119	0.134	0.159	0.205	0.319	NaN	0.289	0.155	0.106	0.082	0.071
0.6	0.100	0.110	0.122	0.140	0.179	0.281	NaN	0.237	0.128	0.087	0.071
0.7	0.087	0.092	0.097	0.105	0.126	0.157	0.239	NaN	0.189	0.098	0.072
0.8	0.079	0.079	0.084	0.087	0.092	0.105	0.128	0.188	NaN	0.135	0.074
0.9	0.073	0.073	0.074	0.075	0.078	0.081	0.088	0.100	0.132	NaN	0.077
1.0	0.069	0.069	0.070	0.071	0.071	0.072	0.073	0.076	0.077	0.078	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.054	0.054	0.055	0.055	0.055	0.054	0.056	0.054	0.054	0.054
0.1	0.054	NaN	0.107	0.081	0.072	0.067	0.064	0.063	0.061	0.061	0.060
0.2	0.053	0.105	NaN	0.158	0.107	0.091	0.081	0.077	0.072	0.069	0.068
0.3	0.053	0.081	0.159	NaN	0.202	0.134	0.109	0.095	0.086	0.082	0.077
0.4	0.056	0.071	0.110	0.202	NaN	0.241	0.158	0.129	0.110	0.098	0.092
0.5	0.055	0.066	0.090	0.133	0.242	NaN	0.280	0.181	0.143	0.122	0.112
0.6	0.055	0.065	0.080	0.107	0.157	0.274	NaN	0.306	0.203	0.155	0.136
0.7	0.054	0.064	0.076	0.096	0.127	0.182	0.305	NaN	0.329	0.224	0.177
0.8	0.055	0.062	0.072	0.089	0.108	0.144	0.202	0.334	NaN	0.360	0.243
0.9	0.055	0.060	0.069	0.081	0.097	0.124	0.158	0.222	0.357	NaN	0.370
1.0	0.055	0.061	0.068	0.079	0.091	0.111	0.136	0.173	0.242	0.374	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.082	0.085	0.089	0.093	0.097	0.102	0.107	0.111	0.119	0.123
0.1	0.080	NaN	0.131	0.111	0.107	0.106	0.108	0.114	0.118	0.124	0.129
0.2	0.082	0.128	NaN	0.182	0.138	0.129	0.123	0.125	0.128	0.130	0.134
0.3	0.084	0.108	0.187	NaN	0.233	0.171	0.150	0.142	0.140	0.143	0.144
0.4	0.088	0.102	0.140	0.234	NaN	0.274	0.199	0.174	0.162	0.158	0.157
0.5	0.092	0.104	0.125	0.169	0.268	NaN	0.314	0.223	0.194	0.179	0.175
0.6	0.096	0.107	0.122	0.149	0.195	0.312	NaN	0.342	0.250	0.219	0.200
0.7	0.100	0.108	0.122	0.139	0.170	0.222	0.340	NaN	0.371	0.272	0.234
0.8	0.105	0.113	0.123	0.138	0.159	0.191	0.246	0.376	NaN	0.398	0.294
0.9	0.112	0.117	0.126	0.136	0.155	0.180	0.211	0.278	0.392	NaN	0.412
1.0	0.117	0.121	0.130	0.142	0.155	0.172	0.199	0.229	0.293	0.418	NaN

Simulation results for $n = 1000$ and $\lambda_c = 200$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.377	0.209	0.144	0.108	0.084	0.068	0.058	0.050	0.046	0.041
0.1	0.372	NaN	0.352	0.195	0.130	0.096	0.075	0.062	0.051	0.046	0.041
0.2	0.215	0.357	NaN	0.318	0.173	0.116	0.085	0.066	0.054	0.046	0.041
0.3	0.143	0.192	0.318	NaN	0.292	0.154	0.100	0.073	0.056	0.048	0.040
0.4	0.105	0.129	0.169	0.292	NaN	0.250	0.131	0.086	0.061	0.049	0.041
0.5	0.083	0.097	0.116	0.153	0.254	NaN	0.215	0.110	0.070	0.051	0.041
0.6	0.068	0.077	0.086	0.102	0.130	0.216	NaN	0.177	0.085	0.055	0.041
0.7	0.058	0.062	0.065	0.074	0.086	0.109	0.177	NaN	0.135	0.062	0.041
0.8	0.050	0.052	0.054	0.057	0.062	0.069	0.088	0.134	NaN	0.088	0.041
0.9	0.044	0.046	0.046	0.047	0.048	0.051	0.054	0.063	0.086	NaN	0.041
1.0	0.040	0.040	0.041	0.040	0.041	0.041	0.040	0.041	0.041	0.041	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.379	0.208	0.146	0.109	0.087	0.074	0.064	0.058	0.056	0.054
0.1	0.371	NaN	0.350	0.191	0.129	0.098	0.078	0.069	0.060	0.056	0.053
0.2	0.215	0.356	NaN	0.322	0.173	0.114	0.087	0.072	0.061	0.057	0.054
0.3	0.140	0.191	0.317	NaN	0.288	0.153	0.102	0.077	0.064	0.057	0.053
0.4	0.106	0.127	0.171	0.290	NaN	0.258	0.131	0.089	0.068	0.059	0.053
0.5	0.085	0.095	0.115	0.153	0.253	NaN	0.217	0.113	0.075	0.060	0.054
0.6	0.070	0.077	0.087	0.102	0.131	0.215	NaN	0.179	0.088	0.062	0.054
0.7	0.061	0.063	0.069	0.076	0.088	0.110	0.177	NaN	0.136	0.069	0.054
0.8	0.054	0.056	0.058	0.061	0.066	0.073	0.090	0.135	NaN	0.093	0.054
0.9	0.050	0.051	0.051	0.053	0.054	0.057	0.061	0.067	0.092	NaN	0.053
1.0	0.047	0.047	0.048	0.048	0.048	0.050	0.051	0.051	0.052	0.053	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.040	0.040	0.040	0.040	0.040	0.041	0.040	0.041	0.040	0.041
0.1	0.04	NaN	0.085	0.061	0.053	0.049	0.048	0.046	0.045	0.044	0.044
0.2	0.04	0.085	NaN	0.128	0.083	0.068	0.061	0.057	0.053	0.052	0.050
0.3	0.04	0.060	0.129	NaN	0.172	0.108	0.083	0.072	0.067	0.061	0.058
0.4	0.04	0.054	0.084	0.174	NaN	0.212	0.130	0.100	0.084	0.075	0.069
0.5	0.04	0.050	0.069	0.107	0.212	NaN	0.246	0.152	0.114	0.095	0.085
0.6	0.04	0.049	0.061	0.086	0.129	0.249	NaN	0.274	0.171	0.130	0.105
0.7	0.04	0.046	0.056	0.073	0.102	0.152	0.280	NaN	0.310	0.189	0.146
0.8	0.04	0.046	0.054	0.065	0.085	0.116	0.178	0.306	NaN	0.334	0.209
0.9	0.04	0.045	0.052	0.061	0.076	0.097	0.131	0.191	0.334	NaN	0.347
1.0	0.04	0.044	0.050	0.057	0.069	0.084	0.108	0.145	0.210	0.352	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.074	0.075	0.076	0.077	0.079	0.080	0.081	0.083	0.084	0.087
0.1	0.073	NaN	0.104	0.088	0.085	0.084	0.085	0.085	0.086	0.087	0.088
0.2	0.073	0.103	NaN	0.143	0.105	0.094	0.093	0.090	0.091	0.091	0.091
0.3	0.073	0.085	0.141	NaN	0.182	0.126	0.107	0.101	0.099	0.098	0.097
0.4	0.073	0.082	0.104	0.184	NaN	0.224	0.148	0.124	0.114	0.107	0.104
0.5	0.074	0.079	0.093	0.124	0.221	NaN	0.257	0.168	0.136	0.122	0.117
0.6	0.073	0.079	0.088	0.106	0.147	0.258	NaN	0.290	0.185	0.155	0.135
0.7	0.074	0.078	0.085	0.096	0.121	0.167	0.291	NaN	0.318	0.208	0.169
0.8	0.073	0.077	0.083	0.092	0.109	0.136	0.190	0.314	NaN	0.344	0.228
0.9	0.074	0.077	0.082	0.089	0.100	0.121	0.150	0.208	0.348	NaN	0.354
1.0	0.074	0.077	0.081	0.086	0.096	0.109	0.130	0.166	0.225	0.363	NaN

Simulation results for $n = 1000$ and $\lambda_c = 1000$:

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.359	0.204	0.136	0.103	0.079	0.065	0.055	0.048	0.042	0.038
0.1	0.372	NaN	0.341	0.186	0.124	0.092	0.070	0.059	0.049	0.042	0.037
0.2	0.204	0.343	NaN	0.309	0.166	0.111	0.082	0.062	0.051	0.043	0.037
0.3	0.138	0.185	0.309	NaN	0.279	0.147	0.095	0.069	0.054	0.044	0.038
0.4	0.105	0.124	0.168	0.279	NaN	0.244	0.126	0.080	0.059	0.045	0.038
0.5	0.081	0.091	0.110	0.145	0.244	NaN	0.209	0.102	0.066	0.048	0.037
0.6	0.065	0.072	0.082	0.095	0.126	0.206	NaN	0.169	0.081	0.051	0.038
0.7	0.055	0.059	0.063	0.070	0.081	0.103	0.172	NaN	0.127	0.058	0.038
0.8	0.048	0.050	0.050	0.054	0.058	0.066	0.080	0.127	NaN	0.081	0.037
0.9	0.042	0.042	0.042	0.044	0.046	0.049	0.051	0.059	0.082	NaN	0.037
1.0	0.037	0.038	0.038	0.038	0.038	0.038	0.038	0.037	0.037	0.038	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_1(t) - S_1(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.354	0.202	0.138	0.106	0.086	0.072	0.064	0.059	0.055	0.052
0.1	0.367	NaN	0.337	0.185	0.124	0.097	0.077	0.067	0.059	0.055	0.052
0.2	0.203	0.336	NaN	0.307	0.166	0.114	0.086	0.070	0.061	0.055	0.052
0.3	0.139	0.184	0.302	NaN	0.275	0.147	0.100	0.075	0.063	0.057	0.052
0.4	0.108	0.125	0.166	0.277	NaN	0.240	0.127	0.086	0.068	0.057	0.052
0.5	0.086	0.095	0.112	0.146	0.241	NaN	0.208	0.106	0.073	0.059	0.052
0.6	0.072	0.078	0.085	0.100	0.128	0.206	NaN	0.172	0.087	0.062	0.052
0.7	0.063	0.066	0.070	0.076	0.086	0.107	0.171	NaN	0.132	0.067	0.052
0.8	0.058	0.059	0.060	0.062	0.066	0.073	0.086	0.128	NaN	0.087	0.052
0.9	0.053	0.054	0.053	0.055	0.056	0.059	0.061	0.067	0.088	NaN	0.052
1.0	0.049	0.050	0.051	0.051	0.051	0.051	0.051	0.051	0.052	0.052	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Ryzhov's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.038	0.039	0.038	0.039	0.038	0.038	0.039	0.038	0.038	0.039
0.1	0.039	NaN	0.081	0.057	0.051	0.048	0.045	0.045	0.044	0.043	0.042
0.2	0.038	0.079	NaN	0.124	0.080	0.067	0.059	0.054	0.052	0.049	0.048
0.3	0.038	0.059	0.125	NaN	0.167	0.104	0.080	0.069	0.063	0.059	0.055
0.4	0.038	0.051	0.082	0.166	NaN	0.203	0.127	0.096	0.081	0.072	0.065
0.5	0.038	0.047	0.066	0.103	0.204	NaN	0.242	0.145	0.109	0.093	0.080
0.6	0.038	0.045	0.060	0.081	0.126	0.235	NaN	0.274	0.166	0.125	0.106
0.7	0.038	0.044	0.054	0.070	0.096	0.145	0.276	NaN	0.302	0.186	0.140
0.8	0.038	0.045	0.051	0.063	0.082	0.111	0.163	0.295	NaN	0.323	0.203
0.9	0.039	0.042	0.049	0.058	0.071	0.094	0.124	0.189	0.323	NaN	0.340
1.0	0.039	0.042	0.047	0.055	0.065	0.081	0.104	0.136	0.201	0.344	NaN

Mean of $\sup_{t\in\mathbb{R}} |\hat{S}_2(t) - S_2(t)|$ with Maiboroda's method :

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	NaN	0.072	0.073	0.073	0.074	0.073	0.074	0.074	0.074	0.074	0.075
0.1	0.073	NaN	0.098	0.083	0.080	0.078	0.077	0.078	0.077	0.076	0.076
0.2	0.073	0.098	NaN	0.134	0.098	0.089	0.084	0.082	0.081	0.079	0.080
0.3	0.072	0.083	0.134	NaN	0.174	0.117	0.100	0.091	0.087	0.086	0.083
0.4	0.072	0.079	0.100	0.173	NaN	0.208	0.136	0.111	0.099	0.094	0.089
0.5	0.072	0.077	0.089	0.117	0.208	NaN	0.244	0.154	0.122	0.109	0.100
0.6	0.072	0.076	0.084	0.099	0.137	0.238	NaN	0.279	0.175	0.137	0.119
0.7	0.072	0.076	0.081	0.090	0.112	0.154	0.278	NaN	0.302	0.192	0.151
0.8	0.072	0.076	0.079	0.086	0.099	0.124	0.171	0.296	NaN	0.323	0.208
0.9	0.073	0.074	0.078	0.083	0.092	0.109	0.136	0.195	0.323	NaN	0.341
1.0	0.073	0.075	0.077	0.081	0.088	0.099	0.118	0.147	0.210	0.341	NaN

Results obtained for the simulations with real data

In this section of the Appendix, we give the results of the estimators obtained using a real database.

Using the real weights $p_1(A) = 0.385819$, $p_2(A) = 0.614181$, $p_1(B) = 0.403643$ and $p_2(B) = 0.596357$, we obtained a sup-norm of 2.489 for the Ryzhov's estimate of S_1 and 1.623 for S_2, and 1.260 for the Maibroda's estimate of S_1 and 1.212 for S_2.

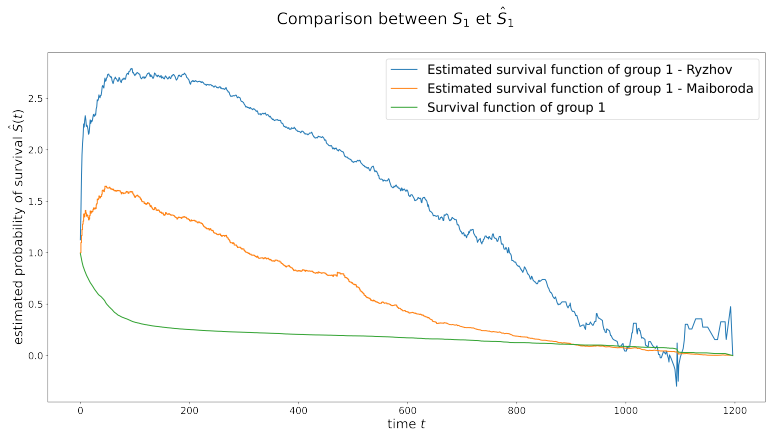

Figure 7: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

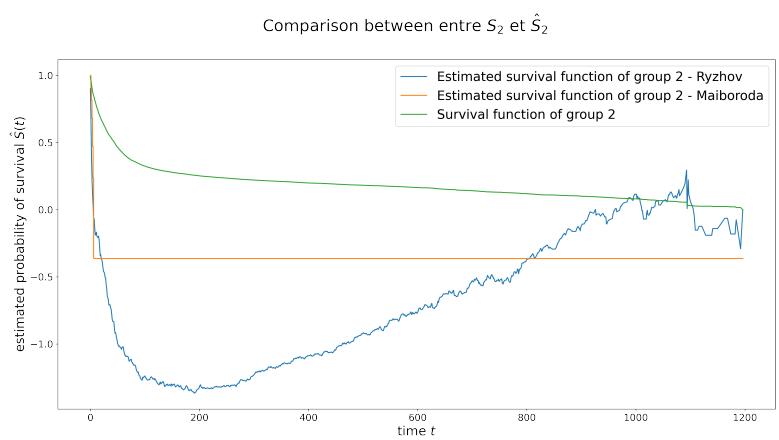

Figure 8: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 2 are here shown without correction.

Now we limit sick leaves at 360 days. We obtained a sup-norm of 1.458 for the Ryzhov's estimate of S_1 and 0.953 for S_2, and 0.765 for the Maibroda's estimate of S_1 and 0.787 for S_2. The distribution in this case is :

Age Group (Sample)	Gender (Group)	Number of people	Proportion
<= 40ans	M	5 675	0.385871
	F	9 032	0.614129
> 40ans	M	5 791	0.405248
	F	8 499	0.594752

Table 11: Proportion of each sample and group for our real datamabe limiting it to 360 days

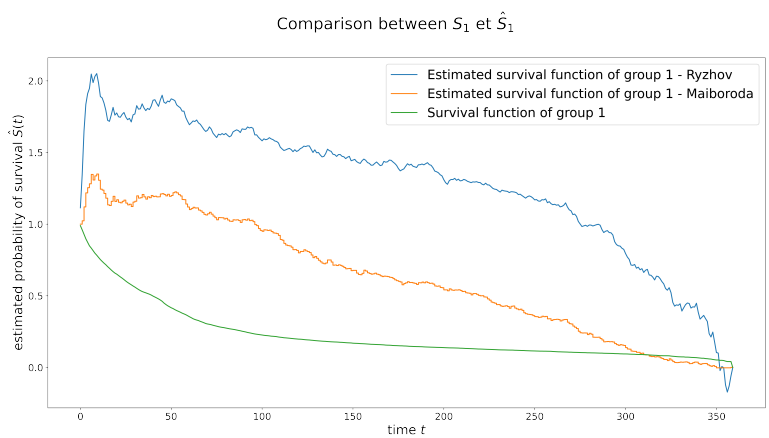

Figure 9: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction, limited at 360 days.

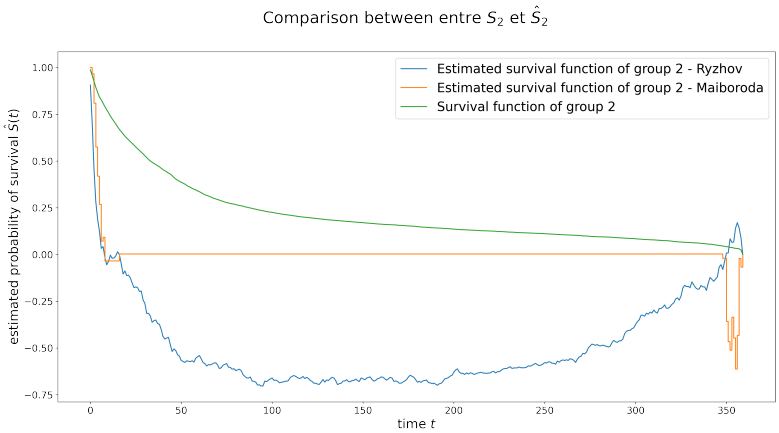

Figure 10: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 2 are here shown without correction, limited at 360 days.

Here are the results after separating the data into 2 samples A and B, in order to choose the proportion of each group (e.g. men/women) in each of these samples. In this way we choose weights that give better results. We have chosen that in sample A there are 6000 women and 10000 men. The proportions are then the following :

Sample	Gender (Group)	Number of people	Proportion
A	M	10 000	0.625
	F	6 000	0.375
B	M	3 098	0.180376
	F	14 077	0.819622

Table 12: Proportion of each sample and group for our arranged database

In this case, we obtained a sup-norm of 0.474 for the Ryzhov's estimate of S_1 and 0.285 for S_2, and 0.396 for the Maibroda's estimate of S_1 and 0.234 for S_2.

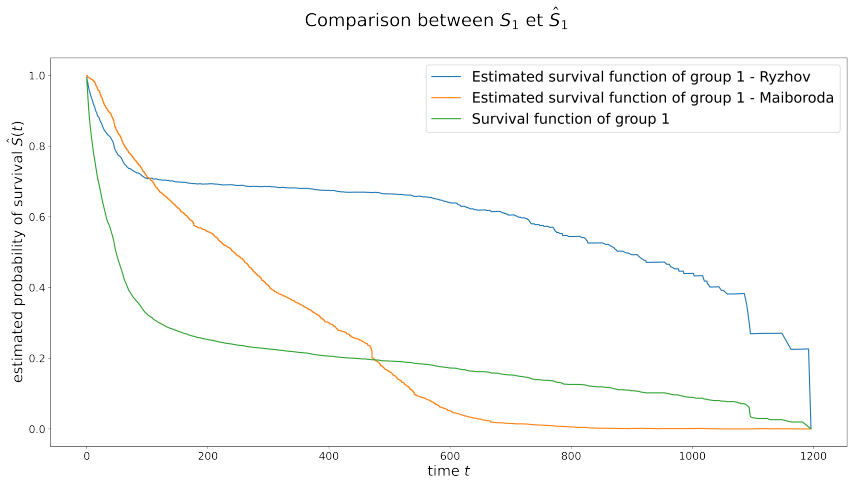

Figure 11: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

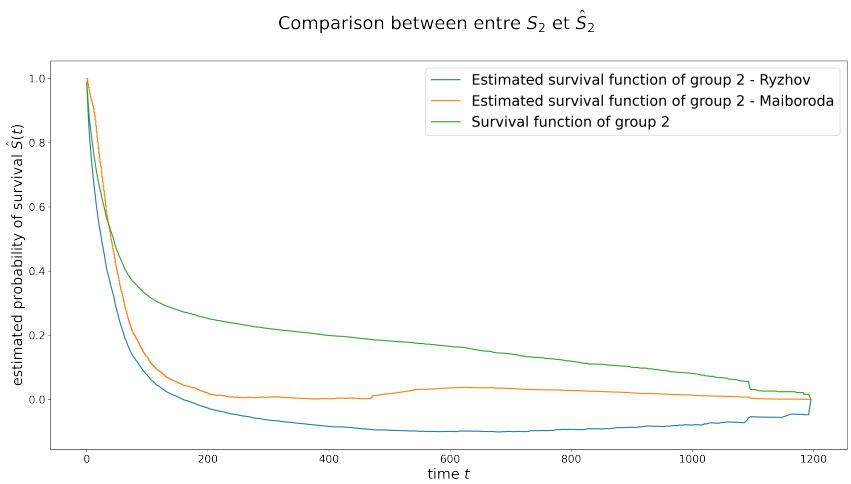

Figure 12: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

As we had better results with a limit of 360 days, we have done the same and then we have chosen that in sample A there are 6000 women and 10000 men. We obtained a sup-norm of 0.215 for the Ryzhov's estimate of S_1 and 0.131 for S_2, and 0.201 for the Maibroda's estimate of S_1 and 0.117 for S_2. The proportions are in this case :

Sample	Gender (Group)	Number of people	Proportion
A	M	10 000	0.625
	F	6 000	0.375
B	M	1 466	0.112795
	F	11 531	0.887205

Table 13: Proportion of each sample and group for our arranged database

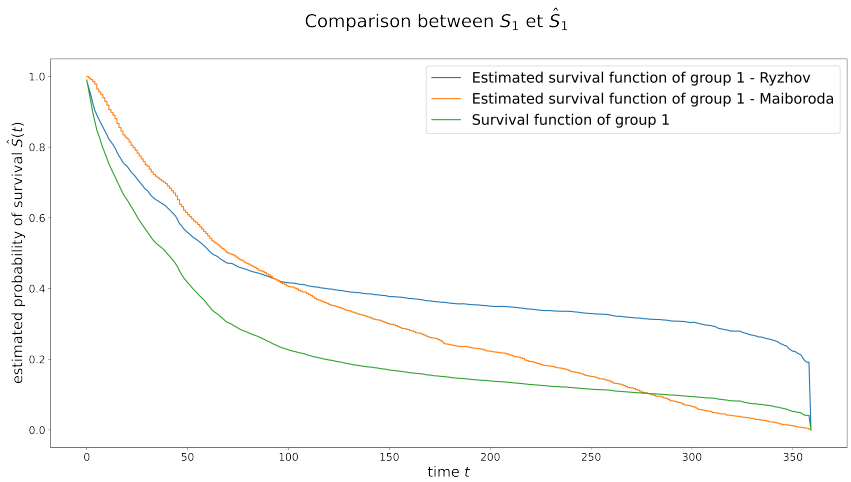

Figure 13: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.

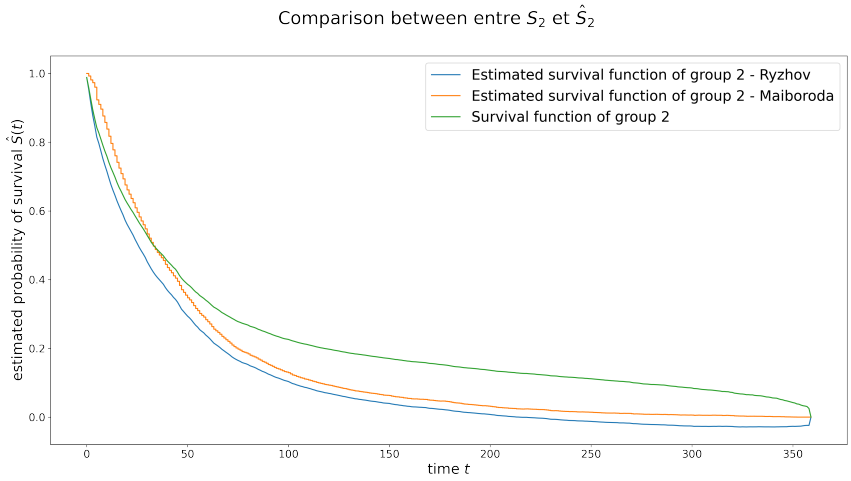

Figure 14: Estimated survival functions using Ryzhov's and Maiboroda's estimators

Note : The estimated survival functions using Ryzhov's and Maiboroda's estimators for group 1 are here shown without correction.