

Efficient computation of modal outgoing Green's kernels in helioseismology

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In this talk, we discuss the computation of modal outgoing Green kernels for the scalar operator L and the simplified Galbrun operator \mathfrak{L} ,

$$L\psi = -\nabla \cdot \left(\frac{1}{\rho_0} \nabla \psi \right) - \frac{\sigma^2}{c_0^2 \rho_0} \psi, \quad \text{and} \quad \mathfrak{L}\xi = -\rho_0 \sigma^2 \xi - \nabla [\gamma \rho_0 \nabla \cdot \xi] + (\nabla \rho_0)(\nabla \cdot \xi) \\ - \nabla(\xi \cdot \nabla \rho_0) + \xi \cdot \nabla \nabla \rho_0 + \rho_0 \xi \cdot \nabla \nabla \phi_0.$$

For application in helioseismology, the coefficients follow in solar interior the spherically-symmetric model S, together with exponentially decaying density ρ_0 and constant sound speed c_0 in the atmosphere. The last term in \mathfrak{L} prescribes effect of background gravity. Denote by G the Green's function of L and \mathbb{G} Green's tensor of \mathfrak{L} . Spherical symmetry reduces the computation of G and \mathbb{G} on each harmonic mode ℓ to that of the modal kernels which are coefficients in spherical harmonic expansion of G and the radial-radial component of \mathbb{G} . Denoted respectively by G_ℓ and \mathbb{G}_ℓ , they solve on $[0, \mathfrak{r}]$ and for each ℓ , $(-\partial_r^2 + V_\ell^{\text{scalar}})G_\ell = \delta(r - s)$, and $(-\partial_r^2 + V_\ell)\mathbb{G}_\ell = \delta(r - s)$ with appropriate conditions at the end points. The full-valued modal kernels contains their values for all r, s .

Obtaining full-valued kernels is already expensive for one mode, since each resolution of the boundary value problems (BVP) of G_ℓ and \mathbb{G}_ℓ gives their values for one source position. We discuss here an algorithm which provides the full-valued kernels by assembling solutions to two BVP with zero source, [3, 2]. The algorithm is particularly powerful for the vector operators, since coupled with a first-order formulation, it provides readily the derivatives ∂_r , ∂_s and $\partial_r \partial_s$ of \mathbb{G}_ℓ , needed to compute the non-radial components of \mathbb{G} , [2]. We will also discuss how the inclusion of gravity introduces non-zero regular singularities in the equations of \mathbb{G}_ℓ , in particular, the potential V_ℓ compared to V_ℓ^{scalar} , [1]. Due to these singularities, the modal kernel \mathbb{G}_ℓ will be computed using the non-Schrödinger form of the equation, which is shown to be numerically more stable, [2].

REFERENCES

- [1] H. Barucq, F. Faucher, D. Fournier, L. Gizon and H. Pham, Outgoing modal solutions for Galbrun's equation in helioseismology, *Journal of Differential Equations*, 2021.
- [2] —, Efficient computation of modal Green's kernels for vectorial equations in helioseismology under spherical symmetry, *Inria Research Report 9433*, 2021.
- [3] —, Efficient and Accurate Algorithm for the Full Modal Green's Kernel of the Scalar Wave Equation in Helioseismology, *SIAM Journal on Applied Mathematics*, 2020.