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► **To cite this version:**

Bahram Djafari-Rouhani, Laurent Carpentier, Yan Pennec. Elastic metasurface made of elliptic shape pillars for acoustic wave focusing. 2022 IEEE International Ultrasonics Symposium, IUS 2022, Oct 2022, Venice, Italy. pp.1-3, 10.1109/IUS54386.2022.9957873 . hal-03885901

HAL Id: hal-03885901

<https://hal.science/hal-03885901>

Submitted on 13 Mar 2023

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Elastic Metasurface made of elliptic shape pillars for acoustic wave focusing

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Abstract – We numerically propose the design of a metasurface for focusing the flexural Lamb waves propagating on a plate. The metasurface consists of a line of pillars of elliptic shape with a gradient in their ellipticity. We first determine an elliptic pillar that allows the superposition of two resonances (one compressional and one bending) in order to obtain a phase shift of 2π around the resonance. Then, the metasurface is built by choosing the ellipticity of each pillar at a given position based on the generalized Snell’s law for the selected focal point. We demonstrate the capacity of this design for subwavelength focusing of the incident plane wave at any targeted point.

Keywords—Metasurface, elastic wave, focusing

I. INTRODUCTION

Metasurfaces are sub-wavelength planar metamaterials that allow the manipulation and control of incident acoustic/elastic waves, in particular to achieve anomalous refraction and reflection scatterings. They have known a great deal of attention during the last decade, in particular for acoustic and elastic waves [1-3]. The metasurface is in general constituted by a repetition of a set of unit cells where each unit produces a specific phase shift of the incident wave such that the phase shifts of all the units span a range of 2π . Therefore, the application of a generalized Snell-Descartes law allows the realization of various functionalities in the behavior of the scattered waves. In this work, we are interested by the capacity of a pillared metasurface for the focusing of flexural Lamb waves propagating along a thin plate. The metasurface consists of a set of pillars, each producing a sufficiently high transmission and a specific phase shift of the incident wave, with a span of all the phase shifts over 2π . We recently proposed such a design exhibiting sub-wavelength focusing by using cylindrical pillars of varying heights [4]. The principle of the design consists of starting from a reference pillar where the geometrical parameters are adapted such as to superimpose two resonances of the pillar, the second bending and the first compressional modes in our case [5]. This is an essential feature of the structure because the transmission phase shift at resonance reaches 2π which cannot be achieved with separated resonances. The functionality of the design was also demonstrated experimentally [6]. However, from a technological point of view, the fabrication of pillars with varying heights is difficult to achieve at the micro or nanoscale. Therefore, in this work, we propose a

different approach based on the utilization of pillars of elliptical shape and of constant height. In this numerical study, we show that the ellipticity can be used as the adjustable parameter to bring the bending and compressional modes in superposition. The design of the proposed metasurface consists of an array of resonant pillars of different ellipticity parameters, allowing to realize a gradient phase shift fully covering a 2π range

II. NORMAL MODES FOR A LINE OF IDENTICAL PILLARS

The numerical analysis was performed with the help of the finite element method (COMSOL MULTIPHYSICS). In these calculations, the plate and pillars are made of silicon and the plate thickness is $e = 145\mu\text{m}$. We determine the frequencies of the lowest resonant modes, hereafter called C_1 for the compressional mode and B_1 and B_2 for the two bending modes, as a function of the pillars parameters by calculating the transmission through a line of identical pillars. The plate is submitted to an incident antisymmetric Lamb wave A_0 whose main component is out-of-plane u_z at the frequency of 7.48MHz, corresponding to a wavelength of $\lambda = 445\mu\text{m}$ (as discussed below, this frequency corresponds to the resonance frequency associated with the superposition of the 2 modes B_2 and C_1). One of the parameters of the ellipse is fixed to the value $e_1 = 20\mu\text{m}$ while the other e_2 can vary along the direction of propagation of the incident wave. All the geometrical parameters are reported in figure 1.

The method we used to compute the resonances is described as follows: an oscillating force is applied normally to the surface of the plate, along a line [AB] parallel to the crystallographic direction (100) of the silicon (referred as to x axis in the following) and the resulting displacements are

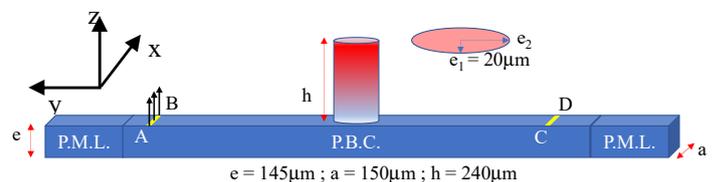


FIG. 1. Model used for finite element calculations of an infinite line of identical pillars. A time-periodic vertical force is applied along [AB]. PML are placed at the ends of the plate and periodic boundary conditions are applied on the lateral sides to simulate an infinite line of pillars. The amplitude of displacement u_z as a function of frequency is monitored along [CD].

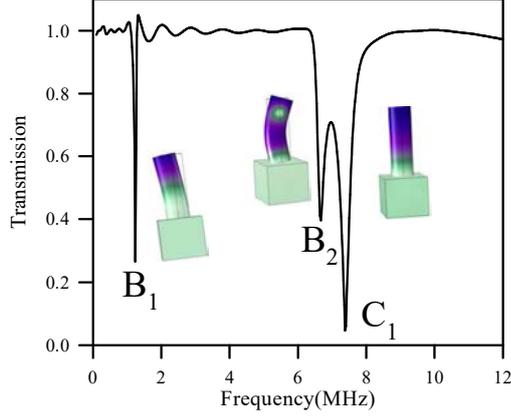


FIG. 2. Resonance frequencies of the pillar appearing as dips in the transmission coefficient through a line of pillars and displacement field of the individual elliptic pillar for $e_2 = 40\mu\text{m}$.

recorded as a function of frequency, on a line [CD] located on the other side of the pillars row. An infinite line of identical pillars is obtained by using the Periodic Boundary Conditions along the direction perpendicular to the incident wave and Perfect Matched Layers (PML) located at each end of the plate. In the explored frequency range (0-10MHz), the three first resonance modes manifest themselves by more or less sharp dips in the transmission curve and can be unambiguously ascribed to eigenmodes corresponding to the two first bending modes B_1 and B_2 , and to the first compressional mode C_1 (figure. 2). While B_1 and C_1 are rather insensitive to the parameter e_2 defining the elliptic shape of the pillar, B_2 mode shifts rapidly toward high frequencies when e_2 increases (figure 3.a). Finally, the B_2 and C_1 modes end up overlapping, at the frequency of 7.48MHz, for the value $e_2 = 50\mu\text{m}$. (figure 3.b)

As a consequence, for this specific value of e_2 for which two resonance modes are superimposed, a span of the phase shift over 2π can be produced by varying the ellipticity of the pillars around the preceding value.

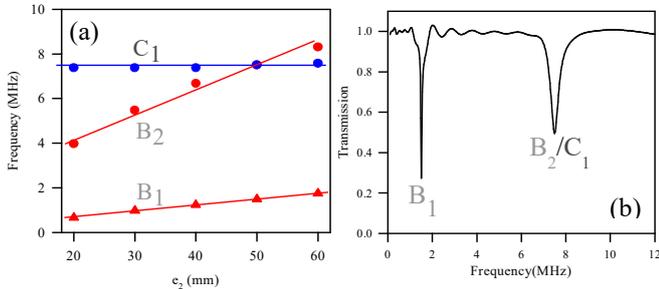


FIG. 3. (a) Resonance frequencies of the elliptic pillar as a function of the parameter e_2 ranging from $20\mu\text{m}$ to $60\mu\text{m}$. (b) For $e_2 = 50\mu\text{m}$, both the second bending mode and the first compressional mode fall at $\nu = 7.48\text{MHz}$ and the corresponding dips in the transmission coefficient overlap.

Therefore, we can now design a gradient-phase meta-line of pillars to realize anomalous transmitted wave fronts based on the generalized Snell law:

$$n_2 \sin\theta_2 - n_1 \sin\theta_1 = \frac{\lambda}{2\pi} \frac{d\Phi(x)}{dx} \quad (1)$$

III. SUBWAVELENGTH FOCUSING

As an illustration of anomalous transmission, we present a structure allowing to focus a transmitted Lamb wave A_0 at any point of the surface $M(x_0, y_0)$. To do that, a line of 21 elliptic pillars spaced by $150\mu\text{m}$ is arranged along the x direction perpendicular to wave propagation. The phase profile $\Phi(x)$ along this direction is determined according to the relation:

$$\begin{aligned} \Phi(x) - \Phi(x=0) \\ = \frac{2\pi}{\lambda} \left(\sqrt{(x-x_0)^2 + y_0^2} - \sqrt{(x_0^2 + y_0^2)} \right) \end{aligned} \quad (2)$$

Where $\lambda = 445\mu\text{m}$ is the wavelength of the A_0 Lamb mode at 7.48MHz. The phase of the central pillar ($x=0$) is set to $-\pi$, corresponding to the strongest resonant status. The discrete theoretical phases $\Phi(x)$ of the 20 other pillars in the line are calculated from the previous equation as a function of their positions x and compared to the numerical calculation of the

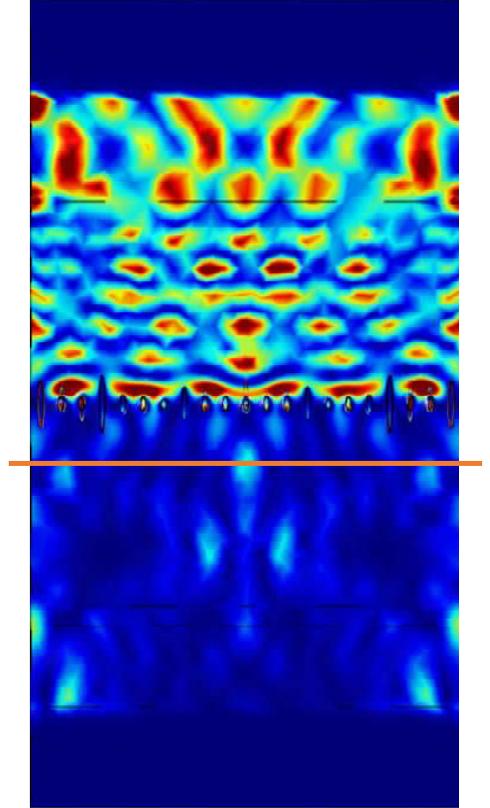


FIG. 4. Illustration of the focusing effect of an incident plane-wave ($\lambda = 445\mu\text{m}$) by an array of 21 pillars with elliptic shape bases. The focal point is targeted at $F = \lambda$.

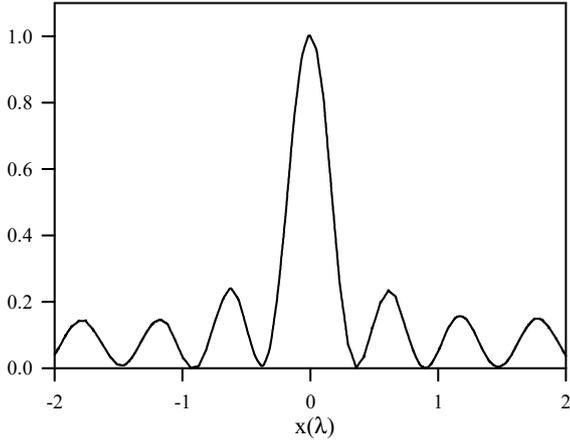


FIG. 5. Intensity of the transmitted wave along the x axis (red line – figure 4) crossing the focusing spot.

phase as a function of ϵ_2 . A span of the phase shift over 2π is obtained for ϵ_2 values ranging from $10\mu\text{m}$ to $210\mu\text{m}$. Figure 4 shows the focusing effect at a point M with coordinates $(0,F)$ where F is the focal length fixed to $F = \lambda = 445\mu\text{m}$. Figure 5 shows the transmitted intensity around the focal point which highlights a significant focusing effect with a sub-wavelength FWHM ($\sim 0.37\lambda$).

VI. CONCLUSION

We proposed a type of pillared elastic metasurface consisting of a line of gradient resonant pillars with elliptic shape base of subwavelength dimension, on a homogeneous plate, which is able to provide a 2π phase range manipulation and high transmissions. A particularly striking advantage of this structure is that, due to their constant height, it is possible to fabricate a microscale sample using nanofabrication facilities. The experimental validation is ongoing.

ACKNOWLEDGMENT

This work was supported by the French National Research Agency ANR as part of the project “RANDOM”, ANR-19-CE24-0014.

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