Distributed observer-based leader-following consensus control robust to external disturbance and measurement sensor noise for LTI multi-agent systems

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Abstract A robust observer-based leader-following consensus control for linear multi-agent systems subject to external disturbance and measurement noise is developed. The H_∞ criterion is implemented to guarantee stability and robustness of the synchronization error and estimation error through the Lyapunov approach. A set of linear matrix inequalities are obtained to compute the control and observer gain matrices. To show its effectiveness, the proposed strategy is carried out to solve the leader-following formation control consensus-based problem in a fleet of unmanned aerial vehicles under the effect of wind turbulence and measurement noise.

1 Introduction

In recent years, multi-agent systems have attracted the attention of researchers [1]. Multi-agent systems in this work are handled as found in the control community,

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where despite the name being shared with the computer science community, the meanings, the objectives, and the tools of common use are adapted to the control area [2]. An agent is defined as an autonomous dynamical system, e.g., Unmanned Aerial Vehicles (UAVs), or satellites, among others. Nevertheless, multi-agent systems can be subject to external disturbance and measurement noise which compromise the synchronization and coordination of the agents. It is therefore necessary to consider these exogenous inputs in the dynamic modeling of the multi-agent system in order to be able to develop a robust control of these external disturbances and measurement noise. This is why it is essential to add robustness to the distributed control and observer because external disturbance (and sensor noise) affect their performance [3]. Different works in the literature have addressed this problem. A robust finite-time problem for a second-order multi-agent system is addressed in [4] for rejecting external disturbances. Also, a sliding mode leader-following control for multiple UAVs is proposed in [5], where the system remains insensitive to external disturbances. Alternatively, in [6], a leader-following sliding mode formation control approach for underactuated surface vehicles considering model uncertainties and external disturbance is developed. In [7], a robust controller using H_2 performance is implemented in a system with communication and input time delays presented in the frequency domain minimizing the error and the disturbance effect. In [8], the robust optimal formation control problem for heterogeneous multi-agent systems considering external disturbances is addressed based on reinforcement learning. An adaptive semi-global bipartite consensus assuming a connected switching topology graph under input saturation and external disturbance is proposed in [9].

In this paper, the main contribution is the design of the distributed leader-following control law based on state estimates of the neighboring agents to coordinated linear multi-agent systems, where the robust control and observer gains are computed simultaneously based on linear matrix inequalities (LMIs). The main difference concerning previous work is the distributed robust observer-based leader-following control for linear multi-agent systems under external disturbance and measurement noise, where the main objective is to follow the trajectories described by a virtual leader and maintain consensus between followers despite the external disturbance and the sensor noise.

2 Problem Statement and system description

Consider a linear homogeneous multi-agent system, which means that A, B, and C matrices are identical for each agent, as follows:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + D_u d_{u_i}(t), \\ y_i(t) &= Cx_i(t) + D_y d_{y_i}(t), \end{aligned} \tag{1}$$

where $i = 1, 2, \dots, N$; N is the total number of agents; $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, $y_i(t) \in \mathbb{R}^p$, $d_{u_i}(t) \in \mathbb{R}^m$, $d_{y_i}(t) \in \mathbb{R}^p$, are the state, input, output, external disturbance and noise vectors respectively; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D_u \in \mathbb{R}^{n\times m}$, and $D_y \in \mathbb{R}^{p\times n}$ are constant matrices of the system. Graph theory [2] can be used for the communication description of multi-agent systems. In this way, the Laplacian matrix $\mathcal{L}$ describes the dynamics of the multi-agent system through the adjacency matrix $\mathcal{A}$. Let us consider a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is a set of nodes (agents), $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $a_{ii} = 0$, $a_{ij} = 1$ if and only if the pair $(i, j) \in \mathcal{E}$ otherwise $a_{ij} = 0$. When the graph is undirected, also, $a_{ij} = a_{ji}$, $\forall i \neq j$ and $\mathcal{A} = \mathcal{A}^T$. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$ is defined as $\mathcal{L}_{ii} = \sum_{j\neq i} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$. The neighbors of i-th agent are denoted as $j \in \mathcal{N}_i$. The following assumptions are held in this paper:

Assumption 1. The pair (A, B) is stabilizable.

Assumption 2. The pair (A, C) is observable.

Assumption 3. The graph $\mathcal{G}$ is undirected and connected.

Lemma 1 *([10]) The matrix $\bar{\mathcal{L}}$ has non negative eigenvalues. The matrix $\bar{\mathcal{L}}$ is positive definite if and only if the graph is connected and undirected.*

The main objective of this work is the design of a robust leader-following control for multi-agent systems consensus-based, where the virtual leader dynamics is presented as follows:

$$\dot{x}_l(t) = Ax_l(t), \tag{2}$$

where $x_l(t) \in \mathbb{R}^n$ is the state vector of the virtual leader. Let $\delta_i = x_i - x_l$, then, the dynamics of the synchronization error between each agent i and the leader is:

$$\dot{\delta}_i(t) = A\delta_i(t) + Bu_i(t) + D_u d_{u_i}(t). \tag{3}$$

In order to estimate the states of each agent, the following distributed observer is proposed:

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + Bu_i(t) + L(y_i(t) - C\hat{x}_i(t)), \\ \hat{y}_i(t) &= C\hat{x}_i(t), \end{aligned} \tag{4}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the estimated state vector, $L \in \mathbb{R}^{n\times p}$ is the observer gain to be designed, and $\hat{y}_i(t) \in \mathbb{R}^p$ is the estimated output vector. The dynamics of the estimation error $e_i = x_i - \hat{x}_i$ is computed as follows:

$$\dot{e}_i(t) = (A - LC)e_i(t) + D_u d_{u_i}(t) - LD_y d_{y_i}(t). \tag{5}$$

The considered observer-based leader-following consensus control law is ([11]):

$$u_i(t) = K\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t)) + \alpha_i(\hat{x}_i(t) - x_l(t))\right], \tag{6}$$

where $K \in \mathbb{R}^{n\times p}$ is the control gain to be designed, a_{ij} are the elements of the adjacency matrix, $\hat{x}_i(t)$ is the estimated state vector of the i-th agent, and $\hat{x}_j(t)$ is the estimated state vector of the neighboring agents, α_i represents the communication between the leader and the followers, where $\alpha_i > 0$ if there is a directed edge from the leader to the i-th agent, otherwise $\alpha_i = 0$. The considered problem is the design of a robust control gain K and a robust observer gain L to steer the multi-agent system (1) subject to external disturbance and sensor noise, to follow the virtual leader's trajectories.

3 Observer-based leader-following consensus robust controller

In this section, the LMI conditions to guarantee the existence of the robust control and observer gains based on the Lyapunov stability analysis are presented. Considering the following new variables $e(t) = [e_1(t)^T,\ e_2(t)^T, \dots, e_N(t)^T]^T$, the disturbance vector $d_u(t) = [d_{u_1}(t)^T, d_{u_2}(t)^T, \dots, d_{u_N}(t)^T]^T$, and noise vector $d_y(t) = [d_{y_1}(t)^T, d_{y_2}(t)^T, \dots, d_{y_N}(t)^T]^T$, then, using the Kronecker product $\otimes$ the dynamic estimation error can be expressed as:

$$\dot{e}(t) = (I_N \otimes (A - LC))e(t) + (I_N \otimes D_u)d_u(t) - (I_N \otimes LD_y)d_y(t). \tag{7}$$

Replace (6) in (3) and let $\delta(t) = \left[\delta_1(t)^T, \delta_2(t)^T, \dots, \delta_N(t)^T\right]^T$, and $\overline{\mathcal{L}} = \mathcal{L} + \Lambda$ where $\mathcal{L}$ is the Laplacian matrix and $\Lambda = diag(\alpha_1, \alpha_2, \dots, \alpha_N)$ is the communication exchange between the virtual leader and the followers. Then, the synchronization error of the multi-agent system is rewritten as:

$$\dot{\delta}(t) = (I_N \otimes A + \overline{\mathcal{L}} \otimes BK)\delta(t) - (\overline{\mathcal{L}} \otimes BK)e(t) + (I_N \otimes D_u)d_u(t). \tag{8}$$

Let $z(t) = [\delta(t)^T, e(t)^T]^T$ then, system (1) can be expressed by:

$$\dot{z}(t) = \widetilde{A}z(t) + \widetilde{D}_u d_u(t) - \widetilde{D}_y d_y(t), \tag{9}$$

where

$$\widetilde{A} = \begin{bmatrix} I_N \otimes A + \overline{\mathcal{L}} \otimes BK & -\overline{\mathcal{L}} \otimes BK \\ 0 & I_N \otimes (A - LC) \end{bmatrix}, \widetilde{D}_u = \begin{bmatrix} I_N \otimes D_u \\ I_N \otimes D_u \end{bmatrix}, \widetilde{D}_y = \begin{bmatrix} 0 \\ I_N \otimes LD_y \end{bmatrix}. \tag{10}$$

The distributed H_∞ criterion is considered as in [11], in order to guarantee the existence of robust control and observer gains able to reject the effect of the external disturbances and sensor noise, then, the following theorem is proposed.

Theorem 1 *Consider the closed-loop system* (9)*. Given the eigenvalues $\lambda_j(\bar{\mathcal{L}})$, $j = 1, 2, \dots, N$; if there exist symmetric matrices $\bar{P}_1 > 0 \in \mathbb{R}^{n \times n}$ and $P_2 > 0 \in \mathbb{R}^{n \times n}$, the tuning scalar variable $\mu > 0$, and minimizing γ by the H_∞ criterion satisfying the following condition:*

$$\begin{bmatrix} \eta_{1_j} & 0 & D_u & 0 & -\lambda_j BN_c & 0 \\ * & \eta_2 & P_2 D_u & -M_o D_y & 0 & I \\ * & * & -\gamma^2 I_N & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I_N & 0 & 0 \\ * & * & * & * & -\mu^{-1}\overline{P}_1 & 0 \\ * & * & * & * & * & -\mu\overline{P}_1 \end{bmatrix} < 0, \tag{11}$$

the synchronization error (8) *and estimation error* (7) *are stable; where* $\eta_{1_j} = He\left\{A\overline{P}_1 + \lambda_j BN_c\right\}$, *and* $\eta_2 = He\{P_2A - M_oC\} + I_N$, *then, the robust control law can be computed with* $K = N_c\overline{P}_1^{-1}$, *and the observer gain with* $L = P_2^{-1}M_o$.

Proof Consider the candidate Lyapunov function as follows:

$$V(t) = z(t)^T \begin{bmatrix} I_N \otimes P_1 & 0 \\ 0 & I_N \otimes P_2 \end{bmatrix} z(t), \tag{12}$$

where, $P_1 \in \mathbb{R}^{n\times n} = P_1^T > 0$, and $P_2 \in \mathbb{R}^{n\times n} = P_2^T > 0$. The derivative of the candidate Lyapunov function along the trajectories of (9) is given by:

$$\begin{aligned} \dot{V}(t) =& 2\delta(t)^T(I_N \otimes P_1A + \overline{\mathcal{L}} \otimes P_1BK)\delta(t) - 2\delta(t)^T(\overline{\mathcal{L}} \otimes P_1BK)e(t) \\ &+ 2e(t)^T(I_N \otimes P_2(A - LC))e(t) + 2\delta(t)^T(I_N \otimes P_1D_u)d_u(t) \\ &+ 2e(t)^T(I_N \otimes P_2D_u)d_u(t) - 2e(t)^T(I_N \otimes P_2LD_y)d_y(t). \end{aligned} \tag{13}$$

Let us perform a spectral decomposition of the matrix $\overline{\mathcal{L}}$, such that $\overline{\mathcal{L}} = TJT^{-1}$ with an invertible matrix $T \in \mathbb{R}^{N\times N}$ and a diagonal matrix $J = diag(\lambda_1, \lambda_2, \ldots, \lambda_N) \in \mathbb{R}^{N\times N}$. Defining the change of coordinates as $\xi = (T^{-1} \otimes I_N)\delta$, $\epsilon = (T^{-1} \otimes I_N)e$, $\omega_u = (T^{-1} \otimes I_N)d_u$, and $\omega_y = (T^{-1} \otimes I_N)d_y$. For reasons of space, the notation of the time dependence of certain variables is omitted. Replacing the new coordinates in (13) leads to:

$$\begin{aligned} \dot{V} =& 2\xi^T(I_N \otimes P_1A + J \otimes P_1BK)\xi - 2\xi^T(J \otimes P_1BK)\epsilon + 2\epsilon^T(I_N \otimes P_2(A - LC))\epsilon \\ &+ 2\xi^T(I_N \otimes P_1D_u)\omega_u + 2\epsilon^T(I_N \otimes P_2D_u)\omega_u - 2\epsilon^T(I_N \otimes P_2LD_y)\omega_y. \end{aligned} \tag{14}$$

Using Lemma 1, (14) can be rewritten as follows:

$$\begin{aligned} \dot{V} =& \sum_{j=1}^{N} \xi_j^T He\left\{P_1A + \lambda_j P_1BK\right\}\xi_j - 2\sum_{j=1}^{N} \xi_j^T \lambda_j P_1BK\epsilon_j + \sum_{j=1}^{N} \epsilon_j^T He\{P_2A - P_2LC\}\epsilon_j \\ &+ 2\sum_{j=1}^{N} \xi_j^T He\{P_1D_u\}\omega_{u_j} + 2\sum_{j=1}^{N} \epsilon_j^T He\{P_2D_u\}\omega_{u_j} + 2\sum_{j=1}^{N} \epsilon_j^T He\left\{P_2LD_y\right\}\omega_{y_j}, \end{aligned} \tag{15}$$

then, considering (15) with H_∞ criterion, $\varphi_j(t) = [\xi_j(t)^T, \epsilon_j(t)^T, \omega_{u_j}(t)^T, \omega_{y_j}^T]^T$, and $J_T \le \sum_{j=1}^{N} \varphi(t)_j^T \Omega_j \varphi(t)_j$, where Ω_j is defined as follows:

$$\Omega_j = \begin{bmatrix} Q_{1_j} + I & -\lambda_j P_1 BK & P_1 D_u & 0 \\ * & Q_2 + I & P_2 D_u & -P_2 L D_y \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{16}$$

where $Q_{1_j} = He\left\{P_1 A + \lambda_j P_1 BK\right\}$, and $Q_2 = He\left\{P_2 A - P_2 LC\right\}$. Then, (16) is pre and post multiplied by $\overline{P}_1$, where $\overline{P}_1 = \overline{P}_1^T = P_1^{-1} > 0$

$$\begin{bmatrix} \Upsilon_{1_j} & 0 & D_u & 0 \\ * & \Upsilon_2 & P_2 D_u & -P_2 L D_y \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + He\left\{ \begin{bmatrix} -\lambda_j BK \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix} \right\} < 0, \tag{17}$$

where $\Upsilon_{1_j} = He\left\{A\overline{P}_1 + \lambda_j BK\overline{P}_1\right\} + \overline{P}_1\overline{P}_1$, and $\Upsilon_2 = He\left\{P_2 A - P_2 LC\right\} + I$. Applying the Young relation [12], the following inequality is obtained:

$$\begin{aligned} &\begin{bmatrix} \Upsilon_{1_j} & 0 & D_u & 0 \\ * & \Upsilon_2 & P_2 D_u & -P_2 L D_y \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \mu \begin{bmatrix} -\lambda_j BK \\ 0 \\ 0 \\ 0 \end{bmatrix} \overline{P}_1^{-1} \begin{bmatrix} (-\lambda_j BK)^T & 0 & 0 & 0 \end{bmatrix} \\ &+ \mu^{-1} \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T \overline{P}_1^{-1} \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix} < 0, \end{aligned} \tag{18}$$

where $\mu > 0$. Using the Schur complement in (18), choosing $N_c = K\overline{P}_1$, and $M_o = P_2 L$, the proof is complete.

4 Simulation results: application to a team of UAVs

In this section, the simulation results for an application to leader-following robust consensus control of a team of homogeneous UAVs are presented. The dynamic model of each UAV is:

$$\begin{aligned} \ddot{x}_i &= (c_{\psi_i} s_{\theta_i} c_{\phi_i} + s_{\psi_i} s_{\phi_i}) \frac{1}{m_i} T_i, & \ddot{\phi}_i &= \dot{\theta}_i \dot{\psi}_i \left(\frac{J_y - J_z}{J_x}\right) + \frac{1}{J_x} R_i, \\ \ddot{y}_i &= (s_{\psi_i} s_{\theta_i} c_{\phi_i} - c_{\psi_i} s_{\phi_i}) \frac{1}{m_i} T_i, & \ddot{\theta}_i &= \dot{\phi}_i \dot{\psi}_i \left(\frac{J_z - J_x}{J_y}\right) + \frac{1}{J_y} P_i, \\ \ddot{z}_i &= -g + (c_{\theta_i} c_{\phi_i}) \frac{1}{m_i} T_i, & \ddot{\psi}_i &= \dot{\theta}_i \dot{\phi}_i \left(\frac{J_x - J_y}{J_z}\right) + \frac{1}{J_z} Y_i, \end{aligned} \tag{19}$$

where the shorthand notation $c_{\{\cdot\}}$ and $s_{\{\cdot\}}$ is $\cos(\cdot)$ and $\sin(\cdot)$ respectively; x_i, y_i, and z_i are the position of the i-*th* UAV in the euclidean space, ϕ_i, θ_i, and ψ_i, are the Euler angles *roll*, *pitch*, and *yaw* respectively, R_i, P_i, and Y_i are the rotor torques, m_i is the total mass of the UAVs, g is the gravitational acceleration, J_x, J_y, and J_z

represents the moments of inertia in their respectively axis, T_i is the thrust of the UAVs rotors. The parameters used in the simulations were taken from [13].

The main objective of the multi-agent system, where each UAV is associated to one agent, is to follow the trajectories of a virtual leader and maintain a desired shape, where $H = [h_1^T, h_2^T, \ldots, h_N^T]^T$ contains the desired distance column vectors h_i of every agent. To solve the formation control problem, a second order representation of each agent [14] is used to find a solution for the LMIs presented in Theorem 1:

$$\dot{p}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t). \tag{20}$$

However, it is noted that for the simulations the complete nonlinear model (19) is used. Considering $p_i(t)$, $v_i(t)$, $u_i(t)$ the position, velocity and acceleration respectively of agent i, $\hat{\hat{s}}_i(t) = [\hat{p}_i(t)^T - h_i^T, \hat{v}_i(t)^T]^T$ the estimated states of agent i, $\bar{s}_l(t) = [p_l(t)^T, v_l(t)^T]^T$ the virtual leader states, h_i is a vector of matrix H of the respective agent, then, the control law (6) is rewritten as:

$$u_i(t) = K \sum_{j \in N_i} a_{ij}(\hat{\hat{s}}_i(t) - \hat{\hat{s}}_j(t)) + \alpha_i K(\hat{\hat{s}}_i(t) - \bar{s}_l(t)), \tag{21}$$

where K is the control gain to be designed. Considering $\hat{\hat{\delta}}_i(t) = \hat{\hat{s}}_i(t) - \bar{s}_l(t)$ equation (21) is rewritten as follows:

$$u_i(t) = K \sum_{j \in N_i} a_{ij}(\hat{\hat{\delta}}_i(t) - \hat{\hat{\delta}}_j(t)) + \alpha_i K \hat{\hat{\delta}}_i(t). \tag{22}$$

Since the dynamic of each UAV is considered as a particle, the desired Euler angles θ_{di}, ϕ_{di}, and ψ_{di} are computed as in [15] to control the position of the UAVs, where the consensus protocol (22) and the desired Euler angles are related as follows:

$$\phi_{di} = \arctan\left(\frac{-u_{2i}}{\sqrt{u_{1i}^2 + (u_{3i} + g)^2}}\right), \quad \theta_{di} = \arctan\left(\frac{u_{1i}}{u_{3i} + g}\right), \quad \psi_{di} = 0, \tag{23}$$

and the thrust of the UAVs rotors is computed by $T_i = m_i(u_{1i}^2 + u_{2i}^2 + (u_{3i} + g)^2)^{1/2}$. The communication topology of the multi-agent system is depicted in Fig. 1 (where a UAV represents an agent and the arrows the flow of information), and the Laplacian matrix $\mathcal{L}$:

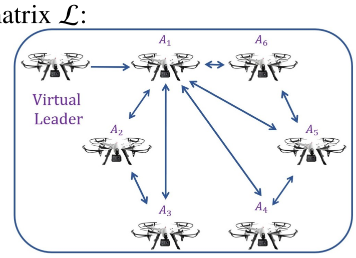

Fig. 1: Directions of the communication links between agents.

$$\mathcal{L} = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ -1 & 0 & 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \tag{24}$$

H and Λ matrices are the following:

$$H = \begin{bmatrix} 0 & 4 & 6 & 4 & 0 & -2 \\ 0 & 0 & 2\sqrt{3} & 4\sqrt{3} & 4\sqrt{3} & 2\sqrt{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{25}$$

The first simulation result in Fig. 2, shows that the formation control problem is solved and the multi-agent system follows the trajectories of the virtual leader. All the UAVs reach the trajectories of the virtual leader while maintaining the rigid formation of a hexagon. This first result does not include external disturbance or measurement sensor noise.

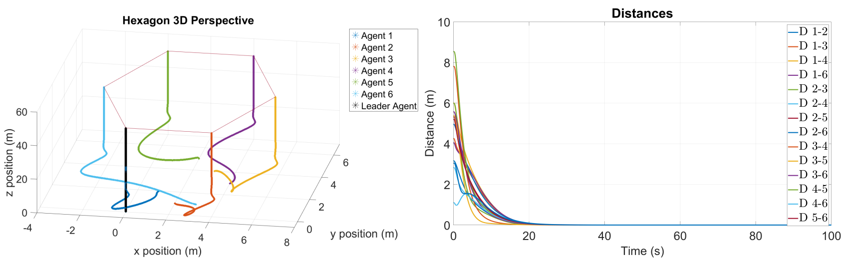

Fig. 2: Formation without disturbance or sensor noise and distances between followers.

The considered external disturbance is modeled as the Dryden wind turbulence presented in [16], and the sensor noise was added as a random function with normal distribution, mean value of zero, and standard deviation of 0.8 m/s, in order to verify the robustness of the control law (22). If the measurement noise or the external disturbance magnitudes increase, the control law (22) is not able to maintain the desired formation or to follow the virtual leader. To overcome the disturbance and noise problem, the LMI presented in (11) is computed with the Sedumi Toolbox [17], where the LMI variable γ value obtained with $\mu = 0.1$ is $\gamma = 1$. The computed control and observer gains are:

$$K = \begin{bmatrix} -0.756 & 0 & 0 & -2.651 & 0 & 0 \\ 0 & -0.756 & 0 & 0 & -2.651 & 0 \\ 0 & 0 & -0.756 & 0 & 0 & -2.651 \end{bmatrix}, L = \begin{bmatrix} 3.208 & 0 & 0 \\ 0 & 3.208 & 0 \\ 0 & 0 & 3.208 \\ 0.836 & 0 & 0 \\ 0 & 0.836 & 0 \\ 0 & 0 & 0.836 \end{bmatrix}. \tag{26}$$

The robustness of the distributed control and estimation implemented in the team of UAVs has been verified by solving the leader-follower formation control problem. The effectiveness of the proposed work is shown in Fig. 3, where each UAV achieves its desired position in the rigid formation of a hexagon while following the trajectory of the virtual leader because the external disturbance and the measurement sensor noise are rejected.

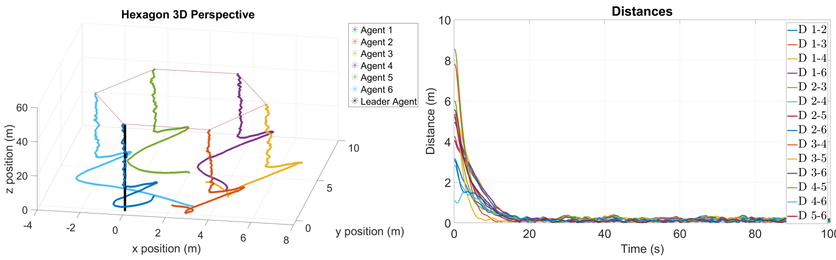

Fig. 3: Formation with disturbance and sensor noise using the control law (22) and observer (4) with gains K, and L (26) computed using Theorem 1.

5 Conclusions

A distributed robust leader-following consensus control observer-based for LTI multi-agent systems was presented. The objective of the multi-agent system is to follow the trajectories described by a virtual leader's dynamics and to maintain a desired rigid formation between followers. This was exemplified with the simulation of a team of UAVs where the agents were able to reject external disturbance and measurement sensor noise. As further work, this approach will be extended to LPV multi-agent systems.

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