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# Modal dynamic equilibrium under different demand management schemes

Louis Balzer<sup>1\*</sup> and Ludovic Leclercq<sup>1†</sup>

<sup>1\*</sup>LICIT-ECO7, Univ Gustave Eiffel, ENTPE, Lyon, 69675, France.

\*Corresponding author(s). E-mail(s): [louis.balzer@univ-eiffel.fr](mailto:louis.balzer@univ-eiffel.fr);

Contributing authors: [ludovic.leclercq@univ-eiffel.fr](mailto:ludovic.leclercq@univ-eiffel.fr);

<sup>†</sup>This author contributed equally to the work.

## Abstract

Congestion pricing and License Plate Rationing (LPR) are classical transportation policies targeting travel demand. Travelers are constrained to reduce their cars' usage to improve traffic conditions or decrease exhaust gases emissions. They have already been proven effective in the field but have also received some criticism for being unfair or badly perceived by users. On the other side, Tradable Credit and Permit Schemes (TCS/TPS) have been investigated in the literature for more than a decade but have never been implemented in practice. In this paper, we present a large-scale dynamic simulation study, corresponding to the morning peak hour for the Lyon city (France) to benchmark pricing, LPR, TCS, and TPS in terms of modal shift from personal cars to Public Transportation (PT), over a horizon of several days and with different charging schemes. Congestion dynamic in the transportation system is reproduced using a trip-based Macroscopic Fundamental Diagram (MFD) framework. We compute the modal assignment at equilibrium by iteratively solving a Quadratic Problem (QP). The initial formulation, presented in previous work for a single day and a uniform TCS, is extended to account for different days, validity cycles, charging schemes, and TPS. The benchmarking shows that the TCS outperforms LPR in terms of both social cost and carbon emissions. Making credit valid over multiple days reduces the price variability while keeping the same system performances and gain distributions.

**Keywords:** tradable credit scheme, tradable permit scheme, trip-based MFD, user equilibrium, mode choice

# 1 Introduction

Most cities around the world are coping with congestion. It induces economic losses, contributes to global warming, and increases the risks of respiratory diseases. Several demand management schemes have been proposed in the literature to reduce the number of individual cars on the network. Among them, the TCS (Tradable Credit Schemes) require users to spend credits they have first to acquire when they want to use specific means of transportation. Those credits are issued by the regulating entity for free but in a limited number. While all users receive equal shares, the total is not sufficient for all to take their favorite transportation mode anytime. Users can trade credits with others to meet their requirements. For example, if, like in this paper, credits are required to take personal cars, users who take other means of transportation, such as Public Transportation (PT), can save credits and sell them. In a sense, virtuous users receive some financial compensation for their effort from other users that have to buy credits. The authority can control how many personal cars can travel every day by regulating the number of credits needed to drive a car to meet social goals. See [Lessan and Fu \(2019\)](#) for an overview of TCS.

Most of the TCS in the literature assumes the credits have to be used on the day they are emitted and cannot be stored/saved. The majority of the contributions, like [Yang and Wang \(2011\)](#), compute network equilibrium independently for each day as no credits transfer happens between days. However, some works introduce a time horizon of several days when credits can be spent. On the one hand, allowing the credits to be transferred increases the TCS's flexibility with regard to the users and can improve its acceptability. On the other hand, it diminishes the regulator's control, and additional measures need to be taken to avoid speculative behaviors. In [Ye and Yang \(2013\)](#), the credits are allocated for several days, and the price is updated each day based on the number of credits still available. [Tian and Chiu \(2015\)](#) defines consumption periods for the TCS and the users need to balance their credit account by the end of the period by using the credit market. If they fail, they need to fill the gap by buying credits at a high price from the regulator. In [Guo et al \(2019\)](#), the credit charges and the allocation are updated between each period. The framework of [Miralinaghi and Peeta \(2016\)](#) allows the users to transfer credits between periods, subject to a fee. In [Miralinaghi and Peeta \(2018\)](#), the interest rates are represented to account for inflation because the periods last several years. [Miralinaghi and Peeta \(2019\)](#) specifies the multi-period TCS to foster the shift from conventional cars to low-emission ones. In [Miralinaghi and Peeta \(2020\)](#), the authors account for the perception of future prices by the network users. To prevent speculative behavior, the authors propose to endorse a rule preventing users from reporting credits they bought on the market. In [Tian and Chiu \(2015\)](#), the focus lies on the trading system, and the congestion model is not detailed. Most contributions use simple static speed/flux functions that predict travel times according to the number of vehicles in each link, e.g., the Bureau of Public Roads (BPR) function. Furthermore, most works about TCS, or more generally about quantity-based

demand management strategies, illustrate the methodology on simplistic case studies, ranging from toy networks to the Sioux-Falls scenario. We found only one contribution using a large case study based on a real city: [Xu and Grant-Muller \(2016\)](#) with Beijing. The authors assess the effects of a distance-based TCS on the total travel time. The estimations of the travel times still rely on the BPR function.

The aim of this paper is to provide a thorough assessment of several TCS implementations on a large and realistic test case using a dynamic representation of traffic states. Furthermore, we will benchmark TCS against more classical Demand Management Schemes (DMS), i.e., congestion pricing and License Plate Rationing (LPR), that have already been applied on the field. While those schemes and TCS have already been studied separately, we believe that a general overview of all their performances considering a large-scale and dynamic simulation environment could provide valuable insights into how TCS position compared to others. Let us recall that under congestion pricing, the car drivers need to pay a toll when crossing cordons or driving in an area during given time slots. It has been implemented in several cities: Singapore (1975), London (2003), and Stockholm (2006/2007), to name a few. The authors of [Gu et al \(2018\)](#) name public acceptability as one of the main challenges to introduce pricing. In the contribution of [Ren and Huang \(2020\)](#), the regulator redistributes the earnings from the toll to ensure the majority of travelers have smaller travel costs than without congestion pricing. LPR restricts the use of personal cars by allowing only cars with odd plate numbers to drive every other day and cars with even plates to drive on the remaining days. It is implemented in several cities and is usually enforced during pollution peaks. However, in [Nie \(2017a\)](#) and [Nie \(2017b\)](#) the author shows for a single Origin-Destination (OD) pair that LPR is ineffective as it fosters in the long-term wealthy people to buy a second car and that TCS can be a good alternative. The observations of [Goddard \(1997\)](#) for Mexico City lead to similar conclusions. [Lian et al \(2019\)](#) mitigates those results when considering transactions costs: for a framework similar to [Nie \(2017a\)](#), the LPR might outperform TCS when the transaction costs are too high.

There is a variation of the TCS named Tradable Permit Scheme (TPS) in the literature. While the distinction between credit and permit is not always consistent in the different contributions, we consider that a credit is a uniform and undifferentiated commodity in this work. On the opposite, a permit is less flexible: it can be specific to a time period, a link, an OD pair, or a destination. [Akamatsu \(2007\)](#), and [Akamatsu and Wada \(2017\)](#) propose a time- and link-based TPS for a network of bottlenecks. The regulation through permits avoids queuing at Vickrey's bottlenecks, as the number of emitted permits matches the fixed capacity of each link. In [Xiao et al \(2021\)](#), a TPS restricts access to parking places. Travelers choose between transit and cars with carpooling possibility in the presence of HOV lane. They do not need permits when riding transit, and car drivers share the burden of the permits when carpooling.

Benchmarking DMS imposes specific requirements to the test case and simulation framework. We have to compute the modal equilibrium, i.e., who drives a car and rides PT, under different DMS acting at the full urban scale, as they apply to every trip. Also, it is important to properly consider congestion dynamics as congestion and carbon emissions are the main Key Performance Indicators (KPI) to assess the performance of such a scheme. Computing indicators at the individual level on top of the system-level KPI is of interest to understand how the different travelers are impacted by the DMS. Static approaches may fail to describe congestion spreading during peak hours accurately. On the other hand, most dynamic traffic models like microscopic simulation are too costly from a computational point of view to calculate large-scale network equilibriums. It explains why we choose in this paper to focus on an intermediate representation level and resort to the concept of Macroscopic Fundamental Diagram (MFD) (Daganzo, 2007). Congestion dynamics are tracked over time by considering that all vehicles share the same mean speed at the same time, described by the speed-MFD curve. Here we use more specifically the trip-based MFD model (Mariotte et al, 2017; Lamotte and Geroliminis, 2018; Jin, 2020), that can differentiate trip length among users, which is crucial for representing trip diversity in an urban context. This modeling framework has also been used in Liu et al (2022) to assess a distance-based TCS to foster departure time shift over one single day.

To compute the network equilibrium of a trip-based MFD model, we have extended our work (Balzer and Leclercq, 2022) to account for different DMS and multiple different days. The demand is elastic as we account for modal choice: car or PT. The need to use the car depends on the days. Indeed, some specific activities might necessitate a personal car, like buying groceries or picking up children at school. To represent the costs of some activity cancellation or modification when a traveler cannot drive its car, we introduce a penalty if it rides PT on given days. Our case study also accounts for travelers not having access to a personal car, i.e., captive PT riders. To drive a car on a day, a user needs to spend the credit charge or the permit corresponding to its route. The credits and permits are issued with a fixed validity period of several days. Users get their credits or permits at the beginning of the period and are free to use or trade them as they want during this period.

To summarize the novelty of this work, we compare the frameworks of the main contributions on DMS based on the congestion representation, the horizon of the credits consumption, the investigated policies, the test case, the charging scheme, and the consideration of pollution on top of congestion in Table 1.

The remainder of this paper is organized as follows. In Sect. 2, we present the initial methodological framework. The proposed TCS and TPS are introduced in Sect. 3 along with LPR and pricing. The computation of the modal equilibrium is detailed in Sect. 4. The different DMS are benchmarked on a large-scale test-case representing the morning commute (about 380 000 trips)

**Table 1:** Comparison of the different contributions on DMS

Article	Congestion model	Validity	Policy	Scenario size	Charging scheme	Pollution
<a href="#">Yang and Wang (2011)</a>	BPR	1 day	TCS	small (4 links)	link-based	no
<a href="#">Ye and Yang (2013)</a>	BPR	days	TCS	small (5 links)	link-based	no
<a href="#">Miralinaghi and Peeta (2016)</a>	BPR	years	TCS	small (10 links)	link-based	no
<a href="#">Miralinaghi and Peeta (2018)</a>	BPR	years	TCS	small (10 links)	link-based	yes
<a href="#">Miralinaghi and Peeta (2019)</a>	BPR	years	TCS	medium (Sioux-Falls - 76 links)	link-based	yes
<a href="#">Miralinaghi and Peeta (2020)</a>	BPR	years	TCS	small (14 links)	link-based	yes
<a href="#">Guo et al (2019)</a>	BPR	days	TCS	medium (Sioux-Falls - 76 links)	link-based	no
<a href="#">Xu and Grant-Muller (2016)</a>	BPR	1 day	TCS	large (Beijing - $10^7$ trips)	distance-based	no
<a href="#">Nie (2017a)</a>	BPR	1 day	LPR, TCS	small (1 link)	uniform	no
<a href="#">Nie (2017b)</a>	BPR	1 day	LPR	small (1 links)	uniform	no
<a href="#">Lian et al (2019)</a>	BPR	1 day	LPR, TCS	small (1 link)	uniform	no
<a href="#">Akamatsu (2007)</a>	Vickrey	1 day	TPS, Pricing	no example	link- and time-based	no
<a href="#">Akamatsu and Wada (2017)</a>	Vickrey	1 day	TPS, Pricing	no example	link- and time-based	no
<a href="#">Balzer and Leclercq (2022)</a>	trip-based MFD	1 day	TCS	large (Lyon - 224 OD)	uniform	yes
This work	trip-based MFD	days	TCS, LPR, Pricing, TPS	large (Lyon - 224 OD)	D- and OD-specific	yes

in Lyon (France) in [Sect. 5](#). [Sect. 6](#) concludes this paper. [Tables A1, A2 and A3](#) sum up the notations.

## 2 Initial methodological framework

Let us start by recalling the framework of [Balzer and Leclercq \(2022\)](#), which serves as the methodological foundation for calculating equilibriums in this paper. The many additions will be described in [Sect. 3](#). The travelers from the same Origin-Destination (OD) pair are aggregated into  $N$  groups with respect to their departure times. Each group  $i$  has a fixed trip length  $l_i$  and departure time  $t_i$ . Among each group, some users take their cars while others take PT. We assume a part of each group does not have access to a personal car, and those travelers are captive PT riders. We note  $r_i$  the ratio of travelers having access to a car. There are  $\gamma_i$  travelers per group.  $x_{d,i}$  is the fraction of car owners from the group  $i$  driving their car on day  $d$ . Thus, the fraction of the group  $i$  driving a car on the day  $d$  is  $r_i x_{d,i}$ . The contribution of group  $i$  to the number of car drivers is  $\gamma_i r_i x_{d,i}$ .  $\mathbf{x}_d$  is the vector of the car shares of all groups for day  $d$ . It means, on day  $d$ ,  $\sum_{i=1}^N r_i \gamma_i x_{d,i}$  travelers drive their cars while the rest,  $\sum_{i=1}^N \gamma_i ((1 - r_i) + r_i(1 - x_{d,i}))$ , ride PT. The ratio  $r_i$  is set to one in the initial work, meaning we assumed everybody could drive a car. Group sizes are flexible and should be tuned to achieve the right balance between computation time (the larger group size, the better) and the dynamic description of the demand level (the number of travelers with the same departure time for the same OD pairs has to fall below a given threshold).

### 2.1 Congestion model

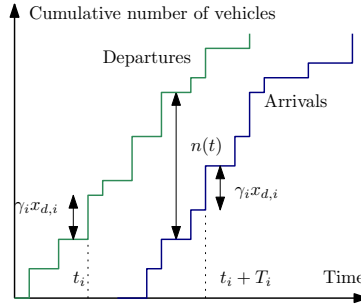
A trip-based MFD framework represents the urban transportation network. In a multi-modal trip-based framework, the mean speed  $V^m$  of each mode  $m$  (car or PT) at a given time depends on the numbers of vehicles of each mode  $n_m$ . The travel time  $T_i^m$  of a traveler from group  $i$  is linked to the departure time  $t_i$  and trip length  $l_i^m$  by:

$$l_i^m = \int_{t_i}^{t_i + T_i^m} V^m(n_{\text{car}}(t), n_{\text{PT}}(t)) dt. \quad (1)$$

We assume PT travel times depend only on the OD pair and departure time in this work. In other words, PT vehicles operate as they usually do, based on historical observations, and then, we do not need to consider their numbers explicitly. Also, as car traffic historical observations include the usual PT vehicles, we can directly focus on the relationship between mean car speed and car accumulation  $V^{\text{car}}(n_{\text{car}}(t))$ . We remove the sub- and superscript 'car' in the sequel to lighten the notation. The car travel time of the group  $i$  is defined by:

$$l_i = \int_{t_i}^{t_i + T_i} V(n(t)) dt. \quad (2)$$

The travel time  $T_i$  is implicitly defined by [Eq. \(2\)](#). The computation of the travel times is based on an event-based framework. We use the fact that the



**Fig. 1:** The accumulation  $n(t)$  is the vertical difference between the cumulative departures and arrivals.

accumulation is constant between two consecutive events (entry or exit of personal cars), so is the speed. The integral given by Eq. (2) is thus split following the events. See [Balzer and Leclercq \(2022\)](#) for details on the computation of the travel times. The accumulation  $n(t)$  at time  $t$  is the difference between the cumulative number of departures and arrivals up to this time, as represented in Fig. 1. The accumulation increases by  $\gamma_i x_{d,i}$  when the car drivers belonging to group  $i$  enter the network at  $t_i$ , and decreases by the same amount when they reach their destination at  $t_i + T_i$ .

## 2.2 Mode choice

Captive PT riders obviously always ride transit. The remaining travelers choose between driving a car or riding PT according to the associated costs. The travel costs for a group  $i$  for a day  $d$  are given by:

$$\begin{cases} C_{i,\text{car}}(d) &= \alpha T_i(\mathbf{x}_d) + P_{k_i}; \\ C_{i,\text{PT}}(d) &= \alpha T_{i,\text{PT}} + \mu_i(d), \end{cases} \quad (3)$$

where  $\alpha$  is the value-of-time (VoT) and  $P_{k_i}$  is the monetary cost associated to the demand management mechanism.  $k_i$  is the class of the trip of the group  $i$ .  $\mu_i(d)$  is the penalty for using PT on one specific day  $d$ . It represents the perceived annoyance of having no other choice than using PT on a specific day when the traveler has a clear need for its personal car because it has to pick up someone or buy groceries. We name those travelers mandatory car users. The initial framework considers a single day. Furthermore, there was no PT penalty, i.e.,  $\mu_i(d)$  is equal to zero for all travelers, and the DMS cost  $P_{k_i}$  is the same for all travelers. The trip-dependent charges are detailed in the next section. They are part of this new contribution.



The decision process is based on the logit model (Daganzo and Sheffi, 1977). The ratio of group  $i$  which *wants* to take the car is:

$$\psi_{d,i}(\mathbf{x}_d, P_{k_i}) = \frac{e^{-\theta C_{i,\text{car}}}}{e^{-\theta C_{i,\text{car}}} + e^{-\theta C_{i,\text{PT}}}}, \quad (4)$$

with  $\theta$  the coefficient of the logit.

### 2.3 Initial DMS

The baseline scenario is without DMS. Costs associated with DMS are set to zero:

$$P_{k_i} = 0. \quad (5)$$

All users who want to take their cars can drive without restriction or additional costs. The modal shares only depend on the differences of car and PT travel times and PT penalties.

The initial work only focuses on TCS. Credits are distributed and traded among all the users. Each user gets  $\kappa$  credits from the regulator. A user needs to spend  $\tau$  credits to drive its car. The credit price  $p^{\text{TCS}}$  is not known a priori and results from the offer and demand during credit trading. In practice, it is an output of the equilibrium process in addition to the modal share of each group.

The TCS specific cost is:

$$P_{k_i} = \tau p^{\text{TCS}}. \quad (6)$$

Note that in Balzer and Leclercq (2022), car drivers spend  $(\tau - \kappa)p$  and PT riders earn  $\kappa p$ . It is equivalent to the current formulation of the logit-based decision by multiplying both sides of the fraction in Eq. (4) with  $e^{-\theta \kappa p}$ .

### 2.4 Equilibrium computation

Since the framework considers a single day, we drop the notation  $d$ . The equilibrium is reached when the modal decisions of all groups  $\psi$  are equal to the corresponding modal shares  $\mathbf{x}$ . Furthermore, the consumed credits cannot exceed the allocated credits. The equilibrium under TCS is given by

$$\begin{cases} \psi & = \mathbf{x}; \\ p^{\text{TCS}} \left( \sum_{i=1}^N \gamma_i (\kappa - \tau x_i) \right) & = 0; \\ \sum_{i=1}^N \gamma_i (\tau x_i - \kappa) & \leq 0. \end{cases} \quad (7)$$

The two last lines of Eq. (7) are specific to TCS: the credit cap and the market-clearing condition (MCC). The MCC constrains the credit price: the credit price is zero, or the credits issued for this cycle are completely consumed. It is a classical assumption for a TCS at equilibrium. Note that captive PT users

only influence the credit market as they sell their allocation to car drivers. They do not impact the traffic conditions.

It is formulated as an optimization problem:

$$J = \frac{1}{2} \sum_{i=1}^N (x_i - \psi_i)^2 + \eta \frac{1}{\sum_{i=1}^N \gamma_i} p^{\text{TCS}} \sum_{i=1}^N \gamma_i (\kappa - \tau x_i), \quad (8)$$

with  $\eta$  the coefficient related to the MCC. It is computationally advantageous to add the MCC in the objective function to avoid non-linear constraints. The global constraints are

$$\begin{cases} 0 \leq x_i \leq 1 \quad \forall i \in [1, N]; \\ p^{\text{TCS}} \geq 0; \\ \sum_{i=1}^N \gamma_i (\tau x_i - \kappa) \leq 0. \end{cases} \quad (9)$$

The optimization problem is linearized and formulated as a Quadratic Problem (QP) to be solved iteratively until convergence is reached:

$$\frac{1}{2} \Delta \tilde{\mathbf{x}}^{\text{T}} \cdot \mathbf{P} \cdot \Delta \tilde{\mathbf{x}} + \mathbf{q} \cdot \Delta \tilde{\mathbf{x}}. \quad (10)$$

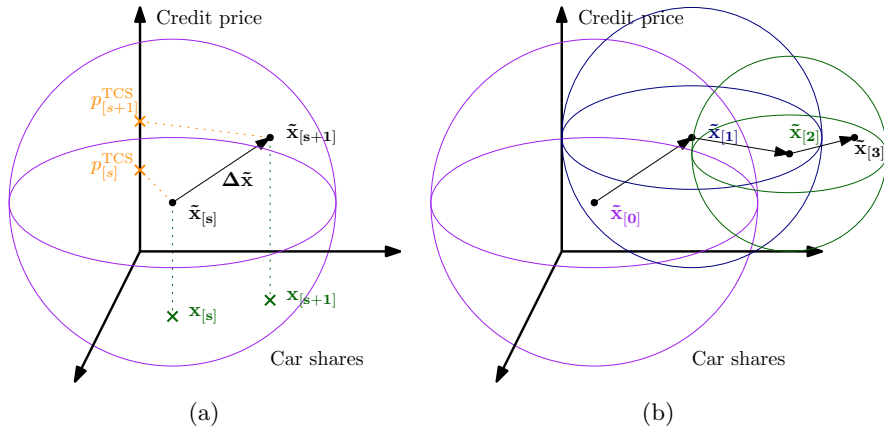
The variable  $\tilde{\mathbf{x}}$  are the modal shares and the credit price:  $\tilde{\mathbf{x}} = [\mathbf{x}; p^{\text{TCS}}]$ , and  $\Delta \tilde{\mathbf{x}}$  its variation. Its size is  $N + 1$ .  $\mathbf{P}$  is a symmetric matrix, and  $\mathbf{q}$  is a vector. The computation of this matrix and vector is based on the linearization of the travel times with respect to the modal shares:

$$T_i = T_{0,i} + \nabla_{\mathbf{x}} \mathbf{T}_i \cdot \Delta \mathbf{x} + o(\Delta \mathbf{x}) \quad \forall i \in [1, N]. \quad (11)$$

One major contribution of our previous work was quantifying the delay induced by one user to the users (a.k.a. marginal external delay) in a trip-based MFD framework for a single day equilibrium. The iterative convergence process is illustrated in Fig. 2. During step  $s$ , the travel times are linearized around the point  $\tilde{\mathbf{x}}_{[s]}$ . It permits to formulate the QP (Eq. (10)) and to solve it in the neighborhood of the reference point. The solution  $\tilde{\mathbf{x}}_{[s+1]}$  is then used as the new reference point. The process is repeated until the value of the cost function  $J$  is below a predefined threshold.

### 3 Demand management strategies

The initial framework needs to be extended in many directions to assess the demand management strategies we have identified. It includes introducing new variables and constraints to handle a time horizon, i.e., the possibility that users can define their strategy over multiple days. We will introduce heterogeneity in user preferences over time to reproduce specific modal constraints that users may have. Also, constraints that reproduce the user behaviors should



**Fig. 2:** The iterative process to find the car shares and credit price at equilibrium. (a) One step and (b) full convergence process.

be tuned to represent not only TCS but also LPR and pricing. Finally, we want to investigate how the DMS can account for spatial adjustments, like OD or destination-specific settings. Again, adequate constraints should be defined before calculating the equilibrium.

First, we allow  $\mu_i(d)$  to be non-zero on some days  $d$  to represent user- and day-specific need for a car on some days over the time horizon. Based on the trip of the group  $i$ , the charging mechanism, i.e., the toll (pricing), the number of credits needed (TCS), or the kind of permit (TPS), depends on  $k_i$ , the class of the trip of the group  $i$ . We assume the cost or restriction of car usage may be distributed over space, for example, depending on the OD pair. The DMS force some travelers to shift from car to PT. The segmentation of the DMS determines the magnitude of this shift: a TPS allows for closer control by setting OD-specific caps, and OD-specific credit charging schemes account for the heterogeneity of the PT network coverage. Some trips are more straightforward to complete with PT than others: it is usually more challenging to shift from car to PT for a trip in the suburbs than downtown. We note  $N_K$  the number of different charges, i.e., the number of different trips classes. For example, in an OD-specific charging scheme,  $N_K$  is the number of OD pairs.

The network regulator can implement different measures that foster modal shifts to manage the demand and decrease total travel time and/or carbon emission. We present here how to integrate each scheme in our modeling framework.

### 3.1 LPR

License Plate Rationing is one of the most basic and easy-to-implement DMS. It has been put in practice multiple times in different cities during pollution peaks. The LPR policy, as it has been implemented in several European cities,

states that a car can be used on odd days if the plate number is odd and on even days if the plate number is even. Some users are exempt from this regulation, like low-emission vehicle owners. We assume each user owns at most one car. Thus, some users cannot drive their cars every two days. For the others, there is no additional cost:

$$\begin{cases} P_{k_i} = \infty & \text{for those not allowed to drive;} \\ P_{k_i} = 0 & \text{for those allowed to drive.} \end{cases} \quad (12)$$

It is similar to the no DMS case, except the car shares of the groups not allowed to drive are set to zero. The ratio of groups not concerned by the LPR is a parameter the authority can use to regulate the number of cars driving in the network. A ratio of 0% is a rigid LPR where the whole population undergoes the LPR, while a ratio of 100% is equivalent to the no DMS case. This ratio permits calibrating the LPR to reach given objectives in terms of congestion and pollution.

### 3.2 Pricing

Users need to pay a toll fixed by the regulator to use their car. So travellers from the group  $i$  face a toll of

$$P_{k_i} = p_{k_i}. \quad (13)$$

Note that given the user decision model and the mode choice, implementing an incentive scheme, i.e., rewarding travelers taking the PT with, let us say 2 EUR instead of charging those who drive their cars, is equivalent to the pricing scheme with a toll of 2 EUR. The modal shares at equilibrium are the same. The only difference is the monetary flow: the regulator would give  $\gamma_i p_{k_i} ((1 - x_i)r_i + 1 - r_i)$  to the group  $i$  with the incentive scheme, whereas the group  $i$  gives  $r_i \gamma_i p_{k_i} x_i$  to the regulator with pricing.

### 3.3 TCS

The credit price is the same for the whole validity period at equilibrium. If the price were higher for a day, credit sellers would sell on this day, and the buyers would buy on another day. For example, if the credits are valid for a week and cost 1 EUR on Monday and 2 EUR on Tuesday, a user would buy credits on Monday and sell them on Tuesday. The demand would thus increase on Monday and the offer on Tuesday till the prices are the same every day, and the market reaches its equilibrium. The TCS specific cost is:

$$P_{k_i} = \tau_{k_i} p^{\text{TCS}}. \quad (14)$$

Note that in opposition to Eq. (6), the TCS specific cost is not the same for all travelers' groups.

### 3.4 TPS

The difference with TCS is that the permits are only valid for specific time periods or regions. The permits depend on the destination or the OD pair in this work. Each user of group  $i$  gets a fraction  $\kappa_{k_i}^{\text{TPS}}$  of a permit (or one permit every  $1/\kappa_{k_i}^{\text{TPS}}$  days). A user needs to spend one permit specific to the class  $k_i$  of its trip to drive its car. The prices of each type of permit  $p_{k_i}^{\text{TPS}}$  are not directly defined by the regulator and not known a priori. They result from permit trading. The main difference with the TCS is that there is one market per permit, so each permit can have a different price. Same as for the TCS, each permit price is constant over the validity cycle. The TPS specific cost is the price of the adequate permit:

$$P_i = p_{k_i}^{\text{TPS}}. \quad (15)$$

### 3.5 Spatial variations of DMS

Every traveler faces the same credit charge in the uniform (U-) TCS, regardless of the corresponding OD-pair. However, every user has a different trip, and the corresponding PT alternative is relatively worse or better than taking the car compared to other users' trips. Thus a U-TCS could be sub-optimal by not considering such heterogeneity and creating spatial inequalities. We propose destination (D-) and OD-pair (OD-) variations for each DMS to account for this spatial heterogeneity. We make the price, credit charge or permit allocation ratio proportional to the quality of the PT alternative over the car option,  $w_k$ .  $w_k$  is computed as the trip length per car over the PT travel time, weighted by the demand. We use the subscript  $k$  as the index for both the D or OD, as the process to compute both spatial DMS are similar.  $w_k$  is homogeneous to a speed. Since instantaneous car speed is the same for every traveler and depends on the mode choices, the car trip length is used instead of the car travel time. Let us explain the spatial DMS design for the case of a D-specific charging scheme.  $k$  refers to D. For the OD-specific case, the computation of the credit charges, permit allocations, and tolls are similar; one only needs to replace D by OD.

$$w_k = \frac{\sum_{i=1}^N \delta_i^k \gamma_i \frac{l_i}{T_i^{\text{PT}}}}{\sum_{i=1}^N \delta_i^k \gamma_i}, \quad (16)$$

with  $\delta_i^k = 1$  if and only if the D of the trip of  $i$  is  $k$ . It is zero otherwise. We define a macro credit charge  $\tau$  for the TCS. We assume the allocation  $\kappa$  is the same for all travelers. The credit cap is set by the ratio of the charge and the allocation, so it is enough to vary one while keeping the other constant to tune the TCS. For D-TCS, the credit charge is proportional to the quality of the PT alternative: it is expensive to drive a car when the PT alternative is good and more affordable when the transit travel time is relatively high. It is

computed using

$$\tau_k = \tau w_k \frac{\sum_{i=1}^N \gamma_i}{\sum_{i=1}^N \gamma_i w_{k_i}}. \quad (17)$$

$k_i$  is the D of the group  $i$ . We define a macro permit ratio  $\kappa^{\text{TPS}}$ . It is the ratio of permits distributed averaged over all the travelers. The permit allocation per class of trip  $k$  is inversely proportional to the quality of the PT. It is more difficult for travelers with good transit alternatives to take their cars. It is defined by:

$$\kappa_k^{\text{TPS}} = \frac{\kappa^{\text{TPS}}}{w_k} \frac{\sum_{i=1}^N \gamma_i}{\sum_{i=1}^N \gamma_i \frac{1}{w_{k_i}}}. \quad (18)$$

Eq. (17) and (18) ensure the DMS is proportional to the quality of the PT alternative and the averaged number of allowed personal cars is  $\frac{\kappa}{\tau}$  or  $\kappa^{\text{TPS}}$  times the number of travelers, i.e.,

$$\left\{ \begin{array}{l} \frac{\sum_i^N \gamma_i \kappa}{\sum_i^N \gamma_i \tau_{k_i}} = \frac{\sum_i^N \gamma_i \kappa}{\sum_i^N \gamma_i \tau} = \frac{\kappa}{\tau}, \\ \frac{\sum_i^N \gamma_i \kappa_{k_i}^{\text{PT}}}{\sum_i^N \gamma_i} = \frac{\sum_i^N \gamma_i \kappa_k^{\text{TPS}}}{\sum_i^N \gamma_i} = \kappa^{\text{TPS}}. \end{array} \right. \quad (19)$$

The macro toll price is  $p$ . For the D-specific tolls, the corresponding pricing for the class of trip  $k$  is  $p_k$ . It is computed similarly to the TCS, proportional to the quality of the PT alternative:

$$p_k = p w_k \frac{\sum_{i=1}^N \gamma_i}{\sum_{i=1}^N \gamma_i w_{k_i}}. \quad (20)$$

It ensures that the averaged faced toll price is  $p$ :

$$\frac{\sum_i^N \gamma_i p_{k_i}}{\sum_i^N \gamma_i} = \frac{\sum_i^N \gamma_i p}{\sum_i^N \gamma_i} = p. \quad (21)$$

## 4 Computing the modal equilibrium

This section presents how the solution method for calculating the SUE under TCS presented in Sect. 2 should be extended to account for the new variables and constraints introduced in Sect. 3. The modifications to account for the new constraints related to LPR and pricing are relatively straightforward. The main contribution here is the modifications related to the time horizon, i.e., the validity cycle. In the initial problem, the SUE was calculated over the same time horizon as the demand management strategy, i.e., over one day. Now, we have a hierarchical problem where prices and quantities are equilibrated over the entire validity cycle while the modal shares are still equilibrated for each day.

The different DMS modify the costs and constraints linked to using private cars. The modal equilibrium is then different following the considered policy. We assume the regulator sets the parameters relative to the different DMS:

travelers exempt from LPR, toll price, credit charge, and permit allocation. The presented framework aims at computing the equilibrium and comparing the DMS. This paper does not optimize the DMS, even though we compare different implementations.

## 4.1 No DMS

As no constraints link the different days, the equilibrium is computed separately for each day. The solutions are different for each day because of the distribution of the PT penalties changes.

The equilibrium is reached when the modal decisions of all groups  $\psi_d$  are equal to the corresponding modal shares  $\mathbf{x}_d$ :

$$\psi_d = \mathbf{x}_d \quad \forall d. \quad (22)$$

With those modifications, the equilibrium computation is the same as with the initial framework, by dropping the terms and constraints linked to the price, the credit cap, and the MCC; or simply by setting  $\tau = 0$  in the Eq. (7) to (10).

## 4.2 LPR

Groups not allowed to drive are removed from the vectors as their modal shares are set to zero, and they do not contribute to the congestion as they ride PT. The equilibrium solution method is the same as for no DMS in Eq. (22), excluding the groups not allowed to drive. The travelers allowed to drive are the ones exempted from the LPR and the ones for which the license plate number matches the ones allowed for this day (odd numbers on odd days and even numbers on even days).

## 4.3 Pricing

The pricing equilibrium method is similar to the no DMS case in Eq. (22). The difference is in the logit decision  $\psi$ , as users account for the toll on top of the travel time. As it is not a quantity-based DMS, no constraints connect the different days. Thus the modal shares are computed by applying the iterative method independently for each day.

## 4.4 TCS

As we now consider that the credits can be valid for a given period, the modal shares and credit price have to be computed over the complete validity cycle  $c$ . In particular, the credit cap applies to the whole validity cycle and not every single day independently. The consumed credits over the cycle cannot exceed the allocated credits during the same period. Furthermore, the credit charge is not the same for all the travelers' groups. We need to reformulate the equilibrium problem to consider several days, the ratio of travelers having

access to a car, and the credit charge heterogeneity. The equilibrium differs from Eq. (7):

$$\begin{cases} \psi_{\mathbf{d}} & = \mathbf{x}_{\mathbf{d}} \quad \forall d \in [1, c]; \\ p^{\text{TCS}} \left( \sum_{i=1}^N \sum_{d=1}^c \gamma_i (\kappa - \tau_{k_i} r_i x_{d,i}) \right) & = 0; \\ \sum_{i=1}^N \sum_{d=1}^c \gamma_i (\tau_{k_i} r_i x_{d,i} - \kappa) & \leq 0. \end{cases} \quad (23)$$

The two last lines of Eq. (23) are specific to TCS: the credit cap and the MCC. The MCC concerns credit consumption over the whole validity cycle.

The optimization problem covers the days forming the validity cycle, and not a single day as in Eq. (8):

$$J = \frac{1}{2} \sum_{d=1}^c \sum_{i=1}^N (x_{d,i} - \psi_{d,i})^2 + \eta \frac{1}{\sum_{i=1}^N \gamma_i} p^{\text{TCS}} \sum_{d=1}^c \sum_{i=1}^N \gamma_i (\kappa - \tau_{k_i} r_i x_{d,i}). \quad (24)$$

The cost function is the sum of the assignment errors over the days plus the MCC, which spans over the validity cycle too. The global constraints are also modified:

$$\begin{cases} 0 \leq x_{d,i} \leq 1 \quad \forall i \in [1, N], d \in [1, c]; \\ p^{\text{TCS}} \geq 0; \\ \sum_{d=1}^c \sum_{i=1}^N \gamma_i (\tau_{k_i} r_i x_{d,i} - \kappa) \leq 0. \end{cases} \quad (25)$$

The optimization problem is linearized and formulated as a QP to be solved iteratively until convergence is reached:

$$\frac{1}{2} \Delta \tilde{\mathbf{x}}^{\text{T}} \cdot \mathbf{P} \cdot \Delta \tilde{\mathbf{x}} + \mathbf{q} \cdot \Delta \tilde{\mathbf{x}}. \quad (26)$$

The quadratic formulation is similar, however, the matrices and vectors are larger to account for the whole validity cycle. The variable  $\tilde{\mathbf{x}}$  are the modal shares for each day of the cycle and the credit price:  $\tilde{\mathbf{x}} = [\mathbf{x}_{\mathbf{1}}; \dots; \mathbf{x}_{\mathbf{c}}; p^{\text{TCS}}]$ , and  $\Delta \tilde{\mathbf{x}}$  its variation. Its size is  $Nc + 1$ . The symmetric matrix  $\mathbf{P}$  and the vector  $\mathbf{q}$  are defined by

$$\begin{cases} \mathbf{P} & = (\tilde{\nabla} \Psi - \mathbf{I}_{\mathbf{x}})^{\text{T}} \cdot (\tilde{\nabla} \Psi - \mathbf{I}_{\mathbf{x}}) + \eta \mathbf{I}_{\mathbf{p}}; \\ \mathbf{q} & = (\tilde{\nabla} \Psi - \mathbf{I}_{\mathbf{x}})^{\text{T}} \cdot (\Psi - \mathbf{I}_{\mathbf{x}} \cdot \tilde{\mathbf{x}}_0) + \eta \mathbf{i}_{\mathbf{p}}. \end{cases} \quad (27)$$

The first terms of  $\mathbf{P}$  and  $\mathbf{q}$  stand for the modal equilibrium and the second ones stand for the MCC.  $\mathbf{I}_{\mathbf{x}}$  the pseudo-identity matrix of size  $(Nc + 1) \times Nc$ , so that  $\mathbf{I}_{\mathbf{x}} \cdot \tilde{\mathbf{x}} = [\mathbf{x}_{\mathbf{1}}; \dots; \mathbf{x}_{\mathbf{c}}]$ .  $\Psi$  is the concatenation of the modal decisions



of every day of the cycle:  $\Psi = [\psi_1; \dots; \psi_c]$ .  $\tilde{\nabla}\Psi$  is defined by:

$$\tilde{\nabla}\Psi = \begin{pmatrix} \nabla_{\mathbf{x}_1}\psi_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \nabla_{p^{\text{TCS}}}\psi_1 \\ \mathbf{0} & \nabla_{\mathbf{x}_2}\psi_2 & \mathbf{0} & \mathbf{0} & \nabla_{p^{\text{TCS}}}\psi_2 \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \nabla_{\mathbf{x}_c}\psi_c & \nabla_{p^{\text{TCS}}}\psi_c \end{pmatrix}. \quad (28)$$

This equation reflects the fact that the logit decision on a day is impacted by the car shares on all days, because it drives the credit consumption, which affects the credit price and thus the car travel costs.  $\nabla_{\mathbf{x}_d}\psi_d$  and  $\nabla_{p^{\text{TCS}}}\psi_d$  are respectively the gradients of the modal decision on day  $d$  with respect to the modal shares on day  $d$  and to the credit price.  $\mathbf{I}_p$  is a symmetric matrix of size  $(Nc + 1)^2$  and  $\mathbf{i}_p$  a vector of size  $Nc + 1$  defined by:

$$\begin{cases} I_{p,N(d-1)+i,Nc+1} & = I_{p,Nc+1,N(d-1)+i} \\ & = -\frac{\gamma^i}{\sum_{j=1}^N \gamma_j} \tau_{k_i} r_i \text{ for } (i, d) \in [1, N] \times [1, c] \\ & \text{and 0 elsewhere;} \\ i_{p,N(d-1)+i} & = -\frac{\gamma^i}{\sum_{j=1}^N \gamma_j} \tau_{k_i} r_i p_0^{\text{TCS}} \text{ for } (i, d) \in [1, N] \times [1, c]; \\ i_{p,Nc+1} & = \frac{1}{\sum_{i=1}^N \gamma_i} \left( \sum_{d=1}^c \sum_{i=1}^N \gamma_i (\kappa - \tau_{k_i} r_i x_{0,d,i}) \right). \end{cases} \quad (29)$$

The constraints of the iterative linearized problem are

$$\begin{cases} \Delta x_{d,i} & \leq \min(1 - x_{0,d,i}, \epsilon_x) \quad \forall i \in [1, N], d \in [1, c]; \\ \Delta x_{d,i} & \geq \max(-x_{0,d,i}, -\epsilon_x) \quad \forall i \in [1, N], d \in [1, c]; \\ \Delta p^{\text{TCS}} & \leq \epsilon_p; \\ \Delta p^{\text{TCS}} & \geq \max(-p_0^{\text{TCS}}, -\epsilon_p); \\ \sum_{d=1}^c \sum_{i=1}^N \gamma_i \tau_{k_i} r_i \Delta x_{d,i} & \leq \sum_{d=1}^c \sum_{i=1}^N \gamma_i (\kappa - \tau_{k_i} r_i x_{0,d,i}), \end{cases} \quad (30)$$

with  $\epsilon_p$  a parameter restricting the search space for the credit price around the current best solution. When a better solution is found, the search space is moved around the new best one and linearizations are performed again.

## 4.5 TPS

As the permits are also issued for a cycle of  $c$  days, the equilibrium in the TPS case is defined over the validity cycle by:

$$\begin{cases} \psi_d & = \mathbf{x}_d \quad \forall d \in [1, c]; \\ p_k^{\text{TPS}} \left( \sum_{i=1}^N \delta_i^k \sum_{d=1}^c \gamma_i (\kappa_k^{\text{TPS}} - r_i x_{d,i}) \right) & = 0 \quad \forall k \in [1, N_K]; \\ \sum_{i=1}^N \delta_i^k \sum_{d=1}^c \gamma_i (r_i x_{d,i} - \kappa_k^{\text{TPS}}) & \leq 0 \quad \forall k \in [1, N_K], \end{cases} \quad (31)$$

with  $\delta_i^k = 1$  if and only if group  $i$ 's trip is part of the  $k$ 's class of trip, and 0 otherwise. The main differences with the TCS are several permit caps and MCC (one per type of permit). The decision vector is larger than the TCS one as there is one price per D or OD. The equilibrium is formulated as an optimization problem:

$$J = \frac{1}{2} \sum_{d=1}^c \sum_{i=1}^N (x_{d,i} - \psi_{d,i})^2 + \eta^{\text{TPS}} \frac{1}{\sum_{i=1}^N \gamma_i} \sum_{d=1}^c \sum_{i=1}^N \gamma_i p_{ki}^{\text{TPS}} (\kappa_{ki}^{\text{TPS}} - r_i x_{d,i}), \quad (32)$$

with  $\eta^{\text{TPS}}$  the coefficient related to the MCC. Note that the coefficient  $\delta_i^k$  does not appear since all the  $N_K$  MCC are summed together, and each group appears exactly once in the MCC concerning its corresponding type of permit. Here again, the MCC are included in the cost function to keep all constraints linear. The global constraints are

$$\begin{cases} 0 \leq x_{d,i} \leq 1 \quad \forall i \in [1, N], \quad d \in [1, c]; \\ p_k^{\text{TPS}} \geq 0 \quad \text{for } k \in [1, N_K]; \\ \sum_{d=1}^c \sum_{i=1}^N \gamma_i \delta_i^k (\kappa_k^{\text{TPS}} - r_i x_{d,i}) \leq 0 \quad \text{for } k \in [1, N_K]. \end{cases} \quad (33)$$

The optimization problem is linearized and formulated as a QP to be solved iteratively around the current best solution:

$$\frac{1}{2} \Delta \bar{\mathbf{x}}^T \cdot \mathbf{P} \cdot \Delta \bar{\mathbf{x}} + \mathbf{q} \cdot \Delta \bar{\mathbf{x}}. \quad (34)$$

The variable  $\bar{\mathbf{x}}$  are the modal shares for each day of the cycle and the permits prices:  $\bar{\mathbf{x}} = [\mathbf{x}_1; \dots; \mathbf{x}_c; p_1^{\text{TPS}}; \dots; p_{N_K}^{\text{TPS}}]$ , and  $\Delta \bar{\mathbf{x}}$  is its variation. Its size is  $Nc + N_K$ . The symmetric matrix  $\mathbf{P}$  and the vector  $\mathbf{q}$  are defined by:

$$\begin{cases} \mathbf{P} &= (\bar{\nabla} \Psi - \mathbf{I}_x)^T \cdot (\bar{\nabla} \Psi - \mathbf{I}_x) + \eta^{\text{TPS}} \mathbf{I}_p; \\ \mathbf{q} &= (\bar{\nabla} \Psi - \mathbf{I}_x)^T \cdot (\psi_0 - \mathbf{I}_x \cdot \bar{\mathbf{x}}_0) + \eta^{\text{TPS}} \mathbf{i}_p. \end{cases} \quad (35)$$

The first terms of  $\mathbf{P}$  and  $\mathbf{q}$  stand for the modal equilibrium and the second ones stand for the MCC.  $\mathbf{I}_x$  is the pseudo-identity matrix of size  $(Nc + N_K) \times Nc$ , so that  $\mathbf{I}_x \cdot \bar{\mathbf{x}} = [\mathbf{x}_1; \dots; \mathbf{x}_c]$ .  $\bar{\nabla} \Psi$  is defined by:

$$\bar{\nabla} \Psi = \begin{pmatrix} \nabla_{\mathbf{x}_1} \psi_1 & 0 & 0 & 0 & \nabla_{p_1^{\text{TPS}}} \psi_1 & \dots & \nabla_{p_{N_K}^{\text{TPS}}} \psi_1 \\ 0 & \nabla_{\mathbf{x}_2} \psi_2 & 0 & 0 & \nabla_{p_1^{\text{TPS}}} \psi_2 & \dots & \nabla_{p_{N_K}^{\text{TPS}}} \psi_2 \\ 0 & 0 & \ddots & 0 & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \nabla_{\mathbf{x}_c} \psi_c & \nabla_{p_1^{\text{TPS}}} \psi_c & \dots & \nabla_{p_{N_K}^{\text{TPS}}} \psi_c \end{pmatrix}. \quad (36)$$

$\nabla_{\mathbf{x}_d} \psi_d$  and  $\nabla_{p_k^{\text{TPS}}} \psi_d$  are respectively the gradients of the modal decision on day  $d$  with respect to the modal shares on day  $d$  and to the permit price of

type  $k$ .  $\mathbf{I}_p$  is a symmetric matrix of size  $(Nc + N_K)^2$  and  $\mathbf{i}_p$  a vector of size  $Nc + N_K$  defined by:

$$\left\{ \begin{array}{l} I_{p,N(d-1)+i,Nc+k_i} = I_{p,Nc+k_i,N(d-1)+i} \\ \quad = -\frac{\gamma^i}{\sum_{j=1}^N \gamma_j} r_i \text{ for } (i, d) \in [1, N] \times [1, c] \\ \quad \text{and 0 elsewhere;} \\ i_{p,N(d-1)+i} = -\frac{\gamma^i}{\sum_{j=1}^N \gamma_j} r_i p_{k_i,0}^{\text{TPS}} \text{ for } (i, d) \in [1, N] \times [1, c]; \\ i_{p,Nc+k} = \frac{1}{\sum_{i=1}^N \gamma_i} \left( \sum_{d=1}^c \sum_{i=1}^N \delta_i^k \gamma_i (\kappa_k^{\text{TPS}} - r_i x_{0,d,i}) \right) \\ \quad \text{for } k \in [1, N_K]. \end{array} \right. \quad (37)$$

The constraints are

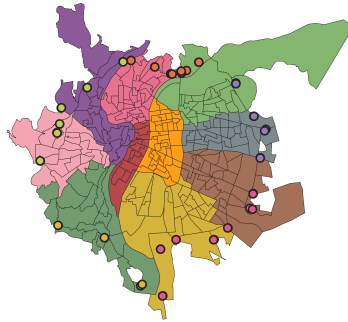
$$\left\{ \begin{array}{l} \Delta x_{d,i} \leq \min(1 - x_{0,d,i}, \epsilon_x) \text{ for } i \in [1, N]; \\ \Delta x_{d,i} \geq \max(-x_{0,d,i}, -\epsilon_x) \text{ for } i \in [1, N]; \\ \Delta p_k^{\text{TPS}} \leq \epsilon_p^{\text{TPS}} \text{ for } k \in [1, N_K]; \\ \Delta p_k^{\text{TPS}} \geq \max(-p_{k,0}^{\text{TPS}}, -\epsilon_p^{\text{TPS}}) \text{ for } k \in [1, N_K]; \\ \sum_{d=1}^c \sum_{i=1}^N \gamma_i r_i \delta_i^k \Delta x_{d,i} \leq \sum_{d=1}^c \sum_{i=1}^N \gamma_i \delta_i^k (\kappa_k^{\text{TPS}} - r_i x_{0,d,i}) \\ \quad \text{for } k \in [1, N_K], \end{array} \right. \quad (38)$$

with  $\epsilon_p^{\text{TPS}}$  a parameter restricting the search space for the permit prices around the current best solution during the iterative process.

The class of trip  $k$  can be the destination D or the OD pair in this work. We compare the different options in Table 2. The initial framework of Sect. 2, i.e., our previous work on TCS with trip-based MFD corresponds to TCS-U for  $c = 1$ .

**Table 2:** Comparison of the different DMS

Strategy	Type	Allocation	Charge	Nb charges	Nb markets	Nb variables
No DMS	-	-	-	-	-	$N$
LPR	rationing	-	-	-	-	$\leq N$
Pri-U	price	-	$p$	1	-	$N$
Pri-D	price	-	$p_k$	$N_D$	-	$N$
Pri-OD	price	-	$p_k$	$N_{OD}$	-	$N$
TCS-U	quantity	$\kappa$	$\tau$	1	1	$cN + 1$
TCS-D	quantity	$\kappa$	$\tau_k$	$N_D$	1	$cN + 1$
TCS-OD	quantity	$\kappa$	$\tau_k$	$N_{OD}$	1	$cN + 1$
TPS-D	quantity	$\kappa_k^{\text{TPS}}$	1	$N_D$	$N_D$	$cN + N_D$
TPS-OD	quantity	$\kappa_k^{\text{TPS}}$	1	$N_{OD}$	$N_{OD}$	$cN + N_{OD}$



**Fig. 3:** The network of Lyon with the IRIS areas merged in ten regions and the five additional regions aggregating the access points (circles).

The computational complexity of the TCS and TPS is substantially higher than the other DMS as the QP size is higher. It is necessary because of the credit/permit cap and MCC over the whole validity cycle. Note that the uniform variant of the TPS is the same as the uniform variant of the TCS since there is a unique market. Thus the TPS-U is not considered because it would be redundant.

## 5 Benchmarking the different demand management policies

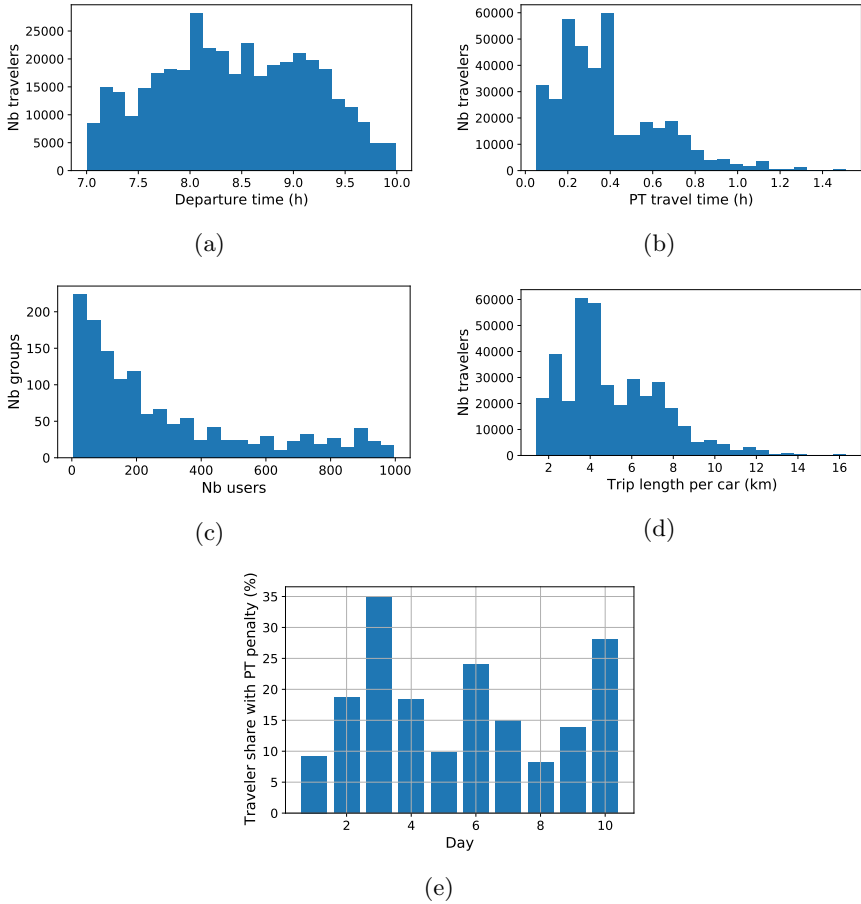
We consider the simulation of the full Lyon Metropolis during the morning peak hour. The MFD comes from the results of [Mariotte et al \(2020\)](#). The relationship between mean car speed and car accumulation was calibrated using loop detectors and taxi GPS data over typical days when buses were operating following the usual timetables. So, the regular circulating buses' effect is already considered in the processed traffic data. Only drastic changes in public transport timetables would require recalibrating the car MFD curve. The initial demand is based on the IRIS areas. Those are areas defined by the French administration for census purposes. They typically include between 1 800 and 5 000 inhabitants for residential areas. See [INSEE \(2021\)](#) for details. We merge the IRIS areas into ten regions to aggregate the demand and regroup travelers starting simultaneously the same trip. Furthermore, five additional regions represent the access to the city to account for travelers coming from or going outside of the network. In total, we have identified 224 macro OD pairs as one macro OD pair has no demand during the considered period. The estimation of the car trip lengths and PT travel times is the average of those values for the IRIS areas weighted by the travel demand. The considered road network, along with the regions and the boundaries forming the 15 origins and destinations, is to be found in [Fig. 3](#). The demand is based on the typical peak hour between 7:00 and 10:00 ([Ameli et al, 2021](#)). For each subperiod of 15 minutes and OD pair, the PT travel times inside Lyon are recovered from the

navigator HERE ([HERE Developer, 2020](#)) at the IRIS level. Those PT travel times are then aggregated into the ten regions to estimate the PT travel time for each macro OD pair. An average PT speed of 10.8 km/h (3 m/s) is used for trips ending or starting outside the city. It is slightly below the PT speed recovered from HERE inside the city to account for the lower PT network coverage outside the city and the burden of switching modes at Park+Ride facilities. The users are aggregated into groups by ensuring at least two groups per hour for the same OD and that groups never gather more than 1 000 users. It is a trade-off between numerical complexity and an accurate representation of the demand dynamics at the OD level. Thus 1 374 groups are formed, representing the total demand of 384 200 travelers. To account for different days, we consider a horizon of two working weeks, i.e.,  $h = 10$  days. We suppose that 10% of the travellers do not have access to a car and that this ratio is homogeneous across the different groups. Table 3 sums up the main parameters. The numerical value for the VoT is based on the work of [Fosgerau](#)

**Table 3:** Parameters used for the simulation

Parameter	Notation	Value
VoT	$\alpha$	10.8 EUR/h
MCC weight TCS	$\eta$	1
MCC weight TPS	$\eta^{TPS}$	1
Logit parameter	$\theta$	1 1/EUR
Horizon length	$h$	10 days
Validity cycle	$c$	{1, 2, 5, 10} days
PT penalty	$\mu_i(d), i \in [1, N], d \in [1, h]$	{0, 10} EUR
Ratio of car access	$r_i, i \in [1, N]$	0.9

[et al \(2007\)](#). The PT penalty is set to 10 EUR. This value ensures that almost all (> 99%) mandatory car users, i.e., travelers willing to take their car the days they face the penalty, are satisfied in the no DMS case. Captive PT riders do not face the PT penalty, as they do not choose their modes. More details about the test case settings can be found in Fig. 4, including the distributions of departure times, PT travel times, trip lengths, group sizes, and PT penalties over the days. The sensibility to the PT penalty distribution is discussed in Appendix B.



**Fig. 4:** Distribution of (a) the departure times, (b) the PT travel times, (c) the group sizes, (d) the car trip length, and (e) days with PT penalty.

To compare the different DMS, we quantify various aspects: travel disutility, users' satisfaction, pollution, equivalent toll, and individual gains.

**Travel disutility** To assess the disutility of the travels, in the sense of travel time and PT penalty, we calculate the average total travel time  $TTT$  and penalty cost  $PC$  over the overall time horizon (ten days). The social cost  $SC$  is defined as the sum of them, with  $TTT$  being weighted by the VoT  $\alpha$ .

$$\begin{cases} TTT &= \frac{1}{h} \sum_{d=1}^h \sum_{i=1}^N \gamma_i (r_i x_{d,i} T_i(\mathbf{x}_d) + (r_i(1 - x_{d,i}) + 1 - r_i) T_i^{\text{PT}}) \\ PC &= \frac{1}{h} \sum_{d=1}^h \sum_{i=1}^N \gamma_i r_i (1 - x_{d,i}) \mu_i(d) \\ SC &= \alpha TTT + PC \end{cases} \quad (39)$$

Note that travelers without access to a car do not face PT penalties. Their travel time and thus social costs remain unchanged by the DMS.

**Satisfaction** Furthermore, to better assess DMS acceptability at an individual level, we compute the satisfaction rate, defined as the ratio of mandatory car users driving their cars. It is a proxy measuring how critical needs for cars can still be fulfilled under the DMS.

**Pollution** We also consider the cars' carbon emissions averaged over the time horizon using a COPERT IV model, i.e., assuming the emission per distance and vehicle depends on the instantaneous mean speed.

**Equivalent toll** To compare the financial consequences for the users of the different DMS, we assess the equivalent toll of the DMS: pricing, TCS, and TPS. The equivalent toll is the out-of-pocket money needed to drive a car. It is the average money spent by car drivers.

- The equivalent toll price is the toll price in a uniform pricing case. However, for D- and OD-specific tolling, the equivalent toll price may differ from the macro toll (weighted by the travel demand) as it is weighted by the realized car demand. The equivalent toll price for D- and OD-pricing schemes is computed as:

$$\frac{\sum_{i=1}^N \gamma_i x_i p_{k_i}}{\sum_{i=1}^N \gamma_i x_i}, \quad (40)$$

with  $k_i$  being the destination (for D-variant) or the OD-pair (for OD-variant) of the group  $i$ .

- It is defined by  $p^{\text{TCS}}(\tau - \kappa)$  for the uniform TCS. For the D- and OD-variants, it is computed with:

$$\frac{\sum_{i=1}^N \gamma_i x_i p^{\text{TCS}}(\tau_{k_i} - \kappa)}{\sum_{i=1}^N \gamma_i x_i}. \quad (41)$$

- For TPS, it is computed in a similar manner:

$$\frac{\sum_{i=1}^N \gamma_i x_i p_{k_i}^{\text{TPS}}(1 - \kappa_{k_i})}{\sum_{i=1}^N \gamma_i x_i}. \quad (42)$$

**Individual gains** The social cost gains are the difference between the perceived costs with the actual DMS and without DMS. It is defined independently of the DMS for a group  $i$  by:

$$r_i \left( x_i^{\text{no DMS}} \alpha T_i^{\text{no DMS}} + (1 - x_i^{\text{no DMS}})(\alpha T_i^{\text{PT}} + \mu_i) - (x_i \alpha T_i + (1 - x_i)(\alpha T_i^{\text{PT}} + \mu_i)) \right). \quad (43)$$

The trade gain is the money a traveler earns with the DMS. When negative, it means the travelers of the corresponding group are losing money, i.e., they spend more money than they earn with the DMS.

- It is negative for pricing as the travelers' money flows to the regulator. It is defined by:

$$-p_{k_i} r_i x_i. \quad (44)$$

- For the TCS, the trade gains are defined by:

$$p^{\text{TCS}}(\kappa - r_i \tau_{k_i}^{\text{TCS}} x_i). \quad (45)$$

- For the TPS, the trade gains are defined by:

$$p_{k_i}^{\text{TPS}}(\kappa_{k_i}^{\text{TPS}} - r_i x_i). \quad (46)$$

In the following, different labels are used to name the different configurations of the DMS in the figures.

- For pricing, it is the macro toll level  $p$  in euro.
- For LPR, it is the ratio of the population exempted from (or non-complying to) LPR.
- For TCS, it is the ratio of the allocation over the macro credit charge  $\kappa/\tau$  (short credit ratio). For the uniform case, this ratio also represents the maximum ratio of car drivers over all the travelers. The maximum car share is unknown for the D- and OD cases, as the groups face different credit charges.
- For TPS, it is the macro permit allocation  $\kappa^{\text{TPS}}$ . It also represents the maximum car share.

Furthermore, to avoid any confusion, the labeling of the DMS follows the conventions [DMS]-[charging], [DMS]-[charging][validity cycle], or [DMS]-[charging][validity cycle][DMS parameter] depending on the context. So TCS-D10-50% refers to the TCS with a D-specific charging scheme with a validity cycle of ten days and a ratio allocation over charge of 50%, i.e., at the maximum, every second traveler can drive a car.

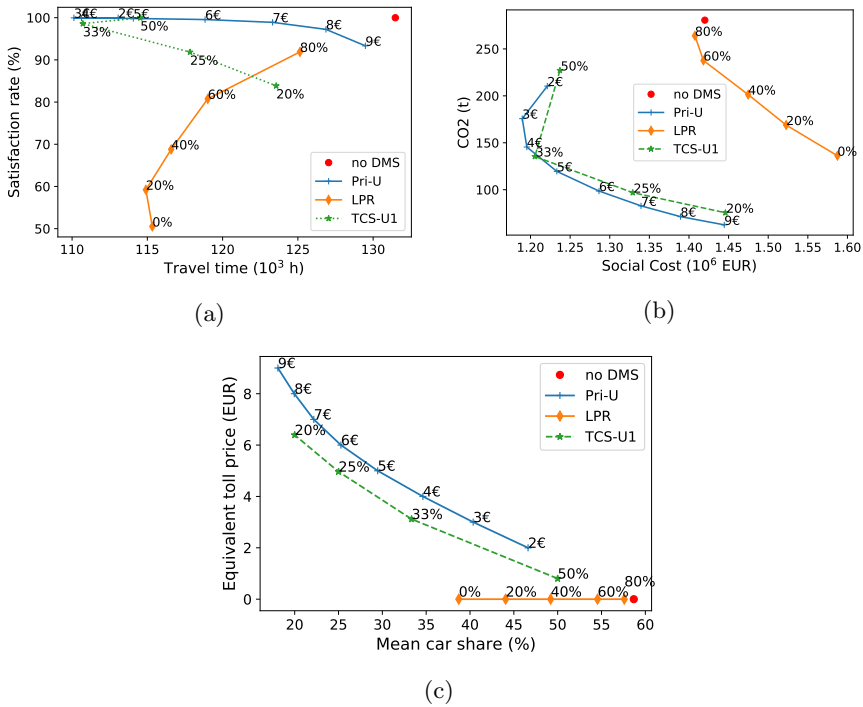
For all the presented results, the convergence quality is measured by the quadratic cost over the validity cycle length  $J/c$ . It is smaller than  $1.4 \times 10^{-2}$ .

## 5.1 Comparing the DMS with uniform charging settings

First, we compare the different DMS: pricing, LPR, and TCS with uniform charging settings. The different DMS are associated with different parameters and settings (credit charge for TCS, toll price for pricing, exemption ratio for LPR), and there is no direct equivalence between them. Comparing two individual scenarios corresponding to two different DMS may appear challenging. However, a fair comparison can be achieved by assessing the different DMS globally using a wide range of parameters covering the most plausible values. We can then compare the network performances at equilibrium by analyzing the relative positioning of the associated curves. Thus, we compare the DMS as a whole, and we do not pair single configurations. The effects on travel time, satisfaction rate, carbon emission, social cost, equivalent toll, and car share of



the DMS with uniform settings are compared in Fig. 5 for a validity cycle of one day.

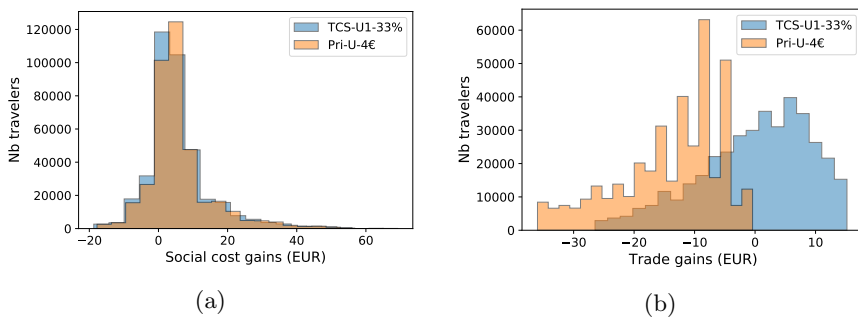


**Fig. 5:** Comparison of the uniform DMS for a cycle of one day: (a) total travel time vs. penalty cost, (b) social cost vs. carbon emission, (c) car share and equivalent toll price.

As expected, implementing a DMS decreases the satisfaction rate because restrictions or increased costs for car travel push users towards PT options. The satisfaction rate decreases rapidly with LPR and falls as low as 50%. Such a policy is myopic and affects users regardless of their actual needs. When fully enforced, half of the mandatory car users cannot use it (exemption rate of 0%). The satisfaction rate only begins to drop for a relatively high toll with pricing. It is only when the toll price reaches the penalty value that mandatory car users start considering PT options. A uniform TCS with an allocation/charge ratio of 33% (maximum one car driver every three travelers) reduces the satisfaction rate by less than 5 points. With the same settings, the network carbon emissions drop by around 50% and the social cost by about 15%. The same effects can be observed with an urban toll of 4 EUR, but the equivalent toll for a TCS of 33% is only about 3.1 EUR, see Fig. 5c. The equivalent toll with TCS is about 1 EUR cheaper than pricing to achieve the

same car share reduction. Indeed, as a part of the needed credits are given by the regulator, car drivers only need to buy the remaining credits. The Pareto fronts formed by the pricing are slightly better than the TCS-U. They are better than the LPR, which reduces carbon emissions at the expense of the satisfaction rate and social costs. They enable better compromises between carbon emissions and social costs. The LPR is limited when reducing the modal share and cannot lead to a car share of less than 39%. The mean car share without any restrictions is 59%.

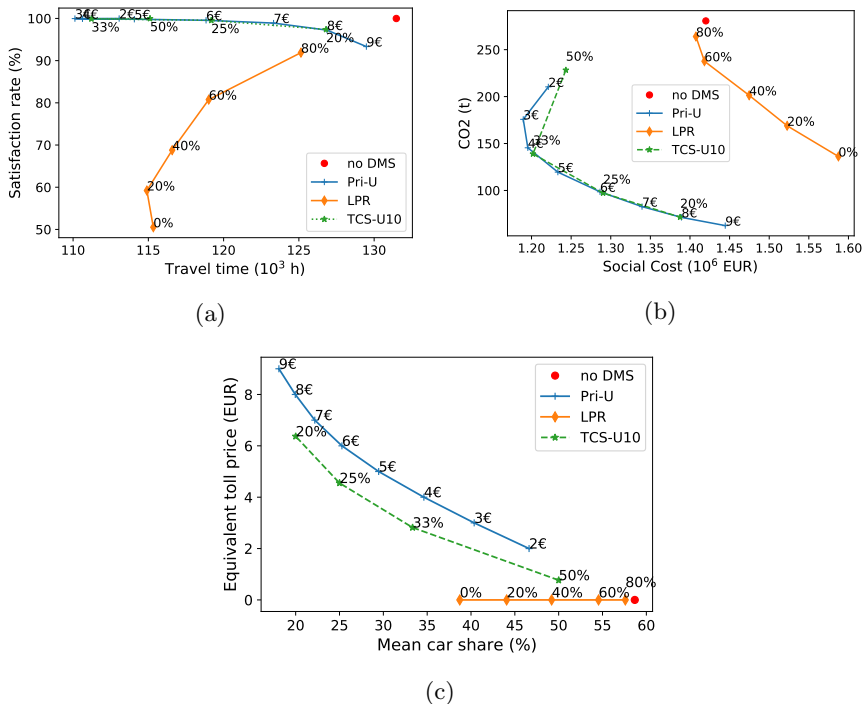
We also look at the distributions of the social gains: the congestion pricing and the TCS give analog distributions. The toll level of 4 EUR and credit ratio of 33% were chosen for the comparison as they give similar social costs and carbon emissions. It permits the comparison of the individual impacts of all DMS, considering the same general output in terms of network performances. We compare those gains in Fig. 6.



**Fig. 6:** Comparison of the distributions of the (a) social and (b) trade gains between uniform TCS with a validity cycle of one day and pricing.

The distribution of social costs, i.e., accounting for the change in travel time and PT penalty, are similar. Some travelers lose the equivalent of 20 EUR, while a few earn up to 70 EUR over the time horizon. However, the bulk of the population earns a social gain between 0 and 10 EUR. So the vast majority of the travelers are better off with TCS or pricing in terms of travel conditions. As the TCS is revenue-neutral, some travelers earn money by selling the credits they do not need. In particular, a specific user can pay some days but get money on other days, reducing its overall balance over the time horizon. With pricing, some travelers spend on average up to 4 EUR per day because of the toll, while they spend a maximum of 3.1 EUR per day for buying credits with TCS. Without redistribution, all users spend money under a pricing scheme. This result illustrates one advantage of TCS over pricing: some travelers are rewarded for their choices.

We compare travel time, satisfaction rate, carbon emission, and social cost for the DMS with uniform setting and a cycle length of ten days in Fig. 7.

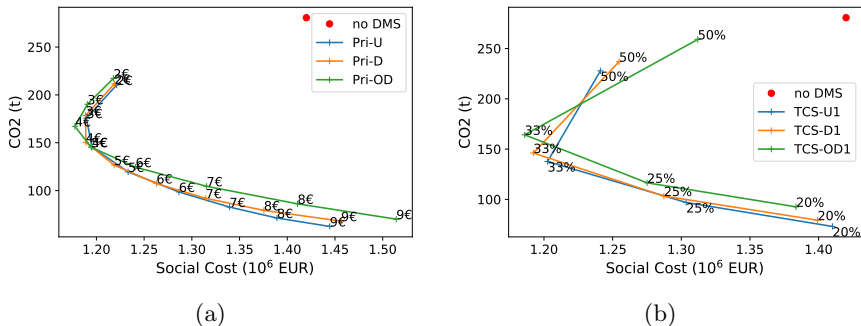


**Fig. 7:** Comparison of the uniform DMS for a cycle of ten days: (a) total travel time vs. penalty cost, (b) social cost vs. carbon emission, (c) car share and equivalent toll price.

The main improvement by increasing the validity cycle from one to several days is that the drop in satisfaction rate with TCS is smaller. Less than 3% of necessary car trips are canceled in the very restricting case of one car trip per working week per traveler (credit ratio of 20%). By giving more flexibility for the credit consumption, it is easier and cheaper for the mandatory car drivers to fulfill their needs. With a validity cycle of ten days, the Pareto fronts of TCS-U and pricing for satisfaction rate vs. total travel time and carbon emissions vs. social costs overlap. It is not surprising, as the credit price is the same every day when the validity cycle equals the horizon under consideration. The modal shares with TCS-U10 are then equivalent to congestion pricing with a toll of  $p = p^{\text{TCS}} \tau$  (the allocation does not matter when it comes to the modal shares, see the remark following Eq. (6)). It thus leads to the same modal shares and same traffic conditions. Note that the equivalent toll price is still 1 to 2 EUR cheaper with TCS (Fig. 7c), thanks to the initial allocation of  $\kappa$  credits.

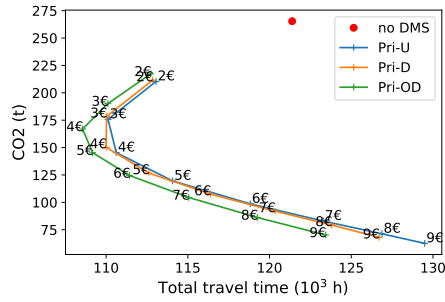
## 5.2 Different spatial charges

We now assess the effect of charging differently the travelers according to their destinations or OD-pairs in Fig. 8 for TCS and pricing.



**Fig. 8:** Social cost vs. carbon emissions for (a) pricing and (b) TCS for a validity cycle of one day.

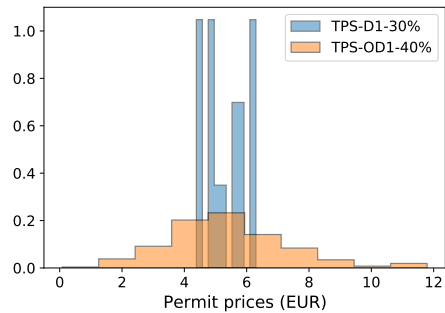
Increasing the spatial resolution from uniform to destination and then OD negatively affects the Pareto fronts of carbon emissions vs. social costs. It means when choosing the macro toll price of credit ratio to reach a desired carbon reduction, the associated social costs with the U-variant are lower than with the OD-variant. In other words, reducing carbon emissions requires a greater sacrifice with the OD-variant than with the U-variant. For pricing, reaching a carbon level of 100 t requires a toll between 6 and 7 EUR for the uniform variant. The corresponding social cost is  $1.3 \times 10^6$  EUR. The OD pricing variant with a macro toll of 8 EUR leads to the same pollution reduction. However, the corresponding social cost is  $1.4 \times 10^6$  EUR, 8% higher. We would expect the opposite since the OD- and D-schemes try to account for the relative burden of switching from car to PT. To understand this difference, we compute the modal equilibrium for pricing without PT penalty (i.e., with  $\mu_i(d) = 0 \forall i, d$ ) in Fig. 9.



**Fig. 9:** Total travel time vs. carbon emissions with pricing without PT penalty.

We represent the total travel time instead of the social cost because without PT penalty, the penalty cost is zero, and thus the social cost is the total travel time weighted by the VoT. It seems that without PT penalty, the D- and OD-specific pricing charging schemes lead to better compromises in terms of congestion and pollution for low total travel time. Especially, the OD variant leads to total travel times below  $105 \times 10^3$  h. However, those specific schemes do not account for the day-specific need to use the car, which appears to play a crucial role when calculating the equilibrium situations, more important than the quality of the PT coverage. Note that the PT penalty represents the same cost as a travel time of about one hour with the chosen parameters.

We compute the distribution of the permit prices for TPS-D and TPS-OD with a macro permit ratio of respectively 30% and 40% (they lead to similar carbon emissions and social costs) in Fig. 10.

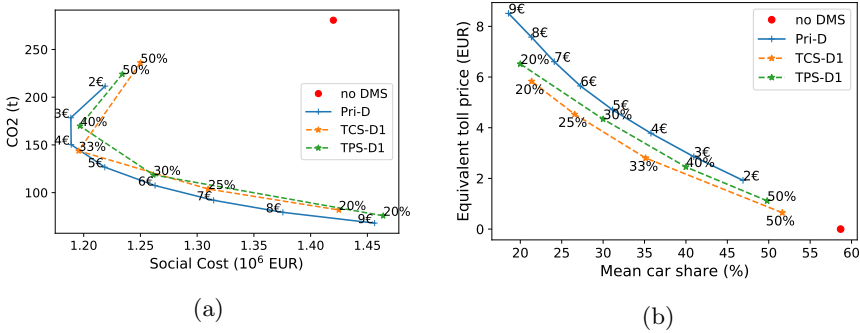


**Fig. 10:** Distribution of the permits prices, averaged over the two working weeks, for a validity cycle of one day.

With the D-variant, the permit prices are only between 4.4 and 6.3 EUR, while they go from 0.1 to 11.8 EUR with the OD setting. It is explained by the smaller number of markets in the D case where larger quantities of permits are traded than the OD-variant. A larger trade quantity stabilizes the market,

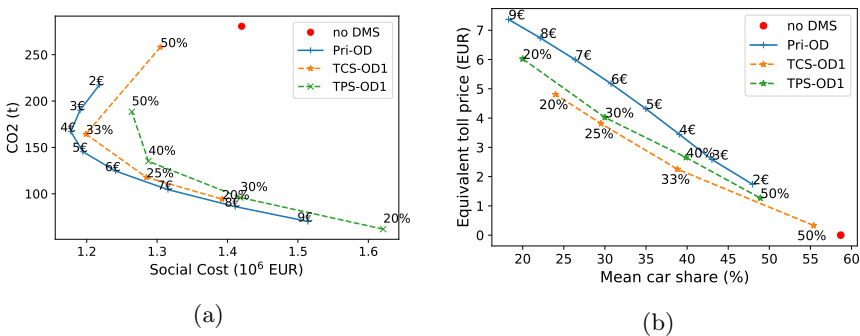
as the effect of the marginal utility of a permit for a traveler (especially when it absolutely needs to drive a car) is less representative.

The performances of D-specific DMS are compared in Fig. 11, and the OD-specific in Fig. 12 for a validity cycle of one day.



**Fig. 11:** (a) Social cost vs. carbon emission and (b) car share and toll equivalent for D-specific DMS for a validity cycle of one day.

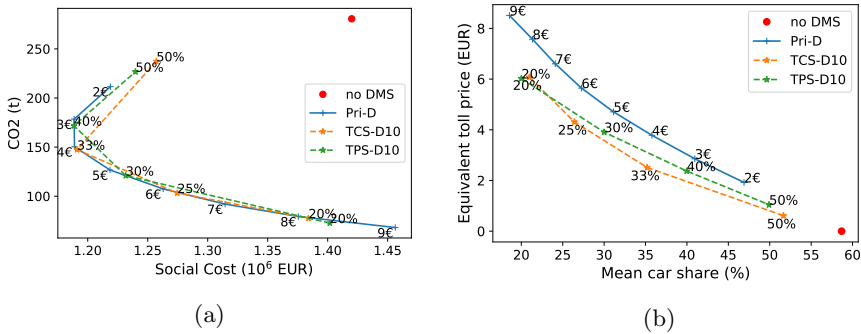
For the D-case, pricing is slightly better than TCS and TPS because its Pareto front dominates the trade-off proposed by TCS-D and TPS-D. For a carbon level of 100 t, the corresponding social cost is about  $1.3 \times 10^6$  EUR with pricing and more than  $1.35 \times 10^6$  EUR with TCS and TPS. For a given car share, the equivalent toll faced by the users is lower with TCS, by 1 to 2 EUR for pricing and some dozens of cents for TPS. Note that the equivalent toll price (y-axis) differs from the macro toll price (labels), as the first is weighted by the travel demand (car and PT), and the realized car demand weights the second.



**Fig. 12:** (a) Social cost vs. carbon emission and (b) car share and toll equivalent for OD-specific DMS for a validity cycle of one day.

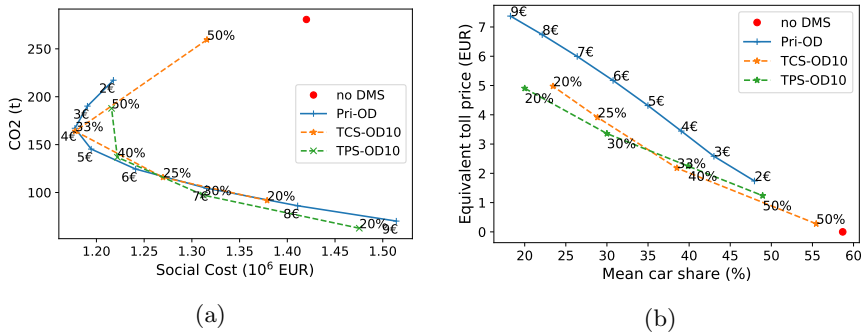
We draw the same conclusions for the OD-specific case as for the D-specific case: better compromises with pricing, especially it can lead to a social cost of less than  $1.2 \times 10^6$  EUR, while it is not the case with TCS or TPS. The TPS leads to similar or worse compromises in the OD case than pricing and TCS. Especially, the TPS-OD is adequate to reduce the carbon emission but not the social cost. This scheme is expected to be the least flexible since there are 224 different types of permits, and travelers trade exclusively along with travelers with the same route.

When the validity cycle is ten days, the results are slightly different, see Fig. 13 for D- and Fig. 14 for OD-specific DMS.



**Fig. 13:** (a) Social cost vs. carbon emission and (b) car share and toll equivalent for D-specific DMS for a validity cycle of ten days.

As noticed and explained before, pricing and TCS are equivalent when the validity cycle is ten days, the number of days under consideration, because the credit price is constant over the days. For the D-specific variants, pricing, TCS, and TPS are equivalent for social costs and carbon emissions. The flexible consumption of credits and permits compensates their drawbacks in comparison to pricing. For the same mean car share over the horizon of ten days, the equivalent toll with TCS and TPS is lower than pricing by 1 to 2 EUR.



**Fig. 14:** (a) Social cost vs. carbon emission and (b) car share and toll equivalent for OD-specific DMS for a validity cycle of ten days.

For the OD-specific cases, TPS leads to better compromises for high carbon emissions reduction. For the same pollution levels, let us say by dropping to 100 t of carbon emissions, the social cost is  $1.4 \times 10^6$  EUR for pricing (7 EUR) and TCS (between 20% and 25%), and only  $1.3 \times 10^6$  EUR for TPS (30%). Furthermore, to reach this carbon reduction objective, the equivalent toll is 3.4 EUR with TPS, 4-5 EUR with TCS, and 6 EUR with pricing. TPS-OD leads to better compromises and is cheaper for travelers than TCS-OD and OD-specific pricing for a validity cycle of ten days. Here again, increasing the cycle improves the quantity-based DMS.

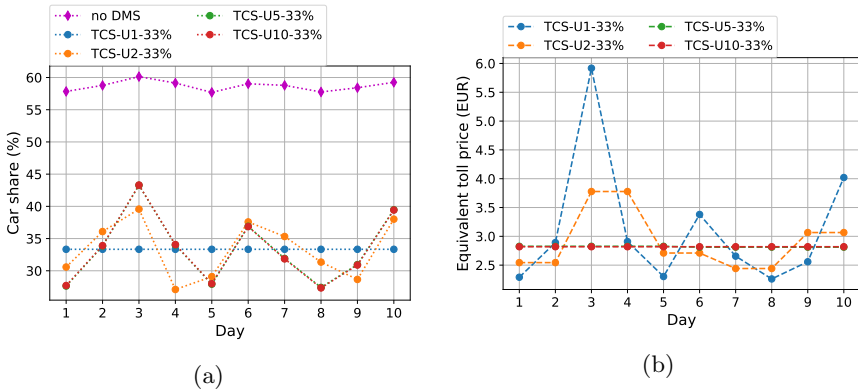
### 5.3 Different cycle lengths

We compare the TCS for different validity cycles: one, two, five, and ten days. We investigate the impacts on mode share, traffic conditions, and credit price.

#### 5.3.1 TCS-Uniform

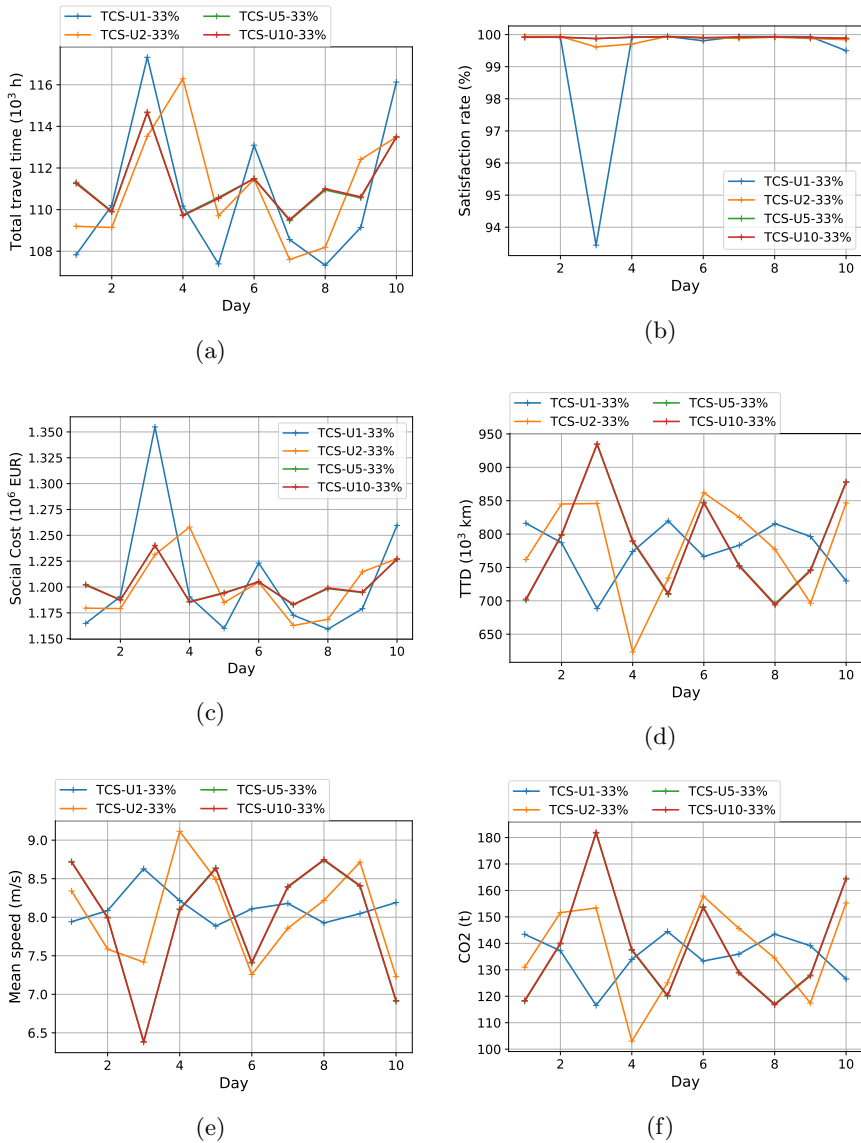
The effects of a longer validity cycle for a uniform TCS for a credit ratio of 33% on car share and equivalent toll price are showed in Fig. 15.





**Fig. 15:** TCS-U with the different cycles: (a) car share and (b) equivalent toll. The credit ratio is 33%.

According to Fig. 15, increasing the validity cycle leads to more variations in the number of cars per day. As expected, with a validity cycle of one day, the car share is 33% every day. For a cycle of two to ten days, it oscillates between 25% and 45%. It is low on days with fewer mandatory car drivers to save them for days with high demand, i.e., lots of mandatory car drivers. This effect is easily observed for two days during the cycle formed by days 3 (high demand) and 4 (low demand). However, a longer validity cycle stabilizes the price. The equivalent toll increases from 2.4 EUR to 6 EUR for a cycle of one day, whereas it is practically constant and equal to 2.8 EUR for a cycle of five days. The Pareto front social cost vs. emission slightly improves as the validity cycle increases. We compute some day-to-day indicators in Fig. 16: total travel time, satisfaction rate, social cost, the total traveled distance by car, mean car speed, and carbon emissions.

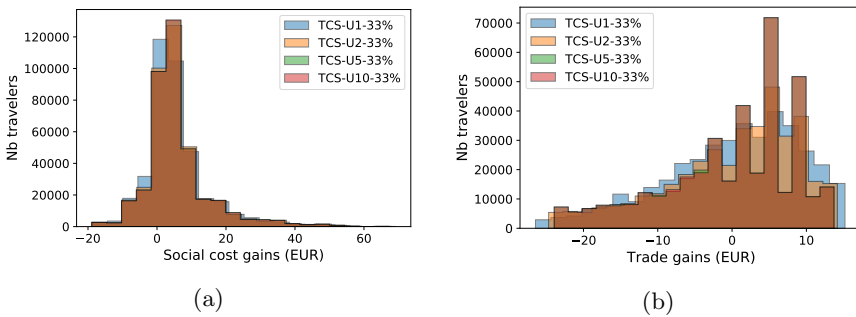


**Fig. 16:** TCS-U with the different cycles: (a) total travel time, (b) satisfaction rate, (c) social cost, (d) total traveled distance, (e) mean car speed, and (f) carbon emissions. The credit ratio is 33%.

The variations of the total travel time because of the cycle length are minimal with respect to their absolute values. The satisfaction rate increases with the validity cycle lengths. On day 3, lots of travelers need to drive their cars. The satisfaction rate is less than 94% for a cycle of one day and almost

100% for five and ten days. The consequence is that the penalty cost, and thus the social costs present a peak on day 3 for a cycle of one day:  $1.35 \times 10^6$  EUR against an average value of  $1.2 \times 10^6$  EUR. The total travel distance, the mean speed, and thus the network carbon emissions present more variations across the days as the cycle length increases. It is because the number of cars per day is not fixed. Only the average over the validity cycle is. The carbon emission varies by about 50% with a cycle length of five or ten days: from less than 120 t on day 1 to a peak around 180 t on day 3 when lots of travelers are mandatory car drivers. It is relatively constant with a cycle of one day. However, the average stays almost the same over the days since the number of cars driving over the horizon of ten days is the same regardless of the validity cycle. Increasing the validity cycle gives more flexibility to travelers. It leads to a better satisfaction rate, almost total satisfaction at the expense of the variability of the traffic conditions. However, the average travel conditions (travel time and emissions) are similar regardless of the validity cycle.

The distribution of the social and trade gains is presented in Fig. 17.

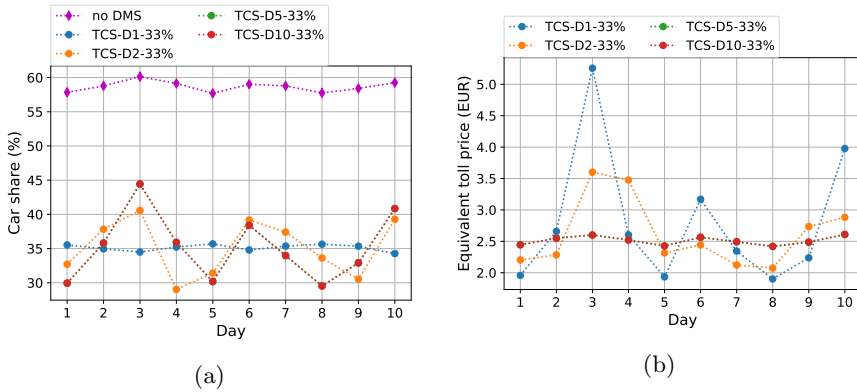


**Fig. 17:** (a) Social and (b) trade gains distribution with TCS-U over the different cycles. The credit ratio is 33%.

The validity cycle has little impact on the distribution of the social and trade gains among the travelers over the two working weeks because the main effects of the validity cycle are the variability of the indicators like prices and total travel times over the days, the average values remain similar. The validity cycle does not affect the distribution of the gains and, in that sense, does not affect the equity of the DMS.

### 5.3.2 TCS-Destinations

The effects on car share and equivalent toll price of a longer validity cycle for a D-specific TCS are showed in Fig. 18 for a macro credit ratio of 33%.

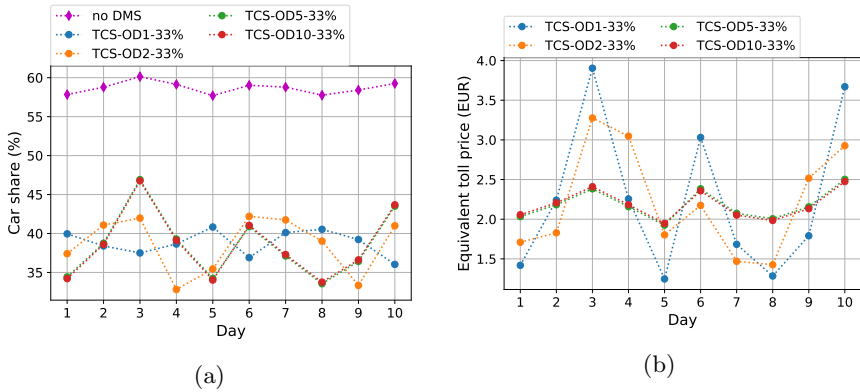


**Fig. 18:** TCS-D with the different cycles: (a) car share and (b) equivalent toll. The macro credit ratio is 33%.

The car share with a validity cycle of one day is around 35%. It is not constant and equal to the macro credit ratio of 33% because as all the users are not facing the same credit charges, the consumption of all the credits does not lead to the number of cars on the network. Especially, the observed car share is higher than the macro credit ratio, as travelers with bad PT alternatives, and thus more prone to drive the car, face lower credit charges. With a higher validity cycle, the car share varies between 30% and 45%. Increasing the validity cycle leads to more variations in the number of cars per day, with up to 15-point changes for cycles of five and ten days, but stabilizes the price. The equivalent toll oscillates between 2 and more than 5 EUR for a validity cycle of one day and stays around 2.5 EUR for five and ten days. The Pareto front social cost/emission slightly improves as the validity cycle increases. Note that for a cycle of ten days, even if the credit price is constant over the days, the equivalent toll is not. As for the similar observation regarding the car shares, since the credit charge is D-specific, the toll equivalent depends on the destination. As different travelers, with different destinations, are driving their cars on different days, the equivalent toll varies because it depends on the car shares (see Eq. (41)). Here again, the conclusions are the same: a large validity cycle stabilizes the price at the expense of the variability of the traffic conditions.

### 5.3.3 TCS-OD pairs

The effects on car share and equivalent toll price of a longer validity cycle for an OD-specific TCS are shown in Fig. 19 for a macro credit ratio of 33%. This credit ratio is chosen as it leads to a similar emission/social cost compromise are the U and D variant for ratios of 33%, see Fig. 8.



**Fig. 19:** TCS-OD with the different cycles: (a) car share and (b) equivalent toll. The macro credit charge is 33%.

The car share varies between 36% and 41% for a cycle of one day. As explained earlier, the car share is not constant and equal to the macro credit ratio for the D-case because all travelers are not facing the same credit charge. Increasing the validity cycle allows more variability for the car share: between 33% and 47%. The equivalent toll oscillates between 1.4 and 4 EUR for a validity cycle of one day and varies only between 2 and 2.5 EUR for five and ten days. The Pareto fronts social cost/emissions and slightly improves as the validity cycle increases. Here again, the conclusions are the same: a large validity cycle stabilizes the price at the expense of the variability of the traffic conditions. The stabilization effect of the validity cycle is also present when investigating the TPS. For conciseness, the results are reported to Appendix C.

## 6 Conclusions

This study provides a deeper look into TCS and TPS performances compared to pricing and LPR when considering congestion dynamics and a time horizon during which the credits/permits can be used. The credits/permits are issued with a validity period of several days. Every day is specific because, even if the demand stays the same, traveling by car brings a different utility for each traveler and day by introducing a penalty term. The framework accounts for captive PT travelers. The developed methodology is applied on a typical morning commute in Lyon. The complexity of the case study (more than 380 000 travelers) permits a realistic benchmarking of the different DMS in a dynamic environment.

The results draw several conclusions with regards to the impacts on the transportation system:

- Pricing and TCS allow for better social cost and carbon emissions compromises than the established LPR or even TPS.

- For similar reductions of social cost and carbon emissions, the equivalent toll faced with TCS is significantly smaller than the pricing toll. Furthermore, TCS is neutral. There is no money flow from the users to the regulator. It is indeed a decentralized policy with a collective bound. A traveler can occasionally drive its car without spending a single euro with a large enough validity cycle, only by stocking its credits. However, a similar money balance at equilibrium can be achieved through toll revenue distribution, but travelers would need to advance the money.
- Charging schemes accounting for the quality of the PT alternative do not reduce even further social costs and carbon emissions for this specific case study. They do not seem robust, as the presence of a day-specific need to drive a car might be more important than the heterogeneity of the PT coverage. However, this observation should be confirmed by studying other case studies and different settings.
- A validity cycle of several days for the credits leads to similar or even better congestion and pollution reduction performances. It stabilizes the credit price and increases the satisfaction rate by providing more flexibility for travelers.

The performances of the congestion pricing are similar to the TCS. However, there are some advantages to the quantity-based DMS. Thanks to the credit cap, the regulator has less uncertainty about the maximum number of vehicles on the network. The TCS defines an overall objective in terms of car usage, while the marketplace defines the credit price. It does not need to find and set the price leading to the desired mode shift. Achieving the same results with congestion pricing requires fine-tuning the tolls to find the targeted equilibrium, which is challenging in practice. The method can easily be transferred to other test cases and scenarios.

To highlight the importance of considering congestion dynamics, we compute the social costs and carbon emissions for LPR with two different departure times distributions in Appendix D. The differences in carbon emissions with the different departure times appear significant. Similar observations can be made when comparing the results with and without trip length heterogeneity by, for example, assuming that all travelers have the same trip length. As the classical Vickrey's bottleneck assumes the same trip length for all travelers, it highlights the limits of the fixed-capacity bottleneck approach. It encourages the use of the trip-based MFD for congestion representation.

Future research directions include allowing travelers to change their departures times under time-varying charging schemes. The modal report is one aspect to address congestion, but fostering travelers driving during the peak to travel off-peak is another valid approach to improve traffic conditions.

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**Authors' contribution.** Conceptualization: LB, LL; Methodology: LB, LL; Formal analysis and investigation: LB; Writing - original draft preparation: LB, LL; Writing - review and editing: LB, LL; Funding acquisition: LL; Resources: LL; Supervision: LL.

**Conflict of interest.** On behalf of all authors, the corresponding author states that there is no conflict of interest.

## Appendix A Notations

**Table A1:** Summary of parameters notations.

Notation	Meaning
$\alpha$	VoT
$\gamma_i$	number of users in group $i$
$\kappa$	credit allocation
$\kappa^{\text{TPS}}$	macro permit allocation ratio
$\kappa_k^{\text{TPS}}$	allocation ratio of permit $k$
$\tau$	macro credit charge
$\tau_k$	credit charge $k$
$\eta$	MCC weight for the QP
$\eta^{\text{TPS}}$	MCC weight for the QP with TCS
$\theta$	logit parameter
$c$	validity cycle length
$C_{i,\text{PT}}$	travel cost of group $i$ by
$N$	number of groups
$N_K$	number of different charges (D or OD)
$r_i$	ratio of travellers in group $i$ having access to a car
$T_{i,\text{PT}}$	travel time per PT of group $i$
$l_i$	trip length of group $i$
$t_i$	departure time of group $i$
$w_k$	quality of the PT alternative

**Table A2:** Summary of variables notations.

Notation	Meaning
$C_{i,\text{car}}$	travel cost of group $i$ by driving its car
$i, j$	index of an group, which represents a group of travelers
$p_k$	price of toll $k$
$p^{\text{TCS}}$	credit price
$p_k^{\text{TPS}}$	price of permit $k$
$n$	accumulation at a given time
$T_i$	travel time per car of group $i$
$TTT$	total travel time
$V$	mean speed in the network at a given time
$\mathbf{x}_d$	shares of groups taking the car
$\tilde{\mathbf{x}}$	concatenation of modal shares for each day and credit price
$\bar{\mathbf{x}}$	concatenation of modal shares for each day and permits prices
$\psi$	modal decisions of the groups
$\Psi$	concatenation of modal decisions for each day

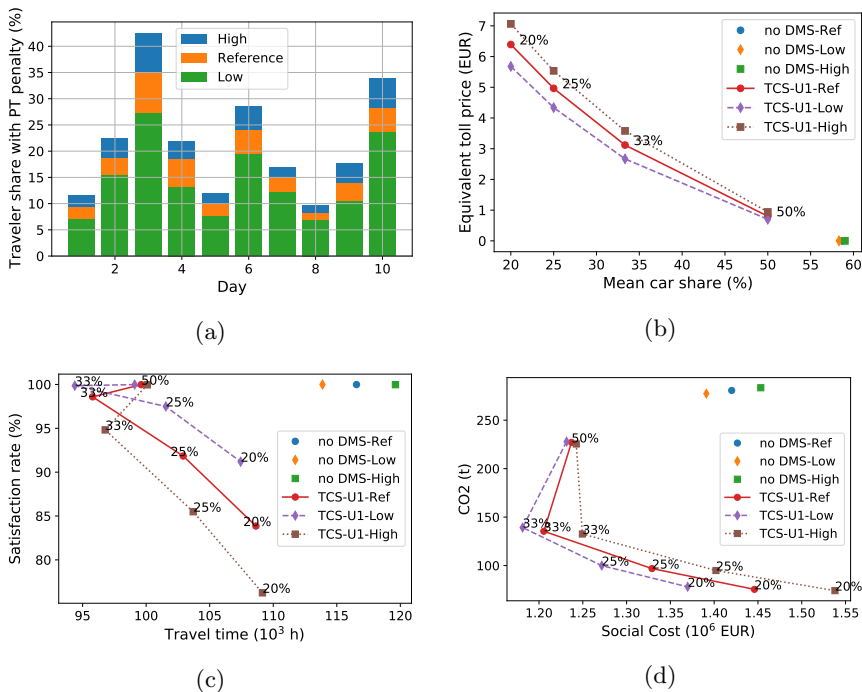


**Table A3:** Other notations.

Notation	Meaning
$\cdot_0$	reference value
$\Delta \cdot$	difference of the value of the variable compared to its reference
$\nabla \cdot$	gradient

## Appendix B Sensitivity of the PT penalty distribution

For actual implementation, the estimate of the distribution of the need to drive a car across the days is prone to uncertainty. To assess the robustness of the TCS, we perform a sensitivity study with regard to the distribution of the PT penalty over the days, we vary it by 20%. We present the penalty distributions, the car shares, equivalent toll prices, total travel times, satisfaction rates, carbon emissions, and social costs in Fig. B1. At first, it is surprising that the

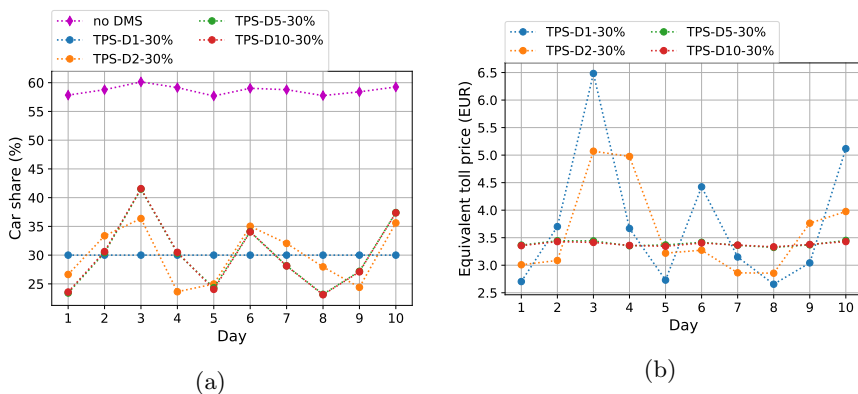


**Fig. B1:** TCS-U with different PT penalty distribution: (a) PT distribution; (b) toll equivalent vs. car share; (c) total travel time vs. satisfaction rate; and (d) social cost vs. carbon emissions.

mean car share without DMS does not change significantly and stays around 60%. The explanation is that travelers who need to drive their cars do so. In the case of a high car demand, the car traffic conditions worsen, and other travelers tend to prefer PT. The toll equivalent increases as more travelers need to drive their cars. Indeed, the credit price increases as the credit demand increases. The satisfaction rate drops with an increasing number of travelers needing to take the car with TCS. For a credit ratio of 25%, the satisfaction rate loses 6 points between the reference and the high demand distribution. The total travel time increases for the no DMS scenario when the number of mandatory car users increases, whereas it stays similar to TCS. The credit cap permits to keep the car shares under control, regardless of the demand, at the expense of the satisfaction rate. As it becomes difficult to satisfy the demand, the cost of driving the car increases (because of the credit price). Some travelers would prefer to face the PT penalty rather than buy the necessary credits. The uniform TCS still reduces both carbon emissions and social costs. Still, the quality of the compromises decreases with a higher traveler share facing PT penalties as the penalty costs increase for a fixed carbon level. With TCS, the regulator needs to sacrifice the satisfaction rate to reach given pollution levels or total travel time.

## Appendix C TPS with different validity cycles

The mean equivalent toll price is defined by Eq. (42). The effect of a longer validity cycle for a D-specific TPS is showed in Fig. C1 for a macro ratio of 30%.

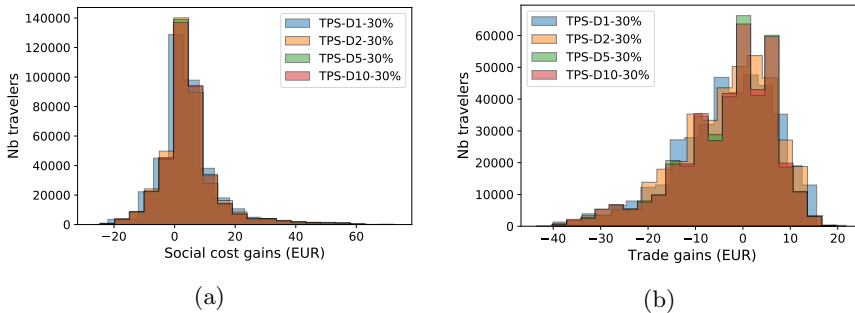


**Fig. C1:** TPS-D with the different cycles: (a) car share and (b) equivalent toll. The macro permit allocation is 30%.

Without DMS, the car share is 30% for a cycle of one day and varies between 24% and 42% with cycles of five and ten days. Increasing the validity

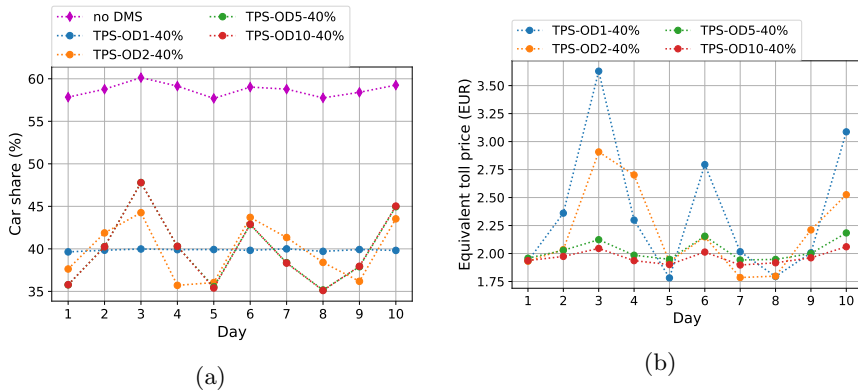
cycle stabilizes the toll equivalent. It more than doubles with a validity cycle of one day. From day 1 to day 3, it rises from 2.7 to 6.5 EUR. For a cycle of five and ten days, the mean equivalent toll is stable at around 3.4 EUR. The Pareto front carbon emissions vs. social costs slightly improve as the validity cycle increases. The conclusion is the same as for TCS: the validity cycle increases the variability of the traffic conditions and stabilizes the toll equivalent. The distribution of the gains is assessed in Fig. C2. For the TPS, the trade gains are defined by:

$$p_{k_i}^{\text{TPS}}(\kappa_{k_i}^{\text{TPS}} - r_i x_i). \quad (\text{C1})$$



**Fig. C2:** (a) Social and (b) trade gains distribution with TPS-D with the different cycles. The macro permit allocation is 30%.

The validity cycle length does not impact the distribution of travelers' social and trade gains. Most travelers benefit from improving their travel conditions as most of them have a social gain from 0 to 20 EUR. Some travelers spend up to 4 EUR per day when it goes to the permits markets, while some earn up to 2 EUR because of the TPS. The bulk of the population spends or earns up to 1 EUR per day on average. The effect of a longer validity cycle for a OD-specific TPS is showed in Fig. C3 for a macro ratio of 40%.

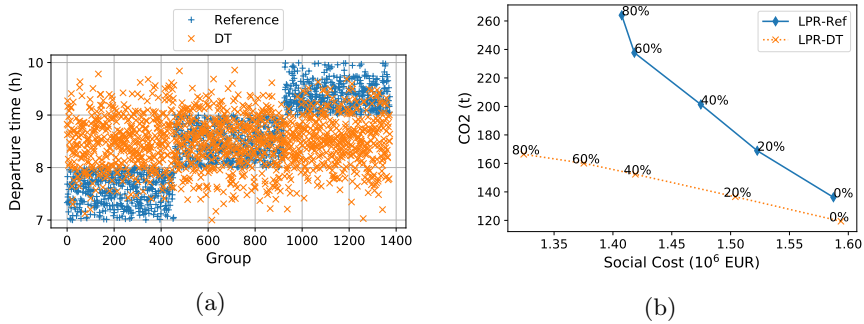


**Fig. C3:** TPS-OD with the different cycles: (a) car share and (b) equivalent toll. The macro permit allocation is 40%.

Without DMS, the car share is 40% for a cycle of one day and varies between 35% and 48% with cycles of five and ten days. The toll equivalent doubles with a validity cycle of one day, between days 5 and 3: from 1.8 EUR to 3.6 EUR. For a cycle of five and ten days, the mean equivalent toll is stable at around 2 EUR. The conclusion is the same as for TCS: the validity cycle increases the variability of the traffic conditions and stabilizes the toll equivalent. Furthermore, the carbon emissions vs. social costs compromises are improved as the validity cycle increases.

## Appendix D Importance of congestion dynamics: example with LPR

Another distribution of the departure times is considered (named DT) to highlight the importance of considering the congestion dynamics. The departure times distribution cannot be considered with static models, such as the BPR function. The alternative departure times are generated following a normal distribution of mean 8:30 and standard deviation 30 minutes. The equilibriums are computed for the LPR with different ratios of exemption. The departure times, social costs, and carbon emissions are presented in Fig. D1. In the alternative network, the departure times are concentrated around 8:30, while they are more spread in the reference case. The traffic conditions are worst since lots of cars enter the network around the same time. It pushes more travelers to ride PT. This effect can be seen by looking at the carbon emissions, which are lower for the alternative scenario DT. For restrictive LPR (exemption of 0%), the carbon emissions and social costs are close. However, for an exemption ratio of 80%, the difference is about 10% for the social costs and 40%



**Fig. D1:** (a) Reference and alternative (DT) departure times and (b) social cost vs. carbon emissions for both departure times distributions.

for carbon emissions. These results underline the relevance of considering congestion dynamics. In this example, it is essential to assess some effects of the DMS, such as carbon emissions.

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**Louis Balzer** is a PhD student at Université Gustave Eiffel. He received his Master’s degree from ENSTA ParisTech and Technical University Vienna in Mechanical Engineering in 2018. His research interest includes demand management and large-scale multimodal traffic dynamics.

**Ludovic Leclercq** is a professor at Université Gustave Eiffel and ENTPE. He received his Ph.D. in 2002 and his habilitation thesis (HDR) in 2009. He is currently the director of the LICIT-ECO7 laboratory. His research interests correspond to multiscale and multimodal dynamic traffic modeling and the related environmental externalities. In 2015, he was awarded the most prestigious research grant in Europe, i.e., an ERC consolidator grant in Social Science and Humanities. In 2019, he was the recipient of the ”Grand Prix de l’Université de Lyon”, a career award for his research in Transportation engineering.