# Fault Detection in Multimode Process based on Hidden Semi-Markov Model and Principal Component Analysis

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***Résumé*** — Cet article présente un processus de détection de défauts pour un processus multimode. Le modèle ~~de~~ semi-Markov caché est utilisé pour détecter le mode de fonctionnement du processus. Chaque mode est estimé par la valeur des statistiques $T^2$ et $Q$ du modèle d'entraînement qui sera utilisé lors de l'étape d'évaluation. Les résultats obtenus sont robustes avec un taux de détection de 99,88%.

***Mots-clefs — analyse en composantes principales, détection des défauts, modèle semi-Markov caché, processus multimode, processus Tennessee Eastman***

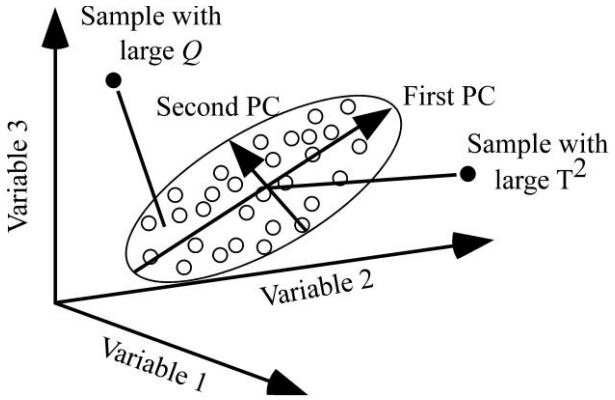
***Abstract—*** This article presents a fault detection process for a multi-mode process. The Hidden semi-Markov model is used to detect the way the process works. Each mode is estimated by the value of the $T^2$ and $Q$ statistics of the training model that will be used in the evaluation step. The results obtained are robust with a detection rate of 98.30%.

***Keywords*** **— fault-detection, HSMM, multimode process, principal component analysis, Tennessee Eastman Process**

## I. INTRODUCTION

Fault detection and process monitoring are very important for estimating abnormal functioning. The objective is to guarantee the reliable operation of a system, a high-quality production in complete safety. With the rise of Industry 4.0, the connected factory makes it possible to have access to a large amount of data (operating history, solicitations, manufacturing recipes, failures, measurements from sensors, etc.). Under these conditions, to detect abnormal process events several multivariate statistical process monitoring (MSPM) approaches were developed [1]. MSPM methods are basically algorithms that can be used for extracting important information from large multivariable data sets such as plant data. For fault detection, the Principal Component Analysis (PCA) model of the process is developed, based on normal operating process data. It is then used to verify the new measurement data. The differences between the new measurement data and their projections to the built model, the residuals, are then subjected to some sort of statistical test to determine if they are significant. Usually the statistic, also called Squared Prediction Error (SPE) or known as $Q$ statistic, and the Hotelling's ($T^2$) statistic are used to represent the variability in the residual subspace and principal component subspace [2]. The $Q$ statistic shows how well a new sample fits into the PCA model built on previous measurement data. It is a measure of the difference (residual) between the sample and its projection onto the principal components retained in the model. As an example, the measurements of three process variables with the result of PCA method are shown in Fig. 1.

Fig. 1. Data projection on two PCs




It should also be noted that numerous industrial processes exhibit multiple operational modes and transitions as a result of a variety of variables, posing a challenge to the Prognostic and Health Management (PHM). Data-driven models such as the Markov model can be adopted for mode identification in the multimode process [3], [4]. This paper combines the Markov model, PCA, $T^2$, and $Q$ statistics to improve the fault detection ability in the multiple mode process.

In this work, we use hidden semi-Markov model for detecting the operation mode or Operation Condition (OC), then PCA is adapted for monitoring the data in each OC. The process starts with data training. First, give the initial parameters of HSMM. Then training the data by the forward-

backward algorithm [5] for HSMM and re-evaluating the parameters. For each mode, we analyse the possible correlations between the different data by the PCA method. We calculate $T^2$, the $Q$-statistic, the adaptive parameters of $T^2$ and the $Q$-statistic. The following work consists of detecting potential defects in the test data. Under these conditions, these statistics can be used for fault detection in multimode processes. The case study focuses on the Tennessee Eastman (TE) process simulation.

The remainder of this paper is structured as follows. In Section II, we describe the preliminaries of HSMM theory, PCA for failure detection, and the parameters of adaptive $T^2$ and $Q$ statistics. The next section outlines the proposed method. Section IV describes the TEP process, multi-operational conditions of TEP, and illustrations of TEP for failure detection. The obtained results are discussed in Section V, and some findings and concluding remarks are drawn in Section VI.

## II. PRELIMINARIES

### A. Hidden Semi-Markov Model

Assume a discrete-time semi-Markov process with a set of (hidden) states $S = \{1,...,M\}$ . The state sequence $Q = \{s_1,...,s_T\}$ is denoted by $s_{1:T}$ , where $s_t \in S$ is the state at time *t*. And the observation sequence $O = \{o_1,...,o_T\}$ by $o_{1:T}$ where $o_t \in V$ is the observation at time *t* with $V = \{v_1,...,v_k\}$ is the set of observable values. This semi-Markov chain is defined by the following parameters:

- Initial probabilities $\pi = \{\pi_m\}$ where $\pi_m = P(s_1 = m)$ with $\sum_m \pi_m = 1$;
- Transition probabilities $A = \{a_{mn}\}$ where $a_{mn} = P(s_{t+1} = n \mid s_t = m)$ with $\sum_{m \neq n} a_{mn} = 1$ and $a_{mm} = 0$;
- Observation or emission probabilities $B = \{b_m(v_k)\}$ where $b_m(v_k) = P(o_t = v_k \mid q_t = s_m)$;
- Duration probabilities $p = \{p_{md}\}$ where $p_{md} = P(s_{t+1:t+d} = m \mid s_{t+1} = m)$ where $d \in (1,...,D)$ and $D$ is maximum duration in state *m*.

The model HSMM is defined as $\lambda_{HSMM} = (A, B, p, \pi)$ . For details, the entire set of model parameters can be estimated using the forward-backward algorithm described in [6].

### B. Principal Component Analysis for Fault Detection

PCA reduces dimensionality by iteratively extracting uncorrelated linear combinations of the original variables, known as Principal Components (PC). They are obtained by using linear combinations of these variables and are orthonormal. Before applying PCA, the process data should be normalized to zero means and unit variance by subtracting the training data's mean and dividing by the training data's standard deviation. Then the normalized process data matrix $X \in R^{n \times s}$ can be decomposed as:

$$\mathbf{X} = \mathbf{TP}^{\mathbf{T}} + \mathbf{E} \tag{1}$$

where: *n* represents the number of samples, *s* represents the number of variables, $\mathbf{T} \in \mathrm{P}^{n \times \rho}$ is score matrix, $\mathbf{P} \in \mathbf{R}^{s \times \rho}$ refers to the loading matrix, and $\mathbf{E} \in \mathbf{R}^{n \times s}$ is residuals matrix.

When PCA is utilized in process monitoring, the retained PCs are frequently summarized using the so-called the Hotelling $T^2$ statistic, and the residuals are summarized using the Q statistic or squared predictive error (SPE). The detection is done by comparing the expected behavior to that provided by the PCA model. Given The PCA model estimated **y,** the $T^2$ and Q statistics can be computed as follows [4]:

$$T^2 = \mathbf{yP}(\mathbf{\Lambda})^{-1}\mathbf{P}^T\mathbf{y}^T \tag{2}$$

$$Q = \mathbf{y}(\mathbf{I} - \mathbf{PP}^T)(\mathbf{I} - \mathbf{PP}^T)\mathbf{y}^T \tag{3}$$

where $\mathbf{I}$ is the identity matrix; $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, ..., \lambda_\rho)$ is the estimated covariance matrix of principal component scores.

The detectability of fault is determined by the thresholds $Q_\alpha$ and $T_\alpha^2$ for the $Q$ and $T^2$ statistics, respectively [7]. These thresholds are derived by applying the appropriate distribution law at a specified confidence level (1- α).

$$Q_\alpha = \theta_1 \left( 1 + \frac{c_\alpha h_0 \sqrt{2\theta_2}}{\theta_1} + \frac{h_0 \theta_2 (h_0 - 1)}{\theta_1^2} \right)^{\frac{1}{h_0}}, \tag{4}$$

$$\theta_i = \sum_{j=r+1}^{\rho} \mathbf{\Lambda}_j^i, \quad i = 1,2,3, \quad and \quad h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}. \tag{5}$$

where $c_\alpha$ is the critical value of the normal distribution for a level of confidence of (1- $\alpha$). And r is number of PCs retain.

The fixed thresholds $T_\alpha^2$ is in formula (6).

$$T_\alpha^2 = \frac{(g^2 - 1)l}{g(g - l)} F_\alpha(l, g - l) \tag{6}$$

where $F_\alpha(l, g-l)$ is the critical value at a significance level $\alpha$ with $l$ and $g-l$ degrees of freedom of the Fisher-Snedecor distribution. Variable *l*, g and $\alpha$ are the number of principal components, number of samples and acceptable false alarm rate, respectively.

### C. Adaptive parameter for Q and $T^2$ statistic

Exponentially Weighted Moving Average (EWMA) was widely used as a statistical method for process monitoring. Suppose that we observe sequence statistics obtained from PCA projections of monitored process measurements $Q = [q_1, q_2, ..., q_n]$ and $T^2 = [t_1, t_2, ..., t_n]$, and suppose the $Q$ statistic is evaluated using a limited window length EWMA control chart, which acts as a backward exponential filter. The filtered $j^{th}$ sample is given by:

$$q_j^{'} = \frac{\sum_{i=1}^{w_q} c_q^i q_{j-w_q+i}}{\sum_{i=1}^{w_q} c_q^i} \quad (7)$$

The parameter $c_q$ is a weighting factor greater than 1, and it determines the rate at which older samples enter into the calculation of $q_j^{'}$. $w_q$ represents the filter window length for $Q$ fault detection index; ie, it is the number of samples used by the filter. By this approach, the $j^{th}$ sample is considered faulty if $q_j^{'} > Q_\alpha$.

The adaptive threshold for the $Q$ statistic [7] at the $j^{th}$ sample is:

$$Q_j^{ad} = \max\left\{\frac{1}{c_q^{w_q}}\left(Q_\alpha \sum_{i=1}^{w_q} c_q^i - \sum_{i=1}^{w_q-1} c_q^i q_{j-w_q+i}\right), 0.2Q_\alpha\right\} \quad (8)$$

Similarly, the adaptive threshold for the $T^2$ statistic [7] at $j^{th}$ sample is:

$$T_j^{2,ad} = \max\left\{\frac{1}{c_t^{w_t}}\left(T_\alpha^2 \sum_{i=1}^{w_t} c_t^i - \sum_{i=1}^{w_t-1} c_t^i t_{j-w_t+i}\right), 0.2T_\alpha^2\right\} \quad (9)$$

where $w_t$ and $c_t$ are the adaptation window length and weighting factor for the $T^2$ statistic, respectively.

## III. PROPOSED METHODOLOGY

The proposed methodology can be seen in Figure 2. The fault detection process begins offline, with the training of normal or healthy data. It can be continued with testing data that contains faults.

The offline model training steps are as follows:

*1) Selection of training data:* healthy data from the system. This data can have many variables.

*2) Clustering:* this divides the data into several segments. Set the number of segments equal to the number of states (M). The clustering method used is Agglomerative.

*3) Setting the initial value of HSMM parameters:* that is, give initial values for *A, B, p,* and $\pi$. *A* vector of $\pi$ and matrix of *A* can be set randomly to satisfy $\sum_m \pi_m = 1$, $\sum_{m \neq n} a_{mn} = 1$, and $a_{mm} = 0$. The results of clustering can be used to assign initial values to matrix *B*. Assume that data in each operation mode obeys unimodal Gaussian distribution, $x_i \square N(\mu_i, \Sigma_i)$, $i = 1, 2, ..., M$. The maximum duration in each state ($D_i$) can be taken from the results of clustering. $D_{max}$ is the longest sojourn time of all states. So, *p* matrix can be set as random vectors satisfying the condition $\sum_{t=1}^{D_{\max}} p_i(t) = 1$.

*4) Forward-backward HSMM training:* it is training healthy data to obtain updated parameter values. The forward variables for HSMM are defined by:

$t = 1, ..., T$:

$$\alpha_{t|t-1}(m,d) = S_{t-1}(m)\, p_m(d) + b_m(o_{t-1|t-2})\alpha_{t-1|t-2}(m,d+1) \quad (10)$$

with the initial value:

$$\alpha_{1|0}(m,d) = \pi_m p_m(d) \quad (11)$$

For convenience in the forward recursion, variables $\varepsilon$ and $\delta$ are defined with respect to the conditional probability of a state ending at *t* given $o_1^t$ and with respect to the conditional probability of a state beginning at t+1 given $o_1^t$.

$$\varepsilon_t(m) = \alpha_{t|t+1}(m,1) b_m^*(o_t) \quad (12)$$

$$\delta_t(m) = \sum\nolimits_n \varepsilon_t(n)\, a_{nm} \quad (13)$$

The ratio of filtered probability by:

$$b_m^*(o_t) = \frac{a_{t|t}(m,d)}{a_{t|t-1}(m,d)} \quad (14)$$

The backward variables is:

$t = T, T-1, ..., 1$:

$$\beta_t(m,d) = \begin{cases} \delta_{t+1}^*(m) b_m^*(o_t), & d = 1 \\ \beta_{t+1}(m, d-1) b_m^*(o_t), & d > 1 \end{cases} \quad (15)$$

For convenience in the backward recursion, denoted variables $\varepsilon^*$ and $\delta^*$ by:

$$\varepsilon_t^* = \sum\nolimits_n p_m(d) \beta_t(m,d) \quad (16)$$

$$\delta_t^* = \sum\nolimits_n a_{mn} \varepsilon_t^*(n) \quad (17)$$

The smoothed probability that a transition from state m to state *n* at *t* occurs, it is defined by:

$$\theta_t(m,n) = \varepsilon_{t-1}(m) a_{mn} \varepsilon_t^*(n) \quad (18)$$

And the probability that state m is entered at *t* and lasts for *d* time units is:

$$\phi_t(m,d) = \delta_{t-1}(m)\, p_m(d) \beta_t(m,d) \quad (19)$$

The marginal probability distribution can be obtain as:

$$\gamma_{t-1|T}(m) = \gamma_{t|T}(m) + \varepsilon_{t-1}(m) \vartheta_t^*(m) - \vartheta_{t-1}(m) \varepsilon_t^*(m) \quad (20)$$

The re-estimation of the model parameters are given below:

$$\hat{a}_{mn} = \frac{\sum_t \theta_{t|T}(m,n)}{\sum_n \sum_t \theta_{t|T}(m,n)} \quad (21)$$

$$\hat{b}_m(v_k) = \frac{\sum_t \gamma_{t|T}(m) I(o_t = v_k)}{\sum_t \gamma_{t|T}(m)} \quad (22)$$

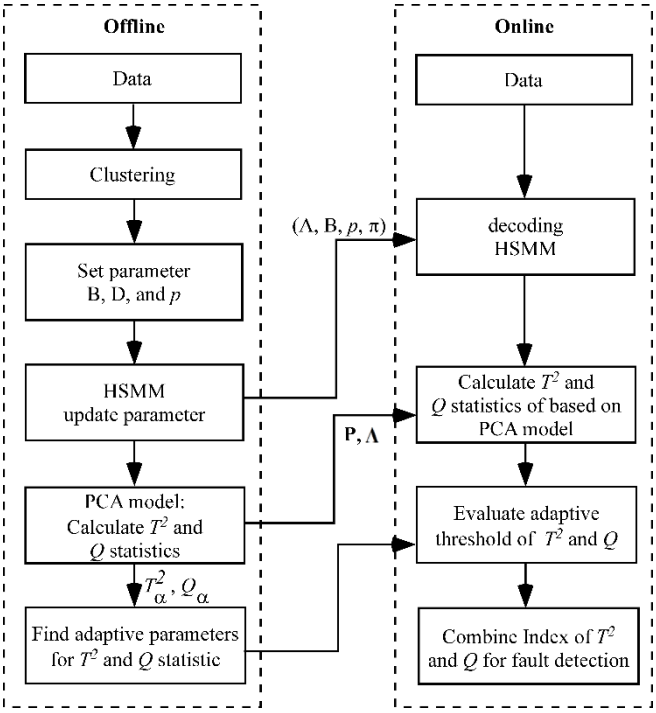
$$\hat{p}_m(d) = \frac{\sum_t \phi_{t|T}(m,d)}{\sum_d \sum_t \phi_{t|T}(m,d)} \tag{23}$$

$$\hat{\pi}_m = \frac{\gamma_{1|T}(m)}{\sum_m \gamma_{1|T}(n)} \tag{24}$$

*5) Calculation of $T^2$ and Q statistics:* it calculates the statistical values of $T^2$, $Q$, $T_\alpha^2$, and $Q_\alpha$ of the training data using the formulas (2), (3), (4), and (6), respectively. We calculate it for each state. The eigen vectors and eigen values from this stage will be used for the calculation of $T^2$ and $Q$ in the online stage with the new test data.

*6) Finding adaptive parameters for $T^2$ and Q:* Adaptation window length that ensures adequate sample averaging and an adaptation weighting factor that produces a false alarm rate of zero percent in the training set. These parameters will be used to evaluate the test data threshold in the online stage.

Fig. 2. Flowchart of proposed method




The steps for test data by online monitoring are:

*1) Input test data:* real-time test data, which may contain healthy and failed data.

*2) Decoding HSMM: g*iven the observation $o_1^T$ , we can estimate the hidden states that start at time $t$ with the Maximum a Posteriori (MAP) using equation:

$$\begin{aligned}(q_t, \theta_t) &\overset{def}{=} \arg\max_{(m,d)} P(s_m \text{ start at } t, \tau_t = d \mid o_1^T) \\ &= \arg\max_{(m,d)} \vartheta_{t|T}(m,d)\end{aligned} \tag{25}$$

*3) Calculation of $T^2$ and Q statistics:* this calculation uses the eigen values and eigenvectors from the training process. This applies to each state's estimate.

*4) Evaluate adaptive threshold of $T^2$ and Q:* it evaluates the $T^2$ and $Q$ of the test data using $T_\alpha^2$, *and* $Q_\alpha$, $T^{2,ad}$, *and* $Q^{ad}$ from the training data. This process is carried out in each estimation state.

*5) Combine index of $T^2$ and Q for fault detection:* combining the two statistics simplifies the fault detection process.

$$CI_{new} = z*\left(\frac{T^2}{T^{2,ad}}\right) + (1-z)*\left(\frac{Q}{Q^{ad}}\right) \tag{26}$$

where $z$ is 0.5 to balance SPE and T. The process is diagnosed as fault, if $CI_{new} > 1$ or $CI_{new} \leq 0$.

## IV. Tennessee Eastman Process

In this section, the proposed methodology is applied to the Tennessee Eastman Process (TEP).

### A. Process description

The TEP was created by the Eastman Chemical Company to provide a realistic industrial process for evaluating process control and monitoring methods [8]. The process consists of five major units: a reactor, condenser, compressor, separator, and stripper; and it contains eight components: A, B, C, D, E, F, G, and H.

The gaseous reactants A, C, D, and E and the inert B are fed to the reactor where the liquid products G and H are formed. The species F is a by-product of the reactions. The reactions in the reactor are:

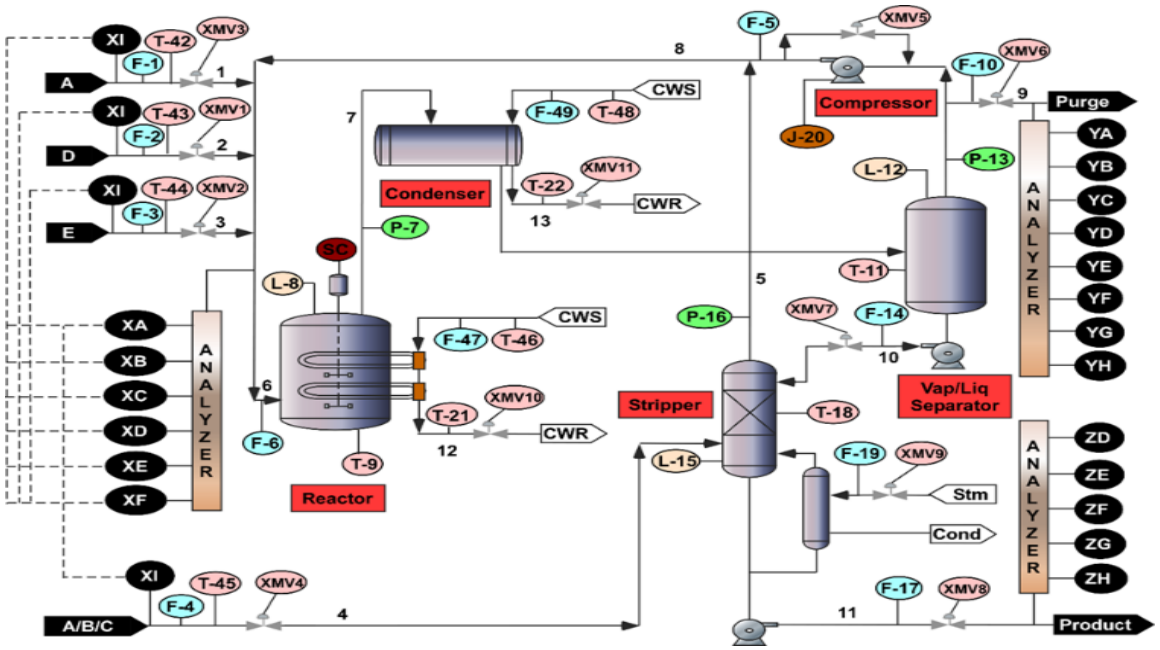
$$\begin{aligned} &A(g) + C(g) + D(g) \rightarrow G(liq), && Product\,1, \\ &A(g) + C(g) + E(g) \rightarrow H(liq), && Product\,2, \\ &A(g) + E(g) \rightarrow F(liq), && Byproduct, \\ &3D(g) \rightarrow 2F(liq), && Byproduct, \end{aligned} \tag{27}$$

The reactions are irreversible, exothermic, and approximately first-order with respect to the reactant concentrations. The reaction rates are Arrhenius functions of temperature where the reaction for G has a higher activation energy than the reaction for H, resulting in a higher sensitivity to temperature.

The reactor product stream is cooled through a condenser and then fed to a vapor-liquid separator. The vapor exiting the separator is recycled to the reactor feed through a compressor. A portion of the recycle stream is purged to keep the inert and byproduct from accumulating in the process. The condensed components from the separator (Stream 10) is pumped to a stripper. Stream 4 is used to strip the remaining reactants from Stream 10, which are combined with the recycle stream via Stream 5. The products G and H exiting the base of the stripper are sent to a downstream process which is not included in the diagram (Fig3).

The revision of the Tennessee Eastman Process provided by [9]. The piping and instrumentation diagram (P&ID) of the process with extended measurements is shown in Fig 3.

Fig. 3. A process flowsheet for the Tennessee Eastman Process (TEP)




## B. *Multi operation condition of TEP*

Generate training data using Tennessee Eastman (TE) process simulation. We utilized the model designed to operate in "Mode 1" conditions. The conventional TE process is a unimode process. This procedure introduces three operation conditions (OC-1, OC-2, and OC-3), which are shown in Table I, to test the multi-mode approaches. While the other setting parameter values are set the same for the three OCs.

TABLE I. FOUR PROCESS OPERATION MODES IN TE PROCESS

| Setpoint Label | OC-1 | OC- 2 | OC- 3 |
|---|---|---|---|
| Production | 22.89 | 22.89 | 18,40 |
| Mol % G | 50 | 60 | 50 |
| Separator level | 40 | 50 | 50 |

The simulated model has 12 manipulated input variables (XMV (1) to XMV (12)) and 73 measured output variables (XMEAS (1) to XMEAS (73)). The considered data consists of 31 variables listed in (Table II).

TABLE II. DESCRIPTION OF THE SELECTED MONITORING DATA VARIABLES

| Variable | Description |
|---|---|
| XMV (1) | D feed flow valve (stream 2) |
| XMV (2) | E feed flow valve (stream 3) |
| XMV (3) | A feed flow valve (stream 1) |
| XMV (4) | Total feed flow valve (stream 4) |
| XMV (6) | Purge valve (stream 9) |
| XMV (7) | Separator pot liquid flow valve (stream 10) |
| XMV (8) | Stripper liquid product flow valve (stream 11) |
| XMV (10) | Reactor cooling water flow |
| XMV (11) | Condenser cooling water flow |
| XMEAS (1) | A feed (stream 1) |
| XMEAS (2) | D feed (stream 2) |
| XMEAS (3) | E feed (stream 3) |
| XMEAS (4) | Total feed (stream 4) |
| XMEAS (5) | Recycle flow (stream 8) |
| XMEAS (6) | Reactor feed rate (stream 6) |
| XMEAS (7) | Reactor pressure |
| XMEAS (8) | Reactor level |
| XMEAS (9) | Reactor temperature |
| XMEAS (10) | Purge rate (stream 9) |
| XMEAS (11) | Product separator temperature |
| XMEAS (12) | Product separator level |
| XMEAS (13) | Product separator pressure |
| XMEAS (14) | Product separator under flow (stream 10) |
| XMEAS (15) | Stripper level |
| XMEAS (16) | Stripper pressure |
| XMEAS (17) | Stripper underflow (stream 11) |
| XMEAS (18) | Stripper temperature |
| XMEAS (19) | Stripper steam flow |
| XMEAS (20) | Compressor work |
| XMEAS (21) | Reactor cooling water outlet Temperature |
| XMEAS (22) | Separator cooling water outlet temperature |

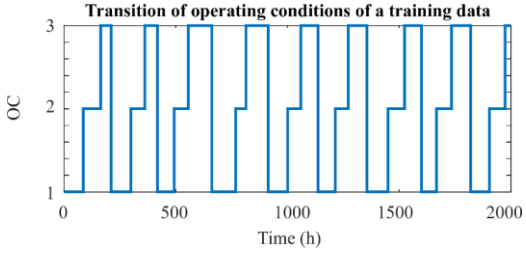
The mode shift probability matrix for three OCs is set as:

$A=$

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 0 | 0 | 1 |
| 0.75 | 0.25 | 0 |

The duration for each OC follows Gaussian distribution $N\left(\mu_i, \sigma_i^2\right)$, where $\mu_i = [80, 60, 70]$, $\sigma_i = [10, 10, 10]$, and $D_{\max} = 100$. The training set consists of one simulation that runs for 2000 hours with normal (healthy) process operation. Data were collected with a sampling interval of 0.5 hours, resulting in 4000 observations. Figure 4 illustrates the transition of OC in respect to time.

Fig. 4. Transition of operating conditions of a training data



### C. TE Process fault detection illustration

The test set corresponds to a healthy process operation that runs for 300 hours (the sampling interval is 0.5 hours). At least 600 samples were observed with the operation condition displacement according to the probability shifting mode. The test set is used to evaluate the obtained PCA model and the constructed monitoring scheme. It is usually used to calculate false alarms contributed by the $Q$ and $T^2$ statistics.

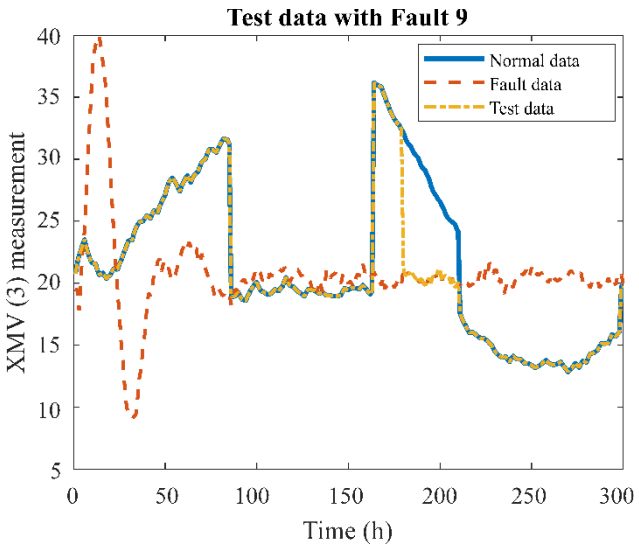
Twenty - two fault sets are generated by different runs and different times of occurrence of fault, each run corresponds to one of the process faults described in Table III.

TABLE III. FAULT DESCRIPTIONS FOR SIMULATIONS

| No | Description | Type – disturbance flags | Fault occurs time |
|---|---|---|---|
| Fault 1 | Feed ratio of A/C, composition constant of B (stream 4) | Step – IDV (1) | 100 h (in mode 2) |
| Fault 2 | Composition of B, ratio constant of A/C (stream 4) | Step – IDV (2) | 100 h (in mode 2) |
| Fault 3 | Feed temperature of D (stream 2) | Step – IDV (3) | 100 h (in mode 2) |
| Fault 4 | Inlet temperature of reactor cooling water | Step – IDV (4) | 100 h (in mode 2) |
| Fault 5 | Inlet temperature of condenser cooling water | Step – IDV (5) | 100 h (in mode 2) |
| Fault 6 | Header pressure loss of C—reduced availability (stream 4) | Step – IDV (7) | 100 h (in mode 2) |
| Fault 7 | Feed composite of A, B, and C on (stream 4) | Random – IDV (8) | 180 h (in mode 3) |
| Fault 8 | Feed temperature of D (stream 2) | Random – IDV (9) | 180 h (in mode 3) |
| Fault 9 | Feed temperature of C (stream 4) | Random - IDV (10) | 180 h (in mode 3) |
| Fault 10 | Inlet temperature of reactor cooling water | Random - IDV (11) | 180 h (in mode 3) |
| Fault 11 | Inlet temperature of condenser cooling water | Random - IDV (12) | 180 h (in mode 3) |
| Fault 12 | Reaction kinetics | Slow drift - IDV (13) | 180 h (in mode 3) |
| Fault 13 | Valve of reactor cooling water | Stiction - IDV (14) | 180 h (in mode 3) |
| Fault 14 | Valve of condenser cooling water | Stiction - IDV (15) | 250 h (in mode 1) |
| Fault 15 | (unknown); deviations of heat transfer within stripper (heat exchanger) | Random - DV (16) | 250 h (in mode 1) |
| Fault 16 | (unknown); deviations of heat transfer within reactor | Random - IDV (17) | 250 h (in mode 1) |
| Fault 17 | (unknown); deviations of heat transfer within condenser | Random - IDV (18) | 250 h (in mode 1) |
| Fault 18 | (unknown); re-cycle valve of compressor, underflow separator (stream 10), underflow stripper (stream 11) and steam valve stripper | Stiction - IDV (19) | 250 h (in mode 1) |
| Fault 19 | (unknown) | Random - IDV (20) | 250 h (in mode 1) |
| Fault 20 | Mode shifts from mode 1 to mode 3 | Step | 250 h (in mode 1) |
| Fault 21 | Mode shifts from mode 1 to mode 3 | Step | 50 h (in mode 1) |
| Fault 22 | Mode shifts from mode 2 to mode 1 | Step | 100 h (in mode 2) |

Figure 5 illustrates test data with fault type 9. The test data is the orange dotted line. This data is based on normal data (solid blue line), then assigned a type 9 fault occurs time at 180 in OC-3.

Fig. 5. Transition of operating conditions of a test data: Fault 9

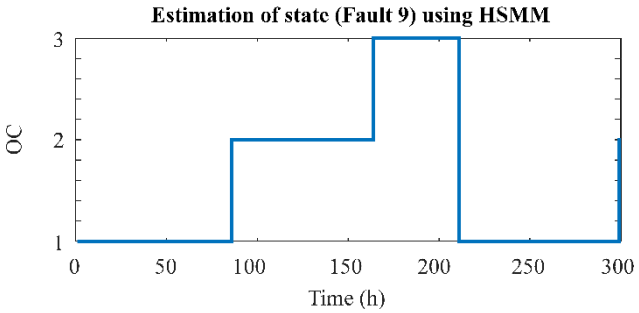



## V. RESULTS AND DISCUSSION

In this section, the proposed methodology is applied to solve the fault detection issue of the Tennessee Eastman Process (TEP). This simulation is named HSMM&PCA because the first part is estimating the operation condition called “state” in HSMM through clustering, parameter initialization, HSMM training and decoding stages. The second process is data modeling using PCA, including calculations of $T^2$, $Q$, $T_\alpha^2$, $Q_\alpha$, $T^{2,ad}$ and $Q^{ad}$. At the end of online monitoring, a combination index called CInew can be used to find faults.

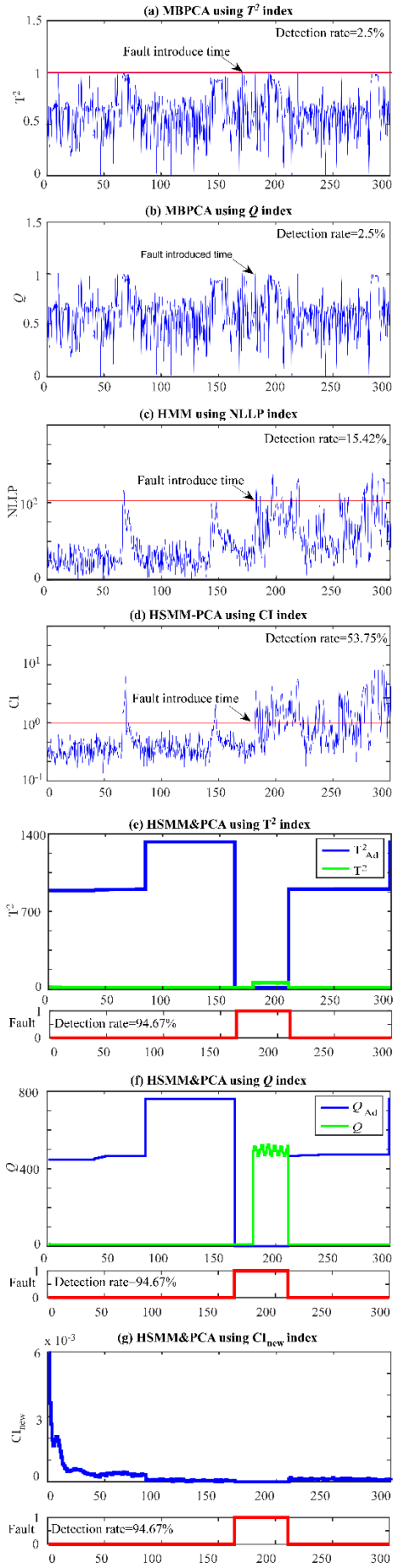
Figure 6 illustrates the result of estimating the state using HSMM for Fault 9 test data. HSMM estimates the state very well for this data.

Fig. 6. State estimation of test data (Fault 9) using HSMM




The TE process data test refers to the data test scenario from Lou and Wang [4]. This study proposes the HSMM-PCA method and compares it with the Mixture Bayesian PCA (MBPCA) [9], and the HMM [3] method. So, we can compare the simulation results with the results in the article. Fault determination using various indexes. HSMM-PCA uses the cumulative index (CI), MBPCA considers $T^2$ and $Q$ values, and HMM uses the NLLP index. We show the results of detecting Fault 9 data from these methods in Figure 7.(a) to Figure 7.(g). At 180 h, the faults occurred.

Fig. 7. Fault detection of Fault 9 data test




The figure shows that the MBPCA and HMM can detect a few faults. The HSMM-PCA method can detect half faults. Meanwhile, the HSMM&PCA method that we propose can detect large corpus of faults. Table IV listed the detection rates of the four methods. If only using the $T^2$, $Q$, and $CI_{new}$ indexes, the maximum fault detection results are for Fault 1-Fault 6, Fault 14-Fault 22 data. Meanwhile, for Fault 7-Fault 13, there are still detection errors. However, for these datasets, the $CI_{new}$ index can increase the detection rate by 5%. Overall, we can conclude that the HSMM&PCA method is the best, with an average detection rate of 98.30%.

TABLE IV. DETECTION RATES (%) OF SIMULATIONS IN TE PROCESS

| Test data | MBPCA | | HMM | HSMM -PCA | HSMM&PCA | | |
|---|---|---|---|---|---|---|---|
| | $T^2$ | $Q$ | NLLP | CI | $T^2$ | $Q$ | $CI_{new}$ |
| Fault 1 | 2.00 | 99.50 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| Fault 2 | 20.25 | 65.75 | 97.25 | 98.50 | 100.00 | 100.00 | 100.00 |
| Fault 3 | 2.00 | 1.50 | 0.75 | 3.50 | 100.00 | 100.00 | 100.00 |
| Fault 4 | 2.25 | 1.50 | 76.25 | 99.75 | 100.00 | 100.00 | 100.00 |
| Fault 5 | 3.00 | 1.50 | 1.50 | 3.75 | 100.00 | 100.00 | 100.00 |
| Fault 6 | 3.00 | 95.75 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| Fault 7 | 15.00 | 59.80 | 96.25 | 98.33 | 94.67 | 94.67 | 94.67 |
| Fault 8 | 3.33 | 2.50 | 2.92 | 4.58 | 94.67 | 94.67 | 94.67 |
| Fault 9 | 2.50 | 2.50 | 15.42 | 53.75 | 94.67 | 94.67 | 94.67 |
| Fault 10 | 2.08 | 3.75 | 80.83 | 97.50 | 94.67 | 94.67 | 94.67 |
| Fault 11 | 2.50 | 2.50 | 2.50 | 12.08 | 94.67 | 94.67 | 94.67 |
| Fault 12 | 72.80 | 91.25 | 96.25 | 97.92 | 94.67 | 94.67 | 94.67 |
| Fault 13 | 2.08 | 2.50 | 78.33 | 95.42 | 94.67 | 94.67 | 94.67 |
| Fault 14 | 2.00 | 6.00 | 1.00 | 9.00 | 100.00 | 100.00 | 100.00 |
| Fault 15 | 2.00 | 6.00 | 1.00 | 9.00 | 100.00 | 100.00 | 100.00 |
| Fault 16 | 3.00 | 9.00 | 71.00 | 89.00 | 100.00 | 100.00 | 100.00 |
| Fault 17 | 6.00 | 10.00 | 57.00 | 77.00 | 100.00 | 100.00 | 100.00 |
| Fault 18 | 1.00 | 6.00 | 17.00 | 83.00 | 100.00 | 100.00 | 100.00 |
| Fault 19 | 29.00 | 70.00 | 97.00 | 97.00 | 100.00 | 100.00 | 100.00 |
| Fault 20 | 5.00 | 5.00 | 2.00 | 92.00 | 100.00 | 100.00 | 100.00 |
| Fault 21 | 2.28 | 12.60 | 1.00 | 62.80 | 100.00 | 100.00 | 100.00 |
| Fault 22 | 2.25 | 7.75 | 6.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| **Mean** | **8.42** | **25.58** | **45.51** | **67.45** | **98.30** | **98.30** | **98.30** |

## VI. CONCLUSIONS

In this study, the HSMM method succeeded in detecting the mode of a system that applies multiple operating conditions. And data monitoring in each OC is done by the PCA model. The use of adaptive parameters $T^2$, $Q$, and a combination of both indexes can increase the fault detection rate. This is verified in the revised model of the TE Process.

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