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# Simplified model for capturing helical vortex interactions with applications to asymmetric rotor wakes

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## 1 Introduction

The wakes of rotors, such as those found on helicopters, wind turbines, and propellers, are characterized by helical vortices. The interaction between these vortices can lead to various types of instabilities, including displacement instabilities, the focus of the current study. Displacement instabilities occur when the vortex core is shifted from its baseline position, causing adjacent vortices to group together (pairing). When the entire helix is shifted, global pairing occurs. This type of instability can be caused by asymmetry in the rotor generating the helical vortex system, as demonstrated for a two-vortex system by Quaranta et al. [1]. Most rotors in industrial applications (and their laboratory-scale counterparts) have three or more blades, substantially increasing the complexity of the interlaced helical vortex system. Furthermore, global pairing occurs frequently in laboratory experiments (e.g., [2]), likely due to small asymmetries that are unavoidable in scaled-down experiments. This phenomenon has not been thoroughly investigated due to this increased complexity.

The current study proposes a simplified model to develop a more complete understanding of the effect of rotor asymmetry on the development of the helical vortices in the wake. The model is based on the behavior of an infinitely repeating periodic strip of point vortices, which represents a cross-section of an infinite helical vortex system. Though highly simplified, a previous study by Delbende et al. [3] for a two-vortex system shows that point vortices can be used to represent helical vortices for low values of helical pitch, as is usually the case for rotor wakes. In addition, comparisons with experiments show that the point vortex model captures rotor wake behavior remarkably well. With this simplified model, the broad parameter space required to fully characterize rotor asymmetry (i.e., variations in length, axial position, and pitch of all blades) can be explored in detail.

## 2 Methods

### 2.1 Point vortex model

The representation of helical vortices as point vortices is based on the assumption that, for small values of helical pitch relative to the radius of the helices, the effects of

curvature and torsion are small [3]. Under these conditions, the interlaced helices can be “unrolled” and represented as an array of straight parallel vortices subject to periodic boundary conditions dictated by the helical geometry. These straight vortices can in turn be represented by periodically-repeating point vortices in two dimensions, whose non-linear dynamics can be calculated easily using the induced velocity of neighboring vortices. For a strip of  $N$  point vortices repeated periodically along the  $x$ -axis, where  $N$  represents the number of helices in the original system, the following formula governs the displacement of the basic strip [4]:

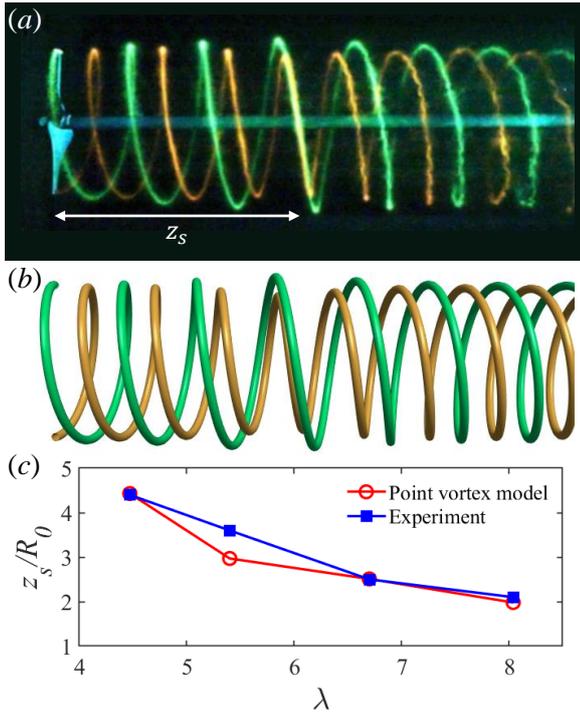
$$\frac{dz_{\alpha}^*}{dt} = \frac{1}{2Li} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^N \Gamma_{\beta} \cot \left\{ \frac{\pi}{L} (z_{\alpha} - z_{\beta}) \right\}$$

where  $z_{\alpha} = x_{\alpha} + iy_{\alpha}$  is the complex position of the point vortex  $\alpha$ ,  $*$  represents the complex conjugate, and  $L$  is the length of the basic strip. If the point vortices are identical and evenly spaced, there will be no displacement. However, the system is unstable, so any asymmetry will perturb the vortices along intertwining trajectories determined by the equation above. Once the trajectories of the point vortices within the  $x - y$  plane are determined, the helical wake can be reconstructed. To transform from the temporal evolution of the helical system to the spatial evolution representing a rotor wake, the point vortices advance along a helical path determined by the advection speed and rotational speed of the rotor. The perturbation deforms the helices, with point vortex displacements in the  $y$ -direction corresponding to radial displacements in the helical system, and  $x$ -direction point vortex displacements having components in the axial and azimuthal directions of the helix.

### 2.2 Experimental set-up

To investigate the accuracy of the point vortex model, water channel experiments visualizing the wake of asymmetric rotors are conducted. The water channel test section is 150 cm  $\times$  38 cm  $\times$  50 cm (length  $\times$  width  $\times$  height). Two- and three-bladed rotors with radii of 8 cm and 9 cm, respectively, are used to generate helical vortex wakes. The vortices are visualized by painting the rotor blade tips with fluorescent dye and illuminating the test section with a laser or LED panel. These visualizations are captured on video for further analysis. The rotation rate and flow speed of the channel are

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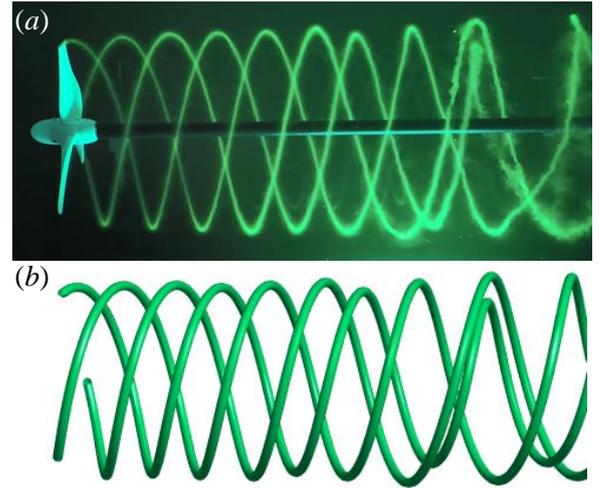


**Figure 1:** (a) Dye visualization of the wake of the asymmetric two-bladed rotor with tip speed ratio  $\lambda = 5.4$ , reproduced from [1]. The white arrow indicates  $z_s$ , the distance to leapfrogging. (b) Helical vortices reconstructed from the point vortex model using the same parameters as the experiment shown in (a). (c) Comparison between  $z_s$  normalized by the rotor radius  $R_0$  obtained from the experiments in [1] and the point vortex model for a range of tip speed ratios.

varied to achieve different values of helical pitch. For the two-bladed rotor, asymmetry is introduced via radial offset relative to the shaft, as described in detail by Quaranta et al. [1]. For the three-bladed rotor, all blades are 3D printed and can be removed and replaced individually.

### 3 Results

Comparisons between predictions from the point vortex model and the experimental results show good agreement. Qualitatively, the key feature of the asymmetric two-bladed rotor wake is the occurrence of leapfrogging, where loops of the two helices swap positions, observed for the experimental case in figure 1(a). When using the values of vortex circulation, helical pitch, and blade asymmetry from the experiment as inputs, the point vortex model reproduces this leapfrogging phenomenon (figure 1b). To quantitatively compare the model to the experiments, the distance  $z_s$  is defined as in [1], i.e., the downstream distance where leapfrogging occurs (figure 1a). Quaranta et al. [1] showed that  $z_s$  depends on tip speed ratio ( $\lambda$ ), the ratio between the speed of the blade tips and the speed of the axial flow approaching the rotor, as  $\lambda$  affects the strength and geometry of vortices in the wake. Figure 1(c) compares  $z_s$  values from the experiments to those obtained from the point vortex model



**Figure 2:** (a) Dye visualization of the wake of the asymmetric three-bladed rotor. (b) Helical vortices reconstructed from the point vortex model.

for a range of values of  $\lambda$ . The results agree well considering the uncertainty in the experimental measurements and the simplicity of the point vortex model. Similar qualitative agreement can be observed in the three-vortex case (figure 2), though a quantitative validation campaign is still underway. A vortex filament model of interlaced helices will also be used for further validation of the point vortex model.

### 4 Conclusion and outlook

The point vortex model for the helical vortex system provides a simple way to predict the behavior of asymmetric rotor wakes. Though it neglects many of the components influencing helical vortices (e.g., curvature, torsion, viscosity, spatial evolution), it is shown to accurately capture key features of rotor wake vortex interaction. As different configurations can be tested quickly, this model has a range of applications. For an experiment investigating undeformed helical vortices, a level of acceptable asymmetry for a given region of interest can be determined. To explore the acceleration of coherent structure breakdown and wake recovery, different rotor asymmetries can be tested to determine the combination of perturbations that most effectively achieves this goal. Furthermore, the agreement between the experimental wake visualizations and the point vortex model suggests that, under the conditions of low helical pitch frequently observed in rotor wakes, helical vortex dynamics are dominated by the interactions between a few neighboring vortex loops.

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