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Fully automated model updating framework for damage detection based on the modified Constitutive Relation Error

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Abstract

Digital twins efficiency lies in fast and representative solutions of inverse problems to accomodate models with physical observations. The quality of the solution of an inverse problem is conditioned by inherent features of the latter, in particular (i) the richness of available data, (ii) the *a priori* experimental and modeling knowledge that allows to regularize the ill-posedness nature of the problem, and (iii) the complexity of the space in which updated parameters are sought. We present in this contribution a fully automated physics-guided model updating framework dedicated to the correction of finite element models from low-frequency dynamics measurements. The proposed methodology is based on the minimization of a modified Constitutive Relation Error (mCRE) functional, whose basic idea is to construct mechanical fields and identify material parameters that are a trade-off between all available information (and associated confidence) but without any further assumption. The dependency into some expert-user's judgment is thus avoided. Dedicated rules are provided to automatically calibrate all mCRE internal tuning parameters as well as a strategy to optimize the space in which parameters are sought, leading to a fully autonomous algorithm. The performance and robustness of the proposed model updating methodology are illustrated using synthetic ground motion tests on a bending plate in which defects of various shapes are identified from noisy acceleration datasets, with inherent limitations due to richness of input loading, sensors sparsity and defect identifiability.

Keywords: Modified Constitutive Relation Error, Automated Model Updating, Low-Frequency Dynamics, Vibration-based damage detection.

1 Introduction

The design, analysis and prediction of dynamical systems require the construction of robust numerical models. These models can be directly built from measurements (*black-box modeling*) or derived after in-depth physical description of the involved phenomena (*white-box modeling*). In each case, as most of modern systems are now equipped with numerous sensors, those models are (at least) assessed by comparison with experimental data to evaluate their degree of representativeness. Indeed, the validation, enrichment and exploitation of numerical models with experimental data are part of the *digital twin* paradigm, whose applications range from simple post-processing to state prediction, control of systems, or even decision-making process [1–3]. Earthquake engineering problems are no exception to the need of using experimental data to build, validate and operate robust numerical models, in particular when it comes to monitor and predict the structural state of civil engineering structures [4].

From the numerical viewpoint, the integration of a digital twin requires a complex numerical framework including a robust model updating algorithm able to operate in real-time. In this paper, we will address the possibilities to perform damage detection from local data collected in an (offline) low-frequency dynamics context, with an emphasis on the trade-off that must be found between:

- the richness of available measurements,
- the complexity of the parameter space,
- the inherent model updating limitations due to the inverse problem itself.

1.1 Vibration-based damage detection at a glance

For structural health monitoring (SHM) applications, the calibration of stiffness parameters from experimental data allows to identify (*i.e.* locate and quantify) structural damage. Throughout the last decades, a wide panel of damage detection methods has been proposed [4–6]. All these techniques derive various model updating approaches and are often dedicated to the exploitation of kinematics measurements. For the sake of conciseness, we will exclusively focus on damage detection from sparse data (obtained from accelerometers, strain gauges, transducers, or Bragg optic fiber measurements for example) although dedicated approaches operating full-field measurements have also been developed, see [7–9] to cite a few. When dealing with spatially sparse datasets, the identification of damage in large structures is inherently difficult because of the relatively reduced amount of available measurements, leading to an ill-posed inverse problem [10].

Due to the fact that many SHM applications directly exploit modal data, a parametrized model is thus not mandatory as the changes in modal features such as eigenfrequencies, damping ratios or modeshapes can provide direct information about the damage state of a structure. Their evolution with time allows the user to track structural damage [11]. In particular, classical

modal analysis techniques aim at identifying modal features through the identification of a state-space model from measurements. One of the fastest and most accurate methods is based on stochastic subspace identification [12, 13]. Since forced vibration testing of large structures in operational conditions is impractical, *Operational Modal Analysis* (OMA) techniques have been considerably developed to process output-only measurements [14–18] obtained from unknown excitations in an operational environment (*i.e.* wind or road traffic for bridges).

However, Finite Element (FE) models can still be updated according to experimental (possibly modal) data. In such a context, Bayesian approaches (whose extended comprehensive review is available in [5]) are classically distinguished from deterministic methods. Among the latter, the sensitivity method is one of the most popular techniques to solve inverse problems. Reviewed in [19], the sensitivity method is based upon linearization of the generally non-linear relationship between measurable outputs and the parameters in need of correction. As most of deterministic inverse problems, regularization techniques are mandatory to prevent the ill-posedness of the problem [20]. Such techniques, although easy to implement, provide identification results that are strongly sensitive to the regularization term (whether it is a L_2 -norm Tikhonov or a L_1 -norm sparse regularization) whose definition totally depends on some user's *a priori* expertise; nevertheless, it is of crucial importance as it conditions the smoothness of the obtained solution [21–23]. Besides, such approaches may also be highly disturbed by measurement noise, with the possibility to obtain divergent or physical-meaningless results.

An alternative consists in using the concept of *modified Constitutive Relation Error* (mCRE) that exploits the reliability of information [24, 25]. This is the main driver behind its selection as a reference method for model updating in this paper. Initially proposed for model updating in dynamics by Ladevèze and co-workers [26, 27], the mCRE functional is defined as a quadratic model-to-measurements distance enriched with a term based on the concept of *Constitutive Relation Error* (CRE) [28], whose enriched physical meaning and strong mechanical content avoid the direct use of regularization terms based on *a priori* expert-user knowledge. This energy-based residual offers interesting advantages. First, local convexity properties are enhanced compared to classical deterministic functionals [29]. Moreover, the CRE part of the residual computed over the whole structure can be exploited as a spatial distribution of the modeling error: it allows for example to restrain the updating process to a few parameters [30], which can be computationally helpful and regularizing (in Tikhonov's sense) when the number of parameters to update becomes important. The relevance and robustness of the mCRE for model updating has been emphasized in many applications. Among other works, the mCRE was shown to be robust to highly noisy and corrupted measurements [31] and able to perform local defect detection [25, 32, 33], or full-field material identification from dense measurements [34, 35]. It has recently been successfully applied to Earthquake Engineering problems, with an implementation in an

industrial FE software [36, 37]. As one can explicitly establish a link between mCRE with other deterministic and stochastic model updating functionals, it is also worth mentioning the comparative study between mCRE, Tikhonov-based and Bayesian damage detection using optical fiber strain measurements investigated in [38].

For the sake of completeness, let us finally mention damage detection techniques based on neural networks and machine learning, that are getting more and more popular due to their capability to process large amount of data [6, 39, 40], exploiting techniques such as statistical pattern recognition that are of growing interest in SHM [41]. However, they still lack of generality in the sense that the learning process conditions the performance of such methods.

1.2 Main contributions

A dedicated mCRE-based model updating framework has been recently developed by the authors in order to perform accurate identification of Finite Element (FE) models from sparse data collected in low-frequency dynamics [42]. Although the robustness to measurement noise of the approach was illustrated by processing actual data from shaking table experiments, the performance of the methodology remains limited by the complex setting of the mCRE framework. To the authors' best knowledge, the design of the parameter space and the tuning procedure that integrates measurements of varying SNR are still open questions in this framework, that have not been clearly addressed in the literature. The ambition of this work is to propose a fully automated mCRE-based model updating algorithm with the objective of identifying damaged areas optimally, *i.e.*, as accurately as possible at minimum computational cost. More precisely, in this contribution, an automated selection of the parameter space is proposed based on the modeling error discrepancies. A clustering strategy is carried out to identify in which areas model updating is of higher importance, generalizing the above-mentioned concept of localization of most erroneous areas. The call to the CRE as model updating indicator makes the tuning of mCRE internal parameters crucial to design an optimal parameter space for damage detection. Dedicated (automated) strategies are implemented and compared for optimal calibration of the confidence in measurements. Finally, physics-inherent model updating limitations are highlighted in this contribution, namely the influence of the sensor placement strategy (particularly when dealing with spatially sparse data) and the identifiability of defects in problems where the strain energy distribution of the tested structure is strongly heterogeneous.

The remainder of the paper is structured as follows. Section 2 recalls basics on mCRE for FE stiffness model updating in low-frequency dynamics. Several strategies regarding the full automation of the algorithm are discussed for the calibration of internal parameters in Section 3. The tuning of the confidence in measurements and the CRE-based design of an optimal parametrization are particularly emphasized. A discussion tackling additional regularization

approaches (whether in L_2 or L_1 metrics) is also included. In order to assess the relevance of the proposed approach, a numerical benchmark in which several defect shapes are identified is discussed in Section 4. Conclusions and prospects are finally drawn in Section 5, suggesting a future implementation of the automated model updating technique for experimental test campaigns and data assimilation.

2 The mCRE for damage detection in low-frequency dynamics

Updating a model is an optimization problem that consists in identifying a set of internal model parameters $\theta \in \Theta$ that enable a model $\mathcal{M}(\theta)$ to better replicate collected data y . In a deterministic approach, in order to overcome the ill-posedness of the inverse problem, a regularization term \mathcal{G} is often added. It guarantees uniqueness of the solution by enforcing local ellipticity properties of the functional to minimize \mathcal{J} . Without loss of generality, a deterministic model updating problem thus reads:

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \left[\mathcal{J}(\theta) \triangleq \alpha \mathcal{D}(\mathcal{M}(\theta), y) + \mathcal{G}(\theta, \theta_0) \right] \quad (1)$$

The cost function \mathcal{J} is built as the weighted sum of a data-to-model distance term $\mathcal{D}(\mathcal{M}(\theta), y)$ enriched with the regularization term. The weighting parameter α thus quantifies the reliability of the data-to-model distance $\mathcal{D}(\mathcal{M}(\theta), y)$ with respect to the *a priori* knowledge θ_0 included in $\mathcal{G}(\theta, \theta_0)$. Its value may be tuned according to several techniques as the L-curve criterion [43, 44] or Morozov's discrepancy principle [45, 46].

In this section, we show how the mCRE-based model updating problem is similar in its formulation to the general case (1) and we briefly recall the fundamentals of this model updating framework for low-frequency dynamics, starting from a linear FE problem written in the frequency domain. Particular attention is paid to the construction of the *Constitutive Relation Error* (CRE) residual from the set of equations defining the reference mechanical problem. Then, details about the *modified Constitutive Relation Error* (mCRE) are explicitly given considering the correction of stiffness parameters. An algorithm allowing to conduct the solution of the associated inverse problem is also given. For additional details, the interested reader is referred to [42, 47] for extended overviews of the mCRE-based model updating framework in the low-frequency dynamics regime.

2.1 FE framework, measurements and stiffness parametrization

Let us consider the general case of a damped elastic structure Ω spatially discretized in (non-overlapping) finite elements such that $\Omega = \cup_{e=1}^E \Omega_e$, and

subjected to a given dynamical loading F . We denote by K, D, M the stiffness, damping and mass FE matrices, respectively, while F_ω and U_ω are the Fourier transforms of nodal loading conditions and displacement field. With these notations, the dynamic equilibrium written in the frequency domain at a given angular frequency ω reads:

$$[-\omega^2 M + i\omega D + K] U_\omega = F_\omega \quad (2)$$

In addition to the dynamic equilibrium, a set of sensors is used to measure the magnitude of some kinematic quantities (displacement, velocity and/or acceleration). In the frequency domain, assuming for the moment that measurements are perfect, such information can be written without loss of generality as:

$$\Pi U_\omega = Y_\omega \quad (3)$$

where Y_ω refers to the Fourier transform of measurements at angular frequency ω and Π denotes a projection matrix that allows to extract the components of U_ω that are measured. Its non-zero entries are integer power of $(i\omega)$.

As mentioned in the introduction, one can model damage as local stiffness loss. Therefore, a convenient manner to parametrize a linear FE problem for damage detection is to parametrize the FE stiffness matrix. Doing so, in this contribution, the n_θ parameters $\theta \in \Theta \subset \mathbb{R}^{n_\theta}$ to update only affect the stiffness matrix K . More precisely, the FE stiffness matrix is decomposed into n_θ non-overlapping subdomains and parametrized as follows:

$$K(\theta) = \sum_{i=1}^{n_\theta} \frac{\theta_i}{\theta_{0,i}} K_{0,i} \quad \text{with} \quad K(\theta_0) = \sum_{i=1}^{n_\theta} K_{0,i} \quad (4)$$

Note that subdomains can encompass several finite elements at once, which allows to reduce the number n_θ of parameters to identify.

2.2 The modified Constitutive Relation Error: a physics-based approach for inverse problems

The fundamental idea of the mCRE concept is to identify mechanical fields and material parameters that are a trade-off between all available information without adding any other *a priori* assumption. Originally intended to perform model verification [28], the key idea for the construction of the CRE residual lies into the distinction between reliable and unreliable information on the reference mechanical problem. The mCRE concept extends this distinction to experimental data as well, with a functional that simultaneously handles measurement error and model uncertainty. Indeed, the redundant information gathered in (2) and (3) means all equations cannot be exactly verified. Some of them must be relaxed (*ie* considered less reliable). They will thus be only verified at best by minimizing the so-called modified CRE functional.

Although this distinction between reliable and unreliable equations is non-unique and deeply relies on the case study and engineering expertise, it is also well-known that, in most applications, constitutive relations are subject to caution. The full separation of equations for the considered damage detection case is given in TAB. 1. Note that the damping modelling will not be subject to caution as damping variations are secondary for damage detection. This choice could be questioned if the stiffness properties were correctly updated. Once modeling equations have been labeled as reliable or unreliable, two admissibility spaces are implicitly defined:

- (i) a kinematic admissibility space \mathcal{U}_{ad} inside which any displacement field U satisfies the reliable kinematic equations of the problem,
- (ii) an auxiliary dynamically admissibility space \mathcal{D}_{ad} defined as the set of displacement fields V derived from the stress field satisfying the dynamic equilibrium.

The distance between \mathcal{U}_{ad} and \mathcal{D}_{ad} can be measured using an energy norm - the CRE - that estimates the relevance of a solution couple $s_\omega = (U_\omega, V_\omega) \in \mathcal{U}_{ad} \times \mathcal{D}_{ad}$ with respect to the mechanical problem. With the above notations, the CRE at a given angular frequency ω reads:

$$\zeta_\omega^2(s_\omega, \theta) = \frac{1}{2}(U_\omega - V_\omega)^H K(\theta)(U_\omega - V_\omega) = \frac{1}{2}\|U_\omega - V_\omega\|_{K(\theta)}^2 \quad (5)$$

with \square^H denoting the Hermitian transpose.

	Reliable	Unreliable
Model	<ul style="list-style-type: none"> • Geometry • Boundary conditions • Equilibrium equations • Dissipative constitutive relations 	<ul style="list-style-type: none"> • Elastic constitutive relations
Experiments	<ul style="list-style-type: none"> • Loading frequencies $\omega/2\pi$ • Sensor locations • Measured inputs F_ω 	<ul style="list-style-type: none"> • Measured outputs Y_ω

Table 1: Distinction between reliable and unreliable information for damage detection from stiffness update in dynamics.

The extension of the CRE concept to unreliable experimental data (see TAB. 1) directly leads to the so-called *modified Constitutive Relation Error* (mCRE). The CRE is completed by a data-to-model distance between the predictions U_ω and the measurements Y_ω :

$$e_\omega^2(s_\omega, \theta, Y_\omega) \triangleq \zeta_\omega^2(s_\omega, \theta) + \frac{\alpha}{2}\|\Pi U_\omega - Y_\omega\|_{\Sigma_y^{-1}}^2 \quad (6)$$

The used Mahalanobis distance between model predictions and data involves the measurement noise covariance matrix Σ_y . The tuning factor $\alpha \in \mathbb{R}^+$ enables to balance both terms by expressing more or less confidence in measurements; large values can be specified when measurements are considered

reliable whereas close-to-zero values are better suited to corrupted or noisy recordings. The choice of α is therefore crucial for providing relevant parameter estimates [48, 49]. Automated techniques for its optimal tuning will be discussed in the following.

As a remark, note that the data-to-model distance in the mCRE literature was often weighted by a matrix G that aimed to ensure that $\|\square\|_G$ is homogeneous to ζ_ω^2 and equivalent in level. The choice of G was not critical as the value of α compensates to integrate the effect of measurement noise [30, 42, 47]. This approach, yet efficient, lacks of robustness in the perspective of automating the mCRE-based model updating process.

The analysis of a single angular frequency is too restrictive in low-frequency dynamics, particularly when the structure response involves several eigenmodes. The model updating procedure must be conducted over a frequency bandwidth D_ω . The mCRE functional \mathcal{J} to be minimized is thus obtained by direct integration over D_ω :

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \left[\mathcal{J}(\theta, Y) \triangleq \int_{D_\omega} z(\omega) e_\omega^2(\hat{s}(\theta, Y_\omega), \theta, Y_\omega) d\omega \right] \quad (7)$$

where $z(\omega)$ is a normalized frequency weighting function that satisfies $\int_{D_\omega} z(\omega) d\omega = 1$. It allows to modulate the importance of specific frequencies of D_ω . An appropriate manner to define $z(\omega)$ in low-frequency dynamics is given in Section 3.1. We denote by $\hat{s}(\theta, Y_\omega)$ the optimal solution in the mCRE sense for given parameters and measurements, which is defined as

$$\forall \omega \in D_\omega, \quad \hat{s}(\theta, Y_\omega) = \arg \min_{[-\omega^2 M + i\omega D]U_\omega + K(\theta)V_\omega = F_\omega} e_\omega^2(s, \theta, Y_\omega) \quad (8)$$

Introducing Lagrange multipliers $\hat{\Lambda}_\omega$ and an augmented cost function, it is easy to show that this constrained minimization problem is equivalent to the solution of the following linear system:

$$\begin{bmatrix} Z(\theta, \omega)^H & \alpha \Pi^H \Sigma_y^{-1} \Pi \\ -K^H(\theta) & Z(\theta, \omega) \end{bmatrix} \begin{bmatrix} \hat{\Lambda}_\omega \\ \hat{U}_\omega \end{bmatrix} = \begin{bmatrix} \alpha \Pi^H \Sigma_y^{-1} Y_\omega \\ F_\omega \end{bmatrix} \quad (9)$$

with $\hat{\Lambda}_\omega = \hat{U}_\omega - \hat{V}_\omega$ and $Z(\theta, \omega) = [K(\theta) + i\omega D - \omega^2 M]$. Its size can be drastically reduced using projection on reduced basis, *e.g.* a truncated modal basis enriched with Krylov vectors [42, 47].

2.3 Solution algorithm

The minimization of the mCRE is more complex than standard deterministic cost functions as it is formed of nested minimization problems addressed on mechanical fields (8) and parameters (7). Herein, the computational burden can be drastically reduced using (i) parallel computing on $\omega \in D_\omega$ and

projection on reduced basis when solving (9), (ii) an analytical expression of the mCRE gradient with respect to the parameters. With the above stiffness parametrization, for a given angular frequency ω , the latter reads:

$$\nabla_{\theta_j} e_{\omega}^2(s_{\omega}, \theta, Y_{\omega}) = \frac{1}{2} \|\Lambda_{\omega}\|_{K_{0,j}}^2 + \Re(\Lambda_{\omega}^T K_{0,j} V_{\omega}) \quad (10)$$

where $\Re(\square)$ denotes the extraction of real part of \square . This result directly derives from the stationarity of the augmented cost function introduced to solve (8). As shown in ALG. 1, typical stopping criterion of the model updating algorithm is based on the stagnation of the updated parameters. The associated threshold ϵ must be chosen as a compromise between accurate enough identification results and reasonable computational time ($\epsilon \approx 10^{-3} - 10^{-5}$ is a typical value for the forthcoming examples). Note that, if correctly normalized, the value of the mCRE itself can be used as complementary stopping criterion [30, 42].

The localization of most erroneous areas (in the sense of the modeling error) is the only regularizing process in Tikhonov's sense as the model updating is limited to a small number of parameters. This principle will be discussed in details regarding the full automation of the algorithm in the next section.

Algorithm 1 Pseudo-code of the mCRE-based model updating strategy.

Require: FE model including mesh and matrices K, D, M , measured outputs $y(t)$, subdomain decomposition and associated initial parameter guess $\theta_0 \in \Theta$, frequency bandwidth D_{ω} , measurement error weight α , threshold ϵ

Initialization:

Data preprocessing in the frequency domain: $Y_{\omega}, \forall \omega \in D_{\omega}$

Computation of the frequency weighting function $z(\omega)$

Model updating algorithm:

while $|\theta_k - \theta_{k-1}| \leq \epsilon |\theta_{k-1}|$ **do**

1. Mechanical fields and localization of erroneous subdomains

 Solution of (9) $\forall \omega \in D_{\omega}$

 Identification of most erroneous areas from $\zeta_{\omega,e}^2 \forall (\omega, e) \in (D_{\omega} \times \llbracket 1; E \rrbracket)$

2. Correction of parameters

 Minimization of $\mathcal{J}(\theta)$ with respect to the identified parameters: $\theta_{k+1} \leftarrow \theta_k$
 (BFGS method with supplied analytical gradient)

3. Convergence test

 Check parameters stagnation: $|\theta_{k+1} - \theta_k| / |\theta_k|$

end while

3 Full automation of the modified CRE for user-friendly model updating

In order to build a fully automated model updating strategy, all internal tuning parameters must be calibrated with rigorous and systematic rules. One must then pay attention to the setting of:

- ▷ the frequency bandwidth D_ω ,
- ▷ the frequency weighting function $z(\omega)$,
- ▷ the measurement error weight α ,
- ▷ the parameter space Θ in which parameters are sought.

As all of these parameters have a significant influence on model updating results, a systematic tuning procedure would then be useful to handle potential non-convincing model updating results. We will briefly recall how to automatically calibrate all these influent internal parameters so as to provide relevant model updating results without requiring any user's *a priori* knowledge or experience (once the CRE has been defined!). The selection of the parameter space and the tuning of α are extensively discussed as no clear contribution has been found in that sense in the literature, out of empirical studies.

3.1 Frequency bandwidth - frequency weighting

The model updating procedure is conducted on a given frequency bandwidth $D_\omega = [\omega_{min}; \omega_{max}]$ which contains the essential part of the mechanical energy of the system. For seismic applications, ground motions usually have a significant frequency content up to 50 Hz, which implies that $D_\omega \subset [0 \text{ Hz}; 50 \text{ Hz}]$. Besides, the frequency step Δf required for the discretization of D_ω must be carefully chosen to capture the frequency content associated to the participating eigenmodes. A common engineering judgment one can recommend is to select Δf such that $3\Delta f \approx \min_i(\xi_i f_i)$ where (ξ_i, f_i) refers to the damping ratio and natural frequency of mode i . In the upcoming earthquake engineering-inspired applications, with typical 5% damping ratios and first eigenfrequencies at around 2-5 Hz, Δf is thus chosen within $[0.1 \text{ Hz}; 0.5 \text{ Hz}]$.

In addition, $z(\omega)$ can be used as a modulation function to favor frequencies deemed to have the largest influence on the model updating procedure. In [42], an automated computation of $z(\omega)$ from experimental datasets was proposed, based on the fact that the essential frequency content of the measurements is gathered around experimental natural frequencies. A normalized version of the *Complex Mode Indicator Function* (CMIF) [50] on transfer functions has been introduced to naturally emphasize experimental eigenfrequencies as it peaks in their vicinity.

3.2 Confidence in measurements

Historically, in addition to the specific continuation scheme proposed in [34], several approaches related to regularization techniques were employed to define an optimal value of α . The influence of α was particularly investigated in [24, 35, 51, 52], with empirical conclusions that an equivalent confidence between the CRE and data-to-model distance terms provided relevant model updating results. In other words, one should tend to calibrate α such that the two terms are of the same order of magnitude upon convergence. This idea is closely related to the L-curve method [44].

Alternatively, another technique to choose α lies in Morozov's discrepancy principle [46, 53, 54]. Therefore, the latter could be used in order to integrate the *a priori* knowledge on measurement noise features appropriately, as it was done in [54, 55] to perform mCRE-based identification from imaging databases.

In this part, two tuning strategies based on the previously mentioned principles are discussed. Both require a preliminary parametric study on α that can be done (at low-cost) in preamble of the model updating procedure (see ALG. 1) and intend to avoid a full parametric study on α that would be prohibitive in terms of CPU time. Their performance will be assessed in Section 4.

3.2.1 A priori balance between modeling and measurement errors

As mentioned above, the (physics-based) CRE term allows to explicitly integrate modeling errors into the updating process. Its relative weight with respect to the modeling error term must be correctly set in order to provide relevant identification results. Without any additional *a priori* information, one can choose to calibrate α to ensure a correct *a priori* balance between measurement error and modeling error. This approach, which is close to the L-curve principle in terms of formulation, has been proposed in recent works [35, 42] and provided relevant model updating results.

$$\hat{\alpha} = \arg \min_{\alpha \in \mathbb{R}^+} \left| \underbrace{\int_{D_\omega} z(\omega) \zeta_\omega^2(\hat{s}(\theta, Y_\omega), \theta) \, d\omega}_{\text{Modeling error}} - \underbrace{\int_{D_\omega} z(\omega) \frac{1}{2} \|\Pi U_\omega - Y_\omega\|_{\Sigma_y^{-1}}^2 \, d\omega}_{\text{Measurement error}} \right| \quad (11)$$

Note that this approach is non-trivial as the mechanical fields that need to be computed (U_ω, V_ω) are indirectly impacted by the value of α following (9).

3.2.2 Morozov's discrepancy principle to integrate the knowledge of measurement noise

Morozov's discrepancy principle permits to calibrate α so that the measurement error term should not be lower than the noise level, denoted δ_s in the

following. We owe to Bonnet and co-workers the only use of Morozov's discrepancy principle from data obtained in the frequency domain within the mCRE framework [52, 55], although its implementation remains questionable. Indeed, as expressed in [55], the measurement noise is assumed to be proportional to the magnitude of measurements, meaning that during the same experiment, sensors measuring low- or high-amplitude data do not have the same noise level. Besides, the implementation of the criterion has only been proposed for harmonic analysis and low-levels of noise.

In this contribution, a Morozov's discrepancy criterion for *a posteriori* optimal choice of α dedicated to the considered mCRE-based model updating framework is proposed. If one (legitimately) assumes that the measurement noise level of the N_s sensors is additive and follows a zero-mean Gaussian probability density function, *i.e.* $\eta(t) \sim \mathcal{N}(0, \delta_s^2) \forall t$, and if Σ_y is proportional to the identity matrix (uncorrelated noise), then one can show that the mCRE measurement error term can be statistically approximated by:

$$\mathbf{E} \left(\int_{D_\omega} z(\omega) \|\Pi U_\omega(\alpha, \hat{\theta}) - Y_\omega\|^2 d\omega \right) = \delta_s^2 N_s \quad (12)$$

as the squared modulus of a zero-mean random process follows a non-centered $\chi^2(2)$ probability distribution conditioned by the its variance δ_s^2 , whatever the value of ω (see Appendix A for the proof). One can thus statistically bound the measurement error term, which directly corresponds to the adaptation of Morozov's discrepancy principle to the mCRE written in the frequency domain (when all sensors are supposed uncorrelated, which is legitimate in practice). Note that the involved mechanical field of the criterion is the one obtained at convergence with $\hat{\theta}$, as shown in ALG. 2. The value of α is tuned using a bisection method after assessing the quality of the solution once having fully minimized the mCRE functional $\mathcal{J}(\theta)$. Doing so, with a few iterations, one could hope to find a relevant value for $\hat{\alpha}$.

Even if Morozov's discrepancy principle is intended to be used *a posteriori*, one can also perform a cheaper *a priori* parametric study on α and define $\hat{\alpha}$ such that:

$$\hat{\alpha} = \arg \min_{\alpha \in \mathbb{R}^+} \left| \underbrace{\int_{D_\omega} z(\omega) \|\Pi U_\omega(\alpha, \theta_0) - Y_\omega\|^2 d\omega}_{\mathcal{R}(\alpha, \theta_0)} - \delta_s^2 N_s \right| \quad (13)$$

Although the provided value is suboptimal, it may be sufficient to get relevant identification results. In particular, it should be useful when one cannot afford to perform several minimization steps to update parameters correctly when performing model updating on-the-fly, as it is our intention in perspective of this work [56]. Illustrations of the *a priori* and *a posteriori* choice of α following (11), (13), and Alg. 2 will be provided in the following applications.

Algorithm 2 A posteriori calibration of the measurement error weight according to Morozov’s discrepancy principle.

Require: FE model including mesh and matrices K, D, M , measured outputs $y(t)$ and noise level of sensors δ_s , subdomain decomposition and associated initial parameter guess $\theta_0 \in \Theta$, frequency bandwidth D_ω , initial value of α (recommended to be small), threshold ϵ

```

while  $|\mathcal{R}(\alpha, \hat{\theta})| > \epsilon$  do
  1. Update the measurement error weight  $\alpha$ 
  if  $\mathcal{R}(\alpha, \hat{\theta}) > 0$  then
    Increase the value of  $\alpha$ 
  else if  $\mathcal{R}(\alpha, \hat{\theta}) < 0$  then
    Decrease the value of  $\alpha$ 
  end if
  2. Minimization of the mCRE functional
  Minimize  $\mathcal{J}(\theta)$  with the updated value of  $\alpha$  starting from  $\theta_0$  using ALG. 1
  3. Assessment
  Compute Morozov’s discrepancy:  $\mathcal{R}(\alpha, \hat{\theta})$ 
end while

```

3.3 CRE-based stiffness parametrization for fully automated model updating

3.3.1 CRE-based localization of most erroneous areas

The CRE provides a direct insight regarding the validity of the model itself, making it a relevant tool for identifying erroneous parts of the model as all finite element contributions to CRE can be computed independently. The CRE per subdomain S_i then reads:

$$\forall i \in \llbracket 1; n_\theta \rrbracket, \quad \zeta_{\omega,i}^2(s_\omega, \theta) = \sum_{e \subset S_i} \frac{1}{2} \|U_\omega - V_\omega\|_{K_e(\theta)}^2 \quad (14)$$

where $K_e(\theta)$ is the stiffness matrix of element e . This asset can be seen as a Tikhonov regularization in the mCRE framework in the sense that a reduced number of parameters can be updated [30, 47]. Indeed, defining a threshold $\beta \in [0; 1]$, one can identify subdomains $i \in \llbracket 1; n_\theta \rrbracket$ to update such that they satisfy the following inequality:

$$\frac{1}{|S_i|} \int_{D_\omega} z(\omega) \zeta_{\omega,i}^2(\hat{s}_\omega, \theta) \, d\omega \geq \beta \max_{i \in \llbracket 1; n_\theta \rrbracket} \left\{ \frac{1}{|S_i|} \int_{D_\omega} z(\omega) \zeta_{\omega,i}^2(\hat{s}_\omega, \theta) \, d\omega \right\} \quad (15)$$

where the normalization by the size of subdomains $|S_i|$ permits to consider subdomains of various sizes. However, if most references consider that $\beta = 0.8$ is a convenient value, no clear parametric study has been performed in order to optimize the choice of β for optimal defect detection based on noisy datasets.

Thus, if the CRE can be a convenient local model indicator, the localization criterion (15) alone is limited to provide an automated parametrization (and associated subdomains). In the remainder of this subsection, we present the potentialities and inherent limitations of a CRE-based clustering technique for automated subdomain definition.

3.3.2 CRE-based clustering of erroneous areas: key ideas

Clustering is one of the most widely used techniques for data analysis and classification [57]. Without going into much details, clustering techniques allow to partition a space in the sense of a given metrics: within a cluster, the distance between objects remains small, whereas the distance between objects of different clusters is larger. For earthquake engineering problems, they have been essentially applied for automated modal analysis purposes [13, 17, 58].

In this work, among the wide range of clustering algorithms that have been proposed, the fuzzy clustering algorithm developed in [13] is reinvested. Briefly, fuzzy clustering algorithms are more effective than standard hard clustering algorithms when dealing with not-well separated and non-spherical clusters. Membership functions μ_{ij} are introduced: they allow to assess the degree of belonging of a data point X_i to a cluster of center C_j . The centers and membership functions are then sought according to the following constrained minimization problem:

$$\{C, \mu\} = \arg \min_{C, \mu} \underbrace{\sum_{j=1}^k \sum_{i=1}^N \mu_{ij}^m \|X_i - C_j\|_A^2}_{J(C, \mu; X)} \quad \text{with} \quad \sum_{j=1}^k \mu_{ij} = 1 \quad \forall i \in \llbracket 1; N \rrbracket \quad (16)$$

where k is the number of clusters, N the amount of data points, and the exponent m reflects the degree of fuzziness of the partition: $m = 2$ is the classical (empirically) chosen value for fuzzy clustering applications. The constraint on $\{\mu_{ij}\}$ states that the sum of cluster membership value per data point should be exactly equal to 1. The Hermitian norm $\|\cdot\|_A$ is defined using a positive definite matrix A , identically chosen as Euclidean for all clusters in our context.

Then, from a given parameter set θ , the objective is to identify a subset of most erroneous elements through the computation of the modeling error (CRE) map. In other words, we perform a two-cluster distinction from the normalized CRE map:

$$X = \frac{\left\{ X_i - \min_{i \in \llbracket 1; n_\theta \rrbracket} \{X_i\} \right\}_{i=1}^{n_\theta}}{\max_{i \in \llbracket 1; n_\theta \rrbracket} \{X_i\} - \min_{i \in \llbracket 1; n_\theta \rrbracket} \{X_i\}}, \quad X_i = \int_{D_\omega} z(\omega) \zeta_{\omega, i}^2(\widehat{s}_\omega, \theta) d\omega \quad (17)$$

which allows to label each finite element as "healthy" or "erroneous". A new parameter space $\hat{\Theta}$ is then defined in which all erroneous elements are updated independently whereas the healthy elements remain unchanged, as concluded in Alg. 3.

If the overall model (in the healthy area) is accurate, this procedure will allow to (at least coarsely) localize the damaged area. This process can be interpreted as a generalization of the localization of most erroneous areas principle that was mentioned earlier, but in a more adaptive manner.

Algorithm 3 Fuzzy clustering algorithm for CRE-based determination of erroneous areas.

Inputs:

Normalized CRE map $X \in \mathbb{R}^{n_\theta}$ from (17), initial centers $\{C_1^0, C_2^0\} = \{1, 0\}$, euclidean cluster metric $A = I$, $m = 2$, threshold ϵ_0

Fixed-point algorithm:

while $J(C, \mu; X) > \epsilon_0$ **do**

Compute membership functions $\mu_{ij} = \left[\sum_{p=1}^2 \left(\frac{\|X_i - C_j\|_A^2}{\|X_i - C_p\|_A^2} \right)^{\frac{1}{m-1}} \right]^{-1}$

Update centers positions $C_j = \frac{\sum_{i=1}^{n_\theta} \mu_{ij}^m X_i}{\sum_{i=1}^{n_\theta} \mu_{ij}^m}$

Compute the cost function $J(C, \mu; X) = \sum_{i=1}^{n_\theta} \sum_{j=1}^2 \mu_{ij}^m \|X_i - C_j\|_A^2$

end while

Clusters identification:

Separate data points $i \in \llbracket 1; n_\theta \rrbracket$ in clusters according to membership values: X_i belongs to cluster j if $\mu_{ij} > 0.5$.

The cluster storing the highest normalized CRE values is the cluster of erroneous elements to which model updating actions should be focused.

Let us point out several subtleties in the approach:

- ▷ Computing the CRE map is not an expensive procedure as it is a (vectorizable) post-processing operation once mechanical fields solution of (9) has been obtained.
- ▷ In standard fuzzy clustering, it is recommended to initialize cluster centers randomly. As one expects that elements having close-to-zero normalized CRE values should not be updated (resp. elements with close to one values should be updated), the location of centers can be initialized at $(C_1, C_2) = (1, 0)$. Note that doing so also avoids the undesired case of equal partitioning.
- ▷ The choice not to consider FE barycentric coordinates in X allows to identify damaged areas at different locations simultaneously. It avoids the complex discussion on the robustness of clustering algorithms when the number of clusters to identify changes from one application to the other

[59]. Besides, as the damage pattern in a structure may not be convex, particularly when several damaged areas have to be identified, it avoids the need for spectral clustering [60].

- ▷ The metric A of each cluster has been chosen Euclidean in Alg. 3, but in practice one can imagine going a step further by recalibrating the metric of each cluster in the fixed-point algorithm, along with center positions and membership functions. In particular, this would enable clusters of elliptic shape to be better dissociated in multidimensional problems - see [13] for more details.

3.4 A sparsity-promoting regularization of the mCRE

Following the developments of [35], one could also imagine adding an explicit Tikhonov regularization term to the mCRE so as to favor some *a priori* knowledge on the parameter estimates that are sought. In particular, when it comes to detect localized damage, sparse regularization would allow to focus the model updating process to a reduced amount of parameters. The sparsity-promoting regularization of the mCRE \mathcal{J}_r reads as follows:

$$\mathcal{J}_r(\theta, Y, \theta^*, \gamma) = \int_{D_\omega} z(\omega) e_\omega^2(\widehat{s}(\theta, Y_\omega), \theta, Y_\omega) \, d\omega + \gamma \|\theta - \theta_0\|_1 \quad (18)$$

where θ^* is the *a priori* information on parameters, γ is the weighting parameter allowing to give more or less importance to the regularization, and $\|\square\|_1$ refers to the L_1 -norm. The minimization of this functional can also be performed using the same algorithmic structure and mathematical developments that have been given previously, except for the analytical gradient formulation that must be slightly modified to integrate the regularization term into consideration. However, the introduction of a new *a priori* information goes against the philosophy of the mCRE which aims to remove any expert-user's judgment. Another disadvantage of this technique is the additional tuning of the weighting parameter γ which affects the quality of the solution [20]. The benefits of sparsity-promoting regularization are assessed in the following section that gathers numerical experiments.

4 Application to damage detection from accelerometer datasets

4.1 Problem setting and objectives of the study

In this section, numerical results are presented and discussed to assess the relevance of the automated mCRE-based model updating strategy for damage detection with typical earthquake engineering applications. Academic examples are considered in which a cantilevered plane wall is subjected to a 60 s-long random ground acceleration input in the outer plane direction (0.1g maximum

acceleration). The reference mesh with the defects one hopes to identify is shown in FIG. 1. Two cases of damage defects are considered to assess the performance of the model updating algorithm (see FIG. 1): (i) a Y-shape defect (representative of a propagated crack with bifurcation) that is modeled with a local 50% Young modulus loss and (ii) a double circular inclusion problem, respectively modeled with local 50% and 30% Young's modulus losses¹. If the first case is much more representative of what could happen in actual experiments involving crack propagation, the second example will allow to emphasize the capability of the clustering algorithm to identify several defects, in areas having heterogeneous sensitivity.

A uniform sensor placement is considered: discrete accelerometers oriented in the outer plane direction are spread over the structure and collect data at the sampling frequency $f_s = 1000$ Hz. The effect of the sensors density is illustrated in the following by modifying the sensor grid cell size d_s . In order to assess the robustness of the methodology with respect to measurement noise, a white noise of known standard deviation is added to simulated data:

$$\ddot{y}_{noisy}(t) = \ddot{y}(t) + \delta_s \eta(t) \quad (19)$$

$\delta_s = \delta \cdot \text{std}(\ddot{u}_d(t))$ represents the noise level with $\delta \in [0; 1]$ and \ddot{u}_d the input ground acceleration, while $\eta(t)$ is a random vector normally distributed with zero mean and unit standard deviation. Therefore, the added measurement noise standard deviation is equal to $\delta \cdot \text{std}(\ddot{u}_d)$.

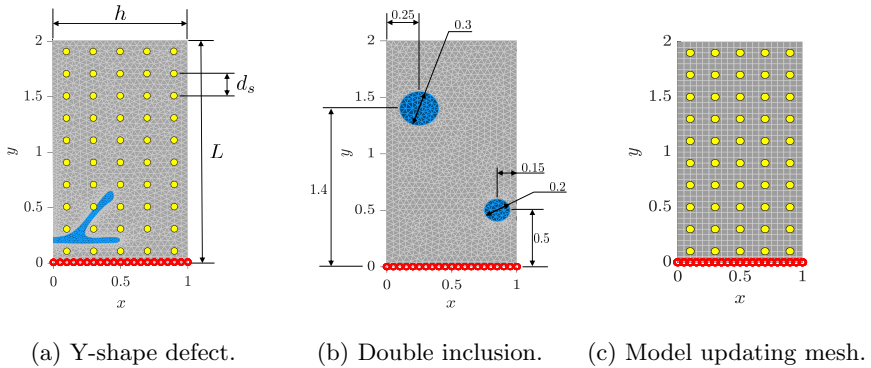


Fig. 1: Reference FE meshes to simulate synthetic data with emphasis on the defect areas Ω_d to identify (blue elements) and FE mesh used for model updating. Locked DOFs are specified with red circles. A possible sensor grid is shown with yellow DOFs.

¹In practice, the following material properties values have been used: $\rho = 3000 \text{ kg/m}^3$, $E = 20 \text{ GPa}$ in healthy areas, $\nu = 0.2$. The dimensions of the geometry are $L = 2 \text{ m}$, $h = 1 \text{ m}$. The wall thickness is equal to 0.07 m .

Before performing any model updating action, it is worth noticing that the input considered herein will limit the accuracy of model updating results as it will only solicitate the first eigenmodes and the low-frequency domain. Of course, one may imagine setting an appropriate input signal (frequency content, location, duration,...) in order to get accurate parameter estimates. However, as it is the case for all inverse problems, the richness of the inputs directly impacts the quality of model updating results.

In practice, meshes and FE matrices (reference and initial guess) are built using the CEA simulation software CAST3M[®] before being uploaded in a MATLAB[®] environment. In both cases, the initial model guess is made of regular quadrangular shell elements whose homogeneous Young's modulus is equal to the non-damaged reference. The knowledge of the expected parameter estimate θ^* enables to assess the model updating accuracy using the following reconstruction errors:

$$\eta(\theta, \theta^*) = \sqrt{\frac{\int_{\Omega} \|\theta - \theta^*\|^2 d\Omega}{\int_{\Omega} \|\theta^*\|^2 d\Omega}} \quad ; \quad \eta_d(\theta, \theta^*) = \sqrt{\frac{\int_{\Omega_d} \|\theta - \theta^*\|^2 d\Omega}{\int_{\Omega_d} \|\theta^*\|^2 d\Omega}} \quad (20)$$

which indicate the closeness of an estimate θ to the expected parameter set θ^* , respectively on the full structure Ω or on the restriction to the damaged area Ω_d (see FIG. 1).

In terms of mCRE calibration, as the first three eigenmodes (whose are the most solicited eigenmodes in the response to random ground motion) have frequencies below 50 Hz²), a frequency bandwidth $D_{\omega} = [1 \text{ Hz}; 50 \text{ Hz}]$ with $\Delta f = 0.1 \text{ Hz}$ has been chosen for the computation of all forthcoming results. The call to a reduced basis made of the first 20 eigenmodes of the structure allowed to achieve convergence in fast CPU times (between 0.5 and 2 minutes per mCRE-minimization with CRE-based clustering) on a personal laptop.

From these typical 2D earthquake engineering academical examples, the aim is to emphasize the main underlying issues of model updating from discrete sensors and to validate the automated strategy that has been presented above. To do so, a complete numerical study is conducted with several objectives:

- ▷ illustrate the limits of the localization criteria for damage detection and the effectiveness of the CRE-based clustering step for automated parametrization;
- ▷ assess both effectiveness and soundness of the criteria proposed in Section 3.2 for automated calibration of α and evaluate the influence of α on parameter estimates;
- ▷ evaluate the robustness of the automated mCRE-based model updating algorithm with respect to measurement noise;

²With the chosen material parameter values, the first eigenfrequencies of the initial modeling guess (non-damaged structure) are then equal to $\{7.37, 32.46, 45.89\} \text{ Hz}$.

- ▷ illustrate the damage detection expectations from limited sensor density with typical earthquake engineering inputs;
- ▷ assess the benefits of additional sparsity-promoting regularization to the mCRE functional.

For the sake of conciseness, as the Y-shaped defect is the most challenging application, the analysis will mainly be focused on the latter, whereas the inclusion example will be used to drive complementary discussions.

4.2 Limits of the localization criterion for optimal damage detection

First, a parametric study on the localization parameter β defined in (15) allows to assess the capabilities of classical mCRE-based model updating to perform optimal damage detection, *i.e.*, accurate identification at low computational cost. A favorable model updating setting is considered: large amount of sensors ($d_s = 0.1$ m), almost noise-free data ($\delta = 0.1\%$). The indicators defined in (20) are computed for each parameter estimate identified at convergence and stored in TAB. 2. Typical model updating results are plotted in FIG. 2.

Localization parameter β	$\eta(\hat{\theta}, \theta^*)$	$\eta_d(\hat{\theta}, \theta^*)$	CPU time [s]
0 (full)	0.1415	1.9578	82.3
0.2	0.1413	1.9589	34.3
0.4	0.1493	1.5645	22.9
0.6	0.1528	2.1865	16.3
0.8	0.1857	4.0000	2.7
with CRE-based clustering	0.1364	1.7147	30.2

Table 2: Emphasis on the limitations of the localization of most erroneous areas for accurate and efficient damage detection in the most favorable damage detection case (noise-free data, dense sensor placement).

One can observe that mCRE-based model updating with a low localization value (assuming the initial guess to be well calibrated) allows to capture accurately the defect shape. However, unexpected corrections occur, mostly overestimations at the boundary between damaged and non-damaged areas, which leads to suboptimal results with respect to η . With high values for β , the CPU time is drastically reduced as very few parameters are updated (the updated area is smaller than the defect shape). The model updating algorithm sometimes overcompensates and diverges by proposing non-acceptable negative stiffness values.

Therefore, the localization of most erroneous areas principle seems to be not sufficient to perform accurate damage detection, as the value of β must be carefully chosen to get acceptable results. A comparison with CRE-based clustering is given in FIG. 3 in the same model updating context. Only a

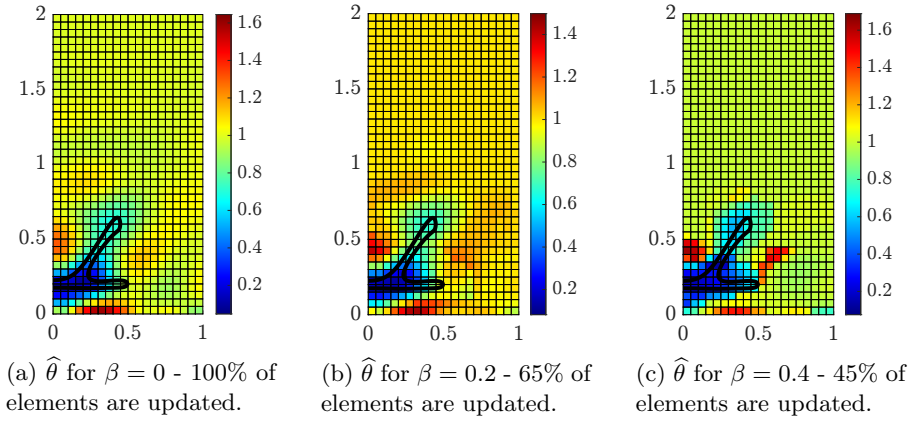


Fig. 2: Limit of the localization of most erroneous areas principle for accurate damage detection - results obtained with a dense sensor placement ($d_s = 0.1$ m) and 0.1% noisy data. The reference defect contour is plotted in black.

reduced group of finite elements is corrected at each iteration according to the modeling error distribution. This group of elements matches with the shape of the damaged area and permits to obtain an accurate estimate, as shown in TAB. 2. Besides, the decrease of the modeling error, from 10^{-2} to 10^{-6} , confirms the efficiency of the proposed approach.

4.3 Automated confidence into measurements

One of the objectives of this study is to check the validity of the criteria allowing to automatically define an optimal confidence into measurements parameter $\hat{\alpha}$. As a reminder, three approaches are compared:

- $\hat{\alpha}_1$ such that the *a priori* model and measurement error are balanced (11),
- $\hat{\alpha}_2$ such that Morozov's discrepancy principle is satisfied *a priori* (13),
- $\hat{\alpha}_3$ such that Morozov's discrepancy principle is satisfied *a posteriori* following ALG. 2 with a dichotomic strategy.

We show in FIG. 4 the natural decrease of measurement error weights ($\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$) with respect to the noise level. This result seems intuitive as the more measurement noise, the less confidence in data. Note that the x -axis of FIG. 4 is weighted by Σ_y^{-1} as it is included in the mCRE data-to-model distance, and thus implicitly conditions the value of $\hat{\alpha}$.

Although similar trends in the evolution of $\hat{\alpha}$ with respect to measurement noise are observed, the values given by both criteria are quite different, and yet one cannot conclude about their validity so far. To assess which of the criteria is the most appropriate to use, a full parametric study of the joint influence of α and δ on the identified estimates is conducted. Associated results are given in FIG. 5 where the accuracy indicators η and η_d are plotted as a

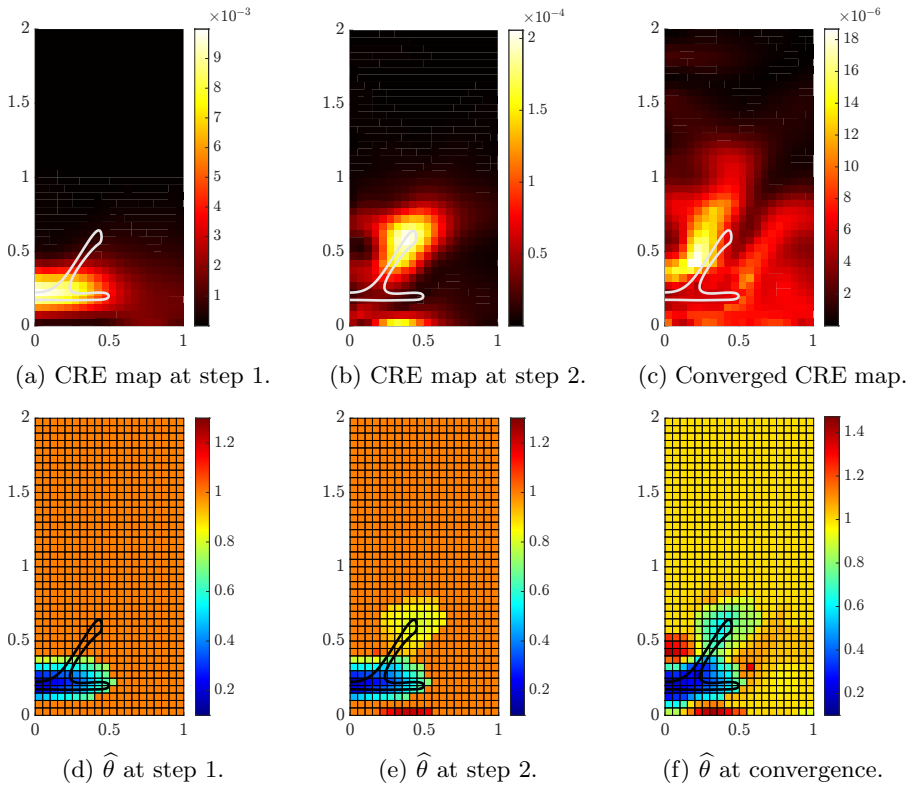
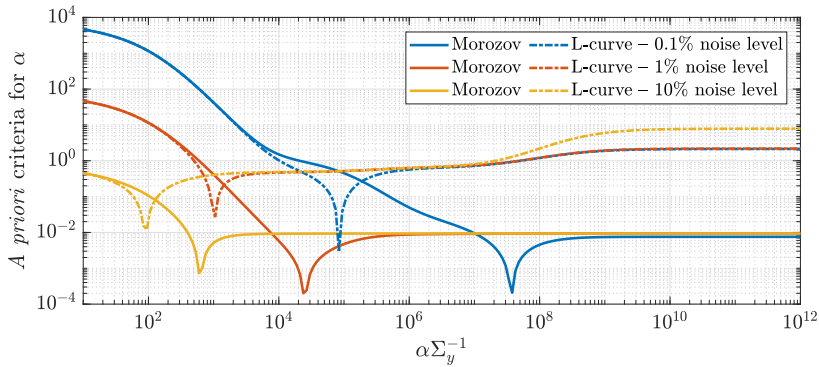


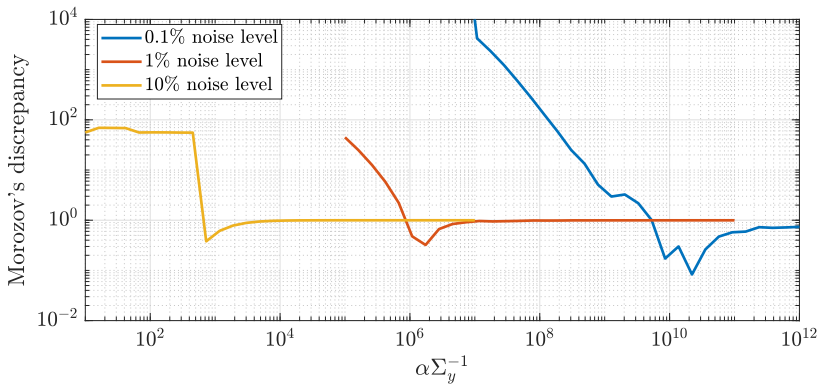
Fig. 3: Fully automated mCRE-based model updating results obtained from noise-free data ($d_s = 0.2$ m). CRE maps and parameter estimates at steps 1, 2 and at convergence (12 iterations are required here) are plotted.

function of δ and α . It allows to appreciate the combined effects of noise level and confidence in measurements on the values of η and η_d for a given sensor placement ($d_s = 0.1$ m). Besides, the values of $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$ are stored in TAB. 3 in order to compare them to the optimal values of α in the sense of the reconstruction errors (denoted $\hat{\alpha}(\eta)$ and $\hat{\alpha}(\eta_d)$).

Their comparison suggests that one should select the *a posteriori* Morozov discrepancy principle to calibrate α as its values are much closer to the optimal ones given in the sense of the reconstruction errors. On the contrary, the *a priori* criteria on α do not provide optimal values for damage detection in the considered case. They can be considered too "conservative" as they systematically underestimate the measurement error weight one should choose in the sense of η or η_d . Although, suboptimal, the "conservative" choice of α avoids the algorithm to diverge as excessive measurement weight may lead to non-acceptable results (with negative stiffness parameter values). However,



(a) L-curve and *a priori* Morozov criteria for the choice of α . Each value of criteria value at given α is obtained after evaluating the mCRE functional *a priori*.



(b) Morozov's discrepancy for the *a posteriori* choice of α . Each value of this criterion is obtained after performing the full mCRE-based model updating procedure.

Fig. 4: Comparison of the criteria for automated tuning of the measurement error weight α . Optimal values $\hat{\alpha}_\bullet$ are located at the minimum point of each curve. The *a priori* criteria are plotted together to emphasize that they provide significantly different values.

the identification results still remain consistent with the location of the defect and naturally lose accuracy with measurement noise, as shown in FIG. 6-7.

4.4 Influence of sensor density on damage detection performance

From the identification results presented in FIG. 8, we observe that the damage detection performance is directly related to the sensor density. As one could have expected, the more sensors are scattered, the more accurate identification

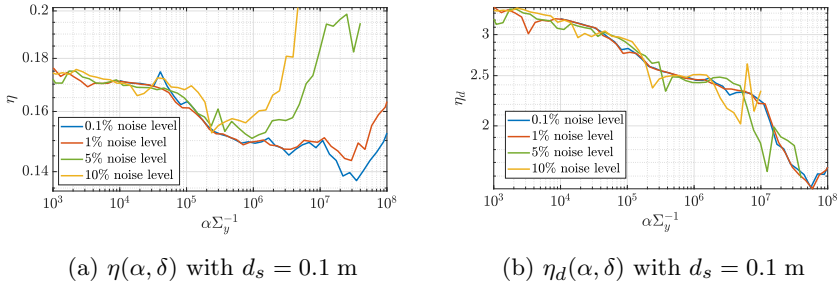


Fig. 5: Accuracy of the automated mCRE-based model updating algorithm using η and η_d according to α , δ at $d_s = 0.1$ m.

Noise level [%]	0.1	1	10
$\hat{\alpha}_1 \Sigma_y^{-1}$	$8.36 \cdot 10^4$	$1.05 \cdot 10^3$	$9.44 \cdot 10^1$
$\hat{\alpha}_2 \Sigma_y^{-1}$	$3.79 \cdot 10^7$	$2.38 \cdot 10^4$	$5.98 \cdot 10^2$
$\hat{\alpha}_3 \Sigma_y^{-1}$	$8.53 \cdot 10^9$	$1.08 \cdot 10^6$	$7.27 \cdot 10^2$
$\hat{\alpha}(\eta) \Sigma_y^{-1}$	$3.50 \cdot 10^7$	$2.94 \cdot 10^7$	$2.38 \cdot 10^5$
$\hat{\alpha}(\eta_d) \Sigma_y^{-1}$	$5.58 \cdot 10^7$	$5.92 \cdot 10^7$	$4.96 \cdot 10^6$

Table 3: Optimal values for $\alpha \Sigma_y^{-1}$ in the sense of the proposed criteria (L-curve, Morozov's discrepancy *a priori*, Morozov's discrepancy *a posteriori*) compared to the optimal values in the sense of the reconstruction errors η, η_d .

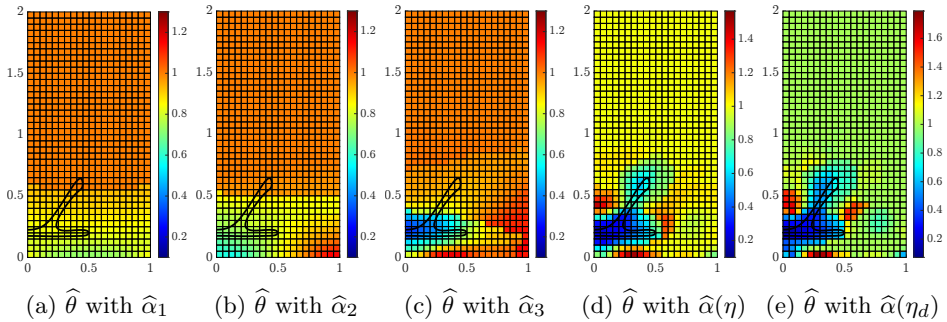


Fig. 6: Influence of α on parameter estimates provided by the automated mCRE-based model updating algorithm with 1% noisy data and $d_s = 0.1$ m.

results are. Note that the parameter estimates have been obtained with α calibrated according to Morozov's discrepancy principle *a posteriori*.

It emphasizes the fact that inverse problem parametrizations should be chosen in accordance with the associated sensor placement configuration.

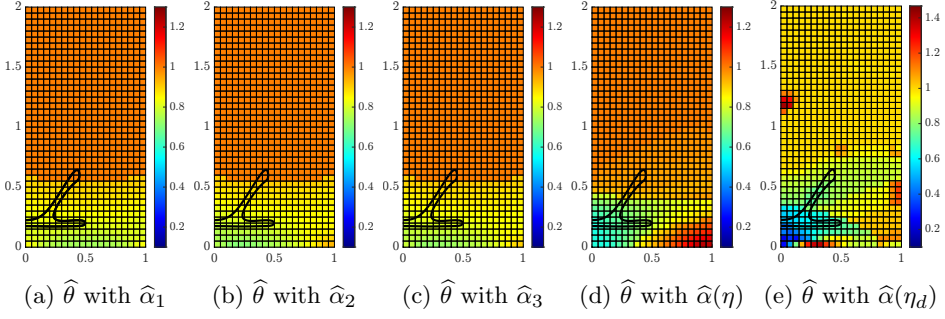


Fig. 7: Influence of α on parameter estimates provided by the automated mCRE-based model updating algorithm with 10% noisy data and $d_s = 0.1$ m.

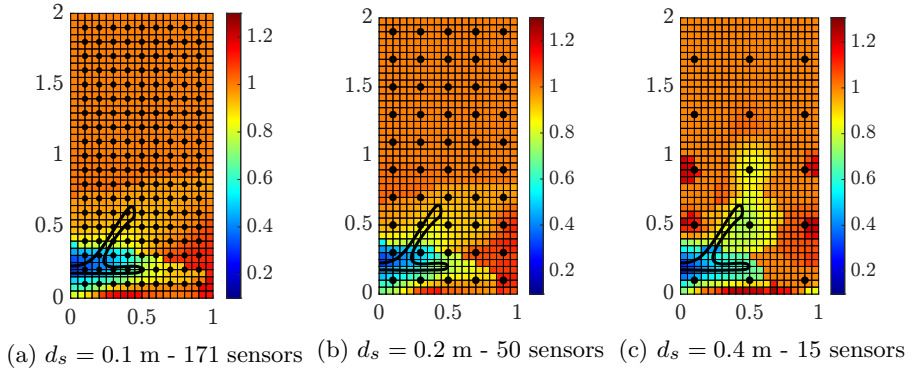


Fig. 8: Automated mCRE-based model updating algorithm - Influence of sensor density on parameter estimates. Results obtained with $\delta = 1\%$ and the optimal value of $\hat{\alpha}_3$. Sensor locations are specified with black dots.

These results also highlight that optimal sensor placement strategies should be applied in order to get the most relevant information from a restricted amount of data. We refer to our recent work [61] for more details on that feature.

As a remark, please note that very similar results have been obtained when identifying the two inclusions of FIG. 1, but as they remain individually convex and that the size of the defects is not locally thinner than the updated element size, reducing the amount of sensors has less effect on parameter estimates accuracy.

4.5 Multiple defect detection

Although the shape of the inclusions makes their identification easier compared to the Y-shape defect, mostly because the size of the defects is not locally thinner than the size of updated elements, the fact that there are two defects

to identify simultaneously is an interesting challenge to face with CRE-based clustering to perform optimal parametrization. The expected parameter values of the inclusions located at $y = 0.5$ m and at $y = 1.4$ m have been respectively chosen at 0.5 and 0.7. We present in FIG. 9 and FIG. 10 identification results obtained from both noise-free and noisy data with a sensor grid such that $d_s = 0.2$ m. The automated procedure with α calibrated *a posteriori* has been used.

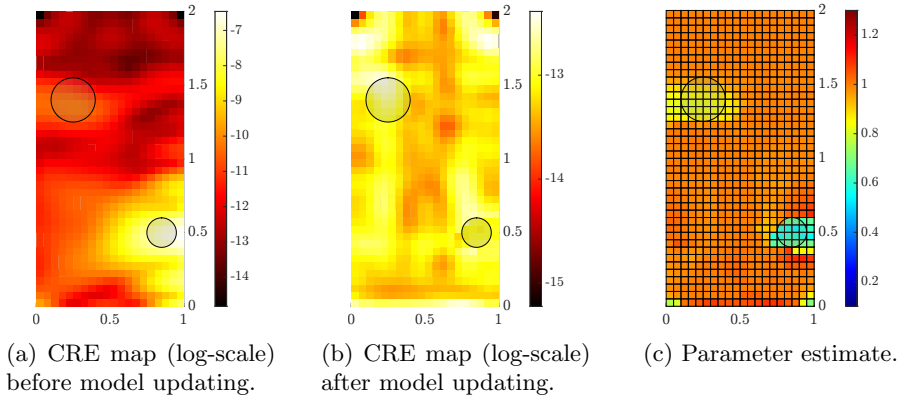


Fig. 9: Identification of the two circular inclusions from noise-free accelerometer data. CRE maps are plotted in log-scale to highlight the heterogeneous sensitivity of inclusions due to their location.

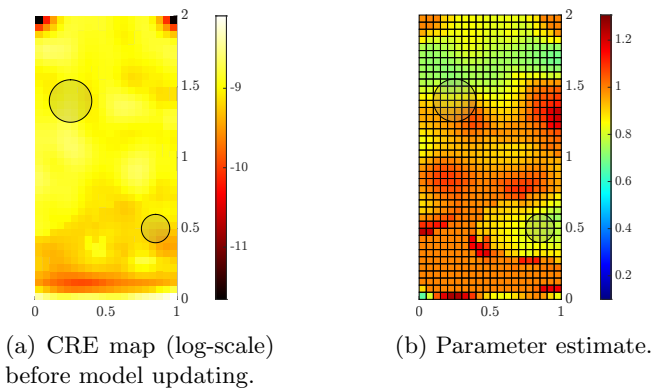


Fig. 10: Identification of the two circular inclusions from 10% noisy accelerometer data. The initial CRE map is already almost uniform due to the presence of measurement noise, making the clustering approach less efficient.

These results illustrate how the methodology is able to identify accurately multiple defects simultaneously, as attested by the emphasis of the initial CRE map on the defects. As mentioned previously, performing clustering without considering the barycentric coordinates of finite elements allows to avoid the (iterative) process on the optimal number of clusters to distinguish. However, when the signal-to-noise ratio is too low (see FIG. 10), the CRE map does not allow the CRE-based clustering to restrict the model updating to damaged areas³.

If the model updating algorithm has been able to identify defects so far, it is worth noticing that the top defect stiffness parameter has not been accurately recovered (0.9 instead of 0.7 for noise-free data, not captured at all with 10% measurement noise). If the presence of measurement noise naturally disrupts the identification, this issue is also related to inherent sensitivity features: as the mCRE is strongly related to strain energy, one could not expect to identify accurately field parameters in areas where little strain energy is present. Considering the fact that the structure is cantilevered, most of the strain energy is stored in the bottom part, and it is harder to properly identify defects in the upper half of the structure. It clearly appears on the CRE map computed before performing model updating, which has been plotted in log-scale to observe that the top defect appears as secondary for the first iteration of the model updating process.

Therefore, it should be kept in mind that the identification is feasible because of the sensor placement, the input solicitation, and the boundary conditions make the updated parameters sensitive enough to the model updating functional. If these conditions are not met, then one should not hope to identify defects with high accuracy. Actually, this remark has to be made for all inverse problem strategies. The authors thus recommend mCRE users to plot relative confidence intervals at convergence [30]. This low-cost operation will allow to assess (at least) the possible difficulties in identifying some parameters.

4.6 Effects of additional sparsity-promoting regularization

Several parameter estimates obtained from the same 10%-noisy measurements are plotted in FIG. 11 to observe the effect of sparsity-promoting regularization on parameter estimates. The *a priori* knowledge is set to $\theta_0 = 1$. The weighting parameter α is calibrated with Morozov's discrepancy principle.

As written above, the addition of a sparse regularization term to the mCRE should be seen as a backup to the method for identifying localized defects more accurately. One can observe that it permits to provide slightly more accurate results when the weighting parameter γ is well calibrated. It is more visible for the bottom inclusion to which the model updating functional is more sensitive.

³Note that the classical localization of most erroneous areas method would not be more efficient either in this situation.

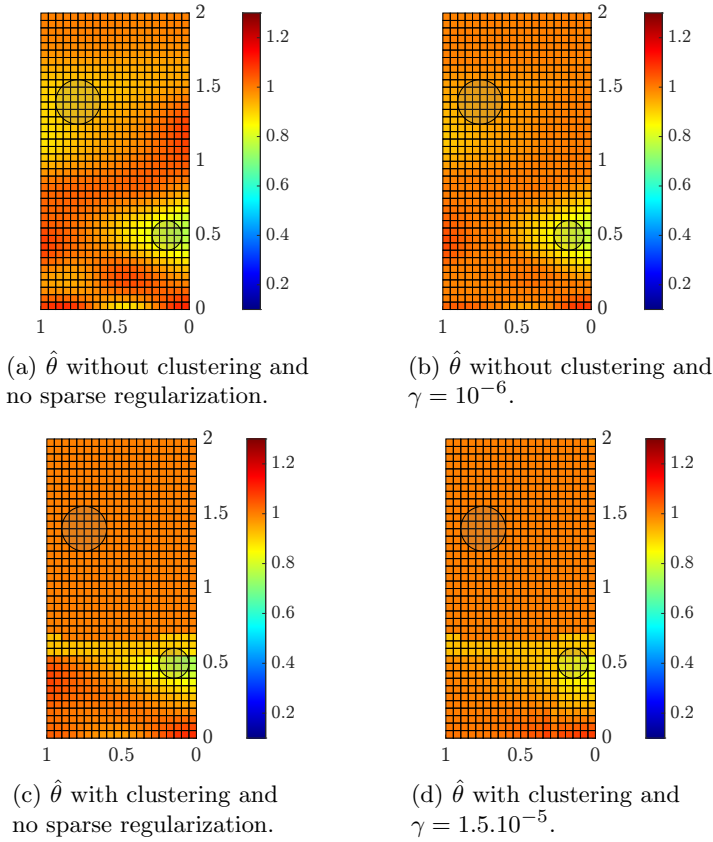


Fig. 11: Effect of sparsity-promoting regularization on the identification of the two inclusions from 10% noisy accelerometer data. Reference results without sparse regularization are given. The combined effect with CRE-based clustering is also considered.

Taking into account CRE-based clustering naturally impacts the optimal value of γ , *i.e.* the one that allows to describe the inclusions without spurious effects in the healthy area. Besides, note that the initial guess perfectly matches with the expected parameter value on non-damaged areas here, thus making the chosen *a priori* value $\theta_0 = 1$ highly relevant. This may not be the case for other applications, in particular considering industrial applications where the stiffness distribution might not be homogeneous.

As one can remark from FIG. 11 as well, the use of the CRE-based clustering step makes the effect of the additional sparse regularization less relevant as correcting actions are already focused on sensitive and erroneous areas.

In spite of these limitations, the additional sparsity-promoting regularization term, if well calibrated, can be beneficial for getting more accurate

parameter estimates. However, due to the fact it requires some *a priori* knowledge, this regularization should be seen as an option that can be considered and calibrated in a case-by-case approach.

5 Conclusions and prospects

In this paper, a model updating framework based on the modified Constitutive Relation Error has been proposed and assessed for vibration-based damage detection. It has the particularity to be fully automated with several guidelines provided for (i) the calibration of the data-to-model distance relative weight and, (ii) the CRE-based design of the parameter space.

As the selection of α , the computation of the CRE map and the clustering steps are fast operations, significant CPU time savings are made without loosing accuracy. The robustness with respect to measurement noise has also been illustrated, particularly due to the selection of $\hat{\alpha}$. CRE-based clustering generalizes and extends the concept of localization of most erroneous areas. It facilitates damage detection as it strongly restrains the number of parameters to update from the same amount of measurements. This is particularly well suited for damage detection as such defects are supposed to appear locally (at the structural scale). Additional sparse regularization has been integrated to increase damage detection accuracy, but requires some *a priori* knowledge and dedicated calibration. Its effect is less significant when combined to CRE-based clustering.

When assessing the proposed approach on academic earthquake engineering-inspired examples, it appeared that the automated procedure does not systematically lead to optimal results, in particular regarding the selection of $\hat{\alpha}$, the density of sensing devices and the sensitivity of parameters with respect to the mCRE functional. We thus provide several recommendations on the application of the model updating algorithm according to the context of use:

- ▷ Before model updating, users should perform a sensitivity analysis so as to observe which areas will be prone to correcting actions. After model updating, confidence intervals should be plotted to analyze which parameters are most uncertain. Due to the energy-based definition of the mCRE, the identification of defects will be difficult in non-sensitive or heterogeneous areas (in the sense of the strain energy distribution). This problem-dependent issue should not be decisive for the case of damage detection from ground motion testings, but it could be the case when trying to identify inherent defects, *e.g.* due to material uncertainty.
- ▷ If real-time constraints exist or if the model updating is exploited for data assimilation, a first precalibration test (with a typical low-magnitude random input) should be made to calibrate all mCRE internal parameters at best. Besides, the initial model guess would be already well calibrated regarding the initial damage state of the specimen under study. This way,

the mCRE-based data assimilation framework developed in [56] could integrate all the proposed improvements. The obtained results will not be optimal (as evidenced before), but they should be relevant enough to provide accurate results in real-time.

- ▷ If there is no particular time constraint, *i.e.* if the algorithm is only exploited as a post-processing tool, then one can afford running several mCRE minimizations around the suboptimal values of $\hat{\alpha}$ to look for optimal results.

As it has been evidenced that sensor placements are critical in the accuracy of identification results, dedicated studies dealing with optimal sensor placement [61] and the influence of uncertain sensor locations are currently being conducted. Other current investigations of the authors deal with mCRE-based model updating using full-field (digital image correlation) or locally rich (optic fibers) measurements. In such a context, not all the available data can be processed at once due to computational limitations. The updated model could then be progressively enriched to integrate data and refine the parameter space only where needed, whether using model selection or mesh adaptation techniques. Coupling the automated model updating algorithm with mesh adaptation and model selection techniques is a promising idea, that will require dedicated efforts in a long-term perspective of this work.

Appendix A Morozov's discrepancy principle for the mCRE framework - mathematical developments

In order to go through technical details without carrying a burden due to notation complexity, please note that the notations used in the following appendices are not necessarily consistent with the ones of the contribution.

A.1 Morozov's discrepancy principle

Let us consider an inverse problem $F(x) = y$ with its associated regularized cost function $\mathcal{J} = \alpha \|F(x) - y^\delta\|^2 + R(x)$, R being the regularization function (in Tikhonov's sense). As explained in [46], assuming that a scalar δ quantifies measurement noise such that noisy data y satisfies

$$\|F(x) - y\|^2 \leq \delta^2 \leq \|y\|^2 \quad (\text{A1})$$

and that R is strictly convex, non-negative, weakly coercive and weakly-lower semi-continuous, then there exists an optimal weighting $\hat{\alpha}(\delta)$ associated to an inverse problem solution $x_{\hat{\alpha}}^\delta$ such that

$$\|F(x_{\hat{\alpha}}^\delta) - y\|^2 = \delta^2 \quad (\text{A2})$$

In other words, it is possible to calibrate the weighting coefficient α according to the noise level quantifier δ .

A.2 Adaptation to the mCRE-based model updating framework

As all the conditions on R are met by the modeling error term of the mCRE functional, some attention must be paid to the definition of the upper bound of the data-to-model distance. Let us start from the fact that time-histories can be legitimately modeled such that:

$$y(t) = y_{ex}(t) + \eta(t) \quad (\text{A3})$$

where $y_{ex}(t)$ is the exact measurement time-series that would have been obtained without any measurement noise and $\eta(t)$ is a white-noise signal, whose value at each acquisition time step follows a standard Gaussian random variable: $\eta(t) \sim \mathcal{N}(0, \delta_s^2) \forall t$ with δ_s the noise amplitude (standard deviation of measurement noise). Then, the frequency domain pre-processing step requires to take the (discrete) Fourier transform of measurements. Focusing on the frequency range D_ω , and using the linearity property of the Fourier transform, one has:

$$Y_\omega = Y_{ex,\omega} + H_\omega, \forall \omega \in D_\omega \quad (\text{A4})$$

where H_ω is the Fourier transform of the random process $\eta(t)$. From here, one can properly introduce the mCRE measurement error by replacing $Y_{ex,\omega}$ with ΠU_ω :

$$\|\Pi U_\omega - Y_\omega\|_{\Sigma_y^{-1}}^2 = \|H_\omega\|_{\Sigma_y^{-1}}^2, \forall \omega \in D_\omega \quad (\text{A5})$$

$$\Rightarrow \int_{D_\omega} \frac{z(\omega)}{2} \|\Pi U_\omega - Y_\omega\|_{\Sigma_y^{-1}}^2 d\omega = \int_{D_\omega} \frac{z(\omega)}{2} \|H_\omega\|_{\Sigma_y^{-1}}^2 d\omega \quad (\text{A6})$$

If one knows the statistics of the square modulus of the discrete Fourier transform of a random process $\|H_\omega\|^2$, then the δ scalar of the Morozov discrepancy theorem would be explicitly made available in a formulation dedicated to the mCRE.

To do so, one can first start from the fact that the discrete Fourier transform of a Gaussian random process is also a random variate whose pdf is a multivariate Gaussian law, both its real and imaginary parts taken alone are zero-mean Gaussian random variables of standard deviation $\sigma\sqrt{N/2}$, where N is the number of sampling points. Therefore, the Fourier transform of $\Re(H_\omega)$ and $\Im(H_\omega)$ are random variable following a centered Gaussian pdf of standard deviation $\sqrt{2}/(\delta_s\sqrt{N})$.

Remark: the dependency on the number of sampling points N seems intuitively logical as the more data points, the more accurate a Fourier transform of a white noise. This dependency can be canceled by normalizing the discrete

Fourier transform by $1/\sqrt{N}$, which is done numerically in the MATLAB[®] `fft` function.

As $\|H_\omega\|^2 = \Re(H_\omega)^2 + \Im(H_\omega)^2$, one can identify that the pdf of $\|H_\omega\|^2$ results from the sum of squared independent Gaussian random variables. Owing to the fact that the squared norm of vector storing d independent centered Gaussian random variables follows a χ^2 distribution of degree d , one can deduce that the pdf of $2N/\sigma^2\|H_\omega\|^2$ is a non-centered χ^2 distribution of degree 2. After variable change, one finally obtains an analytical formulation for the pdf of $\|H_\omega\|^2$:

$$\pi_{\|H_\omega\|^2}(x) = \frac{e^{-x/(\delta_s^2/N)}}{\delta_s^2/N} \quad (\text{A7})$$

Remark: The real and imaginary components of a Fourier transform indeed correspond to the individual Fourier transforms even and odd components of the time domain function. As all functions can be decomposed as a sum of an even and odd function and since the Fourier transform is a one-to-one mapping between the time and frequency domains, the lack of correlation between even and odd parts in the time domain would imply a lack of correlation in the frequency domain too. As we deal with a zero-mean white-noise time series, the real and imaginary parts of its Fourier transform are thus uncorrelated.

Therefore, the squared modulus of a zero-mean random process $\|H_\omega\|^2$ follows a non-centered $\chi^2(2)$ probability distribution conditioned by the measurement noise variance δ_s^2 , whatever the value of ω . In particular, one can note that

$$\mathbf{E}(\|H_\omega\|^2) = \delta_s^2 \quad (\text{A8})$$

where $\mathbf{E}(\square)$ is the mathematical expectation operator. Assuming $\Sigma_y^{-1} = \delta_s^2 I$, i.e. noise is uncorrelated, then one recovers (12), which is the adaptation of Morozov's discrepancy principle to the mCRE framework (in the frequency domain).

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