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## RESEARCH ARTICLE

# Bounded Output-Feedback Consensus-based Formation Control of Nonholonomic Vehicles

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**Abstract**

In multi-agent systems, most commonly, consensus consists in all agents converging to a common equilibrium point. For nonholonomic vehicles, which move on a Cartesian plane and occupy a physical space, it is more appropriate to speak of consensus-based formation, which implies that the vehicles are required to converge to a relative position in a given formation while, distributively, agreeing on the center of such formation and on their orientation. In this paper we solve such a problem via a smooth, time-varying, output-feedback control; more precisely, without velocity measurements. In addition, and this is our main contribution, the controller is designed to satisfy pre-imposed bounds to avoid input saturation. To the best of the authors' knowledge this is the first controller that solves the consensus-based formation problem for multi-agent nonholonomic vehicles via a bounded, output-feedback controller. The paper also provides a simulation comparison between our proposal and its unbounded counterpart.

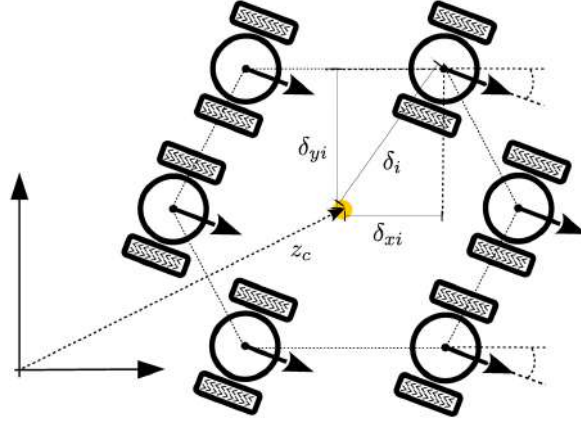
**KEYWORDS:**

Nonholonomic vehicles; consensus; output-feedback control

## 1 | INTRODUCTION

The control objective in the consensus of multiple dynamical agents is to ensure that the states of all agents converge to a common value by sharing (part of) their state with their corresponding neighbors [1, 2, 3]. For nonholonomic vehicles, which obviously cannot occupy the same physical space, the consensus-based formation problem consists in making the positions of all robots converge to a common value, modulo an offset, *i.e.*, a vector originating at the formation's center which determines a predefined position of the robot with respect to that center, while also agreeing on their final orientation value—see Figure 1 for an illustration. This is the problem addressed in this paper. Now, consensus is intrinsically a set-point control problem and the nonholonomic restrictions prevent the use of smooth autonomous controllers to solve a consensus problem [4]. In fact, set-point stabilization is *not* a particular case of tracking control [5]. Consensus-formation control is pertinent as part of a more complex task [6], *e.g.*, one in which scattered robots are, first, required to gather in formation regardless of a particular point and, second, to initiate a scouting mission.

Consensus of nonholonomic mobile robots has been studied, for instance, in [7] where a decentralized feedback control that drives a system of multiple nonholonomic unicycles to a rendezvous point in terms of both position and orientation is proposed, the control law is discontinuous and time-invariant. In [8] necessary and sufficient conditions for the feasibility of a class of position formations are laid. In [9] the position/orientation formation control problem for multiple nonholonomic agents using a time-varying controller that leads the agents to a given formation using only their orientation is proposed. In [10] a consensus



**FIGURE 1** Control goal: for the vehicles to reach a consensus-based formation, that is, to meet around an *a priori* unknown rendezvous set-point and acquire a common orientation.

control law for multi-agent nonholonomic agents, in the presence of bounded disturbances with unknown dynamics in all inputs channels is presented. In [11] a smooth formation control law using a consensus-based approach to drive a group of agents to a desired geometric pattern is proposed. The latter result is extended in [12] by introducing a proportional derivative controller for the velocity dynamics. We remark that the controllers proposed in all of these works rely on velocity measurements, which are often contaminated by noise or affected by sensor defects. Therefore, it is desirable to design controllers that rely only on position and orientation measurements [13].

Even though there is an abundant literature on formation control of autonomous vehicles, there are only a few works that propose output-feedback controllers. For instance, in [14] a discontinuous leader-follower formation-tracking controller is designed, but the problem addressed therein is fundamentally different to that of consensus-based formation since a reference *trajectory* is pre-imposed. In [15] an output-feedback consensus-based formation controller is proposed, but the proposed scheme is prone to saturate the vehicle's actuators (motors).

In concrete applications, however, the actuators are prone to saturation, which may seriously downgrade the overall performance and put the actuators at risk of thermal and mechanical failure. Hence, saturation of the actuators must be always avoided [16]. For instance, in [17, 18] the consensus problems are solved considering actuator saturation without requiring velocities to be measurable. However, none of the results in the latter references apply to nonholonomic vehicles.

In this work we propose a novel dynamic controller that is designed by mimicking a mechanical mass-spring-damper system that is interconnected to the plant through virtual springs. This allows us to inject damping in the controller dynamics and thus to avoid the use of velocity measurements. Through the spring interconnection, the controller steers the second-order plant's positions and also injects damping into the plant. In that regard, it is remarked that the control technique is similar to the one proposed in [19] and as in the latter, our controller is smooth time-varying and relies on persistency of excitation to overcome the restrictions imposed by the nonholonomic constraints—see also [20]. Thus, our work extends previous results in [12, 19, 15] by solving the consensus-based formation control problem under input saturation and without velocity measurements. The main contribution of our work is a novel decentralized output-feedback controller under input constraints that solves the consensus-based formation problem for multi-agent nonholonomic vehicles exchanging information over an undirected graph.

The rest of the paper is organized as follows. In the next section we describe the model and formulate the consensus-based formation control problem. In section 3 we present our main result. Numerical simulation tests are provided in Section 4 and the paper is wrapped up with concluding remarks in Section 5.

Throughout the paper, the following notation is employed.  $\mathbb{R} := (-\infty, \infty)$ ,  $\mathbb{R}_{\geq 0} := [0, \infty)$ .  $\mathbf{1}_N$  is a vector of all elements equal to one of size  $N$ .  $\otimes$  is the standard Kronecker product. For any absolutely continuous function  $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$ , the  $\mathcal{L}_\infty$ -norm is defined as  $\|f\|_\infty := \text{ess sup}_{t \geq 0} |f(t)|$ , and the square of the  $\mathcal{L}_2$ -norm as  $\|f\|_2^2 := \int_0^\infty |f(t)|^2 dt$ . The  $\mathcal{L}_\infty$  and  $\mathcal{L}_2$  spaces are defined as the sets  $\{f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n : \|f\|_\infty < \infty\}$  and  $\{f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n : \|f\|_2 < \infty\}$ , respectively. The subscript  $i \in \bar{N} := \{1, 2, \dots, N\}$ .

## 2 | DYNAMIC MODEL AND PROBLEM FORMULATION

We consider a group of  $N$  nonholonomic vehicles under the assumption that there is no slippage. For each vehicle, the kinematics model is given by

$$\dot{z}_i = \varphi(\theta_i) v_i, \quad \varphi(\theta_i) := [\cos(\theta_i) \ \sin(\theta_i)]^\top \quad (1a)$$

$$\dot{\theta}_i = \omega_i, \quad (1b)$$

where,  $z_i = [x_i \ y_i]^\top \in \mathbb{R}^2$  denotes the Cartesian coordinates of the  $i$ th vehicle on the plane and  $\theta_i \in \mathbb{R}$  denotes its orientation;  $v_i \in \mathbb{R}$  and  $\omega_i \in \mathbb{R}$  are the linear and the angular velocity, respectively.

The vehicles' dynamic behavior follows a torque-controlled second-order system of the form

$$\begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} + \begin{bmatrix} f_{vi} & 0 \\ 0 & f_{\omega i} \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \frac{1}{r_i} \begin{bmatrix} 1 & 1 \\ 2R_i & -2R_i \end{bmatrix} \begin{bmatrix} \tau_{li} \\ \tau_{ri} \end{bmatrix}, \quad (2)$$

where  $I_i$  is the robot inertia,  $m_i$  is the mass,  $f_{vi}$ ,  $f_{\omega i}$  are the friction coefficients,  $\tau_{li}$  and  $\tau_{ri}$  are the left and right wheel torques, respectively,  $r_i$  is the wheel radius, and  $R_i$  is the wheel axle length.

Regarding the input torques, we assume the following.

**Assumption 1.** The left and right wheel torques  $\tau_{li}$ ,  $\tau_{ri}$  may saturate and thus they satisfy  $|\tau_{li}| \leq \bar{\tau}_{li}$  and  $|\tau_{ri}| \leq \bar{\tau}_{ri}$ , with known strictly positive saturation bounds  $\bar{\tau}_{li} > 0$  and  $\bar{\tau}_{ri} > 0$ , respectively.  $\triangleleft$

We also assume that only the Cartesian coordinates and the orientation of the vehicles are available for measurement and hence the linear and the angular velocities,  $v_i$  and  $\omega_i$  cannot be measured. Thus the following assumption.

**Assumption 2.** The Cartesian position  $z_i$  and the orientation  $\theta_i$ , for each vehicle, are available.  $\triangleleft$

In this work, we require the vehicles to meet in a given formation around a *non*-predefined rendezvous point on the plane, denoted  $z_c := (x_c, y_c)$ , and acquire a *non*-predefined common orientation, denoted  $\theta_c$ . The position of the  $i$ th vehicle, modulo a given offset  $\delta_i = [\delta_{xi} \ \delta_{yi}]^\top$ , with  $i \in \bar{N}$ , relative to the unknown center of the formation—see Figure 1, is defined as  $\bar{z}_i := z_i - \delta_i$ . Correspondingly,  $\bar{x}_i := x_i - \delta_{xi}$  and  $\bar{y}_i := y_i - \delta_{yi}$ . Hence, our control goal is to make

$$\lim_{t \rightarrow \infty} v_i(t) = 0, \quad \lim_{t \rightarrow \infty} \bar{z}_i(t) = z_c, \quad (3)$$

$$\lim_{t \rightarrow \infty} \omega_i(t) = 0, \quad \lim_{t \rightarrow \infty} \theta_i(t) = \theta_c \quad \forall i \in \bar{N}, \quad (4)$$

for all initial conditions.

In contrast to the leader-follower formation control problem the control goal defined above is leaderless, since there is no pre-specified reference. That is, neither the coordinates  $(x_c, y_c)$  nor the angle  $\theta_c$  are imposed *a priori*, but they depend on the initial postures, on the systems' *nonlinear* dynamics, and on the graph's topology. This consensus-based formation problem has been successfully solved under different conditions, but the originality of this paper resides in considering the realistic scenario determined by Assumptions 1 and 2 above.

In order to solve the aforementioned problem, it is also assumed that each vehicle exchanges its position  $(x_i, y_i)$  and its orientation  $\theta_i$ —see Figure 1—with a set of neighbors, which we denote by  $\mathcal{N}_i$ . It is naturally assumed that once a communication is set between two vehicles  $i$  and  $j \in \mathcal{N}_i$ , the flow of information is bidirectional and is never lost. Whence the following.

**Assumption 3.** The interconnections are static and are modeled via an undirected and connected graph.  $\triangleleft$

*Remark 1.* In graph theory, a graph is undirected if the nodes exchange information in both directions, it is static if the interconnection is constant, and it is connected if any node is reachable from any other node. The latter is a necessary condition to achieve consensus [21].

In order to establish the proof of our result, it is convenient to define the Laplacian matrix  $L := [\ell_{ij}] \in \mathbb{R}^{N \times N}$ , where

$$\ell_{ij} = \begin{cases} \sum_{k \in \mathcal{N}_i} a_{ik} & i = j \\ -a_{ij} & i \neq j, \end{cases} \quad (5)$$

additionally  $a_{ij} > 0$  if  $j \in \mathcal{N}_i$  and  $a_{ij} = 0$  otherwise. Defining  $\mathbf{1}_N := [1 \cdots 1]^\top$  then, by construction,  $L\mathbf{1}_N = 0$  and, after Assumption 3,  $L$  is symmetric, it has a unique zero-eigenvalue, and all of its other eigenvalues are strictly positive. Thus,  $\text{rank}(L) = N - 1$ .

### 3 | MAIN RESULT

The proposed scheme contains an inner control-loop that is designed as

$$\begin{bmatrix} \tau_{ri} \\ \tau_{li} \end{bmatrix} = \frac{r_i}{2} \begin{bmatrix} 1 & \frac{1}{2R_i} \\ 1 & -\frac{1}{2R_i} \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{oi} \end{bmatrix}. \quad (6)$$

Then, replacing (6) in (2), we obtain

$$\dot{v}_i = \frac{1}{m_i}(u_{vi} - f_{vi}v_i), \quad (7a)$$

$$\dot{\omega}_i = \frac{1}{I_i}(u_{oi} - f_{oi}\omega_i). \quad (7b)$$

The control design exploits the fact that the model of the nonholonomic system consists, essentially, in two interconnected second-order systems given by Eqs. (1a)-(7a) and (1b)-(7b). The first set of equations corresponds to the linear-motion dynamics while the latter corresponds to the angular-motion dynamics. They are interconnected through the term  $\varphi(\theta)$ . In other words, these systems are a pair of double integrators, but one of which is subject to the nonholonomic constraint  $\dot{x}_i^2 + \dot{y}_i^2 - v_i^2 = 0$ . Therefore, the consensus equilibrium is not stabilizable via smooth static feedback. This is because if smooth static feedback is used, the system admits multiple equilibria.

For each robot, we design a dynamic output-feedback controller that mimics the model of two other mechanical second order systems. Then, the controller's systems are coupled, through virtual nonlinear (saturated) springs, to the robots' linear and angular dynamics. To cope with the nonholonomic constraint, we use a so-called  $\delta$ -persistently exciting function [22] that ensures the unicity of the goal equilibrium. Furthermore, to achieve consensus stabilization via output feedback, we employ a control strategy that consists in making the dynamic *controllers* achieve consensus. In turn, due to the virtual-spring coupling, the robots also achieve consensus, both in position and orientation. This control strategy follows up the ideas first laid in [3] for Euler-Lagrange systems with holonomic constraints and used, also, in [19], where an analogy of the controller with a flexible-joint robot is made. With this in mind, the controller for the linear part is given by

$$u_{vi} = -k_{vi}\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi}) \quad (8a)$$

$$\ddot{\vartheta}_{vi} = -d_{vi}\dot{\vartheta}_{vi} - k_{vi} \tanh(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi}, \quad (8b)$$

where  $k_{vi}$ ,  $d_{vi}$ , and  $p_{vi}$  are positive constants and  $e_{vi}$  denotes the consensus error

$$e_{vi} := \sum_{j \in \mathcal{N}_i} a_{ij} (\vartheta_{vi} - \vartheta_{vj}), \quad (9)$$

and  $\tanh(\bar{z}_i - \vartheta_{vi}) := [\tanh(\bar{x}_i - \vartheta_{vix}) \quad \tanh(\bar{y}_i - \vartheta_{viy})]^\top$ .

Equation (8a) corresponds to the control input for the linear-motion dynamics. It is reminiscent of a virtual nonlinear spring coupling, with saturated mechanical force. The inclusion of  $\varphi(\theta_i)$  is motivated by the fact that the linear-motion kinematics corresponds to a nonholonomic integrator. The equation (8b) consists, first, on the closed-loop system of an ordinary second-integrator system—hence, a (unitary-) mass-damper system,

$$\ddot{\vartheta}_{vi} = -d_{vi}\dot{\vartheta}_{vi} - p_{vi} \sum_{j \in \mathcal{N}_i} a_{ij} (\vartheta_{vi} - \vartheta_{vj}), \quad (10)$$

with a rather standard consensus control law, so it is guaranteed to achieve consensus, that is,  $\vartheta_{vi} \rightarrow \vartheta_{vj}$  for all  $i, j \leq N$  [21]. Second, (10) is coupled to the robot system through the term  $-k_{vi} \tanh(\vartheta_{vi} - \bar{z}_i)$ . This yields (8b).

The design of the dynamic output-feedback controller with bounded output for the angular-dynamics follows a similar rationale. We define

$$u_{oi} = -k_{oi} \tanh(\theta_i - \vartheta_{oi}) \quad (11a)$$

$$\ddot{\vartheta}_{oi} = -d_{oi}\dot{\vartheta}_{oi} - k_{oi} \tanh(\vartheta_{oi} - \theta_i) - p_{oi}e_{oi} + \alpha_i, \quad (11b)$$

where  $k_{\omega i}$ ,  $d_{\omega i}$ , and  $p_{\omega i}$  are positive constants and

$$e_{\omega i} := \sum_{j \in \mathcal{N}_i} a_{ij} (\vartheta_{\omega i} - \vartheta_{\omega j}). \quad (12)$$

In contrast to the linear-motion controller, (11b) contains an additional term, denoted  $\alpha_i$ . This is an input to the controller that depends both on time and the plant's states. It is added to ensure that the stabilization of the unique closed-loop system's equilibrium implies that  $e_{vi} \rightarrow e_{\omega i} \rightarrow 0$ . The term  $\alpha_i$  excites the angular dynamics as long as the consensus equilibrium has not been reached; it prevents the multi-agent system to converge to unwanted equilibria. To that end, we define

$$\alpha_i(t, \theta_i, \vartheta_{vi}, \bar{z}_i) := k_{\alpha i} \psi_i(t) \varphi(\theta_i)^{\perp \top} \tanh(\vartheta_{vi} - \bar{z}_i), \quad (13)$$

where  $k_{\alpha i} > 0$ ,  $\varphi(\theta_i)^{\perp} := [-\sin(\theta_i) \quad \cos(\theta_i)]^{\top}$  is the annihilator of  $\varphi(\theta_i)$ , so  $\varphi(\theta_i)^{\perp \top} \varphi(\theta_i) = \varphi(\theta_i)^{\top} \varphi(\theta_i)^{\perp} = 0$ .

The time-varying function  $\psi_i$  is designed such that  $\dot{\psi}_i$  is persistently exciting, that is, there exist  $T$  and  $\mu > 0$  such that

$$\int_t^{t+T} \dot{\psi}_i(s)^2 ds \geq \mu, \quad \forall t \geq 0. \quad (14)$$

Controllers using such persistently-exciting, yet vanishing disturbances are known as  $\delta$ -persistently controllers [23]. The terminology is motivated by the rationale that the function  $\alpha_i$  is persistently exciting as long as the errors are away from a  $\delta$ -neighbourhood of the origin. The induced excitation, however, compels the trajectories to converge.

We are now ready to present our main statement.

**Proposition 1.** The controller defined in (6), (8), (11) and (13) ensures that the consensus-based formation objective (3)–(4) holds globally for any strictly positive gains  $k_{ki}$ ,  $d_{ki}$  and  $p_{ki}$ , with  $k \in \{v, \omega\}$  and  $i \in \bar{N}$ . Additionally, if gains  $k_{vi}$  and  $k_{\omega i}$  are set as

$$\frac{4R_i}{r_i} \min\{\bar{\tau}_{ri}, \bar{\tau}_{li}\} > 2\sqrt{2}R_i k_{vi} + k_{\omega i}, \quad (15)$$

the controller never saturates the actuators and thus  $|\tau_{ki}| < \bar{\tau}_{ki}$ , for  $k \in \{l, r\}$ .  $\triangle$

*Proof.* We start by showing that the actuators do not saturate if (15) holds. To that end, note that from the properties of  $\tanh(\cdot)$  we have  $|u_{vi}| \leq \sqrt{2}k_{vi}$  and  $|u_{\omega i}| \leq k_{\omega i}$ . Therefore, from (6) we have

$$\max\{|\tau_{li}|, |\tau_{ri}|\} \leq \frac{r_i}{2} \left[ \sqrt{2}k_{vi} + \frac{1}{2R_i} k_{\omega i} \right].$$

Hence, setting  $k_{vi}$  and  $k_{\omega i}$  such that (15) holds, it follows that  $|\tau_{ri}| < \bar{\tau}_{ri}$  and  $|\tau_{li}| < \bar{\tau}_{li}$ .

Next, we write the linear- and angular-motion closed-loop equations,

$$\Sigma_{vi} \begin{cases} \dot{z}_i = \varphi(\theta_i)v_i, \\ \dot{v}_i = -\frac{1}{m_i} (f_{vi}v_i + k_{vi}\varphi(\theta_i)^{\top} \tanh(\bar{z}_i - \vartheta_{vi})), \\ \ddot{\vartheta}_{vi} = -d_{vi}\dot{\vartheta}_{vi} - k_{vi} \tanh(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi}, \end{cases} \quad (16)$$

and

$$\Sigma_{\omega i} \begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\frac{1}{I_i} (f_{\omega i}\omega_i + k_{\omega i} \tanh(\theta_i - \vartheta_{\omega i})), \\ \ddot{\vartheta}_{\omega i} = -d_{\omega i}\dot{\vartheta}_{\omega i} - k_{\omega i} \tanh(\vartheta_{\omega i} - \theta_i) - p_{\omega i}e_{\omega i} + \alpha_i. \end{cases} \quad (17)$$

Then, the rest of the proof is divided in two steps. In Step 1 we show boundedness of trajectories; in Step 2 we show that all errors converge asymptotically to zero.

Step 1. Consider the candidate Lyapunov function in the space of the consensus errors,

$$V = \sum_{i=1}^N \left[ \frac{1}{p_{vi}} \mathcal{E}_{vi} + \frac{1}{4} \sum_{j \in \mathcal{N}_i} a_{ij} |\vartheta_{vi} - \vartheta_{vj}|^2 \right], \quad (18)$$

where

$$\mathcal{E}_{vi} := \frac{m_i}{2} v_i^2 + \frac{1}{2} |\dot{\vartheta}_{vi}|^2 + k_{vi} \ln(\cosh(\bar{x}_i - \vartheta_{vix})) + k_{vi} \ln(\cosh(\bar{y}_i - \vartheta_{viy})),$$

$\bar{x}_i := x_i - \delta_{xi}$  and  $\bar{y}_i := y_i - \delta_{yi}$ . Evaluating  $\dot{V}$  along the trajectories of (16) yields

$$\dot{V} = - \sum_{i=1}^N \frac{1}{p_{vi}} [d_{vi} |\dot{\vartheta}_{vi}|^2 + f_{vi} v_i^2] \leq 0, \quad (19)$$

so  $\dot{V} \leq 0$  and, since  $V$  is positive definite and radially unbounded with respect to  $v_i$ ,  $\dot{\vartheta}_{vi}$ ,  $|\vartheta_{vi} - \bar{z}_i|$ , and  $|\vartheta_{vi} - \vartheta_{vj}|$ , it follows by integrating on both sides of  $\dot{V} \leq 0$  that  $v_i$ ,  $\dot{\vartheta}_{vi}$ ,  $|\vartheta_{vi} - \bar{z}_i|$ , and  $|\vartheta_{vi} - \vartheta_{vj}|$  are bounded. Furthermore, integrating on both sides of the first inequality in (19) we conclude that  $\dot{\vartheta}_{vi}, v_i \in \mathcal{L}_2$ .

Next, let

$$W := \sum_{i=1}^N \left[ \frac{1}{p_{\omega i}} \mathcal{E}_{\omega i} + \frac{1}{4} \sum_{j \in \mathcal{N}_i} a_{ij} (\vartheta_{\omega i} - \vartheta_{\omega j})^2 \right],$$

with

$$\mathcal{E}_{\omega i} := \frac{I_i}{2} \omega_i^2 + \frac{1}{2} \dot{\vartheta}_{\omega i}^2 + k_{\omega i} \ln \cosh(\vartheta_{\omega i} - \theta_i).$$

Evaluating  $\dot{W}$  along the trajectories of (17) results

$$\dot{W} \leq - \sum_{i=1}^N \frac{1}{p_{\omega i}} [f_{\omega i} \omega_i^2 + d_{\omega i} \dot{\vartheta}_{\omega i}^2 - \dot{\vartheta}_{\omega i} \alpha_i] \quad (20)$$

Since  $W$  is positive definite and radially unbounded in  $\omega_i$ ,  $\dot{\vartheta}_{\omega i}$ ,  $e_{\omega i}$ , and  $|\vartheta_{\omega i} - \theta_i|$  then (20) implies that  $\Sigma_{\omega i}$  is an output strictly passive map [24] from the input  $\alpha_i$  to the output  $(\omega_i, \dot{\vartheta}_{\omega i})$ . Furthermore, the closed-loop system  $\Sigma_{\omega i}$  has a property reminiscent of zero-state observability, in the sense that if the input and the output are equal to zero, *i.e.*,  $\alpha_i = 0$  and  $(\omega_i, \dot{\vartheta}_{\omega i}) = 0$  respectively, then there exists  $\theta_c \in \mathbb{R}$  such that the consensus errors satisfy  $(\theta_i - \theta_c, \omega_i, \vartheta_{\omega i} - \theta_c) = (0, 0, 0)$ . This last is established invoking Barbashin-Krasovskii's theorem with the fact that, when  $\alpha_i = 0$ ,  $\Sigma_{\omega i}$  is autonomous,  $W$  is positive definite and radially unbounded,  $\dot{W}$  is negative semi-definite, and  $\dot{W} = 0 \Rightarrow (\dot{\vartheta}_{\omega i}, \omega_i) = 0$ . Hence,  $\ddot{\vartheta}_{\omega i} = 0$  and  $\dot{\omega}_i = 0$  so, from the second equation in (17),  $\tanh(\theta_i - \vartheta_{\omega i}) = 0$ . Furthermore, from the third equation in (17), it follows that  $e_{\omega i} = 0$ , so using the properties of the Laplacian matrix this implies that  $L\vartheta_{\omega} = 0$ , *i.e.*, there exists  $\theta_c \in \mathbb{R}$  such that  $\vartheta_{\omega} = 1_N \theta_c$ .

Now, the time-varying function  $t \mapsto \psi_i(t)$  admits the upper bound  $\bar{\psi}_i$ , so we have that, for all trajectories,

$$|\alpha_i| \leq \sqrt{2} \bar{\psi}_i k_{\alpha i}.$$

Therefore it holds that

$$\dot{W} \leq - \sum_{i=1}^N \frac{1}{p_{\omega i}} \left[ f_{\omega i} \omega_i^2 + \left( d_{\omega i} |\dot{\vartheta}_{\omega i}| - \sqrt{2} \bar{\psi}_i k_{\alpha i} \right) |\dot{\vartheta}_{\omega i}| \right]. \quad (21)$$

Note that if  $|\dot{\vartheta}_{\omega i}| \geq \frac{\sqrt{2}}{d_{\omega i}} \bar{\psi}_i k_{\alpha i}$  then, by integration of  $\dot{W} \leq 0$ , and  $W$  is radially unbounded in  $\omega_i$ ,  $e_{\omega i}$ , and  $\dot{\vartheta}_{\omega i}$ , we obtain that  $\omega_i$ ,  $e_{\omega i}$ , and  $\dot{\vartheta}_{\omega i} \in \mathcal{L}_{\infty}$ . In turn, since also  $\alpha_i$  is bounded, we conclude from (17) that  $\dot{\omega}_i, \ddot{\vartheta}_{\omega i} \in \mathcal{L}_{\infty}$ .

Step 2. From the second equation in (16) note that  $\dot{v}_i \in \mathcal{L}_{\infty}$  because  $\varphi(\theta_i)$  and  $\tanh(\bar{z} - \vartheta_{vi})$  are uniformly bounded. Moreover, since  $v_i \in \mathcal{L}_2 \cap \mathcal{L}_{\infty}$  then, by Barbalát's Lemma —see *e.g.*, [25, p. 323],  $\lim_{t \rightarrow \infty} v_i(t) = 0$ . Additionally, since  $\ddot{\vartheta}_{vi}, \dot{\vartheta}_{vi} \in \mathcal{L}_{\infty}$  and  $\dot{\vartheta}_{vi} \in \mathcal{L}_2$  it follows also, from Barbalát's Lemma, that  $\lim_{t \rightarrow \infty} \dot{\vartheta}_{vi}(t) = 0$ .

Note also that  $v_i \in \mathcal{L}_{\infty}$  implies that  $\dot{z}_i \in \mathcal{L}_{\infty}$ , from (16). Hence,

$$\vartheta_{vi}^{(3)} = -d_{vi} \ddot{\vartheta}_{vi} - k_{vi} \begin{bmatrix} \operatorname{sech}^2(\vartheta_{vix} - \bar{x}_i) (\dot{\vartheta}_{vix} - \dot{\bar{x}}_i) \\ \operatorname{sech}^2(\vartheta_{viy} - \bar{y}_i) (\dot{\vartheta}_{viy} - \dot{\bar{y}}_i) \end{bmatrix} - p_{vi} \dot{e}_{vi} \quad (22)$$

is also bounded. The latter and the fact that

$$\lim_{t \rightarrow \infty} \int_0^t \ddot{\vartheta}_{vi}(\sigma) d\sigma = \lim_{t \rightarrow \infty} \dot{\vartheta}_{vi}(t) - \dot{\vartheta}_{vi}(0) = -\dot{\vartheta}_{vi}(0)$$

imply together, by Barbalát's Lemma, that  $\lim_{t \rightarrow \infty} \ddot{\vartheta}_{vi}(t) = 0$ .

Since all signals on the right-hand side of

$$\ddot{v}_i = - \frac{1}{m_i} \left( f_{vi} v_i + k_{vi} \omega_i \varphi(\theta_i)^{\perp \top} \tanh(\bar{z}_i - \vartheta_{vi}) + k_{vi} \begin{bmatrix} \operatorname{sech}^2(\vartheta_{vix} - \bar{x}_i) (\dot{\vartheta}_{vix} - \dot{\bar{x}}_i) \\ \operatorname{sech}^2(\vartheta_{viy} - \bar{y}_i) (\dot{\vartheta}_{viy} - \dot{\bar{y}}_i) \end{bmatrix} \right) \quad (23)$$

are bounded, then  $\ddot{v}_i \in \mathcal{L}_\infty$ . Note, moreover, that

$$\lim_{t \rightarrow \infty} \int_0^t \dot{v}_i(\sigma) d\sigma = \lim_{t \rightarrow \infty} v_i(t) - v_i(0) = -v_i(0),$$

which by Barbalát's Lemma implies that  $\lim_{t \rightarrow \infty} \dot{v}_i(t) = 0$ . Proceeding as above, we also conclude that  $\lim_{t \rightarrow \infty} \ddot{v}_i(t) = 0$ .

Now, convergence to zero of  $\dot{v}_i$ ,  $\ddot{v}_i$ ,  $\dot{\vartheta}_{vi}$ , and of  $v_i$ , implies that

$$\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi}) \rightarrow 0, \quad \omega_i \varphi(\theta_i)^{\perp\top} \tanh(\bar{z}_i - \vartheta_{vi}) \rightarrow 0.$$

On the other hand, the solutions of the equation

$$\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi}) = 0, \tag{24}$$

are of the form  $\tanh(\bar{z}_i - \vartheta_{vi}) = c_1 \varphi(\theta_i)^\perp$  with  $c_1 \in \mathbb{R}$  while the solutions of the equation

$$\omega_i \varphi(\theta_i)^{\perp\top} \tanh(\bar{z}_i - \vartheta_{vi}) = 0, \tag{25}$$

are of the form  $\tanh(\bar{z}_i - \vartheta_{vi}) = c_2 \omega_i \varphi(\theta_i)$  with  $c_2 \in \mathbb{R}$ . Therefore, (24) and (25) hold together if and only if  $c \omega_i \varphi(\theta_i) = \varphi(\theta_i)^\perp$  with  $c := c_1/c_2$ . In turn, the latter may hold only if either  $c = 0$  or  $\omega_i = 0$ .

If (24) and (25) hold simultaneously because  $c = 0$ , then  $\lim_{t \rightarrow \infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$ , hence from the last equation in (16), we have that

$$\lim_{t \rightarrow \infty} e_{vi}(t) = 0,$$

which in turn implies that  $\lim_{t \rightarrow \infty} (L \otimes I_2) \vartheta_v(t) = 0$ . In view of the properties of  $L$ , supports the existence of  $\bar{z}_c \in \mathbb{R}^2$  such that  $\vartheta_v = 1_N \otimes \bar{z}_c$ , or  $\vartheta_{vi} = \bar{z}_c$  for all  $i \in \bar{N}$ . Therefore

$$\lim_{t \rightarrow \infty} \bar{z}_i(t) = \lim_{t \rightarrow \infty} \vartheta_{vi}(t) = \bar{z}_c.$$

Another consequence of  $\lim_{t \rightarrow \infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$  is that  $\alpha_i$  also converges to zero. Thus, it can be seen as a vanishing perturbation of (17) and, in view of the attractivity of  $\{(\theta_i - \theta_c, \omega_i, \vartheta_{\omega i} - \theta_c) = (0, 0, 0)\}$  subject to  $\alpha_i = 0$  for all  $i \in \bar{N}$ , it follows that the equilibrium  $(\omega_i, \theta_i, \vartheta_{\omega i}, \dot{\vartheta}_{\omega i}) = (0, \theta_c, \theta_c, 0)$  is asymptotically stable. Convergence holds globally because the energy-like function  $W$  is radially unbounded. Since  $\alpha_i$  is globally bounded—by construction—and it vanishes, then standard theory for nonlinear systems with vanishing perturbations ensures that the equilibrium  $(\omega_i, \theta_i, \vartheta_{\omega i}, \dot{\vartheta}_{\omega i}) = (0, \theta_c, \theta_c, 0)$  is globally asymptotically stable.

If, alternatively, (24) and (25) hold simultaneously in the limit because  $\lim_{t \rightarrow \infty} \omega_i(t) = 0$ , then

$$\lim_{t \rightarrow \infty} \int_0^t \dot{\omega}_i(\sigma) d\sigma = -\omega_i(0).$$

Then, because

$$\ddot{\omega}_i = -\frac{1}{I_i} [f_{\omega i} \omega_i + k_{\omega i} \operatorname{sech}^2(\theta_i - \vartheta_{\omega i})(\omega_i - \dot{\vartheta}_{\omega i})] \tag{26}$$

is bounded—in view of the boundedness of  $\omega_i$  and  $\dot{\vartheta}$ —it follows that, also,  $\ddot{\omega}_i \in \mathcal{L}_\infty$ . Therefore,  $\lim_{t \rightarrow \infty} \dot{\omega}_i(t) = 0$  and

$$\lim_{t \rightarrow \infty} \int_0^t \ddot{\omega}_i(\sigma) d\sigma = -\dot{\omega}_i(0).$$

In turn, all the functions on the right-hand side of

$$\omega_i^{(3)} = -\frac{1}{I_i} (f_{\omega i} \ddot{\omega}_i - 2k_{\omega i} \operatorname{sech}^2(\theta_i - \vartheta_{\omega i}) \tanh(\theta_i - \vartheta_{\omega i})(\omega_i - \dot{\vartheta}_{\omega i})^2 + k_{\omega i} \operatorname{sech}^2(\theta_i - \vartheta_{\omega i})(\dot{\omega}_i - \ddot{\vartheta}_{\omega i})) \tag{27}$$

are bounded, so  $\omega_i^{(3)} \in \mathcal{L}_\infty$ . After Barbalát's Lemma,  $\lim_{t \rightarrow \infty} \ddot{\omega}_i(t) = 0$ , so, from (26),  $\lim_{t \rightarrow \infty} \dot{\vartheta}_{\omega i}(t) = 0$ . Proceeding as before and invoking systematically Barbalát's Lemma it follows that  $\lim_{t \rightarrow \infty} \ddot{\vartheta}_{\omega i}(t) = 0$  and that  $\lim_{t \rightarrow \infty} \vartheta_{\omega i}^{(3)}(t) = 0$ .

Now, from the third equation of (17) we get

$$\vartheta_{\omega i}^{(3)} = -d_{\omega i} \ddot{\vartheta}_{\omega i} - k_{\omega i} \operatorname{sech}^2(\vartheta_{\omega i} - \theta_i)(\dot{\vartheta}_{\omega i} - \omega_i) - p_{\omega i} \dot{e}_{\omega i} + \dot{\alpha}_i, \tag{28}$$



with

$$\dot{\alpha}_i = k_{\alpha_i} \dot{\psi}_i(t) \varphi(\theta)^{\perp \top} \tanh(\vartheta_{vi} - \bar{z}_i) - k_{\alpha_i} \psi_i(t) \omega_i \varphi(\theta_i)^{\top} \tanh(\vartheta_{vi} - \bar{z}_i) + k_{\alpha_i} \psi_i(t) \varphi(\theta_i)^{\perp \top} \begin{bmatrix} \operatorname{sech}^2(\vartheta_{vix} - \bar{x}_i) (\dot{\vartheta}_{vix} - \dot{\bar{x}}_i) \\ \operatorname{sech}^2(\vartheta_{viy} - \bar{y}_i) (\dot{\vartheta}_{viy} - \dot{\bar{y}}_i) \end{bmatrix}. \quad (29)$$

Convergence to zero of  $\dot{\vartheta}_{\omega_i}$ ,  $\ddot{\vartheta}_{\omega_i}$ ,  $\vartheta_{\omega_i}^{(3)}$  and of  $\omega_i$ ,  $v_i$  imply that

$$\lim_{t \rightarrow \infty} \dot{\psi}_i(t) \varphi(\theta(t))^{\perp \top} \tanh(\vartheta_{vi}(t) - \bar{z}_i(t)) = 0.$$

The fact that  $\dot{\psi}_i(t)$  is persistently exciting ensures that

$$\lim_{t \rightarrow \infty} \varphi(\theta(t))^{\perp \top} \tanh(\vartheta_{vi}(t) - \bar{z}_i(t)) = 0.$$

This last, with the fact that (24) holds, implies that  $\lim_{t \rightarrow \infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$ . Therefore, proceeding as in the previous case, we conclude that  $\lim_{t \rightarrow \infty} \bar{z}_i(t) = \bar{z}_c$ . Furthermore,  $\alpha_i$  converges to zero and thus, because  $\dot{\vartheta}_{\omega_i}$  and  $\ddot{\vartheta}_{\omega_i}$  also converge to zero,  $\lim_{t \rightarrow \infty} e_{\omega_i}(t) = 0$ , which by the properties of the Laplacian matrix establishes the result.  $\square$

## 4 | NUMERICAL SIMULATIONS

To illustrate the good performance of the controller proposed in Section 3, we performed comparative numerical simulations in which we also tested the controller reported in [19], which may be considered as an unbridled version of the controller (8)-(11). In the simulation case-study, we consider a group of six differential-drive mobile robots having three different sets of physical parameters, except for the friction coefficient which we set to the unity for simplicity—see Table 1. The robots are assumed to be interconnected over an undirected graph, whose Laplacian matrix is given by

$$\mathbf{L} = \begin{bmatrix} 3 & -1 & 0 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ -1 & 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{bmatrix},$$

and the consensus control objective is set to making the robots converge to a triangular formation without pre-defined center. For that we set the individual robots' offsets as in Table 2.

**TABLE 1** Physical parameters

index	$m_i$ [Kg]	$I_i$ [Kg m <sup>2</sup> ]	$R_i$ [m]	$r_i$ [m]
1, 2	10	3	0.3	0.05
3, 4	15	3	1.0	0.03
5, 6	12	4	0.7	0.10

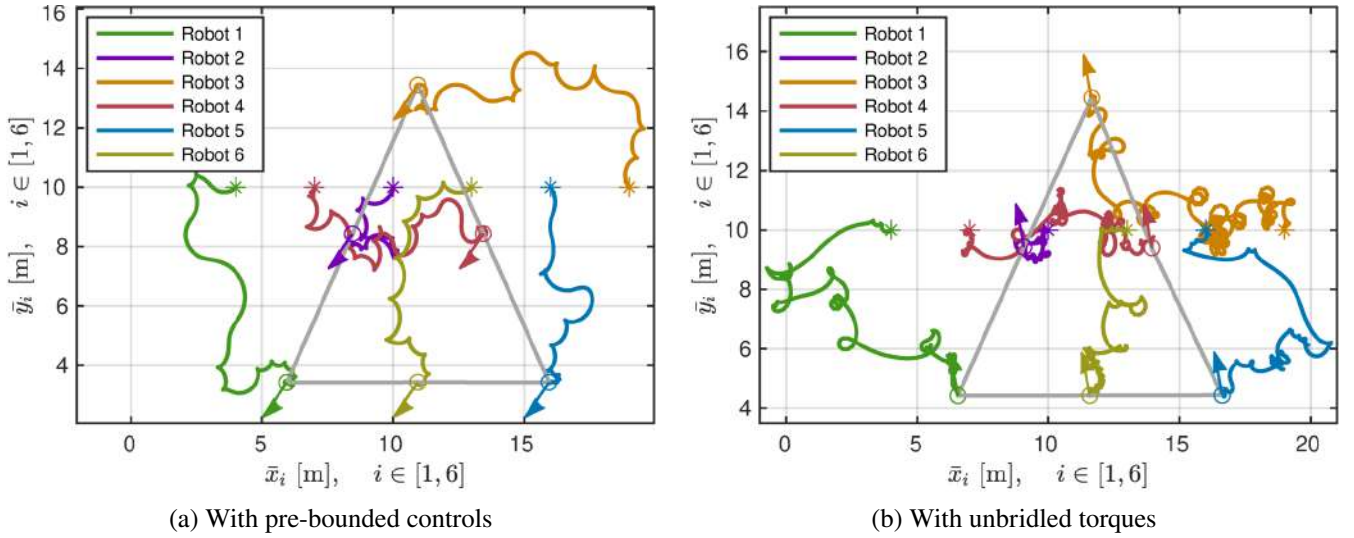
**TABLE 2** Formation offsets and initial conditions

index	$x_i(0)$	$y_i(0)$	$\theta_i(0)$	$\delta_{x_i}$	$\delta_{y_i}$
1	4	10	$-3\pi/4$	-5	-2.7
2	10	10	$-\pi/2$	-2.5	1.7
3	19	10	$3\pi/4$	0	6.7
4	7	10	$\pi/2$	2.5	1.7
5	16	10	$-\pi/4$	5	-2.7
6	13	10	$\pi/4$	0	-2.7

To assess the robustness of our controller, with respect to neglected dynamics, it is assumed that the center of mass does not lay on the wheels axle. Such displacement induces Coriolis terms that are not considered in the dynamic model (2)—see, [26]. Indeed, in this case, Eqs. (7) become

$$\begin{aligned} \dot{v}_i + f_{vi} v_i + \frac{r_i}{3} \omega_i^2 &= \frac{1}{m_i} u_{vi} \\ \dot{\omega}_i + f_{\omega_i} \omega_i - \frac{r_i m_i}{3 I_i} \omega_i v_i &= \frac{1}{I_i} u_{\omega_i}. \end{aligned}$$

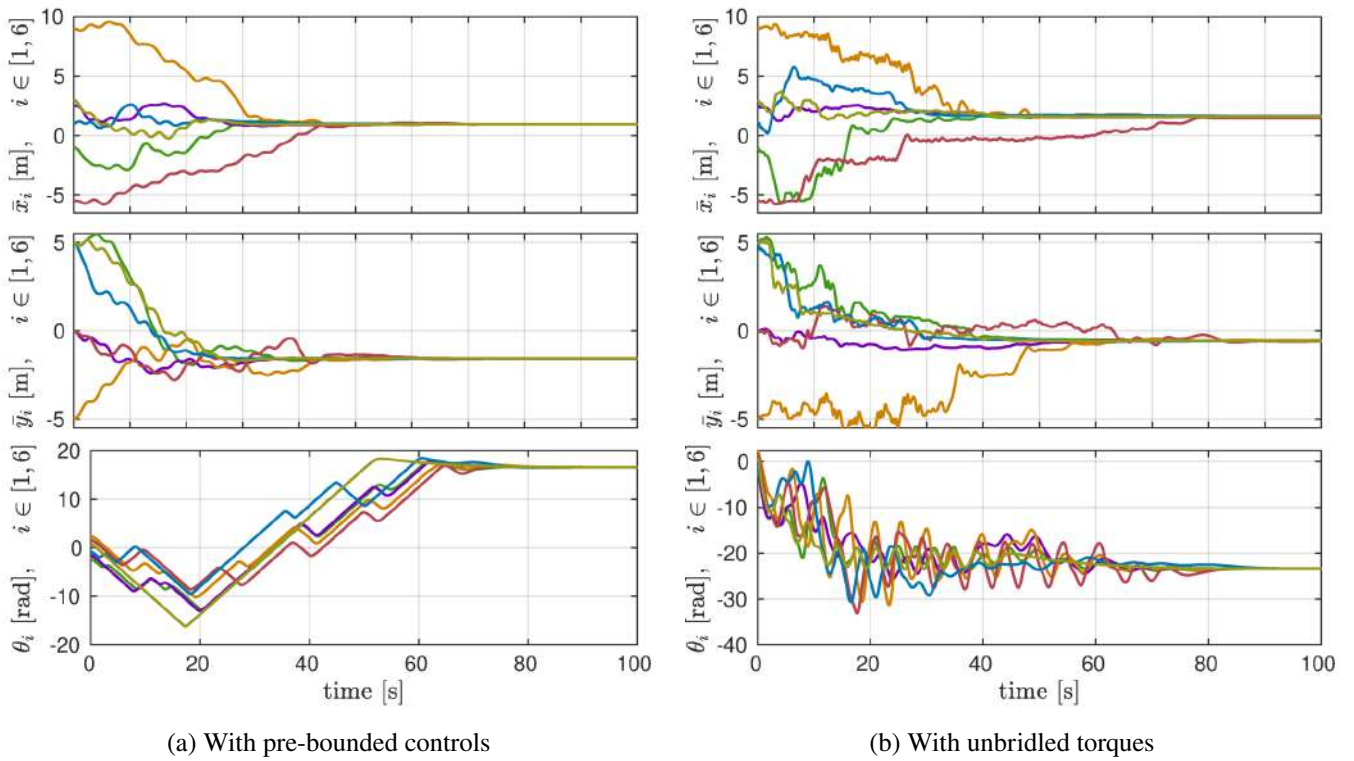
The simulations were carried out using physical parameters similar to those in [27]. We consider three pairs of robots with  $m_i = 1.9c_j$  [Kg],  $r_i = 0.0333c_j$  [m],  $R_i = 0.15c_j$  [m], and  $I_i = 0.5c_j$  [Kgm<sup>2</sup>], with  $i \in \{1, 2, \dots, 6\}$ ,  $j \in \{1, 2, 3\}$ , and  $c_j \in \{0.8, 1, 0.5\}$ . The maximal wheel torque for all the robots is assumed to be  $\bar{\tau}_{r_i} = \bar{\tau}_{l_i} = 0.15$  [N]. The initial postures considered in the simulation are given in Table 2. The simulation results are showed in Figures 2–5.



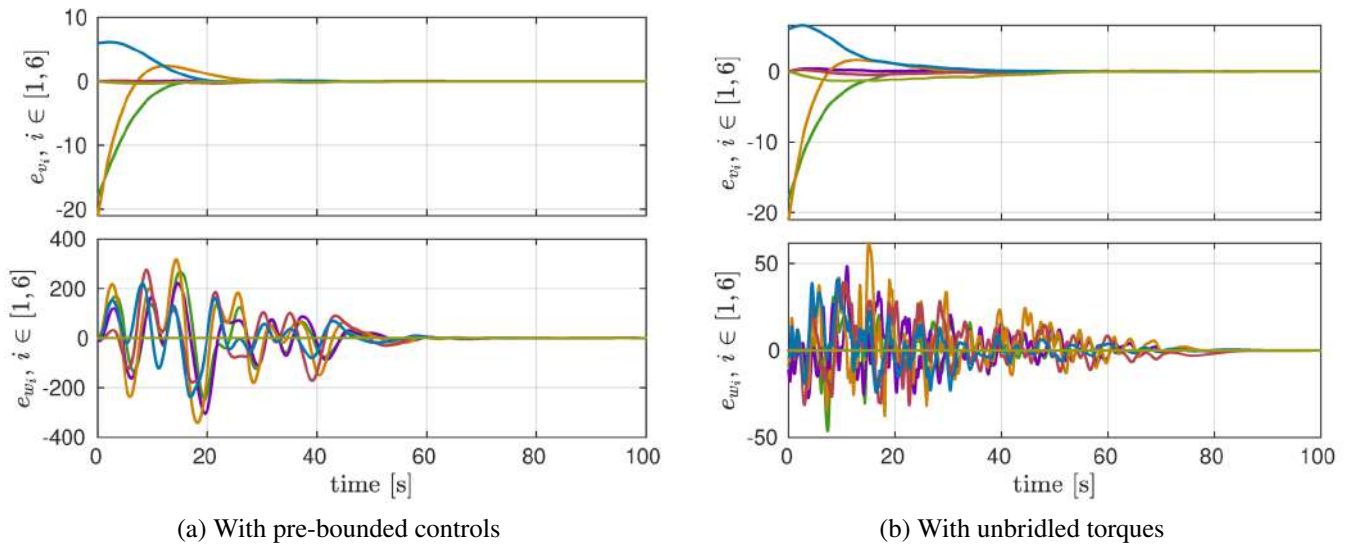
**FIGURE 2** Paths described by the robots on the plane. Final positions in triangle pattern; final orientations represented by arrows

The controller gains are set to  $k_{v_i} = 1$ ,  $k_{\omega_i} = 1$ ,  $d_{v_i} = 50$ ,  $p_{v_i} = 5$ ,  $d_{\omega_i} = 100$ ,  $p_{\omega_i} = 7$ , and  $k_{\alpha_i} = 500$ , for the controller (8)-(11) and  $k_{\alpha_i} = 30$  for the controller from [19]. With these control gains Inequality (15) holds for all the robots. In addition, we used the persistently-exciting function  $\psi_i(t) := 1.25 + \frac{4}{\pi} \sin(0.05t)$ . It is clear that the consensus-based formation objective, both in position and orientation, is achieved under both controllers, with and without pre-bounding the control inputs, and in spite of the neglected dynamics.

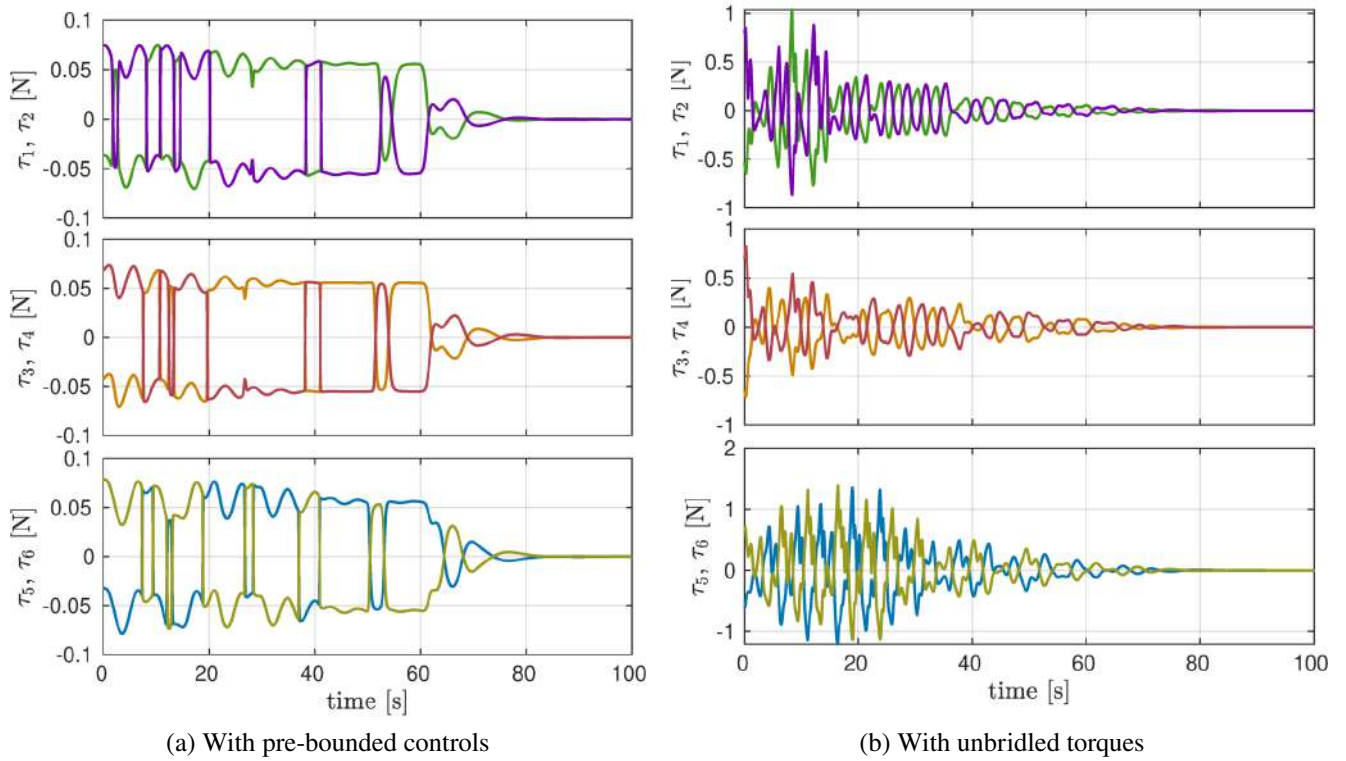
The control gains are the same, except for  $k_{\alpha_i}$ , which determines the amplitude of the “perturbation”  $\alpha_i$ , which is indispensable to stabilize the consensus equilibrium. This parameter is set differently for both controllers for the purpose of obtaining fairly comparable performances. Relatively high values of  $k_{\alpha_i}$  result in faster convergence. However, for the controller [19] this comes at the expense of a highly oscillatory transient behavior, which does not appear when using the controller (8)-(11), due to the presence of the saturations.



**FIGURE 3** Relative Cartesian positions  $\bar{z}$  and angle orientations  $\theta$



**FIGURE 4** Consensus errors as defined in Eqs. (9) and (12)

**FIGURE 5** Wheels' torques for three different pairs of robots

## 5 | CONCLUSIONS

In this work we solve the consensus-based formation problem for multi-agent nonholonomic vehicles. We consider the realistic scenario where the robots are modeled by second order dynamics and where input saturation arises and it must be avoided. The proposed dynamic controller is smooth and time-varying and it does not rely on velocity measurements. The controller injects damping through its second-order dynamics and this damping back-propagates to the plant.

A potential inconvenience of the controller, which is appreciated from the numerical simulations, is that the transient may be long and appear erratic. This is due to the use of persistency of excitation in the controller, which nevertheless is necessary to ensure stabilization. Further simulation tests are needed to characterize the relation of the controller's persistency of excitation to the convergence rate and the oscillatory transient.

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## DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

## CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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