

Bounded Output-Feedback Consensus-based Formation Control of Nonholonomic Vehicles

Emmanuel Nuño, Antonio Loria, Angel Paredes, Tonatiuh Hernandez

▶ **To cite this version:**

Emmanuel Nuño, Antonio Loria, Angel Paredes, Tonatiuh Hernandez. Bounded Output-Feedback Consensus-based Formation Control of Nonholonomic Vehicles. 2022. hal-03870010

HAL Id: hal-03870010
https://hal.science/hal-03870010

Preprint submitted on 24 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Bounded Output-Feedback Consensus-based Formation Control of Nonholonomic Vehicles

Emmanuel Nuño[1] | Antonio Loría[2] | Angel I. Paredes[1] | Tonatiuh Hernández[1]

[1]Department of Computer Sciences, CUCEI, University of Guadalajara, Guadalajara, Mexico

[2]Laboratoire des signaux et systèmes, CNRS, Gif-sur-Yvette, France

Correspondence

*Emmanuel Nuño, Department of Computer Sciences, University Center for Exact Sciences and Engineering, University of Guadalajara, 44430 Guadalajara, JAL, Mexico. Email: emmanuel.nuno@academicos.udg.mx

Abstract

In multi-agent systems, most commonly, consensus consists in all agents converging to a common equilibrium point. For nonholonomic vehicles, which move on a Cartesian plane and occupy a physical space, it is more appropriate to speak of consensus-based formation, which implies that the vehicles are required to converge to a relative position in a given formation while, distributively, agreeing on the center of such formation and on their orientation. In this paper we solve such a problem via a smooth, time-varying, output-feedback control; more precisely, without velocity measurements. In addition, and this is our main contribution, the controller is designed to satisfy pre-imposed bounds to avoid input saturation. To the best of the authors' knowledge this is the first controller of its kind in the context described.

KEYWORDS:

Nonholonomic vehicles; consensus; output-feedback control

1 | INTRODUCTION

The control objective in the consensus of multiple dynamical agents is to ensure that the states of all agents converge to a common value by sharing (part of) their state with their corresponding neighbors [1, 2, 3]. For nonholonomic vehicles, which obviously cannot occupy the same physical space, the consensus-based formation problem consists in making the positions of all robots converge to a common value, modulo an offset, *i.e.*, a vector originating at the formation's center which determines a predefined position of the robot with respect to that center, while also agreeing on their final orientation value —see Figure 1 for an illustration. This is the problem addressed in this paper. Now, consensus being intrinsically a set-point control problem, the nonholonomic restriction prevents the use of smooth autonomous controllers to solve a consensus problem [4]. In fact, set-point stabilization is *not* a particular case of tracking control [5]. Consensus of nonholonomic mobile robots has been studied, for instance, in [6] where a decentralized feedback control that drives a system of multiple nonholonomic unicycles to a rendezvous point in terms of both position and orientation is proposed, the control law is discontinuous and time-invariant. In [7] necessary and sufficient conditions for the feasibility of a class of position formations are laid. In [8] the position/orientation formation control problem for multiple nonholonomic agents using a time-varying controller that leads the agents to a given formation using only their orientation is proposed. In [9] a consensus control law for a network of nonholonomic agents in the presence of bounded disturbances with unknown dynamics in all inputs channels is presented. In [10] a smooth formation control law using a consensus-based approach to drive a group of agents to a desired geometric pattern is proposed. The latter result is extended in [11] by introducing a proportional derivative controller for the velocity dynamics. We remark that the controllers proposed in all of these works rely on velocity measurements, which are often contaminated by noise or affected by sensor defects. Therefore, it is desirable to design controllers that rely only on position and orientation measurements [12].

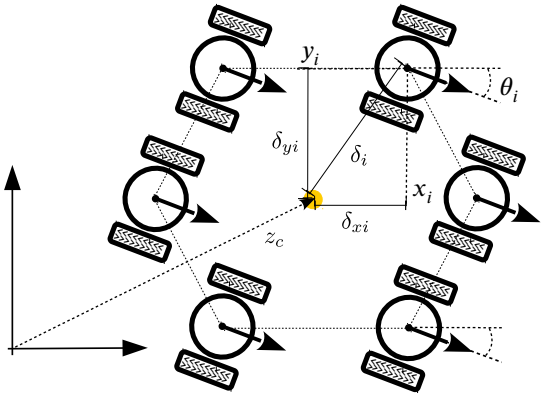


FIGURE 1 Control goal: for the vehicles to reach a consensus-based formation, that is, to meet around an *a priori* unknown rendezvous set-point and acquire a common orientation

Even though there is an abundant literature on formation control of autonomous vehicles, there are only a few works that propose output-feedback controllers. For instance, in [13] a discontinuous leader-follower formation-tracking controller is designed, but the problem addressed therein is fundamentally different to that of consensus-based formation since a reference *trajectory* is pre-imposed. In [14] an output-feedback consensus-based formation controller is proposed, but the proposed scheme is prone to saturate the vehicle's actuators (motors).

In concrete applications, however, the actuators are prone to saturation, which may seriously downgrade the overall performance and put the actuators at risk of thermal and mechanical failure. Hence, saturation of the actuators must be always avoided [15]. For instance, in [16, 17] consensus problems are solved considering actuator saturation without requiring velocities to be measurable. However, none of the results in the latter references apply to nonholonomic vehicles.

The main contribution of our work is a novel decentralized output-feedback controller under input constraints that solves the consensus-based formation problem in undirected networks of nonholonomic vehicles. The controller is dynamic, it is designed by mimicking a mechanical mass-spring-damper system that is interconnected to the plant through virtual springs. Through the latter, the controller steers the second-order plant's positions and also injects damping into the plant. In that regard, it is remarked that the control technique is similar to the one proposed in [18] and as in the latter, our controller is smooth time-varying and relies on persistency of excitation to overcome the restrictions imposed by the nonholonomic constraints—see also [19]. Thus, our work extends previous results in [11, 18, 14] by solving the consensus-based formation control problem under input saturation and without velocity measurements. To the best of the authors' knowledge this is the first article in which such a controller is proposed.

The rest of the paper is organized as follows. In the next section we describe the model and formulate the consensus-based formation control problem. In section 3 we present our main result. Numerical simulation tests are provided in Section 4 and the paper is wrapped up with concluding remarks in Section 5.

Throughout the paper, the following notation is employed. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $\mathbf{1}_N$ is a vector of all elements equal to one of size N. $\otimes$ is the standard Kronecker product. For any absolutely continuous function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|f\|_\infty := ess\,sup\,|f(t)|$, and the square of the $\mathcal{L}_2$-norm as $\|f\|_2^2 := \int_0^\infty |f(t)|^2 dt$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{f : \mathbb{R}_{\geq 0} \overset{t\geq 0}{\to} \mathbb{R}^n : \|f\|_\infty < \infty\}$ and $\{f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|f\|_2 < \infty\}$, respectively. The subscript $i \in \bar{N} := \{1, 2, ..., N\}$.

2 | DYNAMIC MODEL AND PROBLEM FORMULATION

We consider a group of N nonholonomic vehicles under the assumption that there is no slippage. For each vehicle, the kinematics model is given by

$$\dot{z}_i = \varphi\left(\theta_i\right) v_i, \quad \varphi(\theta_i) := [\cos(\theta_i)\ \sin(\theta_i)]^\top \tag{1a}$$

$$\dot{\theta}_i = \omega_i, \tag{1b}$$

where, $z_i = [x_i\ y_i]^\top \in \mathbb{R}^2$ denotes the Cartesian coordinates of the ith vehicle on the plane and $\theta_i \in \mathbb{R}$ denotes its orientation; $v_i \in \mathbb{R}$ and $\omega_i \in \mathbb{R}$ are the linear and the angular velocity, respectively.

The vehicles' dynamic behavior follows a torque-controlled second-order system of the form

$$\begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} + \begin{bmatrix} f_{vi} & 0 \\ 0 & f_{\omega i} \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \frac{1}{r_i} \begin{bmatrix} 1 & 1 \\ 2R_i & -2R_i \end{bmatrix} \begin{bmatrix} \tau_{li} \\ \tau_{ri} \end{bmatrix}, \tag{2}$$

where I_i is the robot inertia, m_i is the mass, f_{vi}, $f_{\omega i}$ are the friction coefficients, τ_{li} and τ_{ri} are the left and right wheel torques, respectively, r_i is the wheel radius, and R_i is the wheel axle length.

Regarding the input torques, we assume the following.

Assumption 1. The left and right wheel torques τ_{li}, τ_{ri} may saturate and thus they satisfy $|\tau_{li}| \leq \bar{\tau}_{li}$ and $|\tau_{ri}| \leq \bar{\tau}_{ri}$, with known strictly positive saturation bounds $\bar{\tau}_{li} > 0$ and $\bar{\tau}_{li} > 0$. ◁

We also assume that only the Cartesian coordinates and the orientation of the vehicles are available for measurement. Thus the following assumption.

Assumption 2. The linear and angular velocities, v_i and ω_i respectively, are not measurable. ◁

It is required that the vehicles meet in formation around a *non*-predefined rendezvous point on the plane, denoted $z_c := (x_c, y_c)$, and acquire a *non*-predefined common orientation, denoted θ_c, modulo a *given* offset $\delta_i = [\delta_{xi}\ \delta_{yi}]^\top$, with $i \in \bar{N}$, which determines the position of the ith vehicle relative to the unknown center of the formation—see Figure 1. In other words, defining, $\bar{z}_i := z_i - \delta_i$ as the relative position of the vehicles with regards to the center of the formation (correspondingly, $\bar{x}_i := x_i - \delta_{xi}$ and $\bar{y}_i := y_i - \delta_{yi}$) the control goal is to make

$$\lim_{t\to\infty} v_i(t) = 0, \quad \lim_{t\to\infty} \bar{z}_i(t) = z_c, \tag{3}$$

$$\lim_{t\to\infty} \omega_i(t) = 0, \quad \lim_{t\to\infty} \theta_i(t) = \theta_c \qquad \forall\, i \in \bar{N}, \tag{4}$$

for all initial conditions.

In contrast to the leader-follower formation control problem the control goal defined above is leaderless, since there is no pre-specified reference. That is, neither the coordinates (x_c, y_c) nor the angle θ_c are imposed *a priori*, but they depend on the initial postures, on the systems' *nonlinear* dynamics, and on network features. This consensus-based formation problem has been successfully solved under different conditions, but the originality of this paper resides in considering the realistic scenario determined by Assumptions 1 and 2 above.

In order to solve the aforementioned problem, it is also assumed that each vehicle exchanges its position (x_i, y_i) and its orientation θ_i—see Fig. 1—with a set of neighbours, which we denote by $\mathcal{N}_i$. It is naturally assumed that once a communication is set between two vehicles i and $j \in \mathcal{N}_i$, the flow of information is bidirectional and is never lost. Whence the following.

Assumption 3. The network interconnection is static and it is modeled as an undirected and connected graph. ◁

Remark 1. In graph theory, a graph is undirected if the nodes exchange information in both directions, it is static if the interconnection is constant, and it is connected if any node is reachable from any other node. The latter is a necessary condition to achieve consensus [20].

In order to establish the proof of our result, it is convenient to define the Laplacian matrix $L := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, where

$$\ell_{ij} = \begin{cases} \sum_{k\in\mathcal{N}_i} a_{ik} & i = j \\ -a_{ij} & i \neq j, \end{cases} \tag{5}$$

additionally $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. Defining $1_N := [1 \cdots 1]^\top$ then, by construction, $L1_N = 0$ and, after Assumption 3, L is symmetric, it has a unique zero-eigenvalue, and all of its other eigenvalues are strictly positive. Thus, $\text{rank}(L) = N - 1$.

3 | MAIN RESULT

The proposed scheme contains an inner control-loop that is designed as

$$\begin{bmatrix}\tau_{ri}\\ \tau_{li}\end{bmatrix} = \frac{r_i}{2}\begin{bmatrix}1 & \frac{1}{2R_i}\\ 1 & -\frac{1}{2R_i}\end{bmatrix}\begin{bmatrix}u_{vi}\\ u_{\omega i}\end{bmatrix}. \tag{6}$$

Then, replacing (6) in (2), we obtain

$$\dot{v}_i = \frac{1}{m_i}(u_{vi} - f_{vi}v_i), \tag{7a}$$

$$\dot{\omega}_i = \frac{1}{I_i}(u_{\omega i} - f_{\omega i}\omega_i). \tag{7b}$$

The control design exploits the fact that the model of the nonholonomic system consists, essentially, in two interconnected second-order systems given by Eqs. (1a)-(7a) and (1b)-(7b). The former set of equations corresponds to the linear-motion dynamics while the latter corresponds to the angular-motion dynamics. Hence, we design a controller of each part of the plant separately. Then, since the velocities v_i and ω_i are not measurable by assumption, we implement a dynamic controller to inject damping into the system. The controller mimics a mass-spring-damper system and its design follows up the ideas first laid in [3] for Euler-Lagrange systems with holonomic constraints and used, also, in [18]. With this in mind, the controller for the linear part is given by

$$\begin{aligned} u_{vi} &= -k_{vi}\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi})\\ \ddot{\vartheta}_{vi} &= -d_{vi}\dot{\vartheta}_{vi} - k_{vi}\tanh(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi}, \end{aligned} \tag{8}$$

where e_{vi} is given by

$$e_{vi} := \sum_{j\in\mathcal{N}_i} a_{ij}\left(\vartheta_{vi} - \vartheta_{vj}\right),$$

and $\tanh(\bar{z}_i - \vartheta_{vi}) := [\tanh(\bar{x}_i - \vartheta_{vix}) \quad \tanh(\bar{y}_i - \vartheta_{viy})]^\top$.

Then, the bounded controller of the angular part is designed as

$$\begin{aligned} u_{\omega i} &= -k_{\omega i}\tanh\left(\theta_i - \vartheta_{\omega i}\right)\\ \ddot{\vartheta}_{\omega i} &= -d_{\omega i}\dot{\vartheta}_{\omega i} - k_{\omega i}\tanh\left(\vartheta_{\omega i} - \theta_i\right) - p_{\omega i}e_{\omega i} + \alpha_i, \end{aligned} \tag{9}$$

where

$$e_{\omega i} := \sum_{j\in\mathcal{N}_i} a_{ij}\left(\vartheta_{\omega i} - \vartheta_{\omega j}\right).$$

All constant gains k_{ki}, d_{ki} and p_{ki}, with $k \in \{v, \omega\}$ and $i \in \bar{N}$, are strictly positive. In addition, α_i is a function defined as

$$\alpha_i(t, \theta_i, \vartheta_{vi}, \bar{z}_i) := k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}\tanh(\vartheta_{vi} - \bar{z}_i), \tag{10}$$

where $k_{\alpha i} > 0$, $\varphi(\theta_i)^\perp := [-\sin(\theta_i) \quad \cos(\theta_i)]^\top$ is the annihilator of $\varphi(\theta_i)$, so $\varphi(\theta_i)^{\perp\top}\varphi(\theta_i) = \varphi(\theta_i)^\top\varphi(\theta_i)^\perp = 0$. The function ψ_i is twice differentiable, bounded, with bounded derivatives and, such that $\dot{\psi}_i$ is persistently exciting, that is, there exist T and $\mu > 0$ such that

$$\int_t^{t+T} \dot{\psi}_i(s)^2 ds \geq \mu, \qquad \forall\, t \geq 0. \tag{11}$$

At this point we are ready to state the main result of this work.

Proposition 1. The controller defined in (6), (8), (9) and (10) ensures that the consensus-based formation objective (3)–(4) holds globally for any strictly positive gains k_{ki}, d_{ki} and p_{ki}, with $k \in \{v, \omega\}$ and $i \in \bar{N}$. Additionally, if gains k_{vi} and $k_{\omega i}$ are set as

$$\frac{4R_i}{r_i}\min\{\,\bar{\tau}_{ri},\ \bar{\tau}_{li}\,\} > 2\sqrt{2}R_i k_{vi} + k_{\omega i}, \tag{12}$$

the controller never saturates the actuators and thus $|\tau_{ki}| < \bar{\tau}_{ki}$, for $k \in \{l, r\}$. △

Proof. We start by showing that the actuators do not saturate if (12) holds. To that end, note that from the properties of $\tanh(\cdot)$ we have $|u_{vi}| \leq \sqrt{2}k_{vi}$ and $|u_{\omega i}| \leq k_{\omega i}$. Therefore, from (6) we have

$$\max\{\,|\tau_{ri}|,\ |\tau_{ri}|\,\} \leq \frac{r_i}{2}\left[\sqrt{2}k_{vi} + \frac{1}{2R_i}k_{\omega i}\right].$$

Hence, setting k_{vi} and $k_{\omega i}$ such that (12) holds, it follows that $|\tau_{ri}| < \bar{\tau}_{ri}$ and $|\tau_{li}| < \bar{\tau}_{li}$.

Next, we write the linear- and angular-motion closed-loop equations,

$$\Sigma_{vi}\begin{cases} \dot{\bar{z}}_i = \varphi(\theta_i)v_i, \\ \dot{v}_i = -\dfrac{1}{m_i}\left(f_{vi}v_i + k_{vi}\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi})\right), \\ \ddot{\vartheta}_{vi} = -d_{vi}\dot{\vartheta}_{vi} - k_{vi}\tanh(\vartheta_{vi} - \bar{z}_i) - p_{vi}e_{vi}, \end{cases} \tag{13}$$

and

$$\Sigma_{\omega i}\begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = -\dfrac{1}{I_i}\left(f_{\omega i}\omega_i + k_{\omega i}\tanh(\theta_i - \vartheta_{\omega i})\right), \\ \ddot{\vartheta}_{\omega i} = -d_{\omega i}\dot{\vartheta}_{\omega i} - k_{\omega i}\tanh(\vartheta_{\omega i} - \theta_i) - p_{\omega i}e_{\omega i} + \alpha_i. \end{cases} \tag{14}$$

Then, the rest of the proof is divided in two steps. In Step 1 we show boundedness of trajectories; in Step 2 we show that all errors converge asymptotically to zero.

Step 1. Consider the candidate Lyapunov function in the space of the consensus errors,

$$V = \sum_{i=1}^{N}\left[\frac{1}{2p_{vi}}\mathcal{E}_{vi} + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}|\vartheta_{vi} - \vartheta_{vj}|^2\right], \tag{15}$$

where

$$\mathcal{E}_{vi} := \frac{1}{m_i}v_i^2 + |\dot{\vartheta}_{vi}|^2 + 2k_{vi}\ln\left(\cosh(\bar{x}_i - \vartheta_{vix})\right) + 2k_{vi}\ln\left(\cosh(\bar{y}_i - \vartheta_{viy})\right),$$

$\bar{x}_i := x_i - \delta_{xi}$ and $\bar{y}_i := y_i - \delta_{yi}$. Evaluating $\dot{V}$ along the trajectories of (13) yields

$$\dot{V} = -\sum_{i=1}^{N}\frac{1}{p_{vi}}\left[d_{vi}|\dot{\vartheta}_{vi}|^2 + f_{vi}v_i^2\right] \leq 0, \tag{16}$$

so $\dot{V} \leq 0$ and, since V is positive definite and radially unbounded with respect to v_i, $\dot{\vartheta}_{vi}$, $|\vartheta_{vi} - \bar{z}_i|$, and $|\vartheta_{vi} - \vartheta_{vj}|$, it follows by integrating on both sides of $\dot{V} \leq 0$ that v_i, $\dot{\vartheta}_{vi}$, $|\vartheta_{vi} - \bar{z}_i|$, and $|\vartheta_{vi} - \vartheta_{vj}|$ are bounded. Furthermore, integrating on both sides of the first inequality in (16) we conclude that $\dot{\vartheta}_{vi}, v_i \in \mathcal{L}_2$.

Next, let

$$W := \sum_{i=1}^{N}\left[\frac{1}{2p_{\omega i}}\mathcal{E}_{\omega i} + \frac{1}{4}\sum_{j\in\mathcal{N}_i} a_{ij}\left(\vartheta_{\omega i} - \vartheta_{\omega j}\right)^2\right],$$

with

$$\mathcal{E}_{\omega i} := \frac{1}{I_i}\omega_i^2 + \dot{\vartheta}_{\omega i}^2 + 2k_{\omega i}\ln\cosh\left(\vartheta_{\omega i} - \theta_i\right).$$

Evaluating $\dot{W}$ along the trajectories of (14) results

$$\dot{W} \leq -\sum_{i=1}^{N}\frac{1}{p_{\omega i}}\left[f_{\omega i}\omega_i^2 + d_{\omega i}\dot{\vartheta}_{\omega i}^2 - \dot{\vartheta}_{\omega i}\alpha_i\right] \tag{17}$$

Since W is positive definite and radially unbounded in ω_i, $\dot{\vartheta}_{\omega i}$, $e_{\omega i}$, and $|\vartheta_{\omega i} - \theta_i|$ then (17) implies that $\Sigma_{\omega i}$ is an output strictly passive map [21] from the input α_i to the output $\dot{\vartheta}_{\omega i}$. Furthermore, the closed-loop system $\Sigma_{\omega i}$ has a property reminiscent of zero-state observability, in the sense that if the input and the output are equal to zero, *i.e.*, $\alpha_i = 0$ and $\dot{\vartheta}_{\omega i} = 0$ respectively, then there exists $\theta_c \in \mathbb{R}$ such that the consensus errors satisfy $(\theta_i - \theta_c, \omega_i, \vartheta_{\omega i} - \theta_c) = (0, 0, 0)$. This last is established invoking Barbashin-Krasovskĭi's theorem with the fact that, when $\alpha_i = 0$, $\Sigma_{\omega i}$ is autonomous, W is positive definite and radially unbounded, $\dot{W}$ is negative semi-definite, and $\dot{W} = 0 \Rightarrow (\dot{\vartheta}_{\omega i}, \omega_i) = 0$. Hence, $\ddot{\vartheta}_{\omega i} = 0$ and $\dot{\omega}_i = 0$ so, from the second equation in (14), $\tanh(\theta_i - \vartheta_{\omega i}) = 0$. Furthermore, from the third equation in (14), it follows that $e_{\omega i} = 0$, so using the properties of the Laplacian matrix this implies that $L\vartheta_\omega = 0$, *i.e.*, there exists $\theta_c \in \mathbb{R}$ such that $\vartheta_\omega = 1_N\theta_c$.

Now, the time-varying function $t \mapsto \psi_i(t)$ admits the upper bound $\bar{\psi}_i$, so we have that, for all trajectories,

$$|\alpha_i| \leq \sqrt{2}\bar{\psi}_i k_{\alpha i}.$$

Therefore it holds that

$$\dot{W} \leq -\sum_{i=1}^{N} \frac{1}{p_{\omega i}} \left[f_{\omega i}\omega_i^2 + \left(d_{\omega i}|\dot{\vartheta}_{\omega i}| - \sqrt{2}\bar{\psi}_i k_{\alpha i} \right) |\dot{\vartheta}_{\omega i}| \right]. \tag{18}$$

Note that if $|\dot{\vartheta}_{\omega i}| \geq \frac{\sqrt{2}}{d_{\omega i}}\bar{\psi}_i k_{\alpha i}$ then, by integration of $\dot{W} \leq 0$, we obtain that $\omega_i, \dot{\vartheta}_{\omega i} \in \mathcal{L}_\infty$. In turn, from (14), we also conclude that $\dot{\omega}_i, \ddot{\vartheta}_{\omega i} \in \mathcal{L}_\infty$.

Step 2. From the second equation in (13) note that $\dot{v}_i \in \mathcal{L}_\infty$ because $\varphi(\theta_i)$ and $\tanh(\bar{z} - \vartheta_{vi})$ are uniformly bounded. Moreover, since $v_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ then, by Barbalăt's Lemma, $\lim_{t\to\infty} v_i(t) = 0$. Additionally, since $\ddot{\vartheta}_{vi}, \dot{\vartheta}_{vi} \in \mathcal{L}_\infty$ and $\dot{\vartheta}_{vi} \in \mathcal{L}_2$ it follows also, from Barbalăt's Lemma, that $\lim_{t\to\infty} \dot{\vartheta}_{vi}(t) = 0$.

Note also that $v_i \in \mathcal{L}_\infty$ implies that $\dot{\bar{z}}_i \in \mathcal{L}_\infty$, from (13). Hence,

$$\vartheta_{vi}^{(3)} = -d_{vi}\ddot{\vartheta}_{vi} - k_{vi} \begin{bmatrix} \operatorname{sech}^2(\vartheta_{vix} - \bar{x}_i)(\dot{\vartheta}_{vix} - \dot{\bar{x}}_i) \\ \operatorname{sech}^2(\vartheta_{viy} - \bar{y}_i)(\dot{\vartheta}_{viy} - \dot{\bar{y}}_i) \end{bmatrix} - p_{vi}\dot{e}_{vi} \tag{19}$$

is also bounded. The latter and the fact that

$$\lim_{t\to\infty} \int_0^t \ddot{\vartheta}_{vi}(\sigma)\, d\sigma = \lim_{t\to\infty} \dot{\vartheta}_{vi}(t) - \dot{\vartheta}_{vi}(0) = -\dot{\vartheta}_{vi}(0)$$

imply together, by Barbalăt's Lemma, that $\lim_{t\to\infty} \ddot{\vartheta}_{vi}(t) = 0$.

Since all signals on the right-hand side of

$$\ddot{v}_i = -\frac{1}{m_i} \left(f_{vi}v_i + k_{vi}\omega_i\varphi(\theta_i)^{\perp\top} \tanh(\bar{z}_i - \vartheta_{vi}) + k_{vi} \begin{bmatrix} \operatorname{sech}^2(\vartheta_{vix} - \bar{x}_i)(\dot{\vartheta}_{vix} - \dot{\bar{x}}_i) \\ \operatorname{sech}^2(\vartheta_{viy} - \bar{y}_i)(\dot{\vartheta}_{viy} - \dot{\bar{y}}_i) \end{bmatrix} \right) \tag{20}$$

are bounded, then $\ddot{v}_i \in \mathcal{L}_\infty$. Note, moreover, that

$$\lim_{t\to\infty} \int_0^t \dot{v}_i(\sigma)\, d\sigma = \lim_{t\to\infty} v_i(t) - v_i(0) = -v_i(0),$$

which by Barbalăt's Lemma implies that $\lim_{t\to\infty} \dot{v}_i(t) = 0$. Proceeding as above, we also conclude that $\lim_{t\to\infty} \ddot{v}_i(t) = 0$.

Now, convergence to zero of $\dot{v}_i$, $\ddot{v}_i$, $\dot{\vartheta}_{vi}$, and of v_i, implies that

$$\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi}) \to 0, \quad \omega_i\varphi(\theta_i)^{\perp\top} \tanh(\bar{z}_i - \vartheta_{vi}) \to 0.$$

On the other hand, the solutions of the equation

$$\varphi(\theta_i)^\top \tanh(\bar{z}_i - \vartheta_{vi}) = 0, \tag{21}$$

are of the form $\tanh(\bar{z}_i - \vartheta_{vi}) = c_1\varphi(\theta_i)^\perp$ with $c_1 \in \mathbb{R}$ while the solutions of the equation

$$\omega_i\varphi(\theta_i)^{\perp\top} \tanh(\bar{z}_i - \vartheta_{vi}) = 0, \tag{22}$$

are of the form $\tanh(\bar{z}_i - \vartheta_{vi}) = c_2\omega_i\varphi(\theta_i)$ with $c_2 \in \mathbb{R}$. Therefore, (21) and (22) hold together if and only if $c\omega_i\varphi(\theta_i) = \varphi(\theta_i)^\perp$ with $c := c_1/c_2$. In turn, the latter may hold only if either $c = 0$ or $\omega_i = 0$.

If (21) and (22) hold simultaneously because $c = 0$, then $\lim_{t\to\infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$, hence from the last equation in (13), we have that

$$\lim_{t\to\infty} e_{vi}(t) = 0,$$

which in turn implies that $\lim_{t\to\infty}(L \otimes I_2)\vartheta_v(t) = 0$. In view of the properties of L, supports the existence of $\bar{z}_c \in \mathbb{R}^2$ such that $\vartheta_v = 1_N \otimes \bar{z}_c$, or $\vartheta_{vi} = \bar{z}_c$ for all $i \in \bar{N}$. Therefore

$$\lim_{t\to\infty} \bar{z}_i(t) = \lim_{t\to\infty} \vartheta_{vi}(t) = \bar{z}_c.$$

Another consequence of $\lim_{t\to\infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$ is that α_i also converges to zero. Thus, it can be seen as a vanishing perturbation of (14) and, in view of the attractivity of $\{(\theta_i - \theta_c, \omega_i, \vartheta_{\omega i} - \theta_c) = (0,0,0)\}$ subject to $\alpha_i = 0$ for all $i \in \bar{N}$, ti follows that the equilibrium $(\omega_i, \theta_i, \vartheta_{\omega i}, \dot{\vartheta}_{\omega i}) = (0, \theta_c, \theta_c, 0)$ is asymptotically stable. Convergence holds globally because the energy-like function W is radially unbounded. Since α_i is globally bounded —by construction— and it vanishes, then standard

theory for nonlinear systems with vanishing perturbations ensures that the equilibrium $(\omega_i, \theta_i, \vartheta_{\omega i}, \dot{\vartheta}_{\omega i}) = (0, \theta_c, \theta_c, 0)$ is globally asymptotically stable.

If, alternatively, (21) and (22) hold simultaneously in the limit because $\lim_{t\to\infty} \omega_i(t) = 0$, then

$$\lim_{t\to\infty} \int_0^t \dot{\omega}_i(\sigma) d\sigma = -\omega_i(0).$$

Then, because

$$\ddot{\omega}_i = -\frac{1}{I_i}\left[f_{\omega i}\omega_i + k_{\omega i}\text{sech}^2(\theta_i - \vartheta_{\omega i})(\omega_i - \dot{\vartheta}_{\omega i})\right] \tag{23}$$

is bounded—in view of the boundedness of ω_i and $\dot{\vartheta}$—it follows that, also, $\ddot{\omega}_i \in \mathcal{L}_\infty$. Therefore, $\lim_{t\to\infty} \dot{\omega}_i(t) = 0$ and

$$\lim_{t\to\infty} \int_0^t \ddot{\omega}_i(\sigma) d\sigma = -\dot{\omega}_i(0).$$

In turn, all the functions on the right-hand side of

$$\omega_i^{(3)} = -\frac{1}{I_i}\left(f_{\omega i}\dddot{\vartheta}_i - 2k_{\omega i}\text{sech}^2(\theta_i - \vartheta_{\omega i})\tanh(\theta_i - \vartheta_{\omega i})(\omega_i - \dot{\vartheta}_{\omega i})^2 + k_{\omega i}\text{sech}^2(\theta_i - \vartheta_{\omega i})(\dot{\omega}_i - \ddot{\vartheta}_{\omega i})\right) \tag{24}$$

are bounded, so $\omega_i^{(3)} \in \mathcal{L}_\infty$. After Barbalăt's Lemma, $\lim_{t\to\infty} \ddot{\omega}_i(t) = 0$, so, from (23), $\lim_{t\to\infty} \dot{\vartheta}_{\omega i}(t) = 0$. Proceeding as before and invoking systematically Barbalăt's Lemma it follows that $\lim_{t\to\infty} \ddot{\vartheta}_{\omega i}(t) = 0$ and that $\lim_{t\to\infty} \vartheta_{\omega i}^{(3)}(t) = 0$.

Now, from the third equation of (14) we get

$$\vartheta_{\omega i}^{(3)} = -d_{\omega i}\ddot{\vartheta}_{\omega i} - k_{\omega i}\text{sech}^2(\vartheta_{\omega i} - \theta_i)(\dot{\vartheta}_{\omega i} - \omega_i) - p_{\omega i}\dot{e}_{\omega i} + \dot{\alpha}_i, \tag{25}$$

with

$$\dot{\alpha}_i = k_{\alpha i}\dot{\psi}_i(t)\varphi(\theta)^{\perp\top}\tanh(\vartheta_{vi} - \bar{z}_i) - k_{\alpha i}\psi_i(t)\omega_i\varphi(\theta_i)^\top\tanh(\vartheta_{vi} - \bar{z}_i) + k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}\begin{bmatrix}\text{sech}^2(\vartheta_{vix} - \bar{x}_i)(\dot{\vartheta}_{vix} - \dot{\bar{x}}_i)\\ \text{sech}^2(\vartheta_{viy} - \bar{y}_i)(\dot{\vartheta}_{viy} - \dot{\bar{y}}_i)\end{bmatrix}. \tag{26}$$

Convergence to zero of $\dot{\vartheta}_{\omega i}, \ddot{\vartheta}_{\omega i}, \vartheta_{\omega i}^{(3)}$ and of ω_i, v_i imply that

$$\lim_{t\to\infty} \dot{\psi}_i(t)\varphi(\theta(t))^{\perp\top}\tanh(\vartheta_{vi}(t) - \bar{z}_i(t)) = 0.$$

The fact that $\dot{\psi}_i(t)$ is persistently exciting ensures that

$$\lim_{t\to\infty} \varphi(\theta(t))^{\perp\top}\tanh(\vartheta_{vi}(t) - \bar{z}_i(t)) = 0.$$

This last, with the fact that (21) holds, implies that $\lim_{t\to\infty} \tanh(\bar{z}_i(t) - \vartheta_{vi}(t)) = 0$. Therefore, proceeding as in the previous case, we conclude that $\lim_{t\to\infty} \bar{z}_i(t) = \bar{z}_c$. Furthermore, α_i converges to zero and thus, because $\dot{\vartheta}_{\omega i}$ and $\ddot{\vartheta}_{\omega i}$ also converge to zero, $\lim_{t\to\infty} e_{\omega i}(t) = 0$, which by the properties of the Laplacian matrix establishes the result. □

4 | NUMERICAL SIMULATIONS

In this section we present some numerical simulations that illustrate the performance of the controller proposed in Section 3 and we compare it to the unsaturated controller reported in [18]. In the simulation case-study, we consider a group of vehicles composed of six differential-drive mobile robots belonging with three different sets of physical parameters—see Table 1. For all the robots we have considered that the friction coefficients are equal to one. The robots are required to converge to a triangular formation, defined by offsets appropriately chosen and from varied initial postures—see Table 2.

The controller gains are set to $k_{vi} = 1$, $k_{\omega i} = 1$, $d_{vi} = 50$, $p_{vi} = 5$, $d_{\omega i} = 30$, $p_{\omega i} = 6$, $k_{\alpha i} = 10$, and the persistence of excitation function is defined as $\psi_i(t) := 1.25 + \frac{4}{\pi}\sin(0.5t) + \frac{4}{3\pi}\sin(1.5t) + \frac{4}{5\pi}\sin(2.5t) + \frac{4}{7pi}\sin(3.5t) + \frac{4}{9\pi}sin(4.5t)$. The robots are assumed to be interconnected over an undirected graph, whose Laplacian matrix is given by

TABLE 1 Physical parameters

index	m_i [Kg]	I_i [Kg m^2]	R_i [m]	r_i [m]
$1,2$	10	3	0.3	0.05
$3,4$	15	3	1.0	0.03
$5,6$	12	4	0.7	0.10

TABLE 2 Initial conditions

index	$x_i(0)$	$y_i(0)$	$\theta_i(0)$	δ_{xi}	δ_{yi}
1	4	10	$-3\pi/4$	5	5
2	10	10	$-\pi/2$	7.5	10
3	19	10	$3\pi/4$	10	15
4	7	10	$\pi/2$	12.5	10
5	16	10	$-\pi/4$	15	5
6	13	10	$\pi/4$	10	5

$$\mathbf{L} = \begin{bmatrix} 3 & -1 & 0 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ -1 & 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{bmatrix}.$$

4.1 | Comparison Between the Novel Bounded and the Unbounded Scheme

The consensus-based formation objective, both in position and orientation, is achieved under both controllers, with and without saturation. The systems' responses are shown in Figures 2 and 3. As shown in Figures 4 and 5, under the bounded controller the transient is longer, consensus takes more time to be reached, than under the action of the unbounded controller. However, the latter is expected, as so is the fact that the unbounded controller demands higher torque values—see Fig. 6. It is to be noted that under the effect of saturation, the torques are upper-bounded by 0.075 [N], 0.08 [N], and 0.068 [N], depending on the set of robots—see Table 2. Clearly, the bounded controller proposed in this paper does not saturate the actuators.

4.2 | Robustness with Regards to Neglected Dynamics

In order to test the robustness of our controller with respect to neglected dynamics, another simulation was performed. In this case, it is assumed that the center of mass does not lay on the wheels axle. Such displacement induces Coriolis terms in the dynamic model that have not been considered in (2)—see, [22], so Eqs. (7) become

$$\begin{aligned} \dot{v}_i + f_{vi} v_i + \frac{r_i}{3}\omega_i^2 &= \frac{1}{m_i} u_{vi} \\ \dot{\omega}_i + f_{\omega i}\omega_i - \frac{r_i m_i}{3 I_i}\omega_i v_i &= \frac{1}{I_i} u_{\omega i}. \end{aligned}$$

The systems' responses under both controllers, with and without input saturation, are shown in Figures 7 and 8. The bounded scheme exhibits a more consistent trajectories than the unbounded scheme. Nevertheless, as shown in Figures 9 and 10 the latter scheme achieves consensus faster than the bounded scheme. This is an expected behavior, due to the input torques shown in Fig. 11, each box corresponds to a subset of robots and each subset has the same saturation bound as the former simulation.

After observing the different simulations reported here, the control objective is fulfilled with the novel control scheme and maintains the actuator forces under the limit imposed in the design. Furthermore, the last simulation successfully illustrates the robustness with respect to neglected dynamic.

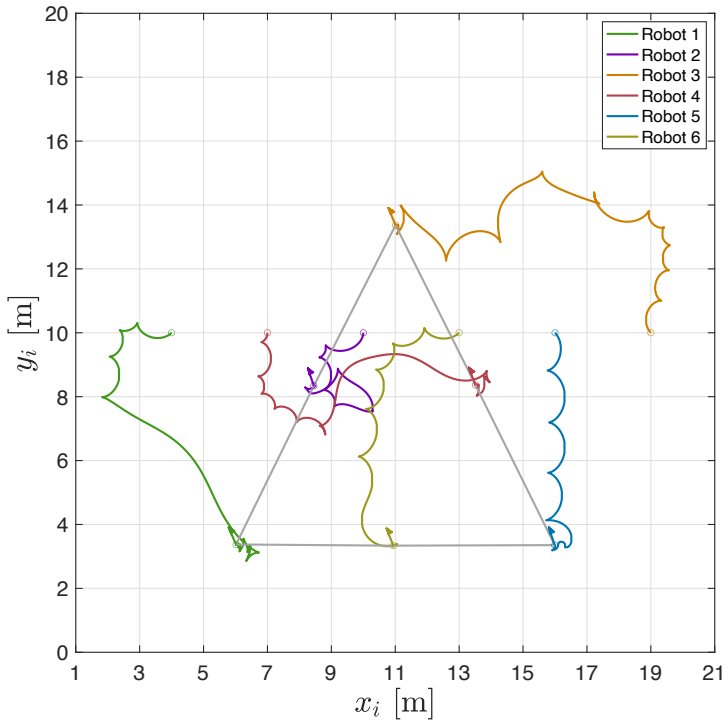


FIGURE 2 Paths followed by the agents reaching the desired formation with orientation consensus with the novel bounded scheme (final orientation is represented with an arrow).

5 | CONCLUSIONS

In this work we solve the consensus-based formation problem in networks of nonholonomic vehicles. We consider the realistic scenario where the robots are modeled by second order dynamics and where input saturation arises and it must be avoided. The proposed dynamic controller is smooth and time-varying and it does not rely on velocity measurements. The controller injects damping through its second-order dynamics and this damping back-propagates to the plant.

A potential inconvenience of the controller, which is appreciated from the numerical simulations, is that the transient may be long and appear erratic. This is due to the use of persistency of excitation in the controller, which nevertheless is necessary to ensure stabilization. Further simulation tests are needed to characterize the relation of the controller's persistency of excitation to the convergence rate and the oscillatory transient.

ACKNOWLEDGMENTS

The work of E. Nuño is supported by the Government of Mexico via the Basic Scientific Research grant CB-282807, sponsored by CONACyT, and by the Government of Spain through the project PID2020-114819GB-I00. The work of A. Loría was supported in part by the French ANR via project HANDY, contract number ANR-18-CE40-0010 and by CEFIPRA under the grant number 6001-A.

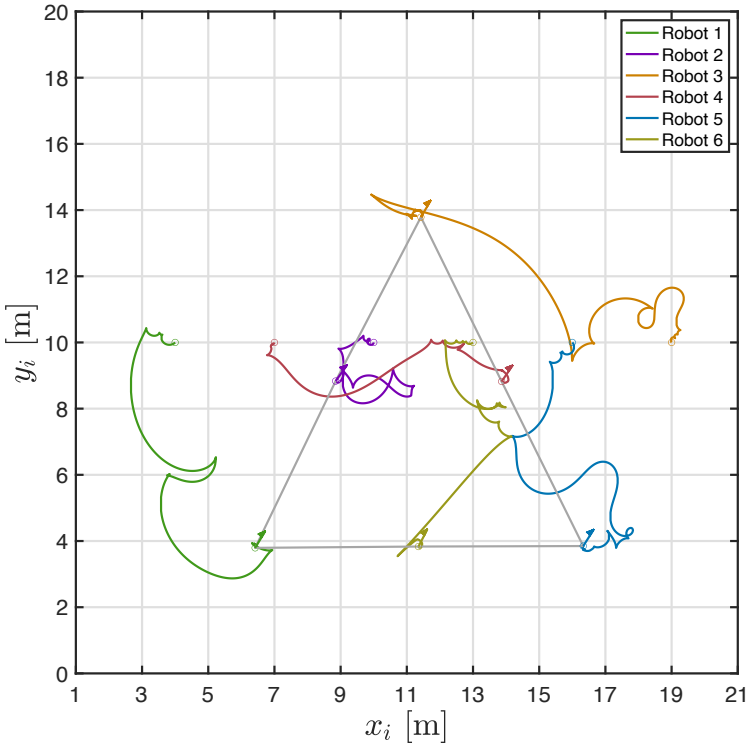


FIGURE 3 Paths followed by the agents reaching the desired formation with orientation consensus with the unbounded scheme (final orientation is represented with an arrow).

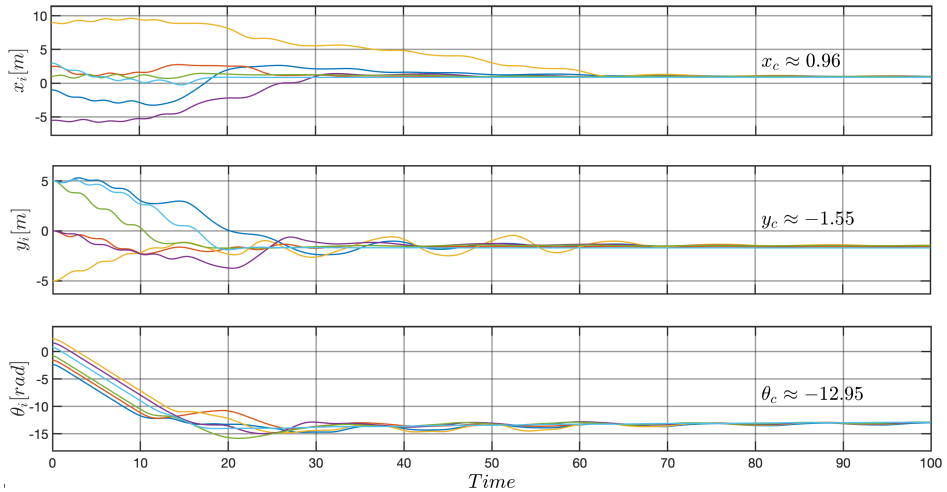


FIGURE 4 Pose consensus for the novel bounded scheme.

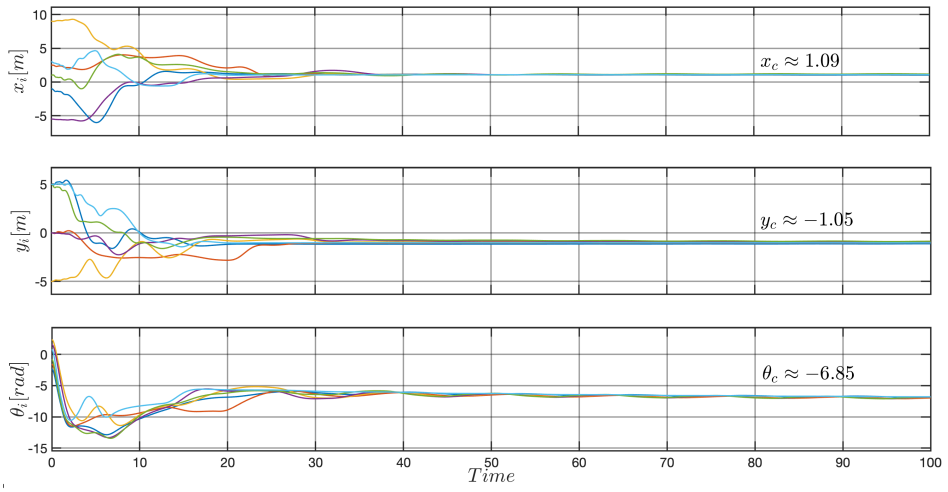


FIGURE 5 Pose consensus for the unbounded scheme.

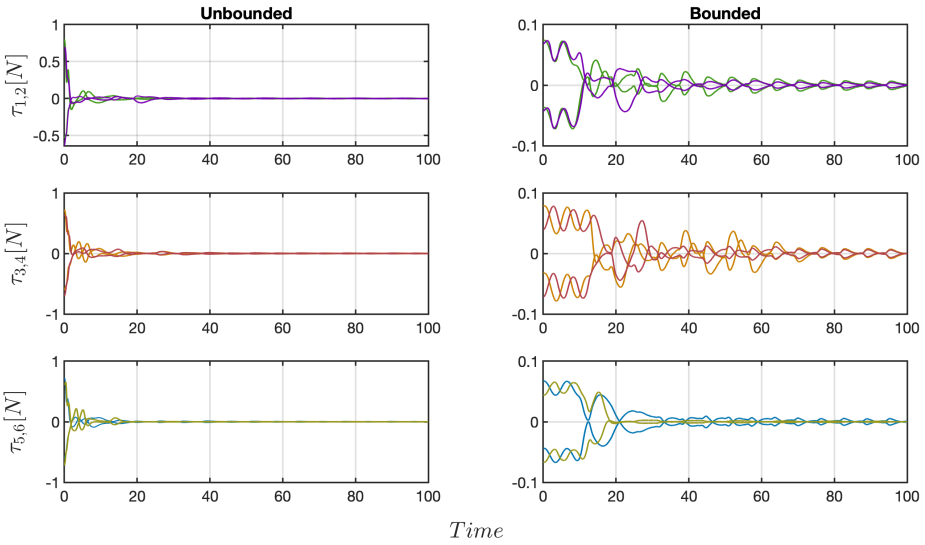


FIGURE 6 Comparison of torque results for both schemes

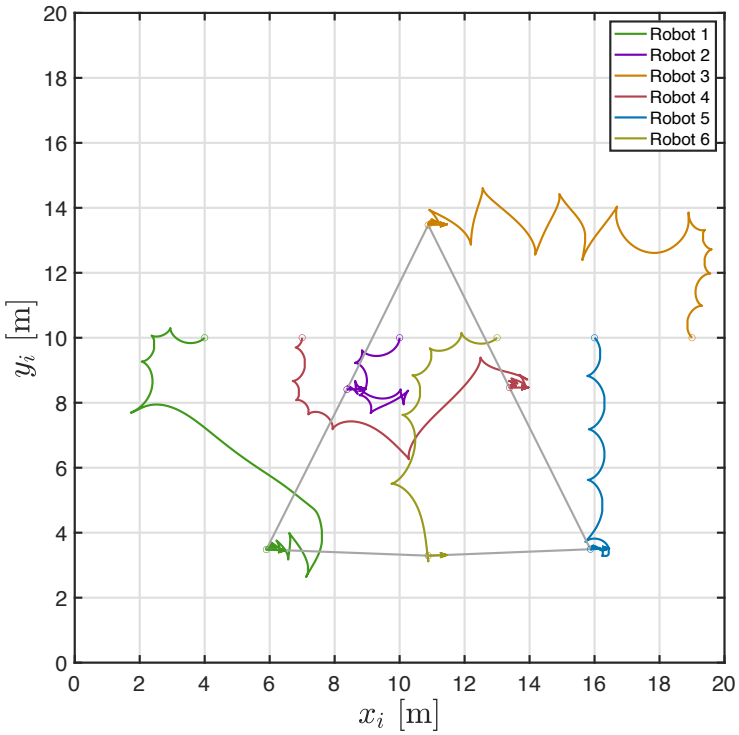


FIGURE 7 Position trajectories in the Cartesian xy-plane for the novel bounded scheme with Coriolis neglected dynamics.

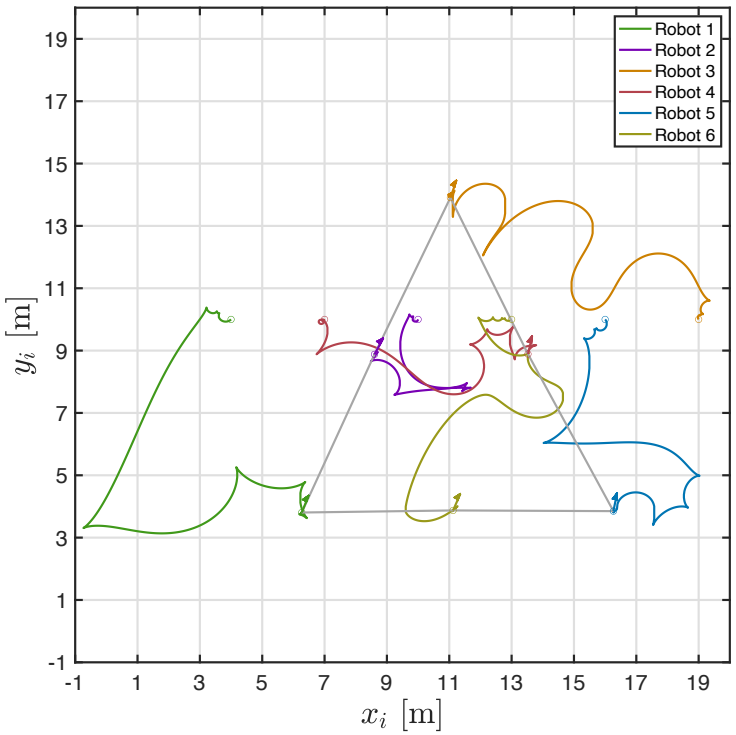


FIGURE 8 Position trajectories in the Cartesian xy-plane for the unbounded scheme with Coriolis neglected dynamics.

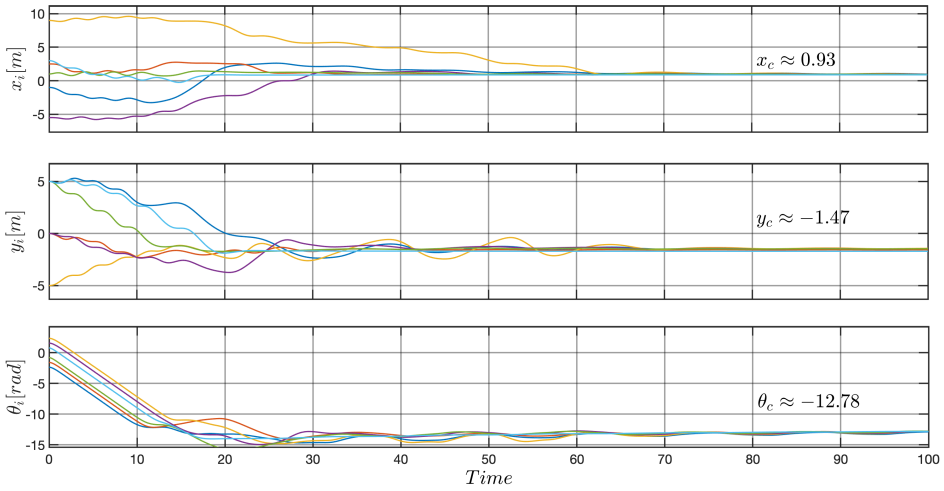


FIGURE 9 Pose consensus for the novel bounded scheme with Coriolis terms.

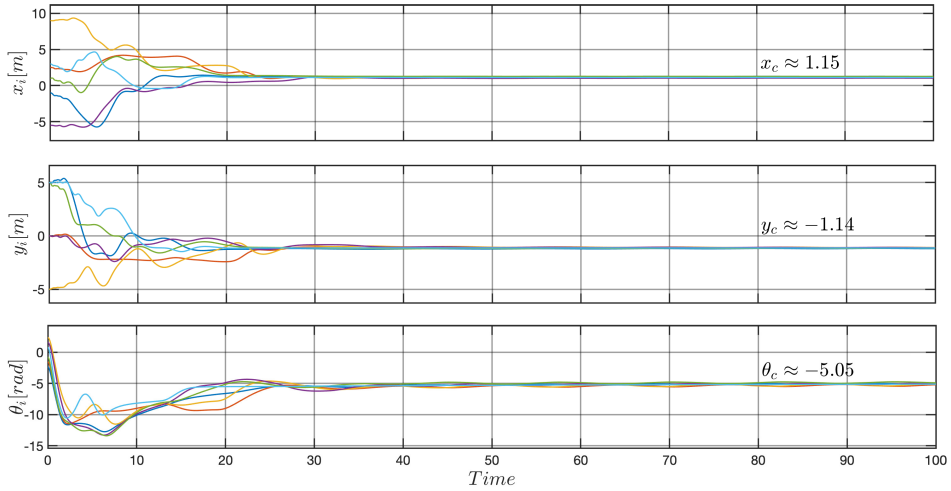


FIGURE 10 Pose consensus for the unbounded scheme with Coriolis terms.

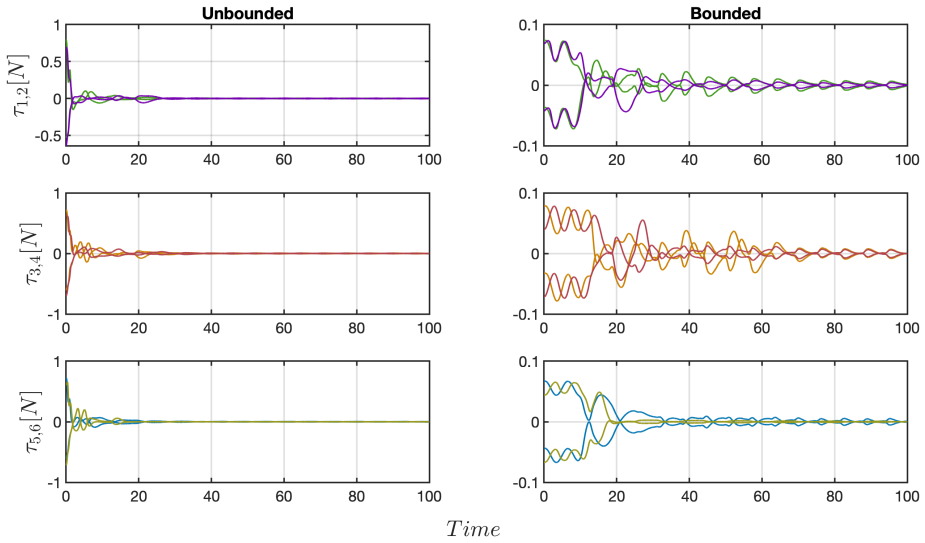


FIGURE 11 Comparison of torque results for both schemes.

References

[1] Jadbabaie A, Lin J, Morse AS. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans. Autom. Control* 2003; 48(6): 988–1001.

[2] Olfati-Saber R, Murray R. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control* 2004; 49(9): 1520-1533.

[3] Nuño E, Ortega R. Achieving Consensus of Euler-Lagrange Agents With Interconnecting Delays and Without Velocity Measurements via Passivity-Based Control. *IEEE Trans. Control Syst. Technol.* 2018; 26(1): 222-232.

[4] Brockett RW. Asymptotic stability and feedback stabilization. *Differential geometric control theory* 1983; 27(1): 181–191.

[5] Lizárraga. DA. Obstructions to the existence of universal stabilizers for smooth control systems. *Math. Control, Sign. Syst.* 2004; 16: 255-277.

[6] Dimarogonas D, Kyriakopoulos K. On the Rendezvous Problem for Multiple Nonholonomic Agents. *IEEE Trans. Autom. Control* 2007; 52(5): 916–922.

[7] Lin Z, Francis B, Maggiore M. Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Trans. Autom. Control* 2005; 50(1): 121–127.

[8] Yang C, Xie W, Lei C, Ma B. Smooth Time-Varying Formation Control of Multiple Nonholonomic Agents. In: Springer. ; 2016: 283–291.

[9] Ajorlou A, Asadi M, Aghdam M, Blouin S. Distributed consensus control of unicycle agents in the presence of external disturbances. *Systems & Control Letters* 2015; 82: 86–90.

[10] Peng Z, Wen G, Rahmani A, Yu Y. Distributed consensus-based formation control for multiple nonholonomic mobile robots with a specified reference trajectory. *International Journal of Systems Science* 2015; 46(8): 1447–1457.

[11] Nuño E, Loría A, Hernández AT, Maghenem M, Panteley E. Distributed consensus-formation of force-controlled nonholonomic robots with time-varying delays. *Automatica* 2020(120): 109114.

[12] Liang X, Wang H, Liu Y, Chen W, Liu T. Formation Control of Nonholonomic Mobile Robots Without Position and Velocity Measurements. *IEEE Trans. Robot.* 2018; 34(2): 434-446.

[13] Cheng Y, Jia R, Du H, Wen G, Zhu W. Robust finite-time consensus formation control for multiple nonholonomic wheeled mobile robots via output feedback. *Int. J. Robust Nonlinear Control* 2018; 28(6): 2082-2096. doi: https://doi.org/10.1002/rnc.4002

[14] Loría A, Nuño E, Panteley E. Observerless Output-feedback Consensus-based Formation Control of 2nd-order Nonholonomic Systems. *IEEE Trans. Autom. Control* 2021 (early access). doi: 10.1109/TAC.2021.3136140

[15] Zavala-Río A, Aguiñaga-Ruiz E, Santibáñez V. Global trajectory tracking through output feedback for robot manipulators with bounded inputs. *Asian Journal of Control* 2011; 13(3): 430-438. doi: https://doi.org/10.1002/asjc.324

[16] Ren W. Distributed leaderless consensus algorithms for networked Euler-Lagrange systems. *Int. Jour. of Control* 2009; 82(11): 2137-2149.

[17] Cruz-Zavala E, Nuño E, Moreno J. Finite-Time Consensus of Euler-Lagrange Agents without Velocity Measurements via Energy Shaping. *International Journal of Robust and Nonlinear Control* 2019; 29(17): 6006-6030.

[18] Nuño E, Loría A, Panteley E. Leaderless consensus formation control of cooperative multi-agent vehicles without velocity measurements. *IEEE Control Systems Letters* 2022; 6: 902–907. doi: 10.1109/LCSYS.2021.3087095

[19] Wang Y, Miao Z, Zhong H, Pan Q. Simultaneous Stabilization and Tracking of Nonholonomic Mobile Robots: A Lyapunov-Based Approach. *IEEE Trans. on Control Systems Technology* 2015; 23(4): 1440-1450.

[20] Ren W, Beard RW. *Distributed consensus in multivehicle cooperative control*. Springer verlag . 2005.

[21] Ortega R, Loría A, Nicklasson PJ, Sira-Ramírez H. *Passivity-based Control of Euler-LagrangeSystems: Mechanical, Electrical and Electromechanical Applications*. Series Comunications and Control EngineeringSpringer Verlag, London . 1998. ISBN 1-85233-016-3.

[22] Tzafestas SG. *Introduction to mobile robot control*. Elsevier Inc, First ed. . 2013.

How to cite this article: E. Nuño, A. Loría, A.I. Paredes and T. Hernández (2022), Bounded Output-Feedback Consensus-based Formation Control of Nonholonomic Vehicles, *2022*.