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# Dynamic consensus and adaptive bias compensation for multi-agent linear systems over directed networks<sup>★</sup>

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**Abstract:** Biased measurements in an inter-networked systems can have severe repercussions in closed-loop stability of the individual systems and decelerate dynamical consensus among the interacting agents. Bias in the measurement, even constant, cannot be dealt with *ad hoc* techniques of robust control, in the presence of additive perturbations, because the control gain amplifies the disturbance. One way to account for the effect of measurement bias is then to rely on adaptive control. This has been done in the literature in the context of individual systems, but to the best of our knowledge not for multi-agent systems, while ensuring consensus control. In this paper we provide a model-reference-adaptive-control scheme to ensure dynamic consensus of generic (stabilizable) linear systems interconnected over directed graphs and under the influence of constant bias measurements. Our controller ensures global asymptotic stability of the synchronization manifold and convergence of the bias estimates.

*Keywords:* Measurement bias, directed graphs, multi-agent systems, linear systems, Lyapunov stability

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## 1. INTRODUCTION

Accuracy of sensor readings plays a vital role in monitoring and maintenance of desired performance in large-scale interconnected systems. Consequently, faulty sensor data may have major repercussions in the closed-loop stability of the physical systems and may lead to severe accidents—see incidents (Lombaerts et al., 2011; Norman, 1980). Short circuited connections, incorrect hardware designs or improper calibration or even low battery level are some of the root causes for occurrence of faults in the data collected by the sensors (Sharma et al., 2007). Sensor faults are of several types: bias, drift, scaling, noise, hard-fault, intermittent (Balaban et al., 2009). Bias fault are persistent offset readings which can affect all the samples gathered in a particular deployment. Such readings can exhibit normal patterns and sometimes go undetected.

There are several available works which handle the problem of consensus of multi-agent systems equipped with sensor whose reading are afflicted with white-noise (Djaidja et al., 2015; Li and Zhang, 2009; Sun et al., 2013; Wu et al., 2021). Li and Zhang (2009) state necessary and sufficient conditions for average consensus for first-order integrator systems communicating via balanced digraph such that the control input for each agent uses relative-position measurements that are corrupted with zero-mean white noises. Several works have also been proposed for agents

communicating under noisy communication network be it for second-order integrator systems (Djaidja et al., 2015) or for general linear systems (Wu et al., 2021) or systems with time delay (Sun et al., 2013) to name a few. Also see (Ni and Li, 2013) which deals with multiplicative noisy measurements.

Symmetric communication network in undirected graphs helps in decoupling the sensor biases from the state measurements as seen in (Shi et al., 2020; Sinha et al., 2022). In reality the influence of one agent with respect its neighbour is not always reciprocal in nature so decoupling of the bias from the state variables measurement can turn out to be a relatively complex problem. On that note, we must take into account some of the major recent strides made for directed graphs—see *e.g.*, (Borzone et al., 2019; Cao et al., 2019; Meng et al., 2020; Sukumar et al., 2018; Tan et al., 2022; Yan et al., 2019) for handling biased measurements. Different techniques have been applied to overcome the hindrance of using biased state measurements that are offset; for instance, via adaptive control (Meng et al., 2020), event-triggered control (Cao et al., 2019), adaptive neural control (Tan et al., 2022), LMI techniques (Yan et al., 2019), to mention a few.

Besides graphical conditions, individual system dynamics can be really influential in the behaviour of the overall collaborating agents, be it for double-integrators (Sukumar et al., 2018), general linear systems (Meng et al., 2020), nonlinear system (Borzone et al., 2019), nonlinear system with non-strict feedback structure (Cao et al., 2019; Tan et al., 2022), or discrete-time linear system (Yan

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et al., 2019). However, in the context of continuous time systems, (Cao et al., 2019; Meng et al., 2020; Tan et al., 2022) deals with leader-follower consensus such that the measurement of the follower state/output variables are unreliable but the measurement of the leader states is bias-free. Also, (Borzzone et al., 2019) uses a distributed piecewise constant impulsive reference. A general connected directed graph can either be or contain spanning tree so leader-follower consensus is applicable if the communication topology is a spanning tree with the root node behaving as a leader. In order to achieve leader-less consensus, assuming clean measurement of the state-variables of the root nodes or the nodes which influence the behaviour of the emergent dynamics is not a feasible option. Even though Sukumar et al. (2018) tackle the leaderless consensus problem using measurements affected by *constant* biases in a more general context, the synchronisation error is shown to be only uniformly ultimately bounded.

Model reference adaptive control (MRAC) consists in designing a desired plant model depending on performance requirements and to design a feedback control law that can suitably morph the dynamics of the actual plant so that its input-output properties matches with the reference model (Ioannou and Sun, 1996). Taking inspiration from classical MRAC for individual systems, there have been a few works on distributed model reference adaptive control for cooperative tracking (Liu and Jia, 2012; Peng et al., 2012; Sumizaki et al., 2010) and cooperative regulation (Mei et al., 2021) for different communication topology- whether it is undirected (Liu and Jia, 2012; Peng et al., 2012; Sumizaki et al., 2010) or directed (Mei et al., 2021). Application of MRAC for leader-follower consensus generally portrays the leader node as the singular reference model (Liu and Jia, 2012; Peng et al., 2012; Sumizaki et al., 2010). Mei et al. (2021) provide a framework for adaptive leaderless consensus of linear (homogeneous) agents with uncertain dynamics using model reference adaptive consensus whereby each agent has to track the output generated by their corresponding reference linear model with relative state measurement as input. Although all of these papers use MRAC-based techniques to deal with uncertain systems, none of them address the issue of handling biased relative position measurements. In continuous time domain, study of leader-less dynamical consensus among agents exchanging biased position measurements (even for root nodes) through unidirectional network is rather limited. Patre and Joshi (2011) presents a MRAC-based controller to accommodate for constant sensor bias compensation, but not in a multi-agent context.

In this paper, we address the leader-less dynamical consensus problem for general linear systems equipped with sensor providing state measurement tampered with unknown bounded biases by using MRAC based consensus procedure. These agents can communicate over a directed network that is or may contain a spanning tree. Also, we enlist the condition under which complete bias estimations can be ensured. The contributions of this manuscript is as follows:

- (1) We propose an MRAC-based control law for leaderless consensus among agents which can override the effect of biased measurements relative to (Liu and Jia,

2012; Mei et al., 2021; Peng et al., 2012; Sumizaki et al., 2010).

- (2) We do not assume that the position sensor deployed on the leader/root nodes provides bias-free measurements, as in (Cao et al., 2019; Meng et al., 2020; Tan et al., 2022).
- (3) Unlike (Sukumar et al., 2018), where the synchronisation error is only uniformly ultimately bounded, we propose conditions under which dynamic consensus and bias estimation may be achieved asymptotically in spite the presence of constant biases in the available measurement.

The remainder of the paper is organised as follows. In the following section we present the problem formulation, including our main hypotheses. In Section 3 we provide our main statements and in Section 4 we present some illustrative simulation results. The paper is wrapped up with some concluding remarks in Section 5.

## 2. PROBLEM FORMULATION: SYSTEM SETUP

We consider  $N \in \mathbb{N}$  general linear autonomous systems of the form,

$$\dot{x}_i = Ax_i + Bu_i, \quad \forall i = \{1, 2, \dots, N\} \quad (1)$$

where  $x_i \in \mathbb{R}^n$ ,  $u_i \in \mathbb{R}^p$  is the control input and  $A, B$  are constant matrices of compatible dimensions. We address the consensus problem which will essentially guarantee  $\lim_{t \rightarrow \infty} \|x_l(t) - x_m(t)\| = 0, \forall l \neq m, l, m \in \{1, 2, \dots, N\}$  under the following assumptions:

- (A1) the pair  $(A, B)$  is stabilizable;
- (A2) the matrix  $A$  is of full rank;
- (A3) the agents interact over a directed connected network;
- (A4) the  $i^{th}$  agent has access to its own state and the state of its neighbours but these measurements are tampered with unknown constant biases. The agent in question also has the estimate of its own and neighbour's sensor-biases.

For the sake of illustration, we can assume  $N$  autonomous vehicles communicating unilaterally such that each vehicle is equipped with a position sensor which gives tampered measurements unbeknownst to the user. Through the wireless communication network, a particular vehicle has access to the position measurements of its neighbours which are also afflicted with heterogenous biases. As a result, the relative position data available to each agent is in turn affected.

In short, the  $i^{th}$  agent has access to the measurements  $m_i = \bar{x}_i = (x_i + \delta_i)$ ,  $m_j = \bar{x}_j = (x_j + \delta_j)$ ,  $j \in \mathcal{N}_i$  where  $\mathcal{N}_i$  represents the index set for the neighbours of the  $i^{th}$  agent, so the relative state data available is of the form  $m_{ij} = \bar{x}_i - \bar{x}_j = ((x_i + \delta_i) - (x_j + \delta_j)) = (x_i - x_j) + (\delta_i - \delta_j)$  for all  $i \neq j, j \in \mathcal{N}_i$ .

The intensity of the one-way interaction is captured by the elements  $a_{ij} \geq 0$  such that  $a_{ij} > 0$  if  $i^{th}$  agent receives information from the  $j^{th}$  agent else  $a_{ij} = 0$ . As is customary, it is assumed that there are no self loops, *i.e.*,  $a_{ii} = 0$ . Hence, the structure of the communication graph  $\mathcal{G}$  is portrayed by a so-called Laplacian matrix  $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ , whose elements are

$$l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}, \quad l_{ij} = -a_{ij}. \quad (2)$$

Naturally the Laplacian matrix  $\mathcal{L}$  of a connected directed network is almost always non-symmetric. This hinders the construction of suitable Lyapunov functions to study the stability of such networked systems. In this paper we rely on two technical Lemmata that are recalled below for the sake of completeness.

*Lemma 1.* (Ren and Beard, 2005, 2008) If a directed network has a directed spanning tree, then the Laplacian matrix  $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$  has a singular zero eigenvalue and  $N-1$  eigenvalues have strictly positive real part. That is,

$$\sigma_1(\mathcal{L}) = 0, \quad \Re\{\sigma_k(\mathcal{L})\} > 0, \quad k = \{2, \dots, N\}.$$

The right eigenvector of the zero eigenvalue is given by  $\mathbf{1}_N = [1 \ 1 \ \dots \ 1]^\top$  and the left eigenvector  $v_l$  satisfies the properties  $\sum_{k=1}^N v_{lk} = 1$  and  $v_l^\top \mathcal{L} = 0^\top$ .

*Lemma 2.* (Panteley et al., 2020) Let us consider a directed graph  $\mathcal{G}$  of order  $N$  containing a spanning tree and its Laplacian matrix is  $\mathcal{L} \in \mathbb{R}^{N \times N}$ . Then, for any  $Q_{\mathcal{L}} \in \mathbb{R}^{N \times N}$ , such that  $Q_{\mathcal{L}} = Q_{\mathcal{L}}^\top > 0$ , and  $\alpha \in \mathbb{R}^+$ , there exists a matrix  $P_{\mathcal{L}} \in \mathbb{R}^{N \times N}$ , such that  $P_{\mathcal{L}} = P_{\mathcal{L}}^\top > 0$  and

$$P_{\mathcal{L}}\mathcal{L} + \mathcal{L}^\top P_{\mathcal{L}} = Q_{\mathcal{L}} - \alpha[P_{\mathcal{L}}\mathbf{1}_N v_l^\top + v_l \mathbf{1}_N^\top P_{\mathcal{L}}]. \quad (3)$$

In the following section, we present our distributed MRAC-based controller, which borrows inspiration from (Mei et al., 2021), a recent work devoted to leaderless consensus among uncertain systems.

### 3. MAIN RESULTS: ROBUST CONSENSUS SCHEME

The control strategy relies on the construction of a reference model that can act as a yardstick for each of the interacting agent to emulate. Each of these reference models is designed to synchronise with each other, using its own ‘‘measurements’’ (computed data), and the biased measurements of interconnected neighbours. In addition, the reference model dynamics includes a bias compensation, computed using an adaptation law. Then, dynamic consensus of all agents is achieved provided their respective reference models synchronise.

We introduce  $N$  reference-models given by

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i + \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{x}_i - \bar{x}_j) \\ &\quad - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{\delta}_i - \hat{\delta}_j) \\ &= A\hat{x}_i + \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}((x_i + \tilde{\delta}_i) - (x_j + \tilde{\delta}_j)) \\ &= A\hat{x}_i + \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) \end{aligned} \quad (4)$$

for some coupling weight  $\epsilon \in \mathbb{R}^+$ , and  $\tilde{\delta}_i = \delta_i - \hat{\delta}_i$  where  $\delta_i$  is a constant measurement bias associated with the state of the  $i^{\text{th}}$  agent and  $\hat{\delta}_i$  is its corresponding estimate.

In light with the control strategy, the synchronisation objective is split into three goals that need to be satisfied

to ensure dynamic consensus among the interacting agents in the presence of measurement bias. These are:

$$(CO1) \quad \lim_{t \rightarrow \infty} \|x_i(t) - \hat{x}_i(t)\| \rightarrow 0, \forall i \in \{1, \dots, N\}$$

$$(CO2) \quad \lim_{t \rightarrow \infty} \|\tilde{\delta}_i(t)\| \rightarrow 0, \forall i \in \{1, \dots, N\}$$

$$(CO3) \quad \lim_{t \rightarrow \infty} \|\hat{x}_i(t) - \hat{x}_j(t)\| \rightarrow 0, i \neq j, i, j \in \{1, \dots, N\}.$$

Now, the decentralised practical tracking error available to the  $i^{\text{th}}$  agent is

$$\tilde{e}_i = \bar{x}_i - \hat{x}_i - \hat{\delta}_i = (x_i + \delta_i) - \hat{x}_i - \hat{\delta}_i = x_i - \hat{x}_i + \tilde{\delta}_i. \quad (5)$$

Now, the total derivative of  $\tilde{e}_i$  along (1), (4) yields

$$\begin{aligned} \dot{\tilde{e}}_i &= Ax_i + Bu_i - A\hat{x}_i - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) \\ &\quad - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) + \dot{\tilde{\delta}}_i \\ &= A\tilde{e}_i - A\tilde{\delta}_i + Bu_i - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) \\ &\quad - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) + \dot{\tilde{\delta}}_i. \end{aligned} \quad (6)$$

Naturally, we want the practical tracking error  $\tilde{e}_i$  to gradually decrease. With this intention, we choose the distributed feedback control input  $u_i$  as

$$\begin{aligned} u_i &= \epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{x}_i - \bar{x}_j) - \epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{\delta}_i - \hat{\delta}_j) + F\tilde{e}_i \\ &= \epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + \epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) + F\tilde{e}_i, \end{aligned} \quad (7)$$

where  $F \in \mathbb{R}^{p \times n}$ , is given by  $F = -B^\top M$  is the feedback matrix such that  $M = M^\top > 0$  solves the Algebraic Riccati equation given by

$$MA + A^\top M - MB B^\top M = -Q, \quad Q = Q^\top > 0. \quad (8)$$

As explained, the model-reference-based estimator (4) is complemented by an adaptive estimation law given by

$$\dot{\hat{\delta}}_i = -(M^{-1}A^\top M)\tilde{e}_i. \quad (9)$$

Since the bias estimate for the bias associated with the measurements of the  $i^{\text{th}}$  sensor depends on the corresponding available tracking error  $\tilde{e}_i$ , so we can surmise that the assigned dynamics is distributed.

*Proposition 3.* Consider  $N$  identical collaborating linear systems as in (1), equipped with sensors whose readings are offset by constant unknown biases, and satisfying Assumptions (A1)–(A4). Consider, in addition, the distributed consensus control input given by (7) with  $\epsilon \geq \sigma_{\max}(P_{\mathcal{L}})$ , where  $P_{\mathcal{L}}$  solves (3) for  $Q_{\mathcal{L}} = I_N$ , the feedback matrix  $F = -B^\top M$ , where  $M$  solves (8), and the bias estimation law (9). Then, all the agents reach dynamic consensus and, moreover, the synchronization manifold is globally asymptotically stable.

*Proof:* The proof consists in showing that the intermediary objectives (CO1)–(CO3) are met. To that end, we start by writing the closed-loop equations. For the tracking errors  $\tilde{e}_i$ , we have:

$$\begin{aligned}\dot{\tilde{e}}_i &= A\tilde{e}_i - A\tilde{\delta}_i + B\left(\epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) \right. \\ &\quad \left. + \epsilon F \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) + F\tilde{e}_i\right) - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) \\ &\quad - \epsilon BF \sum_{j \in \mathcal{N}_i} a_{ij}(\tilde{\delta}_i - \tilde{\delta}_j) + \dot{\tilde{\delta}}_i \\ &= A\tilde{e}_i - A\tilde{\delta}_i + BF\tilde{e}_i + \dot{\tilde{\delta}}_i \\ &= (A + BF)\tilde{e}_i - A\tilde{\delta}_i + \dot{\tilde{\delta}}_i.\end{aligned}\quad (10)$$

In compact form, (9) and (10) can be written as

$$\dot{\tilde{e}} = (I_N \otimes (A + BF))\tilde{e} - (I_N \otimes A)\tilde{\delta} + \dot{\tilde{\delta}} \quad (11a)$$

$$\dot{\tilde{\delta}} = (I_N \otimes (M^{-1}A^\top M))\tilde{e}. \quad (11b)$$

The above dynamical system admits the Lyapunov function candidate

$$V(\tilde{e}, \tilde{\delta}) = \tilde{e}^\top (I_N \otimes M)\tilde{e} + \tilde{\delta}^\top (I_N \otimes M)\tilde{\delta},$$

which is both positive definite and decrescent in the space of the practical tracking error  $\tilde{e} \in \mathbb{R}^{Nn}$  and error in estimation of the biases  $\tilde{\delta} \in \mathbb{R}^{Nn}$ . Furthermore, its time-derivative along the trajectories of (11a)–(11b) yields

$$\begin{aligned}\dot{V}(\tilde{e}, \tilde{\delta}) &= 2\tilde{e}^\top (I_N \otimes M)\left((I_N \otimes (A + BF))\tilde{e} - (I_N \otimes A)\tilde{\delta} \right. \\ &\quad \left. + \dot{\tilde{\delta}}\right) + 2\tilde{\delta}^\top (I_N \otimes M)\left((I_N \otimes (M^{-1}A^\top M))\tilde{e}\right) \\ &= \tilde{e}^\top (I_N \otimes (MA + A^\top M - 2MBB^\top M))\tilde{e} \\ &\quad - 2\tilde{e}^\top (I_N \otimes (MA))\tilde{\delta} + 2\tilde{e}^\top (I_N \otimes (A^\top M))\tilde{e} \\ &\quad + 2\tilde{\delta}^\top (I_N \otimes (A^\top M))\tilde{e} \\ &= 2\tilde{e}^\top (I_N \otimes (MA + A^\top M - MBB^\top M))\tilde{e} \\ &= -2\tilde{e}^\top (I_N \otimes Q)\tilde{e} \leq 0.\end{aligned}\quad (12)$$

We see that  $\dot{V}$  is negative definite for all  $\tilde{e} \in \mathbb{R}^{Nn}$  and there does not exist any other positive semi-trajectory contained in the set  $\{\dot{V} = 0\}$  than the origin. In other words, setting  $\tilde{e} = 0$  in (11) we see that, under Assumption (A2), the only solution is  $\tilde{\delta} = 0$ , so the largest invariant set contained in  $\{\dot{V} = 0\}$  is the origin. Global asymptotic stability follows after Barbashin-Krasovskii's theorem (Barbashin and Krasovskii, 1952), also (wrongly) known as LaSalle's (Loría and Panteley, 2017).

Now, to prove that the last control objective, (CO3) is met, we will check the resulting compact form of the reference systems as  $\tilde{e}, \tilde{\delta} \rightarrow 0_{Nn}$

$$\begin{aligned}\dot{\hat{x}} &= (I_N \otimes A)\hat{x} - \epsilon(\mathcal{L} \otimes BB^\top M)x - \epsilon(\mathcal{L} \otimes BB^\top M)\tilde{\delta} \\ &= (I_N \otimes A)\hat{x} - \epsilon(\mathcal{L} \otimes BB^\top M)\hat{x}\end{aligned}\quad (13)$$

Using Proposition 1 in (Dutta et al., 2022), it can be shown that the reference models in (13) too reaches consensus among themselves provided the Assumptions (A1)–(A4) hold true and  $\epsilon \geq \sigma_{max}(P_{\mathcal{L}})$  where  $P_{\mathcal{L}}$  solves (3) for some

$Q_{\mathcal{L}} = Q_{\mathcal{L}}^\top > 0$ . Following (Panteley and Loría, 2017), we can apply a linear transformation that can map the reference systems into dichotomous system: mean-field/emergent system which is the weighted average of all the reference systems and synchronisation error system which dictates the distance of all the individual reference systems from the mean-field system. In particular,

$$\begin{bmatrix} \hat{x}_m \\ s_{\hat{x}} \end{bmatrix} = \begin{bmatrix} (v_l^\top \otimes I_n)\hat{x} \\ ((I_N - 1_N v_l^\top) \otimes I_n)\hat{x} \end{bmatrix} \quad (14)$$

The directional derivative of (14) along (13) is given by

$$\begin{bmatrix} \dot{\hat{x}}_m \\ \dot{s}_{\hat{x}} \end{bmatrix} = \begin{bmatrix} A\hat{x}_m \\ ((I_N \otimes A) - \epsilon(\mathcal{L} \otimes BB^\top M))s_{\hat{x}} \end{bmatrix} \quad (15)$$

where we have used the general properties of the Laplacian matrix  $\mathcal{L}$  and the left eigenvector  $v_l$  of the zero eigenvalue. In other words, this transformation converts the consensus problem of the reference models into stabilization problem of the system with respect to the manifold  $\mathcal{S} = \{s_{\hat{x}} \in \mathbb{R}^{Nn} | s_{\hat{x}} = 0_{Nn}\}$ . The exponential stability of  $\mathcal{S}$  can be proved by a positive definite and decrescent Lyapunov function  $V(s_{\hat{x}}) = s_{\hat{x}}^\top (P_{\mathcal{L}} \otimes M)s_{\hat{x}}$ . The time derivative of  $V(s_{\hat{x}})$  can be summarised in few steps as follows,

$$\begin{aligned}\dot{V}(s_{\hat{x}}) &= s_{\hat{x}}^\top (P_{\mathcal{L}} \otimes (MA + A^\top M)) \\ &\quad - \epsilon(P_{\mathcal{L}}\mathcal{L} + \mathcal{L}^\top P_{\mathcal{L}}) \otimes (MBB^\top M)s_{\hat{x}}.\end{aligned}\quad (16)$$

Next, we replace (3) in (16) and set  $Q_{\mathcal{L}} = I_N$ . Then we use  $v_l^\top 1_N = 1$  to see that  $\alpha(P_{\mathcal{L}} 1_N v_l^\top \otimes I_n)s_{\hat{x}} = \alpha(P_{\mathcal{L}} 1_N v_l^\top \otimes I_n)((I_N - 1_N v_l^\top) \otimes I_n)\hat{x} = 0_{Nn}$  and apply the orthogonal decomposition of  $P_{\mathcal{L}} = P_{\mathcal{L}}^\top > 0$  to get  $P_{\mathcal{L}} = T\Lambda T^\top$  where  $T \in \mathbb{R}^{N \times N}$  is an orthogonal matrix. Thus, from (16) we obtain that

$$\dot{V}(s_{\hat{x}}) = p_M s_{\hat{x}}^\top (TT^\top \otimes (MA + A^\top M - \frac{\epsilon}{p_M} MBB^\top M))s_{\hat{x}}.$$

where  $p_M I \leq P_{\mathcal{L}} \leq p_M I$ . For  $\epsilon \geq p_M$ ,  $\dot{V}(s_{\hat{x}})$  is upper bounded as

$$\begin{aligned}\dot{V}(s_{\hat{x}}) &\leq p_M s_{\hat{x}}^\top (I_N \otimes (MA + A^\top M - MBB^\top M))s_{\hat{x}} \\ &\leq -p_M s_{\hat{x}}^\top (I_N \otimes Q)s_{\hat{x}} < 0\end{aligned}\quad (17)$$

where  $Q$  is defined in (8). This, implies the global exponential stability of manifold  $\mathcal{S}$ . Since  $s_{\hat{x}} = 0$  implies that  $\hat{x}_i = \hat{x}_m = \hat{x}_j$  for all  $i, j \leq N$ , it follows that the reference models reach dynamic consensus, that is, they synchronise with the mean-field system  $\hat{x}_m = A\hat{x}_m$ . The statement follows.

#### 4. SIMULATION RESULTS

For the purpose of illustration we provide some numerical simulation results generated using Matlab R2021a, for the case-study of five harmonic oscillators, modelled as in (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (18)$$

Clearly, the pair  $(A, B)$  is stabilizable and  $A$  is a full rank matrix, so Assumptions (A1) and (A2) hold.

The initial conditions assigned for the five agents is

$$\begin{aligned} x_1(0) = \hat{x}_1(0) &= [3 \ -1 \ 1.5 \ -1.5 \ 2]^\top, \\ x_2(0) = \hat{x}_2(0) &= [2 \ -2 \ 0.5 \ -0.5 \ 2.5]^\top. \end{aligned}$$

Additionally, the initial conditions for bias estimates are

$$\begin{aligned} \hat{\delta}_1(0) &= [3 \ -1 \ 1.5 \ -1.5 \ 2]^\top, \\ \hat{\delta}_2(0) &= [1 \ -2 \ 0.5 \ -0.5 \ 2.5]^\top. \end{aligned}$$

The constant unknown biases are taken all different, defined as

$$\delta_i = \left[ \frac{\pi i}{10} \ \frac{\pi i}{12} \right]^\top, \quad i \in \{1, \dots, 5\}. \quad (19)$$

Then, to compute the feedback matrix  $F$ , we use a matrix  $M \in \mathbb{R}^{2 \times 2}$  which solves the algebraic Riccati equation (8) for  $Q = I_2$ . The resulting matrix  $M$  is

$$M = \begin{bmatrix} 1.9123 & 0.4142 \\ 0.4142 & 1.3522 \end{bmatrix}. \quad (20)$$

The case-study involves five harmonic oscillators interconnected in a directed spanning tree graph as the one shown in Figure 1, so Assumption (A3) holds.

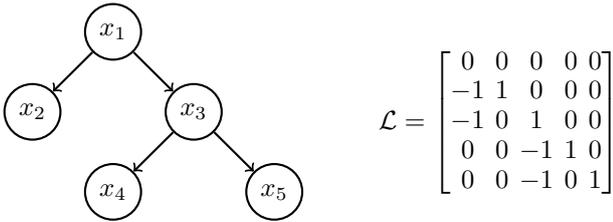


Fig. 1. A spanning-tree graph and its corresponding Laplacian

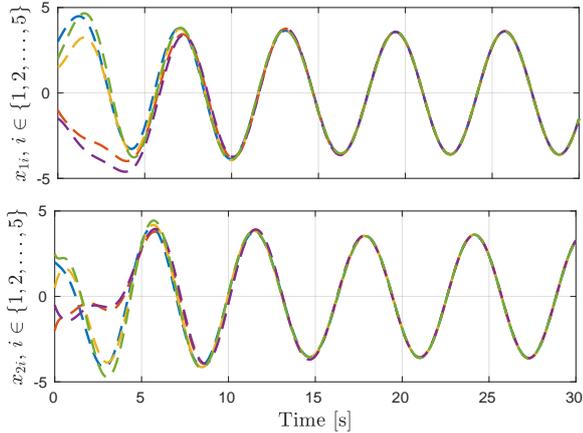


Fig. 2. Responses of five harmonic oscillators in full synchronization

The simulation results are depicted in Figures 2–4. The blue plot corresponds to that of the first agent, which coincides with the mean-field dynamics in this case. As expected, it can be seen that state variables  $x_1$  and  $x_2$  for the remaining four agents synchronise with that of the mean-field dynamics—see Figure 2. In turn, the responses of the reference models, which also achieve dynamic consensus, is depicted in Figure 3. Finally, in Figure 4, we show the bias estimation errors, which clearly converge to zero asymptotically.

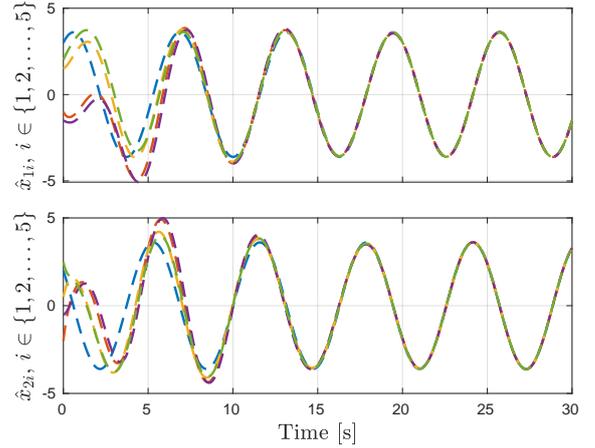


Fig. 3. Responses of the five corresponding reference models in full synchronization

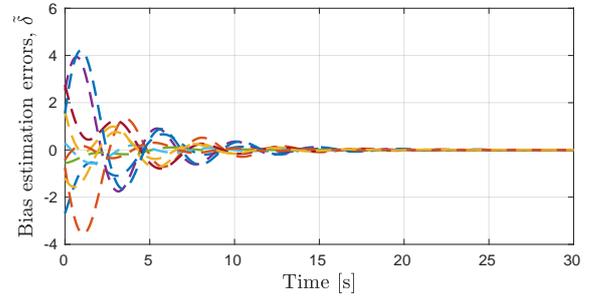


Fig. 4. Bias estimation errors for all five agents

## 5. CONCLUSION

In network with unilateral exchange of biased measurements, agents with linear homogeneous stabilizable dynamics can still reach dynamic consensus. That is, their motions may be controlled to synchronise with that of an average dynamical system. This may be achieved using a dynamic distributed consensus controller that borrows inspiration from model-reference adaptive control. Asymptotic decay of *constant* bias estimation error can ensure that each of the individual agent tracks its corresponding reference systems which in turn can reach consensus among themselves. This work albeit restricted to interaction of linear systems through a static graph, can provide a gateway for handling even more complex scenarios: such a linear systems exchanging biased data through time varying graph or agents with generic nonlinear dynamics or systems with time delay. Such scenarios are currently under study.

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