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# Context, content effects, and flexibility (*author version*)

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## 1. Introduction

Can cognitive flexibility be considered independently of the context in which it is applied, and of the contents upon which it operates? Although many studies lead us to believe that there are general mechanisms of cognitive flexibility which account for an individual's inclination to adopt a new perspective of a situation, this does not mean that this flexibility can be conceived of as an absolute and decontextualized ability whose effectiveness remains constant in any situation. To the contrary, in this chapter we explore the major impact that the content upon which reasoning operates and the context in which the reasoning occurs have on the ability of an individual to flexibly approach the situation. We thus propose to study cognitive flexibility in its interaction with the environment, by describing how the latter can, in turn, hinder or favor the adoption of a new perspective on a given concept, situation, or problem.

This idea that human reasoning is not exclusively driven by logic and may also be influenced by context is a relatively recent assumption in the history of thought. In his *Nicomachean Ethics*, Aristotle stated that the main characteristic distinguishing animals from humans consists in the latter's ability to carry out logically valid reasoning, which gives humans the status of rational animals, guided above all by the rules of logic (Aristotle 1999). For Descartes (1984), God's gift to man is a reliable intellect which, if used correctly, enables him to avoid all errors in reasoning. Finally, for Inhelder and Piaget (1958, p. 1), "reasoning is nothing but propositional calculus". Thus, it has long been considered that the ability to reason according to the rules of logic was one of the specificities of the human mind.

It was only in the second half of the 20<sup>th</sup> century that reasoning became the subject of experimental studies. Until then, it was considered that human reasoning was mainly a matter of logic, based on mathematics and philosophy, but was not of a part of human psychology. The mere idea of analyzing human reasoning without resorting to the study of logic would have seemed ludicrous, since the terms “reasoning” and “logic” were considered synonymous. In this respect, Boole’s choice to name his algebra book which presented a formalism for describing logical relationships *A Study of the Laws of Thought* (1854) is particularly informative.

In the last fifty years, however, considerable progress has been made on these issues through experimental psychology, and we have gained a more detailed understanding of the role of context in human reasoning. In particular, the work of Wason (1960, 1968) initiated a paradigm shift that opened up a new line of research on the reasoning biases that affect adults. In a now-famous series of studies, Wason investigated the comprehension of conditional statements of the form “if P then Q” using a Wason selection task. In this task, participants are typically given four cards, each with a number on one side and a letter on the other. The visible sides of the four cards show “3”, “8”, “A”, and “D” respectively. Participants are asked, “Which card(s) do you need to turn over to test if the proposition “if a card has an even numbered noun on one side, then its opposite side bears a vowel?” is true.

The correct answer on this task is to turn over the “8” card (following *modus ponens*, since 8 is an even number) and the “D” card (following *modus tollens*, since D is not a vowel). However, Wason showed that the correct answer was found by less than 10% of the participants (Wason 1968). In contrast, the majority of adults interviewed reported that they would turn over card A and card 8. This systematic error highlighted the fact that participants tend to ignore the *modus tollens* rule (if it is true that P implies Q, and if the opposite of Q is true, then the opposite of P is also true). This seeming inability to apply a fundamental rule of propositional logic has since been widely replicated, calling out the idea that human reasoning strictly follows the laws of logic (Ragni *et al.* 2018).

In addition, subsequent work has shown that participants’ ability to select the correct cards depends directly on the context of the situation. For example, when the relatively abstract rule that “if even, then vowel” is replaced by a more concrete rule relevant to a real social context, performance on this task significantly increases. In particular, Cosmides and Tooby (1992) showed that when the rule “if I drink alcohol, then I must be over 18” was presented with four cards labelled “16”, “25”, “drinks beer”, and “drinks soda”,

respectively, most participants had no difficulty selecting the correct “16” and “drinks beer” cards. In other words, although adults have difficulty understanding certain logical relationships, their difficulties can be overcome by changing the semantic content of the situation. Several versions of this task have been proposed, and although there is no current consensus regarding the adequate explanation of the context effects that the findings highlight, these effects have been widely replicated and their existence is undisputed (see, for example, Cheng and Holyoak 1985; Cosmides 1989; Girotto *et al.* 2001; Stenning and van Lambalgen 2001; Hilton *et al.* 2005; Klauer *et al.* 2007 or Ragni *et al.* 2018 for a meta-analysis). Thus, human reasoning seems not to settle solely for the laws of logic, but it also seems to depend on the context in which it operates; human beings are at times logical, at times irrational, depending on the subject of their reasoning. This leads us to formulate a new line of study: to what extent can the context mediate the capacity of individuals to adopt different perspectives on the situations they encounter? Can we construct situations that are more favorable than others for cognitive flexibility to emerge? Can the content on which one reasons favor or limit this flexibility? Given the wide variety of situations in which individuals are expected to reason flexibly, it is important to raise these questions.

## **2. Context, concepts and flexibility**

### **2.1. *The categorical ambiguity of concepts***

In order to understand how contextual elements can influence the flexibility of the perspectives adopted by individuals, we must first look at the multitude of perspectives that can be adopted on the concepts that underlie our thinking. We thus propose to examine how different views of the same concept can be constructed, activated, coexist and follow one another depending on the contexts that individuals encounter. In this respect, Rosch (1978) has described concepts as entities organized in the form of mental categories whose structure is not strictly delimited but gradual. Rosch explains that each concept has at its center a so-called “prototypical” element, which is the most representative of the category in question. The other entities belonging to the same category would be more or less distant from this prototype, depending on the degree to which one considers them as belonging to the conceptual category. Thus, in most Western cultures, a dog or cat is more prototypical of the category “animal” than is the coral, for example, just as a blackbird or sparrow is more prototypical of the category “bird” than is the ostrich. In this way, a more prototypical entity will be more easily recognized as a member of a category than will an atypical

entity. The more an object conforms to the prototypical attributes of the category, the more likely it is to be quickly recognized as a member of that category.

This approach, which considers concepts as gradual structures rather than delimited storage boxes, has the advantage of accounting for the plurality of possible interpretations for a given concept. The category “animal” can refer to an orangutan or a sea sponge, and different characteristics of the category will be activated depending on the meaning. In line with this approach, many works thus emphasize that different properties of a concept may be activated depending on the context in which the concept is evoked (see for example (Rips *et al.* 1973; Barsalou 1982, 1983; Barsalou and Medin 1986; Lebois *et al.* 2015; Yee and Thompson-Schill 2016; Zwaan 2016)). The influence that context has on the activation of specific characteristics of concepts thus illustrates a certain flexibility of the conceptual system (Pecher and Zwaan 2017).

Barsalou (1982) was interested in the extent to which the activation of different properties of concepts depends on the context in which the concept is activated. To study this question, he asked participants to comment on the properties of several entities presented in sentences. For example, the sentence “the roof was renovated before the rainy season” was presented, and participants had to decide whether the property “you can walk on it” was associated with the concept “roof” or not. The author showed that participants’ attribution of properties was context-dependent. In the presented sentence, participants took a certain amount of time to identify that it is indeed possible to walk on a roof. However, when the sentence “the roof creaked under the weight of the repairman” was presented, participants were quicker to identify that it was indeed possible to walk on a roof. In other words, the context in which the concept was presented influenced how quickly participants accessed its properties.

This finding was further supported by a second study Barsalou (1982) conducted, in which he asked participants to judge the similarity between two entities belonging either to the same common category or to an ad hoc category. For example, the pair “robin/eagle” belonged to the common category “birds”, while the pair “jewellery/slaves” belonged to the ad hoc category “plunder taken by conquerors”. Half of the participants first saw the names of the categories before the pairs of entities were presented. The other half simply saw the entity pairs, without their category labels. Participants were asked to judge, on a scale of 1 to 9, how similar the properties of the two entities in each pair seemed to them. The hypothesis was that entities belonging to ad hoc categories would only be perceived as similar if their membership in the same category was emphasized by the context (presentation of the

category label beforehand), whereas these contextual cues would have no effect on pairs of entities belonging to common categories. Not surprisingly, the results showed that perceived similarity was more pronounced for common categories than for ad hoc categories. Nevertheless, consistent with the author's hypothesis, contextual cues had a significant influence on participants' judgments of similarity. Perceived similarity for ad hoc category pairs was greater when category names were presented before each pair than when pairs were presented without labels. The author thus concluded that the context made certain features of the presented concepts accessible, even though these features would not necessarily be activated in other contexts. Thus, the perspective adopted on a given concept is intrinsically linked to the context in which the concept is presented: different properties are highlighted thanks to contextual cues, and the perspective of individuals thus adapts to this information. The switch from one perspective to another thus seems to depend in part on contextual cues extrinsic to the encountered concept.

## ***2.2. The role of prior knowledge***

The context in which a concept is encountered may thus activate a broader spectrum of knowledge than the simple mention of the concept on its own: each context calls upon prior knowledge, which influences the individual's perspective. However, it may happen that not all the properties of certain concepts are known to the individuals, who must then resort to knowledge from other domains to make inferences about these properties. This is particularly the case for children when faced with unfamiliar concepts for which they only have a limited repertoire of knowledge. How do contextual cues help them enrich their point of view on the encountered concept?

This question has been the subject of numerous studies, particularly in the field of biology learning. Piaget (1960) had already noted that young children have a strong tendency to animism and personification – that is, in the absence of sufficiently precise biological knowledge, they attribute human characteristics to non-living entities. Later, Inagaki and Hatano (1987, 1991) studied how pre-schoolers use their knowledge about humans to make hypotheses about the behaviors of animate entities, including animals and planets, about which they have no explicit knowledge. In their study, 5- to 6-year-olds were presented with situations in which they had to predict the behavior of plants or animals. In some situations, the expected behavior of the plant or animal was similar to what a human being might do. In other situations, the expected behavior differed from what might be expected of a human being. For example, in the following situation: “Imagine that

someone receives a baby grasshopper[tulip] and wants to keep it the same size forever, because it is small and cute. Is this possible?”, the expected answers regarding questions about the grasshopper were the same as if the question had been asked about a baby: it is impossible to keep it small, because the growth of living things is inevitable. In this example, children can infer the animal’s behavior by using their knowledge about humans. In contrast, in the situation: “Imagine that a grasshopper has been raised in a small cage. One day, someone places a larger cage next to it. Will the grasshopper think anything?” The child’s knowledge about human thought processes cannot be directly applied to the grasshopper’s thoughts. Indeed, in a similar situation, a human might have felt jealousy, whereas the consensual answer among adults who were asked the same question was that the grasshopper will not feel any particular emotion.

The analysis of the children’s responses revealed that their predictions about animal and plant behavior were valid when the behavior of a human in the same situation would have been identical. However, when the expected behavior of the grasshopper was different from what a human might do, and when children did not have the appropriate knowledge about the biology of grasshoppers, then the children’s predictions were based on their understanding of humans, and therefore did not make the same predictions as those that adults might make. In other words, in the absence of explicit knowledge about the biology of animals and plants, children tend to use their knowledge of human functioning to infer properties of the animate entities they encounter.

These studies highlight the fact that in the absence of relevant knowledge about the target concept, individuals tend to use their knowledge from other domains to make sense of the concepts and situations they encounter. Thus, the use of knowledge from other domains allows individuals to flexibly consider certain notions for which their repertoire of knowledge is limited. This process of inferring properties is based on analogical reasoning, which consists of grasping a novelty using previous knowledge about familiar concepts (Holyoak and Thagard 1995). Analogical reasoning is based on a mapping between an unknown or less familiar entity (a target) and a more familiar entity (a source) (Gentner 1989; Holyoak and Thagard 1995). Hofstadter and Sander (2013) even defend the idea that it is analogy that allows humans to create mental categories that shape thinking. Thus, our view of a given concept can be influenced by our prior knowledge of other, more or less distant concepts. Thanks to certain contextual cues, it can be possible to identify similarities between two concepts and thus to infer the properties of new “target” concepts that are not well known thanks to our previous knowledge of “source” concepts. The context can thus not only facilitate access to the characteristics of concepts that would

not be spontaneously retrieved, but it can also call upon knowledge from other domains and allow us to adopt a new perspective on the encountered concepts.

### **3. Representing situations and tasks**

#### **3.1. *Problem solving in the service of the study of reasoning***

Beyond the interpretation of the concepts themselves, there is the question of the perspective adopted on the situations, within which several concepts may be mobilized. Indeed, if the context influences the perspective of individuals on precise notions, it seems likely that it can also influence the encoding of complex situations, and thus interfere with the reasoning of individuals in very varied circumstances. According to Evans (1991), the issue of context is a crucial aspect that any theory of human reasoning must address. As mentioned in the introduction of this chapter (see section 1), in the field of inferential reasoning, it has frequently been pointed out that variations in context may influence an individual's ability to make logical inferences (see, for example, Byrne 1989; Thompson 1994; Chao and Cheng 2000; De Neys *et al.* 2002; Daniel and Klaczynski 2006; Johnson-Laird 2006; Bonnefon and Villejoubert 2007; Douven *et al.* 2018)) to name a few of the many studies on propositional reasoning).

However, reasoning is not limited to making logical deductions from clearly stated propositional sentences, and most of the time individuals have to reason based on complex situations, which may be subject to several interpretations. In order to estimate the influence of context on the reasoning of individuals in their daily life, the study of problem solving may be particularly informative.

Indeed, one of the advantages of problem solving is that it allows one to engage in reasoning processes on a wide variety of situations, closer to real-life concerns than abstract logic problems – although, arguably, still far from truly ecological settings. The study of problem solving thus gained momentum in the early 19<sup>th</sup> century, with behavioral researchers interested in the abilities of animals to solve certain problems (Thorndike 1898; Watson 1930; Hull 1943). These researchers proposed that learning to solve problems was the result of a gradual process based solely on a trial-and-error approach. According to them, animals would find the correct solution to puzzle-like problems by chance, and then reinforce the behavior that led to that solution by positive conditioning, until that behavior was fully automatized. However, this point of view was quickly criticized for its inability to account for “intelligent” problem-solving strategies, in which intermediate goals must



be set in order to find the solution (Köhler 1917, 1925). By refusing to consider the representations of problems constructed by individuals, behaviorism faced a dead-end when explaining the use of such strategies.

### 3.2. The notion of problem representation

On the other hand, the advocates of Gestalt psychology have put the notion of problem representation at the heart of their thinking, and have argued that problem solving is a productive process during which reconstructions (*insights*) of the representation can occur and lead to a sudden revelation of its solution (Wertheimer 1959; Mayer 1983). A crucial advantage of this approach was that it could account for the negative impact that past experiences may have on the ability to find the solution to a new problem (see for example (Luchins 1939, 1942; Katona 1940)). Thus, Gestalt psychology has notably led to the discovery of functional fixity, a cognitive bias limiting the capacity of individuals to imagine unusual uses of common objects. In a famous study, Duncker (1945) designed different tasks to evaluate how often individuals could reuse an object provided to them to solve a problem. For example, in his “Candle problem”, he asked participants to attach a candle to the wall so that its wax would not drip to the floor, using only a box of thumbtacks, matches, and the candle itself (see Figure 1a). Very few participants were able to come up with the solution of turning the empty thumbtack box into a candleholder and attaching it to the wall (see Figure 1b).

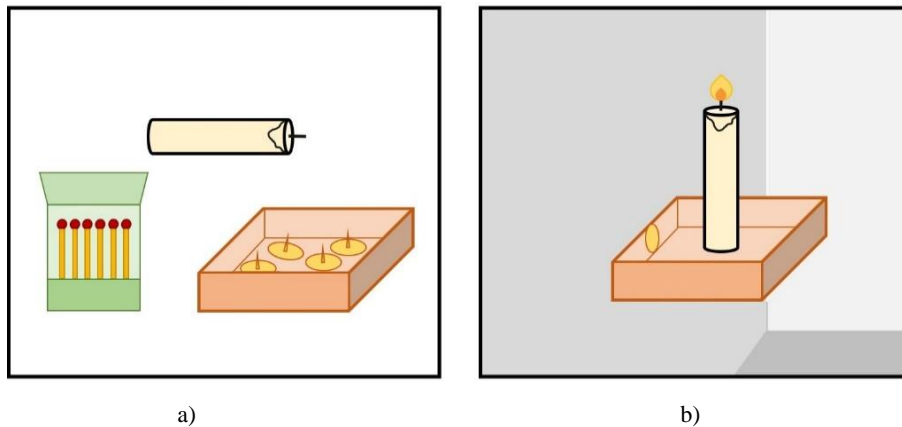


Figure 1. Initial state (a) and final state (b) of the candle problem

However, when participants were presented with a handful of thumbtacks and an empty box separately, significantly more participants used the box as a candlestick to solve the problem. In other words, participants encountered more difficulties when the box had an initial function that could not be used to solve the problem (thumbtack container), than when the box was presented alone, without a contextually predefined function. This experiment is a striking example of the influence that context - in this case prior information about an object's function - can have on the ability of individuals to be flexible in a given situation.

Thus, a crucial contribution Gestalt psychology made was the introduction of the notion of an internal representation of a problem, which can be influenced by prior knowledge and experience. However, this approach has since been criticized for its lack of specificity, and because it says too little about the processes underlying the phenomenon of *insight*, making it difficult to evaluate empirically (Clément 2009). To address these shortcomings, the notion of problem space emerged from the work of Newell and Simon (1972), in which they defined problem solving as the exploration of a problem space which consists of all possible states and transitions between problem states, depending on the interpretation the person attempting to solve the problem makes. This idea that problem solving depends on one's interpretation of the problem is a great improvement when accounting for variations in performance between problems that share the same structure but have dissimilar surface features elements (Simon and Newell 1971; Kotovsky *et al.* 1985). Take, for example, the famous Hanoi Tower problem, defined as follows:

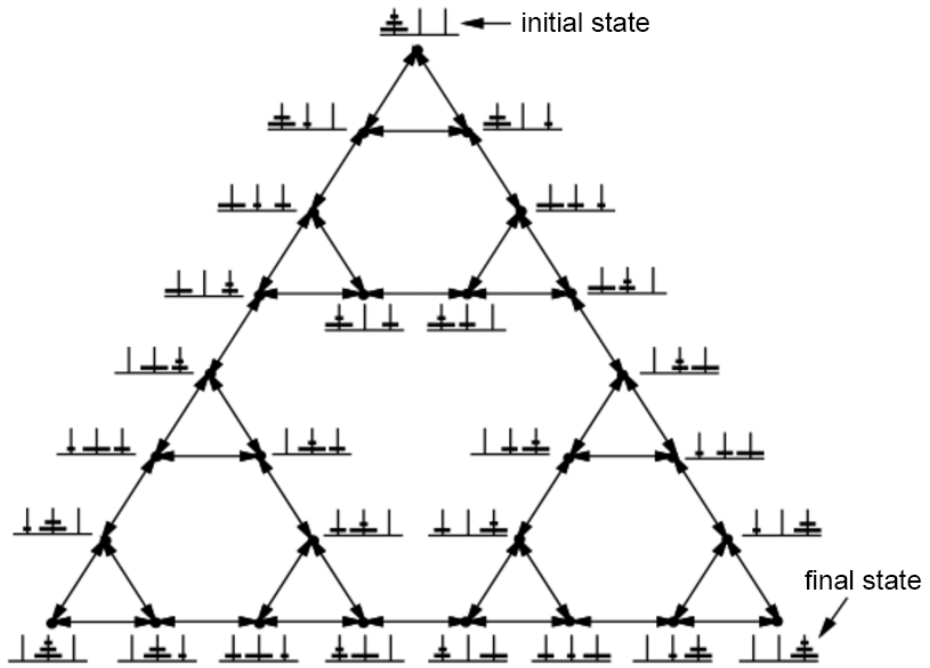
“

There are three discs of unequal sizes, positioned on the leftmost of three pegs, such that the largest disc is at the bottom, the middle-sized disc is in the middle, and the smallest disc is on the top. Your task is to transfer all three discs to the rightmost peg, using the middle peg as a stationing area, as needed. You may move only one disc at a time, and you may never move a larger disc on top of a smaller disc..” (Pretz *et al.* 2003, p. 7)

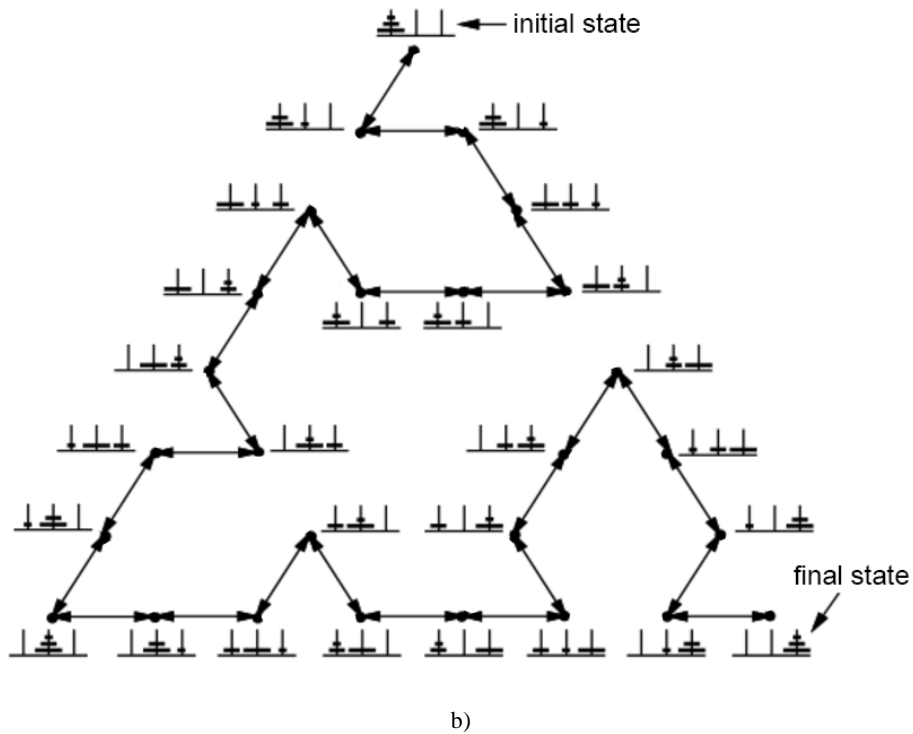
The entire problem space of this problem includes all possible transitions from the initial to the final state, and admits the use of a 7-step solving strategy (see Figure 2a). However, some individuals (notably children aged 6 and 7) add an unnecessary constraint to their interpretation of the problem by making the (erroneous) assumption that only moves between two neighboring pegs are allowed (Richard *et al.* 1993). Thus, these children

construct a reduced problem-space with a “no peg-jumping” rule, which greatly increases the number of steps (26) needed to find the solution (see Figure 2b). Interestingly, Clement and Richard (1997) showed that even adults can be lead to construct a reduced problem-space with a “no jump” rule by changing the surface features of the problem. For example, if instead of moving disks between pegs, the problem is framed in terms of individuals moving between floors using an elevator, then participants are more likely to believe that movement is only allowed between neighboring floors, since they know that an elevator usually does not “skip” floors.

By focusing on the learners’ interpretation of a situation, these studies on problem solving have provided critical insights about the role context plays in adults’ reasoning . This idea that the way in which a problem is presented can influence the representation that will be constructed and therefore the ability of individuals to envision the solution has direct consequences for the understanding of cognitive flexibility. Thus, despite our ability to conceive abstract ideas, human reasoning is always embedded in a concrete context – be it real-life situations or laboratory experiments – and it therefore seems relevant to characterize how contextual information interferes with our ability to take different perspectives on the same situation.



a)



**Figure 2.** Complete (a) and truncated (b) problem space of the Tower of Hanoi problem

### 3.3. Content effects as mediators of flexibility?

In addition to the abovementioned problem solving tasks, there are also other situations, similar to those that can be encountered in real-life school settings, that are suitable to study the conditions in which the cognitive flexibility of individuals may be tested. For instance, consider the learning of expansion and factorization in mathematics. Understanding the mathematical equivalence between an expanded and a factorized equation requires adopting two distinct points of view of the same situation, which is not a trivial endeavour. In order to study the conditions that make it possible for this change of perspective to take place, Coquin-Viennot and Moreau (2003) designed an experiment in

which they asked third- and fifth-grade students to solve problems of the following type: “For a contest, a florist prepares 5 roses and 7 tulips for each of the 14 candidates. How many flowers will the florist use in total?” This statement was primarily solved by the students with an expansion strategy:  $(14 \times 5) + (14 \times 7) = 168$ . However, there is another strategy that makes it possible to solve this problem by performing only one multiplication, and thus limiting the risk of error. This is the factorization strategy:  $14 \times (5 + 7) = 168$ . To help students identify the relevance of the latter solving strategy, the researchers designed a slightly modified version of the problem: “For a contest, a florist prepares, for each of the 14 candidates, a bouquet of 5 roses and 7 tulips. How many flowers will the florist use in total?” They then observed that the mere mention of “a bouquet” in the problem statement led students to first attempt the factorization strategy, because it encouraged them to represent the 5 roses and the 7 tulips as an inseparable set, which then had to be multiplied by the number of candidates. Thus, this experiment showed that it was possible to promote a certain type of cognitive flexibility in students by modifying the wording of the statement so that a different representation of the situation is created.

Along these lines, researchers have investigated what influences the construction of certain interpretations of given mathematical situations. They manipulated the objects mentioned in the statement of a problem in order to favor an alternative point of view, without even having to enrich the problem statement with more details. For example, Bassok *et al* (1998) conducted an experiment on the influence everyday knowledge has on individuals’ choices between additive or multiplicative situations. The authors showed that when they asked individuals to conceive problems involving tulips and roses, most imagined situations in which flowers of both varieties are added and should be counted all together (e.g., 6 tulips + 2 roses = 8 flowers). Conversely, when these same individuals were asked to make up problem statements involving vases and tulips, this time the majority designed situations in which the total number of tulips is divided by the number of vases (e.g., 6 tulips ÷ 2 vases = 3 tulips per vase). In other words, the simple fact of replacing roses by vases influenced the individuals to imagine entirely different situations, and thus to create arithmetic statements requiring the adoption of radically opposed perspectives to be solved. Thus, reference to certain objects or entities may be sufficient to lead an individual to adopt a specific perspective, which may have favorable or deleterious effects on the flexibility with which individuals can transfer their knowledge from one situation to another. This idea that transfer of learning can be conditioned by such content effects was studied by Bassok *et al.* (1995), who conducted an experiment with mathematical permutation problems. In their study, participants first studied a permutation problem with its solution, and then had to apply the same solving strategy to a new problem with the

same mathematical structure. All the problems presented involved situations with assignments: in some cases, objects were assigned to individuals (e.g., students received prizes), in other cases, individuals were assigned to objects (e.g., secretaries were assigned to computers), and finally, in some cases, individuals were assigned to other individuals (e.g., doctors from two hospitals worked in pairs). The results of the study showed that the participants' ability to apply the solution learnt on the first problem to the new problem depended on the type of assignment made within the statements. Indeed, if the solution was learnt on a problem in which objects were assigned to individuals, then 89% of the participants were able to apply it correctly to the other problems in which objects were assigned to individuals (for example, cars assigned to mechanics). On the other hand, when participants had to apply this solution on problems describing opposite assignments, in which individuals are assigned to objects, then none of the participants managed to use the learnt solving strategy correctly. In other words, their understanding of the algorithm for solving the training problem was deeply linked to the content of the problem statement, and when the statement changed in the new problem, then the participants were unable to adapt their solving strategy. Here, the content of the problems thus both promoted the transfer of strategy use from one situation to another (when both problems had assignments of objects to individuals), and hindered this transfer (when the new problem had an assignment of individuals to objects). Furthermore, when participants were asked to use the algorithm on problems in which individuals were assigned to other individuals, then participants were only able to correctly apply the solution in half of the cases, highlighting that they were neither particularly disadvantaged nor particularly aided by the content in this situation. In sum, the inclination of individuals to adopt a relevant perspective on a problem seems directly dependent on the content of the statements, which acts as a mediator of flexibility.

### **3.4. Robust influences**

It thus seems true that the perspective adopted by certain individuals may depend on the content of the encountered situations, yet the question whether this influence is really present in all populations, or whether it is only present among the youngest or those who are less expert in the field remains open. To address it, a study by Gros *et al.* (2017) compared the performance of about 60 fifth-grade children and 50 adults on an arithmetic problem-solving task. They were interested to see how the use of certain variables (durations, heights, number of floors) within a problem would elicit an ordinal view of the situation (where a representation of the numerical values presented in the problem can be

ordered along an axis), whereas the use of other variables (collections of elements, weights, prizes) would elicit a cardinal view of the situation (where the representation would take the form of set representing the different quantities). For example, problems mentioning durations should elicit an ordinal view of the situation, it is customary for participants to create representations of temporal values along an axis (as on a timeline, for example). Conversely, problems involving marbles that should be counted should elicit a cardinal viewpoint, as marbles are more often represented grouped in bags rather than aligned along an axis. However, the specificity of the problems presented to the participants was that they all admitted two distinct solving strategies: a three-step strategy, and a one-step strategy. The authors hypothesized that the participants would use one or the other of these two strategies depending on the perspective (cardinal or ordinal) that they adopted on the situation described in the problem. Indeed, the three-step solution was compatible with a cardinal representation of the situation, whereas it was necessary to adopt an ordinal point of view on the problems in order to use the one-step solution. Yet, from a mathematical point of view, it is possible to use both strategies on all problems. In order to assess the impact of content on the adopted point of view, participants were asked to draw a diagram of each problem and then to solve them using as few operations as possible. The diagrams drawn by the participants made it possible to identify which point of view they adopted on each problem. The results revealed that both children and adults tended to adopt the point of view compatible with the content of the statements: duration problems were represented with axes, scales, intervals, etc., whereas on cardinal problems, the drawings represented mainly sets or groupings of elements. In other words, the perspective adopted in both groups of participants depended on the content of the statements, despite the fact that all problems shared the same underlying mathematical structure.

Furthermore, analysis of the calculations reported by the participants made it possible to identify which strategy they had primarily used to solve the problems. In both populations, responses varied according to the cardinal/ordinal perspective elicited by the content of the statements: both children and adults preferred to use the three-step strategy on marble problems, whereas they preferred to use the one-step strategy on duration problems. Despite the explicit instruction to use as few calculation steps as possible, taking an ordinal view of a cardinal problem was too costly for them in the majority of cases, due to the influence of the problem statement's content. Both groups of participants were thus unable to move beyond the content of the statements to flexibly adopt either of the two possible perspectives. The mention of durations or marbles in the wording of the problems constrained their representation of the described situations, suggesting that the content



interfered with the cognitive flexibility of both children and adults, despite adults' arithmetic skills.

Even though this influence of content remains present during development, an important question remaining open regards whether this influence persists even at a high level of expertise, or whether it disappears as soon as individuals attain sufficient mastery of a domain? A recent study explored this issue by comparing the performance of general adults and expert mathematicians on problems similar to those just described (Gros *et al.* 2019). The presented task consisted of cardinal and ordinal problems for which only the one-step strategy could be used. Each problem was accompanied by its solution (a simple subtraction) and participants had to decide whether this solution actually solved the problem, or whether the problem was unsolvable. The idea was to test whether or not, when only one solution is possible, participants are able to change their perspective on the problem in order to perceive the relevance of the proposed solution. The results showed that lay adults accepted the proposed solution on more than 80% of ordinal problems, whereas they were able to identify its relevance on only 47% of cardinal problems. For expert mathematicians, although their performance was superior to that of the lay adult, a similar difference emerged: in 24% of cases, they incorrectly stated that cardinal problems had no solutions, whereas they made this mistake in only 5% of ordinal problems. Moreover, participants who were able to understand the relevance of the one-step solution on cardinal problems took more time than for ordinal problems, implying that the change of perspective they had to make had a significant cognitive cost. Thus, even experts in mathematics were fell prey to content effects: they made more errors and took more time to answer when the perspective solicited by the content of the statements was not directly compatible with the proposed solution. This study thus illustrates the widespread influence of context on the perspective that individuals – even experts – adopt to solve problems.

#### **4. Semantic recoding and cognitive flexibility**

##### **4.1. *Semantic recoding, a lever for flexibility in school?***

Given the strong influence content has on the perspectives adopted by individuals, it is important to ask by which mechanisms such effects may be attenuated and how to favor a new, more effective perspective when the initial one is inappropriate or inefficient. Indeed, cognitive flexibility implies alternating between several distinct perspectives on the same situation, which means that the process by which this switch is carried out can be considered as a lever of flexibility, an indispensable tool for flexibility to take place. The

idea that it is possible to re-represent a problem or situation has been put forward several times in the literature (Pretz *et al.* 2003; Vicente *et al.* 2007) and the term semantic recoding has been proposed to describe this process (Clément 2009; Gamo *et al.* 2010; Gros *et al.* 2020; Gvozdic and Sander 2020). Thus, alternating between two strategies in a flexible way would require a semantic recoding of the constructed representation, a crucial step making it possible to adopt a new point of view on the situation.

How, then, can such a recoding be elicited? Changes in the problem statement can facilitate the adoption of a new perspective on a situation (Ibarra and Lindvall 1982; Hudson 1983; Riley *et al.* 1983). For example, De Corte *et al.* (1985) showed that reformulating arithmetic statements to make the described semantic relations more explicit without changing the structure of the problem facilitated its resolution by young children. However, it also seems possible to facilitate a recoding of the initial point of view without having to directly modify the problem. Indeed, a study by Gamo *et al.* (2010) showed that it was possible to train third and fifth graders to switch between two points of view on the same problem. Using cardinal and ordinal problems similar to those mentioned earlier (see section 3.4), through semantic recoding the authors tried to get students to consider either of the two possible solving strategies. In two one-hour sessions, students were asked to compare several problems that shared the same mathematical structure, but whose statements solicited different points of view. The idea was to encourage students to shift from one perspective to another by pointing out the similarities between the problems. In this process, the students constructed an abstract schema of the mathematical structure of the problems, in order to emphasize that all the problems shared a common foundation, despite the divergent perspectives that the problems might lead to. This training on the semantic recoding of problems was followed by a test phase to measure its effectiveness. The results showed that the training was successful: students who participated in both sessions were more likely to find the solution to cardinal problems with one-step strategies, than were students who did not receive training on semantic recoding. Thus, by highlighting the similarities between the problems and emphasizing the common structure they shared, it was possible to train the students to switch from one point of view to another, and therefore to achieve greater cognitive flexibility with regards to the demands of the task. Hence, it appears that semantic recoding can be practiced in a school setting to help students overcome content effects and approach the situations they encounter with flexibility. But can content effects sometimes play a role in favor of learning?

As we have seen, students regularly draw on prior, daily-life knowledge to interpret the content that is less familiar to them (Inagaki and Hatano 1987, 1991). When they arrive to

school, students are confronted with academic concepts for the first time in a formal setting. Informal interpretations of the taught concepts may sometimes be consistent with the formal explanations recognized in the studied field, but often inconsistencies between intuitions based on previous experience and the studied academic concepts will arise. When there is a conflict between the informal and formal components of learning, an epistemological barrier emerges, leading to misinterpretations and systematic errors (Fischbein 1987; Hofstadter and Sander 2013). One of the main challenges of school-based learning is therefore to enable students to go beyond the informal interpretation of a situation and adopt a new perspective, one that is closer to the formally recognized notion.

These misconceptions arise in all school subjects and have been studied in particular in the learning of mathematics. Indeed, ranging from the implementation of mental calculation strategies to the interpretation of arithmetic problems, the systematic use of partly erroneous intuitions can lead to difficulties (Fischbein 1987). As an example, let us consider the learning of subtraction. When we present a subtraction problem to 6–7-year-old students such as “Mehdi has 12 marbles, he loses 3. How many marbles does Mehdi have now?”, students will interpret it as a “taking-away” situation, in which marbles are lost, which will lead them to use a strategy of taking away 3 from 12. Now let us consider a different problem statement: “Nadia had 9 marbles, then she won some more. Now she has 12 marbles. How many marbles did she gain?”. Even though it is still a subtraction problem, this new statement will lead the students to interpret the problem as a “searching for the distance” situation where they will proceed by addition with missing numbers ( $9 + \dots = 12$ ) rather than by direct subtraction (De Corte and Verschaffel 1987; Brissiaud and Sander 2010).

Here, the two possible views of the problem (taking away or searching for the distance) are appropriate for finding the solution. However, some strategies can be more costly than others for students, and taking on one perspective rather than another can sometimes lead students to fail. This is particularly the case in the problem: “Mehdi has 12 marbles, he loses 9. How many marbles does Mehdi have now?”. In this statement, the student risks adopting the “taking away” point of view, and thus trying to find the solution by subtracting 9 from 12. In this case, the direct subtraction strategy is more costly than the indirect addition strategy, and therefore leads to an increased error rate.

In order to help students go beyond the initial encoding prompted by the content of such statements and adopt perspectives more favorable for success, the ACE-Arithm'École intervention program has implemented an arithmetic problem-solving curriculum based on semantic recoding (Vilette *et al.* 2017; Fischer *et al.* 2018). In the lessons, the teachers

showed the students how to compare different problem statements and solving strategies for a wide range of problems (e.g., combination problems: “I have 7 blue marbles and 4 red marbles, how many do I have in total?”; comparison: “I have 7 roses and 11 daisies in a bouquet, how many more daisies are there than roses?”; change: “I have 4 euros and I earn more. I now have 11 euros. How much did I earn? How much did I earn?”, etc.). These comparisons were made to identify the similarity between problems which lead to different perspectives despite their common mathematical structure. Different representation tools were used during the sessions: diagrams in the form of number lines or arithmetic boxes, which they had to fill in with the values present in the problem statements. In a study designed to measure the effectiveness of this intervention, Gvozdic and Sander (2020) showed that students who received the ACE-Arithm’École intervention focused on semantic recoding were able to switch between indirect addition and direct subtraction more frequently than students who followed the regular arithmetic curriculum. This study has thus shown that it is possible to provide students with a greater repertoire of relevant problem-solving strategies and understand their range of applicability, allowing them to flexibly choose the strategy suitable for solving encountered problems. Semantic recoding training thus appears to be a promising method for increasing the flexibility with which students approach school notions.

#### **4.2. New perspectives to explore**

Another interesting perspective for fostering semantic recoding and transfer in a school setting comes from recent studies on the role of argumentation in the spreading of counterintuitive beliefs. According to the argumentative theory of reasoning, individuals are more objective and less biased when evaluating arguments made by other individuals (Mercier and Sperber 2011). Thus, this theory predicts that individuals are more likely to change their minds when they engage in discussions with others, than when they are simply presented with statements that challenge their views (Mercier 2016). The underlying idea is that argumentation might play a specific role in helping a person change their viewpoint and adopt a new, yet counterintuitive perspective. For instance, Claidière *et al.* (2017) conducted an experiment in which they asked groups of participants to solve problems with counterintuitive solutions in either a “discussion” or “silence” situation. After an initial individual phase in which participants were asked to solve a problem on their own, they were allowed to communicate with each other. Participants in the discussion condition were told that they could discuss their answers with their immediate neighbors, while groups in the silence condition were only allowed to look at their neighbors’ answers, without speaking or discussing the problem. Participants were asked to note their responses

at regular intervals to determine if they changed their minds about the answer to the problem. The results showed that in the free discussion condition, the majority of the participants in each group reached a global consensus on the correct answer, whereas in the silent condition, participants who found the correct answer communicated it only to a limited number of people, with most participants remaining – erroneously – convinced of the validity of their initial answer. This inability to change one’s point of view in the absence of discussion echoes with the difficulty that individuals have in demonstrating cognitive flexibility in a given situation. The authors concluded that argumentation was a powerful tool to help students acquire counterintuitive concepts when other methods fail, and such an approach is promising for overcoming the most deceitful effects of contents.

Finally, another path of study aimed at encouraging the adoption of a new perspective on a given situation comes from the theoretical framework of conceptual metaphors (Lakoff and Johnson 1999). According to Lakoff and Johnson, conceptual metaphors allow us to grasp abstract concepts by relating them to knowledge which arises from our senses. This approach considers that the metaphors conveyed by language are not only figures of speech at the service of rhetoric, but that they actually reflect the conceptual system of their speakers. Thus, conceptual metaphors would be a central cognitive mechanism that structures concepts (Lakoff and Johnson 1999). They are embodied correspondences that preserve the inferences drawn from our sensory experiences and allow us to establish a correspondence between concepts belonging to semantically different domains (Bowdle and Gentner 2005). They reflect the sensory system from which they originate and leverage it metaphorically. For example, the idea that “to know is to see” makes it possible to use expressions such as “you see what I mean”. This phrase conveys a metaphorical content (the act of knowing) rather than the literal meaning (the act of seeing). Although the embodied nature of metaphors is still debated today, the use of conceptual metaphors for understanding abstract concepts is undeniably recognized (Gibbs 2009).

Moreover, conceptual metaphors make it possible for individuals to reason about different topics, regardless of their inherent complexity, even when they have limited understanding of those topics (Thibodeau and Boroditsky 2011). A series of studies by Thibodeau and Boroditsky ((2011, 2013, 2015); see also (Robins and Mayer 2000; Thibodeau *et al.* 2017)) have examined how metaphors shape reasoning. They hypothesized that different metaphors should activate different semantic networks, which lead to different inferences. In their studies they asked participants, for instance, to read and think about a crime problem that was introduced with the use of one among two possible metaphors: either crime was presented as a wild beast attacking the city or as a virus

spreading through the city. The choice of the metaphor that framed the problem had an impact on the solutions proposed by the participants. When participants were confronted with a crime presented *via* the virus metaphor, they most often chose preventive solutions. When faced with a crime presented via the wild beast metaphor, they more often chose repressive measures. Consistent with other studies conducted on the subject, these results indicate that the chosen metaphorical framing results in the activation of knowledge consistent with the metaphor, which influences the point of view adopted on the situation and leads to different solutions. Conceptual metaphors are thus a means by which we can use different contents to prompt changes in viewpoints on a given task or situation. Identifying the metaphors underlying individuals' points of views and proposing alternative metaphors can thus facilitate the expression of cognitive flexibility, by presenting and choosing among two alternative ways of looking at the same situation.

For example, consider a persistent educational myth that education professionals sometimes struggle to dispel: the theory of learning styles. This theory, which postulates that some learners are more “visual,” others more “auditory,” and still others more “kinaesthetic,” advocates that each student should be given learning materials in accordance with his or her preferred sensory modality. Thus, visual students would learn best by looking at pictures, auditory students would need to listen to lessons, while kinaesthetic students would need to manipulate objects in order to integrate new concepts. Although extremely popular, this theory is actually a “neuromyth,” a unfounded claim that is not supported by research findings (Howard-Jones 2014; Sander *et al.* 2018). Indeed, although preferences for one sensory modality over another may be robust among students (Henson and Hwang 2002), receiving course materials that correspond only to that sensory modality has no benefit for the student, and may even be deleterious (Massa and Mayer 2006; Cook *et al.* 2009). Here, a metaphor that would fit the learning styles perspective might be: different students benefit differently from different formats of presentations, because they have different needs, like plants. Not all plants should be treated the same, because what makes one plant grow may kill another. Therefore, it is necessary to adapt the environment of each plant to meet its specific needs. Yet, we actually know that the most beneficial approach for students is to provide them with a wide variety of learning materials: visual, auditory, kinaesthetic, etc. (Gellevij *et al.* 2002; Geake 2008). How is it then possible to abandon ineffective and even deleterious educational practices, and favor the adoption of a point of view that is more suitable for attaining learning objectives? The use of another metaphorical description might allow for such a semantic recoding: teaching students exclusively on the basis of their learning style preferences is like only giving them the dish they prefer to eat at each meal, ignoring the balance of their diet. For example, feed them

only mashed potatoes, because that is their favorite food. With the new framework provided by this metaphor, the shortcomings of such a stance suddenly become clear: settling for the students' preferred learning modalities does not guarantee optimal learning, and it is preferable to provide more diverse materials that alternatively solicit different sensory modalities. Thus, the use of conceptual metaphors as a way of fostering semantic recoding seems to be a promising outlook for increasing the cognitive flexibility of individuals.

## **5. Conclusion**

At the beginning of this chapter, we opened the question regarding the role of context in the face of cognitive flexibility (see sections 3 and 4). From the few works that we discussed, it appears that context plays a role at several stages of reasoning. First of all, the very apprehension of the concepts that underlie each human thought seems to be dependent on the context in which individuals find themselves. On the one hand, different properties of a single concept may be activated according to the context in which it is presented. On the other hand, in the absence of sufficient knowledge, certain contextual elements can help individuals to call upon knowledge from another domain in order to apprehend a concept that is insufficiently grasped. Furthermore, beyond the simple apprehension of concepts within a given situation, the perspective that individuals adopt when the situation is complex is also influenced by contextual cues, as well as the previous knowledge solicited by the very content of these situations. This gives contextual elements a pivotal role in the reasoning individuals engage in: it is limited by the initial perspective they adopt and from which they have difficulties spontaneously disengaging. These observations, made among both young learners and experts, highlight the importance of enhancing the flexibility with which individuals approach a context as a way of informing their reasoning. While some inferences from contextual cues may be extremely useful in approaching new situations (Gentner 1989), others can lead to dead ends in which individuals invariably fail. Thus, training semantic recoding seems to be a useful step in promoting a change in perspective, and thus support cognitive flexibility. Explicit teaching of alternative viewpoints that may be adopted could be one way to achieve this goal, just as the use of argumentation or conceptual metaphors could make it possible to develop a greater flexibility in individuals of all levels.

It is important to remember that learning and teaching do not take place in isolation but are always embedded in the real world. It is therefore only natural that there should be

interference between school notions and contextual insights. This interdependence can sometimes be misleading, but, if used appropriately, may also be a driving force in helping individuals understand complex abstract ideas with little effort. By using the right examples, it is possible to foster a deeper understanding of complex concepts, making it possible for individuals to gradually extract the abstract structure underlying different situations. In sum, we believe it is time to abandon the idea that humans must first and foremost strive to act as purely rational beings, guided by reason alone. Rather than trying to prevent all forms of contextual interference, it seems suitable to make use of the countless inferences that each person makes at each moment to guide them towards a flexible use of knowledge from their surroundings.

## 6. Bibliography

- Aristotle, R. (1999). *Nicomachean Ethics*. In *Nicomachean Ethics*, 2<sup>nd</sup> edition, Irwin, T. (ed.). Hackett Pub, Indianapolis.
- Barsalou, L.W. (1982). Context-independent and context-dependent information in concepts. *Memory and Cognition*, 10, 82–93.
- Barsalou, L.W. (1983). Ad hoc categories. *Memory and Cognition*, 11, 211–227.
- Barsalou, L.W., Medin, D.L. (1986). Concepts: static definitions or context-dependent representations?. *Cahiers de psychologie cognitive*, 6, 187–202.
- Bassok, M., Wu, L., Olseth, L.K. (1995). Judging a book by its cover: interpretative effects of content on problem-solving transfer. *Memory and Cognition*, 21(6), 1522.
- Bassok, M., Chase, V.M., Martin, S.A. (1998). Adding apples and oranges: Alignment of semantic and formal knowledge. *Cognitive Psychology*, 35(2), 99–134.
- Bonnefon, J.F., Villejoubert, G. (2007). Modus Tollens, Modus Shmollens: Contrapositive reasoning and the pragmatics of negation. *Thinking & reasoning*, 13(2), 207–222.
- Boole, G. (1854). *An investigation of the laws of thought: on which are founded the mathematical theories of logic and probabilities*. Dover Publications, New York.
- Bowdle, B.F., Gentner, D. (2005). The Career of Metaphor. *Psychological Review*, 112(1), 193–216.
- Brissiaud, R., Sander, E. (2010). Arithmetic word problem solving: A situation strategy first framework. *Developmental Science*, 13(1), 92–107.



- Byrne, R.M. (1989). Suppressing valid inferences with conditionals. *Cognition*, 31(1), 61–83.
- Chao, S.J., Cheng, P.W. (2000). The emergence of inferential rules: The use of pragmatic reasoning schemas by preschoolers. *Cognitive Development*, 15(1), 39–62.
- Cheng, P.W., Holyoak, K.J. (1985). Pragmatic reasoning schemas. *Cognitive psychology*, 17(4), 391–416.
- Claidière, N., Trouche, E., Mercier, H. (2017). Argumentation and the diffusion of counter-intuitive beliefs. *Journal of Experimental Psychology: General*, 146(7), 1052.
- Clément, E. (2009). *La résolution de problèmes*. Armand Colin, Paris.
- Clément, E. Richard, J.-F. (1997). Knowledge of domain effects in problem representation: The case of Tower of Hanoi isomorphs. *Thinking & Reasoning*, 3(2), 133–157.
- Cook, D.A., Thompson, W.G., Thomas, K.G., Thomas, M.R. (2009). Lack of interaction between sensing-intuitive learning styles and problem-first versus information-first instruction: A randomized crossover trial. *Advances in Health Sciences Education*, 14(1), 79–90.
- Cosmides, L. (1989). The logic of social exchange: Has natural selection shaped how humans reason? Studies with the Wason selection task. *Cognition*, 31(3), 187–276.
- Coquin-Viennot, D., Moreau, S. (2003). Highlighting the role of the episodic situation model in the solving of arithmetical problems. *European Journal of Psychology of Education*, 3, 267–279.
- Cosmides, L., Tooby, J. (1992). Cognitive adaptations for social exchange. In *The adapted mind: Evolutionary psychology and the generation of culture*, Barlow, J.H., Cosmides, L., Tooby, J. (eds.). Oxford University Press, Oxford.
- Daniel, D.B., Klaczynski, P.A. (2006). Developmental and individual differences in conditional reasoning: Effects of logic instructions and alternative antecedents. *Child Development*, 77(2), 339–354.
- De Corte, E., Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education*, 18(5), 363–381.
- De Corte, E., Verschaffel, L., De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solution. *Journal of Educational Psychology*, 77, 460–470.

- De Neys, W., Schaeken, W., D'ydewalle, G. (2002). Causal conditional reasoning and semantic memory retrieval: A test of the semantic memory framework. *Memory & cognition*, 30(6), 908–920.
- Descartes, R. (1984). *Meditations on first philosophy*. In *The Philosophical Writings of René Descartes*, volume 2, Cottingham, J., Stoothoff, R., Murdoch, D. (eds.). Cambridge University Press, Cambridge.
- Douven, I., Elqayam, S., Singmann, H., van Wijnbergen-Huitink, J. (2018). Conditionals and inferential connections: A hypothetical inferential theory. *Cognitive psychology*, 101, 50–81.
- Duncker, K. (1945). On solving problems. *Psychological Monographs*, 58(5), 270.
- Evans, J.S.B.T. (1991). Theories of Human Reasoning: The Fragmented State of the Art. *Theory & Psychology*, 1(1), 83–105.
- Falavarjani, M.F., Yeh, C.J. (2018). The impact of acculturation identification and acculturative stress on creativity among Iranian immigrants living in Malaysia. *Journal of Ethnic and Migration Studies*, 44(13), 2219–2239.
- Fischbein, E. (1987). *Intuition in science and mathematics. An educational approach*. Reidel, Dordrecht.
- Fischer, J.-P., Sander, E., Sensevy, G., Vilette, B., Richard, J.-F. (2018). Can young students understand the mathematical concept of equality? A whole-year arithmetical teaching. *European Journal of Psychology of Education*, 34(2), 439–456.
- Gamo, S., Sander, E., Richard, J.-F. (2010). Transfer of strategy use by semantic recoding in arithmetic problem solving. *Learning and Instruction*, 20, 400–410.
- Geake, J. (2008). Neuromythologies in education. *Educational Research*, 50(2), 123–133.
- Gellevij, M., van der Meij, H., De Jong, T., Pieters, J. (2002). Multimodal versus unimodal instruction in a complex learning context. *The Journal of Experimental Education*, 70(3), 215–239.
- Gentner, D. (1989). The mechanisms of analogical learning. In *Similarity and analogical reasoning*, Vosniadou, S., Ortony, A. (eds.). Cambridge University Press, Cambridge.
- Gibbs, R.W. (2009). Why do some people dislike conceptual metaphor theory? *Cognitive Semiotics*, 5(1/2), 14–36.

- Giroto, V., Kimmelmeier, M., Sperber, D., van der Henst, J.B. (2001). Inept reasoners or pragmatic virtuosos? Relevance and the deontic selection task. *Cognition*, 81(2), B69–B76.
- Gros, H., Thibaut, J.-P., Sander, E. (2017). The nature of quantities influences the representation of arithmetic problems: evidence from drawings and solving procedures in children and adults. In *Proceedings of the 39th Annual Meeting of the Cognitive Science Society*, Granger, R., Hahn, U., Sutton, R. (eds.). Cognitive Science Society, Austin.
- Gros, H., Sander, E., Thibaut, J.-P. (2019). When masters of abstraction run into a concrete wall: Experts failing arithmetic word problems, *Psychonomic Bulletin & Review*, 26(5), 1738–1746.
- Gros, H., Thibaut, J.-P., Sander, E. (2020). Semantic Congruence in Arithmetic: A New Conceptual Model for Word Problem Solving. *Educational Psychologist*, 55, 1532–6985.
- Gvozdic, K., Sander, E. (2020). Learning to be an opportunistic word problem solver: going beyond informal solving strategies. *ZDM Mathematics Education*, 52, 111–123.
- Henson, R.K., Hwang, D.Y. (2002). Variability and prediction of measurement error in Kolb's learning style inventory scores a reliability generalization study. *Educational and Psychological Measurement*, 62(4), 712–727.
- Hilton, D.J., Kimmelmeier, M., Bonnefon, J.F. (2005). Putting ifs to work: goal-based relevance in conditional directives. *Journal of Experimental Psychology: General*, 134(3), 388.
- Hofstadter, D.R., Sander, E. (2013). *Surfaces and essences: Analogy as the fuel and fire of thinking*. Basic Books, New York.
- Holyoak, K., Thagard, P. (1995). *Mental Leaps: Analogy in Creative Thought*. MIT Press, Cambridge.
- Howard-Jones, P.A. (2014). Neuroscience and education: myths and messages. *Nature Reviews Neuroscience*, 15(12), 817.
- Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. *Child Development*, 54(1), 84–90.
- Hull, C.L. (1943). *Principles of Behavior: An Introduction to Behavior Theory*. Appleton Century, Boston.

- Ibarra, C.G., Lindvall, C.M. (1982). Factors associated with the ability of kindergarten children to solve simple arithmetic story problems. *The Journal of Educational Research*, 75(3), 149–156.
- Inagaki, K., Hatano, G. (1987). Young children's spontaneous personification as analogy. *Child Development*, 58(4), 1013–1020.
- Inagaki, K., Hatano, G. (1991). Constrained person analogy in young children's biological inference. *Cognitive Development*, 6(2), 219–231.
- Inhelder, B., Piaget, J. (1958). The growth of logical thinking from childhood to adolescence: An essay on the construction of formal operational structures. *Psychology Press*, 22.
- Johnson-Laird, P.N. (2006). *How we reason*. Oxford University Press, Oxford.
- Katona, G. (1940). *Organizing and Memorizing: Studies in the Psychology of Learning and Teaching*. Columbia University Press, New York.
- Klauer, K.C., Stahl, C., Erdfelder, E. (2007). The abstract selection task: New data and an almost comprehensive model. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 33(4), 680.
- Köhler, W. (1917). *Intelligenzprüfungen an Menschenaffen [Intelligence tests on anthropoid apes]*. Preußische Akademie der Wissenschaft, Berlin.
- Köhler, W. (1925). *The mentality of apes*. Harcourt Brace Jovanovich, New York.
- Kotovsky, K., Hayes, J.R., Simon, H.A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive psychology*, 17(2), 248–294.
- Lakoff, G., Johnson, M. (1999). *Philosophy in the flesh: the embodied mind and its challenge to Western thought*. Basic Books, New York.
- Lebois, L.A.M., Wilson-Mendenhall, C.D., Barsalou, L.W. (2015). Are automatic conceptual cores the gold standard of semantic processing? The context-dependence of spatial meaning in grounded congruency effects. *Cognitive Science*, 39, 1764–1801.
- Luchins, A.S. (1939). The Einstellung Effect in learning by repetition. Unpublished PhD thesis, New York University, New York.
- Luchins, A.S. (1942). Mechanization in problem solving. *Psychological Monographs*, 54, 248.

- Massa, L.J., Mayer, R.E. (2006). Testing the ATI hypothesis: Should multimedia instruction accommodate verbalizer-visualizer cognitive style?. *Learning and Individual Differences*, 16(4), 321–335.
- Masson, M.E.J. (1995). A distributed memory model of semantic priming. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 21, 3–23.
- Mayer, R.E. (1983). *Thinking, problem solving, cognition*. Freeman, New York.
- McRae, K., De Sa, V.R., Seidenberg, M.S. (1997). On the nature and scope of featural representations of word meaning. *Journal of Experimental Psychology: General*, 126, 99–130.
- Murphy, G.L. (2002). *The big book of concepts*. The MIT Press, Cambridge.
- Mercier, H. (2016). The argumentative theory: Predictions and empirical evidence. *Trends in Cognitive Sciences*, 20(9), 689–700.
- Mercier, H., Sperber, D. (2011). Why do humans reason? Arguments for an argumentative theory. *Behavioral and Brain Sciences*, 34(2), 57.
- Murphy, G.L. (2002). *The big book of concepts*. The MIT Press, Cambridge.
- Newell, A., Simon, H.A. (1972). *Human problem solving*. Prentice-Hall, Englewood Cliffs.
- Pecher, D., Zwaan, R.A. (2017). Flexible concepts: a commentary on Kemmerer (2016). *Language, Cognition and Neuroscience*, 32(4), 444–446.
- Piaget, J. (1960). *The child's conception of the world*. Littlefield, Totowa.
- Plaut, D.C. (2002). Graded modality specific specialisation in semantics: A computational account of optic aphasia. *Cognitive Neuropsychology*, 19, 603–639.
- Pretz, J.E., Naples, A.J., Sternberg, R.J. (2003). Recognizing, defining, and representing problems. *The Psychology of problem solving*, 30(3).
- Ragni, M., Kola, I., Johnson-Laird, P.N. (2018). On selecting evidence to test hypotheses: A theory of selection tasks. *Psychological Bulletin*, 144(8), 779–796.
- Richard, J.-F., Poitrenaud, S., Tijus, C. (1993). Problem-solving restructuring: Elimination of implicit constraints. *Cognitive Science*, 17(4), 497–529.
- Riley, M.S., Greeno, J.G., Heller, J.I. (1983). Development of children's problem-solving ability in arithmetic. In *The development of mathematical thinking*, Ginsburg, H.P. (ed.). Academic Press, New York.

- Rips, L.J., Shoben, E.J., Smith, E.E. (1973). Semantic Distance and the Verification of Semantic Relations. *Journal of Verbal Learning and Verbal Behavior*, 12, 1–20.
- Robins, S., Mayer, R.E. (2000). The metaphor framing effect: Metaphorical reasoning about text-based dilemmas. *Discourse Processes*, 30(1), 57–86.
- Rosch, E. (1978). Principles of Categorization. In *Cognition and Categorization*, Rosch, E., Lloyd, B.B. (eds.). Erlbaum, Hillsdale.
- Ross, B.H., Bradshaw, G.L. (1994). Encoding effects of reminders. *Memory & Cognition*, 22(5), 591–605.
- Sander, E. (2000). *L'analogie, du naïf au créatif*. L'Harmattan, Paris.
- Sander, E., Gros, H., Gvozdic, K., Scheibling-Sève, C. (2018). *Les Neurosciences en Éducation*. Retz, Paris.
- Simon, H.A., Newell, A. (1971). Human problem solving: The state of the theory in 1970. *American Psychologist*, 26(2), 145.
- Stenning, K., van Lambalgen, M. (2001). Semantics as a foundation for psychology: A case study of Wason's selection task. *Journal of Logic, Language and Information*, 10(3), 273–317.
- Thibodeau, P.H., Boroditsky, L. (2011). Metaphors we think with: The role of metaphor in reasoning. *PloS one*, 6(2).
- Thibodeau, P.H., Boroditsky, L. (2013). Natural language metaphors covertly influence reasoning. *PloS one*, 8(1).
- Thibodeau, P.H., Boroditsky, L. (2015). Measuring effects of metaphor in a dynamic opinion landscape. *PloS one*, 10(7).
- Thibodeau, P.H., Hendricks, R.K., Boroditsky, L. (2017). How linguistic metaphor scaffolds reasoning. *Trends in Cognitive Sciences*, 21(11), 852–863.
- Thompson, V.A. (1994). Interpretational factors in conditional reasoning. *Memory & Cognition*, 22(6), 742–758.
- Thorndike, E.L. (1898). Animal intelligence: An experimental study of the associative processes in animals. *Psychological Monographs: General and Applied*, 2(4), 1–109.
- Vicente, S., Orrantia, J., Verschaffel, L. (2007). Influence of situational and conceptual rewording on word problem solving. *The British Journal of Educational Psychology*, 77, 829–848.

- Vilette, B., Fischer, J.-P., Sander, E., Sensevy, G., Quilio, S., Richard, J.-F. (2017). Peut-on améliorer l'enseignement et l'apprentissage de l'arithmétique au CP ? Le dispositif ACE. *Revue Française de Pédagogie*, 201, 105–120.
- Wason, P.C. (1960). On the failure to eliminate hypotheses in a conceptual task. *Quarterly journal of experimental psychology*, 12(3), 129–140.
- Wason, P.C. (1968). Reasoning about a rule. *Quarterly journal of experimental psychology*, 20(3), 273–281.
- Watson, J.B. (1930). *Behaviorism*, revised edition. University of Chicago Press, Chicago.
- Wertheimer, M. (1959). *Productive Thinking*. University of Chicago Press, Chicago.
- Yee, E., Thompson-Schill, S.L. (2016). Putting concepts into context. *Psychonomic Bulletin and Review*, 23, 1015–1027.
- Zwaan, R.A. (2016). Situation models, mental simulations, and abstract concepts in discourse comprehension. *Psychonomic Bulletin & Review*, 23, 1028–1034.