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Alessandro Colotti [b], Dominique Monnet [a], Alexandre Goldsztejn [b], Franck Plestan [a]

[a] *École Centrale de Nantes – LS2N UMR CNRS 6004, 1 rue de la Noë, 44300 Nantes, France*

[b] *Centre National de la Recherche Scientifique – LS2N UMR CNRS 6004, 1 rue de la Noë, 44300 Nantes, France*

Abstract

A simple proof is proposed that extends Seeber and Horn's recent sufficient conditions (Automatica 2017) for the finite time convergence of the super twisting algorithm to a larger class of perturbations. The new convergence conditions are compared to state of the art conditions, and turn out to accept sensibly smaller gains.

Key words: Finite time stability, super twisting control, nonsmooth Lyapunov function.

1 Introduction

Lyapunov functions is one key approach for proving the finite time stability of sliding mode controllers [6,5,9]. They often offer some insights: e.g., the twisting controller can be assimilated to a nonsmooth harmonic oscillator with dissipation that enjoys an energy related Lyapunov function (see, e.g., Example 3 of [2, p.63]). The Lyapunov approach is easier when no or specifically structured uncertainties are involved. Several works have used a Lyapunov analysis on systems with no or special uncertainty and extended it to reach more general uncertainties, e.g., [6,8]. The convergence conditions [9] for the super twisting algorithm (STA) with perturbation given in (1) is used to obtain new convergence conditions for the more generally perturbed STA (2).

Seeber and Horn investigated in [9] the finite time stability of the STA

$$\dot{x}_1(t) = -k_1\,\phi(x_1(t)) + x_2(t) \tag{1a}$$
$$\dot{x}_2(t) = -k_2\,\operatorname{sign} x_1(t) + \delta(t), \tag{1b}$$

where $\phi(x) = |x|^{\frac{1}{2}}\,\operatorname{sign} x$, subject to the absolutely continuous bounded perturbation $\delta(t) \in [-L, L]$. Here and in the rest of the paper, trajectories are defined in the sense of Filippov [3]. Finite time stability of the STA (1) means converging to $x_1(t) = 0$ and $x_2(t) = 0$ in finite time. They provided the following sufficient conditions for finite time stability:

Theorem 1 ([9]) *The system* (1) *is finite time stable if* $k_2 > L$ *and* $k_1^2 > k_2 + L$.

We build on this result and derive new finite time stability sufficient conditions for a larger class of perturbations (modeled with functions $g(t)$, $h(t)$ and $d(t)$), that commonly appear when modeling physical systems controlled by STA:

$$\dot{x}_1(t) = h(t) + g(t)\big(-k_1\,\phi(x_1(t)) + x_2(t)\big) \tag{2a}$$
$$\dot{x}_2(t) = -k_2\,\operatorname{sign} x_1(t) + d(t). \tag{2b}$$

We assume that the perturbations $g(t)$, $h(t)$ and $d(t)$ satisfy $0 < \underline{g} \le g(t) \le \overline{g}$, $|h(t)| \le \hbar$ and $|d(t)| \le d_{\mathrm{M}}$ and are continuously differentiable. As a consequence, $\gamma(t) = h(t)/g(t)$ is also continuously differentiable, and we assume $|\dot{\gamma}(t)| \le M$. If $|\dot{g}(t)| \le G$ and $|\dot{h}(t)| \le H$, then $M = H/\underline{g} + G\hbar/\underline{g}^2$ is a valid bound for $|\dot{\gamma}(t)|$. The integral correction $x_2(t)$ does not converge to zero as (1) does, therefore the finite time stability of the STA (2) is understood here as only $x_1(t)$ converges to zero in finite time, we say (2) is x_1-finite time stable. Our main result is the following theorem:

Theorem 2 *The system* (2) *is* x_1*-finite time stable if* $k_2 > M + d_{\mathrm{M}}$ *and* $\underline{g}\,k_1^2 > k_2 + M + d_{\mathrm{M}}$.

⋆ Corresponding author A. Colotti.

Email addresses: alessandro.colotti@ls2n.fr (Alessandro Colotti), dominique.monnet@ec-nantes.fr (Dominique Monnet), alexandre.goldsztejn@ls2n.fr (Alexandre Goldsztejn), franck.plestan@ec-nantes.fr (Franck Plestan).

A slightly more restrictive condition is compactly stated as $\frac{1}{2}\underline{g}\,k_1^2 > k_2 > M + d_M$.

The perturbations $g(t)$, $h(t)$ and $d(t)$ wider the scope of [9]. Both $g(t)$, $h(t)$ allow tackling more practical situations, where the system is affine with respect to the input and presents uncertain functions $g(t)$ and $h(t)$. The perturbation $d(t)$ has no real practical usage but is kept here for the sake of generality. The differentiability with bounded derivative is typically assumed for the investigation of the finite time convergence of the STA, e.g., in [10,6]. In [5] Theorem 3 and in [9] Section 3, $g(t) = 1$ but $h(t)$ is not assumed to be bounded or differentiable, and finite time convergence is proven only under some technical additional assumption entailing that $h(t)$ vanishes at the origin [1]. Theorem 2 could be extended to handle the perturbation $\eta(t)\sqrt{|x_1|}$ used in [9] but details are not provided here for the sake of simplicity and clarity [2]. Dealing with unbounded uncertainties that are allowed to grow linearly with x_1, typically satisfying $|h(t)| \leq \alpha_1 + \alpha_2|x_1|$, increases the practical scope of the investigation. Up to the knowledge of the authors, it requires modifications of the STA, e.g., generalizing the command law [1]. Other results on output feedback sliding mode controllers also achieve handling unbounded perturbations, e.g., [7].

In Section 3 two consecutive changes of variables will transform (2) into (1) with a time-varying integral gain $k_2(t)$. The extension of stability properties to time variable gains is not trivial (see, e.g., Section 6 in [4]). The extension of Theorem 1 to time variable integral gain $k_2(t)$ is performed in Section 2. Finally, the conditions of Theorem 2 is compared to the classical conditions [10,6] in Section 4.

2 Extension to a time-varying integral gain

We consider here that the integral gain k_2 in system (1) is a time-varying gain $k_2(t)$, and we generalize Seeber and Horn's sufficient conditions to accept a time-varying integral gain $k_2(t)$. Its proof is an extension of Seeber and Horn's original proof: their argument is based on the (time-invariant) Lyapunov function,

$$V(x) = \begin{cases} 2\sqrt{x_2^2 + 3\alpha^2 k_1^2 x_1} - x_2 & \text{if } x \in \Sigma_1 \\ 2\sqrt{x_2^2 - 3\alpha^2 k_1^2 x_1} + x_2 & \text{if } x \in \Sigma_2 \\ 3\,|x_2| & \text{otherwise,} \end{cases} \tag{3}$$

where $\Sigma_1 = x_1 \geq 0 \cap x_2 \leq \alpha k_1\sqrt{x_1}$ and $\Sigma_2 = x_1 \leq 0 \cap x_2 \geq \alpha k_1\sqrt{-x_1}$. The argument readily extends to a variable integral gain since k_2 does not influence the Lyapunov function, preserving the negativeness of the Lie derivative where it exists. However, their proof cannot be extended to time-varying gain $k_1(t)$ because (3) depends on k_1.

We deal with the states where the locally Lipschitz Lyapunov function is not differentiable by using the non-smooth version of Lyapunov's stability theorem (see, e.g., [2]). Let $\tilde{\mathcal{L}}_f V(x)$ denote the set-valued Lie derivative of V w.r.t. f evaluated at x, where f represents the system's vector field. Intuitively, at the points where $\dot{V}(x)$ does not exist $\tilde{\mathcal{L}}_f V(x)$ is the convex hull of the limit points of neighbor Lie derivatives. We recall that, in analogy with Lyapunov's stability theorem, one needs to investigate the negativeness of $\max \tilde{\mathcal{L}}_f V(x)$ to assess the system's stability (see, e.g., [2] for details). Before stating Theorem 1, we introduce Lemma 1 which, when f is continuous, allows to assess the negativeness of $\tilde{\mathcal{L}}_f V(x)$ by considering only the values of $\dot{V}(x)$ where $V(x)$ is differentiable (the proof of Lemma 1 is given in Appendix A).

Lemma 1 *Consider an open subset $\mathcal{N}$ of the state space. We assume that $f(t,x)$ is continuous w.r.t. x for all $x \in \mathcal{N}$ and all t, and that $V(x)$ be locally Lipschitz. Let $r(x)$ be a real valued function continuous on $\mathcal{N}$. If, for all t, $\dot{V}(t,x) = \nabla V(x)^T f(t,x) \leq r(x)$ on every point where V is differentiable on $\mathcal{N}$, then, for all t, $\max \tilde{\mathcal{L}}_f V(t,x) \leq r(x)$ on $\mathcal{N}$.*

Remark 1 *Lemma 1 allows for any function $r(x)$ to be used, in practice $r(x) = -\epsilon$ or $r(x) = -V(x)^\alpha$, $0 < \alpha < 1$, can be used to assess finite time stability.*

The following theorem extends Seeber and Horn's theorem to a time-varying integral gain. The assumption that $k_2(t)$ is strictly greater than $|\delta(t)|$ can be stated in two natural ways: first by using a lower bound $\underline{k}_2 \leq k_2(t)$ and enforcing $\underline{k}_2 \geq L$ as in Theorem 1. Second, by enforcing the inequality $k_2(t) \geq |\delta(t)| + \epsilon$ at each time, with an arbitrary $\epsilon > 0$. The first condition is sensibly stronger than the second, and leads to a more restrictive finite time stability condition than Theorem 2 (with an additional factor $\overline{g}/\underline{g}$). This is why the second statement is used in the following time-varying integral gain version of Theorem 1:

Theorem 3 *The system (1) with time-varying integral gain $k_2(t)$ is finite time stable if there exists $\epsilon > 0$ such that $k_2(t) \geq |\delta(t)| + \epsilon$ and $k_1^2 \geq k_2(t) + |\delta(t)| + \epsilon$ holds for all times.*

Proof. The same Lyapunov function (3) is used here, except for the choice of the parameter $\alpha < 1$ that needs

[1] The condition $|h(t)| \leq \eta(t)\sqrt{|x_1|}$ is used in [9] and the condition $|h(t)| \leq \eta(t)\sqrt{|x_1| + x_1^2}$ in [5]. Both allow unbounded $|h(t)|$ but entail $h(t)$ converges to zero when $x_1(t)$ does.

[2] The constraint in Theorem 2 would then be $\underline{g}\,k_1^2 > k_2 + M + d_M + \eta_M$, where η_M would be a bound on $\eta(t)$.

to account for ϵ in the statement: here one can choose α such that

$$(\alpha k_1)^2 \geq k_2(t) + |\delta(t)| + \tfrac{1}{2}\epsilon \tag{4}$$

Wherever the Lyapunov function is differentiable, the upper bound on the Lie derivative computed in Eq. (14) in [9] still holds when k_2 varies in time, meaning that

$$\dot{V} \leq 3 \max\left\{k_2(t) - \delta(t) - (\alpha k_1)^2,\ \tfrac{1}{2}\alpha(\alpha-1)k_1^2\right\}. \tag{5}$$

As in the original proof, Eq. (4) entails $k_2(t) - \delta(t) - (\alpha k_1)^2 \leq -\frac{1}{2}\epsilon$ and $0 < \alpha < 1$ entails $\alpha(\alpha-1) < 0$, showing that the maximum of them is strictly negative.

Now, we need to investigate the states where V is not differentiable. It is well known that no sliding motion can occur along the set $S_1 = \{x | \, x_1 = 0,\ x_2 \neq 0\}$, whether k_2 is fixed or time-varying. The other set where V is not differentiable is $S_2 = \{x | \, x_2 = \alpha k_1 \sqrt{|x_1|},\ x_1 \neq 0\}$. Since system (1) is continuous on $\mathcal{N} := \mathbb{R}^2 \backslash S_1$, we can apply Lemma 1 with $r(x) = 3\max\left\{-\frac{1}{2}\epsilon, \frac{1}{2}\alpha(\alpha-1)k_1^2\right\}$ to show that the same strictly negative upper bound that holds when V is differentiable also holds on the whole set $\mathcal{N}$, which includes the set S_2. □

3 Extension to practical uncertainties

3.1 Centered system

The STA (2) aims $x_1(t) = 0$ and $\dot{x}_1(t) = 0$, the integral correction $x_2(t)$ being foreseen not to converge. Indeed, the value of $x_2(t)$ enforced at steady state can be computed with respect to uncertainties: if $x_1(t) = 0$ and $\dot{x}_1(t) = 0$ then $x_2(t) = -h(t)/g(t)$. Classically, by defining the system $y_1(t) = x_1(t)$ and $y_2(t) = x_2(t) + h(t)/g(t)$, we obtain a system that is expected to converge to zero. It satisfies the following ODE:

$$\dot{y}_1(t) = g(t)\big(-k_1\,\phi(y_1(t)) + y_2(t)\big) \tag{6a}$$
$$\dot{y}_2(t) = -k_2\ \mathrm{sign}\, y_1(t) + d(t) + \dot{\gamma}(t), \tag{6b}$$

with

$$\dot{\gamma}(t) = \frac{d}{dt}\frac{h(t)}{g(t)} = \frac{\dot{h}(t)}{g(t)} - \frac{\dot{g}(t)\,h(t)}{g(t)^2}. \tag{7}$$

The change of variable $x \mapsto y$ is a diffeomorphism for all times, therefore Theorem 2 of [3, p.99] proves that Filippov solutions $x(t)$ map to Filippov solutions $y(t)$ and vice-versa. Since $y_1(t) = x_1(t)$ the y_1-finite time stability of the (6) is equivalent to the x_1-finite time stability of (2).

3.2 Time shifted system

We introduce a rescaling of time $z(t) = y(\tau(t))$, that is going to move the impact of the perturbation $g(t)$ from (6a) to (6b), so as to match the perturbation structure of (1). We define $\tau(t)$ by the following ODE:

$$\dot{\tau}(t) = \frac{1}{g(\tau(t))}, \tag{8}$$

with $\tau(0) = 0$. Since $g(t) \in [\underline{g}, \overline{g}]$ is bounded away from zero by $\underline{g} > 0$ and is differentiable with bounded derivative $\dot{g}(t) \in [-G, G]$, the function $1/g(\tau)$ is continuously differentiable with bounded derivative. As a consequence the ODE (8) has a unique solution, which is globally defined in $\mathbb{R}$. Furthermore, from (8) and the bounds on $g(t)$ we obtain $1/\overline{g} \leq \dot{\tau}(t) \leq 1/\underline{g}$, which proves that $\tau(t)$ is strictly increasing and satisfies,

$$\frac{t}{\overline{g}} \leq \tau(t) \leq \frac{t}{\underline{g}}. \tag{9}$$

This makes $\tau(t)$ a valid rescale of time. The time-rescaled system $z(t) = y(\tau(t))$ satisfies the following relation:

$$\dot{z}(t) = \dot{\tau}(t)\,\dot{y}(\tau(t)) = \frac{1}{g(\tau(t))}\,\dot{y}(\tau(t)), \tag{10}$$

obtained using the chain rule and (8). Finally using (6) we obtain the ODE governing $z(t)$:

$$\dot{z}_1(t) = -k_1\,\phi(z_1(t)) + z_2(t) \tag{11a}$$
$$\dot{z}_2(t) = -\frac{k_2}{\tilde{g}(t)}\ \mathrm{sign}\, z_1(t) + \frac{\tilde{d}(t) + \tilde{\gamma}(t)}{\tilde{g}(t)}, \tag{11b}$$

where $\tilde{g}(t) = g(\tau(t))$, $\tilde{\gamma}(t) = \dot{\gamma}(\tau(t))$ and $\tilde{d}(t) = d(\tau(t))$. The system (11) is in the scope of Theorem 3 with

$$k_2(t) = \frac{k_2}{\tilde{g}(t)} \quad \text{and} \quad \delta(t) = \frac{\tilde{d}(t) + \tilde{\gamma}(t)}{\tilde{g}(t)}. \tag{12}$$

Its z_1-finite time stability is equivalent to y_1-finite time stability of (6) because of (9). The bounds $g(t) \in [\underline{g}, \overline{g}]$, $\dot{\gamma}(t) \in [-M, M]$ and $d(t) \in [-d_\mathrm{M}, d_\mathrm{M}]$ hold for their time-shifted counterparts $\tilde{g}(t)$, $\tilde{\gamma}(t)$ and $\tilde{d}(t)$.

3.3 Proof of Theorem 2

Suppose the conditions $k_2 > M + d_\mathrm{M}$ and $\underline{g}\,k_1^2 > k_2 + M + d_\mathrm{M}$ of Theorem 2 hold. Since $\tilde{\gamma}(t) \in [-M, M]$, $\tilde{g}(t) > \underline{g}$ and $\tilde{d}(t) \in [-d_\mathrm{M}, d_\mathrm{M}]$, there exists $\epsilon > 0$ such that both $k_2 \geq |\tilde{\gamma}(t) + \tilde{d}(t)| + \epsilon$ and $\tilde{g}(t)\,k_1^2 \geq k_2 + |\tilde{\gamma}(t) + \tilde{d}(t)| + \epsilon$ hold for all times. Dividing both inequalities by $\tilde{g}(t) < \overline{g}$, we prove that

$$k_2(t) \geq |\delta(t)| + \frac{\epsilon}{\overline{g}} \quad \text{and} \quad k_1^2 \geq k_2(t) + |\delta(t)| + \frac{\epsilon}{\overline{g}}, \tag{13}$$

where $k_2(t)$ and $\delta(t)$ are defined in (12). This allows applying Theorem 3 to the STA (11) with (12), hence proving its z_1-finite time stability.

4 Comparison to state-of-the-art conditions

Moreno [5] handles the case $g(t) = 1$ and $h(t)$ bounded with no differentiability assumption, but does not prove x_1-finite time stability in this case (he does handle successfully more structured perturbations proportional to $\sqrt{|x_1| + x_1^2}$). Castillo, Fridman and Moreno [1] handle more general perturbations than here, with a generalized STA (GSTA). However, the finite time stability conditions for the GSTA do not apply to the STA [3].

In order to handle the same differentiable perturbations $g(t)$ and $h(t)$ as in (2), Shtessel, Edwards, Fridman and Levant [10] have modified the STA by adding a switching law to keep u bounded in $[-U_m, U_m]$. They proved some sufficient conditions on k_1, k_2 and U_m for the x_1-finite time stability, see [10, p.156]. After some algebraic manipulations [4], one can prove that these conditions imply

$$k_2 > M \quad \text{and} \quad \underline{g}\, k_1^2 > 2\left(\frac{\overline{g}}{\underline{g}}\right)^2 (k_2 + M), \tag{14}$$

where $M = H/\underline{g} + G\,\hbar/\underline{g}^2$ uses the bounds defined in Section 1. The first bound on k_2 is the same as in Theorem 2. However, the second bound on k_1^2 is seen to be similar but more restrictive that the one in Theorem 2 because of the factor $2(\overline{g}/\underline{g})^2$. In particular, if the interval $[\underline{g}, \overline{g}]$ is large and/or $\underline{g}$ is close to zero then $(\overline{g}/\underline{g})^2$ is sensibly greater than 1 and the conditions of [10] become sensibly more restrictive than Theorem 2. In addition to allowing for sensibly lower gains, the new condition of Theorem 2 does not require the switching law to keep $u \in [-U_m, U_m]$, which simplifies the usage of the STA.

Polyakov and Poznyak in [6] proved that the conditions $32H < k_1^2 < 8(k_2 - H)$ entails the x_1-finite time convergence of the STA (2) in the restricted case $g(t) = 1$. In this restricted case, $M = H$ holds and our conditions become $2H < k_2 + H < k_1^2$. None of these two conditions imply the other: gain values are accepted by Polyakov and Poznyak's condition and rejected by Theorem 2 if and only if $32H < k_1^2 \leq k_2 + H$ (the black boundary area in Figure 1). Theorem 2 accepts pairs of small gains greater but arbitrarily close to the limit pair of gains $k_1 = \sqrt{2H}$ and $k_2 = H$ (see the lower-left point in Figure 1), while pairs of gains accepted by Polyakov and Poznyak must satisfy $k_1 \geq \sqrt{32H}$ and $k_2 \geq 5H$ (see the upper-right point in Figure 1), so the new conditions of Theorem 2 accept sensibly lower gains, the greater H the more sensitive the improvement. Both conditions are therefore seen to be complementary. In particular, they prove together that the STA (2) is x_1-finite time stable for all positive gains in the case where $g(t) = 1$ and $\dot{h}(t) = 0$, which was not proved previously up to the authors knowledge. The new condition of Theorem 2 are valuable because they allow for sensibly smaller gains.

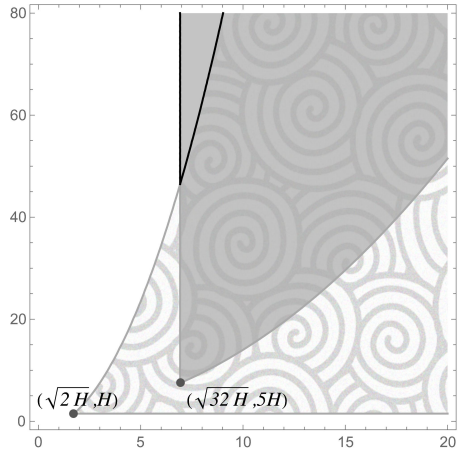

Figure 1. Gains (k_1, k_2) satisfying the conditions of Theorem 2 in the textured area, and satisfying the conditions of Polyakov and Poznyak [6] in the gray area. The graphic displays the numerical value $H = 1.5$.

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A Proof of Lemma 1

Let t be an arbitrary time instant and let Ω_V be the set of non-differentiability points of V inside $\mathcal{N}$. On $\mathcal{N} \backslash \Omega_V$, $\tilde{\mathcal{L}}_f V(t,x) = \{\dot{V}(t,x)\}$, so the lemma is trivially true. For $x \in \Omega_V$, by the continuity of f we have that $\tilde{\mathcal{L}}_f V(t,x)$ consists of the convex combinations of limit points of $\dot{V}(t,x_i)$ at neighboring points x_i where V is differentiable (this is a direct consequence of the definition of $\tilde{\mathcal{L}}_f V$ and a few of its basic properties which can be found, e.g., in [2]). Since $\tilde{\mathcal{L}}_f V(t,x)$ is a compact interval of real numbers, its max is a vertex of the one dimensional convex hull, meaning that there exists a converging sequence $x_i \to x$ such that $\dot{V}(t,x_i) \to \max \tilde{\mathcal{L}}_f V(t,x)$. But $\dot{V}(t,x_i) \leq r(x_i)$ for every point in the sequence, meaning that, by continuity of r, we have $\max \tilde{\mathcal{L}}_f V(t,x) \leq r(x)$, which holds for all times since t is arbitrary. □

[3] The STA is obtained from the GSTA in [1] by setting β to zero, but this is incompatible with the conditions of Theorem 1 [1, p. 5 Eq. (15)] because they involve a factor $1/\beta$. For strictly positive β, their conditions also contain a factor $1/(k_m - k_M)$, which is not applicable in the case where $g(t)$ is constant, i.e., $k_m = k_M$.

[4] Among other basic manipulations, setting q and U_m to the best cases $q \in \{0, 1\}$ and $U_m = C$ in order to weaken the conditions, and using the fact that $0 < \underline{g}k_2 - C \leq \underline{g}k_2 + C$ with $C = H + G\,\hbar/\underline{g} = \underline{g}\,M$ being the smallest upper bound to $|\dot{h}(t) + \dot{g}(t)\,u(t)|$ subject to $|u(t)| \leq U_m$ that can be obtained using bounds on the perturbations given in Section 1.

References

[1] I. Castillo, L. Fridman, and J. A. Moreno. Super-twisting algorithm in presence of time and state dependent perturbations. *Int. Journ. of Control*, 91(11):2535–2548, 2018.

[2] J. Cortes. Discontinuous dynamical systems. *IEEE Control Systems Magazine*, 28(3):36–73, 2008.

[3] Aleksej F. Filippov. *Differential Equations with Discontinuous Righthand Sides: Control Systems.* Kluwer, 2010.

[4] J. A. Moreno. *Lyapunov Approach for Analysis and Design of Second Order Sliding Mode Algorithms.* Springer, 2012.

[5] J. A. Moreno and M. Osorio. Strict lyapunov functions for the super-twisting algorithm. *IEEE Trans. on Automatic Control*, 57(4):1035–1040, 2012.

[6] A. Polyakov and A. Poznyak. Reaching time estimation for "super-twisting" second order sliding mode controller via lyapunov function designing. *IEEE Trans. on Automatic Control*, 54(8):1951–1955, 2009.

[7] Victor Hugo Pereira Rodrigues and Tiago Roux Oliveira. Global adaptive hosm differentiators via monitoring functions and hybrid state-norm observers for output feedback. *International Journal of Control*, 91(9):2060–2072, 2018.

[8] R. Seeber, M. Horn, and L. Fridman. A novel method to estimate the reaching time of the super-twisting algorithm. *IEEE Trans. on Automatic Control*, 63(12):4301–4308, 2018.

[9] Richard Seeber and Martin Horn. Stability proof for a well-established super-twisting parameter setting. *Automatica*, 84:241–243, 2017.

[10] Y. B. Shtessel, C. Edwards, L. Fridman, and A. Levant. *Sliding mode control and observation.* Springer, 2014.