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► **To cite this version:**

Fabio D'andreagiovanni, Hicham Lakhlef, Antonella Nardin. Green design of Single Frequency Networks by Multiband Robustness and a hybrid metaheuristic. 16th Artificial Life and Evolutionary Computation (WIVACE 2022), Aug 2022, Gaeta, Italy. pp.1-7. hal-03866775

**HAL Id: hal-03866775**

**<https://hal.science/hal-03866775>**

Submitted on 22 Nov 2022

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# Green design of Single Frequency Networks by Multiband Robustness and a hybrid metaheuristic

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**Abstract.** The passage to a second generation of broadcasting Single Frequency Networks has generated the need for reconfiguring and re-designing existing networks. In this work, we present a robust optimization model for the green design of such networks based on the digital television DVB-T2 standard. Our robust model pursues protection against uncertainty of signal propagation in a complex real-world environment. As reference model, we adopt Multiband Robustness and we propose to solve the resulting model by a hybrid metaheuristic that combines mathematically strong formulations of the optimization model, with an exact large neighborhood search. We report computational tests based on realistic instances, showing that the multiband model grants highly protected solutions without reducing service coverage and without leading to a high price of robustness.

**Keywords:** Green Telecommunications Networks · Robust Optimization · Hybrid Metaheuristics

## 1 Introduction

Smart cities are nowadays enjoying a wide range of brand new telecommunication services enabled by last generation 5G networks [20]. However, within this ultramodern context, older and more traditional broadcasting services like television and radio are still considered critical by many governments, since they support a cheap, easy and democratic access to information to all the population. A very important advancement in offering broadcasting services of higher quality has been represented by the switch from analogue to digital transmission, which has allowed to implement the paradigm of Single Frequency Networks (SFNs). An SFN is a broadcasting network in which all transmitting stations emit the same data on the same frequency. This allows to improve spectrum usage, a

very important goal considering that radio resources are scarce and that the expansion of telecommunications services has rapidly lead to the congestion of frequencies (see e.g., [14]). Major examples of SFNs are represented by Digital Video Broadcasting - Terrestrial (DVB-T) networks, which broadcast television content in a digital format and are based on the DVB-T standard adopted in more than 140 countries worldwide [14]. In the last years, SFNs have gained new attention because of the release of a second generation of standards like DVB-T2. The new standards offer higher performance, by supporting improved modulation scheme. Moreover, thanks to an improved spectrum efficiency, they enable a higher number of broadcasting operators to operate simultaneously. The entrance of new operators and the need for reconfiguring older networks have renewed the interest in software for designing SFN networks, in particular those exploiting mathematical optimization models and algorithms.

In this work, we address the question of developing a robust optimization model for designing DVB-T2 networks while taking into account the uncertainty that naturally affects wireless signal propagation. Specifically, our original contributions are:

1. We present a binary linear programming robust counterpart based on signal-to-interference formulas, which is typically adopted to represent DVB-T design. The counterpart is defined according to the principles of Multiband Robust Optimization [6], a refined version of the classical  $\Gamma$ -Robust Optimization model by Bertsimas and Sim [4], which has been widely used to deal with data uncertainty in optimization problems (see e.g., [2, 3]).
2. Since the resulting robust optimization model may result very challenging even for a state-of-the art optimization software like IBM ILOG CPLEX, we define a hybrid metaheuristic for its solution, proposing to combine a probabilistic variable fixing procedure with an exact large variable neighborhood search. The probabilistic fixing exploits the precious information that can be derived from a tight linear relaxation of the MILP model adopted to represent the DVB-T design problem, whereas the exact search consists of exploring a solution neighborhood formulating the search as an optimization problem that is solved at the optimum.
3. We highlight the performance of our new modelling and algorithmic approach by means of tests conducted on realistic DVB-T instances, showing the superior performance of the multiband approach with respect to both a benchmark robust and deterministic model.

We remark that, while the deterministic (i.e., not considering data uncertainty) optimal design of wireless networks based on signal-to-interference ratios has received wide attention, the use of optimization under uncertainty techniques, such as Robust Optimization and Stochastic Programming, has received less attention and has especially considered the effects of traffic uncertainty. This is also true for the case of DVB-T, in which optimization approaches have neglected data uncertainty (e.g., [10, 11, 16, 18]). To the best of our knowledge, this is the first work that discusses the adaption of a robust optimization approach

to DVB-T and presents a hybrid metaheuristic for the solution of the resulting complex problem.

## 2 Optimal SFN design

To derive an optimization model, we refer to an SFN network based on the DVB-T standard: in such network, all broadcasting stations synchronously transmit identical data on the same frequency channel according to the OFDM modulation scheme [14]. Broadcasting services are spread over a target territory to reach the receiving devices of a population. Following the recommendations of telecommunications regulatory bodies (e.g., [1, 15]), we discretize the target territory into a raster of pixels: each pixel represents a fragment of territory so small that the signal strength measured at its center (testpoint) can be considered representative for the strength of signals in any other point of the pixel. If we denote by  $S$  the set of broadcasting stations and by  $T$  the set of testpoints, the network design problem can be essentially described as that of i) setting the power emission of every station and b) selecting the serving station of each pixel/testpoint, with the objective of minimizing the total emitted power (green perspective) under the condition of granting service to a given fraction of the people located in the target territory.

The previous problem belongs to the family of *Wireless Network Design Problems* (e.g., [11, 18]) and, in particular, it constitutes a variant of the Scheduling and Power Assignment Problem, known to be NP-Hard [7, 18]. We can model the two decisions taken in the considered problem by means of two sets of binary variables, namely:

- to represent the power emission of a station  $s \in S$ , we introduce a set of equally spaced discrete power values  $P = \{p_1, p_2, \dots, p_n\}$  on which each station may emit. We also introduce the set  $L = \{1, 2, \dots, n\}$  to represent the index of the discrete power values, which we call power levels. Given the set  $P$ , we introduce a set of binary variables  $y_{s\ell} \in \{0, 1\}$  to represent whether a station  $s \in S$  emits on power value  $p_\ell \in P$ , such that  $y_{s\ell} = 1$  when  $s$  emits on  $p_\ell$  and  $y_{s\ell} = 0$  otherwise (in what follows, we also alternatively write that a station  $s \in S$  emits on power level  $\ell \in L$ ).
- to represent whether a testpoint  $t \in T$  is served by a station  $s \in S$ , we introduce a binary variable  $x_{ts} \in \{0, 1\}$  such that  $x_{ts} = 1$  when  $t$  is served by  $s$  and  $x_{ts} = 0$  otherwise.

Canonically, in order to assess the coverage of a testpoint, it is evaluated the ratio of the total useful signal (i.e., the sum of signals from stations that contribute to increase the quality of service) to the total interfering signal (i.e., the sum of signals from stations that contribute to decrease the quality of service), leading to the definition of the so-called Signal-to-Interference Ratio (SIR) [19]. In the case of DVB-T-based networks, a testpoint picks up signal from all stations and the distinction between useful and interfering signals depends upon whether the signals are received within a detection time interval of

the receiver/testpoint [14, 17, 18]. For practical modelling purposes, the detection window of a testpoint  $t \in T$  is considered starting when the signal of its serving station  $s \in S$  is received: the set of useful stations  $U(t, s)$  and the set of interfering stations  $I(t, s)$  then clearly depends upon the station  $s$  chosen as server of  $t$ . We remark that the sets  $U(t, s)$  and  $I(t, s)$  constitute a partition of the set of stations  $S$  and that, in general, changing server of a testpoint implies that the sets  $U(t, s)$  and  $I(t, s)$  change as well. The power  $P_{ts}$  that a testpoint  $t \in T$  receives from a station  $s \in S$  is proportional to the power  $P_s$  emitted by  $s$  by a coefficient  $a_{ts} \in [0, 1]$ , (thus,  $P_{ts} = a_{ts} \cdot P_s$ ), which is commonly called fading coefficient and represents the reduction to which a signal is subject while propagating from its emitter  $s$  to its receiver  $t$  [19].

We now proceed to formally define the SIR formulas, which are at the basis of the optimization model for designing the considered SFN network based on DVB-T. For a testpoint  $t$ , served by station  $s$ , the SIR is written as follows:

$$SIR_{ts} = \frac{\sum_{\sigma \in U(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{\sigma \ell} \right)}{N + \sum_{\tau \in I(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{\sigma \ell} \right)} \geq \delta$$

in which i)  $N > 0$  is the noise of the system, ii)  $\delta > 0$  is the minimum SIR value requested for considering the testpoint served, iii) the power emitted by a station  $\sigma \in S$  is expressed by the combination of the power values  $p_{\ell}$  by the corresponding binary power activation variable  $y_{\sigma \ell}$  (i.e.,  $\sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{\sigma \ell}$ ). The SIR lies at the core of every wireless network design problem and for the specific case that we consider, the model is:

$$\min \sum_{s \in S} \left( \sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{s \ell} \right) \quad (1)$$

$$\sum_{\sigma \in U(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{\sigma \ell} \right) - \delta \sum_{\tau \in I(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell=1, \dots, n} p_{\ell} \cdot y_{\sigma \ell} \right) + M(1 - x_{ts}) \geq \delta \cdot N \quad t \in T, s \in S \quad (2)$$

$$\sum_{\ell=1, \dots, n} y_{s \ell} = 1 \quad s \in S \quad (3)$$

$$\sum_{s \in S} x_{ts} \leq 1 \quad t \in T \quad (4)$$

$$\sum_{t \in T} \sum_{s \in S} \pi_t \cdot x_{ts} \geq \alpha \cdot \sum_{t \in T} \pi_t \quad (5)$$

$$y_{s \ell} \in \{0, 1\} \quad s \in S, \ell \in L. \quad (6)$$

$$x_{ts} \in \{0, 1\} \quad t \in T, s \in S. \quad (7)$$

in which, a) the objective function (1) pursues the minimization of the total power emission; b) the quality-of-service conditions are expressed by the SIR

formulas (easily reorganized by simple algebra operations) and including the big-M term  $M(1 - x_{ts})$  to activate or deactivate the constraint depending on whether testpoint  $t$  is served by station  $s$  (see [10, 11] for details); c) constraints (3) imposes that each station emits by exactly one power value whereas (4) imposes that each testpoint is served by at most one station. The constraints (5) impose that at least a fraction  $\alpha \in [0, 1]$  of the total population of the region must be covered with service (in the left-hand-side, each service decision variable is multiplied by a coefficient  $\pi_r$  that equals the population of the pixel associated with receiver  $r$ , while in the right-hand-side the total sum of the population over all the pixels is multiplied by the value  $\alpha$ ). Finally, (6) and (7) are the decision variables previously defined.

### 3 Protecting against propagation uncertainty

The fading coefficients  $a_{ts}$  that are part of the SIR constraints (2) are naturally subject to uncertainty because of the wide range of factors that influence signal propagation in a real environment (e.g., landscape, obstacles, weather, etc.) and that are hard to precisely assess [19]. These coefficients are commonly computed by (empirical) propagation models that, using extensive field propagation measurements, provide a formula for computing the coefficient values on the basis of factors like the distance between the communicating points, the portion of the spectrum adopted for transmissions, and the characteristics of the propagation environment (e.g., with many obstacles like tall buildings or in line of sight). As well-known by telecommunication professionals, the actual propagation values may be sensibly different from the values returned by the propagation models and it is thus very important to protect design solutions from possible fluctuations in these values.

Since the fading coefficients constitute *uncertain data*, i.e. data whose value is not exactly known when the problem is solved, we protect against fluctuations in their value that could cause infeasibility or sub-optimality of produced solutions by Robust Optimization (RO). RO is one of the most successful methodology for protecting against data uncertainty optimization and is based on the following main assumptions [3]:

- the *actual value* of each uncertain coefficient is unknown to the decision maker;
- the decision maker has at disposal a *nominal value* of each uncertain coefficient, representing an estimation of its actual value;
- deviations against which solutions must be protected are specified by an *uncertainty set*;
- the problem that is solved is a *robust counterpart*, a modified version of the original deterministic problem, including only robust feasible solutions, namely solutions that remain feasible for all the deviations values of the uncertainty set applied to the nominal values;
- a robust optimal solution offers the best objective value under the worst data deviations;

- since the robust counterpart excludes a subset of feasible solution of the deterministic problem, the robust feasible set is a subset of the deterministic feasible set; as a consequence, a robust optimal solution grants protection against deviations, but generally presents a worse value than a deterministic optimal solution. This reduction in optimality constitutes the so-called *price of robustness* [4].

As Robust Optimization model, we propose here to adopt *Multiband Robust Optimization* (MB) introduced in [6] to generalize and refine classical  $\Gamma$ -Robustness [4]: MB uses multiple deviation bands for better modeling arbitrary discrete distributions, under the form of histograms, which are commonly considered by professionals to analyze deviations in the input data in real-world optimization problems (as also illustrated in [2]). As basis, we assume that the actual value of a generic uncertain fading coefficient  $a_{ts}$  belongs to the symmetric interval  $[\bar{a}_{ts} - d_{ts}, \bar{a}_{ts} + d_{ts}]$  (here,  $\bar{a}_{ts}$  is the nominal value of the uncertain coefficient, while  $d_{ts}$  is its maximum allowed deviation). Practically,  $\bar{a}_{ts}$  could be the value provided by a propagation model, while  $d_{ts}$  could be set as the maximum deviation that the network planner wants to consider according to its risk aversion. Following the principles of MB, for the uncertain fading coefficient we define the following Multiband Uncertainty Set (MBUS):

1. we partition the overall deviation range  $[-d_{ts}, d_{ts}]$  into  $K$  bands, defined on the basis of  $K$  deviation values:  
 $-d_{ts} = d_{ts}^{K^-} < \dots < d_{ts}^{-1} < d_{ts}^0 = 0 < d_{ts}^1 < \dots < d_{ts}^{K^+} = d_{ts}$ ;
2. through these deviation values,  $K$  deviation bands are defined, namely: a set of positive deviation bands  $k \in \{1, \dots, K^+\}$  and a set of negative deviation bands  $k \in \{K^- + 1, \dots, -1, 0\}$ , such that a band  $k \in \{K^- + 1, \dots, K^+\}$  corresponds to the range  $(d_t^{k-1}, d_t^k]$ , and band  $k = K^-$  corresponds to the single value  $d_t^{K^-}$ . Note that  $K = K^+ \cup K^-$ ;
3. we define a lower and upper bound on the number of values that may experience a deviation of value in each band: for each band  $k \in K$ , two bounds  $l_k, u_k \in \mathbb{Z}_+$ :  $0 \leq l_k \leq u_k \leq |T| \cdot |S|$  are introduced. Furthermore, the number of coefficients that may deviate in the zero-deviation band  $k = 0$  is not limited (i.e.,  $u_0 = |T| \cdot |S|$ ) and we impose that  $\sum_{k \in K} l_k \leq |T| \cdot |S|$ , so as to ensure that there exists at least one feasible assignment of coefficients to deviations bands.

An MB uncertainty set is particularly suitable for modelling histograms. Furthermore, it also considers bands associated with beneficial and non-adversarial deviations: this is done since, in real-world applications, our main objective is to be protected against adversarial data deviations that lead to infeasibility, but at the same time we want to take into account also beneficial deviations which may take place and compensate the adversarial deviations, therefore reducing the price of robustness.

The linear robust counterpart of an uncertain SIR constraint defined for a couple  $(s, t)$  is obtained according to the theoretical results of Multiband Robust Optimization (in particular Theorem 1 of [6] about the mathematical form

of a linear and compact multiband robust counterpart). Specifically, the single deterministic SIR constraint (2) of  $(s, t)$  is replaced by the following set of constraints:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot \left( \sum_{\ell=1,\dots,n} p_{\ell} \cdot y_{s\ell} \right) - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot \left( \sum_{\ell=1,\dots,n} p_{\ell} \cdot y_{s\ell} \right) - \left( \sum_{k \in K} \theta_{ts}^k \cdot w_{ts}^k + \sum_{s \in S} z_{ts} \right) + M(1 - x_{ts}) \geq \delta \cdot N \quad (8)$$

$$w_{ts}^k + z_{ts} \cdot \left( \sum_{\ell=1,\dots,n} p_{\ell} \cdot y_{s\ell} \right) \geq d_{ts}^k \left( \sum_{\ell=1,\dots,n} p_{\ell} \cdot y_{s\ell} \right) \quad k \in K \quad (9)$$

$$w_{ts}^k \geq 0 \quad k \in K \quad (10)$$

$$z_{ts} \geq 0 \quad (11)$$

which includes the additional constraints (9) and variables (10), (11) to linearly reformulate the original (non-linear) robust SIR constraints.

The robust optimization problem that we consider and that we denote by Robu-DVB-MILP is obtained by DVB-MILP substituting each SIR constraints with (8) and the auxiliary dual constraints and variables (9), (10), (11).

## 4 A hybrid solution algorithm

The previous robust problem results challenging to be solved even for a state-of-the-art optimization solver like CPLEX, especially because of the presence of the complicating (robust) SIR constraints. To solve it, we thus propose a hybrid metaheuristic that combines heuristic exploration of the feasible set with the adoption of exact optimization methods (i.e., guaranteeing convergence to an optimal solution) for suitable subproblems of the complete problem. Specifically, we propose a metaheuristic that follows the algorithmic principles presented in [9, 12], to which we refer the reader for more details. It is mainly based on a *probabilistic variable fixing* procedure integrated with an *exact large neighborhood search*.

The probabilistic fixing procedure combines an a-priori and an a-posteriori fixing measures. In our case, the a-priori measure is provided by a linear relaxation of the robust model (model (1)-(7) with SIR constraints (2) replaced by (8)-(11)), denoted by Robu-DVB-MILP, while the a-posteriori measure is given by a (tighter) linear relaxation of the model (1)-(7), denoted by DVB-MILP (where a subset of variables has been fixed in value). At the end of each cycle of variable fixing, the a-priori fixing measure is updated, evaluating how good were the applied fixing. Once a time limit is reached, the fixing cycle stops and an exact large neighborhood search is executed for trying to improve the best solution found.



In the probabilistic fixing procedure, a number of solutions are built iteratively: at every iteration, a partial solution (i.e., a solution where only a subset of variables has its value fixed) is available and we can fix the value of an additional variable. Once the value of all the variables has been fixed, we obtain a complete solution whose quality is evaluated by means of its objective value. The fixing procedure is based on the observation that once the power emission variables have been fixed in value, it is possible to easily check which testpoints are covered with service by some station and compute the value of the objective function. We thus base the fixing procedure on deciding the values assumed by the power variables. At a generic iteration of the construction cycle of a feasible solution, we have at disposal a partial solution to the problem (obtained by having chosen the power emissions of a subset of stations  $S^{\text{FIX}} \subseteq S$  by fixing their variables  $y_{sl}$  while respecting (3)). We probabilistically choose the next station whose power emission is fixed by means of the following formula, defined  $\forall s \in S \setminus S^{\text{FIX}}, l \in L$ :

$$p_{sl} = \frac{\alpha \tau_{sl} + (1 - \alpha) \eta_{sl}}{\sum_{s \in S \setminus S^{\text{FIX}}} \sum_{\lambda \in L} [\alpha \tau_{s\lambda} + (1 - \alpha) \eta_{s\lambda}]}, \quad (12)$$

which expresses the probability of fixing the power emission of station  $s \in S \setminus S^{\text{FIX}}$  to power level  $P_l$  by considering all the couples  $\sigma \in S \setminus S^{\text{FIX}}, \lambda \in L$  of stations whose emission is not yet fixed. In the formula,  $\tau_{sl}$  is the a-priori attractiveness measure obtained from the optimal value of Robu-DVB-MILP including power-indexed variables, while  $\eta_{sl}$  is given by the value of a tight linear relaxation of DVB-MILP including fixing of variables done in previous iterations. The two measures are combined by a coefficient  $\alpha \in [0, 1]$ . After having fixed  $y_{sl} = 1$  for some couple  $(s, l)$ , because of constraint (3) we can set  $y_{s\lambda} = 0$  for all  $\lambda \in L : \lambda \neq l$ .

After having defined the power emissions of all stations (assume this is denoted by a binary power vector  $\bar{y}$ ), all the SIR ratios can be easily computed. On the basis of the value of these ratios, we can also easily check which testpoints are covered with service and thus derive a valorization of the server assignment variables  $\bar{x}$ . The resulting solution  $(\bar{y}, \bar{x})$  which is feasible for DVB-MILP is accepted as robust when it maintains its feasibility also when the fading coefficients are deviating to their worst value.

Once a round of construction of feasible solutions has been operated, the a-priori measures are updated using the following formula:

$$\begin{aligned} \tau_{sl}(h) &= \tau_{sl}(h-1) + \sum_{\text{SOL}=1}^{\gamma} \Delta\tau_{sl}^{\text{SOL}} \\ \Delta\tau_{sl}^{\text{SOL}} &= \tau_{sl}(0) \cdot \left( \frac{OG(v^{\text{AVG}}, u) - OG(v^{\text{SOL}}, u)}{OG(v^{\text{AVG}}, u)} \right) \end{aligned} \quad (13)$$

where  $\tau_{sl}(h)$  is the a-priori measure of fixing station  $s$  at power level  $P_l$  at the  $h$ -th execution of the cycle and  $\Delta\tau_{sl}^{\text{SOL}}$  is the modification to the value of the a-priori measures, computed over a summation that considers the last  $\gamma$

solutions that have been constructed. Moreover,  $u$  is an upper bound on the optimal value of the problem,  $v^{\text{SOL}}$  is the value of the SOL-th feasible solution built in the last construction cycle,  $v^{\text{AVG}}$  is the average of the values of the last  $\gamma$  solutions that have been constructed. The optimality gap  $OG(v,u)$  measures how far is the value  $v$  of a solution from the upper bound  $u$  and is defined as  $OG(v,u) = (u - v)/v$ . The role of formula (13) is to update the a-priori measure rewarding (penalizing) those fixing that have lead to a solution with lower (higher) optimality gap in comparison to the moving average value  $v^{\text{AVG}}$ .

At the end of the construction cycle, with the aim of improving the best robust solution found, an exact neighborhood search is conducted, i.e. we explore a (very large) neighborhood of the best solution, formulating the search as an optimization problem which is optimally solved by a state-of-the-art solver (see e.g., [5,13]). The adoption of exact searches is motivated by the fact that, while it can be difficult and long for a solver to solve the complete problem, it is instead possible to efficiently solve to optimality some subproblems. The large neighborhood that we define is built from a robust solution  $(\bar{y}, \bar{x})$  allowing to change the power emission of all stations by either 1) turning off a station  $s$  (i.e., setting  $y_{s0} = 1$  or 2) allowing a modification of the power emission to the adjacent power level set by  $\bar{y}$  (i.e., if  $y_{sl} = 1$  then it is allowed to set  $y_{sl-1} = 1$  or  $y_{sl+1} = 1$ ). The exact search is then conducted by expressing the previous conditions as linear constraints that are added to Robu-DVB-MILP and the resulting problem is solved by an exact solver.

The pseudocode of the matheuristic for solving Robu-DVB-MILP is presented in Algorithm 1. The first step consists of solving the linear relaxation of Robu-DVB-MILP including the power fixing of each couple  $(s,l)$  with  $s \in S$  and  $l \in L$ . The obtained optimal values are employed to initialize the a-priori measures  $\tau_{sl}(0)$ . Then a solution construction cycle is executed until reaching a time limit. In each execution of the cycle, a number of feasible solutions are built first by fixing the power emission binary variables through formula (12), then deriving the corresponding valorization of variables  $x$  and finally checking their robustness. At the end of each execution of the cycle, the a-priori measures  $\tau$  are updated on the basis formula (13). As last step, once the construction time limit is reached, the exact large neighborhood search is conducted, using as basis the best robust feasible solution defined during the construction cycle.

## 5 Preliminary computational results

The robust optimization approach was tested on 15 instances including realistic data defined from regional DVB-T networks deployed in Italy, including up to about 300 stations and 4000 testpoints. The revenue associated with covering a testpoint is represented by the population of the testpoint, so, in what follows, the value of the best solution found by an algorithm is expressed as the percentage of the population covered with service. As optimization software, we used IBM ILOG CPLEX [8] and the algorithms were tested on a Windows machine with 2.70 GHz Intel i7 and 8 GB of RAM. The hybrid metaheuristic of

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**Algorithm 1**

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1: compute the linear relaxation of the power-indexed version of Robu-DVB-MILP
   for all  $y_{sl} = 1$  and initialize the values  $\tau_{sl}(0)$  with the corresponding optimal values
2: let  $(x^*, y^*)$  be the best robust feasible solution found
3: while a global time limit is not reached do
4:   for  $SOL := 1$  to  $\gamma$  do
5:     construct a feasible power vector  $\bar{y}$  using the probabilistic fixing formula
       (12)
6:     compute the set of station-testpoint couples associated with satisfied SIR
       quantities and derive the corresponding  $\bar{x}$  vector
7:     check the robustness of the feasible solution  $(\bar{x}, \bar{y})$ 
8:     if the coverage granted by  $(\bar{x}, \bar{y})$  is better than that of  $(x^*, y^*)$  then
9:       update  $(x^*, y^*)$  with  $(\bar{x}, \bar{y})$ 
10:    end if
11:   end for
12:   update  $\tau$  according to (13)
13: end while
14: execute the exact large neighborhood search using  $(x^*, y^*)$  and the modified power-
   indexed version of Robu-DVB-MILP as basis
15: return  $(x^*, y^*)$ 

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Algorithm 1 ran with a time limit of 1 hour (50 minutes are devoted to the solution construction and 10 minutes are reserved to the execution of the exact neighborhood search). The parameters  $\alpha$  and  $\gamma$  are set equal to 0.5 and 5, respectively. The robust model takes into account a deviation range that allows deviation up to 20% of the value of the fading coefficients and that is partitioned into 5 deviation bands. In order to evaluate the performance of the multiband robustness model, we considered the coverage of the population that is able to guarantee and the corresponding Price of Robustness (PoR), which we recall to be the reduction in solution optimality that we must pay in order to guarantee protection against uncertain coefficients. We also generated 1000 scenarios of realizations of the uncertain fading coefficients for evaluating the protection that the best found robust solution is able to guarantee. The preliminary results of the computational tests are presented in Table 1, where: i) ID identifies the instance; ii) *COV* is the percentage coverage of the population associated with the best solution found within the time limit and is reported for three models of the design problem, namely *Det*, which is the model not considering the presence of uncertain fading coefficient, *Full*, which considers the model including all the fading coefficients set to their worst value, and *Multi*, which is the Multiband Robust Optimization model; ; iii) *PROT* is the percentage of scenarios in which the best solution found results feasible (specified for the three considered models); iv) *PoR%* is the price of robustness, expressed as percentage increase in the total power value emitted by all stations (specified for the three considered models).

Looking at the table, a first observation that can be made is that the percentage coverage granted by the solution associated with full robustness is much

lower than those by the deterministic and multiband models (on average only an unsatisfying 83% of the population is covered). This is not so surprising, since imposing full robustness forces the model to take into account all worst data deviations occurring simultaneously and this leads to a substantial shrinkage of the feasible set and to the identification of robust solutions that are unnecessarily conservative (it is indeed highly unlikely that all data jointly deviate to their worst value). In contrast, multiband robustness allows to guarantee a percentage coverage of the population that is very close to that of the deterministic model (on average 96.0% granted by the deterministic model versus 95.9% of the multiband model). This (superior) performance of the multiband model must be observed also taking into account the protection that is offered: the multiband model is able to offer the same full 100% protection of the full robustness model, which is much higher than that associated with the deterministic model, whose solutions turn out to be infeasible for about 12% of the cases. Finally, if we look at the price of robustness, the multiband model is able to entail a percentage increase in total power which is about halved on average with respect to full robustness (naturally, the deterministic model is associated with null price of robustness since it does not provide any protection). Looking jointly at the three performance indicators, multiband robustness is thus able to guarantee a full protection against deviations in propagation while maintaining the same level of coverage of the deterministic model and granting a substantial reduction in the price of robustness with respect with the full robustness model.

As future work, we intend to widen the computational experience to a larger set of instances, also conducting a study about the impact of parameter tuning. Moreover, we intend to also better study the impact of different characterization of the uncertainty set on the robustness of solutions.

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**Table 1.** Computational results

ID	COV%			PROT%			PoR%		
	Det	Full	Multi	Det	Full	Multi	Det	Full	Multi
1	96.5	81.6	95.7	83.4	100	100	0	32.5	15.5
2	96.7	83.0	96.2	86.0	100	100	0	24.8	13.7
3	95.6	84.2	95.3	88.1	100	100	0	21.3	12.8
4	95.4	83.4	95.9	86.7	100	100	0	30.2	14.3
5	96.3	85.5	96.4	87.5	100	100	0	24.8	12.9
6	95.8	82.9	95.4	87.6	100	100	0	25.7	13.2
7	96.9	83.3	96.3	86.7	100	100	0	25.2	12.4
8	95.6	84.7	95.7	88.8	100	100	0	28.7	14.1
9	95.7	81.5	95.4	87.4	100	100	0	29.3	16.0
10	96.3	85.6	96.8	86.3	100	100	0	27.1	13.7
11	96.2	83.3	96.2	92.4	100	100	0	25.5	14.6
12	95.8	81.0	96.1	90.3	100	100	0	24.2	12.7
13	96.4	84.8	95.7	87.9	100	100	0	27.9	11.6
14	95.5	82.9	95.6	89.6	100	100	0	28.8	13.6
15	95.8	84.6	96.3	88.9	100	100	0	25.7	16.8

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