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# Green and robust optimal design of Single Frequency Networks by min-max regret and ACO-based learning

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**Abstract**—Notwithstanding the introduction of brand new 5G-based wireless services, single frequency networks supporting digital television and radio broadcasting still represent a major source of telecommunications services in modern smart cities. In this work, we propose a robust optimization model for the green design of second generation single frequency networks based on the digital television DVB-T standard, whose ongoing adoption requires to reconfigure and redesign existing networks. Our robust model aims at protecting design solutions against the data uncertainty that naturally affect propagation of signals in a real environment. For reducing conservatism of solutions, we refer to a heuristic min-max regret paradigm and to solve the resulting problem we propose to adopt a hybrid exact-heuristic algorithm based on the combination of an Ant Colony Optimization-like learning procedure, exploiting tight formulations of the optimization model, with an exact large neighborhood search. Results of computational tests considering realistic instances show that the heuristic min-max regret approach can produce solutions characterized by a substantially lower price of robustness without sacrificing protection against data uncertainty.

**Index Terms**—Single Frequency Networks, DVB-T, Network Design, Robust Optimization, Metaheuristics

## I. INTRODUCTION

Smart cities are nowadays enjoying a wide range of brand new telecommunication services enabled by last generation 5G networks [10], [33], [35], [37], [39], [40]. However, within this ultramodern context, older and more traditional broadcasting services like television and radio are still considered critical by many governments, since they support a cheap, easy and democratic access to information to all the population. A very important advancement in offering broadcasting services of higher quality has been represented by the switch from analogue to digital transmission, which has allowed to implement the paradigm of Single Frequency

Networks (SFNs). An SFN is a broadcasting network in which all transmitting stations emit the same data on the same frequency. This allows for improving spectrum usage, a crucial goal considering that radio resources are scarce and that the expansion of telecommunications services has rapidly led to the congestion of frequencies (see e.g., [21]–[23], [26]). Major examples of SFNs are represented by DVB-T (Digital Video Broadcasting-Terrestrial) networks, which broadcast television content in a digital format and are based on the DVB-T standard adopted in more than 140 countries worldwide [22]. In the last years, SFNs have gained new attention because of the release of a second generation of standards like DVB-T2 [22]. The new standards supports improved modulation scheme and offer higher performance. In addition, thanks to an improved spectrum efficiency, they enable a higher number of broadcasting operators to operate simultaneously. The entrance of new operators and the need for reconfiguring older networks have renewed the interest in software for designing SFN networks, in particular those exploiting mathematical optimization models and algorithms.

In this work, we focus on developing a robust optimization model and algorithm for designing SFN networks taking into account the uncertainty that naturally affects propagation of signals in a real environment. Neglecting such uncertainty may lead to critical issues, such as infeasibility and sub-optimality of design solutions, due to unexpected deviations of the input data representing signal propagation. We remark that, while the deterministic optimal design (i.e., not including the modelling of uncertainty) of SFN networks has received substantial attention (e.g., [3], [12], [14], [19], [29], [30], [32], [34]), the robust design based on using optimization under uncertainty techniques has received less coverage. In this work, we thus propose a new optimization under data uncertainty approach based on min-max regret. Specifically, our main original contributions are:

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- 1) we propose a mixed integer linear programming problem for representing a green SFN optimal design problem with the objective of minimizing the total power emission of the transmitters, while serving a target fraction of the population with a fixed level of quality of service;
- 2) we deal with the question of modelling the uncertainty of propagation of wireless signals, proposing a min-max regret model which offers less conservative design solutions than a classical robust optimization techniques;
- 3) since min-max regret reduces solution conservatism at the price of computational tractability, requiring the solution of more complex optimization problems, we propose to adopt a heuristic min-max regret solution approach, based on the identification of a peculiar median scenario that attempts at capturing the features of all the possible (potentially exponential) scenarios of realizations of the uncertain data. The resulting model is solved with the support of a metaheuristic that combines an Ant Colony Optimization-inspired learning algorithm, based on exploiting stronger formulations available for the model, with an exact large neighborhood search;
- 4) we test the performance of the heuristic min-max regret approach on realistic DVB-T instances, showing that we obtain a substantial reduction in the price of robustness without reducing protection against data uncertainty.

The remainder of the paper is organized as follows: the optimal DVB-T network design model is presented in Sect.II; the heuristic min-max regret model is derived in Sect.III, while Sect.IV presents the hybrid exact-heuristic ACO-inspired learning algorithm. Finally, results of computational tests are presented in Sect.V and conclusions are presented in Sect.VI.

## II. OPTIMAL SFN DESIGN

As reference SFN network, we consider a DVB-T network in which all the transmitters constituting the network transmit the same data on the same frequency in a synchronized way, by means of Orthogonal Frequency Division Multiplexing (OFDM) [22]. The transmitters offer broadcasting services to a region in which a number of users are located. The users constitute the receivers of the DVB-T telecommunication service. The task of designing the network consists of a) setting the power emitted by each transmitter and b) choosing the serving transmitter of each user/receiver, with the goal of minimizing the total power emission of transmitters, while guaranteeing service coverage to a target percentage of the population above a given level of quality of service.

For modelling purposes, we adopt the recommendation of international and national regulatory bodies like ITU and AGCOM to model the region where DVB-T services are provided through a rasterization process [1]: the region is discretized into a grid of sufficiently small pixels and the strength of the DVB-T signal picked up in the point at the center of each pixel (the testpoint) is considered representative of the strength of signal picked up in all the other points of the pixel. Given such rasterization of the region to be covered with DVB-T services, we identify the set of transmitters  $T$ ,

which correspond to the DVB-T transmitting stations, and the set of receivers  $R$ , which correspond to the testpoints at the center of the pixels identified in the region.

The optimal design of an SFN network based on the DVB-T standard can be traced back to a family of optimization problems known as *Wireless Network Design Problems* (WNDPs), in which the configuration of a set of wireless transmitting stations must be chosen in order to provide services to users located in a target region, (see e.g., [14], [18], [28], [32]). Specifically, referring to the hierarchy of WNDPs, the problem that we consider corresponds to a variant of a Scheduling and Power Assignment Problem, which has been proved to be NP-Hard [32].

The two kind of decisions that must be taken in the optimal network design problem that we consider, namely choosing a power emission for the transmitters and choosing the serving transmitter of a receiver can be modelled by continuous and binary decision variables, as follows:

- for each transmitter  $t \in T$ , a continuous variables  $p_t \in [0, P_{\max}]$  is defined to represent the power emission of the transmitter;
- for each transmitter  $t \in T$  and receiver  $r \in R$ , a binary variable  $x_{tr} \in \{0, 1\}$  is defined to represent the fact that receiver  $r$  is assigned to transmitter  $t$  (specifically,  $x_{tr} = 1$  if  $r$  is assigned to  $t$  - we say in this case that  $t$  is the serving transmitter of  $r$  - and  $x_{tr} = 0$  otherwise).

The power  $P_{tr}$  that a receiver  $r \in R$  obtains from any transmitter  $t \in T$  is proportional to the power emitted by  $t$  according to a coefficient  $a_{tr} \in [0, 1]$ , i.e.:  $P_{tr} = a_{tr} \cdot p_t$ . The coefficient  $a_{tr}$  sums up the reduction in power that a signal suffers when propagating from  $t$  to  $r$  in a real-world environment and commonly goes under the name of fading coefficient (see e.g., [36]). Once clarified how power behaves, we can proceed to discuss how signals combine when picked up by a receiver: in an OFDM-based DVB-T system, each receiver picks up signals from all the transmitters and their distinction in useful signals, which contribute to increase the quality of service, and interfering signals, which contribute to decrease the quality of service, depends upon whether they fall in a detection time window (see e.g., [31], [32] for details). According to a common design practice, in our model the detection window of a receiver starts in the instant in which the signal of the serving station is received. This allows identifying the useful and interfering transmitters easily once the serving transmitter is chosen: if  $t$  is the serving transmitter of  $r$ , then we denote by  $U(t, r)$  the set of useful transmitters and by  $I(t, r)$  the set of interfering transmitters for receiver  $r$  (note that in this setting a transmitter is either useful or interfering for a receiver).

The quality of service for receiver  $r$  served by transmitter  $t$  depends upon the Signal-to-Interference Ratio (SIR), namely the ratio of the sum of the power of the useful signals to the sum of the power of the interfering signals [36]:

$$SIR_{tr} = \frac{\sum_{\tau \in U(t,r)} a_{\tau r} \cdot p_{\tau}}{N + \sum_{\tau \in I(t,r)} a_{\tau r} \cdot p_{\tau}} \geq \delta. \quad (1)$$

which involves the power emissions  $p_\tau$  of all transmitters and in which  $N > 0$  and  $\delta > 0$  are the noise of the system and the minimum SIR value that guarantees the desired quality of service, respectively. It is easy to rewrite SIR inequality (1) as

$$\sum_{\tau \in U(t,r)} a_{\tau r} \cdot p_\tau - \delta \sum_{\tau \in I(t,r)} a_{\tau r} \cdot p_\tau \geq \delta \cdot N \quad (2)$$

which allows to highlight the linear nature of the constraint with respect to the power decision variables  $p_\tau$ . The next step in the modelling procedure is to express the other decision of identifying the serving transmitter of each receiver. To do so, we link the binary variable  $x_{tr}$ , denoting whether receiver  $r$  is served by transmitter  $t$ , to the corresponding SIR inequality (2) defining the following modified SIR constraint:

$$\sum_{\tau \in U(t,r)} a_{\tau r} \cdot p_\tau - \delta \sum_{\tau \in I(t,r)} a_{\tau r} \cdot p_\tau + M \cdot (1 - x_{tr}) \geq \delta \cdot N \quad (3)$$

that constitutes a so-called big-M constraint, in which the inclusion of the binary variable  $x_{tr}$  multiplied by a suitably large coefficient  $M \geq 0$  (the big-M) is needed to activate or deactivate the constraints. Indeed, when  $x_{tr} = 0$ ,  $r$  is not served by  $t$  and it is not necessary to satisfy the corresponding SIR inequality: the term  $M(1 - x_{tr})$  is then equal to the large value  $M$ , which is added to the left-hand-side of the constraint (3) and the constraint is thus satisfied by any combination of the decision variables  $p_\tau$ . If instead  $x_{tr} = 1$ ,  $r$  is served by  $t$  and the corresponding SIR constraint must be satisfied - in this case  $M(1 - x_{tr}) = 0$  and thus (3) reduces to (2). The complete model can then be written as:

$$\min \sum_{t \in T} p_t \quad (4)$$

$$\sum_{\tau \in U(t,r)} a_{\tau r} \cdot p_\tau - \delta \sum_{\tau \in I(t,r)} a_{\tau r} \cdot p_\tau + M(1 - x_{tr}) \geq \delta \cdot N \quad t \in T, r \in R \quad (5)$$

$$\sum_{t \in T} x_{tr} \leq 1 \quad r \in R \quad (6)$$

$$\sum_{r \in R} \sum_{t \in T} \pi_r \cdot x_{tr} \geq \alpha \cdot \sum_{r \in R} \pi_r \quad (7)$$

$$0 \leq p_t \leq P^{\max} \quad t \in T \quad (8)$$

$$x_{tr} \in \{0, 1\} \quad t \in T, r \in R. \quad (9)$$

in which, a) the objective function (4) pursues the minimization of the total power emission; b) the quality-of-service conditions are expressed by the SIR constraints (5) accompanied by constraints (6) imposing that at most one transmitter can act as server of each receiver. The constraints (7) impose that at least a fraction  $\alpha \in [0, 1]$  of the total population of the region must be covered with service (in the left-hand-side, each service decision variable is multiplied by a coefficient  $\pi_r$  that equals the population of the pixel associated with receiver  $r$ , while in the right-hand-side the total sum of the population over all the pixels is multiplied by the value  $\alpha$ ). Finally, (8) and (9) are the decision variables previously defined.

### III. PROTECTING FROM PROPAGATION UNCERTAINTY

The optimization problem (4)-(9) is a deterministic model in which it is neglected the fact that the fading coefficients  $a_{tr}$  constitute uncertain data: propagation of signals in a real-world environment is indeed affected by many factors (presence of obstacles, complexity of the lay of the land, etc.) and it is thus very complicated to accurately estimate the values of  $a_{ts}$  over large territories like those involved in DVB-T services. The coefficients are then commonly estimated through empirical propagation models [36], which specify formulas to compute the coefficients. However, as telecommunication professionals know well, the actual values of the propagation coefficients may (even largely) vary with respect to those provided by the models. In our design problem, issues may arise both in cases of over- and under-estimation: if the fading coefficient of an interfering signal is under-estimated, the actual interference will result higher than expected, thus decreasing the quality of service; in contrast, in the case of a useful signal, issues arise in case of over-estimation, since the actual useful signal will result lower than expected. Consequently, the fading coefficients represent uncertain data in an optimization problem, i.e. data whose value is not exactly known when the problem is solved. The presence of uncertain data cannot be neglected, since the occurrence of unexpected deviations in the value of coefficients may cause infeasibility and suboptimality of solutions recognized as feasible and optimal (e.g., [4]–[7]).

For tackling the uncertainty of fading coefficients, we assume that for each transmitter-receiver  $(t, r)$  we know a set of possible values that the coefficient may assume: we denote such set by  $A_{tr} = \{a_{tr}^1, a_{tr}^2, \dots, a_{tr}^k\}$  and we assume that the  $k$  possible values are sorted increasingly. We then introduce the set  $\mathcal{A} = A_{11} \times \dots \times A_{tr} \times \dots \times A_{|T||R|}$  to denote all the possible  $|T||R|$ -tuples, which represent the set of all possible combinations of fading coefficients when considering all the transmitter-receiver couples.

If we follow the paradigm of Robust Optimization (e.g., [6]), the aim is to find a *robust* optimal solution, namely a solution which guarantees the best objective value for the worst realization of the uncertain coefficients. If we denote the feasible set of solutions of problem (4)-(9) by  $\mathcal{F}$  and its objective function by  $f(x, p, a)$ , highlighting that it does not only depends upon the value of the service assignment variables  $x$  and power variables  $p$ , but also on the realization of the fading coefficients  $a \in \mathcal{A}$ , the robust version of the optimal design problem can be written as:  $\min_{(x,p) \in \mathcal{F}} \max_{a \in \mathcal{A}} f(x, p, a)$ , which pursues the identification of an optimal solution  $(x^*, p^*)$  that offers the minimum power emission while protecting against the worst fading coefficient realization of  $\mathcal{A}$  [2]. It is important to stress that such an optimal solution remains feasible for all the realizations of data of  $\mathcal{A}$  and is thus fully protected against data uncertainty. However, at the same time, this full protection may entail a too high *price of robustness*, i.e. a too high reduction in optimality due to excluding non-robust solutions that are not feasible for all fading realizations in  $\mathcal{A}$ .

Since the price of robustness may result too high and look-

ing for full protection may result excessive, we investigate the adoption of an alternative robust methodology, *min-max regret*, which is less conservative [2]. This methodology is based on evaluating the effects of taking a non-optimal decision through a *regret function*  $R(x, p, a)$ , which in our case is defined as the following difference of the objective value of a solution  $(x, p) \in \mathcal{F}$  with respect to the value of the optimal solution  $(x^*, y^*)$  for a fading realization  $a \in \mathcal{A}$ :

$$R(x, p, a) = f(x, p, a) - f(x^*, y^*, a) \quad (10)$$

The *maximum regret*  $R^{\max}(x, p)$  for a solution  $(x, p)$  is then equal to the maximum of the regret function for all  $a \in \mathcal{A}$ :

$$R^{\max}(x, p) = \max_{a \in \mathcal{A}} R(x, p, a) \quad (11)$$

and the min-max regret version of (4)-(9) can be written as the problem of finding the feasible solution  $(x, p)$  that minimizes the maximum regret for all the fading realizations:

$$\min_{(x,p) \in \mathcal{F}} \max_{a \in \mathcal{A}} R(x, p, a). \quad (12)$$

Since the min-max regret problem needs to identify the maximum regret of every solution  $(x, y)$ , therefore requesting to execute the impractical task of potentially verifying exponentially many realizations of the fading coefficients and optimally solving the associated optimization problems, we rely on a heuristic min-max regret method introduced in [24] for a wireless sensor network application. This method is centered around what we call a *pessimistic median* fading coefficient realization scenario: we first define a median fading coefficient scenario  $a^{med}$  that, for every transmitter-receiver couple  $(t, s)$ , includes the fading value  $\bar{a}_{tr} \in A_{tr}$  that is closest to the median value of the values in the set  $A_{tr}$ . Solving the design problem (4)-(9) for the median scenario  $a^{med}$  would allow to identify a solution that is on average of good quality and reliable. However, we could still take a chance on facing infeasibility in case of substantial deviations from the median values. Consequently, for reducing this risk, we identify a more robust fading realization scenario by introducing the concept of super-transmitters, namely transmitters that, favoured by their location with respect to the population to be covered, tend to contribute more decisively to the coverage of a target region (see [16] for more details) and the setting of their power emission is more critical. For these super-transmitters, denoted by  $S \subseteq T$  the fading coefficient is set to the lowest value when act as serving transmitter and to the highest value when acting as interfering transmitter. We call Pessimistic Median (PM) scenario the resulting fading realization scenario and we denote the corresponding fading realization by  $a^{PM}$ . A robust solution for problem (4)-(9) is then identified by optimally solving the problem for the pessimistic median scenario  $a^{PM}$ .

#### IV. AN ACO-BASED LEARNING SOLUTION APPROACH

Being a Mixed Integer Linear Program, problem (4)-(9) could be solved through an optimization software like IBM ILOG CPLEX [11]. However, since it is hard to optimally solve it within the limited time budget that is typically at

disposal of network professionals, especially because of the presence of the complicating big-M SIR constraints, we propose to accelerate the convergence of a software like CPLEX by defining a warm-start heuristic (i.e., a heuristic algorithm that fast identifies good quality solutions to be passed as initial solutions to CPLEX). Specifically, we propose to adopt a hybrid exact-heuristic approach originally proposed in [15], [16] that draws inspiration from Ant Colony Optimization (ACO - see e.g., [8], [9], [20]), as a kind of reinforcement learning algorithm that is based on rewarding value settings of decision variables that have led to the definition of good quality feasible solutions and on penalizing value settings that have led to bad quality solutions (see e.g., [25], [38]). More precisely, the solution approach combines a *probabilistic variable fixing* procedure integrated with an *exact large neighborhood search*.

In the probabilistic fixing procedure, a number of solutions are built iteratively (according to the ACO metaphor, each of these solutions corresponds with an ant): at every iteration, we have at disposal a partial solution, namely a solution in which just a subset of decision variables has been assigned a value, and we can further complete the solution by setting the value of one or more additional variables. When all the variables have been assigned a value, we finally have a complete solution and its quality can be assessed using the corresponding value of the objective function.

In the case of our wireless network design problem, we can note that if we set the value of the power variable  $p_t$  of each transmitter, we can then readily compute the coverage of receivers that is granted by checking which SIR constraints (5) are satisfied. We thus recognize the power variables as critical decision variables in our hybrid solution procedure. To this end, we identify equally spaced power values in the power emission range of each transmitter: we therefore characterize a set  $\mathcal{P} = P_1 = 0, P_2, \dots, P_n = P_{\max}$  of discrete power values in the range  $[0, P_{\max}]$  and each transmitter must be assigned a power value belonging to this discrete set. This allows to derive a power-indexed model of the network design problem, which provides stronger mathematical formulations (see [14]).

When we are at a generic iteration of the cycle that has the task of defining a feasible solution, we have a partial power solution in which a subset of transmitters  $T^{\text{FIX}} \subseteq T$  has been assigned a power emission value of  $\mathcal{P}$ . The next transmitter whose power is fixed is then chosen on the basis of probability formula that represents an improved formula of the classical ACO (see [15] for details):

$$P(p_t = P_\ell) = \frac{\beta \cdot \phi_{t\ell} + (1 - \beta) \cdot \eta_{t\ell}}{\sum_{\tau \in T \setminus T^{\text{FIX}}} \sum_{l \in L} [\beta \cdot \phi_{\tau l} + (1 - \beta) \cdot \eta_{\tau l}]}, \quad (13)$$

that computes the probability of setting the power of transmitter  $t \in T \setminus T^{\text{FIX}}$  equal to  $P_\ell \in \mathcal{P}$ , taking into account all the transmitters  $\tau \in T \setminus T^{\text{FIX}}$  and power emission levels  $l \in L$  whose power has not yet been set.

In (13), two measures of value fixing attractiveness are combined:  $\phi_{\tau l}$  constitutes an a-priori attractiveness measure that resembles the pheromone trails of ACO and is obtained from a

tight-relaxation of the model (in particular from model (4)-(9) strengthened through the (strong) valid inequalities proposed in [13], [14]), whereas  $\eta_{\tau l}$  is an a-posteriori attractiveness measure obtained from the linear relaxation of (4)-(9) (i.e., a version of the model in which the integrality requirement of variables is removed). The two measures are combined by a parameter  $0 \leq \beta \leq 1$ .

The probability measure is used in the parallel construction of  $\gamma > 0$  solutions and, once that a round of solution construction is completed, we proceed to reward/penalize the single construction moves that have been operated, on the basis of the quality of the solutions that have been defined. Specifically, the update rule of the a-priori measure is:

$$\phi_{\tau l}(i) = \phi_{\tau l}(i-1) + \sum_{\text{SOL}=1}^{\gamma} \Delta \phi_{\tau l}^{\text{SOL}} \quad (14)$$

$$\Delta \phi_{\tau l}^{\text{SOL}} = \phi_{\tau l}(0) \cdot \left( \frac{OG(v^{\text{AVG}}, u) - OG(v^{\text{SOL}}, u)}{OG(v^{\text{AVG}}, u)} \right) \quad (15)$$

in which  $\phi_{\tau l}(i)$  is the a-priori attractiveness measure of setting the power of transmitter  $\tau \in T$  equal to  $P_\ell$  in the  $i$ -th execution of the cycle and  $\Delta \phi_{\tau l}^{\text{SOL}}$  is the correction to this measure, computed considering a moving average over the last  $\gamma$  solutions that have been constructed. Furthermore,  $u$  is an upper bound on the optimal value,  $v^{\text{SOL}}$  is the value of the SOL-th feasible solution built in the last construction cycle,  $v^{\text{AVG}}$  is the average of the values of the last  $\gamma$  solutions that have been constructed. The value  $OG(v, u)$  is an optimality gap and assesses the difference between the value  $v$  of a solution and an upper bound  $u$  (formally,  $OG(v, u) = (u - v)/v$ ). Concretely, after having defined all its elements, formula (14) is introduced to update the a-priori measure with the aim of rewarding (penalizing) the setting of decision variables to values which have conducted to solutions associated with lower (higher) optimality gap, when compared to a moving average  $v^{\text{AVG}}$  that involves the last  $\gamma$  solutions.

After the cycle of solution construction has ended, we conduct an improvement phase in which we rely on an exact large neighborhood search for improving the best solution found. Specifically, the strategy is to formulate the search in a large neighborhood of solutions of the best solution as an optimization problem which is solved by an optimization software like CPLEX [8]. This is motivated by noticing that the software may be unable to find the optimal solution of the complete problem, but may instead efficiently identify optimal solutions of appropriate subproblems. In our specific case, we explore solution neighborhoods defined by allowing to modify the power emission of the transmitters: each transmitter can be either switched off (thus its power emission is set to zero) or its power emission can be increased to the closest higher discrete power level or decreased to the closest lower power level. This can be easily expressed through linear constraints involving binary variables representing the power modifications and that can be added to problem (4)-(9), similarly to the procedure illustrated in [17]. The resulting model is then passed to CPLEX to be solved to an optimum. For not letting CPLEX

waste too much time on closing the optimality gap, we impose a time limit on the solution of the problem corresponding to exploring the solution neighborhood (high quality solutions are typically fast obtained).

We provide the pseudocode of the hybrid ACO-based learning heuristic in Algorithm 1. The best solution found by the algorithm is used to warm-start the solver CPLEX, i.e. it is passed to CPLEX as starting feasible solution to accelerate its convergence to an optimum. In Algorithm 1, problem (4)-(9) is solved referring to the pessimistic median scenario  $a^{PM}$  and the first step corresponds with solving the strengthened power-indexed optimal design model and use it for initializing the values of the a-priori fixing measure. After this, the algorithm is centered around running a cycle for building solutions, with a time limit as stopping criterion. Each run of the cycle leads to the definition of  $\gamma$  solutions, built by first setting the value of the power decision variables by means of the probability measure (13) and then by evaluating the obtained population coverage, checking which SIR constraints are satisfied by the power setting. Once a run of the cycle is completed, the next step is to update the a-priori attractiveness measure  $\phi$  by means of formula (14), learning from the quality of the value fixing of decision variables that has been operated. Finally, once the construction time limit has been reached, the best solution found is used as initial solution in the conclusive exact large neighborhood search, to further improve it.

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#### Algorithm 1 (Hybrid ACO-based learning heuristic)

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- 1: define the pessimistic median scenario  $a^{PM}$
  - 2: compute the linear relaxation of the strengthened power-indexed version of (4)-(9) for  $a^{PM}$  and initialize the values  $\phi_{\tau l}(0)$  with the corresponding optimal values
  - 3: let  $(p^*, x^*)$  be the best solution found
  - 4: **while** a time limit is not reached **do**
  - 5:   **for**  $\text{SOL} := 1$  to  $\gamma$  **do**
  - 6:     iteratively set the power emission  $p_t$  of every transmitter  $t \in T$  using formula (13) and let  $\bar{p}$  be the corresponding fixed power vector
  - 7:     check which SIR constraints (3) are satisfied by  $\bar{p}$  and let  $\bar{x}$  be the corresponding service coverage vector
  - 8:     compute the total population coverage of solution  $(\bar{p}, \bar{x})$
  - 9:     **if** the coverage satisfies constraint (7) and the total power emission of  $(\bar{p}, \bar{x})$  is lower than that of  $(p^*, x^*)$  **then**
  - 10:       update  $(p^*, x^*)$  with  $(\bar{p}, \bar{x})$
  - 11:     **end if**
  - 12:   **end for**
  - 13:   update the a-priori measure  $\phi$  according to (14)
  - 14: **end while**
  - 15: run the exact large neighborhood search using  $(p^*, x^*)$  as incumbent solution
  - 16: return  $(p^*, x^*)$
- 

## V. COMPUTATIONAL RESULTS

We test the performance of the heuristic min-max regret approach, based on considering the pessimistic media scenario, on a set of realistic DVB-T instances corresponding to region-wide networks deployed in Italy. The fading coefficients are computed according to propagation models defined taking into

account recommendation of regulatory bodies [1], [27]. These recommendations also specify how to model propagation stochasticity of DVB-T signals and we considered discussion held with professionals working in the field of single frequency networks for defining a range of deviation for the standard robust optimization model that does not result too conservative and include unlikely deviations. The instances that we consider refer to 20 Italian DVB-T regional networks that involve up to about 300 stations and 4000 pixels and for each pixel the population is derived from demographic data. In order to solve the considered mixed integer linear programming problem, we rely on the software IBM ILOG CPLEX [11], running on a Windows machine with 2.70 GHz Intel i7 and 8 GB of RAM. In the warm-start hybrid heuristic presented in Algorithm 1, we allow the parallel construction of  $\gamma = 10$  solutions and we assign a weight  $\beta = 0.5$  so as to assign the same weight to the combination of the a-priori and a-posteriori measure. A time limit of 600 seconds is imposed to the final exact search.

The results of the preliminary computational tests that we have conducted are presented in Table I in which: 1) ID identifies the regional DVB-T network instance; 2) columns 2 and 3 report the Price of Robustness (PoR%) for the case of the standard robust optimization approach (SR) and for the case of the min-max regret approach (mMR) - we note that the PoR% is computed as the percentage increase in the total power that must be faced with respect to the deterministic version of the design problem which neglects the presence of uncertain fading coefficients; 3) column  $\Delta$ PoR% reports the percentage decrease in the price of robustness that the use of the less conservative min-max regret approach grants with respect to the standard robust optimization approach; 4) the last three columns report measures Prot% about the protection granted by standard robustness SR and min-max regret mMR. More in detail, for each instance, we generate 1000 realizations of the uncertain fading coefficients and we then compute the percentage of realizations in which the robust solutions of SR and mMR are feasible. The percentage difference between the two measures of protection is expressed by  $\Delta$ Prot%.

If we analyze the data reported in Table I, a first important remark that can be made is that granting protection against fading coefficient uncertainty leads to a remarkable increase in the total power needed to cover the population in the considered territory. Indeed, for both SR and mMR the total power emission of the transmitters increase of more than 10% on average. However, at the same time, it can be clearly seen that for all instances the increase in power and thus the price of robustness is substantially higher for the standard robust optimization model: SR on average requires 16% more of average power emission, while mMR entails a lower increase in power that allows to save 26% of power emission. This is of course a remarkable result, since a greener and less power consuming network does not only lead to lower operational costs, but also limits electromagnetic pollution, exposing people to lower electromagnetic fields. The reduction of the total power emission needed to handle uncertainty results particularly significant in the case of instances like

I17 and I19, for which the price of robustness and thus the total power emission is reduced of more than 30% and even reaches more than 40%. A natural question that then arises is whether this reduction in the price of robustness granted by the min-max regret approach is accompanied also by a significant reduction in protection. However, this is not the case as it can be seen from the column *Prot%*: the min-max regret solutions are feasible for more than 93% of the 1000 scenarios of fading realizations that we generated, leading to a very small decrease in protection with respect to standard robustness (which offers on average protection for about 95% of the scenarios). The small reduction in protection is thus much more than compensated by the very relevant reduction in total power. This shows the expediency and advantages of adopting the min-max regret approach, which, for the considered wireless network design problem, is able to return optimal solutions that, at the same time, are well-protected against uncertainty and grant sensibly lower price of robustness.

TABLE I  
COMPUTATIONAL RESULTS

ID	PoR% (SR)	PoR% (mMR)	$\Delta$ PoR%	Prot% (SR)	Prot% (mMR)	$\Delta$ Prot%
I1	17.2	13.1	23.6	93.2	91.5	1.8
I2	13.4	11.0	17.4	95.7	93.4	2.4
I3	14.9	12.7	14.7	97.7	94.8	2.9
I4	11.6	7.8	31.9	97.3	95.7	1.6
I5	13.2	9.5	27.8	92.6	91.9	0.7
I6	18.8	14.0	25.5	94.2	90.7	3.7
I7	21.5	15.3	28.8	96.4	94.2	2.2
I8	16.5	13.6	17.3	92.9	91.0	2.0
I9	17.4	13.1	24.7	96.8	94.5	2.3
I10	14.9	10.7	28.1	95.2	91.4	3.9
I11	14.5	9.9	31.6	97.4	95.3	2.1
I12	19.7	13.8	29.6	95.9	94.9	1.0
I13	15.3	11.3	26.1	94.8	92.8	2.1
I14	14.2	10.7	24.4	96.0	92.0	4.1
I15	18.4	14.7	19.6	96.1	93.6	2.6
I16	16.0	10.9	31.5	95.7	92.6	3.2
I17	17.0	9.7	42.9	93.8	93.2	0.6
I18	14.7	10.6	27.4	97.0	95.4	1.6
I19	18.8	12.3	34.3	96.2	93.7	2.5
I20	21.3	15.7	26.1	95.1	94.1	1.0

## VI. CONCLUSION AND FUTURE WORK

We have considered the problem of optimally designing a single frequency network while taking into account the uncertainty that naturally affects the propagation of wireless signals in a real-world environment. Since neglecting such uncertainty may lead to wrong design solutions affected by unexpected holes in the coverage of users, we have proposed to adopt a robust optimization approach. Robust optimization is a major methodology adopted in optimization to deal with the presence of uncertain data, namely data whose value is not exactly known when the problem is solved. Since canonical robustness may result too conservative, leading to

robust solutions that entails a too high price of robustness (i.e., a too high deterioration in the objective value to guarantee protection), we have investigated the adoption of an alternative way to guarantee protection, namely a (heuristic) min-max regret approach, which grants less conservatism and higher computational tractability. The solution of the resulting problem is accelerated by a hybrid heuristic that combines an Ant Colony Optimization (ACO)-inspired learning algorithm with exact large neighborhood search. Computational results considering realistic instances show that the heuristic min-max regret approach can identify substantially less conservative solutions, without sacrificing protection. This encourages to further develop the approach, in particular by investigating the definition of refined median scenarios that could better capture the stochastic behaviour of signal propagation and lead to robust optimal solutions associated with even lower price of robustness. Furthermore, we plan to investigate a multi-objective version of the problem, pursuing joint coverage maximization and power minimization, adopting an iterative solution procedure as in [41].

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