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The dual influence of the reed resonance frequency and tonehole lattice cutoff frequency on sound production and radiation of a clarinet-like instrument

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The internal and external spectra of woodwind reed instruments are partially determined by the tonehole lattice cutoff and reed resonance frequencies. Because they can impact the spectrum in similar ways, a study of one without accounting for the other risks incomplete or false conclusions. Here, the dual effects of the cutoff and reed resonance frequencies are investigated using digital synthesis with clarinet-like academic resonators. It is shown that the odd and even harmonics have similar amplitudes at and above the cutoff frequency or reed resonance frequency, which ever is lowest. However, because the resonators radiate efficiently at the cutoff, it has the additional role of reinforcing the amplitude of both the odd and even harmonics in the external spectrum. The spectra are analyzed using the single value descriptors playing frequency, spectral centroid, odd/even ratio, and brightness, as a function of the musician mouth pressure. Higher reed resonances correspond to higher values for all descriptors. The odd/even ratio and brightness increase with resonator cutoff frequency, whereas the spectral centroid exhibits more complicated trends. The reed resonance has a larger impact on the 'playing condition oscillation threshold', implying that it may have a more important role in sustaining auto-oscillation.

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[XYZ]

1 I. INTRODUCTION

Like all instruments, the timbre of a woodwind re-2 ³ sults from a number of interacting parameters. Some of ⁴ these, such as the blowing pressure within the mouth, are ⁵ controlled by the musician, while others are innate physi-⁶ cal characteristics of the instrument such as the geometry 7 of the air column. Because it is difficult to independently ⁸ vary only one aspect of an instrument/musician combi-⁹ nation, it is often unclear how different parameters affect ¹⁰ the resulting spectrum. This is especially true when two ¹¹ or more parameters affect the spectrum in a similar man-¹² ner. For cylindrical single-reed instruments, two such pa-¹³ rameters, the lowest reed resonance frequency $f_{\rm r}$ and the tonehole lattice (THL) cutoff frequency f_c , can lead to 14 similar effects on the internal and external spectrum of an 15 instrument; disentangling their dual effects is the subject 16 ¹⁷ of the current article. Indeed, the ambiguous results of a previous study¹ using real instruments demonstrates the 18 difficulty of separating these two variables and motivates 19 a systematic approach to the problem. 20

The THL of many woodwind instruments have a certain acoustic periodicity which leads to pass- and stopbands within the air column²⁻⁶. At low frequencies, an acoustic wave traveling from the mouthpiece towards the toneholes will largely radiate and reflect from the closest open tonehole. The strong reflection from the first open ²⁷ tonehole allows for the interference that determines the ²⁸ low frequency modes which facilitate auto-oscillation. In ²⁹ contrast, higher frequencies are able to propagate past ³⁰ the first open tonehole at which point they radiate from ³¹ subsequent toneholes and the bell, as well as reflect from ³² various locations within the lattice. The cutoff frequency ³³ f_c that separates these two regimes is determined by the ³⁴ specific geometry (tonehole radii, chimney heights, inter-³⁵ hole spacing, and main bore radius) of the THL and, for ³⁶ a clarinet, is approximately 1500 Hz⁵.

Under normal playing conditions, the oscillation of 37 ³⁸ the reed favors frequencies that correspond to the first ³⁹ several peaks of the input impedance. These low fre-40 quency peaks are shifted in frequency by changing the ⁴¹ fingering of the THL, and therefore sounding pitch, of ⁴² the note. The reed has its own natural frequencies, the ⁴³ lowest of which is considerably higher than the frequen-⁴⁴ cies in the first register of the clarinet. For simplicity, $_{45}$ only the lowest reed resonance frequency $f_{\rm r}$ is considered 46 in the current model. It is difficult to experimentally ⁴⁷ measure $f_{\rm r}$, although it may be estimated using various ⁴⁸ tools such as instrumented mouthpieces or artificial blow-⁴⁹ ing machines⁷⁻⁹; classically reported values are between ⁵⁰ 2000-3000 Hz^{1,10}. Values between 1000-2000 Hz are to ⁵¹ be expected when the reed is supported by the lip¹¹, 52 and previous values used in a numerical continuation ap-⁵³ proach include 1500 Hz¹². However, one complication

reed is that the reed/mouthpiece/lip system is inherently 113 the results and conclusions are covered in Section V. 55 nonlinear, including the nonlinear characteristic linking 56 the pressure in the mouthpiece to the flow through into 57 the instrument, nonlinear contact forces between the reed 58 and the lay of the mouthpiece, and the nonlinear mate-115 59 60 61 62 63 64 nomena. 65

66 67 68 69 70 71 72 73 induces a "region of reinforced spectrum," for which the 130 a musically appropriate hall^{1,14,18}. 74 odd and even harmonics are of similar amplitude and are 131 75 76 77 78 79 80 81 of the clarinet 11,15 . 82

83 84 cause neither one can be varied without potentially im-85 pacting other aspects of the instrument. This is espe-86 cially true for f_c because any alteration requires a mod-87 ification of the lattice geometry, likely damaging the in-88 89 90 91 92 94 95 96 97 98 99 effects of each model parameter. 100

101 102 103 104 is provided in the current article. Section III shows the 159 a control, is a simple cylinder devoid of toneholes. 105 effect of f_c and f_r on general characteristics of the inter-106 107 108

⁵⁴ that arises in determining the dynamical properties of the ¹¹² threshold, are provided in Section IV. A discussion of

114 II. METHODOLOGY

Digital synthesis is used to compare the competing rial properties of the musicians lips. There is a trade-off 116 effects of the cutoff frequency and reed resonance on both between using a simple model for which the results can 117 the internal and external pressure spectra of simplified be easily interpreted, versus a more complex model that 118 clarinet-like resonators. This is preferred over other availmore accurately captures the nonlinear physical system, 119 able methodologies such as an instrumented mouthpiece but which may result in unnecessary higher-order phe- 120 or an artificial blowing machine because it allows to un-¹²¹ ambiguously isolate the variables of interest, notably the The THL $f_{\rm c}$ and $f_{\rm r}$ are both known to impact the ¹²² reed resonance frequency. Similarly, the musician conspectrum of the clarinet¹. However, their relative im- ¹²³ trol parameters relating to the blowing pressure and emportance has not been systematically studied. Clarinets, 124 bouchure can be fixed to precise values, which may be which may be idealized as a quarter-wave resonator at 125 achieved with a blowing machine, but is generally not low frequencies, generally have high-amplitude odd har- 126 possible for real musicians to attain when using an inmonics corresponding to the input impedance peaks and 127 strumented mouthpiece. Furthermore, it obviates comlow-amplitude even harmonics due to the lack of support 128 plications relating to spectra measurements in real enviat the input impedance valleys. However, the THL f_{c}_{129} ronments, be it a laboratory space, anechoic chamber, or

While digital synthesis generally does not perfectly efficiently radiated into the surrounding environment¹³. 132 match experimental results, it is a robust method for This phenomena also occurs at the nominal cutoff of real 133 evaluating the qualitative differences that arise when instruments^{1,14}. The f_r may also induce a reinforced $_{134}$ some aspect of an instrument is modified¹⁹. For this amplitude for harmonics that are close in frequency¹⁰. ¹³⁵ reason, it is an appropriate tool for the current study Other aspects related to the f_r , such as the lip force and $_{136}$ which seeks to isolate the effects of two specific variables. lip position, affect the pitch, sound level, and spectrum 137 Similarly, it may be useful to instrument manufacturers ¹³⁸ who wish to evaluate a hypothetical modification of an Determining the influence of $f_{\rm c}$ and $f_{\rm r}$ on the spec- 139 existing or non-existing instrument, without accruing the trum is challenging in the case of real instruments be-140 cost and time associated with manufacturing prototypes.

141 A. Resonator design

The current experiment utilizes four cylindrical resstrument. In order to investigate the dual influence of 143 onators that are designed to resemble a simplified verthe THL f_c and f_r , it is necessary to develop a method- 144 sion of the clarinet, originally developed by some of the ology that allows the two parameters to be varied in- $_{145}$ current authors for a different study¹³. Three of the resdependently. While it is relatively straightforward to 146 onators include a geometrically regular THL comprising devise and construct resonators with a given cutoff fre-147 10 toneholes, the geometry of which are chosen to inquency^{6,16}, the practical challenge of controlling the reso-148 duce a THL cutoff frequency at 1.0, 1.5, and 2.0 kHz, see nance frequency of a physical reed¹⁷ remains a challenge. ¹⁴⁹ Table I, hereafter referred to as $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$, re-Therefore, the phenomenon is explored numerically in the 150 spectively. The inter-hole spacing is kept constant so that current article. Choosing to use a numerical model allows 151 equivalent fingerings produce approximately the same for precise control of the variables of interest, although 152 sounding frequencies for each resonator. The upstream the results are largely valid for comparing the qualitative 153 length, or the section that is between the mouthpiece and ¹⁵⁴ the first hole of the lattice, is chosen to account for the The methodology of the numerical experiment is pre-155 THL length correction such that the note corresponding sented in Section II. Because much of the protocol has 156 to all the toneholes being open has the same first input been previously published (citations in relevant sections), 157 impedance peak frequency for each resonator. The fourth only a truncated account sufficient to interpret the results $_{158}$ "resonator" (see below), referred to as \mathcal{R}_{cyl} and used as

In order to generate enough data to make general nal and radiated spectrum. Analysis highlights how each 161 comments regarding the spectrum, it is necessary to anparameter can influence different aspects of the spec- 162 alyze the harmonic content of multiple notes. There-¹⁰⁹ trum. Single value descriptors of the spectrum (playing $_{163}$ fore, the highest six notes of resonators $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}$, and ¹¹⁰ frequency, spectral centroid, odd/even ratio, and bright- $_{164} \mathcal{R}_{2.0}$, out of the 11 available notes excluding forked fin-¹¹¹ ness), as well as an analysis relating to the oscillation ¹⁶⁵ gerings, are considered in the analysis. This ensures that 166 the lowest note considered still retains a lattice with five

Resonator	L (cm)	b (mm)	$f_{\rm c}~({\rm Hz})$	Impedance peak frequency (Hz)	Inharmonicity (cents)
$\mathcal{R}_{1.0}$	39.9	2.5	1000	185.2; 172.8; 161.9; 152.3; 143.7; 136.1	2.2; 4.3; 6.4; 7.9; 10.4; 11.4
$\mathcal{R}_{1.5}$	41.7	4.0	1500	185.1; 172.6; 161.8; 152.2; 143.6; 136.0	9.3; 11.0; 10.7; 11.3; 12.8; 13.1
$\mathcal{R}_{2.0}$	42.6	5.8	2000	185.0; 172.6; 161.7; 152.1; 143.6; 136.0	11.2; 11.7; 12.4; 12.9; 13.2; 13.1
$\mathcal{R}_{\mathrm{cyl}}$	L (cm):	45.2; 48	.5; 51.8; 55.0; 58.3; 61.5	185.1; 172.6; 161.7; 152.1; 143.6; 136.0	11.8; 12.7; 13.5; 14.0; 14.0; 13.9

TABLE I. Summary of the geometry and acoustical characteristics of the four resonators used in the current article. All resonators have a common internal radius a = 6.5 mm. Resonators $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$ have a common inter-hole spacing $\ell = 32.6$ mm (center to center) and tonehole heights h = 9.8 mm, where the six notes are produced by closing the first five consecutive toneholes. In the bottom row are the six lengths used for \mathcal{R}_{cyl} (no THL), chosen to have similar first input impedance peak frequencies as $\mathcal{R}_{1.0}$, $\mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$, but no THL cutoff. The right columns provide the first input impedance peak frequency f_1 in Hz and the inharmonicity $1200\log_2(f_2/(3f_1))$ in cents for each of the six notes. It is worth noticing that, contrary to a real clarinet, the inharmonicity between the first two peaks is positive, but it does not change the results of the current study.

168 quency is maintained throughout the scale. The purely 211 rent work because the resonators are designed using a 169 170 ent lengths, each with a first input impedance peak fre- 213 waves that occur above the approximate cutoff for a real ¹⁷¹ quency corresponding to those of the 6 fingerings of the ²¹⁴ instrument (traversing the entire length of the resonator, 172 other three resonators. The maximum difference for a 215 rather than confined between the first open tonehole and 173 four resonators is less than 3 cents. 174

175 176 the 'realistic' case where the volume of the closed tone- 219 mouthpiece. The relaxed definition of a cutoff frequency 177 holes (the closed THL²⁰) is included in the calculations 220 in this work corresponds more closely to the case of a real 178 ¹⁷⁹ such that the section between the mouthpiece and the ²²² periodic media, especially considering that the tonehole 180 181 182 183 184 185 186 187 188 189 $_{190}$ of the odd harmonics tend to be lower while the ampli- $_{233}$ impedance²¹. Further complicating this topic, clarinet 191 192 193 194 when there is additional inharmonicity. Including the 238 this study. tonehole volumes has a lesser effect above the cutoff be-196 cause the input impedance peaks are no longer harmon-197 ically related to the low frequency resonances, and are 198 ¹⁹⁹ sampled more or less at random by the harmonics of ²⁴⁰ 200 201 202 203 204 fect of the cutoff and reed resonance frequencies without 246 nal Interactions (TMMI)²². 205 the added complication of increased inharmonicity. 206

It is important to recall that the cutoff f_c is only 248 impedance 207 208 defined for an infinite, lossless lattice, and therefore is ²⁰⁹ always approximate for a finite, lossy resonator². The

¹⁶⁷ open holes, and therefore that the intended cutoff fre-²¹⁰ implications of this assumption do not impact the curcylindrical resonator \mathcal{R}_{cyl} is simulated using six differ- 212 physical model, which accurately simulates the standing given note's input impedance peak frequency across the 216 the mouthpiece). Indeed, the results would be different 217 if the resonator had a true cutoff (lossless, infinite lat-Simulations were performed under two conditions: ²¹⁸ tice) above which no energy is reflected back towards the and the 'academic' case where the volume is excluded, 221 instrument than the theoretical pass- and stop-bands in first open tonehole is perfectly cylindrical for all finger- ²²³ lattice of a real clarinet is not strictly periodic⁵. Another ings. The realistic case introduced complications because 224 difference between the academic resonators used here and the inharmonicity between the first and second input 225 a real clarinet is the absence of a bell, which will tend to impedance peaks was not the same for the different res- 226 efficiently radiate frequencies above the tonehole lattice onators. Specifically, the inharmonicity becomes larger 227 cutoff, weakening the standing waves within the full bore for the higher cutoff frequency resonators used in this 228 of the instrument. Indeed, the standing waves above cutstudy due to the larger tonehole radii. It was found that 229 off for the academic resonators can be seen in the transthis affected both the internal and external spectra, al- 230 fer functions shown in Sect. III C; the weakened standing though the effect is subtle. When the tonehole volumes 231 waves above cutoff for real clarinets, in part due to the are included, at frequencies below cutoff, the amplitude 232 bell, may be observed on the measured or simulated input tude of the even harmonics tend to be higher. This is 234 bells have their own cutoff response at around 1500 Hz. expected because the harmonics of the playing frequency 235 Therefore, while it would be possible to include a bell in will not sample as close to the maxima (odd harmonics) 236 the resonator simulations, the academic solution is again and minima (even harmonics) of the input impedance 237 prioritized to better isolate the two factors of interest in

239 B. Simulation of the resonator characteristics

The passive acoustic response corresponding to the the playing frequency. Additionally, excluding the closed 241 six notes on all four resonators is simulated using an tonehole volumes ensures that there is no closed THL fil- 242 extension of the Transfer Matrix Method (TMM) that tering between the internal and external sound fields²⁰. 243 is modified to include external interactions of apertures To minimize the complexity of the study, closed toneholes 244 (toneholes and the termination) radiating into the same are not included in the simulation to better isolate the ef- 245 space, known as the Transfer Matrix Method with exter-

> This calculation was devised to simulate the input 247

$$Z_0(\omega) = P_0(\omega)/U_0(\omega), \qquad (1)$$

²⁴⁹ where $P_0(\omega)$ and $U_0(\omega)$ are the frequency domain ($\omega = 296$ $_{250}$ $2\pi f$) acoustic pressure and flow at the input of the res- $_{297}$ respond to the mouth pressure γ and embouchure ζ en- $_{251}$ onator. One advantage of the TMMI algorithm is that it $_{298}$ acted by the musician²⁶, $_{252}$ also provides the pressure and flow in the *n*th radiating ²⁵³ aperture $P_n(\omega)$ and $U_n(\omega)^{13}$. For the resonators with a THL comprising 10 holes, n = 11 including the termina-254 tion of the resonator; n = 1 for the simple cylinders. 255

256 $_{257}$ useful relations, such as the transfer function between the $_{300}$ units and w is the width of the reed channel. The reed $_{258}$ input pressure or flow quantities to those in the radiating $_{301}$ is treated as a single degree of freedom oscillator²⁷: 259 apertures:

$$H_{u,n}(\omega) = U_n(\omega)/P_0(\omega).$$
(2)

260 These transfer functions (any combination of $P_0(\omega)$, 302 where f_r and q_r are the resonance frequency and damping $_{261} U_0(\omega), P_n(\omega), \text{ and } U_n(\omega))$ can be used to understand $_{303}$ coefficient of the reed. It is possible to include higher reed ²⁶² basic characteristics of the resonator beyond what is ³⁰⁴ modes: the next lowest, which is also flexural, is approxi-²⁶³ available from the input impedance alone. In particu- ³⁰⁵ mately 2.6 times the first resonance⁷. However, harmon-²⁶⁴ lar, Eq. (2) can be used used to calculate the far field ³⁰⁶ ics in this frequency range (2600-5200 Hz, depending on $_{265}$ pressure at $P_{\text{Ext},m}(\omega)$ due to a given mouthpiece pres- $_{307}$ the first resonance) are generally more than 40 dB below $_{266}$ sure $P_0(\omega)$, treating each nth aperture as a monopole $_{308}$ the fundamental, and are therefore of secondary impor-²⁶⁷ source and ignoring diffraction:

$$P_{\text{Ext},m}(\omega) = \sum_{n=1}^{N} j\omega \rho \frac{H_{u,n}(\omega)P_0(\omega)}{4\pi r_{n,m}} e^{-jkr_{n,m}}.$$
 (3)

268 Here, j is the imaginary unit, ρ is the density of the ²⁶⁹ medium, $k = \omega/c$ is the wavenumber in free space with $_{270}$ the speed of sound c, and $r_{n,m}$ is the distance between $_{271}$ the *n*th radiating aperture and the *m*th observation po-272 sition. This simplified radiation model corresponds well $_{273}$ to measurements performed in an anechoic chamber²³.

274 C. Digital synthesis

A previously^{24,25} developed digital synthesis model 275 276 is used to simulate the discrete time domain behavior of each resonator coupled to a single reed mouthpiece, as it 277 would be played by a musician. The synthesis results of 278 an earlier version of the algorithm was validated against 279 acoustic recordings produced by an artificial mouth as 280 part of a timbre perception study¹⁵. 281

The three primary variables describing the dynami-282 cal behavior of the instrument are the reed displacement 283 $_{284} x(t)$, acoustic pressure $p_0(t)$, and acoustic flow $u_0(t)$ at 285 the input of the resonator

$$x(t) = \frac{\hat{x}(t)}{H}, p_0(t) = \frac{\hat{p}_0(t)}{p_M}, u_0(t) = z_c \frac{\hat{u}_0(t)}{p_M}, \quad (4)$$

287 is the pressure required to close the reed channel com- 331 for the academic resonators used in the current study, it $_{288}$ pletely, and z_c is the characteristic impedance of the res- $_{332}$ may become important with realistic instrument geome-289 onator. The three primary variables are dimensionless, 333 tries, particularly with the addition of narrow register while the hat notation is used for the physical value of 334 holes. 290 each variable. The subscript used for the acoustic pres- 335 291 292 sure and flow indicate that they are the discrete time 336 malism that is computed from the inverse Fourier trans-293 domain representation of the frequency domain variables 337 form of the reflection coefficient, which is itself deter-²⁹⁴ defined in Eq. (1) corresponding to the plane at the input ³³⁸ mined by the input impedance³². This is preferential 295 of the resonator.

The dimensionless musician control parameters cor-

$$\gamma = \frac{p_m}{p_M}, \zeta = w H z_c \sqrt{\frac{2}{\rho p_M}},\tag{5}$$

These quantities can be used to calculate a number of 299 where p_m is the musician mouth pressure in physical

$$\frac{1}{2\pi f_{\rm r}}\ddot{x} + \frac{q_{\rm r}}{2\pi f_{\rm r}}\dot{x} + x = p_0(t) - \gamma + F_{\rm c}(x), \qquad (6)$$

309 tance compared to the effect of the first reed resonance $_{\rm 310}$ and THL cutoff frequency. A contact force $F_{\rm c}$ between $_{311}$ the reed and the mouthpiece lay²⁸ is defined by

$$F_{\rm c} = K_{\rm c} \left(\frac{x+1-|x+1|}{2}\right)^2 (1+\beta \dot{x}), \qquad (7)$$

³¹² where $K_c = 100$ and $\beta = 5e-4$ are nonlinear stiffness and 313 damping coefficients, respectively. Note that the contact $_{314}$ force is only activated when x < -1, corresponding to 315 the condition for which the reed is in contact with the $_{316}$ lay of the mouthpiece^{28,29}.

The flow through the reed channel is linked to the 318 pressure differential between the bore and the mouth of 319 the player:

$$u_0(t) = \zeta \left(\frac{x(t) + 1 + |x(t) + 1|}{2}\right) \operatorname{sign}(\gamma - p_0(t)) \sqrt{\gamma - p_0(t)}$$
(8)

 $_{320}$ In order to regularize the nonsmooth functions of Eqs. (7) $_{321}$ and (8), each absolute value term $|\Lambda|$ (for an arbitrary ³²² argument Λ) is replaced by $\sqrt{\Lambda^2 + \eta}$, where $\eta = 0.01$ is ³²³ a small regularization quantity³⁰. Note that this model is inherently nonlinear, and that the regularization term 324 325 likely captures a more realistic reed/lay interaction than ³²⁶ the simplest single degree of freedom reed model^{17,27,31}. $_{327}$ It has been checked that the specific value chosen for η 328 does not alter the qualitative trends observed in the syn-329 thesized spectra. However, the values may impact the $_{286}$ where H is the height of the reed channel at rest, p_M $_{330}$ emergence of higher registers. While this is not a concern

> The resonator is modeled by a reflection function for-339 to a truncated modal representation because it includes

³⁴⁰ high frequencies in the synthesis, which is of particular ³⁹⁰ and the top to bottom order of reed resonances is main-³⁴¹ importance when considering the effect of the THL cutoff ³⁹¹ tained in Fig. 2.) The square and circular data markers 342 frequency.

343 D. Numerical experiment

The six notes for each resonator are synthesized with 396 reference equal to unity. 344 345 a duration of 10 seconds using different reed resonance 397 ³⁴⁷ dio examples provided in online supplementary material). ³⁹⁹ appear to induce a boost in the amplitude of the even 348 349 clarinet, but with a wide enough range to observe their 401 lower. The effect of the cutoff can be observed by com-351 $_{352}$ sure and embouchure are set to central values of $\gamma = 0.7$ $_{404}$ in the left panels, 1500 Hz in the middle-left panels, and 353 354 355 musician. Only first register notes were observed, likely 407 resonance is more subtle and is most easily observed for 357 regimes may be produced for other combinations of mu- 409 in the even harmonics at each reed resonance. It can also 358 sician control parameters.²⁹ The reed quality factor is 410 be observed for resonators $\mathcal{R}_{1.5}, \mathcal{R}_{2.0}$. For example, the ³⁵⁹ fixed at $q_r = 0.4$. These values, corresponding to a 'forte' ⁴¹¹ top panel for $\mathcal{R}_{1.5}$ shows a gap of approximately 20 dB at ³⁶⁰ playing level, were chosen in order to ensure synthesized ⁴¹² 1000 Hz while the bottom has a gap greater than 30 dB. 361 harmonics. 362

363 364 365 ₃₆₆ are treated with a Hann window. Similarly, the inter- ⁴¹⁸ harmonics, the cutoff appears to be the stronger effect. ³⁶⁷ nal flow spectrum $U_0(\omega)$ can be calculated from $u_0(t)$. Equation. (2) is used to determine the resulting flow in $_{419}$ B. External spectrum 368 each aperture. 369

The external pressure $P_{\text{Ext},m}(\omega)$ is calculated using 420 370 $_{371}$ Eq. (3) at M = 100 far field locations ($|r_{1,m}| = 10$ m $_{421}$ onators and three reed resonances are shown in Fig. 2, $_{372}$ $\forall m$), evenly distributed on a circle in the plane of the res-³⁷³ onator with the first open tonehole at the origin. There ⁴²³ in Fig. 1. The vertical axis is proportional to Fig. 1. As 374 is no need to account for observation positions outside 424 with the internal spectrum, odd and even harmonics be-375 the plane because each tonehole is assumed to radiate 425 come more similar in amplitude at and above the reed 376 as a monopole and diffraction is ignored, resulting in an 426 resonance. The effect of the cutoff frequency, however, 377 axis-symmetric directivity pattern. The total radiated 427 is more pronounced: in addition to boosting the ampli-378 pressure spectrum is approximated by the sum of the 428 tude of the even harmonics, it introduces a 'region of ³⁷⁹ pressures at all 100 external observation locations:

$$\bar{P}_{\text{ext}}(\omega) = \sum_{m=1}^{M} P_{\text{Ext},m}(\omega).$$
(9)

380 This method is chosen in order to avoid problems as-³⁸¹ sociated with the strong directivity of the resonator at ³⁸² frequencies above the cutoff.

383 III. RESULTS I: GENERAL CHARACTERISTICS

384 A. Internal spectrum

385 $_{386} \mathcal{R}_{1.0}, \mathcal{R}_{1.5}, \text{ and } \mathcal{R}_{2.0}$ in the left three columns and \mathcal{R}_{cyl} $_{443}$ and/or reed resonance. Higher reed resonances will not 387 in the right column, where the reed resonances are, from 444 have the same degree of impact as the first reed resonance $_{388}$ top to bottom, $f_{\rm r} = 1000, 1500$, and 2000 Hz. (The left $_{445}$ because the odd and even harmonics already have equal 389 to right order of the resonators is maintained in Figs. 2-6 446 amplitudes at frequencies above the first resonance and

392 (online: black and red) denote the odd and even har-³⁹³ monics, respectively. The vertical axes are represented ³⁹⁴ in normalized decibels: $20\log_{10}(|P_0(\omega)|)$, where the di-³⁹⁵ mensionless pressure $P_0(\omega)$ is normalized by an implicit

The dual effects of the THL cutoff and reed resonance values equal to 1000 Hz, 1500 Hz, and 2000 Hz (au- 398 on the internal spectrum can be observed in Fig. 1: both These values are chosen to be approximately those of a 400 harmonics at, and above, either f_c or f_r , whichever is effect on the resulting waveform. The dimensionless mu- 402 paring the gap between odd and even harmonics at the sician control parameters corresponding to mouth pres- 403 cutoff of each resonator: they tend to merge at 1000 Hz and $\zeta = 0.4$, and are both slowly varied by approximately $_{405}$ 2000 Hz in the middle right panels, corresponding to the 15 % in order to mimic the natural instability of a real $_{406}$ cutoffs of $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$. The effect of the reed due to the resonators simple geometry, although other 408 \mathcal{R}_{cyl} in the right column, where there is a distinct bump tones that have substantial energy in the high frequency $_{413}$ A similar comment may be made for $\mathcal{R}_{2.0}$. It is less ⁴¹⁴ clear in the panels corresponding to $\mathcal{R}_{1,0}$, for which f_c The frequency domain mouthpiece pressure $P_0(\omega)$ is $_{415}$ is less than or equal to f_r for all combinations. Figure 1 calculated from the time domain pressure $p_0(t)$ using the 416 demonstrates that, while both the THL cutoff frequency built in MATLAB fft function in 0.5 s intervals that 417 and the reed resonance cause a reinforcement of the even

The external pressure spectrum for the four res-⁴²⁹ reinforced spectrum' at the cutoff frequency comprising 430 both odd and even harmonics. Notice that the envelop $_{431}$ of the spectrum around cutoff is highest for $\mathcal{R}_{1.0}$, and ⁴³² drops by 5-10 dB for $\mathcal{R}_{1,5}$ and $\mathcal{R}_{2,0}$. This is due to the ⁴³³ natural decrease in harmonic envelop characteristic of a clarinet-like system¹. 434

Note that including higher reed resonances would ⁴³⁶ likely have a subtle affect on the spectral envelop shown ⁴³⁷ in Figs. 1 and 2. However, any influence would occur ⁴³⁸ at frequencies (2600-5200 Hz) where the harmonics are 439 already very weak. More importantly, the characteris-440 tics of the clarinet spectrum are particularly important ⁴⁴¹ between 1000 and 2000 Hz, where the amplitude of the Figure 1 shows the results for the resonators 442 odd and even harmonics become equal due to the cutoff



FIG. 1. (color online) Power spectra of synthesized non-dimensional pressure in the mouthpiece $P_0(\omega)$ for resonators with THLs $\mathcal{R}_{1.0}$, $\mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$ (left three columns; f_c marked by vertical line) and no THL \mathcal{R}_{cvl} (right column) for varying reed resonances $f_{\rm r} = 1000, 1500, 2000$ Hz, from top to bottom. Each panel contains the spectra of six different notes with fundamental frequencies ranging from 136 Hz to 185 Hz, see Table I. Odd and even harmonics denoted by square (black) and circular (red) data markers, respectively.



FIG. 2. (color online) Synthesized external non-dimensional pressure $\bar{P}_{\text{Ext}}(\omega)$ power spectra. Panel organization and pressure amplitudes relative to Fig. 1.

447 THL cutoff frequencies. That the spectrum has two prin- 456 pears to be more strongly impacted by the THL cutoff 448 ciple frequency ranges (strong odd/weak even harmonics 457 frequency. Here, the effect of the THL as a filter is ex-449 at low frequencies, parity at high frequencies) is likely an 458 plored to better understand the distinction between the ⁴⁵⁰ important perceptual characteristic of the clarinet.

451 C. The lattice as a filter

As seen in Sections III A and III B, the THL cutoff 452 453 and reed resonance frequencies both impact the inter-⁴⁵⁴ nal and external spectra by reinforcing the amplitude of ⁴⁵⁵ the even harmonics. However, the external spectrum ap-

⁴⁵⁹ internal and external spectra.

Figure 3 shows the transfer function between the 460 ⁴⁶¹ pressure in the mouthpiece and the summed time deriva-⁴⁶² tives of the flow in each radiating aperture:

$$H_{\rm rad}(\omega) = \sum_{n=1}^{N} j\omega U_n(\omega) / P_0(\omega).$$
 (10)



FIG. 3. Transfer function between the pressure at the input of the resonator and the sum of time derivative of the flow at each radiating aperture for, from left to right, resonators $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}, \mathcal{R}_{2.0}$, and \mathcal{R}_{cyl} . This is an approximation of the transfer function between the mouthpiece pressure and the external sound field. The transfer functions are continuous in frequency and different for each fingering: for readability, only the highest fingering of each resonator is shown (grey line). The transfer function is sampled at integer multiples of the first input impedance peak, corresponding approximately to the harmonics in the spectrum, denoted by large diamonds (odd) and large circles (even) data markers. To reinforce the trends associated with cutoff (regardless of fingering), integer multiples of the first input impedance peak for the other five fingerings are depicted by small data markers.

⁴⁶³ To within a constant scaling factor, this expression is pro-⁴⁹⁷ where * indicates the complex conjugate operation. All 464 465 466 separation, and spherical spreading, and is a reasonable 500 value at high frequencies. There is a distinct increase in $_{467}$ approximation of general radiation characteristics that $_{501}$ radiation efficiency at the cutoff of resonators $\mathcal{R}_{1,0}, \mathcal{R}_{1,5},$ do not include the effects of directivity. Note that this $_{502}$ and $\mathcal{R}_{2.0}$, which is not observed for \mathcal{R}_{cyl} . is a passive characteristic of the resonator that depends 503 469 470 471 472 473 474 475 476 477 478 479 480 481 function of the highest note, shown in grey. 482

483 484 485 486 487 488 489 490 491 even harmonics is maintained even at high frequencies. 492 493

power flowing through the plane at the input of each 528 coincide with a peak or valley. resonator and the sum of the power flowing through each 529 ⁴⁹⁶ radiating aperture:

$$\mathbf{H}_{\Pi,n}(\omega) = \sum_{n=1}^{N} \operatorname{Re}\left[\frac{P_n(\omega)U_n^*(\omega)}{P_0(\omega)U_0^*(\omega)}\right],\qquad(11)$$

portional to the transfer function between the internal 498 four resonators have approximately the same radiation efpressure and farfield pressure ignoring diffraction, source 499 ficiency below the cutoff and tend to converge to a similar

These passive radiation characteristics, demonsolely on the geometry: the results are the same regard- 504 strated by Figs. 3 and 4, provide the link between the less of reed properties (such as resonance frequency) and 505 internal and external spectra depicted in Figs. 1 and 2. other musician control parameters. The transfer func- 506 While the reed resonance induces reinforced even hartion is calculated for all six notes of each resonator, but 507 monics in the nearby frequency band, it has no effect for clarity, only the highest note is shown in totality by 508 on the ability of the resonator to efficiently radiate these the thin grey line. The first input impedance resonance 509 frequencies to the surrounding environment. The THL frequency (corresponding approximately to the sound- 510 also induces reinforced even harmonics, but within a freing frequency) is extracted for each fingering, and the 511 quency band for which the resonator radiates efficiently. transfer function $H_{rad}(\omega)$ is sampled by integer multi- ⁵¹² Therefore, both even and odd harmonics are reinforced ples of this frequency with odd and even harmonics de- 513 in the radiated spectra. That the cutoff induces both of noted by diamonds and circles, respectively. The large 514 these effects is not to be viewed as a coincidence, but diamond/circle data markers correspond to the transfer ⁵¹⁵ rather the dual results of an intertwined phenomena: be-516 cause the resonator radiates efficiently at and above the The panels in Figure 3 demonstrate the impact of the 517 cutoff frequency, the internally reflected waves that result THL on some radiation properties of the resonators. Be- 518 in constructively interfering odd and destructively interlow the cutoff, odd harmonics are radiated weakly com- ⁵¹⁹ fering even resonances no longer correspond to a quarterpared to even harmonics, in accordance with the quarter- 520 wave resonator model. This allows even harmonics to be wave low frequency approximation of the resonator in 521 reinforced internally in addition to a strong radiation of which the lattice is ignored. At and above the cutoff, 522 both even and odd harmonics externally. Note that this both the even and odd harmonics are efficiently radiated ⁵²³ is analogous to the interpretation that the harmonics of into the surrounding space. Resonator \mathcal{R}_{cvl} demonstrates 524 the playing frequency sampling the input impedance: bethe extreme case, where the distinction between odd and 525 low the THL cutoff, the odd harmonics tend to fall on or ⁵²⁶ near peaks, while the even harmonics correspond primar-Figure 4 shows the transfer function between the 527 ily to valleys, while above cutoff, they are equally likely

> As seen in Fig. 3 (especially in the right panel which $_{530}$ shows the lattice-less \mathcal{R}_{cyl}), the behavior deviates from ⁵³¹ this simple description at high frequencies. One cause ⁵³² is inharmonicity: the peaks and valleys of the input ⁵³³ impedance do not coincide at perfect integer multiples of ⁵³⁴ the first peak due to increased viscothermal losses at high



FIG. 4. Transfer function between the power flowing through the input of the resonator and the sum of the power flowing through the radiating apertures for, from left to right, resonators $\mathcal{R}_{1.0}$, $\mathcal{R}_{1.5}$, $\mathcal{R}_{2.0}$, and \mathcal{R}_{cyl} .

⁵³⁵ frequencies. This is also why the closed tonehole volumes ⁵³⁶ are excluded in the simulations of the resonator; includ-⁵³⁷ ing those volumes increases the inharmonicity. Another reason is that higher frequencies radiate more efficiently 538 than lower frequencies, so the transfer function increases 530 up to the THL cutoff frequency of resonators $\mathcal{R}_{1,0}$, $\mathcal{R}_{1,5}$, 540 ⁵⁴¹ and $\mathcal{R}_{2,0}$, and across the entire frequency band for \mathcal{R}_{cvl} . 542 Above the THL cutoff, there is no simple application of this observation due to increased viscothermal losses as the sound wave reflects back and forth throughout the 544 entire length of the resonator. Note that, under playing 545 conditions, the playing frequency is always slightly lower 547 than the first input impedance peak frequency so, even 548 ignoring losses, the harmonics do not sample the input ⁵⁴⁹ impedance exactly on the peaks and valleys.

550 IV. RESULTS II: SINGLE VALUE DESCRIPTORS OF THE 551 SPECTRUM

The relative effect of the THL cutoff and the reed res-552 onance frequencies on the internal and external spectrum 553 is evaluated using objective descriptors. The purpose of 554 this section is to demonstrate how the qualitative effects 555 observed in Sect. III map to numerical descriptors that 556 are often used for describing timbre. Here, the spectrum 557 for each cutoff/resonance frequency combination is sim-558 ulated for a wide range of normalized musician mouth 559 pressures γ and fixed $\zeta = 0.4$, providing some insight on 560 how the spectrum varies with the musician mouth pres-561 sure control parameter. Each signal is simulated for a 562 duration of two seconds using a constant value for γ (in 563 contrast to Section III), sufficiently long to extract the spectrum descriptors. Signals that are either unstable in 565 pitch or sound outside of the first register are discarded. 566 The MIRtoolbox³³ is used to process the spectral 567 characteristics of the internal and external soundfields. 568 The external signal $\bar{P}_{ext}(\omega)$ is computed following the 569 same procedure as in Section IIB, from which the temporal signal is determined by the inverse Fourier trans-571 form assuming Hermitian symmetry in order to be pro-572 cessed as an audio file by the MIRtoolbox. A peak finding scheme (mirpeaks) is applied to identify the har-574 ⁵⁷⁵ monic components of each signal. The spectral centroid, 576 odd/even ratio, and brightness are computed from the



FIG. 5. Playing frequency as a function of musician mouth pressure γ for, from left to right, resonators $\mathcal{R}_{1.0}$, $\mathcal{R}_{1.5}$, $\mathcal{R}_{2.0}$, and \mathcal{R}_{cyl} . The thick black, thin black, and thick grey lines correspond to reed resonances $f_r = 1000, 1500$, and 2000 Hz, respectively.

577 harmonic power spectrum of the signal up to 9500 Hz, 578 although choosing a lower value such as 5000 Hz does not 579 change the qualitative trends because most of the energy 580 in the signals is below 2000 Hz.

581 A. Playing frequency

Figure 5 shows the playing frequency as a function frequency as a function γ for the six notes considered on each of the four restion on the six notes considered on each of the four resset on ators. The thick black, thin black, and thick grey lines for correspond to reed resonance frequencies $f_r = 1000, 1500,$ and 2000 Hz, respectively. With a maximum discrepancy for of approximately 17 cents for a given fingering, the six set note 'scale' available to each resonator are reasonably similar in frequency, especially for the higher fingerings. This provides confidence that a comparison of spectrum characteristics across resonators is not likely to be com⁵⁹² plicated by a difference in intonation, although it could ⁶³⁶ all three descriptors regardless of resonator cutoff. It may 593

594 595 596 597 598 599 600 peak frequencies is likely primarily due to the inhar- 644 account for different playing frequencies. However, be- $_{601}$ monicity between the first and second input impedance $_{645}$ cause they quantify the high frequency content (B₃₀₀₀ is 602 peaks and the reed induced flow, although the reed dy- 646 relative to 3000 Hz), it is unlikely that small variations in ⁶⁰³ namics also play a role.²⁴ It is interesting to note that ⁶⁴⁷ the playing frequency will influence the global qualitative ⁶⁰⁴ higher reed resonance frequencies result in higher play- ⁶⁴⁸ trends. ⁶⁰⁵ ing frequencies for all four resonators.

606 B. Spectrum descriptors

Single value descriptors are extracted from the wave-607 ⁶⁰⁸ forms to determine global spectra differences due to the THL cutoff and reed resonance. The descriptors estimate 609 610 the relative distribution of power between different sets 611 of frequencies, calculated from harmonics extracted from ⁶¹² the steady portion of the tone, discarding the transient. ⁶¹³ The first descriptor is the spectral centroid (SC),

$$SC = \frac{\sum_{n=1}^{N} f_n A_n^2}{\sum_{n=1}^{N} A_n^2},$$
 (12)

⁶¹⁴ which is often linked to the perceived 'brightness' of ⁶¹⁵ a tone³⁴. The second descriptor, the odd/even ratio ₆₁₆ (OER), is the ratio between the power in the odd and 617 even harmonics:

$$OER = \frac{\sum_{n=1}^{N} A_{2n-1}^2}{\sum_{n=1}^{N} A_{2n}^2}.$$
 (13)

⁶¹⁸ A third descriptor, known as brightness (B₃₀₀₀),

$$B_{3000} = \frac{\sum_{n(f \ge 3000)}^{N} A_n^2}{\sum_{n=1}^{N} A_n^2},$$
(14)

⁶²⁰ fixed cutoff frequency. Common values are 1000 Hz³⁵, ₆₇₇ have the most flexibility for finding an adequate timbre 1500 Hz^{33} , and 3000 Hz^{36} . For this article, the bright-621 622 ness calculation is in relation to a cutoff at 3000 Hz in order to be well above the THL cutoff and reed reso-624 nance frequencies. Therefore, this descriptor provides an 625 indication of the variables effect on the high frequency ₆₂₆ portion of the spectrum, above the region of reinforced ₆₂₇ spectra (due to the THL) and above the frequency by which the odd and even harmonics have reached parity 628 (due to either the THL or reed resonance). 629

Figure 6 shows the descriptors applied to the same 630 THL cutoff and reed resonance configurations as in 631 Figs. 1 and 2 as a function of the excitation pressure. The ⁶³³ values shown correspond to the average of all six finger-634 ings, thereby emphasizing the global spectra characteris-⁶³⁵ tics. The reed resonance categorically raises the values of

induce difficulties in a subjective listening experiment. $_{637}$ be pointed out that higher reed resonances f_r also result Despite the significant differences in resonator ge- 638 in higher playing frequencies, which should naturally lead ometries used in the current study compared with a real 639 to higher descriptors of the spectra. However, it has been clarinet, the general trends seen in Fig. 5, such as the 640 checked that the dimensionless SC (such that it has been decrease in pitch up to $\gamma = 0.5$ followed by an increase in $_{64}$ normalized by the playing frequency) exhibits the same pitch for higher γ , is consistent with previous results^{24,26}. ⁶⁴² trends as the normal SC defined in Eq. (12). The OER The deviation in pitch from the first input impedance ⁶⁴³ and B₃₀₀₀ descriptors cannot be equivalently modified to

649 1. Spectral centroid

As previously shown by some of the current authors, 650 ⁶⁵¹ the SC of the internal spectrum increases with increasing ⁶⁵² THL cutoff frequency¹⁶. In contrast, there is not a monotonic increase in SC for increasing $f_{\rm c}$ for the external 653 ⁶⁵⁴ spectra. This may initially seem counter-intuitive: if it ⁶⁵⁵ increases with internal spectra one might expect the same ⁶⁵⁶ for the external spectra. Furthermore, Fig. 2 shows a re-⁶⁵⁷ gion of reinforced spectra at the cutoff, so it is sensible to ⁶⁵⁸ expect this to shift the SC higher for higher cutoffs. How-⁶⁵⁹ ever, for a clarinet-like instrument, the vast majority of acoustical energy is contained in the low ranking harmon-⁶⁶¹ ics. A low THL cutoff will create the region of reinforced ⁶⁶² spectra for harmonics that have a relative large fraction ⁶⁶³ of the total energy compared to those in the region of ⁶⁶⁴ reinforced spectra for the higher THL cutoffs. There is a ⁶⁶⁵ complicated balance in the SC calculation between inher-666 ently high-energy low-ranking harmonics and boosting ⁶⁶⁷ the inherently low-energy high-ranking harmonics. This ⁶⁶⁸ indicates that the SC may not be a very useful descrip-⁶⁶⁹ tor to distinguish between the competing effects of the ⁶⁷⁰ THL cutoff and the reed resonance frequency, especially ⁶⁷¹ in relation to human perception which is very sensitive to ⁶⁷² frequencies around 2000 Hz. However, it is worthwhile to $_{673}$ note that $\mathcal{R}_{1.5}$, which has a cutoff that is most similar to ⁶⁷⁴ real clarinets, has the largest range of SC available when 675 considering different reed resonance frequencies. This $_{619}$ is the fraction of the total power that is above some $_{676}$ suggests that an instrument with $f_{\rm c} \approx 1500$ Hz might ⁶⁷⁸ by using reeds with different mechanical properties.

679 2. Odd/even ratio

The OER is shown in the middle row of panels in 680 ₆₈₁ Fig. 6. At low γ , this descriptor has very large values ₆₈₂ because the signal is primarily composed of the plaving ⁶⁸³ frequency, with very little energy in higher harmonics. ⁶⁸⁴ As the pressure is increased, the value drops to a mini-685 mum value at approximately $\gamma = 0.5$ as other harmonics 686 emerge. Above $\gamma = 0.5$, the value once again increases 687 as the signal approaches the square-wave shape charac-⁶⁸⁸ teristic of a clarinet played at a high level^{37,38}.

Generally, the OER increases for resonators with 689 690 higher THL cutoffs, and is the highest for the lattice-⁶⁹¹ less cylinder. Similarly, the OER also increases with



FIG. 6. Objective descriptors calculated from the harmonic power spectra of the synthesized waveforms as a function of blowing pressure γ for each THL cutoff and reed resonance frequency combination, averaged from the highest six fingerings of each resonator. Top: spectral centroid; Middle: odd/even ratio of harmonics; Bottom: Brightness relative to a cutoff at 3000 Hz. Left to right: $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}, \mathcal{R}_{2.0}$, and \mathcal{R}_{cyl} . Reed resonance frequencies $f_r = 1000, 1500$, and 2000 Hz are denoted by thick black, thin black, and thick grey lines, which are solid for the external field and dotted for the internal field.

⁶⁹² the reed resonance frequency. However, this narrative ⁶⁹³ is challenged by the details: at high γ , the OER cor-⁶⁹⁴ responding to $f_{\rm r} = 1500$ Hz is higher for $\mathcal{R}_{1.0}$ than for ⁶⁹⁵ $\mathcal{R}_{1.5}$. An interesting result is that the presence (or not) ⁶⁹⁶ of a THL, regardless of it's cutoff frequency, appears to ⁶⁹⁷ have the greatest impact on the OER. Resonators $\mathcal{R}_{1.0}$, ⁶⁹⁸ $\mathcal{R}_{1.5}$, and $\mathcal{R}_{2.0}$ all have much lower OER than $\mathcal{R}_{\rm cyl}$. For a ⁶⁹⁹ lattice-less cylinder, the OER is strongly impacted by the ⁷⁰⁰ reed resonance frequency, which is a much smaller effect ⁷⁰¹ for the other resonators. Although outside the scope of ⁷⁰² this study, it would be interesting to study how the pres-⁷⁰³ ence of a bell, which is sometimes treated as a surrogate ⁷⁰⁴ THL²¹, would impact these results.

705 3. Brightness

The bottom panels of Fig. 6 show the brightness as 707 defined by the MIR toolbox, not to be confused with 708 the subjective use of this word to describe timbre. The 709 brightness descriptor is perhaps the easiest to interpret: 710 it increases with cutoff frequency and it increases with 711 reed resonance. The conclusion is that both of these vari-712 ables enable energy in the higher harmonics for both the 713 internal and external spectra. The lattice-less resonator 714 has brightness values that are greater than the resonators 715 with a THL.



FIG. 7. Playing condition threshold of oscillation as a function of reed resonance frequency $f_{\rm r}$ for resonators $\mathcal{R}_{1.0}, \mathcal{R}_{1.5}, \mathcal{R}_{2.0}$, and $\mathcal{R}_{\rm cyl}$.

716 C. Playing condition oscillation threshold

The threshold of oscillation, a parameter that is rer18 lated to the 'playability' of an instrument, is known to be r19 impacted by the reed resonance frequency^{39,40}, while the r20 effect of the THL cutoff has, to the authors' knowledge, r21 never been studied. Here, the phenomena is explored usr22 ing a quantity that may be considered a 'Playing Condir23 tion Oscillation Threshold' (PCOT). Rather than studyr24 ing the oscillation threshold using a slow pressure ramp r25 or continuation scheme¹², the PCOT assumes that the 727 728 results in a stable first regime oscillation for a given reed 785 reliable tool to characterize the sounds of clarinet-like 730 731 732 733 734 $_{735}$ disregard the γ values that result in this phenomenon. It $_{792}$ to the 'ease of playing' of a particular instrument. is noted that a full bifurcation analysis is not the goal of 793 736 737 of f_c and f_r on the internal and external spectra. 738

739 740 nance and THL cutoff frequency, the PCOT is computed 797 mechanical properties of the reed is not easily achieved 741 742 743 745 746 747 onance frequency. It is interesting to note that, above 804 and observing trends in the amplitude of the even har- $_{748}$ 1300 Hz, $\mathcal{R}_{1.5}$, $\mathcal{R}_{2.0}$, and \mathcal{R}_{cyl} have similar values while $_{805}$ monics in the internal spectrum in comparison with syn- $_{749} \mathcal{R}_{1.0}$ has a uniformly higher value. The deviation for $\mathcal{R}_{1.0}$ 806 thesis results. An additional challenge for *in situ* exper-751 752 754 755 termining the PCOT. 756

757 758 759 reed resonance that is not evident when only considering 816 should be made cautiously due to their more complicated $_{760}$ the spectrum. While f_c appears to affect the spectrum $_{817}$ THL geometries, which inherently have ambiguous cut- $_{761}$ more than $f_{\rm r}$, it is the latter that may have the dominant $_{818}$ off frequencies. However, the THL cutoff is known to be 762 role in sound production. This indicates that the 'ease 819 an important characteristic of woodwind instruments, so 763 ⁷⁶⁴ linked to the reed resonance than the THL cutoff.

V. CONCLUSION 765

Digital synthesis using clarinet-like academic res-766 767 onators shows that the tonehole lattice cutoff frequency $f_{\rm c}$ and reed resonance $f_{\rm r}$ impact the internal and ex-⁷⁶⁹ ternal spectra. While they both lead to an increase in parity between the odd and even harmonics, the THL 827 770 771 772 773 774 troid does not monotonically increase or decrease with ⁸³² (ANR16-LCV2-0007-01 Liamfi project). 775 the cutoff frequency. The ratio between the odd/even 776 harmonics increases with both increasing tonehole lat-777 tice cutoff frequency and reed resonance. However, the 833 778 presence of any tonehole lattice, regardless of it's cutoff, ⁸³⁴ 779 780 appears to be the greatest effect when compared with the 835 781 lattice-less resonator. The brightness, corresponding to 836 ⁷⁸² the ratio of power above 3000 Hz to the power below, does ⁸³⁷

 $_{726}$ mouth pressure undergoes a rapid jump from $\gamma = 0$ to $_{783}$ increase with cutoff frequency. These conflicting trends its final value. The PCOT is the lowest value of γ that $_{784}$ demonstrate that single value descriptors may not be a resonance frequency. Perhaps due to the relative simplic- 786 instruments. An analysis of the 'playing condition osity of the resonators considered in this study, the second 787 cillation threshold' demonstrates that the reed resonance regime occurs very rarely, always close to the PCOT, 788 has a larger impact than the cutoff frequency on the minand develops from an unstable first regime within sev- $_{789}$ imum blowing pressure γ necessary for the emergence of eral seconds. Therefore, the signals are synthesized for 790 first register oscillations. This indicates that the reed a duration of 10 seconds (as in Section II D) in order to 791 resonance may be the more important variable relating

A future study that would make these results practithe current study, which is mainly interested in the effect 794 cal for instrument manufacturers could involve listening f_c and f_r on the internal and external spectra. In order to study the relative effects of the reed reso- f_{795} experiments to link f_c and f_r to subjective impressions of the timbre. This poses two challenges: (1) controlling the as a function of reed resonance frequency ($\zeta = 0.4$) for real instruments; (2) digitally synthesized tones do the four resonators. Figure 7 shows the results for the 799 not always produce realistic sounds, which could hinder highest note of each resonator. The PCOT has a rel- 100 their use in such a study. One possible solution may atively high value when the reed resonance is less than ⁸⁰¹ be to estimate the mechanical properties of a reed us-1200 Hz, drops rapidly to a minimum at approximately $_{802}$ ing an artificial mouth or instrumented mouthpiece^{9,41}, 1300 Hz and then rises slowly with increasing reed res- 803 perhaps coupled to a cylindrical, lattice-less resonator, may be due to the small number of low frequency input 807 iments arises due to the variation in playing frequency impedance peaks that facilitate in the auto-oscillation of $\frac{1}{2}$ caused by f_r ; even a difference of 2 Hz (about 17 cents the reed, making it harder to achieve oscillation. How- ⁸⁰⁹ for the highest note of the resonators used in the curever, because the general trends are the same for all four suo rent study) can complicate a subjective impression of resonators including the cylinder without a THL, it is 111 'brightness.' One could use a variable length L for the reasonable to conclude that f_c is secondary to f_r in de- $_{312}$ upstream portion of the resonator that can be adjusted ^{\$13} to match the tuning of each note and reed resonance The PCOT demonstrates a fundamental difference ^{\$14} frequency, although the practicality of this solution is between the roles of the THL cutoff frequency and the ⁸¹⁵ questionable. Finally, generalizations to real instruments of playing' and possibly articulation may be more closely ⁸²⁰ the results in the current article likely remain applicable. ⁸²¹ Because the cutoff frequency has a strong impact on the ⁸²² external spectrum of an instrument, but only a minimal ⁸²³ effect on the production of sound, it is conceivable that it ⁸²⁴ may be utilized as a design parameter relating primarily ⁸²⁵ to timbre.

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The authors appreciate Tom Colinot and Jean Keralso induces increased radiation at its cutoff frequency. 228 gomard for their comments on the manuscript, and Jean-The single value descriptors of playing frequency, spec- 229 Pierre Dalmont for his insightful comments during an tral centroid, odd/even ratio, and brightness all increase and early stage of the project. This work has been partly supfor higher reed resonance frequencies. The spectral cen- ⁸³¹ ported by the French Agence Nationale de la Recherche

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