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► **To cite this version:**

Lucrezia Carboni, Michel Dojat, Sophie Achard. Nodal statistics-based structural pattern detection for graph collections characterization. Conference of Complex System, Oct 2022, Palma de Mallorca, Spain. . hal-03866319

**HAL Id: hal-03866319**

**<https://hal.science/hal-03866319>**

Submitted on 22 Nov 2022

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# Nodal statistics-based structural pattern detection for graph collections characterization

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*This work has been partially supported by MIAI@Grenoble Alpes (ANR 19-P3IA-003)*



<http://arxiv.org/abs/2210.01053>

## Motivations

- Human consciousness states can be differentiated by nodal organization in brain functional connectivity networks [1]
- structural pattern definition
- extend node role discovery to graph collections [2, 3]
- graph collections comparison based on structural pattern

## Nodal statistics-based structural pattern on single graph

### Def. Nodal statistics based equivalence relation.

We consider undirected unweighed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  and refer to a nodal statistics  $s : \mathcal{V} \rightarrow s(\mathcal{V})$  any function of the adjacency matrix.

The **equivalence relation**  $\sim_s$  associated with a nodal statistics  $s$ , on the nodes set  $\mathcal{V}$  of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The **equivalence relation associated with any collection of statistics**  $\mathcal{S} = \{s_i\}_{i=1,\dots,n}$ , is defined as:

$$a \sim_{\mathcal{S}} b \iff a \sim_{s_1} b, a \sim_{s_2} b, \dots, a \sim_{s_n} b. \quad (1)$$

**Def. Structural Pattern.** Its induced partition  $P$  on  $\mathcal{V}$ ,

$$P_{\mathcal{S}} = \frac{\mathcal{V}}{\sim_{\mathcal{S}}} = \{[a], \forall a \in \mathcal{V}\} \quad (2)$$

defines the **structural pattern** of  $\mathcal{G}$  associated with the statistics collection  $\mathcal{S}$ .

**Def. Node role.** The class of equivalence

$[a] = \{b \in \mathcal{V} | a \sim_s b \iff s(a) = s(b)\}$  corresponds to **node role**.

## Structural patterns for graph collections characterization

**Def. Correspondence structural pattern score.** Let  $\mathcal{G}, \mathcal{G}'$  be two graphs having same vertices  $\mathcal{V}$  and let  $\mathcal{S}$  be a statistics collection whose associated partitions are  $P_{\mathcal{S}}, P'_{\mathcal{S}}$  on  $\mathcal{G}, \mathcal{G}'$  respectively. Given bijective mapping from  $P_{\mathcal{S}}, P'_{\mathcal{S}}$  to an initial segment of the natural numbers as enumerations, let  $c(v_i), c'(v_i)$  be the enumeration of the classes of  $v_i$ , the correspondence structural pattern score between  $\mathcal{G}, \mathcal{G}'$  is defined as:

$$C(\mathcal{G}, \mathcal{G}') = \max_{\pi \in \Pi} \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \mathcal{X}(\pi(c(v_i)) = c'(v_i)) \quad (3)$$

where  $\Pi$  is the set of all coupling between the elements in  $P_{\mathcal{S}}$  and the elements in  $P'_{\mathcal{S}}$  and  $\mathcal{X}$  is the indicator function.

**Def. Nodal-percentage of participation.**

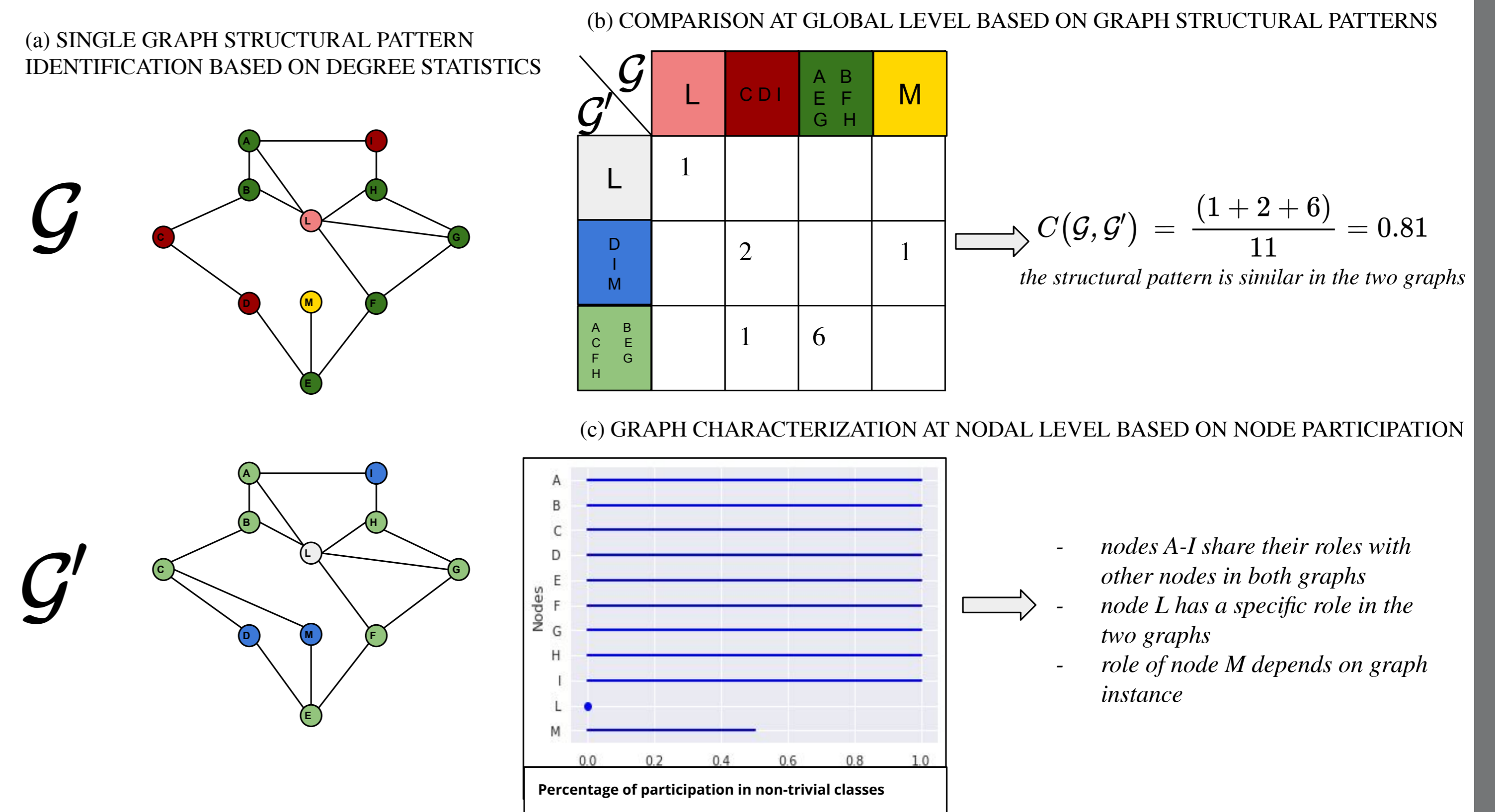
Given a graphs collection  $G = \{\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k) \text{ s.t. } \mathcal{V}_k = \mathcal{V}\}$ , and a statistics collection  $\mathcal{S}$  we count the percentage of participation of each node of  $\mathcal{V}$  in non-trivial classes:

$$\forall v \in \mathcal{V} \quad PP_{\mathcal{G}}^{\mathcal{S}}(v) = PP_{\mathcal{G}}(v) = \frac{|\{\mathcal{G}_k \in G \text{ s.t. } \#[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k} \neq 1\}|}{|G|} \quad (4)$$

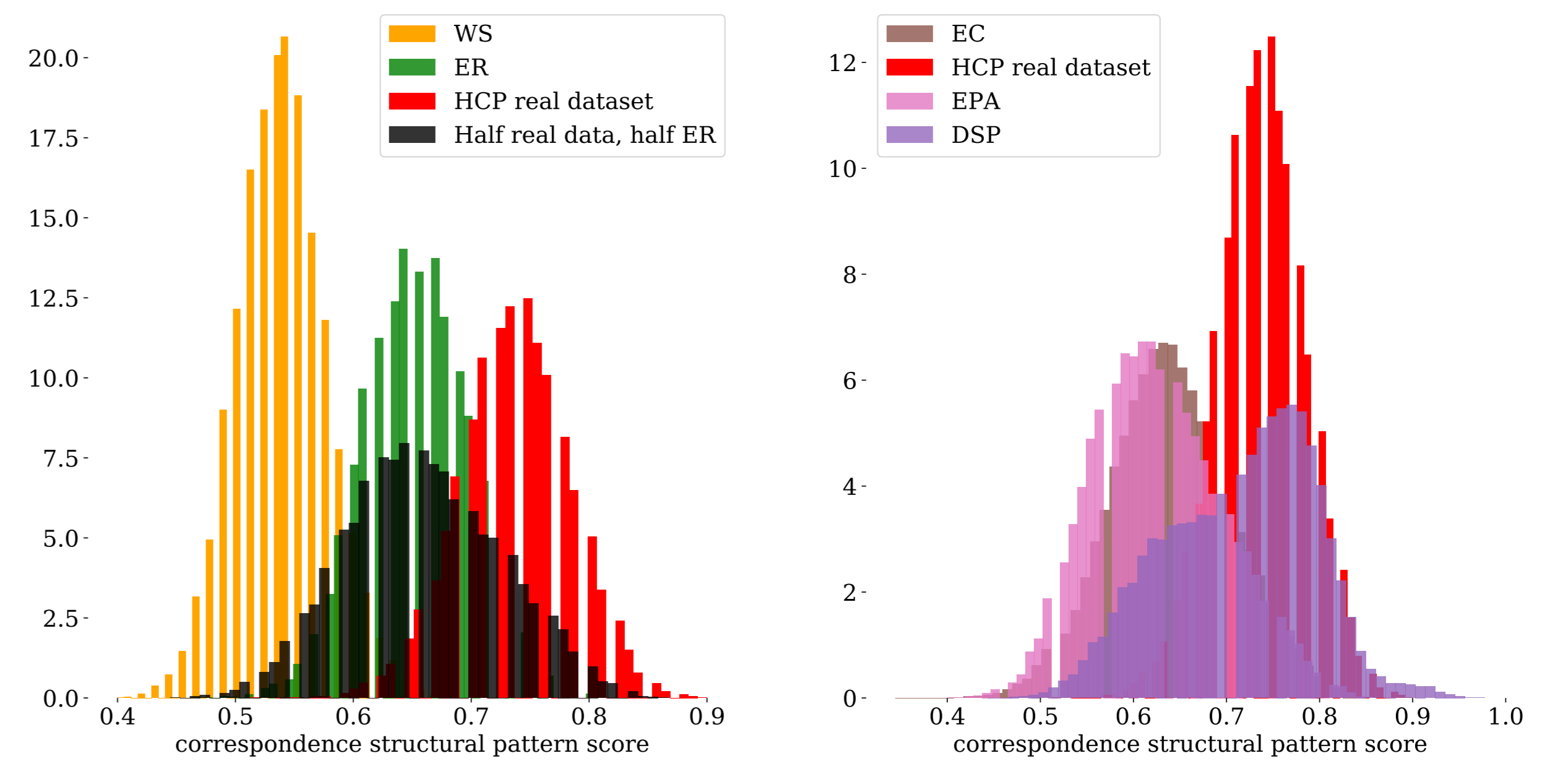
with  $[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k}$  the class of  $v$  in  $\mathcal{G}_k$  in the partition induced by  $\mathcal{S}$ .

## References

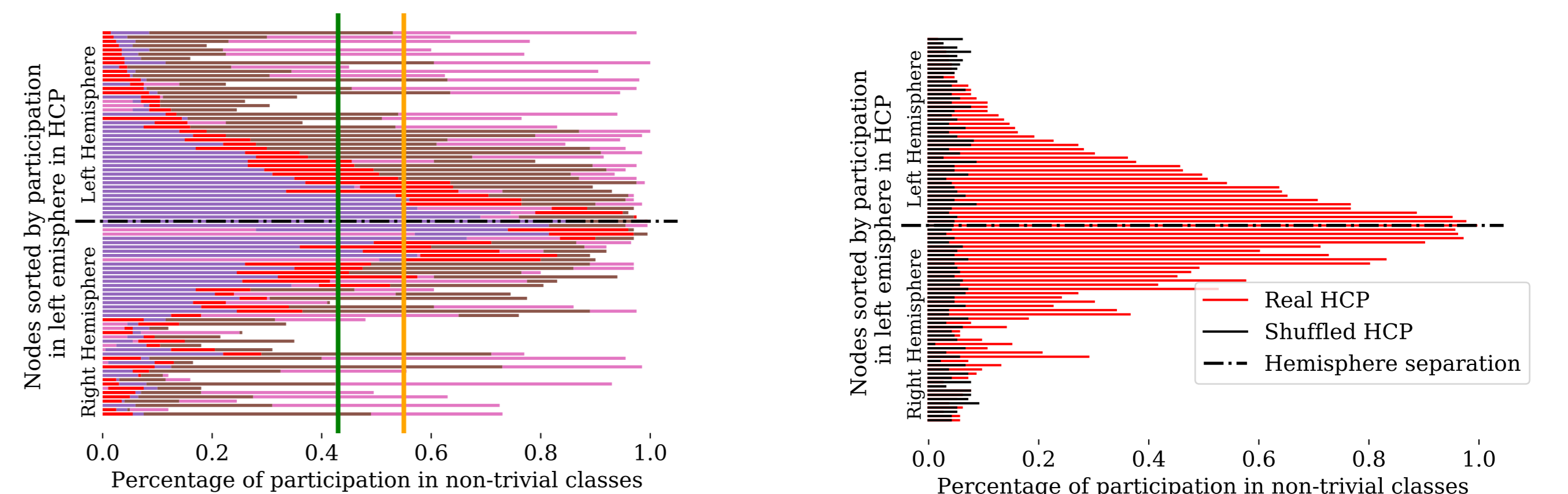
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## Correspondence structural pattern score differentiates models



## Nodal-percentage of participation captures brain homotopy



## Conclusion

New mathematical method for exploration and characterization of **network collections**.

New way to **combine** many graph **nodal** statistics and to associated relative **structural patterns**.

Correspondence structural pattern score to **compare graphs at global level** based on their structural pattern similarity.

Nodal-percentage of participation for nodal level characterization.

Validation on generative models and real clinical data.