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Nodal statistics-based structural pattern detection for graph collections characterization

Lucrezia Carboni^{1,2}

lucrezia.carboni@inria.fr

Michel Dojat¹, Sophie Achard²

¹ Univ. Grenoble Alpes, Inserm, U1216, Grenoble Institut Neurosciences, 38000, FR

² Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LJK, 38000 Grenoble, FR

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<http://arxiv.org/abs/2210.01053>

Motivations

- Human consciousness states can be differentiated by nodal organization in brain functional connectivity networks [1]
- structural pattern definition
- extend node role discovery to graph collections [2, 3]
- graph collections comparison based on structural pattern

Nodal statistics-based structural pattern on single graph

Def. Nodal statistics based equivalence relation.

We consider undirected unweighed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and refer to a nodal statistics $s : \mathcal{V} \rightarrow s(\mathcal{V})$ any function of the adjacency matrix.

The **equivalence relation** \sim_s associated with a nodal statistics s , on the nodes set \mathcal{V} of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The **equivalence relation associated with any collection of statistics** $\mathcal{S} = \{s_i\}_{i=1,\dots,n}$, is defined as:

$$a \sim_{\mathcal{S}} b \iff a \sim_{s_1} b, a \sim_{s_2} b, \dots, a \sim_{s_n} b. \quad (1)$$

Def. Structural Pattern. Its induced partition P on \mathcal{V} ,

$$P_{\mathcal{S}} = \frac{\mathcal{V}}{\sim_{\mathcal{S}}} = \{[a], \forall a \in \mathcal{V}\} \quad (2)$$

defines the **structural pattern** of \mathcal{G} associated with the statistics collection \mathcal{S} .

Def. Node role. The class of equivalence

$[a] = \{b \in \mathcal{V} | a \sim_s b \iff s(a) = s(b)\}$ corresponds to **node role**.

Structural patterns for graph collections characterization

Def. Correspondence structural pattern score. Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices \mathcal{V} and let \mathcal{S} be a statistics collection whose associated partitions are $P_{\mathcal{S}}, P'_{\mathcal{S}}$ on $\mathcal{G}, \mathcal{G}'$ respectively. Given bijective mapping from $P_{\mathcal{S}}, P'_{\mathcal{S}}$ to an initial segment of the natural numbers as enumerations, let $c(v_i), c'(v_i)$ be the enumeration of the classes of v_i , the correspondence structural pattern score between $\mathcal{G}, \mathcal{G}'$ is defined as:

$$C(\mathcal{G}, \mathcal{G}') = \max_{\pi \in \Pi} \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \mathcal{X}(\pi(c(v_i)) = c'(v_i)) \quad (3)$$

where Π is the set of all coupling between the elements in $P_{\mathcal{S}}$ and the elements in $P'_{\mathcal{S}}$ and \mathcal{X} is the indicator function.

Def. Nodal-percentage of participation.

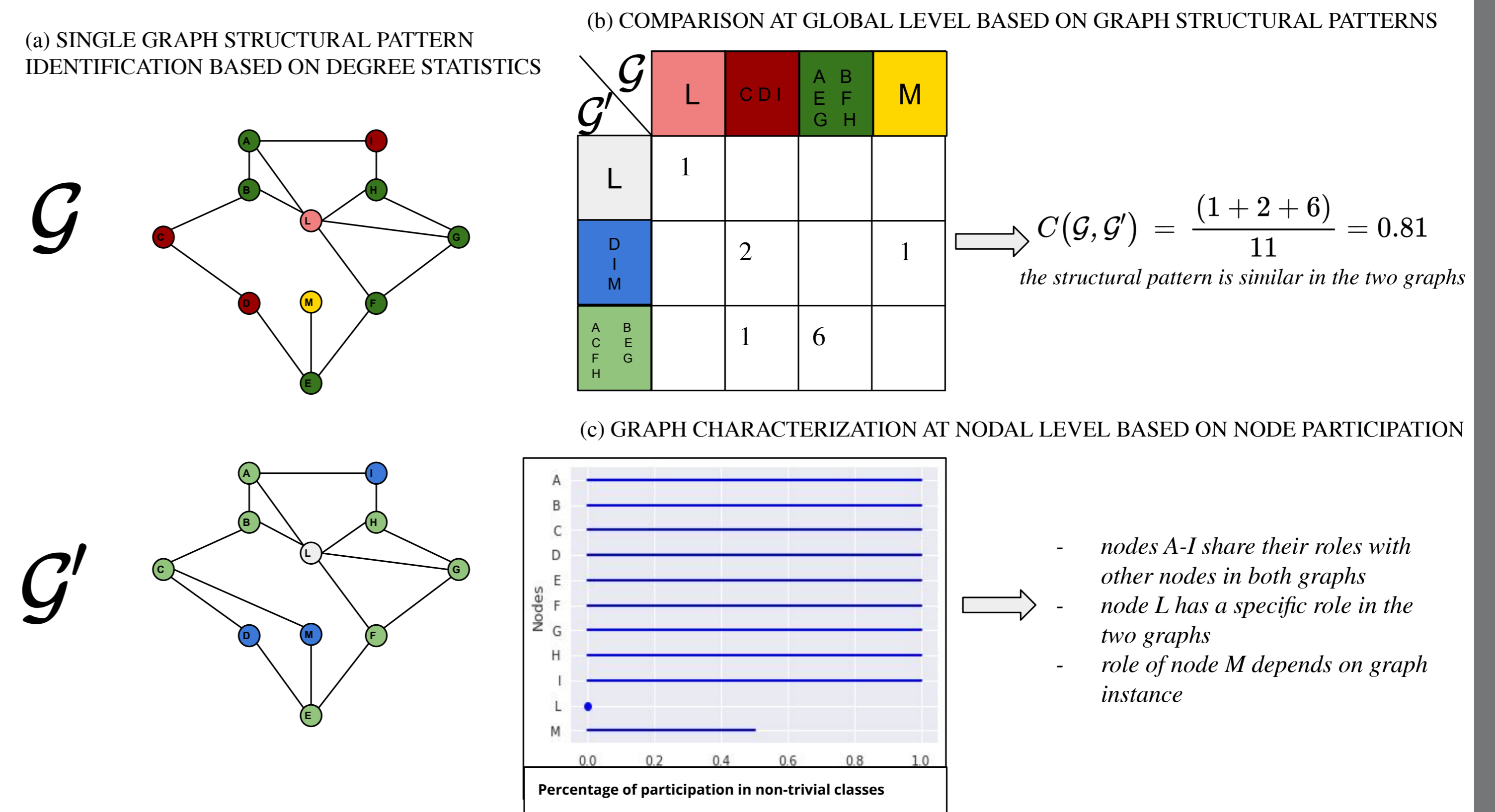
Given a graphs collection $G = \{\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k) \text{ s.t. } \mathcal{V}_k = \mathcal{V}\}$, and a statistics collection \mathcal{S} we count the percentage of participation of each node of \mathcal{V} in non-trivial classes:

$$\forall v \in \mathcal{V} \quad PP_{\mathcal{G}}^{\mathcal{S}}(v) = PP_{\mathcal{G}}(v) = \frac{|\{\mathcal{G}_k \in G \text{ s.t. } \#[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k} \neq 1\}|}{|G|} \quad (4)$$

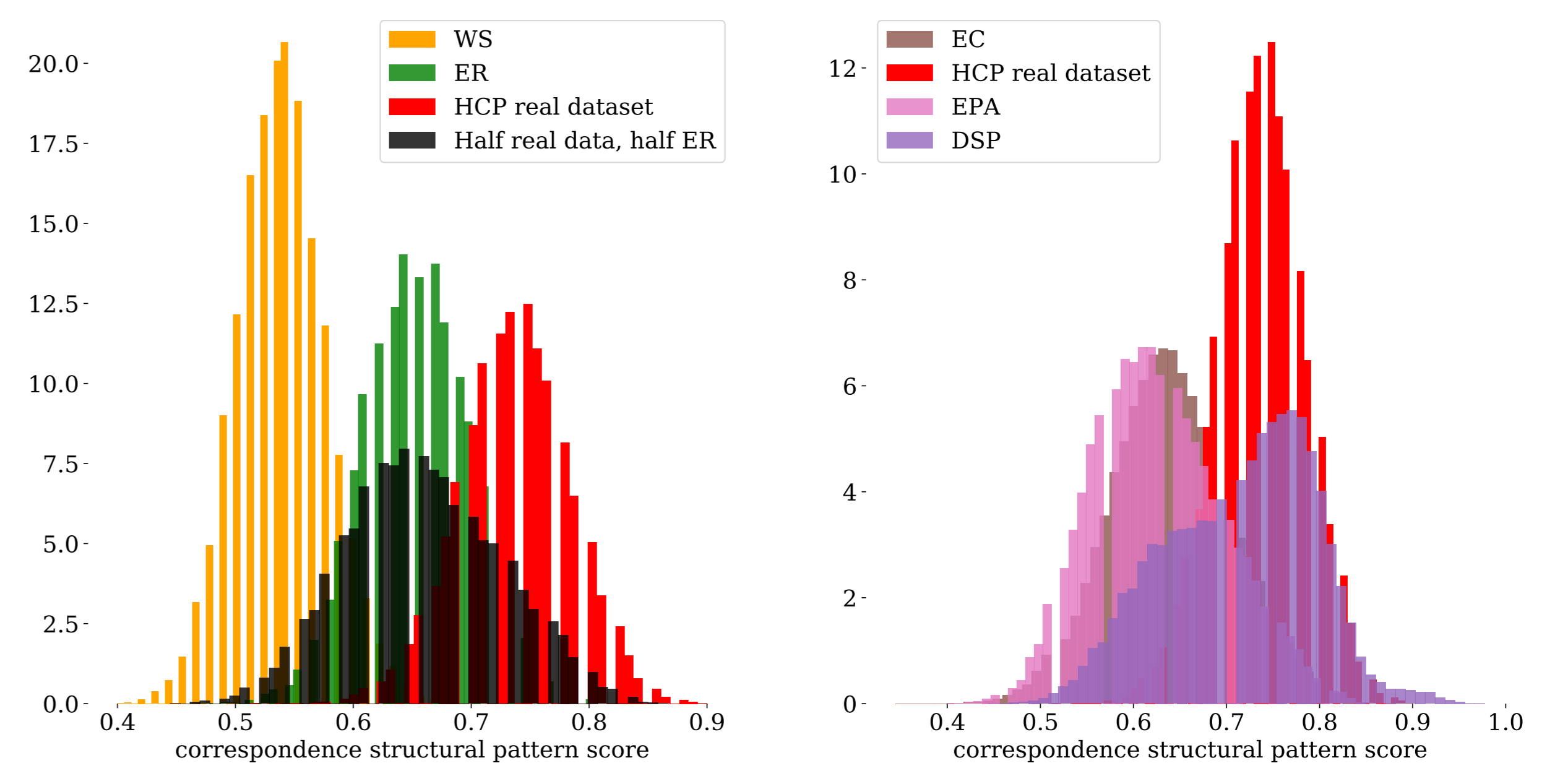
with $[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k}$ the class of v in \mathcal{G}_k in the partition induced by \mathcal{S} .

References

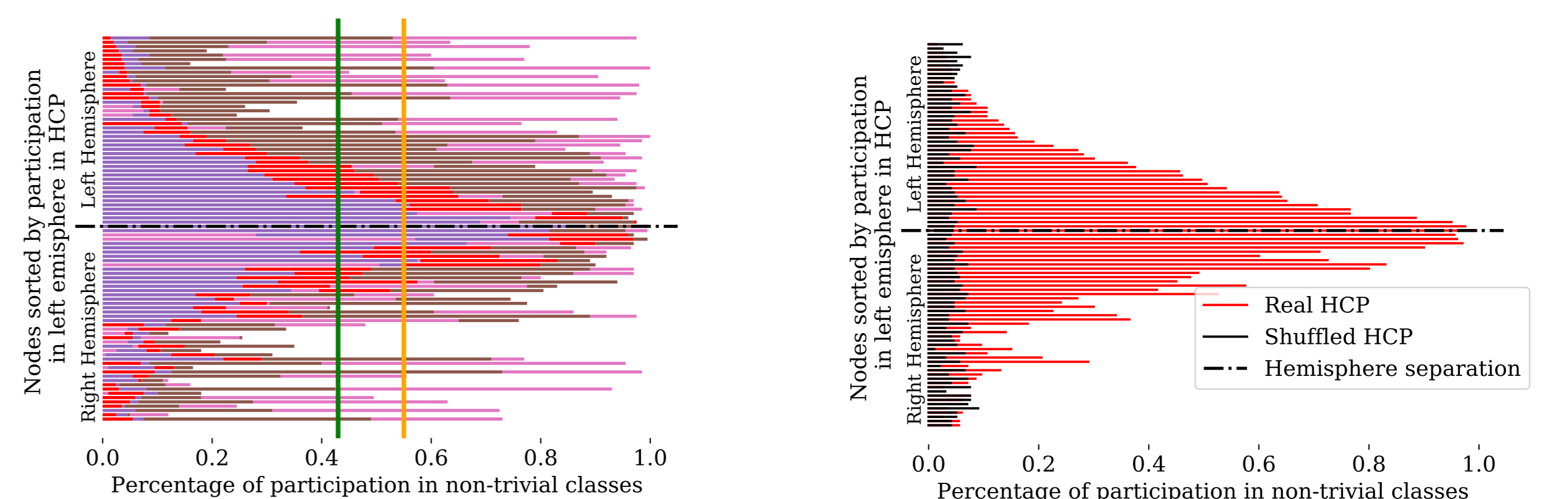
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Correspondence structural pattern score differentiates models



Nodal-percentage of participation captures brain homotopy



Conclusion

New mathematical method for exploration and characterization of **network collections**.

New way to **combine** many graph **nodal** statistics and to associated relative **structural patterns**.

Correspondence structural pattern score to **compare graphs at global level** based on their structural pattern similarity.

Nodal-percentage of participation for nodal level characterization.

Validation on generative models and real clinical data.