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Nodal statistics-based structural pattern detection for graph collections characterization

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Motivations

- Human consciousness states can be differentiated by nodal organization in brain functional connectivity networks [1]
- Structural pattern definition
- Extend node role discovery to graph collections [2, 3]
- Graph collections comparison based on structural pattern

Nodal statistics-based structural pattern on single graph

Def. Nodal statistics based equivalence relation.
We consider undirected unweighted graph $G = (\mathcal{V}, \mathcal{E})$ and refer to a nodal statistics $s : \mathcal{V} \rightarrow s(\mathcal{V})$ any function of the adjacency matrix. The equivalence relation $\sim_s$ associated with a nodal statistics $s$, on the nodes set $\mathcal{V}$ of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The equivalence relation associated with any collection of statistics $S = \{s_j\}_{j=1, \ldots, n}$ is defined as:

$$a \sim S b \iff a \sim_{s_1} b, a \sim_{s_2} b, \ldots, a \sim_{s_n} b. \quad (1)$$

Def. Structural Pattern. Its induced partition $P$ on $\mathcal{V}$,

$$P_S - \frac{\mathcal{V}}{\sim_S} = \{[a], \forall a \in \mathcal{V}\} \quad (2)$$

defines the structural pattern of $G$ associated with the statistics collection $S$.

Def. Node role. The class of equivalence $[a] = \{b \in \mathcal{V} | a \sim_S b\} \iff s(a) = s(b)$ corresponds to node role.

Structural patterns for graph collections characterization

Def. Correspondence structural pattern score. Let $G, G'$ be two graphs having same vertices $\mathcal{V}$ and let $S$ be a statistics collection whose associated partitions are $P_S, P'_S$ on $G, G'$ respectively. Given bijective mapping from $P_S, P'_S$ to an initial segment of the natural numbers as enumerations, let $c(v), c'(v)$ be the enumeration of the classes of $v$, the correspondence structural pattern score between $G, G'$ is defined as:

$$C(G, G') = \max \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \lambda(c(v_i)) = c'(v_i)) \quad (3)$$

where $\Pi$ is the set of all coupling between the elements in $P_S$ and the elements in $P'_S$ and $\lambda$ is the indicator function.

Def. Nodal-percentage of participation. Given a graphs collection $G = \{G_i = (\mathcal{V}_i, \mathcal{E}_i) s.t. \forall i = \mathcal{V}_i\}$, and a statistics collection $S$, we count the percentage of participation of each node of $\mathcal{V}$ in non-trivial classes:

$$\forall v \in \mathcal{V} \quad PP_S(G) = \frac{|\{G_i \in G s.t. #v|^{\mathcal{V}_i}_{\sim_S} \neq 1\}|}{|G|} \quad (4)$$

with $|v|^{\mathcal{V}_i}_{\sim_S}$ the class of $v$ in $G_i$ in the partition induced by $S$.

References


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