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Nodal statistics-based structural pattern detection for graph collections characterization

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<http://arxiv.org/abs/2210.01053>

Motivations

- Human consciousness states can be differentiated by nodal organization in brain functional connectivity networks [1]
- structural pattern definition
- extend node role discovery to graph collections [2, 3]
- graph collections comparison based on structural pattern

Nodal statistics-based structural pattern on single graph

Def. Nodal statistics based equivalence relation.

We consider undirected unweighed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and refer to a nodal statistics $s : \mathcal{V} \rightarrow s(\mathcal{V})$ any function of the adjacency matrix.

The **equivalence relation** \sim_s associated with a nodal statistics s , on the nodes set \mathcal{V} of a graph is:

$$v \sim_s u \iff s(u) = s(v).$$

The **equivalence relation associated with any collection of statistics** $\mathcal{S} = \{s_i\}_{i=1,\dots,n}$, is defined as:

$$a \sim_{\mathcal{S}} b \iff a \sim_{s_1} b, a \sim_{s_2} b, \dots, a \sim_{s_n} b. \quad (1)$$

Def. Structural Pattern. Its induced partition P on \mathcal{V} ,

$$P_{\mathcal{S}} = \frac{\mathcal{V}}{\sim_{\mathcal{S}}} = \{[a], \forall a \in \mathcal{V}\} \quad (2)$$

defines the **structural pattern** of \mathcal{G} associated with the statistics collection \mathcal{S} .

Def. Node role. The class of equivalence

$[a] = \{b \in \mathcal{V} | a \sim_s b \iff s(a) = s(b)\}$ corresponds to **node role**.

Structural patterns for graph collections characterization

Def. Correspondence structural pattern score. Let $\mathcal{G}, \mathcal{G}'$ be two graphs having same vertices \mathcal{V} and let \mathcal{S} be a statistics collection whose associated partitions are $P_{\mathcal{S}}, P'_{\mathcal{S}}$ on $\mathcal{G}, \mathcal{G}'$ respectively. Given bijective mapping from $P_{\mathcal{S}}, P'_{\mathcal{S}}$ to an initial segment of the natural numbers as enumerations, let $c(v_i), c'(v_i)$ be the enumeration of the classes of v_i , the correspondence structural pattern score between $\mathcal{G}, \mathcal{G}'$ is defined as:

$$C(\mathcal{G}, \mathcal{G}') = \max_{\pi \in \Pi} \frac{1}{|\mathcal{V}|} \sum_{i=1}^{|\mathcal{V}|} \mathcal{X}(\pi(c(v_i)) = c'(v_i)) \quad (3)$$

where Π is the set of all coupling between the elements in $P_{\mathcal{S}}$ and the elements in $P'_{\mathcal{S}}$ and \mathcal{X} is the indicator function.

Def. Nodal-percentage of participation.

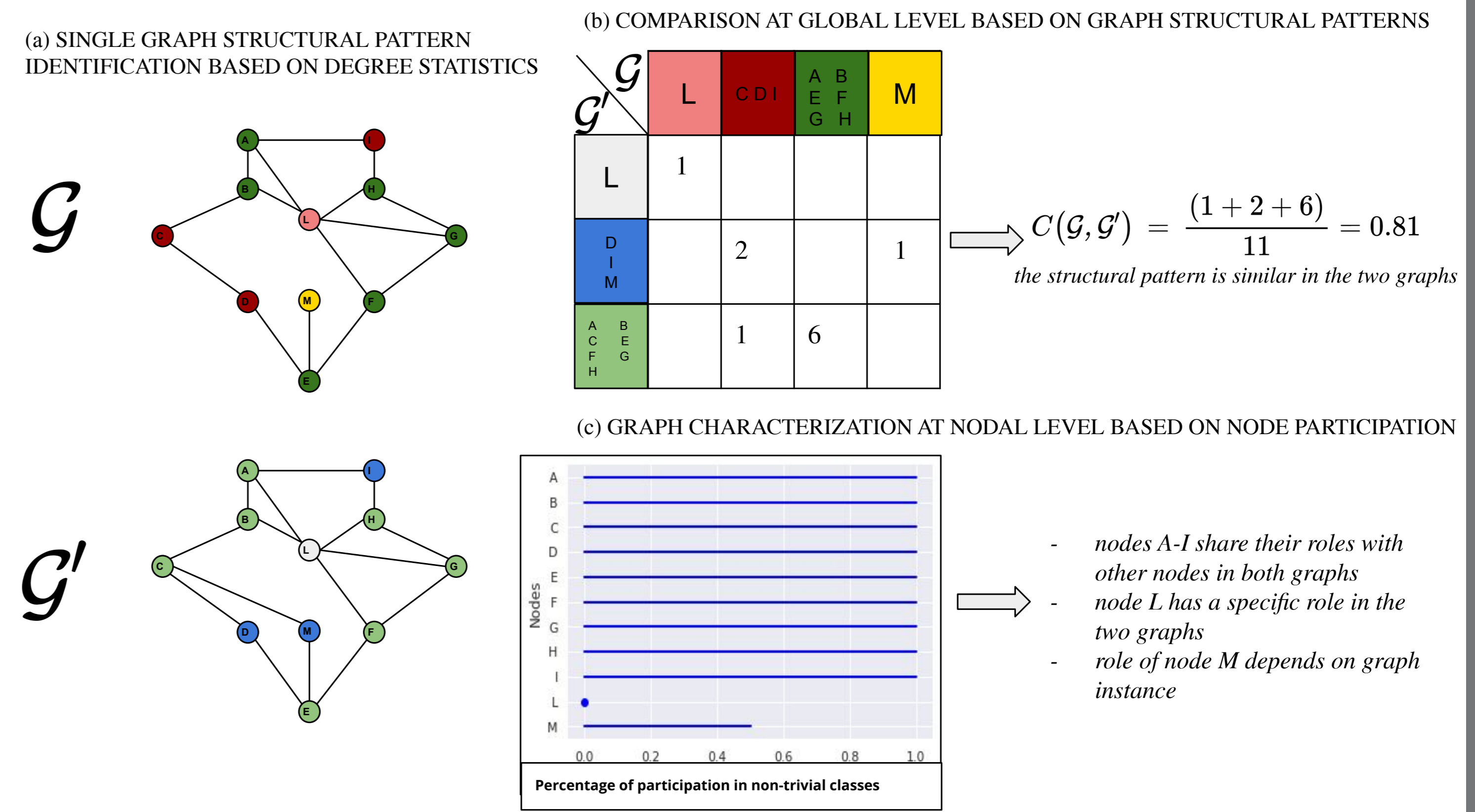
Given a graphs collection $G = \{\mathcal{G}_k = (\mathcal{V}_k, \mathcal{E}_k) \text{ s.t. } \mathcal{V}_k = \mathcal{V}\}$, and a statistics collection \mathcal{S} we count the percentage of participation of each node of \mathcal{V} in non-trivial classes:

$$\forall v \in \mathcal{V} \quad PP_{\mathcal{G}}^{\mathcal{S}}(v) = PP_{\mathcal{G}}(v) = \frac{|\{\mathcal{G}_k \in G \text{ s.t. } \#[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k} \neq 1\}|}{|G|} \quad (4)$$

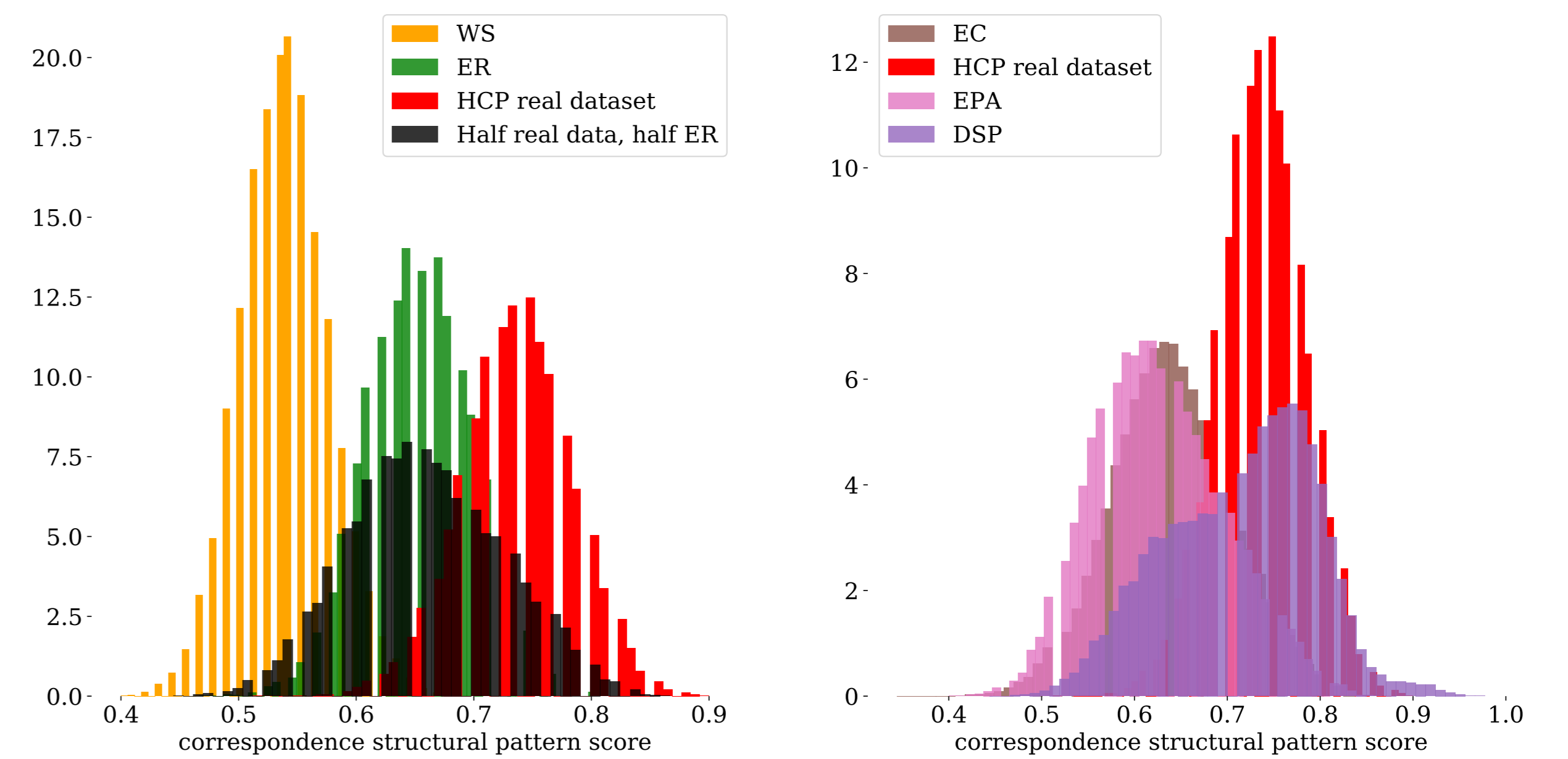
with $[v]_{\sim_{\mathcal{S}}}^{\mathcal{G}_k}$ the class of v in \mathcal{G}_k in the partition induced by \mathcal{S} .

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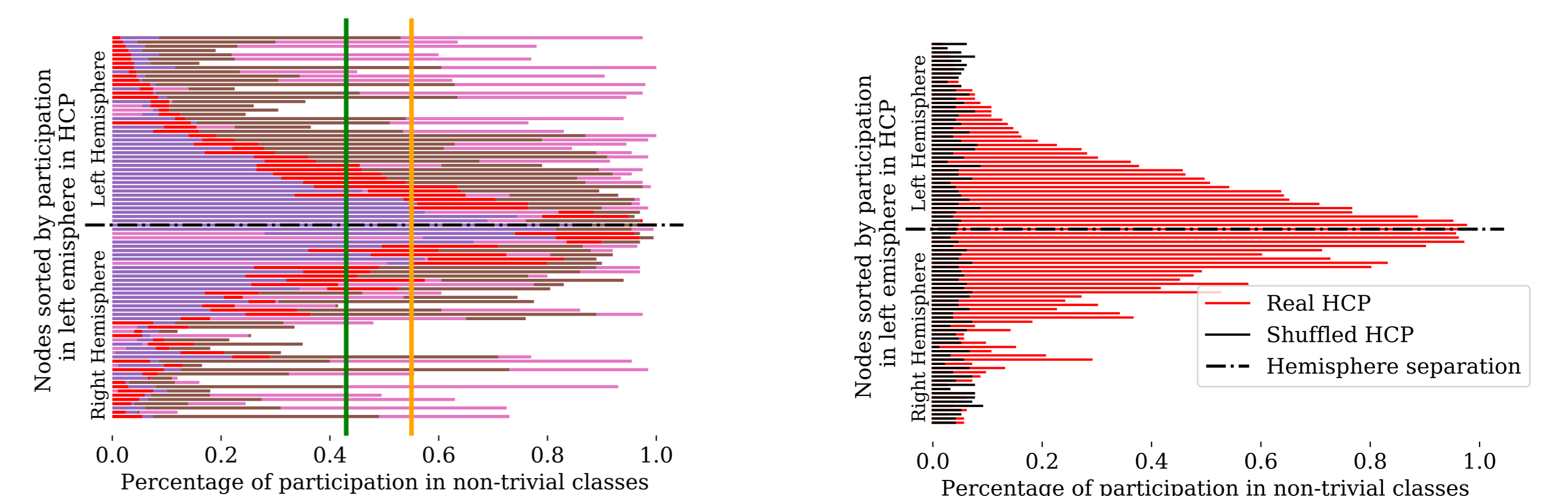


Correspondence structural pattern score differentiates models



(Left) WS: Watt-Strogatz model, ER: Erdős-Rényi model, HCP: Human Connectome Project real networks [4]; (Right) Brain inspired models.[5] DSP: Degree sequence preserving model; EPA: Economical preferential attachment model; EC: Economical clustering model.

Nodal-percentage of participation captures brain homotopy



The symmetry in real data and brain models reveals the expected hemisphere similarity in the participation of analogue regions.

Conclusion

New mathematical method for exploration and characterization of **network collections**.

New way to **combine** many graph **nodal** statistics and to associated relative **structural patterns**.

Correspondence structural pattern score to **compare graphs at global level** based on their structural pattern similarity.

Nodal-percentage of participation for nodal level characterization.

Validation on generative models and real clinical data.