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Novel Composite Clock Algorithm for the Generation of Galileo Robust Timescale

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Biographies

Pedro Roldan works on time and frequency systems and their application to GNSS. He holds a master degree in Aerospace Engineering and in Meteorology and Geophysics, and is currently following PhD studies. He has participated in the design, development, monitoring and operation of the Galileo system, focused on the application of algorithms of time transfer, generation of timescales and steering. Currently, he is in charge of time and frequency activities in Thales Alenia Space.

Sébastien Trilles is an expert in navigation algorithms and performances. He received his Ph.D. degree in Pure Mathematics from the Paul Sabatier University and an Advanced M.S. in Space Technology from SUPAERO. He heads the Performance and Processing department where high precise algorithms are designed as the navigation, integrity and ionosphere modelling algorithms of SBAS systems, the orbitography solutions and the composite time reference generation for Galileo.

Xavier Serena Alos is a senior GNSS system engineer and architect. He holds a master degree in Aerospace Engineering from Universitat Politècnica de Catalunya. Since 2012 he has been involved in the Galileo project holding different roles: from implementation and development of improvements in the orbit determination and time synchronization process (ODTS) and related barriers up to system level engineering activities for the Galileo Second Generation, always focusing on the navigation and performance aspects. Currently, he is managing the engineering of Galileo Mission Segment in Thales Alenia Space.

Abstract

The synchronization of satellite clocks is one of the key functions for the satellite geolocation provided by the navigation systems. This time synchronization is carried out with respect to a system time, which must meet very demanding criteria. Among these criteria, the reference time is usually required to be tightly linked to a physical timescale, such as the International Atomic Time (TAI), to allow its use in timing and metrological applications. It must be also stable enough to allow an accurate predictability of the offsets between satellite clocks and the system time, a key factor for ensuring a good level of performance in the user positioning. In addition, the system time must be always available, since any interruption of its generation or dissemination to the end-users could lead to the unavailability of the system. Finally, this timescale must be resilient to any event that could jeopardize its availability and stability. Generally two techniques are possible for the generation of a system time: either this timescale is given by a single physical clock, and then we speak of a master clock, or it is given as a well-made average of a set of physical clocks, and then we speak of a composite clock or paper time. This paper presents a novel composite clock algorithm designed by Thales Alenia Space, based on the IGS algorithm but modified to maximize the robustness to the typical operational conditions found in the generation of GNSS system times. The integration of this algorithm within the orbit and clock estimation and the navigation message generation functions is also considered, to fully analyze the use case of GNSS system time generation. The performance of the algorithm is finally presented, as well as its behavior in presence of feared events like clock jumps, degraded stability or removal of clocks.

1. INTRODUCTION

Both master clock and composite clock approaches are currently used by GNSS systems, being both able to comply with the very demanding stability and availability levels required by GNSS applications. In the master clock concept, this is granted by the use of very stable master clocks based on hydrogen technologies and the definition of the necessary redundancies to cope with anomalies in the clocks and maintenance and operational activities, while in the paper time concept it is ensured by using composite clock algorithms capable of detecting anomalies in the contributing clocks and adapting the list of participants and their characterization

when anomalies are detected. Algorithms of composite clock allow to combine different clocks to obtain a timescale more stable than any of the input clocks and insensitive to anomalies and gaps in them.

Most algorithms of composite clocks are based on estimating the state of the clocks with a Kalman filter, considering the noises associated to the measurements and the clock states. The time given by the individual clocks is not directly accessible for the composite clock generation, but only the time difference between pairs of clocks. The use of time differences as input measurements to the filter implies that the number of measurements is always smaller than the number of states. If N clocks are considered, at least N different states should be estimated with only $N-1$ independent measurements, obtained as differences $CLK_i - CLK_j$. In case this observability problem is not addressed, the covariance associated to these N states would indefinitely increase and a drift could appear in the estimated states. This can be solved by limiting the covariance to the part associated to the independent measurements, as done in the algorithms of Brown (1992) and Greenhall (2006, 2007), or by introducing constraints as additional observations, making the number of observations equal to the number of states, as done by the algorithms of IGS, version 1 (Senior et al., 2003) and version 2 (Senior & Coleman, 2017).

The differences of clocks used as measurements can be considered in phase, as done by the algorithms of Brown, Greenhall and IGS version 2, or in frequency, as done by IGS version 1. For modelling the clocks, different states can be considered. Generally, the three states of phase, frequency and drift are considered when processing measurements in phase, and two states of frequency and drift are considered when processing measurements in frequency. For the case of IGSv2, an additional state for the total phase, including a white noise, is also included.

In a general problem, the measurements would be those of Equation (1), being phase or frequency depending on the algorithm, while the estimated states would be those of Equation (2), with some of the states potentially set to zero and with the possibility of adding other states depending on the modelling of the clocks.

$$Z_{ij} = (CLK_i - CLK_j)_{ph/fr} \quad (1)$$

$$X_i = \begin{bmatrix} (CLK_i - CC)_{ph} \\ (CLK_i - CC)_{fr} \\ (CLK_i - CC)_{dr} \end{bmatrix} \quad (2)$$

This implies that certain states are directly observable with the measurements, while some others only appear when propagating the observable states. Focusing on the case with processing of phase measurements and estimation of three states, the measurements are directly related to the phase states, while the frequency and drift states are indirectly observed through the propagation step of the Kalman filter. The presence of frequency and drift states increases the stability of the solution in the long-term, since this indirect observability makes the estimation of these states varying more slowly than the estimation of the phase states. Despite this benefit related to the increase of stability, the slower estimation of frequency and drift states may generate an anomalous behavior in the composite timescale in case the frequency and drift of some of the input clocks suddenly change, since it would not be able to properly reflect this change in the estimated states.

This limitation when facing sudden changes in the clocks has been identified and analyzed by Galleani & Tavella (2010), together with other limitations of a Kalman-based estimation of clocks, like the behavior in case of addition and removal of clocks. The estimation of not directly observed states may also produce unexpected behaviors during the convergence of the Kalman filter. As shown by Wright (2007a, 2007b), accumulated errors in the estimation of frequency and drift states during the convergence can lead to an arbitrary offset in frequency and drift in the resulting timescale.

For the application as a GNSS system time, the robustness against operational events shall be the main design driver of a composite clock algorithm. For this reason, the algorithm described in this paper has been designed to mitigate as much as possible these limitations. The non-observable states have been removed with the addition of constraints linking the estimated states to the predicted ones and allowing a change from epoch to epoch in the level of the noise of each particular clock, following the approach of IGS version 2 (Senior & Coleman, 2017). The indirectly observable states have been limited to those cases in which they provide a clear benefit for the modelling of the clocks. To do that, the possibility of adding steered clocks contributing to the composite has been considered, being the state vector for these clocks only composed by phase states, directly observed with the clock differences used as measurements in the filter. To avoid the impact of reconfigurations of the ensemble, all the input clocks are aligned before being

injected in the filter, so that the phase, frequency and drift offset between them is never too large to avoid a smooth transition in the composite timescale. To avoid any dependency with a particular clock, the approach of injecting differences with respect to a single reference clock used in the algorithm of IGS has been here replaced by the injection of a set of differences without any privileged clock. Among the available clock differences, most of them would depend on the others and would not really add new information to the filter. A set of independent differences could be selected instead, according to particular criteria like the residuals of the estimation.

2. ROBUST COMPOSITE CLOCK

The composite clock algorithm considered in this paper contains the steps of prediction and update of the Kalman filter, together with additional functions for preprocessing of measurements and anomaly detection based on the residuals of the estimation. A high level architecture of this algorithm is included in Figure 1.

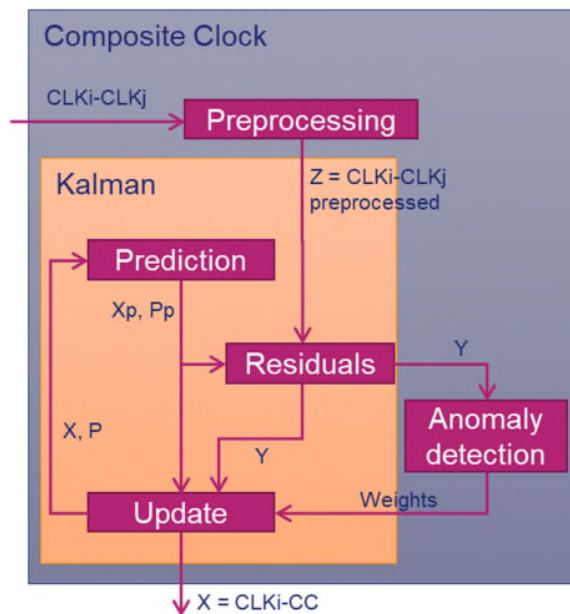


Figure 1 Architecture of robust composite clock

The preprocessing function analyses the input differences to detect and correct clock anomalies and operational events like outliers, phase and frequency jumps. To do that, it compares the clock differences for one epoch with the clock differences predicted from the previous epochs. In case an event is detected, two strategies would be possible: rejecting the clock differences or correcting the event. If the accuracy is the main driver, the strategy of measurement rejection would be very likely the best one, since the correction of events like phase or frequency jumps could be not accurate enough to allow for an accurate estimation of the clocks in the filter. However, this strategy could degrade the availability, especially in scenarios with a large amount of events. If the availability is the main driver or there are too frequent events, the option of correcting the events and include the corrected measurements in the filter could provide better results.

Even if the algorithm considers constraints to limit the change in the composite timescale from one epoch to the next one, in case very large phase, frequency or drift offsets exists among the different clocks, a significant discontinuity may appear in case clocks are added or removed from the ensemble. To mitigate this problem, an alignment of the input clocks is also performed in the preprocessing step, being its goal not to accurately model the input clocks, but only to ensure that the differences between them do not exceed what can be absorbed by the Kalman filter constraints.

In the prediction step, the state vector of Equation (2) and the associated covariance matrix P are propagated a time step τ according to the Equation (3). In this equation, the transition matrix F is obtained by composing the transition matrices of each clock F_i , as per Equation (4), and the process noise matrix Q is generated with the matrices of each clock Q_i , following Equation (5).

$$\begin{aligned} X &= FX \\ P &= FPF' + Q \end{aligned} \quad (3)$$

$$F_i = \begin{bmatrix} 1 & \tau & \tau^2/2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$Q_i = \begin{bmatrix} q1_i \cdot \tau + q2_i \cdot \frac{\tau^3}{3} + q3_i \cdot \frac{\tau^5}{20} & q2_i \cdot \frac{\tau^2}{2} + q3_i \cdot \frac{\tau^4}{8} & q3_i \cdot \frac{\tau^3}{6} \\ q2_i \cdot \frac{\tau^2}{2} + q3_i \cdot \frac{\tau^4}{8} & q2_i \cdot \tau + q3_i \cdot \frac{\tau^3}{3} & q3_i \cdot \frac{\tau^2}{2} \\ q3_i \cdot \frac{\tau^3}{6} & q3_i \cdot \frac{\tau^2}{2} & q3_i \cdot \tau \end{bmatrix} \quad (5)$$

The noises $q1_i$, $q2_i$ and $q3_i$ respectively represent the contributions of Random Walk PM, Random Walk FM and Random Walk DM to the clock noise. These noises can be obtained based on the Hadamard variance of the clocks, following the approach in Hutsell (1995).

With the predicted state, the pre-fit residuals Y can be computed as in Equation (6). In this equation, the measurements Z are compared against the reconstructed measurements, obtained by applying the design matrix H to the state vector X . To solve the observability problem derived from the estimation of N clocks with $N-1$ clock differences, the estimated state is constrained to the predicted one, following the approach in Senior & Coleman (2017). Taking this into account, the vector of measurements Z is composed by the clock differences themselves Z_{ij} and additional rows for the constraints Z_{const} , as per Equation (7). In the same way, the design matrix H contains one row H_{ij} for each clock difference included in the filter and additional rows H_{const} for the constraints, as shown in Equation (8).

$$Y = Z - HX \quad (6)$$

$$Z_{ij} = (CLK_i - CLK_j)_{ph}$$

$$Z_{const} = \begin{bmatrix} \sum_{k=0}^n w_{k,ph} \cdot (CLK_k - CC)_{ph,pred} \\ \sum_{k=0}^n w_{k,fr} \cdot (CLK_k - CC)_{fr,pred} \\ \sum_{k=0}^n w_{k,dr} \cdot (CLK_k - CC)_{dr,pred} \end{bmatrix} \quad (7)$$

$$H_{ij} = [\dots \ 1 \ 0 \ 0 \ \dots \ -1 \ 0 \ 0 \ \dots]$$

$$H_{const} = \begin{bmatrix} w_{1,ph} & 0 & 0 & \dots & w_{N,ph} & 0 & 0 \\ 0 & w_{1,fr} & 0 & \dots & 0 & w_{N,fr} & 0 \\ 0 & 0 & w_{1,dr} & \dots & 0 & 0 & w_{N,dr} \end{bmatrix} \quad (8)$$

These constraints allow to define the evolution of the composite timescale, by constraining a weighted average of the estimated states $(CLK_k - CC)_{ph}$, $(CLK_k - CC)_{fr}$ and $(CLK_k - CC)_{dr}$ to a weighted average of the predicted states $(CLK_k - CC)_{ph,pred}$,

$(CLK_k - CC)_{fr,pred}$ and $(CLK_k - CC)_{dr,pred}$. Those clocks with a larger weight in the constraint will have a larger contribution to the evolution of the composite timescale. This approach, even if strongly dependent on the selection of weights, allows to easily remove the contribution of a particular clock to the composite timescale by reducing its weight. It is therefore a suitable approach for an operational application like a GNSS system time, in which it shall be possible to totally remove the contribution of clocks impacted by anomalies or involved in operational and maintenance activities.

The residuals are computed for all the available clock differences $CLK_i - CLK_j$, but the processing of all these clock differences in the composite clock would not be useful to improve the estimation of the clock states, since they provide redundant information. For example, if the measurements Z_{ij} , Z_{ik} and Z_{jk} are included in the Kalman filter, only two of them would efficiently contribute to the estimation, since the third one could be obtained from the others as $Z_{jk} = Z_{ik} - Z_{ij}$, potentially with a different measurement noise but with the same link to the estimated parameters. The residuals of the clock differences could be used then as a criterion to select those differences that better allow to characterize the behavior of the clocks, and only include these differences in the update step of the Kalman filter.

In the update step, the Kalman gain K is obtained based on the predicted covariance P and the measurement noise R . The residuals Y of a set of independent measurements are used then to update the state vector X and the associated covariance, according to Equation (9).

$$\begin{aligned} S &= HPH' + R \\ K &= PH'S^{-1} \\ X &= X + KY \\ P &= (I - KH)P(I - KH)' + KKK' \end{aligned} \quad (9)$$

The measurement noise matrix R can be obtained by composing the measurement noise of each measurement R_{ij} and the measurement noises associated to the constraints R_{const} . Since the constraints are based on a weighted average of the predicted states, a weighted average of the associated covariance could be a good choice, as described in Equation (10).

$$R_{const} = \begin{bmatrix} R_{ij} = q0_i + q0_j & & & \\ \sum_{k=0}^n w_{k,ph} \cdot P_{kk,ph} & 0 & & 0 \\ 0 & \sum_{k=0}^n w_{k,fr} \cdot P_{kk,fr} & & 0 \\ 0 & 0 & \sum_{k=0}^n w_{k,dr} \cdot P_{kk,dr} & \end{bmatrix} \quad (10)$$

The noise of the measurements R_{ij} could be modelled as a white noise $q0_i$ and $q0_j$, even if additional contributions may be needed, depending on the source of the measurements, to take into account the noise introduced by the measuring system.

The residuals can be also used as a second barrier to detect anomalies that have not been detected in the preprocessing. For this purpose, the residual of the measurement Y_{ij} is normalized with the covariance of the clocks $P_{ii,ph}$ and $P_{jj,ph}$ and the measurement noise $q0_i + q0_j$ to obtain the normalized residual N_{ij} , as per Equation (11).

$$N_{ij} = \frac{Y_{ij}}{\sqrt{P_{ii,ph} + P_{jj,ph} + q0_i + q0_j}} \quad (11)$$

Those clock differences showing a large normalized residual are rejected, and in case several differences are rejected for the same clock, the clock is excluded from the filter. In a first step, the clock is de-weighted, by progressively reducing its weight in the constraints of Equations (7), (8) and (10). If the clock is not recovered, it is de-correlated from the other clocks by removing the crossed terms of the covariance matrix and using the clock differences involving that clock only to estimate its own state. Like this,

the estimation of the state of the other clocks is not contaminated by the differences between them and the anomalous clock. If the situation is not automatically recovered, the state vector and the covariance matrix for the anomalous clock are finally reinitialized, allowing for a new estimation from scratch.

Apart from the presence of anomalies, other factors like the noise and the availability of data also contribute to define the weight of the clock, as shown in Equation (12). The weights are obtained as the inverse of the process noise for each state, combined with multiplicative factors to reduce the weight in case of data gaps or anomalies.

$$\begin{aligned}
 w_{i,ph} &= \frac{Avail_i \cdot Anom_i}{q1_i \cdot \tau + q2_i \cdot \frac{\tau^3}{3} + q3_i \cdot \frac{\tau^5}{20}} \\
 w_{i,fr} &= \frac{Avail_i \cdot Anom_i}{q2_i \cdot \tau + q3_i \cdot \frac{\tau^3}{3}} \\
 w_{i,dr} &= \frac{Avail_i \cdot Anom_i}{q3_i \cdot \tau}
 \end{aligned} \tag{12}$$

The factors $Avail_i$ and $Anom_i$ would be 1 in case the clock has a nominal behaviour, and would smoothly reduce in case no more data is available for a particular clock, or in case an anomaly is detected when analysing its residuals.

3. PROPOSED INTEGRATION IN GALILEO

The use of a composite clock as a GNSS system time can be put in place by setting it as the reference time of the Orbit Determination and Time Synchronization (ODTS) process that generates the predicted orbits and clocks included in the navigation message of the system. To do that, the composite clock could be integrated with the ODTS, in a tightly coupled approach, or could be done separately from the ODTS, in a loosely coupled approach. The tightly coupled approach would be optimized in terms of performance, since both the composite clock and the ODTS are included in the same estimator. However, this solution may be not optimal from a robustness point of view. The loosely couple approach would allow for a more modular design, that could prevent the propagation of errors from the ODTS to the composite clock and from the composite clock to the ODTS. For this reason, a loosely coupled approach, in which the ODTS and the composite clock are executed in totally independent modules, have been selected. With this approach, the ODTS can be also independently configured to use as reference time the composite clock, or to switch to a master clock, for example in case the composite clock needs to be reinitialized.

If the composite clock is executed outside the ODTS, an alternative source of clock differences would be needed. The use of predicted clocks from the ODTS could be considered, but to avoid a degradation of the composite clock performance the use of predictions at a very short time would be needed, compromising the separation of composite clock and ODTS modules. An additional module of time transfer is therefore considered, that allows to perform a preliminary estimation of the clock difference between satellite and station clocks independent from the ODTS. This module uses a GNSS time transfer with Galileo satellites, based on the same GNSS observations used in the ODTS and fixing the orbits to those predicted by the ODTS. Even if the use of predicted orbits from the ODTS introduces a dependency between composite clock and ODTS, the validity of the predicted orbits is much longer than that of the predicted clock, so in case the quality of the new predicted orbits cannot be ensured, the previous ones could be kept.

This logics is summarized in Figure 2. The observations from a station network and the predicted orbits of the ODTS are used to perform time transfer and obtain the differences $CLK_i - CLK_j$, including differences between satellite clocks, between station clocks and between satellite and station clocks. These differences are injected as measurements in the composite clock, that estimates models for the differences $CLK_i - CC$. These model are used by the ODTS to refer the estimated orbits and clocks to the composite timescale. Finally, the predicted orbits and clocks from the ODTS are used to generate the navigation message broadcast to the users.

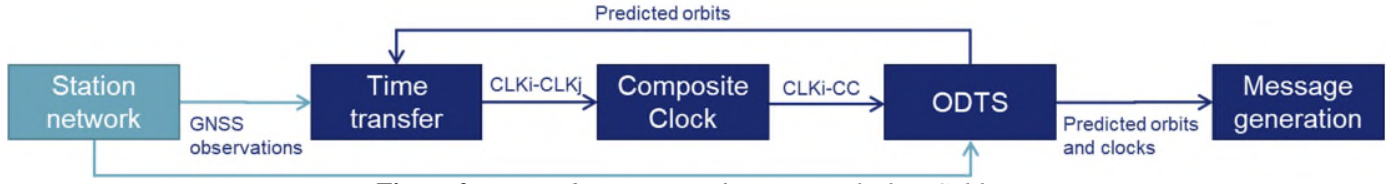


Figure 2 Proposed integration of composite clock in Galileo

In a master clock mode, the reference time in the ODTS is defined by setting one of the estimated clocks $CLK_{\text{master}} - ODTS_{\text{ref}}$ to zero, as per Equation (13). In a composite clock mode, the reference time would be defined by applying constraints to all the estimated clocks, as per Equation (14), so that the estimation of $CLK_i - ODTS_{\text{ref}}$ performed in the ODTS is similar to the values of $CLK_i - CC$ provided by the composite clock.

$$CLK_{\text{master}} - ODTS_{\text{ref}} = 0 \quad (13)$$

$$(CLK_i - ODTS_{\text{ref}}) - (CLK_i - CC) = 0 \quad (14)$$

4. EXPERIMENTATION RESULTS

The composite clock algorithm proposed in Section 2 and the integration within a GNSS system proposed in Section 3 have been tested with data from a station network of 16 stations for the whole year 2020. Even if the observations from 16 stations have been considered for the time transfer, the ensemble of the composite clock only contained the Galileo satellite clocks and two stations, equipped with hydrogen masers and steered to UTC.

As discussed in Section 1, to reduce as much as possible the indirectly observed states, the station clocks have been modelled only with a phase state, being their frequency and frequency drift removed by the steering. For the satellite clocks, not steered, the three states of phase, frequency and frequency drift have been considered.

This scenario includes several operational events, like the outages in the satellite E19 in March (NAGU 2020006) and June (NAGU 2020012), in the satellite E24 in September (NAGU 2020015), and in the satellite E11 in November (NAGU 2020017). For the station clocks, a jump has been considered in one of the stations (STA1) in February, while the other station (STA2) was impacted by a gap in November and a jump in December.

Figure 3 shows the Allan Deviation of the clocks resulting from this experimentation. It can be observed that the stability of stations is well in line with the stability of the hydrogen masers, showing that the composite clock is not degrading the stability of the differences $CLK_i - CC$.

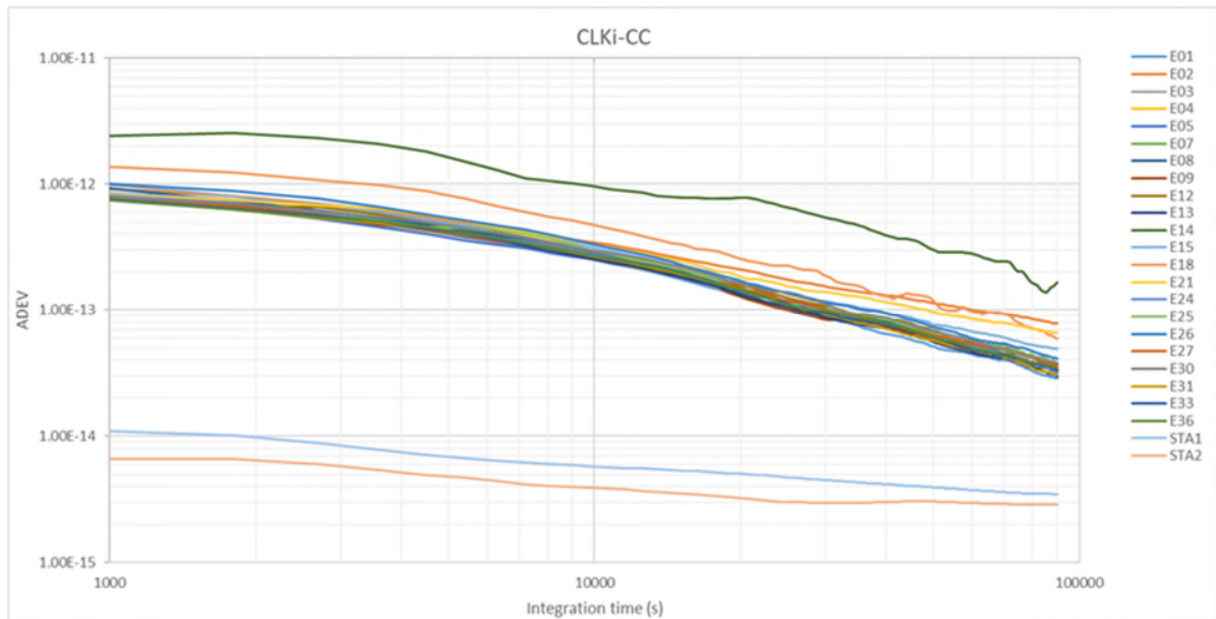


Figure 3 Allan Deviation of the clock differences CLK_i-CC

The stability of the station clocks is much better than that of satellite clocks, leading to a larger weight for the stations, as shown in Figure 4. The weight associated to the satellite clocks is only relevant in case of gap or jump in one of the stations, like in February, November and December. In those cases, the weight of the other station was limited to 0.7, to prevent that a single clock drives the composite timescale.

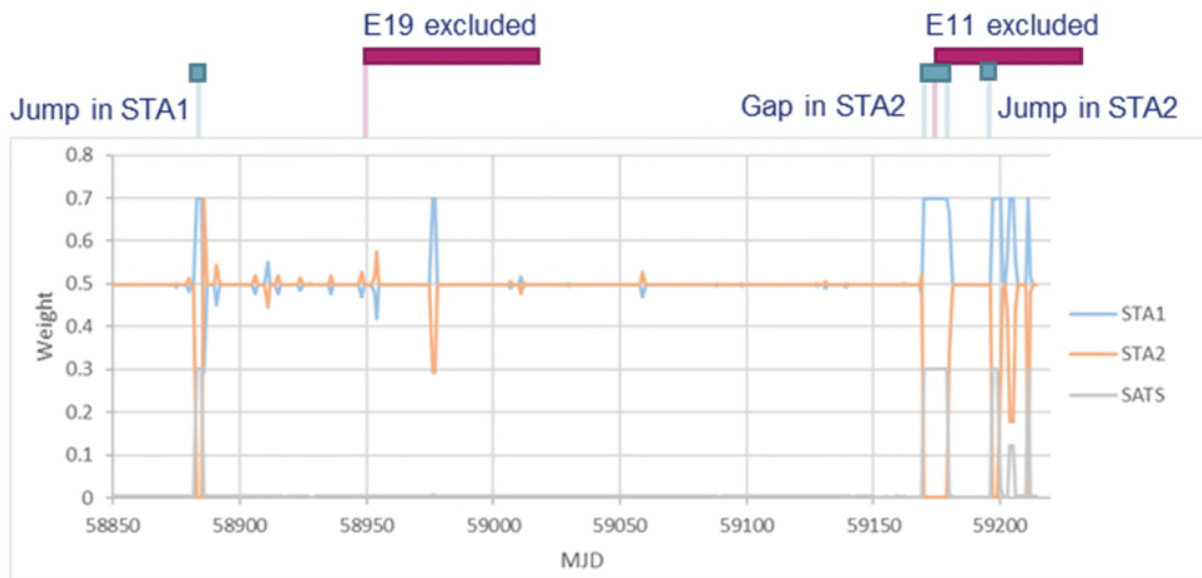


Figure 4 Operational events during 2020 and weights of station and satellite clocks

This strategy of weighting allowed to obtain a continuous and stable composite timescale, despite the events in the station and the satellite clocks. Figure 5 shows that the composite timescale remained aligned to the station clocks within 5ns during the whole year. For the periods when both stations were available, the composite timescale remained close to the average of the station clocks. In

November, when one of the stations was impacted by a large gap, the composite clock smoothly transitioned to a different phase, in line with the remaining station.



Figure 5 Time series of station clocks with respect to CC

The jumps in the station clocks were properly detected and corrected, and no impact on the composite timescale was observed. Similarly, the exclusion of some satellites like the E19 and the E11, and the frequency jumps in the satellite clocks, as those observed in Figure 6, did not lead to any alteration of the composite timescale.

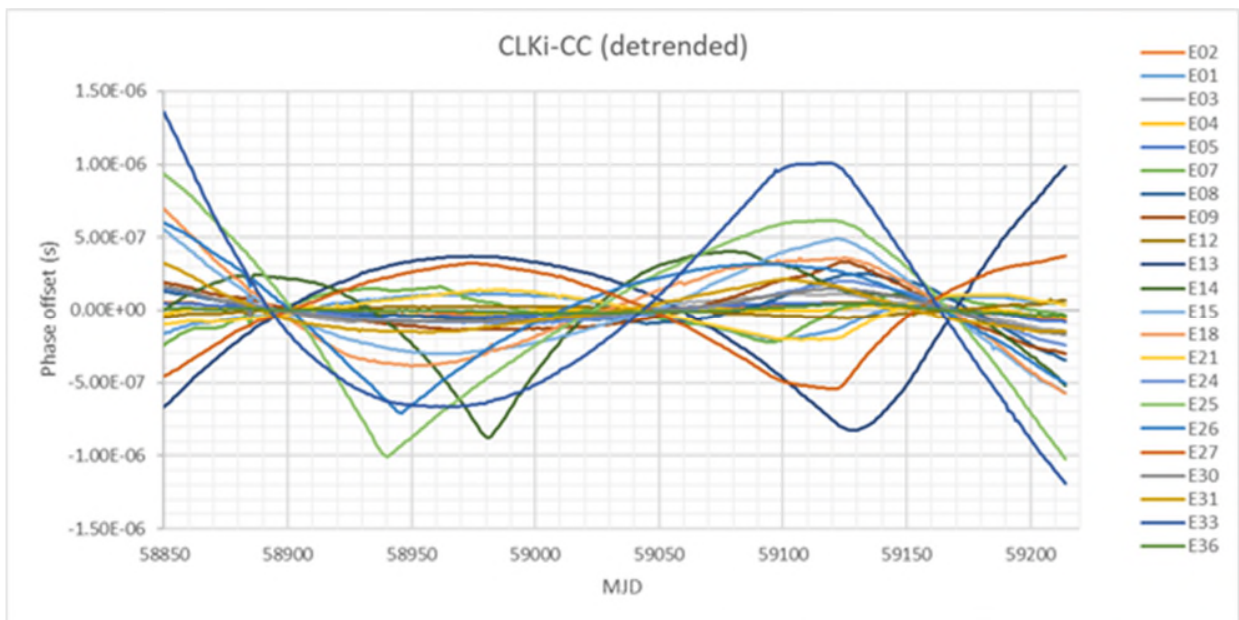


Figure 6 Time series of Galileo satellite clocks with respect to CC, after removing a quadratic model

5. CONCLUSIONS

An algorithm of composite clock based on a Kalman filter can be a good solution for generating a GNSS system time. However, mechanisms to ensure the robustness of the algorithm in case of operational events, like gaps, outliers or jumps in the contributing clocks, are required. The proposed solution, based on:

- the processing of a set of independent clock differences without any privileged clock,
- the addition of constraints in the filter following the IGS approach to remove the non-observable states,
- the use of indirectly observable states only for those clocks that require them,
- the detection and correction of outliers and jumps in a preprocessing step,
- the alignment of input clocks to allow for smooth reconfigurations of the composite clock, and
- the anomaly detection based on the residuals of the estimation,

is able to provide good results in a long-term scenario representative of the real operation of a GNSS system, using the satellite clocks and a limited set of station clocks.

The integration with the other functions of a GNSS system in a modular way allows to isolate as much as possible any potential problem, increasing the robustness of the system. It also allows to keep both the composite clock and the master clock modes, and to be able to switch from one to the other only by changing the constraints in the ODTs.

From a performance point of view, the stability of the composite timescale can be better than that of the most stable clocks, and in any case much better than that of the satellite clocks. This implies that in case the composite timescale is used as reference time for the broadcast satellite clocks, the predictability of the offsets would be driven by the predictability of the satellite clocks, minimizing the ranging error at user level.

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