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Nonlinear wave attenuation in strongly-confined falling liquid films sheared by a laminar counter-current gas flow

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Abstract:	<p>Experiments are conducted in water films falling along the bottom wall of a weakly-inclined rectangular channel of height 5mm in the presence of a laminar counter-current air flow. Boundary conditions have been specifically designed to avoid flooding at the liquid outlet, thus allowing to focus on the wave dynamics in the core of the channel. Surface waves are excited via coherent inlet forcing before they come into contact with the air flow. The effect of the air flow on the height, shape, and speed of two-dimensional travelling nonlinear waves is investigated and contrasted with experiments of Kofman (Int. J. of Multiphase Flow, 2017, vol. 95) which were performed in a weakly-confined channel. We observe a striking difference between these two cases.</p> <p>In our strongly-confined configuration, a monotonic stabilizing effect or a non-monotonic trend (i.e. the wave height first increases and then diminishes upon increasing the gas flow rate) is observed, in contrast to the weakly-confined configuration where the gas flow is always destabilizing. This stabilizing effect implies the possibility of attenuating waves via the gas flow and it confirms recent numerical results obtained by Laval (J. Fluid Mech. Rapids, 2021, vol. 919) in a superconfined channel.</p>

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Nonlinear wave attenuation in strongly-confined falling liquid films sheared by a laminar counter-current gas flow

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Experiments are conducted in water films falling along the bottom wall of a weakly-inclined rectangular channel of height 5 mm in the presence of a laminar counter-current air flow. Boundary conditions have been specifically designed to avoid flooding at the liquid outlet, thus allowing to focus on the wave dynamics in the core of the channel. Surface waves are excited via coherent inlet forcing before they come into contact with the air flow. The effect of the air flow on the height, shape, and speed of two-dimensional travelling nonlinear waves is investigated and contrasted with experiments of Kofman *et al.* (Int. J. of Multiphase Flow, 2017, vol. 95) which were performed in a weakly-confined channel. We observe a striking difference between these two cases. In our strongly-confined configuration, a monotonic stabilizing effect or a non-monotonic trend (i.e. the wave height first increases and then diminishes upon increasing the gas flow rate) is observed, in contrast to the weakly-confined configuration where the gas flow is always destabilizing. This stabilizing effect implies the possibility of attenuating waves via the gas flow and it confirms recent numerical results obtained by Lavallo *et al.* (J. Fluid Mech. Rapids, 2021, vol. 919) in a superconfined channel.

1. Introduction

We consider a falling liquid film in contact with a laminar counter-current gas flow within a narrow rectangular channel inclined at an angle β (figure 1). This configuration can be considered as a prototype for compact devices found in engineering applications, such as reflux condensers and distillation columns with structured packings (Valluri *et al.* 2005). It is well known that the interaction between the wavy film and the gas flow can promote flooding events leading to a deterioration in the process performance. Depending on the geometry and/or ratio of liquid and gas flow rates (or superficial velocities) we can observe film atomization (Zapke & Kröger 2000), wave or liquid flow reversal (Tseluiko & Kalliadasis 2011), or obstruction of the channel (Vlachos *et al.* 2001). In order to prevent or delay such events, it is crucial to characterize the dynamics of interfacial waves and in particular their linear and nonlinear responses to the gas flow.

Many theoretical and experimental works dedicated to the linear stability of falling films are reported in the literature since the seminal work of Kapitza (1948) on the

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41 different wavy flow regimes. The linear stability analysis of a liquid film in a passive
42 atmosphere was initiated by Benjamin (1957) and Yih (1963). They found that the film
43 is unstable to long-wave disturbances when the Reynolds number is greater than $5/6 \cot \beta$
44 through an inertia-driven mechanism. This threshold for the so-called Kapitza instability
45 was confirmed experimentally by Liu *et al.* (1993). Recent theoretical investigations
46 (Lavalle *et al.* 2019) have shown that the impact of the gas phase on the linear stability
47 threshold becomes non negligible in weakly inclined and/or strongly confined channels by
48 causing a stabilizing effect and ultimately the full suppression of the Kapitza instability.
49 This linear stabilization due to confinement was confirmed experimentally in Lavalle
50 *et al.* (2019) for a water film in contact with quiescent air in a slightly inclined channel
51 ($\beta = 1.69^\circ$) of height $H = 5$ mm. The main ingredient involved in the stabilization
52 mechanism was found to be the tangential viscous stress exerted by the gas on the film
53 surface, as suggested by Tilley *et al.* (1994). The strong stabilizing effect induced by a
54 counter-current air flow on a wavy water film is reproduced by Trifonov (2019) at small
55 inclination and strong confinement. Kushnir *et al.* (2021), via a comprehensive parametric
56 study, have recently established that regimes of linear stabilization in the case of a zero
57 net gas flow occur for channel heights $H \leq 10$ mm at $\beta = 1^\circ$ and $H \leq 2$ mm at $\beta = 10^\circ$
58 in water/air system.

59 Furthermore, when the gas flows counter-currently, it was shown in Lavalle *et al.* (2019)
60 that the linear stabilization is intensified over the entire range of unstable wave numbers
61 when the confinement is sufficiently strong. For intermediate confinement levels, the
62 linear effect of the gas flow can become non-monotonic, i.e. the cut-off wave number of
63 the Kapitza instability first decreases and then increases with the gas flow rate (Vellingiri
64 *et al.* 2015; Trifonov 2017; Lavalle *et al.* 2019). The stabilizing effect observed at low gas
65 flow rate arises at all wave numbers. By contrast, when the gas flow rate is large, short
66 waves are attenuated while long waves are amplified, in accordance with the experiments
67 of Alekseenko *et al.* (2009) and Vellingiri *et al.* (2015) on linear waves in a vertically-falling
68 liquid film, which were conducted in a vertical tube with a turbulent counter-current gas
69 flow. Finally, for weak confinement, a monotonic increase of the cut-off wave number is
70 observed and the destabilization of the film arises at all wave numbers.

71 The nonlinear response of surface waves has also been studied in a number of works,
72 usually involving a turbulent counter-current gas flow in the context of flooding (Trifonov
73 2010; Tseluiko & Kalliadasis 2011; Drosos *et al.* 2006; Vlachos *et al.* 2001; Kofman *et al.*
74 2017). These studies, which concerned weak confinement levels or vertical configurations
75 revealed that the amplitude of nonlinear waves increases and steepens with increasing
76 gas flow rate while their speed decreases. The authors also observed that capillary ripples
77 in front of the solitary waves are damped, in line with the linear stabilization of short
78 waves observed by Alekseenko *et al.* (2009). In the weakly-confined setting, flooding
79 manifests itself in experiments as wave reversal or wave breaking phenomena (Kofman
80 *et al.* 2017; Drosos *et al.* 2006) emerging from the interfacial shear stress imposed by the
81 gas flow. By contrast, in strongly-confined vertical channels, flooding is usually associated
82 with a wave-induced local obstruction of the channel cross section (Vlachos *et al.* 2001;
83 Dietze & Ruyer-Quil 2013), which can coincide with wave reversal events (Lavalle *et al.*
84 2020). In such configurations, the counter-current gas flow amplifies the wave height and
85 thus increases the flooding risk. Recent numerical investigations suggest that this trend
86 could be inverted in weakly-inclined channels. For example, Trifonov (2019) identified
87 a non-monotonic variation of the interfacial velocity, mean film thickness and inter-
88 phase friction coefficient with increasing counter-current gas velocity, although the trend
89 of the wave amplitude remained monotonic and increasing. Lavalle *et al.* (2021) have
90 identified regimes where the nonlinear wave height and the linear growth rate both

91 decrease with increasing gas flow rate. Such regimes, if confirmed experimentally, could
 92 avoid catastrophic flooding events, while exploiting the benefits of surface waves (e.g.
 93 intensification of heat and mass transfer).

94 The main aim of the current manuscript is to confirm the existence of such regimes
 95 through experiments. We study the effect of a counter-current laminar air flow on forced
 96 two-dimensional solitary waves resulting from the Kapitza instability in a weakly-inclined
 97 ($\beta = 4.9^\circ$) strongly-confined channel of 5 mm height. A particular attention has been paid
 98 to the boundary conditions, specifically designed to avoid flooding due to outlet effects
 99 at the end of the test section, thus allowing to focus on the wave dynamics in the core of
 100 the channel. Temporal forcing is applied at the liquid inlet to force surface waves within
 101 a buffer zone before they come into contact with the air flow. The influence of the gas
 102 shear stress on the shape, amplitude and velocity of these nonlinear waves is investigated
 103 and contrasted with the experiments of Kofman *et al.* (2017), which were performed in a
 104 weakly-confined channel ($H = 19$ mm) where the gas flow was turbulent. Furthermore,
 105 we are interested in the effect of the surface waves on the mean film thickness when the
 106 gas velocity is increased.

107 The paper is organised as follows. The experimental set-up and the measuring tech-
 108 niques are presented in § 2. Results are presented in § 3, where we discuss the response of
 109 nonlinear surface waves to an increase in counter-current gas flow rate. In § 3.1, we focus
 110 on the role of the confinement level, by contrasting our results with the weakly-confined
 111 experiments of Kofman *et al.* (2017). **In § 3.2, we focus on the effect of the counter-current**
 112 **gas flow on the dynamics of precursory capillary ripples. In § 3.3, we discuss the role of**
 113 **the liquid Reynolds number and finally in § 3.4 we assess the effect of surface waves on**
 114 **the mean film thickness.** Conclusions are drawn in § 4.

115 2. Experimental set-up and measurement methods

116 A general view of the experimental apparatus is sketched in figure 1: a liquid film falling
 117 on the bottom plate of an inclined rectangular channel in contact with a counter-current
 118 gas flow. Figure 2 displays a cross-sectional view of the channel. The liquid-related part
 119 is the same as the one used in Lavallo *et al.* (2019). In subsection 2.1, we recall briefly
 120 its main characteristics. Then in subsection 2.2 we describe the part related to the gas
 121 phase.

122 2.1. Liquid loop

123 The liquid-related part consists in an inclined glass plate ($L = 150$ cm long, $W = 27$ cm
 124 wide and 5 mm thick) placed on a massive framework mounted on rubber feet to dampen
 125 environmental vibrations. The inclination angle β can be changed in the range $0^\circ - 20^\circ$
 126 and is measured using an inclinometer with a precision of 0.05° . In this work β is fixed to
 127 4.9° . At the channel exit, the glass plate is extended with a porous medium that drains
 128 the liquid away, allowing to avoid flooding events due to exit effects. A gear pump brings
 129 the liquid from the outlet tank located at the exit of the plane to an inlet tank, from
 130 which the liquid overflows and runs onto the plane. The volumetric liquid flow rate, q_l , is
 131 changed by varying the pump power and is measured by using a magnetic-induced flow
 132 meter. The liquid Reynolds number is calculated as:

$$133 \quad Re_l = \frac{q_l}{\nu_l W}, \quad (2.1)$$

133 where ν_l is the kinematic viscosity of the liquid.

134 In this study, Re_l is varied in the range 20 - 49. A temporal periodic forcing of the
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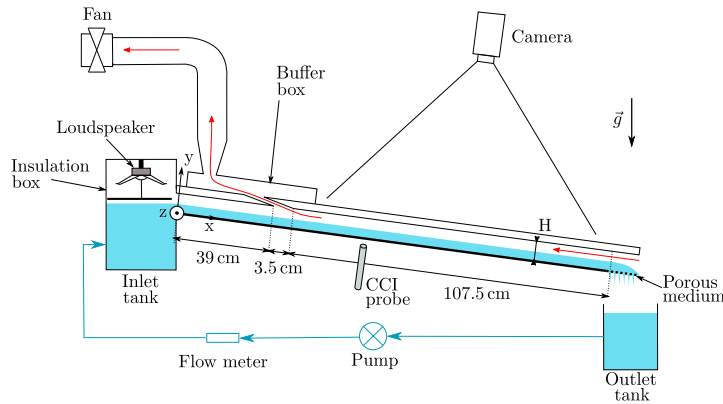


Figure 1: Sketch of the experimental set-up. The liquid loop appears in blue and the gas path is highlighted by red arrow.

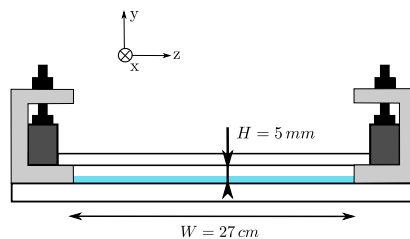


Figure 2: Sketch of the channel cross-section.

135 film is introduced at the inlet to trigger surface waves of prescribed frequency f . This
 136 is achieved through a thin aluminium plate fixed to the membrane of two loudspeakers
 137 thus generating harmonic vibrations above the liquid surface across the whole width of
 138 the inlet tank (Kofman *et al.* 2017). **The choice of the forcing frequency depends on**
 139 **Re_l and is based on the aerostatic case (without counter-current gas flow). We adjust it**
 140 **such that 2D saturated travelling waves are observed at the end of the unshered zone**
 141 **while avoiding secondary subharmonic or side-band instability (Liu & Gollub 1994). The**
 142 **waves then form a regular wave train over the working area and appear as a large hump**
 143 **preceded by capillary ripples.**

144 Water is used as working liquid. The temperature of the liquid is measured in the outlet
 145 tank and upstream of the inlet tank. The surface tension is regularly monitored by taking
 146 a water sample then using a drop shape analyser. A one-point temporal measurement of
 147 the film thickness based on the CCI (Confocal Chromatic Imaging) technique (Cohen-
 148 Sabban *et al.* 2001) is performed through the bottom glass plate with a spatial precision
 149 of 250 nm and a temporal resolution up to 2 kHz. The CCI probe is mounted on a linear
 150 translation stage in order to perform measurements along the streamwise axis of the
 151 glass plate. In this study, the film thickness is measured at mid-width and at $x = 60$ cm
 152 from the inlet (see figure 1). An example of a CCI time trace is shown in figure 3a. From
 153 such temporal measurements, we can extract: the time-averaged film thickness, h_m (blue
 154 line) and the film thickness range, given by h_{max} and h_{min} , which correspond to the
 155 statistical mean of the maximum and minimum wave height values measured over the
 156 duration of the signal (red line and green line). **The temporal fluctuations of h_{max} and**
 157 **h_{min} are quantified from the standard deviation of the statistical measurements and are**

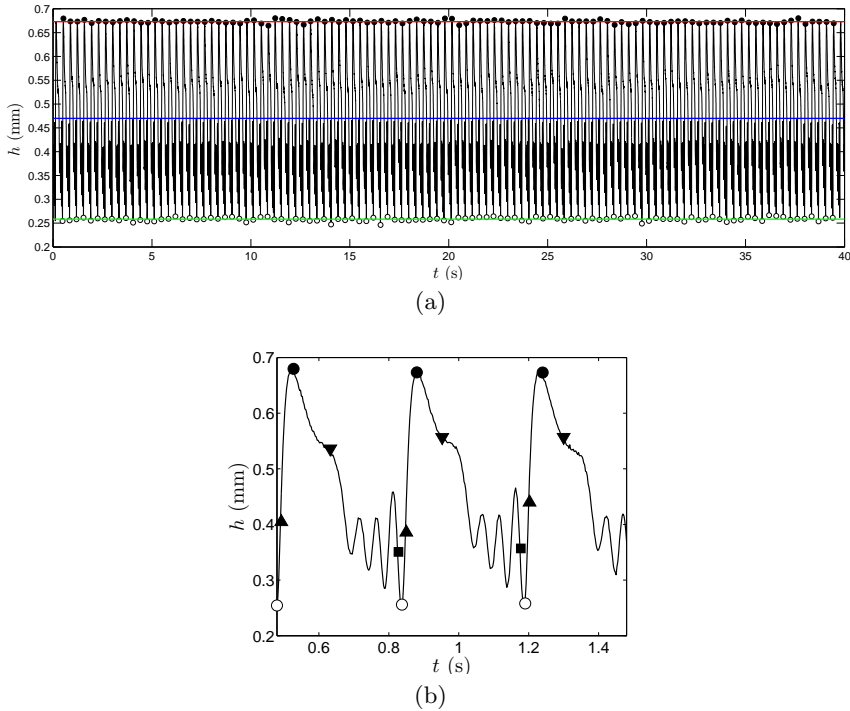


Figure 3: Film thickness time trace measured with the CCI technique. $Re_l = 35$, $f = 2.8$ Hz, $\eta_0 = 11.1$, without counter-current gas flow. (a) The blue line corresponds to the time-averaged film thickness over the duration of the signal. Open and filled circles correspond to the minimal and maximal height of each wave in the signal, respectively. (b) Enlargement of the signal presented in (a). Squares and downward triangles mark the maximum positive free-surface slope magnitude, located at the back of the first capillary wave and at the main wave tail. Upward triangles mark the maximum negative free-surface slope magnitude, located at the main wave front.

158 used to include error bars in the relevant figures. In addition, we can extract the local
 159 free-surface temporal slope, $\partial_t h$, and calculate the statistical mean of its maximum and
 160 minimum values over the duration of the signal, as well as the related standard deviation.
 161 Figure 3b represents a blown up view over three wave periods, where we have highlighted
 162 the maximum (positive) and minimum (negative) free-surface slope magnitude, located
 163 at the back of the first capillary wave (square) and at the main front (upward triangle),
 164 respectively. The maximum free-surface slope magnitude is also indicated downward
 165 triangle).

166 For a given liquid Reynolds number, we quantify the relative confinement of the liquid
 167 film with the global parameter:

$$\eta_0 = \frac{H}{h_m^0}, \quad (2.2)$$

168 where h_m^0 is the time-averaged film thickness measured in the case of a quiescent gas
 169 (superscript 0), i.e. without imposing a counter-current gas flow.

170 In addition to the pointwise CCI film thickness measurement, we visualize the film
 171 surface with shadowgraphy. For this, the film is illuminated with an oblique white light

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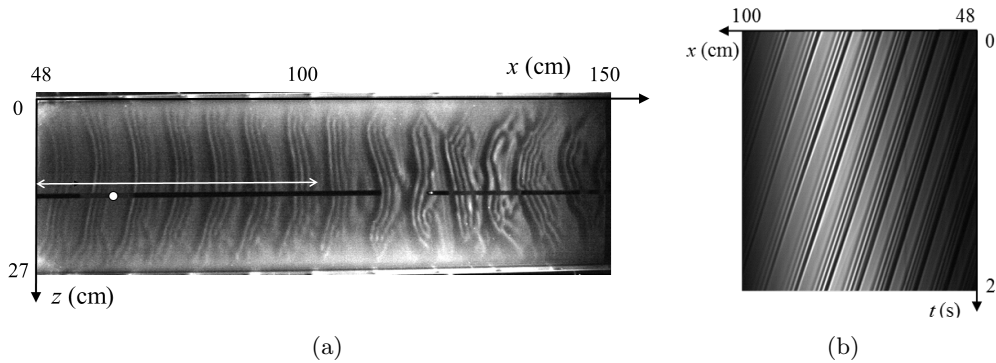


Figure 4: Typical experiment: $Re_l = 35$, $f = 2.8$ Hz, $\eta_0 = 11.1$, without counter-current gas flow (aerostatic case). (a) Visualisation of the gas/liquid interface over the entire test section. The white arrow marks the length over which spatiotemporal diagrams are constructed. The filled white circle marks the location for the CCI measurements ($x = 60$ cm), (b) Spatiotemporal diagram obtained using a linear camera located in the midplane ($z = 13.5$ cm).

172 sheet and imaged from the top either by a 2D camera to provide shadowgraphs over whole
 173 width and length of the working area (figure 4a) or by a linear CCD camera focused on
 174 the central axis ($z = 0.5W$) to obtain spatio-temporal diagrams (figure 4b) from which
 175 the wave speed, c , is determined. **At least two spatio-temporal diagrams are acquired for**
 176 **each experiment and we measure the velocity of several waves around $x = 60$ cm on each**
 177 **diagram. c is then the mean of the set of measurements and the related error is given by**
 178 **corresponding the standard deviation.** The wave speed can also be used to convert the
 179 temporal slope $\partial_t h$ of the film surface to the spatial slope $\partial_x h = \partial_t h / c$.

180

2.2. Gas loop

181 The gas flow is confined between the surface of the falling liquid film, which flows on
 182 the bottom glass plate, and an upper 5 mm thick glass plate placed at a distance H from
 183 the bottom one. The uniformity of H is regularly checked both in the streamwise and the
 184 transverse direction, based on measurements with the CCI method. From one experiment
 185 to another, H can vary from 5 mm to 5.2 mm depending on the force applied by the
 186 screws to fix the top plate, but its variation for a given experiment is less than 0.5%.
 187 Ambient air is sucked through the channel using a fan (see figure 1). The gas flow enters
 188 the channel at the lower end and leaves through an outlet slot spanning the entire width
 189 of the top plate. This slot ranges from $x = 39$ cm to $x = 42.5$ cm and opens into a buffer
 190 box to which the fan is connected via a flexible pipe. Upstream of the slot ($x < 39$ cm),
 191 the liquid film is allowed to develop without being disturbed by the gas flow.

192 The air flow rate, q_g , is controlled by the fan power and is quantified based on a
 193 calibration curve, which was obtained by measuring velocity profiles over the channel
 194 height with a hot-wire anemometer, at different fan powers. The procedure is detailed
 195 in the appendix A. These calibration measurements were performed in the absence of a
 196 liquid film and we define a gas Reynolds number as:

$$Re_g = \frac{q_g}{\nu_g W}, \quad (2.3)$$

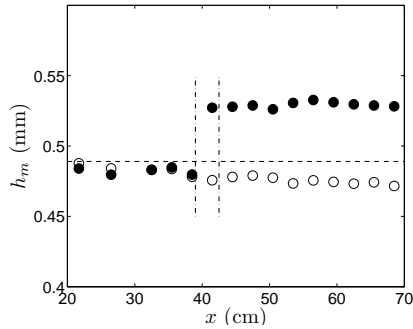


Figure 5: Streamwise evolution of the time-averaged film thickness, h_m along the central axis of the channel, in the aerostatic case ($Re_g = 0$, empty circles) and with counter-current gas flow ($Re_g = 942$ ($u_g = 3.05$ m/s), filled circles). The horizontal dashed-line indicates the Nusselt thickness (flat-surface film thickness without gas flow) associated to the prescribed liquid Reynolds number. The opening slot through which ambient air is sucked spans from $x = 39$ cm to $x = 42$ cm (highlighted by vertical dot-dash lines). $Re_l = 35$, $f = 2.8$ Hz, $\eta_0 = 11$.

197 where ν_g is the kinematic viscosity of the gas and q_g is the gas flow rate measured in a
 198 dry channel at the same fan power.

199 For the range of parameters used in this study, we assume that for a given fan power,
 200 the air flow rate is not significantly affected by the presence of the liquid film in the
 201 channel and thus the Re_g values are representative of the gas flow rate in the falling
 202 liquid film experiments. This statement is discussed at the end of appendix A.

203 In this paper, Re_g was varied up to 1350, beyond which flooding occurs in the channel
 204 for the liquid Reynolds number values considered. Experiments performed by Patel &
 205 Head (1969) in a rectangular channel of aspect ratio $W/H = 48$, which is comparable to
 206 the confined channel used in this paper ($W/H = 54$), showed that the flow remains
 207 laminar up to $Re_g = 1300$ and becomes fully turbulent for $Re_g > 2800$. Velocity
 208 measurements conducted in our channel and presented in appendix A confirmed that
 209 the laminar-turbulent transition starts at $Re_g > 1200$. We can thus conclude that the
 210 gas flow is laminar in the experiments presented in this paper.

211 The mean gas velocity, u_g , is defined as the spatially averaged velocity over the gas
 212 cross-section and is determined from q_g :

$$u_g = \frac{q_g}{W(H - h_m)}, \quad (2.4)$$

213 where h_m is the time-averaged film thickness measured in the presence of the counter-
 214 current gas flow. h_m remains almost constant along the central axis of the channel in
 215 the sheared region (i.e. downstream of the slot) for a given gas flow rate, as shown in
 216 figure 5 (filled circles). So we can consider that the mean gas velocity does not depend
 217 on the streamwise position. Figure 5 also shows that the liquid film is not affected by
 218 the counter-current gas flow in the unsheared zone upstream of the slot ($x < 39$ cm).
 219 Finally, this figure highlights the influence of the waves on the mean film thickness. We
 220 observe that the measured mean film thickness without imposed counter-current gas flow
 221 (empty circles) matches the primary-flow solution (dashed line) near the liquid inlet, then
 222 it decreases in the streamwise direction as the surface waves develop due to secondary
 223 instability.

8

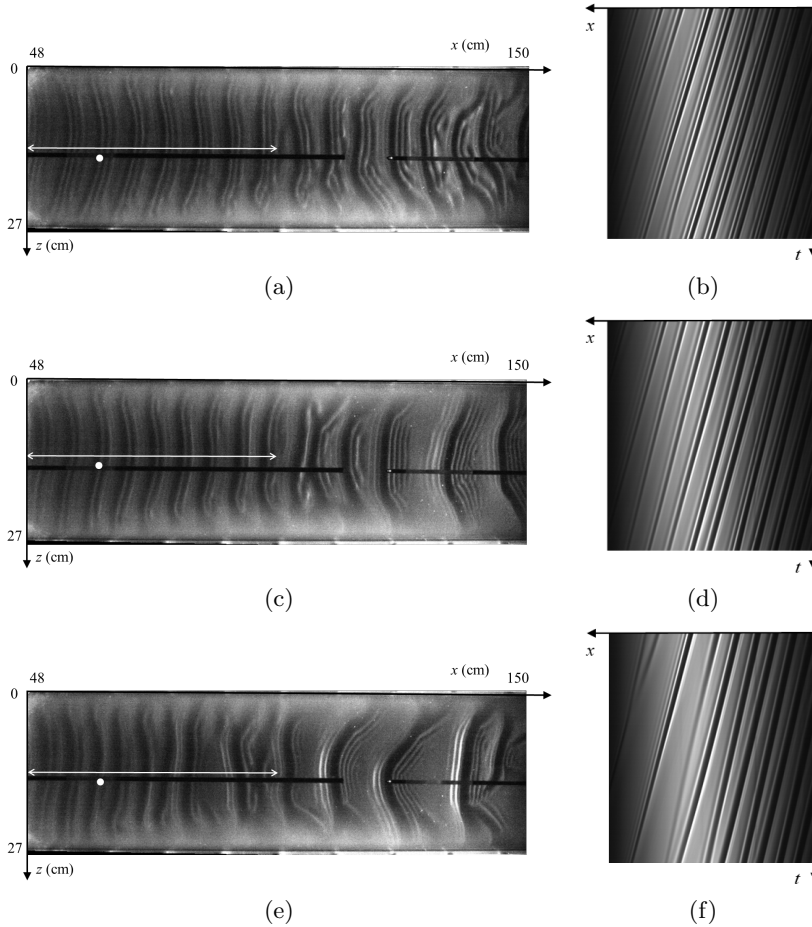


Figure 6: (a, c, e) Shadowgraphs of the gas/liquid interface over the entire test section for three different gas velocities, (b, d, f) associated spatio-temporal diagrams. From top to bottom: $u_g = 1.9$ m/s ($Re_g = 587$), $u_g = 3.4$ m/s ($Re_g = 1023$), $u_g = 4.1$ m/s ($Re_g = 1242$). These figures are the continuation of the aerostatic case ($u_g = 0$) presented in figure 4. The white dot and arrow are the same as those described in the caption of figure 4. $Re_l = 35$, $f = 2.8$ Hz, $\eta_0 = 11.1$.

3. Results

Figure 6 displays shadowgraphs of the film surface and the corresponding spatio-temporal diagrams for different gas velocities at $Re_l = 35$ and $f = 2.8$ Hz. Increasing the gas velocity generates interactions between waves that can induce coalescence events, clearly visible in figure 6f, resulting in solitary waves of high amplitude preceded by numerous capillary ripples in the downstream portion of the plane (figures 6c and 6e). The initial regular wavetrain is progressively disrupted by the counter-current gas flow as one moves downstream, but it remains globally two-dimensional before merging occurs. In this study, we are interested in this region of 2D travelling waves, so measurements are performed at the location indicated by the white mark in figures 6a, 6c and 6e. In particular, we wish to know the nonlinear response of these waves to the strongly-confined counter-current gas flow.

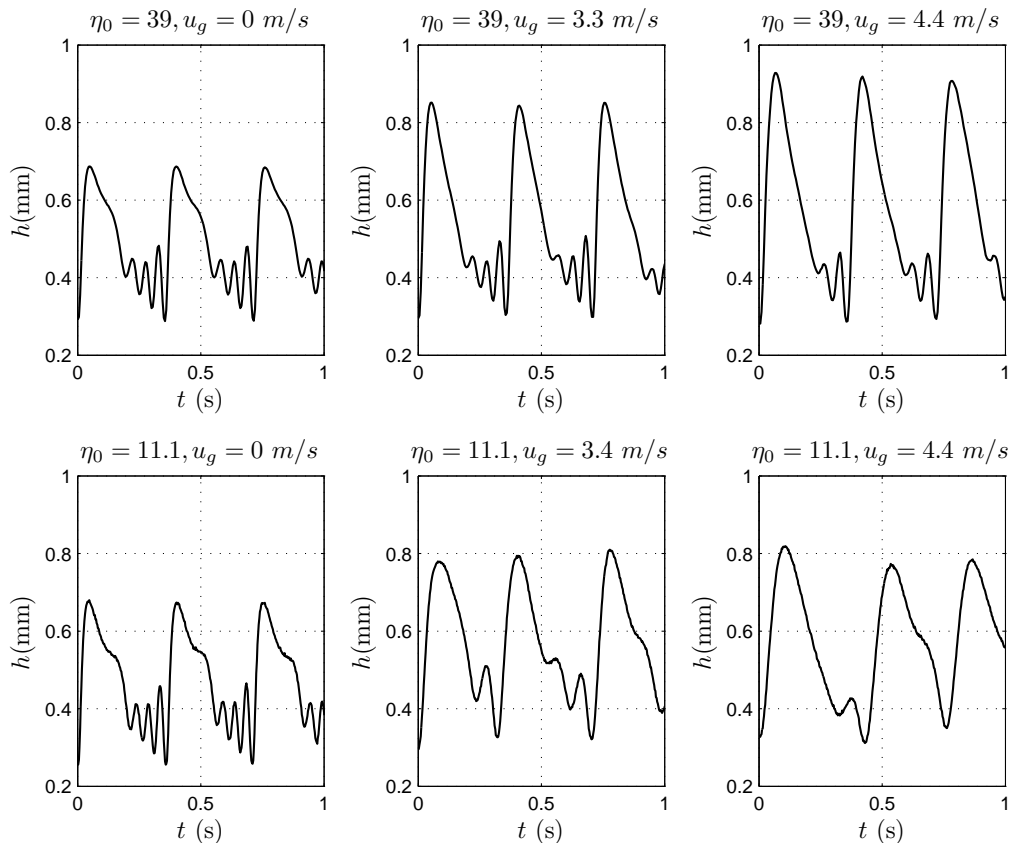


Figure 7: Influence of the gas velocity u_g on nonlinear wave profiles for two confinement levels: $\eta_0 = 39$ ($H = 19$ mm) and $\eta_0 = 11.1$ ($H = 5.2$ mm). $Re_l = 35$, $f = 2.8$ Hz, $x = 60$ cm.

3.1. Effect of confined counter-current gas flow on 2D saturated travelling waves

In this section, we confront the results obtained in our strongly-confined channel with previous experiments conducted in a larger channel (Kofman *et al.* 2017). We consider a liquid film flowing at $Re_l = 35$ on which we force travelling waves of frequency $f = 2.8$ Hz through our coherent inlet forcing. We are interested in studying the effect of the counter-current gas flow on these waves, and, in particular, how this effect changes depending on the channel height.

Figure 7 confronts time records of the film thickness for the $H = 19$ mm channel (top panels), which corresponds to a relative confinement $\eta_0 = 39$ (Kofman *et al.* 2017), with those measured in our strongly-confined $H = 5.2$ mm channel (bottom panels), where $\eta_0 = 11.1$. The measurement point was located at $x = 60$ cm from the liquid inlet in both experiments. Without imposed gas flow ($u_g = 0$, left panels in figure 7) the wave train is identical in both cases, consisting in waves with a large-amplitude asymmetric hump preceded by three capillary ripples. Thus, the level of confinement is inconsequential for the aerostatic configuration. Starting from this reference state, we compare the response of solitary waves to a gradual increase of the gas velocity for both confinements (central and right panels in figure 7). The striking difference between the two cases is that the maximum film height gradually increases with u_g in the large channel whereas it levels off

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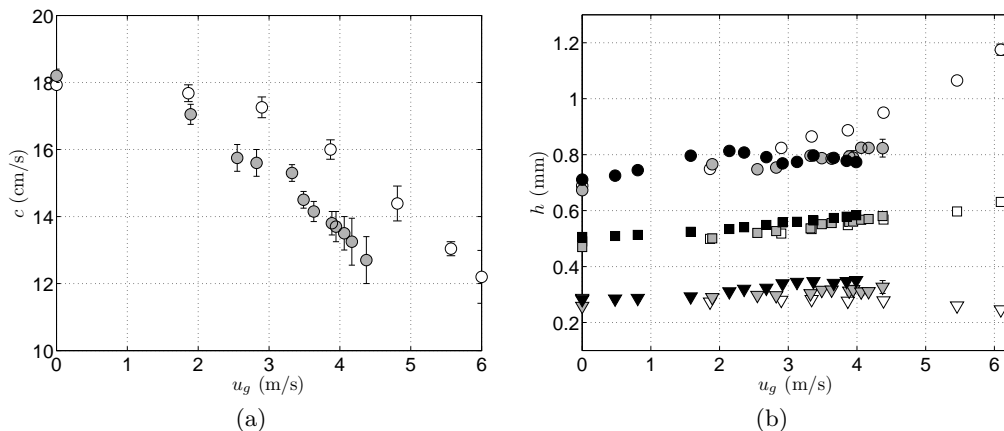


Figure 8: Influence of the gas velocity u_g on: (a) wave celerity c , (b) minimum h_{min} (downward triangles), mean h_m (squares) and maximum h_{max} (circles) film thickness for $\eta_0 = 39$, $\eta_0 = 11.1$ and $\eta_0 = 9.9$ (white, grey and black symbols, respectively). $Re_l = 35$, $f = 2.8$ Hz, $x = 60$ cm.

254 in the strongly-confined case. We also observe that the counter-current air flow attenuates
 255 the capillary ripples ahead of the main hump, as discussed in (Kofman *et al.* 2017). This
 256 effect is much more pronounced in the strongly-confined channel.

257 Figure 8 quantifies the gas effect on the wave characteristics for these two confinement
 258 levels. Figure 8a displays the effect of the gas on the wave velocity, c , and figure 8b
 259 its effect on the minimal (h_{min}), averaged (h_m) and maximal (h_{max}) values of the film
 260 thickness. We find that the slowing of the waves caused by the counter-current flow
 261 is more pronounced in the strongly-confined channel than in the larger one. Figure 8b
 262 shows that h_{min} and h_{max} are influenced by the confinement. h_{min} slightly increases
 263 with u_g in the strongly-confined channel while it remains constant in the large channel,
 264 or even slightly decreases for high gas velocities. As shown in figure 7, h_{max} in the
 265 strongly-confined channel increases with u_g in a first stage ($u_g < 2.1$ m/s for $\eta_0 = 11.1$)
 266 then saturates, whereas it continuously increases in the large channel. By contrast, h_m
 267 increases by the same amount with u_g in all experiments, regardless of the confinement
 268 level. In order to account for this increase in the mean film thickness, we report in figure
 269 9 the normalized quantities:

$$\delta_{max} = \frac{h_{max}}{h_m} \quad (3.1)$$

270 based on the data from figure 8b.

271 For $\eta_0 = 39$ ($H = 19$ mm), δ_{max} monotonously increases with u_g (open circles). Thus,
 272 increasing the gas flow rate amplifies the nonlinear waves, which implies a destabilizing
 273 (nonlinear) effect. This behaviour has been reported in several numerical studies (Tri-
 274 fonov 2010, 2019; Tseluiko & Kalliadasis 2011). By contrast, for the strongly-confined
 275 channel, δ_{max} first increases but then clearly decreases with u_g (filled circles), in partic-
 276 ular for $\eta_0 = 9.9$. These results confirm experimentally the numerical finding of Lavalle
 277 *et al.* (2021) that increasing the counter-current gas flow can attenuate nonlinear waves.

278 For the strongly-confined channel, the attenuation of nonlinear waves observed in figure
 279 9 sets in at $u_g > 2$ m/s. As shown in figure 8a, this coincides with a sharp drop in
 280 wave celerity under the effect of the gas which is much more pronounced in the strongly

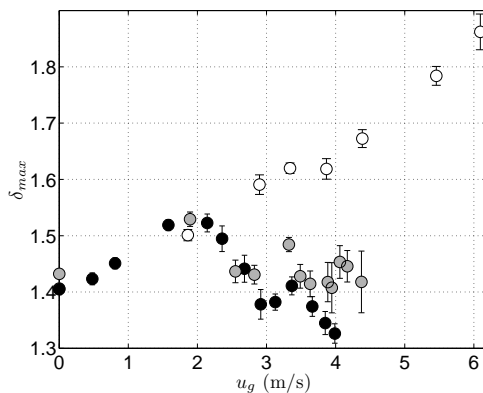


Figure 9: Influence of the gas velocity u_g on δ_{max} (3.1) for $\eta_0 = 39$, $\eta_0 = 11.1$ and $\eta_0 = 9.9$ (white, grey and black symbols, respectively). $Re_l = 35$, $f = 2.8$ Hz, $x = 60$ cm.

281 confined channel than in the large channel. In figure 9, the wave amplitude curve $\delta_{max}(u_g)$
 282 exhibits a saw-tooth shape during the stabilizing stage ($u_g > 2$ m/s) that will be discussed
 283 in the next subsection. For $\eta_0 = 11.1$ (grey symbols) the increase in $\delta_{max}(u_g)$ which
 284 reflects a re-destabilization of the waves between $u_g = 2.9$ m/s and $u_g = 3.3$ m/s is
 285 correlated to the modulation in the wave celerity depicted in figure 8a (grey symbols)
 286 which means that the stabilization mechanism is very sensitive to the wave celerity.

3.2. Effect of the counter-current gas flow on precursory capillary ripples

288 The saw-tooth shape exhibited in figure 9 for the strongly-confined channel at $u_g >$
 289 2.1 m/s is correlated to the wave celerity and results in a sequential suppression of
 290 capillary ripples under the effect of an increasing gas flow, as shown in figure 10, where
 291 wave profiles at different gas velocities are displayed for $\eta_0 = 9.9$ (black symbols in
 292 figure 9). For $u_g < 2.1$ m/s the effect of the counter-current gas is to increase the wave
 293 amplitude without significantly affecting the wave speed and the number of capillary
 294 ripples remains unchanged (in this case, 3 capillary ripples are observed ahead of the main
 295 hump). For $u_g > 2.1$ m/s, the wave speed drops significantly, which reduces the
 296 wavelength of the main waves ($f = 2.8$ Hz is constant here). Both effects are known
 297 to reduce the number and amplitude of precursory capillary ripples (Dietze 2016), and
 298 this is observed in figure 10 ($\eta_0 = 9.9$): we move from 3 to 2 capillary ripples between
 299 $u_g = 2.9$ m/s (alternation of 2 or 3 capillary ripples) and $u_g = 3.1$ m/s (2 capillary
 300 ripples) and from 2 to 1 at $u_g = 3.8$ m/s just before flooding. For $\eta_0 = 11.1$ (grey
 301 symbols in figure 9) the transition 3 – 2 and 2 – 1 occurs at $u_g \simeq 2.6$ m/s and 3.6 m/s.
 302 The change in the number of capillary ripples correspond to the troughs of the saw-tooth
 303 shape in figure 9.

304 To confirm that the number of capillary ripples is indeed governed by the wave speed,
 305 figures 11 compares the maximum positive slope, $\max\{\partial_x h\}$, for the two confinement
 306 levels $\eta_0 = 11.1$ (grey symbols) and $\eta_0 = 39$ (white symbols). This maximum is observed
 307 at the back of the first capillary ripple (squares in figure 3b) and is an indicator of the
 308 compression of the capillary wave train due to the decrease of the wave length. Figure 11a
 309 displays $\max\{\partial_x h\}$ (square symbols) as a function of u_g , and Figure 11b as a function
 310 of the normalized wave velocity \hat{c} :

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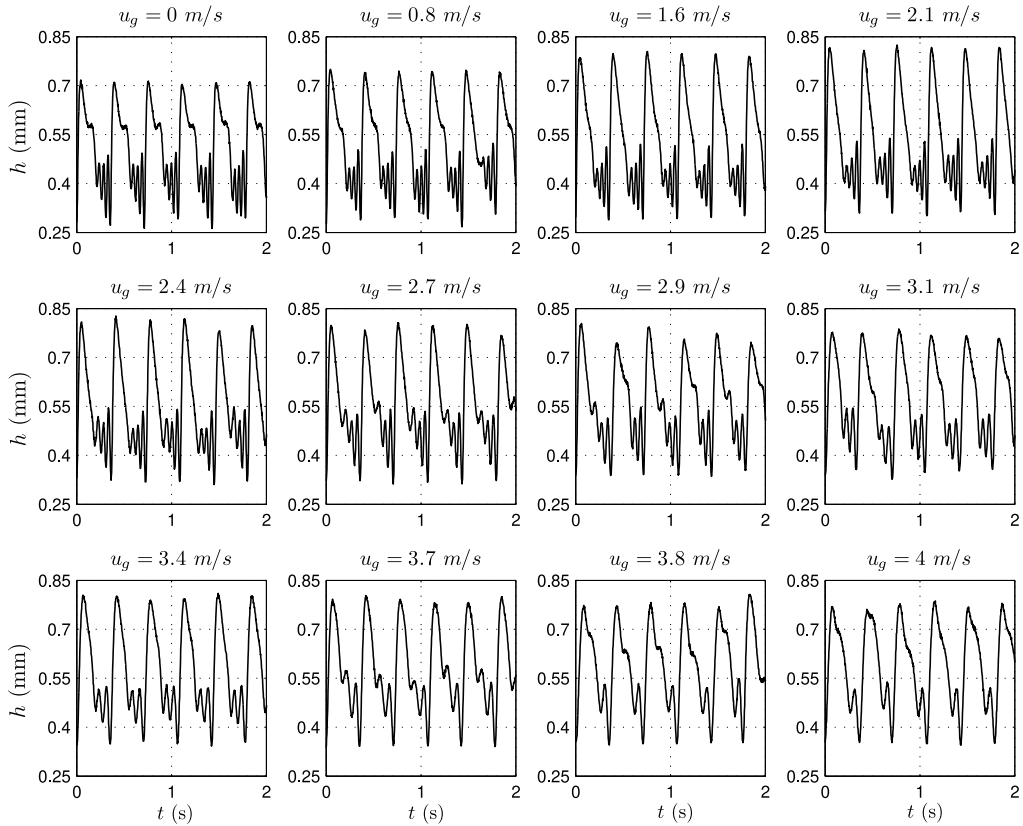


Figure 10: Influence of the gas velocity u_g on nonlinear wave profiles measured at $x = 60$ cm for $Re_l = 35$, $f = 2.8$ Hz, $\eta_0 = 9.9$, corresponding to the experiment presented in figure 9 (black symbols).

$$\hat{c} = \frac{c_0 - c}{c_N}, \quad (3.2)$$

311 where $c_N = \frac{gh_N^2 \sin \beta}{\nu_l}$ is the speed of kinematic waves in a passive atmosphere, c_0
 312 corresponds to the measured wave speed in the case of a quiescent gas, and $h_N =$
 313 $\left(\frac{3q_l \nu_l}{\sin \beta g}\right)^{1/3}$ is the Nusselt thickness (the flat-surface film thickness in the passive-gas
 314 limit).

315 Figure 11a shows that for a given value of $u_g \neq 0$, the slope magnitude at the back
 316 of the first capillary ripple as well as the number of capillary ripples depend on the
 317 confinement. By contrast, figure 11b shows that for a given value of \hat{c} these quantities
 318 are identical regardless of the confinement, which is also clearly visible from wave profiles
 319 presented in figure 12 for $\hat{c} = 0.13$ and $\hat{c} = 0.23$ (marked by vertical dashed lines in
 320 figure 11b). This behaviour implies that the capillary region is not directly affected by
 321 the gas shear-stress but is controlled by the indirect effect of the gas-induced slowing
 322 of the large waves. In figure 11 we have also plotted the maximum negative slope $\min \{\partial_x h\}$
 323 at the main hump front (upward triangles) which is an indicator of the steepness of the
 324 wave, and the maximum (positive) slope at the wave tail (downward triangles). In the

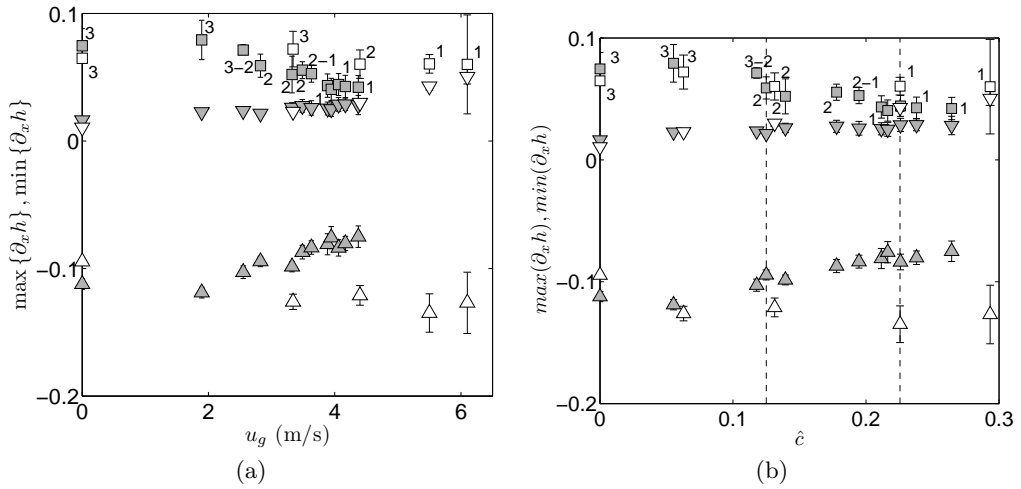


Figure 11: Influence of the gas velocity u_g in (a) and the normalized gas velocity \hat{c} (3.2) in (b), on the maximum positive free-surface slope $\max\{\partial_x h\}$ at the back of the first capillary ripple (squares) and at the main hump tail (downward triangles), and on the maximum negative slope $\min\{\partial_x h\}$ at the front of the main hump (upward triangles), for $\eta_0 = 39$ (white symbols) and $\eta_0 = 11.1$ (grey symbols). The number of capillary ripples is indicated. Vertical dashed lines in (b) indicate the values of \hat{c} related to the wave profiles presented in figure 12. $f = 2.8$ Hz, $Re_l = 35$, $x = 60$ cm.

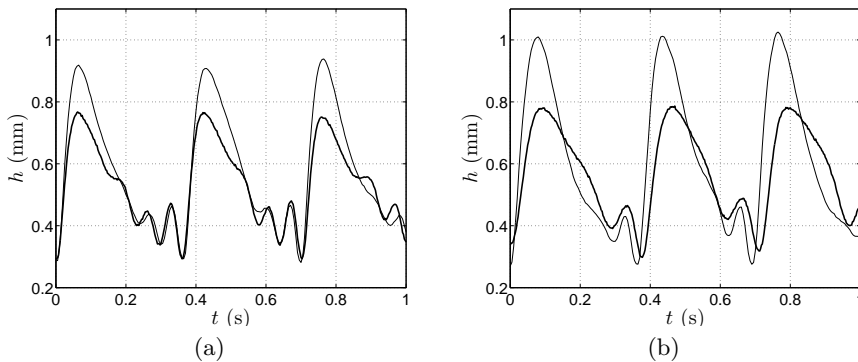


Figure 12: Influence of the confinement on wave profiles at given values of \hat{c} (3.2) marked by vertical dashed lines in figure 11b. (a) $\hat{c} = 0.13$, (b) $\hat{c} = 0.23$. Thin solid line: $\eta_0 = 39$ ($u_g = 4.4$ m/s in (a) and 5.5 m/s in (b)), thick solid line: $\eta_0 = 11.1$ ($u_g = 2.8$ m/s in (a) and 4 m/s in (b)). Same parameters as in figure 11.

325 large channel (white symbols) the front inclination remains unchanged as the gas velocity
 326 is increased, while the tail steepens, thus resulting in a more symmetrical wave (thin solid
 327 curves in figure 12). By contrast in the strongly-confined channel (grey symbols), the
 328 front inclination weakens while the tail slope remains unchanged under the action of the
 329 counter-current flow, and the resulting wave is less symmetric than in the large channel
 330 (thick solid curves in figure 12).

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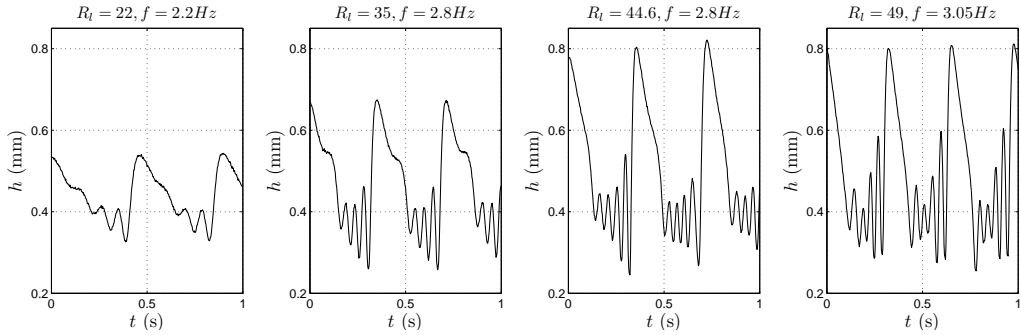


Figure 13: Wave profiles for different values of the liquid Reynolds number Re_l and forcing frequency f in the strongly-confined channel for the aerostatic case. From left to right, the resulting wavelength is 6.5 cm ($Re_l = 22$), 6.5 cm ($Re_l = 35$), 7.3 cm ($Re_l = 44.6$), not available for $Re_l = 49$.

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3.3. Influence of the liquid Reynolds number

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We now investigate the effect of the counter-current gas flow on the response of nonlinear waves for different values of the liquid Reynolds number Re_l and the forcing frequency f , in the strongly-confined channel. As shown in figure 13 for the quiescent-gas reference case, these parameters clearly affect the mean film thickness and consequently the confinement level η_0 (2.2), the maximum film thickness h_{max} , and the number of precursory capillary ripples. The forcing frequency has been empirically adjusted from the aerostatic case according to the liquid Reynolds number, in order to generate 2D saturated quasi-solitary waves in the working zone. The resulting wavelength of the unshered wave train is specified in the caption of figure 13 provided that wave velocity measurements are available.

We report in figure 14 the variation of c (figure 14a) and δ_{max} (figure 14b) in terms of Re_g for different Re_l and forcing frequency f . For $Re_l = 44.6$ (grey triangles), we observe the same overall behaviour as for $Re_l = 35$, which was presented in the previous section. In a first stage, δ_{max} increases with Re_g (the gas flow is nonlinearly destabilizing) then decreases when c is significantly reduced (the gas flow is nonlinearly stabilizing), with saw-tooth modulations associated with the change in the number of capillary ripples, whereas the variation of c with Re_g remains monotonous. For a higher liquid flow rate, $Re_l = 49$ (crosses), the destabilizing stage at low gas velocity is replaced by a stabilizing one. For the lowest Reynolds number, $Re_l = 22$ (open triangles), there are only two capillary waves to start with (see first panel in figure 13) and we observe a weak stabilization until $Re_g \approx 600$, associated with the suppression of one capillary wave. δ_{max} then increases to reach a constant value while maintaining one capillary wave until the experiment breaks down due to flooding downstream.

We now introduce the local relative confinement parameter η_{max} as:

$$\eta_{max} = \frac{H}{h_{max}} \quad (3.3)$$

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Figure 15 displays η_{max} as a function of Re_g for the same parameters as figure 14. The gas-induced stabilization at $Re_l = 22$ is associated with a relative confinement $\eta_{max} \approx 9$. For $Re_l = 35, 45$ and 49 the stabilizing effect occurs for smaller confinement levels.

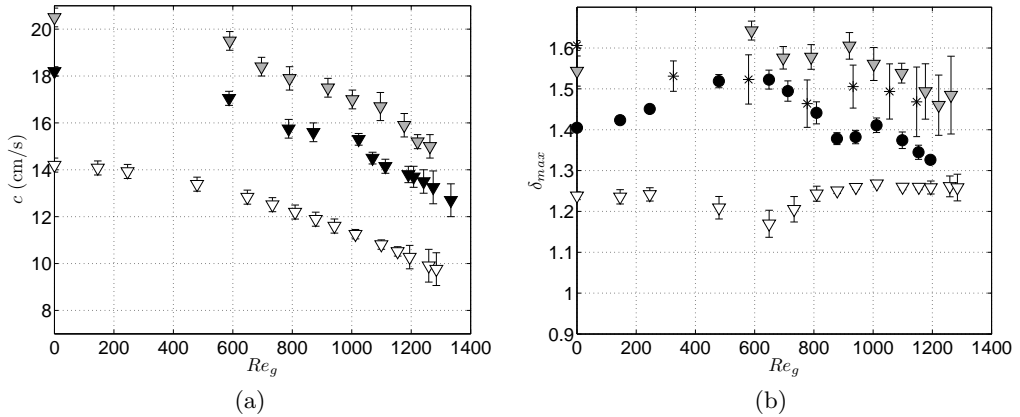


Figure 14: Influence of the liquid Reynolds number Re_l on: (a) wave celerity: $Re_l = 22$, $\eta_0 = 11.5$ (white triangles) - $Re_l = 35$, $\eta_0 = 11.1$ (black triangles) - $Re_l = 44.6$, $\eta_0 = 10.3$ (grey triangles), (b) $\delta_{max} = h_{max}/h_m$: same parameters as (a) for white and grey triangles, $Re_l = 35$, $\eta_0 = 9.9$ (black circles) - $Re_l = 49$, $\eta_0 = 10.3$ (crosses). Strongly-confined channel.

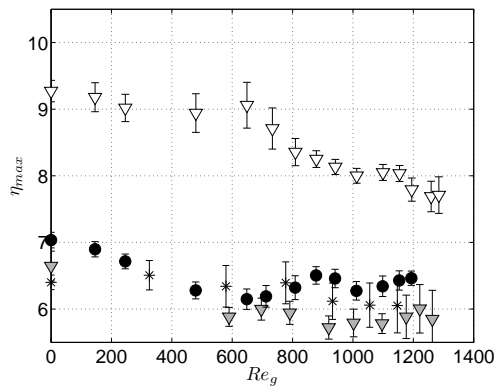


Figure 15: Influence of the liquid Reynolds number Re_l on the local confinement parameter η_{max} (3.3) in the strongly-confined channel. Same parameters as in figure 14b.

3.4. Effect of the surface waves on the mean film thickness

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360 We now wish to gauge the effect of the gas-sheared surface waves on the mean film
 361 thickness h_m . In particular, we wish to know whether h_m is decreased or increased
 362 versus the primary-flow thickness. In the case of a passive gas or a quiescent unconfined
 363 atmosphere, it is well known that h_m is smaller than the primary-flow thickness. This
 364 is known as wave-induced effective film thinning (Miyara 1999). In our current case of a
 365 counter-current gas flow, it turns out that the wave effect depends on the confinement
 366 level. To group our measurement data for the $H=5$ mm channel, where the gas flow is
 367 laminar, and the data of Kofman *et al.* (2017) for an $H=19$ mm channel, where the gas
 368 flow is turbulent, we introduce the Froude number $Fr_{g,l}$, which relates interfacial shear
 369 stress ($\sim \rho_g u_g^2$) to gravitational effects:

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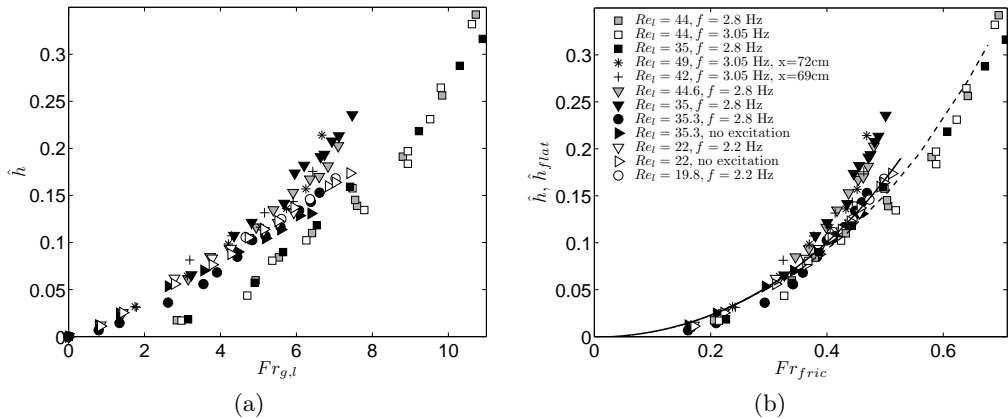


Figure 16: Relative mean film thickness \hat{h} (3.5) measured in the $H \approx 5$ mm channel and in the $H \approx 19$ mm channel (Kofman *et al.* (2017), square symbols), as a function of: (a) the Froude number $Fr_{g,l}$ (3.4), (b) the modified Froude number, Fr_{fric} (3.6). The solid and dashed curves correspond to the flat-film solution \hat{h}_{flat} according to B13 with $\epsilon = 0$ ($\hat{h}_{flat} = \tilde{h}_{flat} - 1$).

$$Fr_{g,l} = \sqrt{\frac{\rho_g u_g^2}{\rho_l g \sin \beta h_N}}, \quad (3.4)$$

370 and the normalized film thickness \hat{h} :

$$\hat{h} = \frac{h_m}{h_m^0} - 1 \quad (3.5)$$

371 Figure 16a displays \hat{h} as a function of $Fr_{g,l}$, for all experiments conducted in the
 372 $H = 5$ mm channel (different Re_l and f), as well as the experiments of Kofman *et al.*
 373 (2017) conducted in a $H = 19$ mm channel (square symbols). The experimental data
 374 collapse onto two different trends, one for the narrow channel and another for the wide
 375 channel. This is due to the different flow regimes in the two configurations, the gas flow
 376 being laminar in the former case and turbulent in the latter.

377 To account for this, we improve our approximation of the interfacial shear stress used
 378 in (3.4) in a modified gas-liquid Froude number, Fr_{fric} , based on the friction velocity,
 379 u_{fric} :

$$Fr_{fric} = \sqrt{\frac{\rho_g u_{fric}^2}{\rho_l g \sin \beta h_N}} = \sqrt{\frac{C_{f,i}}{2}} Fr_{g,l}, \quad (3.6)$$

380 where u_{fric} is based on the tangential interfacial gas shear stress of the primary flow (no
 381 waves), τ_i , as:

$$u_{fric} = \sqrt{\frac{\tau_i}{\rho_g}} \quad (3.7)$$

382 τ_i is related to the global skin-friction coefficient, $C_{f,i}$:

$$C_{f,i} = \frac{\tau_i}{1/2 \rho_g u_g^2} \quad (3.8)$$

383 In the strongly-confined channel, the gas flow is laminar and we approximate $C_{f,i}$ by
 384 the relation for Poiseuille flow through a plane dry channel:

$$C_{f,i} = 12Re_g^{-1} \quad (3.9)$$

385 In the large channel the gas flow is fully turbulent and we use an experimental
 386 correlation established by Patel & Head (1969) in a wide rectangular horizontal dry
 387 channel:

$$C_{f,i} = 0.0376Re_g^{-1/6} \quad (3.10)$$

388 Using these correlations for $C_{f,i}$, we assume that the liquid-gas interface can be con-
 389 sidered as immobile. To validate this motionless interface assumption, we have compared
 390 the film thickness of a flat-surface film subject to a counter-current laminar gas flow as
 391 obtained by numerical solution of the fully-coupled two-phase primary flow (Lavalle *et al.*
 392 2019) with the thickness obtained by assuming an inter-phase skin friction coefficient
 393 according to (3.9). Details on this flat-film model are given in appendix B. The two
 394 predictions are in excellent agreement and this holds over the entire range of experimental
 395 conditions studied in this work (see figure 23 in appendix B).

396 In figure 16b, \hat{h} (3.5) is plotted as a function of Fr_{fric} , according to (3.6). We observe
 397 that the new scaling is more appropriate to describe the effect of the shear stress exerted
 398 by the gas on the interface, as the laminar and turbulent data are now gathered. The
 399 two additional solid and dashed curves correspond to the flat-film model \hat{h}_{flat} (according
 400 to B 13 with $\epsilon = 0$, $\hat{h}_{flat} = \tilde{h}_{flat} - 1$) for the laminar gas flow in the strongly-confined
 401 channel (solid line) and for the turbulent gas flow in the large channel (dashed line). **This**
 402 **model represents a reference solution for comparison with the wavy experimental data.**

403 **The mean film thickness results from two opposing intricate effects: the well known**
 404 **effective thinning due to nonlinear surface waves, illustrated in figure 5 for the aerostatic**
 405 **case, and the thickening effect of the counter-current gas flow produced by the interfacial**
 406 **shear-stress. The relative competition of these two effects determines the mean film**
 407 **thickness ultimate behaviour.**

408 In the large channel (square symbols) the experimental data lie slightly below the flat-
 409 film prediction. Thus, surface waves tend to reduce the mean film thickness, as in the
 410 case of a quiescent atmosphere.

411 In the strongly-confined channel, at high liquid Reynolds number values Re_l and
 412 high Froude numbers ($Fr_{fric} \geq 0.35$), experimental data are clearly above the flat-
 413 film prediction which means that the mean wavy film thickness grows faster than the
 414 reference flat-film thickness. This behaviour occurs in the nonlinear stabilizing regime for
 415 strong relative confinement levels, where surface waves promote a significant variation of
 416 the gas cross-section, which in turn affects the local tangential shear stress. This is the
 417 basis for the linear stabilization mechanism described by Lavalle *et al.* (2019). In this
 418 regime, the gas effect tends to reduce the nonlinear amplitude of the main hump and thus
 419 the thinning effect of the waves, while it increases the residual flat film thickness between
 420 two humps. In figure 14b, we observe that during this stage, the interaction between the
 421 liquid film and the counter-current gas flow generates strong disturbances of the wave
 422 train as highlighted by the large temporal fluctuations measurements for $Re_l = 44.6$ and
 423 49.

424 By contrast, when Re_l and thus the wave amplitude is small, experimental data are well
 425 predicted by the flat-film model over the entire range of the Froude number. In this case,
 426 the mean thickness of the wavy film increases at the same rate as the flat film thickness
 427 when the gas flow rate is increased, indicating that the thinning effect of the surface
 428 waves balances the overall thickening effect due to the gas shear stress. **This balance**

429 can also be observed at higher Re_l and moderate Froude number values ($Fr_{fric} \leq 0.35$,
 430 crosses) for experiments concerned with the monotonously nonlinear stabilizing regime
 431 (see crosses in figure 14b).

432 Further, we conclude that the interfacial shear stress effect on the nonlinear surface
 433 waves changes by increasing the gas velocity in the strongly confined channel. This
 434 behaviour could be related to the observations of Trifonov (2019), who performed simu-
 435 lations of nonlinear waves for a set of parameters similar to our experimental conditions
 436 ($\beta = 5^\circ$, $H = 5$ mm, $Re_l = 5, 15$ and 40 , see the supplementary material associated with
 437 the cited article). The author demonstrates the existence of a local minimum of the mean
 438 film thickness and of the gas friction coefficient in terms of the gas superficial velocity.
 439 More specifically, at $Re_l = 40$, he found that the decrease of the friction coefficient
 440 deviates from the decreasing trend of the waveless solution ($\propto 1/Re_g$) at a gas velocity
 441 comparable to the experimental values associated to the deviation of h_m from the flat-film
 442 solution observed in figure 16b.

443 The experimental data corresponding to the black filled circles deviates from the other
 444 data at similar liquid Reynolds numbers. For this experiment the Kapitza number is
 445 lower (by 13% due to a different ambient temperature) and the channel height H is
 446 slightly lower (by 4%). These variations do not significantly affect the primary flow,
 447 as can be inferred from the right-pointing triangles which correspond to an experiment
 448 performed the same day without inlet forcing, and which lie on the solid curve related to
 449 the primary flow solution. However, the destabilizing effect of the gas on the nonlinear
 450 waves at moderate Froude number values may be enhanced, which could favor the wave-
 451 induced effective film thinning effect (these data lie below the solid curve).

452 4. Conclusion

453 In this paper, experiments were conducted on water films falling along the bottom wall
 454 of a weakly-inclined ($\beta = 4.9^\circ$) rectangular channel of height $H \approx 5$ mm interacting with
 455 a laminar counter-current air flow. Surface waves were excited via coherent inlet forcing
 456 before they come into contact with the air flow to promote travelling waves consisting in
 457 a large-amplitude hump preceded by capillary ripples. We focused on the sheared region
 458 of the film where the waves were two-dimensional without interaction between them.
 459 Local film thickness time traces and wave celerity measurements have been performed
 460 together with visual observations.

461 The effect of the air flow on the height, shape, and speed of the waves was investigated
 462 and contrasted with results obtained by Kofman *et al.* (2017) in a channel of height
 463 19 mm, where the counter-current gas flow was turbulent. The stronger confinement
 464 level studied here versus the experiments of Kofman *et al.* (2017) not only changes the
 465 geometrical constriction of the gas flow but also leads to a laminar flow regime in the
 466 gas instead of a turbulent one. In experiments, these two effects, which play their own
 467 distinct roles, cannot be decoupled. However, Trifonov (2010) has shown via numerical
 468 simulations that the main effect of turbulence is to shift the onset of wave-induced critical
 469 events (e.g. the flooding onset) toward lower superficial gas velocities.

470 In our experiments, no confinement-induced effect was detected on the wave dynamics
 471 in the case of a quiescent gas. Indeed, the shape, amplitude and velocity of the nonlinear
 472 waves excited at a prescribed frequency are almost identical for the two confinement
 473 levels. When a counter-current gas flow is imposed, striking differences are observed.
 474 For all the experiments conducted in the weakly confined channel, the effect of the gas
 475 on the travelling nonlinear waves was destabilizing (see Kofman *et al.* (2017)), whereas

476 a monotonic stabilizing effect or a non-monotonic trend was observed in the strongly-
477 confined channel, depending on the liquid Reynolds number Re_l .

478 For small Re_l in the strongly confined channel, inertia is weak and the relative
479 confinement is quite low without gas flow ($\eta_{max} > 9$). Under these conditions, the wave
480 amplitude first slightly diminishes as the gas velocity is increased and then increases to
481 reach a plateau sustaining one capillary ripple until onset of flooding. In that case, the
482 stabilizing mechanism induced by the variation of the gas shear stress at the interface is
483 quite weak.

484 For moderate Re_l , the wave amplitude first increases with the gas velocity while
485 the number of capillary ripples ahead of the main hump and the wave celerity remain
486 unchanged. Upon increasing the gas velocity further, the wave amplitude diminishes and
487 a gradual reduction of the number of capillary ripples is observed. For higher values of
488 Re_l , a monotonically gas-induced wave attenuation is observed.

489 These experimental observations are in line with recent numerical results obtained by
490 Lavalley *et al.* (2021) in a superconfined channel, where monotonic gas-induced amplifica-
491 tion (attenuation) of the nonlinear wave amplitude was detected for larger (smaller) H ,
492 and a non-monotonic trend was observed for intermediate H .

493 To quantify the effect of surface waves on the mean film thickness in a universal form,
494 a modified Froude number has been constructed which relates viscous drag based on the
495 skin friction coefficient and gravity. We have compared the normalized measured mean
496 film thickness \hat{h} with the primary-flow solution \hat{h}_{flat} obtained in the limit of a frozen
497 (immobile) liquid-gas interface. In the case of a turbulent gas flow, which corresponds to
498 experiments conducted in the large channel, \hat{h}_{flat} is greater than \hat{h} . In these experiments,
499 waves are significantly amplified by the counter-current flow and the wave-induced film
500 thinning effect (Miyara 1999) is dominant compared to the thickening effect of the gas
501 shear stress. In the laminar configuration, which corresponds to experiments conducted
502 in the strongly confined channel, \hat{h} and \hat{h}_{flat} correspond when Re_l is small, which implies
503 that the thinning effect of the waves on the mean thickness is balanced by the shear-
504 stress thickening effect, and the experimental behaviour is well predicted by the model.
505 For higher liquid Reynolds number values, $\hat{h}_{flat} < \hat{h}$ in the range of parameters where
506 the nonlinear stabilization occurs.

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512 Declaration of Interests: the authors report no conflict of interest.

513 Appendix A. Measurement of the gas flow rate

514 In this section we describe the procedure that has been used to quantify the air flow rate
515 q_g generated by the fan in the inclined strongly-confined channel of figure 1. This quantity
516 is used to evaluate the gas Reynolds number according to equation (2.3). This was done
517 based on a calibration curve between a given fan power and the corresponding gas flow
518 rate obtained from velocity measurements in the dry channel, i.e. without the liquid film.
519 The porous medium at the end of the bottom glass plate was replaced by a Plexiglas
520 plate with an opening to allow access for a hot-wire anemometer probe (see figure 17).
521 With this probe, we measured the wall-normal profile of the streamwise velocity $u(y)$

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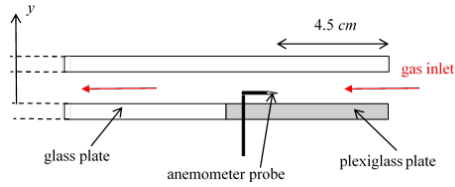


Figure 17: Location of the anemometer used to measure the air velocity at the end of the channel (gas inlet).

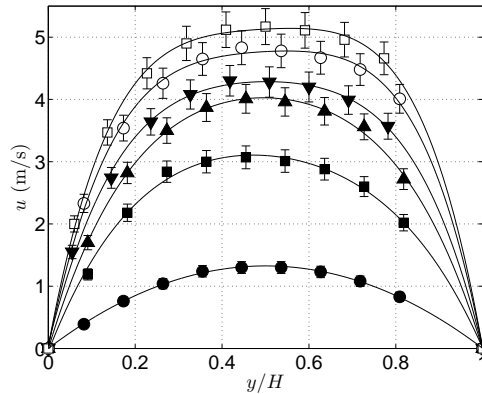


Figure 18: Air velocity profiles measured for different fan powers (in Watt) in the dry channel: $P_f = 1.3$ (circles), 3.9 (squares), 6.5 (upward triangles), 7.8 (downward triangles), 10.4 (open circles) and 13 (open squares). Solid lines correspond to polynomial fits. $\beta = 4.9^\circ$.

522 at mid-width of the channel and 4.5 cm from the air inlet. Figure 18 displays profiles
 523 obtained for different fan powers and for the inclination angle used in the experiments,
 524 $\beta = 4.9^\circ$. Each dot corresponds to a time-averaged measurement over 30 s and the error
 525 bars are included according to the uncertainty of the probe given by the manufacturer.
 526 A polynomial fit is performed for each profile from which we compute the air flow rate:

$$q_g = W \int_0^H u(y) dy, \quad (\text{A } 1)$$

527 where $W = 27$ cm is the channel width. The calibration curve presented in figure 19
 528 gives the air flow rate per unit width as a function of the imposed fan power, P_f . The
 529 gas Reynolds number, Re_g , is then calculated using (2.3). When the fan is switched
 530 off ($P_f=0$), there is no gas flow in the dry channel, and, thus, $Re_g=0$ according to our
 531 procedure, even though a small non-zero gas flow would be generated in the presence of
 532 a falling liquid film.

533 In figure 18, we have plotted several examples of measured velocity profiles for laminar
 534 flow conditions (black symbols), i.e. $Re_g < 1300$ according to the laminar-turbulent
 535 transition threshold determined by Patel & Head (1969) in a high-aspect-ratio channel.
 536 Table 1 reports the corresponding values of the velocity ratio u_g/u_{max} , where the
 537 mean velocity u_g has been defined in (2.4) and u_{max} is the maximum velocity. The
 538 aerodynamic entry length, L_d , is also reported, which is defined as the length needed for
 539 the boundary layers along the two horizontal channel walls to join at $y = H/2$. Based

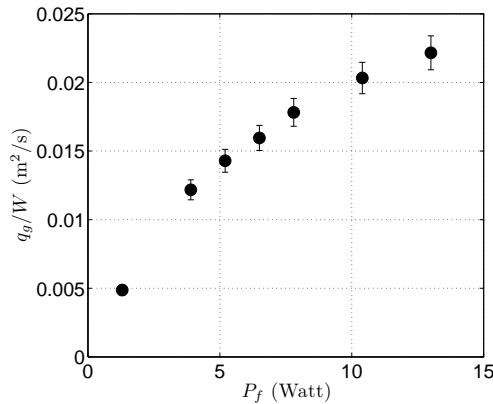


Figure 19: Calibration curve used to quantify the gas flow rate q_g (A 1) per unit of channel width from the imposed fan power P_f . $\beta = 4.9^\circ$.

Table 1: Ratio between the mean and the maximum air velocity issued from the measured velocity profiles (black symbols in figure 18) and the corresponding aerodynamic entry lengths given by (A 2).

Re_g	u_g/u_{max}	L_d (mm)
325	0.667	17
810	0.721	42
1065	0.723	50
1190	0.757	55

540 on the presumption that air enters the channel with a uniform velocity profile and using
 541 the Blasius correlation, it is straightforward to show that:

$$\frac{L_d}{H} = 0.01 Re_g \quad (\text{A } 2)$$

542 Based on this data, it is clear that the gas flow is fully developed in the entire region
 543 of the channel used for our falling liquid film experiments.

544 A specific velocity measurement series has been performed to confirm the laminar-
 545 turbulent transition threshold determined by Patel & Head (1969). The top glass plate
 546 of the channel was replaced by a Plexiglas plate with an opening to allow access for the
 547 probe far from the air entrance (about 1 m away). In this campaign, only the maximum
 548 air velocity, u_{max} , has been measured for different fan powers. By assuming that the flow
 549 is laminar and fully-developed at this location, i.e. $u_g \approx 0.667u_{max}$, we can calculate a
 550 flow rate and then a Reynolds number, Re_g^{lam} . For a given fan power, we compare Re_g^{lam}
 551 with Re_g issued from the complete vertical velocity profiles (using (A 1) then (2.3)).
 552 Figure 20 displays Re_g^{lam} and Re_g against the imposed fan power. It can be seen that
 553 for $Re_g \leq 1200$ the points collapse, which means that $Re_g = Re_g^{lam}$ and thus the flow is
 554 confirmed to be laminar. The discrepancy observed for $Re_g > 1200$ means that the flow
 555 is no longer laminar. From these measurements we can conclude that in our inclined
 556 channel, the laminar-turbulent transition occurs for Re_g slightly greater than 1200.

557 The gas Reynolds number, Re_g , defined in (2.3) can be used to characterize the counter-
 558 current air flow in the experiments provided that the gas flow rate circulating into the

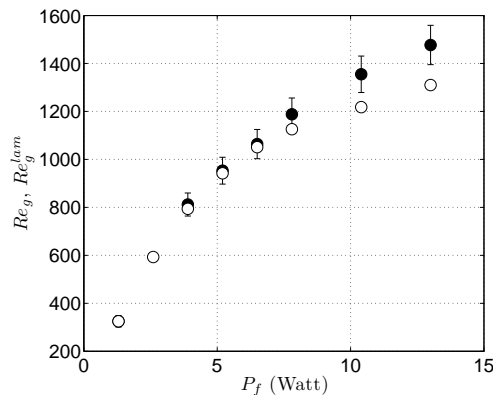


Figure 20: Re_g (filled circles) and Re_g^{lam} (open circles) as a function of the fan power P_f . Re_g is calculated by integrating velocity profiles measured at the end of the channel (figure 18), and Re_g^{lam} from the maximum velocity measurements performed in the fully-developed region and assuming a laminar parabolic profile i.e. $u_g = 2/3u_{max}$.

559 experimental gas loop is not affected by the presence of the liquid film in the channel at
 560 a fixed fan power. The validity of this assumption is discussed below.

561 For a gas flowing in a dry rectangular channel with a large transversal aspect ratio
 562 of length L and height H (in our configuration $W/H = 54 \gg 1$), the pressure drop
 563 is given by the analytic relation $\Delta p_{channel} = 4C_f \frac{L}{2H} \frac{1}{2} \rho_g (\frac{q}{H})^2$ where $q = u_g H$ is the
 564 volumetric gas flow rate (per unit width, i.e. $q = q_g/W$) and C_f is the skin friction
 565 coefficient, with $C_f = 12/Re_g$. Thus, $\Delta p_{channel} \propto q/H^3$ and the corresponding power
 566 $P_{channel} = q \Delta p_{channel} \propto q^2/H^3$. For a given power value $P_{channel}$, decreasing the channel
 567 height by 10% (typical to the reduction due to the presence of the liquid film in the
 568 experiments) leads to a 15% decrease in the flow rate. However, $\Delta p_{channel}$ constitutes
 569 only a small part of the total pressure drop, which is dominated by the resistance in
 570 the section between the outlet slot and the fan. Thus, the actual decrease in q_g upon
 571 decreasing H by 10% is relatively low. For the range of parameters considered in this
 572 study, we have estimated it to be less than 5%.

573 Appendix B. Flat-film model for laminar and turbulent gas flow

574 In panel 16b, we have estimated the liquid film thickness of the flat-surface primary
 575 flow based on the skin friction coefficient $C_{f,i}$. This approach allows to account for a
 576 laminar or turbulent gas flow within the flat-film model. For laminar flow, this coefficient
 577 $C_{f,i}$ is given by (3.9) and for turbulent flow by (3.10), according to the experiments of
 578 Patel & Head (1969). However, these experiments were performed in a dry channel, and,
 579 thus, our flat-film model implies an immobile liquid-gas interface. The current appendix
 580 aims to verify this assumption. We do this based on the laminar case, which is the limiting
 581 one, and for which we can easily obtain a reference solution for the flat-surface film based
 582 on the fully-coupled primary flow (Lavalle *et al.* 2019).

583 The gas velocity profile for the fully-coupled primary flow is given by:

$$u(Y) = \frac{-K_g}{2\mu_g} H_g^2 (Y - Y^2) + u_0 (1 - Y), \quad (\text{B1})$$

584 where K_g reads:

$$K_g = \left(\partial_x p - \rho_g g \sin \beta \right), \quad (\text{B2})$$

585 where $\partial_x p$ is the driving longitudinal pressure gradient, positive in our case for a counter-
 586 current gas flow, and $Y = (y - h_{flat})/H_g$, with h_{flat} denoting the flat-surface film
 587 thickness and $H_g = H - h_{flat}$ the height of the gas cross-section. u_0 is the liquid velocity
 588 at the film surface ($u_0 > 0$).

589 The gas velocity averaged over H_g can be written as:

$$u_g^+ = \frac{-K_g}{12\mu_g} H_g^2 + \frac{u_0}{2} = u_g^0 + \frac{u_0}{2} = u_g^0 \left(1 + \frac{\epsilon}{2} \right), \quad (\text{B3})$$

590 where u_g^0 corresponds to the average gas velocity in the limit $u_0 = 0$ and $\epsilon = u_0/u_g^0$.

591 The gas Reynolds number is defined as:

$$Re_g^+ = \frac{u_g^+ H_g}{\nu_g} = \frac{u_g^0 H_g}{\nu_g} + \frac{u_0 H_g}{2\nu_g} = Re_g^0 + Re_0 = Re_g^0 \left(1 + \frac{\epsilon}{2} \right) \quad (\text{B4})$$

592 We can write the tangential shear-stress at the liquid-gas interface and at the top wall,
 593 τ_i and τ_w , as:

$$\tau_i = -\frac{\mu_g}{H_g} \partial_Y u|_{Y=0} = \frac{K_g}{2} H_g + \mu_g \frac{u_0}{H_g} = \tau_i^0 \left(1 - \frac{\epsilon}{6} \right), \quad (\text{B5})$$

$$\tau_w = \frac{\mu_g}{H_g} \partial_Y u|_{Y=1} = \frac{K_g}{2} H_g - \mu_g \frac{u_0}{H_g} = \tau_i^0 \left(1 + \frac{\epsilon}{6} \right), \quad (\text{B6})$$

594 where $\tau_i^0 = H_g K_g / 2$ is the limit of τ_i and τ_w for a static interface (i.e. for $\epsilon = 0$).

595 We define the global skin-friction coefficient at the liquid-gas interface, $C_{f,i}$ as:

$$\begin{aligned} C_{f,i} &= \frac{\tau_i}{1/2\rho_g u_g^{+2}} \\ &= \frac{\tau_i^0}{1/2\rho_g u_g^{02}} \frac{1 - \epsilon/6}{(1 + \epsilon/2)^2} = C_{f,i}^0 \frac{1 - \epsilon/6}{(1 + \epsilon/2)^2} \end{aligned} \quad (\text{B7})$$

596 In a similar way we can define $C_{f,w}$ at the top wall as:

$$C_{f,w} = C_{f,i}^0 \frac{1 + \epsilon/6}{(1 + \epsilon/2)^2} \quad (\text{B8})$$

597 $C_{f,i}^0$ is the skin friction coefficient in the case of static boundaries (i.e. for $\epsilon = 0$). In

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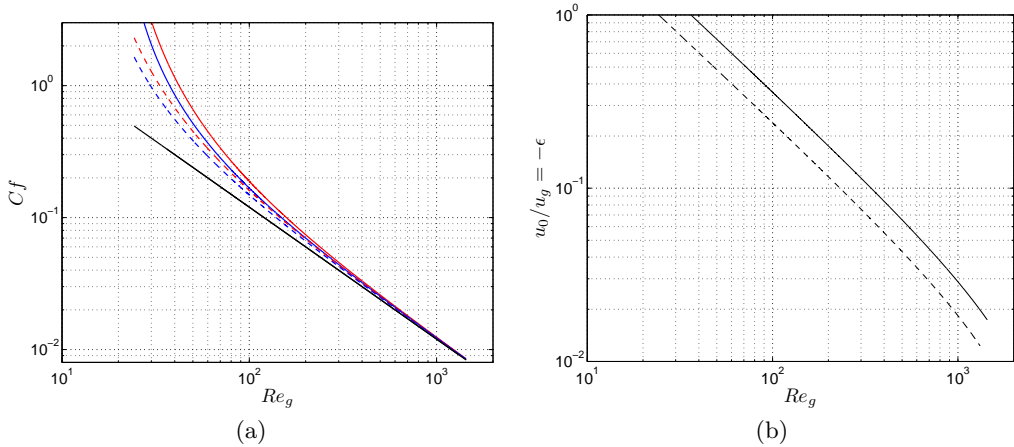


Figure 21: (a) Global skin-friction coefficients at the liquid-gas interface $C_{f,i}$ (B 7) (red line) and at the upper wall of the channel $C_{f,w}$ (B 8) (blue line) as a function of Re_g , calculated from the fully-coupled primary flow solution. The black line corresponds to a skin-friction coefficient in the case of a static interface (i.e. $\epsilon = 0$). (b) Ratio between the interface velocity and the gas velocity $u_0/u_g = -\epsilon$. $Re_l = 22$ (dashed lines) and $Re_l = 44.6$ (solid lines), $\beta = 4.9^\circ$, $H = 5.2$ mm.

598 the case of a laminar gas flow, it is given by:

$$C_{f,i}^0 = \frac{-12}{Re_g^0} \quad (\text{B 9})$$

599 For convenience, in the experiments u_g is always taken as positive, therefore $u_g = -u_0^0$
600 and $Re_g = -Re_g^0$ in our counter-current case.

601 Now, we compute $C_{f,i}$ and $C_{f,w}$ based on (B 7) and (B 8), using the liquid velocity at
602 the interface u_0 (which enters ϵ) from the fully-coupled primary-flow solution. Figure 21a
603 displays $C_{f,i}$ (red lines), $C_{f,w}$ (blue lines) and $C_{f,i}^0$ (black line) as a function of the gas
604 Reynolds number $Re_g = -Re_g^0$ over the range $25 < Re_g < 1500$, for two liquid Reynolds
605 numbers considered in our experiments with $H = 5.2$ mm: $Re_l = 22$ (dashed lines) and
606 $Re_l = 44.6$ (solid lines). The associated variation of $u_0/u_g = -\epsilon$ is presented in figure 21b.
607 To specify the range of parameters where the static interface approximation based on the
608 skin-friction coefficient criteria can be considered as valid, we focus on the gas Reynolds
609 values such as the difference between $C_{f,i}$ and $C_{f,i}^0$ is less than 10%. From figure 21a
610 we can conclude that this criterion is met for $Re_g > 250$ ($Re_l = 22$) and $Re_g > 400$
611 ($Re_l = 44.6$), which corresponds to $u_0/u_g \leq 0.08$ in figure 21b. Figure 21a also shows
612 that $C_{f,i} \approx C_{f,w}$ over a range of u_0/u_g where the static interface assumption is no longer
613 valid. For example the discrepancy between $C_{f,i}$ and $C_{f,w}$ is around 10% in figure 21a for
614 $(Re_l, Re_g) = (22, 80)$ and $(44.6, 125)$, corresponding to $u_0/u_g \approx 0.3$ in figure 21b. This
615 behaviour can be explained by the change in the the gas velocity distribution with the
616 gas flow rate. Indeed, the location of the maximum gas velocity is shifted towards the
617 top wall when u_0/u_g is increased by decreasing Re_g , as shown in figure 22. Consequently,
618 the tangential shear stress is reduced at the liquid/gas interface while it increases at the
619 top wall and the dissymmetry arising from the moving interface is partially attenuated.

620 To go further in testing the validity of this frozen-interface approximation, we will
621 compare the film thickness obtained from the fully-coupled primary flow solution with

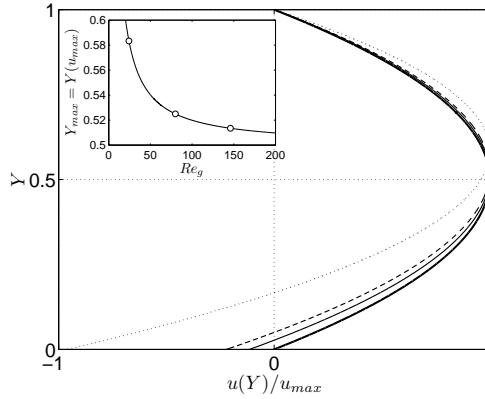


Figure 22: Gas velocity profiles calculated from the fully-coupled primary flow solution for $Re_l = 22$: $u(Y)/u_{max}$ with $u(Y)$ according to (B 1) and $u_{max} = \min(u(Y)) < 0$. Dash-dot line, $u_0/u_g = 1$ ($Re_g = 24.3$); dashed line, $u_0/u_g = 0.3$ ($Re_g = 80$); solid line, $u_0/u_g = 0.16$ ($Re_g = 145$); thick solid line, $u_0 = 0$ (Poiseuille flow). The location of u_{max} in the channel, $Y_{max} = Y(u_{max})$, is plotted in the inset as a function of Re_g ($Y_{max} = 0.5$ for a Poiseuille flow). Open circles mark Y_{max} for the profiles represented in the figure (except the Poiseuille flow). $\beta = 4.9^\circ$, $H = 5.2$ mm.

622 the thickness obtained by assuming an inter-phase skin friction coefficient according to
623 (B 9) using a simple model described below.

624 We start by writing the velocity profile in the liquid film:

$$u_l(y) = \frac{K_l}{\mu_l} \left(h_{flat} y - \frac{y^2}{2} \right) - \frac{\tau_i}{\mu_l} y \quad (\text{B } 10)$$

625 K_l reads:

$$K_l = \left((\rho_l - \rho_g) g \sin \beta - \frac{\tau_i + \tau_w}{H - h_{flat}} \right), \quad (\text{B } 11)$$

626 where τ_i and τ_w are given by (B 5) and (B 6), respectively. Therefore, $\tau_i + \tau_w = 2\tau_i^0 =$
627 $\rho_g u_g^0 C_{f,i}^0 = \rho_g u_g^0 C_{f,i}^0$.

628 Integrating (B 10) across the liquid layer gives the liquid flow rate:

$$Q_l = \frac{K_l}{3\mu_l} h_{flat}^3 - \frac{\tau_i}{2\mu_l} h_{flat}^2 \quad (\text{B } 12)$$

629 By expressing the flowrate $Q_l = (\rho_l g \sin \beta)(3\mu_l)^{-1} h_N^3$ in terms of the Nusselt thickness
630 h_N , i.e. the thickness of an equivalent fully-developed flat-surface film falling in a passive
631 atmosphere, we obtain the following equation for the flat-surface film thickness sheared
632 by a counter-current gas flow:

$$\left(1 - \frac{\rho_g}{\rho_l} - \underbrace{Fr_{g,l}^2 C_{f,i}^0 \left(\frac{H}{h_N} - \tilde{h}_{flat} \right)^{-1}}_{\Lambda_p} \right) \tilde{h}_{flat}^3 - \underbrace{\frac{3}{4} Fr_{g,l}^2 C_{f,i}^0 (1 - \epsilon/6) \tilde{h}_{flat}^2 - 1}_{\Lambda_i} = 0, \quad (\text{B } 13)$$

633 where $\tilde{h}_{flat} = h_{flat}/h_N$. The contribution of pressure gradient in the gas phase is included
634 in Λ_p and the contribution of the interfacial shear-stress is included in Λ_i . The inter-phase

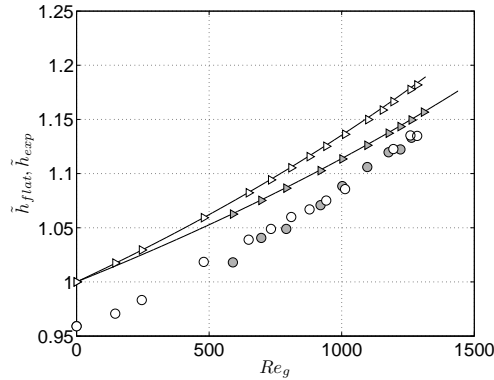


Figure 23: Thickness of a flat-surface film (normalized by the Nusselt thickness h_N) subject to a counter-current laminar gas flow, obtained from the flat-film model (triangles) (according to (B 13) with $\epsilon = 0$) compared with the numerical solution of the fully-coupled two-phase primary flow (solid lines). The normalized mean film thickness $\tilde{h}_{exp} = h_m/h_N$ measured in the experiments is also reported (circles). $Re_l = 22$ (white symbols) and $Re_l = 44.6$ (grey symbols), $\beta = 4.9^\circ$, $H = 5.2$ mm.

635 skin friction coefficient $C_{f,i}^0$ is given by (B 9) in the laminar case and $Fr_{g,l}$ is given by
 636 (3.4).

637 The contribution of the moving interface is represented by $\epsilon = u_0/u_g^0$, the ratio between
 638 the interface velocity and the gas velocity ($\epsilon < 0$ in our counter-current case). By
 639 assuming $\epsilon = 0$, i.e. the liquid/gas interface is static, the liquid and gas phases are
 640 coupled only via the interface position h_{flat} .

641 In figure 23, we compare the solution given by the model from solving (B 13) assuming
 642 $\epsilon = 0$ (symbols) with the film thickness obtained from solving the fully-coupled two-phase
 643 primary flow (solid lines), for $Re_l = 22$ and $Re_l = 44.6$, and for gas Reynolds number
 644 values according to the experiments. The results obtained from these two approaches are
 645 in excellent agreement over the entire range of experimental conditions, even for small
 646 gas Reynolds numbers for which $|\epsilon|$ is not so small. Indeed, if we go back for example to
 647 the aforementioned experiment conducted with $Re_l = 22$ and $Re_g = 145$, $\epsilon = -0.3$ (see
 648 figure 21b) which implies that the effect of the interfacial shear stress (Λ_i in (B 13)) is
 649 underestimated by only $0.3/6 = 5\%$ by assuming an immobile liquid/gas interface. From
 650 this analysis, we can conclude that this approximation is admissible in all the experiments
 651 presented in the current paper.

652 Finally, the normalized mean film thickness $\tilde{h}_{exp} = h_m/h_N$ measured in the experi-
 653 ments is also reported in figure 23, for $Re_l = 22$ and $Re_l = 44.6$. We observe that
 654 $\tilde{h}_{exp} < \tilde{h}_{flat}$ reflecting the effective film thinning due to the presence of interfacial waves
 655 in experiments. For $Re_l = 22$, \tilde{h}_{exp} increases in the same amount as \tilde{h}_{flat} when the gas
 656 flow is increased, whereas \tilde{h}_{exp} increases faster than \tilde{h}_{flat} for $Re_l = 44.6$. This behaviour
 657 is discussed in §3.3.

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