

P. Laure, J. Férec, L. Silva

MINES ParisTech, PSL Research University, CEMEF, UMR CNRS 7635, CS 10207, 06904 Sophia Antipolis cedex (France)

IRD, CNRS, Université Bretagne Sud, Rue de Saint Maude, 56321 Lorient, France
Ecole Centrale de Nantes, 1 Rue de la Noë, 44300 Nantes, France

Objective

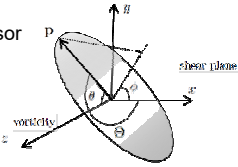
Influence on the particle shape and fluid behavior on the validity of Jeffery' equation

$$\dot{\mathbf{p}} = \mathbf{W} \cdot \mathbf{p} + \xi (\mathbf{D} \cdot \mathbf{p} - [\mathbf{D} : \mathbf{p}\mathbf{p}] \mathbf{p})$$

\mathbf{W} vorticity tensor; \mathbf{D} deformation rate tensor

$$\xi = \frac{\beta^2 - 1}{\beta^2 + 1} \text{ Shape factor}$$

$$\beta = \frac{L}{D} \text{ Aspect ratio}$$



Prolate and oblate ellipsoids; cylinders

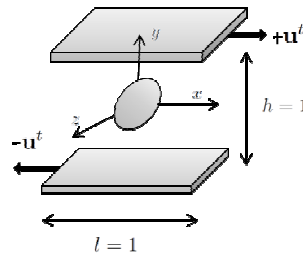
Direct numerical simulations

Fictitious domain method:

- particle defined by a level set
- Lagrange multiplier for rigidity condition
- Velocity imposed on upper and lower walls
- Finite element solution

Navier-Stokes + extra-stress

→ velocity and pressure on the whole domain



Particle position update:

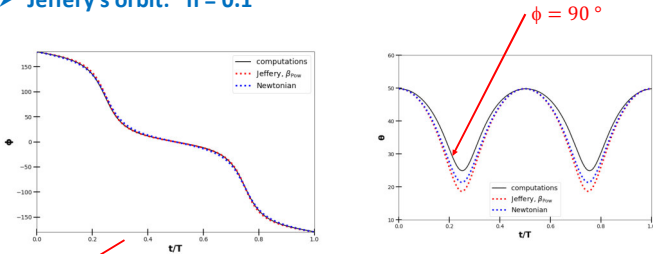
- Velocity of center of mass: $\mathbf{u}(X_c)$
- Rotation vector: $\vec{\omega} = \frac{1}{2} \nabla \times \mathbf{u}(X_c)$
- Adams-Bashfort scheme

$$\frac{d\mathbf{X}_c}{dt} = \mathbf{u}(X_c); \frac{d\mathbf{p}}{dt} = \vec{\omega} \times \mathbf{p}$$

Power Law fluid

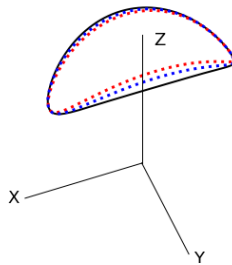
$$\eta = K \dot{\gamma}^{n-1}$$

Jeffery's orbit: $n = 0.1$



$$T = \frac{2\pi(\beta^2 + 1)}{\dot{\gamma}\beta} \rightarrow \beta_{\text{equivalent}}$$

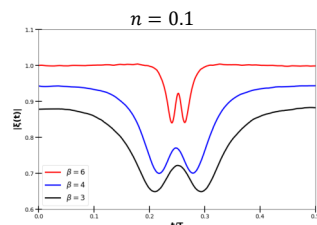
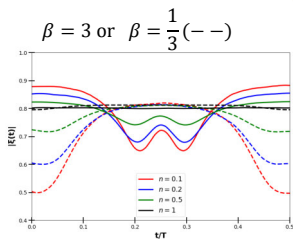
- prolate $T_{\text{Newt}} < T_{\text{Pow}} \rightarrow \beta_{\text{equivalent}} \nearrow$
- oblate $T_{\text{Newt}} > T_{\text{Pow}} \rightarrow \beta_{\text{equivalent}} \searrow$



n	$\beta = 3$				$\beta = 1/3$			
	T_{Pow}	β_{Pow}	ξ_{Pow}	$\langle \ \Delta \mathbf{p}\ \rangle$	T_{Pow}	β_{Pow}	ξ	$\langle \ \Delta \mathbf{p}\ \rangle$
1	21.08	3.02	0.8029	0.007	21.94	0.320	0.8139	0.004
0.5	21.85	3.16	0.8181	0.02	21.34	0.326	0.8082	0.04
0.2	22.76	3.32	0.8338	0.04	20.92	0.334	0.7995	0.08
0.1	23.83	3.51	0.8497	0.05	20.83	0.336	0.7975	0.11

$\beta_{\text{equivalent}} = \text{Cte} ?$

$$\|\xi(\mathbf{p})\| = \frac{\sqrt{\omega_x^2 + \omega_y^2 + (\omega_z + \frac{\dot{\gamma}}{2})^2}}{\frac{\dot{\gamma}}{2} \sqrt{(p_x p_z)^2 + (p_y p_z)^2 + (1 - p_y^2 - p_z^2)^2}}$$



Oldroyd-B fluid

$$\sigma_f = -p\mathbf{I} + 2\eta_f \varepsilon(\mathbf{u}) + \tau; \quad \tau = \frac{\eta_e}{\lambda} (3\mathbf{s} - \mathbf{I})$$

$$\frac{\partial \mathbf{s}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{s} - (\nabla \mathbf{u} \cdot \mathbf{s} + \mathbf{s} \cdot \nabla \mathbf{u}^t) + \frac{1}{\lambda} (3\mathbf{s} - \mathbf{I}) = 0$$

λ relaxation time from 0.1 to 5 → Main axis // vorticity axis

