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# Multi Item Capacitated Lot Sizing Problem with Stochastic Demand Timing and Setup Times

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## 1 Introduction

Production planning under uncertainty has always been a challenging task for practitioners and a frequently visited research avenue for researchers. In order to draw attention to the growing trend on this subject and to indicate possible research directions, two surveys on stochastic lot sizing problems can be found in [1] and [4]. These reviews show that the uncertainty in demand quantity is the main focus of researchers. We believe that a major drawback of previous studies is that uncertain demands in different periods are uncorrelated and treated as independent random variables, which is often unrealistic in practical settings. To answer this criticism, [2] have recently proposed a novel way of modelling the uncertainty on demand in the single-item dynamic lot sizing problem. In their approach, the quantities of demands are deterministic but their timing might be stochastic, i.e. they might fully occur in a given window of multiple periods, with a given probability for each period. Since demand quantities are known but their occurrences are stochastic, demands are naturally correlated.

In our paper, we consider the multi-item capacitated case, to consider more practical settings than in [2], by relying on some of the contributions of [2] for the single-item uncapacitated case.

## 2 Problem statement and mathematical model

In this study, the multi-item capacitated lot sizing problem with stochastic demand timing is addressed. On a production horizon discretized in  $T$  periods, the deterministic demands  $D_{kt}$  of item  $k$  in period  $t$  are known, for which no backlog or lost sale is allowed. Moreover, stochastic demands  $d_i$  of item  $k(i)$  are also given, for which the timing is stochastic with probability  $p_t^i \geq 0$  for period  $t$  within a given time interval  $[l^i, u^i] \subseteq [1, T]$ . The demand quantity  $d_i$  is not stochastic, and backlog is allowed until period  $u^i$ . The objective of the problem is to determine a production plan that minimizes the total expected cost and that for which both the deterministic and randomly occurring stochastic demands must be satisfied.

Let  $[l^i, u^i] \subset [1, T]$  be an interval indexed by  $i$ , where the stochastic demand  $d^i$  of item  $k(i)$  fully occurs in a single period, with a probability of  $p_t^i \geq 0$  for each period  $t \in [l^i, u^i]$  and such that  $\sum_{t=l^i}^{u^i} (p_t^i) = 1$ .

The expected cost is calculated as follows:

$$EC^i(t) = \sum_{l=t}^{u^i} h_{k(i)l} \sum_{m=l+1}^{u^i} p_m^i + \sum_{l=l^i}^{t-1} b_{k(i)l} \sum_{m=l^i}^l p_m^i \quad (1)$$

where  $h_{k(i)l}$  is the holding cost of item  $k(i)$  in period  $l$ , and  $b_{k(i)l}$  is the backlog cost of item  $k$  in period  $l$ .

Since demand  $d^i$  is stochastic, the inventory variable is also stochastic, which makes the modelling our problem challenging. To deal with it, [2] propose to model the problem with binary variables  $z_{lt}$ , which is the fraction of the deterministic demand  $D_t$  produced in period  $l \leq t$ , and  $z_t^i$ , the fraction of the stochastic demand quantity  $d_i$  produced in period  $l \leq u^i$ . Continuous variables  $x_{k,t}$ , resp. binary  $y_{k,t}$ , model the production quantity, resp. the setup state, of item  $k$  in period  $t$ . The mathematical model is formalized below:

$$\begin{aligned} \min z = & \sum_{k=1}^K \sum_{t=1}^T f_{kt} y_{kt} + \sum_{k=1}^K \sum_{t=1}^T \sum_{l=t}^T (c_{kt} + \sum_{m=t}^{l-1} h_{km}) z_{klt} D_{kt} \\ & + \sum_{i \in I} \sum_{t=1}^{u^i} (c_{k(i)t} + EC^i(t)) z_t^i d^i \end{aligned} \quad (2)$$

$$\sum_{l=1}^t z_{klt} = 1 \quad \forall t = 1, \dots, T, k = 1, \dots, K \quad (3)$$

$$\sum_{l=1}^{u^i} z_l^i = 1 \quad \forall i \in I \quad (4)$$

$$\sum_{k=1}^K a_{kt} \left( \sum_{l=t}^T z_{ktl} D_{kl} \right) + \sum_{i \in I; t \leq u^i} a_{k(i)t} z_t^i d^i + \sum_{k=1}^K su_{kt} y_{kt} \leq cap_t \quad \forall t = 1, \dots, T \quad (5)$$

$$\sum_{l=t}^T z_{ktl} D_{kl} + \sum_{i \in I; t \leq u^i} z_t^i d^i \leq \left( \sum_{l=t}^T D_{kl} + \sum_{i \in I; t \leq u^i, k=k(i)} d^i \right) y_{k,t} \quad \forall t = 1, \dots, T, \forall k = 1, \dots, K \quad (6)$$

$$y_{kt} \in \{0, 1\} \quad \forall t = 1, \dots, T, \forall k = 1, \dots, K \quad (7)$$

$$0 \leq z_{ktl}, z_t^i \leq 1 \quad \forall i \in I, \forall t = 1, \dots, T, \forall l = t, \dots, T, \forall k = 1, \dots, K \quad (8)$$

The objective function (2) minimizes the total expected cost including set up, production, holding and backlog costs. Constraints (3) and (4) ensure that both deterministic and stochastic demands are satisfied, respectively. The limited production capacity is ensured in Constraints (5), where  $a_{kt}$  is the capacity consumed by one unit of item  $k$  in period  $t$ ,  $su_{kt}$  is the set up time of item  $k$  in period  $t$  and  $cap_t$  is the capacity available in period  $t$ . Constraints (6) link the setup and production quantity variables, where  $(\sum_{l=t}^T D_{kl} + \sum_{i \in I; t \leq u^i, k=k(i)} d^i)$  is the upper bound of  $x_{k,t}$ .

### 3 Numerical results

To validate and analyze the mathematical model, a set of instances defined from the instances generated by [3] are tailored according to the settings of our problem. In particular, the probability of occurrence of a stochastic demand in any period  $t \subset [l^i, u^i]$  is normally distributed. The instances include 100 products and 24 periods, and each product has 10 stochastic demands. The standard solver IBM ILOG CPLEX 12.9 is used.

The preliminary results (Table 1) show that a feasible solution cannot be reached, displayed by “Nf” for “No feasible”, for most instances after 60 seconds. When limiting the computational time to 120 and 300 seconds, a feasible solution can be reached but the optimality gap is significant. These results motivate us to develop heuristic approaches to solve the problem for large instances.

### 4 Conclusions and perspectives

In this work, inspired from the work of [2], a mixed integer linear model is proposed for the multi-item capacitated lot sizing problem when the demand timing is stochastic. The model has been tested on various instances to show its limitations.

A Lagrangian relaxation heuristic is being developed that relies on the dynamic programs proposed in [2]. Constraint (5) in the model is relaxed to solve uncapaci-

Table 1: Numerical results obtained after 60, 120 and 300 seconds

<b>Ins</b>	<b>Gap (%)</b>			<b>Ins</b>	<b>Gap (%)</b>		
	<b>60</b>	<b>120</b>	<b>300</b>		<b>60</b>	<b>120</b>	<b>300</b>
<b>1</b>	Nf	Nf	0.85	<b>12</b>	Nf	2.29	1.45
<b>2</b>	Nf	5.44	4.59	<b>13</b>	Nf	Nf	4.37
<b>3</b>	Nf	Nf	1.09	<b>14</b>	Nf	2.37	0.84
<b>4</b>	Nf	2.51	1.28	<b>15</b>	Nf	37.87	37.87
<b>5</b>	Nf	8.24	3.72	<b>16</b>	Nf	1.18	0.83
<b>6</b>	54	54	54	<b>17</b>	Nf	Nf	2.26
<b>7</b>	Nf	2.01	1.87	<b>18</b>	38.75	2.41	2.16
<b>8</b>	Nf	3.37	1.19	<b>19</b>	7.12	3.07	0.66
<b>9</b>	Nf	3.63	1.23	<b>20</b>	Nf	2.40	1.80
<b>10</b>	Nf	4.26	0.69	<b>21</b>	9.86	2.55	2.55
<b>11</b>	Nf	3.71	3.66	<b>22</b>	8.54	1.86	1.25

tated single-item problems. In the workshop, we plan to present this approach and the corresponding numerical results.

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