



HAL
open science

Drag regimes of circular discs at different inclinations

Sukruth Satheesh, Andreas Spohn, Franck Kerhervé, Laurent Cordier

► **To cite this version:**

Sukruth Satheesh, Andreas Spohn, Franck Kerhervé, Laurent Cordier. Drag regimes of circular discs at different inclinations. *Ocean Engineering*, 2022, 266, pp.112931. <10.1016/j.oceaneng.2022.112931>. <hal-03861718>

HAL Id: hal-03861718

<https://hal.science/hal-03861718v1>

Submitted on 20 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Drag regimes of circular discs at different inclinations

Sukruth Satheesh^{a,*}, Andreas Spohn^a, Franck Kerhervé^a, Laurent Cordier^a

^a*Département Fluides, Thermique et Combustion, Institut Pprime, UPR 3346 CNRS,
ENSMA, Université de Poitiers, 86360 Futuroscope-Chasseneuil, France*

Abstract

Experiments on circular and square plate models were performed in a low-speed water tunnel at a fixed Reynolds number [$\mathcal{O}(10^4)$] for inclinations between 0° and 90° with respect to the flow direction. The experiments focus on drag force evolution and the associated flow structures to predict the drag regimes across the range of inclinations considered. Measurements show that drag increases rapidly with inclination for small angles, while at high angles it being nearly independent of inclination. The change between these drag regimes takes place at an inclination of 54° , with hydrogen bubble and PIV flow visualizations indicating this to be accompanied by a change in the wake flow structure as well. Trailing vortices at low angles suggests the existence of induced drag, while shedding of shear layers from all around model circumference at high angles points to the dominance of pressure drag as in the case of bluff bodies.

Keywords: Drag force, PIV, inclination, Cross-flow disc, bluff body wake

1. Introduction

Understanding the drag behaviour of thin flat plates and cylinders in cross-flow is one of the fundamental problems of interest in fluid dynamics, and has been studied for a long time using theory, numerics, and experiments. It is a problem rich in physics, involving several flow phenomena including wake transition which are dependent on Reynolds number (Re , ratio of inertial force to viscous force, based on characteristic body length), along with other conditions such as orientation, geometry, amongst others. The dependency of forces, motion and wake

*Corresponding author

Email address: sukruth.satheesh@univ-poitiers.fr (Sukruth Satheesh)

14 transitions on Re , fineness ratio (χ , ratio of disc diameter to thickness) and inclination are even
15 more important in the case of flat circular plates in cross-flow. While flow around freely falling
16 discs has been studied both experimentally and numerically at low Reynolds numbers [$O(10^2)$]
17 ([1, 2, 3] and others), studies of low aspect ratio flat plates in a cross-flow with inclinations varying
18 between 0° and 90° at these intermediate Re remain scarce. The goal of this study is to better
19 understand the synergy of various drag contributing physical mechanisms, and for improving
20 the design of many engineering systems. These range from their potential use in Autonomous
21 Underwater and Micro Aerial Vehicles (AUV's, MAV's) for control surfaces and wings, marine
22 structures to energy harvesters or even ground-based structures such as billboards, solar panels and
23 floodlights wherein the operational Re is intermediate [$O(10^3 - 10^5)$].

24 Fail et al. [4] conducted one of the earliest set of experiments on understanding the effect of
25 plate geometry on drag forces in cross-flow. They found the generated wake to be not dependent
26 on the plate profile, especially for axisymmetric bodies. Calvert [5] performed experiments
27 involving pressure and wake velocity measurements on circular discs with fineness ratio of 32
28 and $Re = (3.5 - 5) \times 10^4$ at different inclinations, and found that the vortex shedding frequency
29 increases with disc inclination (angle measured w.r.t disc normal), and that the plane of vortex
30 shedding is not fixed but varies azimuthally with time when the disc face is perpendicular to the
31 flow. This latter finding has also been verified and explained in detail using hot-wire measurements
32 and dye-flow and laser-induced fluorescence snapshots by Miao et al. [6]. Torres and Mueller [7]
33 have performed experiments over different wing planforms at aspect ratios (AR , ratio of square of
34 wingspan to the wing area) for MAV applications. They observed very high lift forces and stall
35 angles in $AR \leq 1.5$ planforms, and a drop in drag after wing stall even as the angle of attack (AOA,
36 measured as the angle subtended by the model and flow direction) increases. But this feature has
37 not been explored further. Shields and Mohseni [8] have performed a series of experiments to
38 study the effect of side-slip on flat plates of different aspect ratios and configurations, over a range
39 of Reynolds numbers ($Re = (5 - 10) \times 10^4$) and AOA. The reduction in drag after stall for low AR
40 plates is attributed to the post-stall detachment of the trailing vortices leading to a drop in induced
41 drag. Although the force measurements have been corrected for blockage, it is not clear if the very
42 high blockage in these cases also affect the stall angle and drag reduction. DeVoria and Mohseni

43 [9] have conducted force and flow visualization experiments on low AR rectangular plates at
44 $Re = 8 \times 10^4$ over a range of inclinations, and report sustained high lift generation with inclination
45 occurring only in models of $AR < 1.5$ by reattachment of mean-flow due to the downwash of tip
46 vortex. It is also indicated that the closer tip vortices occurring in low AR plates delays the flow
47 reattachment point on the trailing edge compared to that observed on high AR plates.

48 In terms of AOA (denoted by α), available literature focuses mostly on lift and drag forces of
49 non-circular plates ranging from 0° to 45° . To our best knowledge, investigations covering details
50 of flow in the entire range of inclinations ($45^\circ \leq \alpha < 90^\circ$ in particular) remain scarce. Chen
51 and Fang [10] conducted a series of experiments to study the vortex shedding characteristics of
52 wall-to-wall mounted flat plates ($\chi = 1 - 10$) at several α , Reynolds numbers and plate chamfer
53 angle, and mention that chamfer angle has a stronger effect on the shedding phenomena than Re
54 due to the shift of flow separation point along the chamfer direction (from the front edge to rear
55 edge for reducing α). This shift was found to be dependent on the inclination especially for cases
56 $\alpha \leq 40^\circ$. Earlier studies [4] have identified minimal effect of geometry in low AR , high Re
57 scenarios. However, it has been reported by a few works [8, 9, 11, 12] that drag force has a weak
58 dependence on Reynolds number over a specific range of low aspect ratios ($0.75 \leq AR \leq 1.33$)
59 at $\alpha = 90^\circ$. For inclined circular plates, neither is it completely clear if the same dependency
60 holds, nor is the flow physics behind the post-stall drag behaviour well established. With flat plates
61 aligned with the flow direction, skin-friction drag dominates over the pressure drag component,
62 whereas in cross-flow plates, it is the opposite. They thus behave as so-called bluff bodies. Over
63 the intermediate angles, both pressure drag and skin-friction drag exist alongside induced drag. It
64 is not known if there exists or even what constitutes the limiting angle for a body to be deemed bluff.
65 Former studies have not identified the demarcation between these regimes. This work focuses on
66 identifying the inclinations at which this transition of drag regimes occur. In discs, the origin of
67 trailing vortices is not imposed by the plate geometry whereas in square plates the formation of
68 trailing vortices along the lateral sides is imposed. The aim of this work is on understanding and
69 tackling these questions by correlating the forces generated by discs over a range of inclinations to
70 the wake characteristics, and comparing them with that presented by square plates of equal area
71 operating at same conditions.

72 The paper is organized as follows. Section 2 describes the experimental setup and the utilized
73 instrumentation. Force measurements and flow visualization results are presented in section 3,
74 finally ending with an overarching discussion in section 4 that correlates forces to the visualizations
75 and details the underlying drag regimes at different angles of attack.

76 2. Experimental Setup

77 Experiments were conducted in a low-speed water tunnel to facilitate the combination of force
78 measurements, PIV and flow visualization. Figure 1 shows a schematic of the test section and
79 model arrangement. The overall test section is 2.1 m long and $0.51 \times 0.33 \text{ m}^2$ in cross-section,
80 the freestream velocity (U_∞) ranging from 0.05 to 0.45 ms^{-1} . Optically transparent circular disc
81 models of diameter (d) 0.11 m and thickness (t) 2.2 mm were fabricated from polycarbonate,
82 ensuring the fineness ratio ($\chi = 50$) is similar to that observed in literature. Model edges were
83 chamfered by 60° in order to fix the flow separation locations and minimize the effect of flow
84 reattachment on the generated forces. The model was mounted in the tunnel such that is equidistant
85 from all the boundaries, and connected to a force sensor located outside via carbon fibre sting.
86 Interference effects was minimized by ensuring that the horizontal strut length of the sting was
87 greater than $2d$. The disc inclination is measured as the angle subtended by the disc surface with
88 flow direction (α), as highlighted in figure 1. Therefore, $\alpha = 90^\circ$ corresponds to the case when
89 disc face is normal to the flow, while $\alpha = 0^\circ$ to when the disc face is along flow direction. In
90 this work, AOA and inclination correspond to the same variable and are used interchangeably.
91 With disc inclination being the primary variable of interest, and to reduce model swap during
92 experimentation, a hinge-based mechanism was developed in order to continuously vary the disc
93 inclination in steps of 10° . Comparison with less invasive fixed angle model mounts were also
94 conducted in order to assure that the hinge mechanism did not affect the force measurements.
95 The blockage ratio, computed as the ratio of disc area to the water tunnel cross-sectional area
96 was around 5.5% when $\alpha = 90^\circ$, and reduces with reducing α . The hinge/pivot arm length of
97 the mechanism was very small [$O(1 \text{ mm})$], ensuring that the change in gap between model and
98 boundaries do not influence the drag measurements. Square plates with exposed area equal to that
99 of discs were similarly fabricated and tested.

100 A single component piezoelectric load-cell with a resolution of 0.002 N and peak load capacity
101 of 10 N was utilized to measure the total drag. Calibration using known weights was performed to
102 achieve a high load resolution. Particle Image Velocimetry (PIV) [13] was undertaken to visualize
103 the flow quantitatively. A 532 nm green continuous wave laser and equipped with ancillary optics
104 was used to generate a planar laser sheet and aligned onto the inclined disc for wake visualization.
105 A 2048×2048 pixels resolution high speed camera with a f/1.2 12.5-75 mm focal length lens
106 and neutrally buoyant seeding particles of 20 μm diameter was used for experiments. The laser
107 sheet was horizontal and imaging undertaken through a mirror mounted underneath the tunnel at
108 45°, the image acquisition rate being 80 fps. The captured images were processed by intensity
109 thresholding with the non-illuminated regions masked. PIV analysis was done iteratively using
110 multi-pass technique, the starting and final interrogation window sizes being 128×128 and 54×54
111 pixels respectively, and overlap of 50% to yield 4575 vectors in each velocity field, covering
112 approximately a field of view of around 36.86×36.86 cm. The outlier vectors obtained from the
113 cross-correlation scheme were identified after applying a threshold and replaced with new values
114 obtained from averaging the vectors neighbouring the outlier. Hydrogen bubble [14] sheet was
115 generated in the horizontal plane located approximately mid-body to visualize the wake evolution.
116 A 50 μm stainless steel wire mounted taut between 2 supports is used as cathode– source of
117 hydrogen bubbles when supplied with a voltage of 80 V from a regulated power supply. The wire
118 was located just upstream of the model and in contact with its leading edge in order to seed the
119 flow, especially the model base region. Pulsed or continuous bubble sheets can be generated using
120 a computer-controlled electronic switch which enable time lines to be marked outside the central
121 recirculation region of the wake.

122 3. Results

123 The circular disc was mounted within the water tunnel such that the disc normal is in the middle
124 of test section, thus ensuring that the model was equidistant to the tunnel walls and the free surface.
125 With the expected disc drag forces at this Re being very low [$O(10^{-1}) N$], the effect of noise and
126 drift would be significant and is essential to minimize their effect. Thus, the force data from the
127 piezoelectric sensor was sampled at 5 kHz, and the measured signal filtered and compensated for

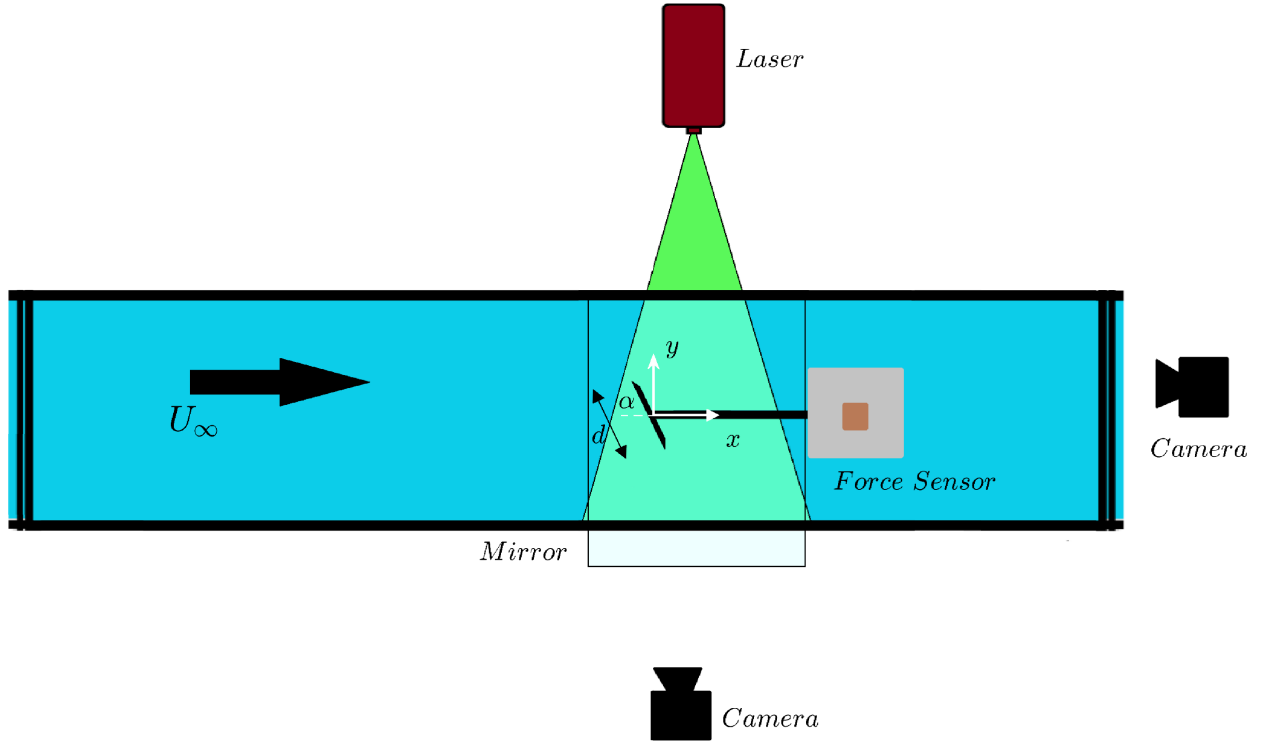


Figure 1: Top view of the experimental setup

128 drift. The drag generated only by the sting was subtracted from the total drag in order to calculate
 129 the model drag. The filtered drag signal was averaged (F) and non-dimensionalized using dynamic
 130 pressure and the maximum exposed disc area (A , at $\alpha = 90^\circ$), as seen in equation 1 to obtain the
 131 drag coefficient (C_D) with ρ being fluid density. The data was acquired for 180 seconds at each
 132 inclination, the acquisition undertaken after flow unsteadiness due to model rotation is minimized.

$$C_D = \frac{F}{\frac{1}{2}\rho U_\infty^2 \cdot A} \quad (1)$$

133 Reynolds number ($Re_d = U_\infty d / \nu$) is calculated with disc diameter as the characteristic length
 134 and ν the kinematic viscosity of water, is fixed at 1×10^4 .

135 3.1. Effect of inclination on drag force

136 Figure 2 presents the variation of drag coefficient for disc and square plate as a function of
 137 inclination, the y-axis plotted in logarithmic scale to highlight the change in order of magnitude of
 138 drag coefficient. By using a fixed value of exposed disc area for non-dimensionalization, the trend

139 of drag coefficient with inclination is in essence that of the drag force. This plot indicates that C_D
 140 (plotted as ● and ■, for disc and square plate, respectively) is nearly constant at ~ 1.3 over a small
 141 range of inclinations ($\alpha \sim 70^\circ - 90^\circ$). However, the reduction in C_D with decreasing α is rapid
 142 post 30° , reaching a value of $\simeq 0.015$ at $\alpha = 0^\circ$. The near similar C_D trends for disc and square
 143 plate indicates the model geometry to have negligible bearing on the drag forces.

144 With discs and square plates being canonical structures, a significant number of studies have
 145 been performed over the years spanning a wide range of Reynolds numbers, blockage ratios, model
 146 fineness ratios and other test conditions. For the sake of brevity, only a few results from literature
 147 have been plotted here. The current measurements are in good agreement with those of Satheesh
 148 and Huera-Huarte [12] and Maskell [15] for square plates and discs, respectively in cross-flow.
 149 Results by Blevins [16] for a plate aligned with the flow ($\alpha = 0^\circ$) have also been plotted here only
 150 for the sake of completeness, and are very close to the obtained results. This confirms the capability
 151 of force sensor to measure very low force magnitudes [$O(10^{-1}) N$]. The work of DeVoria and
 152 Mohseni [9] involving lift and drag measurements for square plates over $\alpha = 0^\circ - 45^\circ$ at Reynolds
 153 number of 8×10^4 has also been included in the same plot. It can be seen that our results are in good
 154 agreement over most of the inclinations. Hoerner [17] presents an empirical formulation for drag
 155 coefficient– $C_D = 1.17 \cdot \sin(\alpha)$, indicating its origins to be from sailing and applicable at $\alpha \geq 45^\circ$.
 156 This equation seems to have been updated by incorporating more results by NASA [18], indicated
 157 as $C_D = 1.28 \cdot \sin(\alpha)$, and has been used in the figure. It can be observed that there is a very good
 158 match for $40^\circ \leq \alpha \leq 90^\circ$, both discs and square plates.

159 In order to account for the change in blockage with model inclination and the consequent
 160 wake realignment, correction technique developed by Gould [19] for plates in cross flow has been
 161 implemented, given by equation 2. According to this equation, the corrected drag coefficient (C_{D_c})
 162 is given as a function of measured drag coefficient (C_D) and blockage ratio (B), which in turn
 163 depends on the models' projected area in streamwise direction.

$$C_{D_c} = 1.163 \cdot [1 + 2.81 \cdot (C_D \cdot B) - 0.96 \cdot (C_D \cdot B)^2] \quad (2)$$

164 The model projected area is directly related to its inclination $0.25\pi(d \sin \alpha)^2$ and $(a \sin \alpha)^2$,

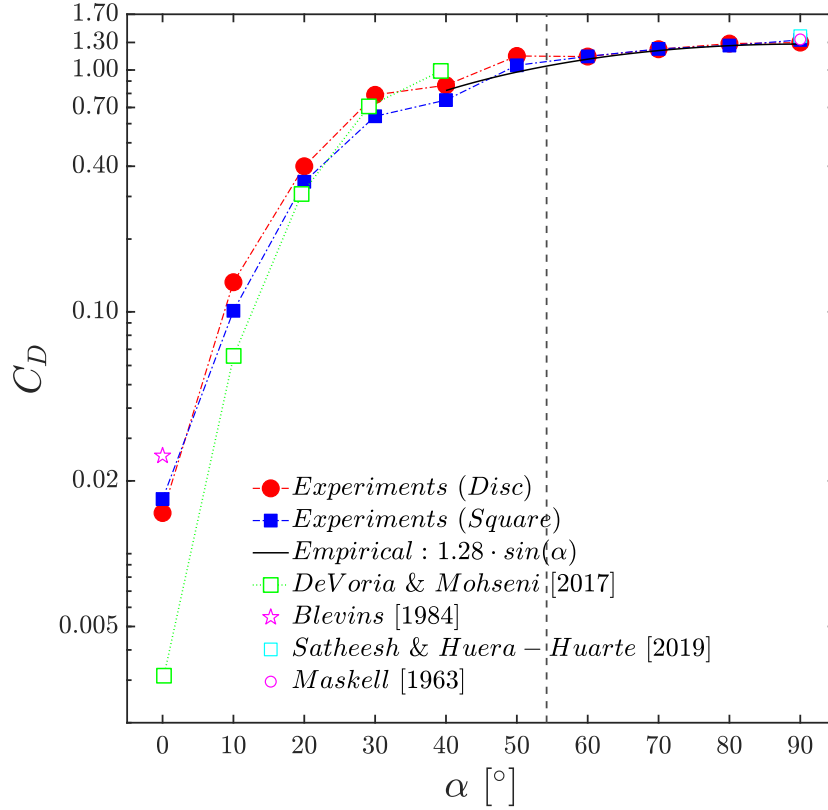


Figure 2: Variation of measured drag coefficient with inclination for disc and square plate. Dashed vertical line marks the critical angle of transition between the induced and pressure drag dominated regimes.

165 for discs and square plates respectively. Thus, the projected area is maximum when $\alpha = 90^\circ$ and
 166 reduces with α . Figure 3a presents the variation of corrected drag coefficient with inclination,
 167 showing a high correlation coefficient ($C_{D_c} = -0.01572\alpha^2 + 0.06443\alpha + 1.344, R^2 = 0.996$). It
 168 can be observed that C_{D_c} increases in almost a linear manner with increasing α , but the rate of
 169 increase in drag is significantly lowered once α is over 60° . To corroborate this, the empirical drag
 170 formulation along with the projected area relation (for B calculation) was substituted into equation
 171 2, generating a trigonometric relation given by–

$$C_{D_c} = 1.163 + 0.2399 \cdot \sin^3(\alpha) - 0.006 \cdot \sin^6(\alpha) \quad (3)$$

172 To identify the change in trend/curvature of the drag plot, the inflection point is calculated for
 173 equation 3 using the $\partial^2 C_{D_c} / \partial \alpha^2 = 0$ relation, yielding $\alpha_{crit} = 54.2^\circ$. This has been plotted as

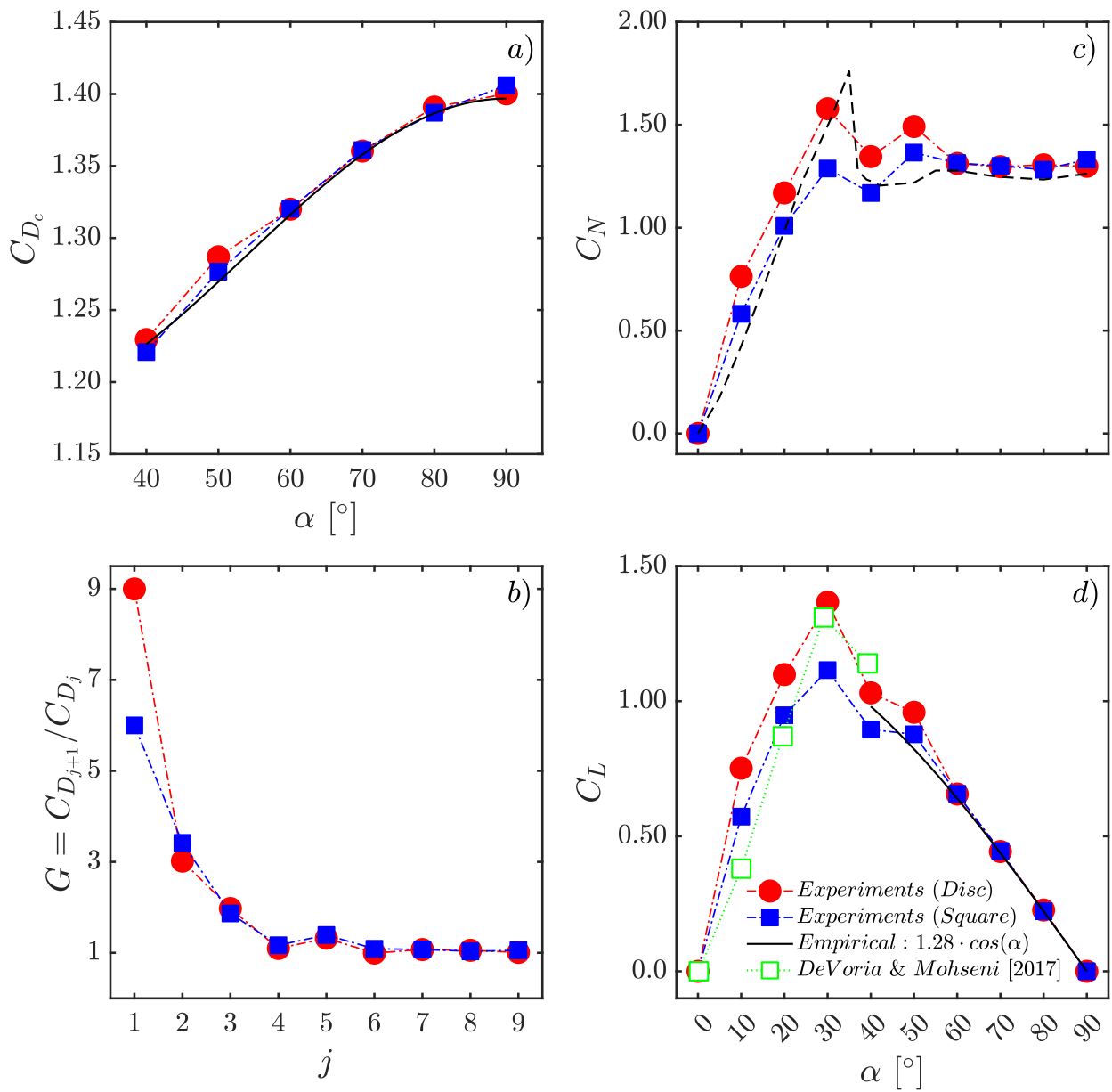


Figure 3: Variation of (a) corrected drag coefficient, (b) drag gain, (c) normal force coefficient, and (d) lift coefficient with inclination

174 a dashed vertical line in figure 2. This information, obtained purely from empirical drag relation
 175 and current model/test section dimensions explicitly validates the aforementioned observations—
 176 that the gain in drag coefficient ($G = C_{D_{j+1}}/C_{D_j}$) with inclination reduces after α_{crit} to a value
 177 around 1. This can also be seen in figure 3b where G remains nearly constant after $j = 6$

178 ($C_{D_{60}}/C_{D_{50}}$), implying that drag coefficient is nearly independent of inclination. To reinforce
 179 the observations regarding α_{crit} , curve-fitting using a quartic polynomial ($C_{D_{circ}} = 0.08368\alpha^4 -$
 180 $0.02684\alpha^3 - 0.3507\alpha^2 + 0.5024\alpha + 1.024$, $R^2 = 0.99$; $C_{D_{sq}} = 0.07636\alpha^4 - 0.05447\alpha^3 - 0.2915\alpha^2 +$
 181 $0.5658\alpha + 0.9431$, $R^2 = 0.99$) was employed on the current experimental data over the entire range
 182 of inclinations. On solving the expressions for inflection point, the resulting critical angle for discs
 183 and squares are very close to that obtained from the empirical relation.

184 For inclined thin flat plates in a flow, the suction peak does not exist on the plate surface.
 185 Therefore, the contribution of axial force component on lift and drag is small and can be neglected.
 186 This assumption allows utilizing the C_D and α information to estimate normal force ($C_N \approx$
 187 $C_D/\sin \alpha$) and lift force ($C_L \approx C_N \cos \alpha$) coefficients, which have been plotted in figures 3c and
 188 3d respectively. The results in figure 3c clearly highlight the rapid rise in C_N with α followed by
 189 an abrupt drop to a value of ~ 1.3 , the drop occurring around 40° . Similarly, figure 3d presents
 190 lift curves for discs and squares. At high inclination ($\alpha \rightarrow 90^\circ$), the data for squares and discs
 191 match very well with the empirical formulation ($C_L = 1.28 \cdot \cos \alpha$), deviating only when $\alpha < 60^\circ$.
 192 This confirms the assumption of very low axial force magnitudes. At low α , the trends are in line
 193 with results of DeVoria and Mohseni [9] for squares. Even though exact magnitudes of C_N and C_L
 194 are not obtained through this method, they capture the trends extremely well, especially the stall
 195 location in lift curve.

196 3.2. Flow visualization

197 PIV and hydrogen-bubble flow visualizations were undertaken at specific inclinations in the
 198 near wake to analyze in greater detail the transition process observed in force measurements. The
 199 2D-2C PIV results are time averaged in order to describe the mean-flow characteristics at each
 200 inclination. The time window (Δt) of 0.25 s for averaging was selected after analyzing the results
 201 over several other window sizes, with this window size providing a balance between the observable
 202 flow structures and their magnitude comparable to that presented in literature [9, 11, 12].

203 Figure 4 presents a montage of instantaneous hydrogen bubble and time-averaged PIV results
 204 (xy plane). Vorticity (ω_z) obtained through PIV has been quantified based on the coordinate
 205 axes setup, with clockwise rotation being negative and counter-clockwise positive, denoted by

206 red and blue respectively in the colormap, and non-dimensionalized using disc diameter and the
207 flow velocity. Each row corresponds to a specific inclination, the first column presenting the
208 instantaneous hydrogen bubble snapshots and the second column vorticity from PIV. The plots
209 have been arranged in decreasing inclination, starting from the cross-flow condition ($\alpha = 90^\circ$) to
210 $\alpha = 30^\circ$ case, the lowest inclination of interest in this scenario. Figures 4a and 4b present flow
211 separation from the edges and visible wake evolution, which is highly symmetric. The measured
212 vorticity extends approximately two disc diameters downstream. Figure 4a shows the various
213 Kelvin-Helmholtz (K-H) vortices that are generated by instabilities in the separated flow which
214 appear as extended shear layers in figure 4b. At $\alpha = 60^\circ$ (figures 4c, 4d), differences in the separated
215 flow from the leading (LE) and trailing (TE) edges are clearly noticeable, with separated flow from
216 the LE presenting K-H vortices and extending $1.75d$ downstream as before, while that from TE
217 being smaller ($1.5d$). In figures 4e and 4f, the LE separated flow appears to be similar as before but
218 with higher deflection compared to the former cases, resulting in increased proximity of the shear
219 layer to the disc base region. Meanwhile at TE, a vortex evolves and traverses downstream, which
220 appears to be fixed/pinned at $x \sim 0.5d$ in the time-averaged snapshot, unlike the observations in
221 previous inclinations. At $\alpha = 40^\circ$ (figures 4g, 4h), the recirculating flow from LE is in close
222 proximity to the disc base while the TE vortex appears to be pinned as in figure 4f, but is not
223 prominent. Finally, at $\alpha = 30^\circ$ (figures 4i, 4j), the recirculating flow from LE completely envelops
224 the disc and flow exits smoothly from TE with no vortex formation.

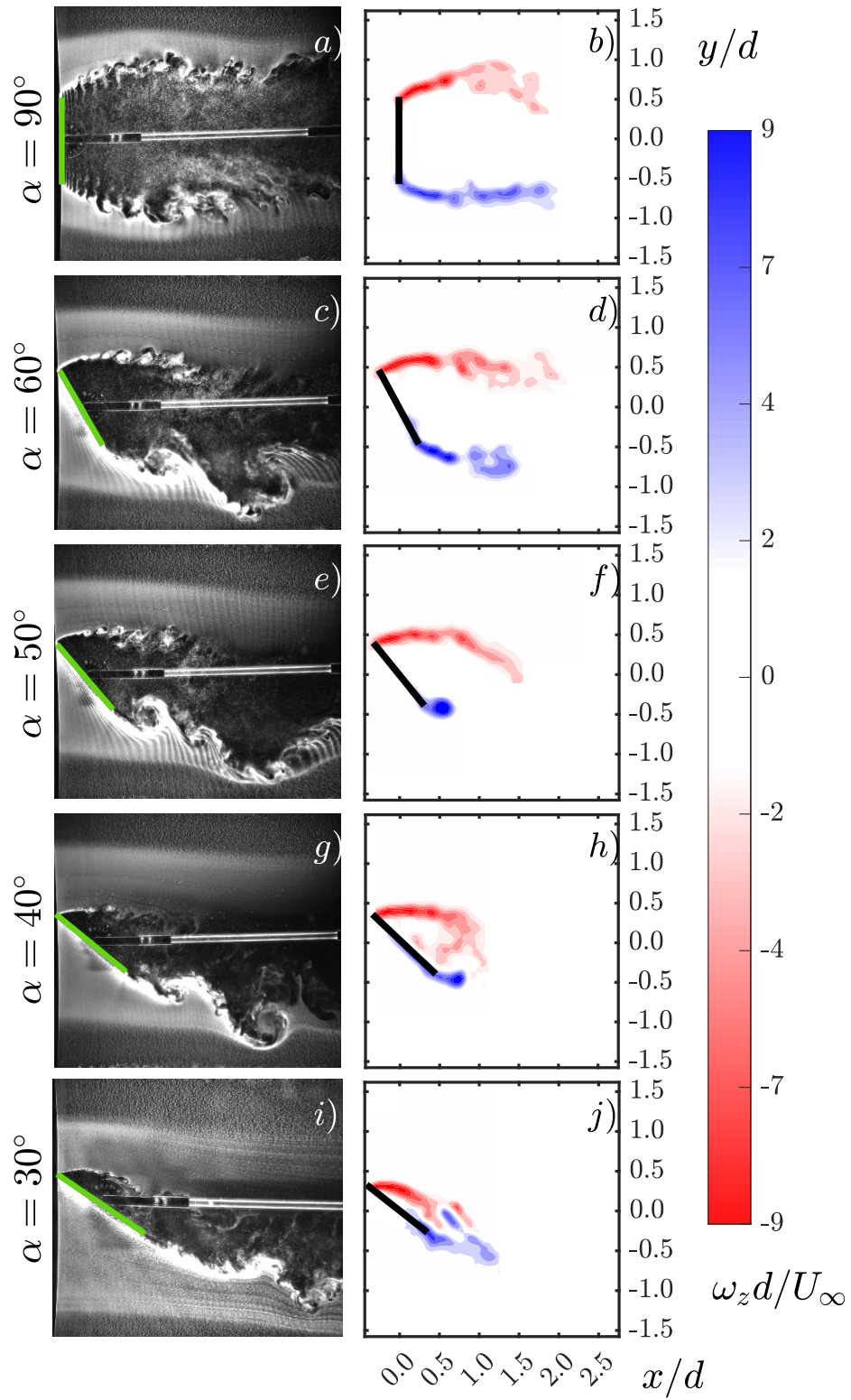


Figure 4: Instantaneous hydrogen bubble and PIV disc flow visualizations at different inclinations

225 These results precisely indicate the various flow structures existing on discs at different incli-
226 nations. The plots at $\alpha = 90^\circ$ is as expected involving symmetric separated flows with minimal
227 differences in non-dimensional vorticities as well. This has been observed by several other re-
228 searchers in different geometries ranging from discs, squares to even elliptical and rectangular
229 plates ([12, 11, 20], amongst several others) in cross-flow. The plots for α of 40° and 30° are
230 completely in line with those obtained by DeVoria and Mohseni [9] for square plates, validating
231 the absence of TE vortex at the lowest visualized inclination, and the current flow visualization
232 setup. Similarly, the flow structures observed by Linehan and Mohseni [21] for square plates' LE
233 and TE at α of 30° and 40° without sideslip matches well with the current observations.

234 Even though figure 4 highlights the specific flow structures at α of 60° and 50° that have not been
235 observed in literature till now, circulation ($\Gamma = \iint \omega_z \cdot ndS$) has also been evaluated to complement
236 the observations. Figure 5 presents the evolution of LE and TE circulation over inclinations of
237 interest. Time-averaged circulation values in the regions about LE and TE are represented by \blacktriangle
238 and \blacktriangledown respectively, and non-dimensionalized using the disc diameter and flow velocity. Based on
239 the sign convention, LE circulation values are negative but have been presented in the same plot to
240 enhance ease of comparison with the corresponding values at TE. The plot indicates that the TE
241 circulation monotonically decreases with decreasing α , starting from a value of approximately 4
242 and ending at nearly 0. Meanwhile, the LE circulation also starts at a value of around 4 as expected
243 at $\alpha = 90^\circ$, and decreases with α before presenting a distinct step in circulation at $\alpha = 50^\circ$
244 reaching a final value of approximately 1.2 at the minimum inclination visualized here. LE and
245 TE circulation values from DeVoria and Mohseni [9] for square plates at $\alpha \leq 45^\circ$ have also been
246 included in the same plot (non-dimensionalized using effective diameter instead of chord as in their
247 work), and it can be seen that their magnitudes and trends are well in agreement with the current
248 measurements.

249 Finally, time-averaged hydrogen bubble visualizations have been performed for both discs and
250 squares in the near wake to visualize the flow structures emanating from the lateral edges. This
251 is to verify that the observed changes in circulation can indeed be attributed to the differences
252 observed in the drag force. To visualize yz plane, a vertical laser sheet was generated at $x = d/2$
253 downstream of the model. This visualization angle aligns the flow direction to be outward from

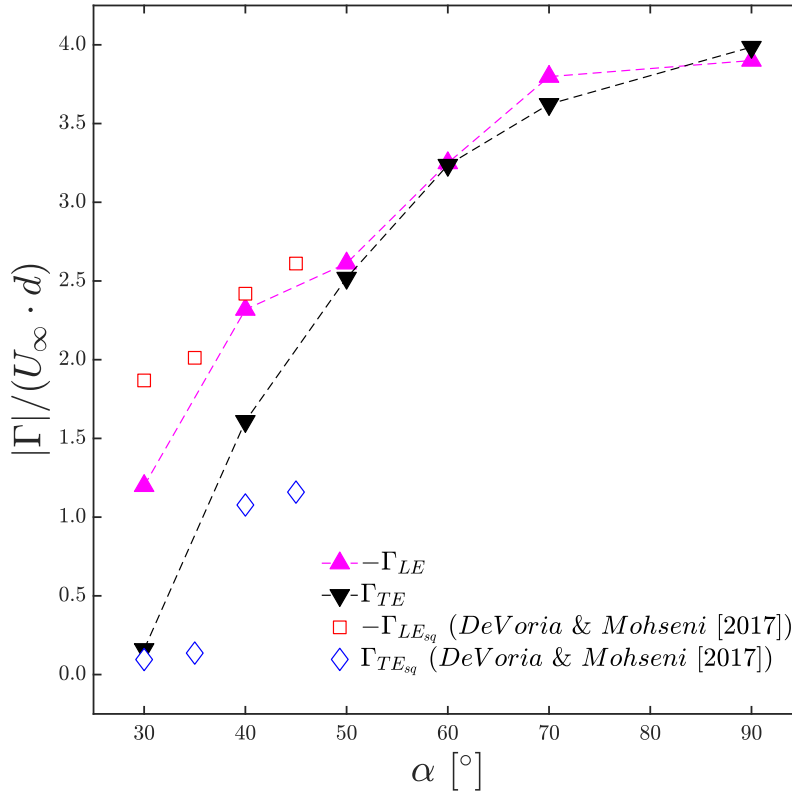


Figure 5: Variation of Leading Edge and Trailing Edge non-dimensional circulation for discs with inclination

254 the plane. The central white region highlights the model support. The model's axis of inclination
 255 is vertical. Model LE exists to right of the support and TE to the left, emphasized by red and
 256 blue outlines respectively. Figure 6 presents this scenario for specific inclinations. Sectors of
 257 higher light intensity indicate regions wherein flow is steady, while the diffuse ones are regions of
 258 increased skewing and/or flow unsteadiness. A horizontal hydrogen bubble sheet was generated
 259 upstream of the model and closer to the lateral edge, which appears as intense bright horizontal
 260 sectors located on either side of the model cutting across the vertical laser sheet in all the averaged
 261 images.

262 From figures 6a and 6b ($\alpha = 90^\circ$), the near wake for discs and squares are remarkably similar
 263 and symmetric about the central support. This indicates that the flow separation process is blind
 264 to the location of edge in this condition. At $\alpha = 60^\circ$, a bright sector exists downstream and below
 265 the model that corresponds to the flow from the lateral edge, with diffuse regions occurring in the
 266 direction towards LE. Figure 6c also shows that the separated flow for disc is more curved than to

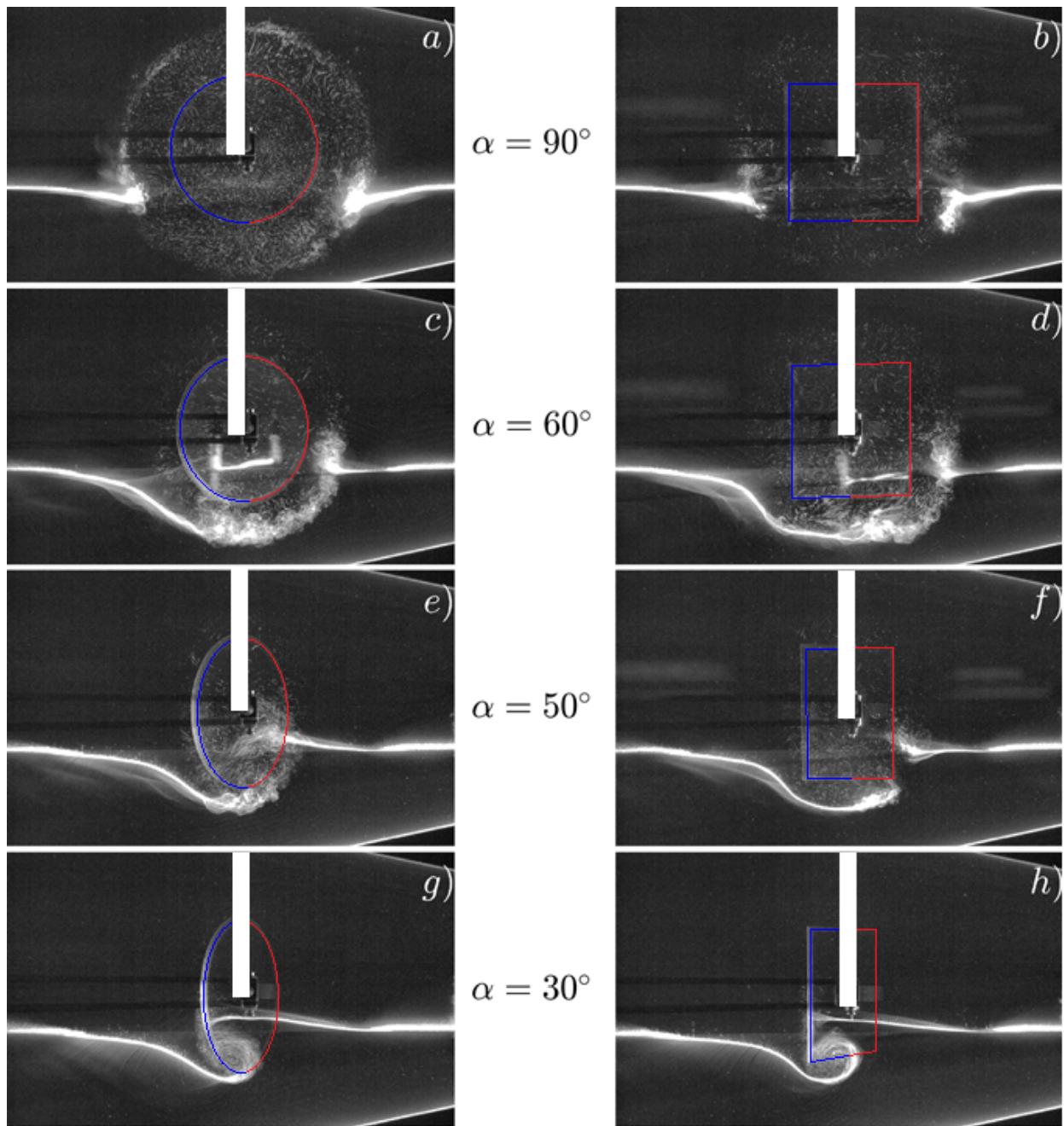


Figure 6: Time-averaged hydrogen bubble flow visualizations of lateral edge in near wake at different inclinations for discs and squares. Model LE (red) and TE (blue) is to the right and left of central support, respectively.

267 that in 6d for squares, the flow following more closely the model geometry. Further decrease in
 268 inclination (figures 6e, 6f) results in an increase in the length of steady flow region closer to the TE,
 269 while increasing their curvature as well. Finally at $\alpha = 30^\circ$ (figures 6g, 6h), the bubble sheet has

no significant distortion and presents a prominent vortex emanating from the lateral edge (wing-tip vortex), located approximately below the vertical support for both discs and squares. The size of wing-tip vortex is also significant and extends approximately 25-30% of the model diameter at this downstream location. There also exists a bright horizontal sector above the vortex highlighting the extent of the recirculating bubble that exists on the model base region, which was also observed in PIV. In all, consolidating the circulation results from figure 5 and lateral flow visualizations of figure 6 presents a comprehensive picture of the flow around discs. At $\alpha = 90^\circ$, the symmetric flow condition implies circulation magnitudes to be near similar as well. With reducing inclination, the LE and TE circulation values diminish, while wing-tip vortices which are manifestations of a lifting surface ($\alpha = 30^\circ$) increase in prominence.

4. Discussion and final remarks

The results from drag force measurements and flow visualization have provided a detailed view regarding the drag regimes and their associated flow structures. The force measurements indicated that the changes in C_D magnitudes are almost negligible at high inclinations ($\alpha \sim 60^\circ - 90^\circ$), below which the drop in drag values is rapid. Although the work by Gould [19] is developed for cross-flow plates mounted centrally or in close proximity to a wind tunnel wall and specific test conditions ($0 < C_D \cdot B \leq 0.3$), the current work highlights that this correction technique is applicable for inclined models as well. However, it must be observed that there exists a range of inclinations over which this correction technique is valid, namely $45^\circ \leq \alpha \leq 90^\circ$. At lower inclinations the drag coefficient predicted by this equation is not comparable to the measured values. Using the empirical formulations ([17, 18]) turned out to be more accurate for comparing the current drag measurements, for they are independent of model geometry and developed for plates of finite aspect ratio. The visualizations elucidate reason for the observed drag behaviour—the reorganization process occurring between α of 60° and 50° of the separated flow from LE to a recirculating flow in close proximity to the disc base, and the separated flow from TE to a slow vortex shedding process that appears as a pinned vortex in PIV. This transition process is not dependent on plate geometry. Some of the flow structures observed by Gao et al. [22], Zhao et al. [23] in their numerical studies on discs and squares at $Re = 300$ and different inclinations are similar to that

298 observed here experimentally, indicating the effect of Re to be not very significant on flow physics
299 even though Re magnitudes are vastly different. However, changes in plate aspect ratio or Re may
300 have certain effects on α_{crit} owing to the change in strength and proximity of wing-tip vortices to
301 the mean-flow, and is beyond the scope of this work.

302 The current work in fact presents a potential origin for the empirical formulations– the aspect
303 of minimal effect of axial force on lift and drag due to the non-existence of the suction peak on
304 inclined thin plates. Besides, even by using the C_L , C_D results of DeVoria and Mohseni [9] to
305 perform an order of magnitude estimation of the induced drag ($C_{D_i} \sim (C_L)^2/\pi$, for square plates)
306 yields that the contribution of induced drag to the total drag (C_{D_i}/C_D , assuming the effect of wing
307 efficiency factor e to be not relevant for calculating this ratio) is dominant only for $0^\circ \leq \alpha \leq 40^\circ$,
308 after which it drops precipitously. This, along with the knowledge of skin-friction and pressure
309 drag components being the dominant contributors to total drag at $\alpha = 0^\circ$ and $\alpha = 90^\circ$ respectively,
310 reveals the various drag regimes. As α increases from 0° , the low AR plates function as lifting
311 surfaces. Thus, the contribution of induced drag to the total drag increases which peaks around
312 40° , and is mainly due to vortical wake generated by lift. Further increase in α results in lift
313 reduction due to stall, consequently reducing the induced drag. In the second regime ($\alpha > 50^\circ$),
314 drag is nearly independent of inclination. The slow drag evolution in this regime is well described
315 by the empirical laws given by Hoerner [17] and NASA [18]. Flow visualizations confirm the
316 absence of strong trailing vortices in this regime, suggesting that flow separation across the disc
317 circumference leads to recirculation zone formation with low base pressure. The transition region
318 between the flow regimes dominated by lift and flow separation ($40^\circ \leq \alpha \leq 60^\circ$) is marked by
319 the development of a vortex shedding process from the trailing edge and the absence of wing-tip
320 vortices. At the same time the difference in circulation magnitudes shed from leading and trailing
321 edges reduces. This points to lift reduction while total drag continues to rise, implying that the
322 contribution of pressure drag increases. In all, the current experiments highlight the various flow
323 regimes and expound the transition process. At α of 90° , the separated flow is symmetric, with low
324 base pressure. Flow continues to remain separated from model LE and TE, but with an asymmetry
325 induced by α till around 60° . This regime corresponds to pressure drag. Further reduction in
326 angle of attack results in a change in shedding process, resulting in separated flow from model

327 LE to approach closer to model base, while a slow vortex shedding is apparent at model TE, with
328 this process occurring till $\alpha = 40^\circ$. Further reduction in α results in separated flow from LE to
329 encapsulate the base region and no flow separation at TE, with strong tip vortices emanating from
330 lateral edges confirming strong lift forces and the consequent induced drag. These observations
331 reaffirm the current results of the bluff body definition being apt for models at inclination greater
332 than 50° .

333 Acknowledgements

334 The present work is funded by the French organization ANR under the project COWAVE
335 ANR-17-CE22-0008.

336 References

- 337 [1] S. B. Field, M. Klaus, M. G. Moore, F. Nori, Chaotic dynamics of falling disks, *Nature* 388 (6639) (1997)
338 252–254.
- 339 [2] P. Ern, F. Risso, D. Fabre, J. Magnaudet, Wake-induced oscillatory paths of bodies freely rising or falling in
340 fluids, *Annual Review of Fluid Mechanics* 44 (2012) 97–121.
- 341 [3] X. Tian, Z. Hu, H. Lu, J. Yang, Direct numerical simulations on the flow past an inclined circular disk, *Journal*
342 *of Fluids and Structures* 72 (2017) 152–168.
- 343 [4] R. Fail, J. A. Lawford, R. C. W. Eyre, Low-Speed-Experiments on the Wake Characteristics of Flat Plates normal
344 to an Air Stream, Tech. rep., Aeronautical Research Council R&M 3120 (1959).
- 345 [5] J. R. Calvert, Experiments on the flow past an inclined disk, *Journal of Fluid Mechanics* 29 (4) (1967) 691–703.
- 346 [6] J. J. Miao, T. S. Leu, T. W. Liu, J. H. Chou, On vortex shedding behind a circular disk, *Experiments in fluids*
347 23 (3) (1997) 225–233.
- 348 [7] G. E. Torres, T. J. Mueller, Low-aspect-ratio wing aerodynamics at low Reynolds number, *AIAA Journal* 42 (5)
349 (2004) 865–873.
- 350 [8] M. Shields, K. Mohseni, Effects of sideslip on the aerodynamics of low-aspect-ratio low-Reynolds-number
351 wings, *AIAA journal* 50 (1) (2012) 85–99.
- 352 [9] A. C. DeVoria, K. Mohseni, On the mechanism of high-incidence lift generation for steadily translating low-
353 aspect-ratio wings, *Journal of Fluid Mechanics* 813 (2017) 110–126.
- 354 [10] J. M. Chen, Y. C. Fang, Strouhal numbers of inclined flat plates, *Journal of Wind Engineering and Industrial*
355 *Aerodynamics* 61 (2-3) (1996) 99–112.

- 356 [11] J. N. Fernando, D. E. Rival, On vortex evolution in the wake of axisymmetric and non-axisymmetric low-aspect-
357 ratio accelerating plates, *Physics of Fluids* 28 (1) (2016) 017102.
- 358 [12] S. Satheesh, F. J. Huera-Huarte, Effect of free surface on a flat plate translating normal to the flow, *Ocean*
359 *Engineering* 171 (2019) 458–468.
- 360 [13] C. E. Willert, M. Gharib, Digital particle image velocimetry, *Experiments in fluids* 10 (4) (1991) 181–193.
- 361 [14] F. A. Schraub, S. J. Kline, J. Henry, P. W. Runstadler Jr, A. Littell, Use of hydrogen bubbles for quantitative
362 determination of time-dependent velocity fields in low-speed water flows, *Journal of Basic Engineering*.
- 363 [15] E. C. Maskell, A theory of the blockage effects on bluff bodies and stalled wings in a closed wind tunnel, Tech.
364 rep., Aeronautical Research Council R&M 3400 (1963).
- 365 [16] R. D. Blevins, *Applied fluid dynamics handbook*, New York.
- 366 [17] S. F. Hoerner, *Fluid Dynamic Drag*, Hoerner Fluid Dynamics, 1965.
- 367 [18] <https://www.grc.nasa.gov/WWW/k-12/airplane/kiteincl.html>, last Updated May 2021.
- 368 [19] R. W. F. Gould, Wake Blockage Corrections in a Closed Wind Tunnel for One or Two Wall-Mounted Models
369 Subject to Separated Flow, Tech. rep., Aeronautical Research Council R&M 3649 (1969).
- 370 [20] A. Hemmati, D. H. Wood, R. J. Martinuzzi, Effect of side-edge vortices and secondary induced flow on the wake
371 of normal thin flat plates, *International Journal of Heat and Fluid Flow* 61 (2016) 197–212.
- 372 [21] T. Linehan, K. Mohseni, Leading-edge flow reattachment and the lateral static stability of low-aspect-ratio
373 rectangular wings, *Physical Review Fluids* 2 (11) (2017) 113901.
- 374 [22] S. Gao, L. Tao, X. Tian, J. Yang, Flow around an inclined circular disk, *Journal of Fluid Mechanics* 851 (2018)
375 687–714.
- 376 [23] Y. Zhao, S. Gao, X. Zhang, X. Guo, X. Li, X. Tian, Direct numerical simulations on the flow past a thin square
377 plate, *Physics of Fluids* 33 (3) (2021) 034128.