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# Big Questions in Mathematics Education: A “Representations in Mathematics Teaching and Learning” Perspective

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*I summarize the perspective that I presented, as leader of TWG24 “Representations in Mathematics Teaching and Learning”, bringing attention to a shift in how groups of researchers in Mathematics Education think and talk about mathematical objects and their representations. I argue that such a shift has theoretical and practical implications that should be taken into account in future research, especially when addressing some “Big Questions in Mathematics Education”.*

*Keywords: Digital extension, mathematical objects, representations.*

## **A shift in how we think about mathematical objects and their representations**

In a recent paper (Baccaglini-Frank et al., 2022), we discussed what we see as a shift away from a Platonic conception of mathematical objects and their representations, characterizing various current lines of research in Mathematics Education. Such a shift opens new venues to how we think and talk about representations, which, in turn, influences how we use (and study) them in the teaching and learning of mathematics. In Plato’s allegory of the cave, representations of mathematical objects are seen as reflections of natural things; such reflections inspire artificial objects, of which we only get to see shadows; such shadows are the imperfect forms that allow indirect access to the “real” perfect objects behind them. An implication is that whoever seeks mathematical knowledge should strive to obtain *mental representations* as close as possible to the ideal (non-physical) forms. The Platonic philosophical stance is at the basis of much research on mathematical representations. Many theories on learning mathematics claim the importance of understanding mathematical objects by somehow tapping on their “true” meanings that can be constructed by abstracting from their representations. However, researchers in Mathematics education, including members of the CERME12 community, have also started exploring different theoretical perspectives on the meanings and representations of mathematical objects (e.g., Miragliotta & Lisarelli, 2022; Palatnik & Abrahamson, 2022).

A very important shift away from the Platonic perspective has been initiated by Anna Sfard in her Commognitive Framework (2008). Taking a Vygotskian socio-constructivist perspective, and following Wittgenstein, Sfard sees mathematical objects as no longer residing in some hyper-reality, but in discourse itself, being part of an autopoietic system, a system that defines its own objects. Hence, their meanings stem from the ways in which realizations of a mathematical object are used discursively. An implication of such a shift is that the term “representation” is inappropriate: Sfard rejects the Platonic view of mathematical objects existing “out there” and being re-presented in discourse; rather, for her, mathematical objects “come to life” as part of a discourse of certain human communities.

Another perspective, supported by cumulative data from various fields (neurobiology, robotics, kinesiology) is casting doubt on the Platonic view, and in particular on its implication that bodily experiences are separate from the ideal “mental representations” discussed above. Indeed, the “embodied” turn in cognitive science rejects the hierarchical mind–body separation and stresses that

perception and action are formatively constitutive of our thinking. In the Mathematics Education field, the embodied paradigm has been taken to suggest that learning new concepts begins with discovering new ways to act in the environment, using new instruments to perform tasks on discovered affordances (e.g., Abrahamson & Bakker, 2016). Working with the things themselves, students develop a capacity to act efficiently; they learn to describe the world mathematically to coordinate collaborative actions; they iteratively encounter more complex problems; and ultimately they modify the environments to solve emergent problems (Abrahamson & Abdu, 2021).

A specific interest of mine concerns learning mathematics with or through digital tools. Turning away from the Platonic perspective, these can be conceived as “extensions” of our mind-and-body selves, and we can explore new ways of thinking, talking and using representations of mathematical objects. This is a line opened by posthuman discourse that describes the blending of human and technology as a “triumphant overcoming” the “natural” limitations of the human body, leading to the fascinating notion of “digital extension” (see the discussion on Merleau-Ponty in Dolezal, 2020).

### **My three questions**

Based on the shift discussed above, on discussions in TWG24 over past meetings, and on my personal research interests, I am particularly interested in thinking about the three following questions.

**Question 1:** *How can we (and will we) produce and share representations/realizations of mathematical objects to make teaching-learning processes truly inclusive?* Indeed, producing and sharing representations/realizations of mathematical objects are fundamental processes to consider and study in the context of inclusive mathematics education. Research in this domain has highlighted the importance of using multi-modal channels of communication, perhaps also supported by Artificial Intelligence artifacts (e.g., Lew & Baccaglini-Frank, 2021). Given the new perspectives on what representations/realizations of mathematical objects might be, we should explore ways to share and appropriate others’ thoughts and personal experiences with such representations.

**Question 2:** *How does (and will) learning occur (a) through making representations/realizations of mathematical objects or (b) through making artifacts that make these?* A very interesting (to me) direction of research has been opened by research on “learning as making”, in a constructionist perspective, where, for example, shapes in space are constructed using a 3D pen or a 3D printer (e.g., Ng & Sinclair, 2018; Ng & Tsang, 2021), or sketches of figures are produced on the plane by drawing robots that can be programmed by young children using a graphical block coding language (e.g., Baccaglini-Frank et al., 2020; Baccaglini-Frank & Mariotti, 2022).

**Question 3:** **Finally, since learning experiences can be very different and involve many different factors, I ask: How can we capture and study students’ experiences with representations/realizations more holistically?** I ask this because in most of the research studies I am familiar with, we attend to only a small part of the “whole picture”, focusing, for example, either on cognitive aspects or affective ones, or on a certain small “bit”, of a student’s larger and more complex interaction with an artifact. However, I believe that it would be beneficial to have analytical tools that allow us to see more of the bigger picture. Perhaps we could work harder on trying to integrate results from studies that each looked at a small bit, but that together can provide new insights into students’ learning processes.

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